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Quantum feedback of solid-state qubits

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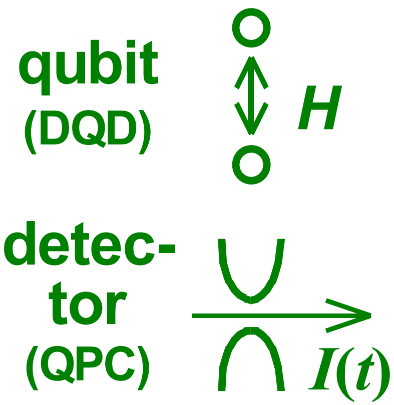
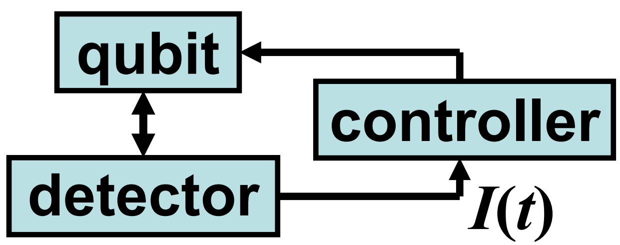
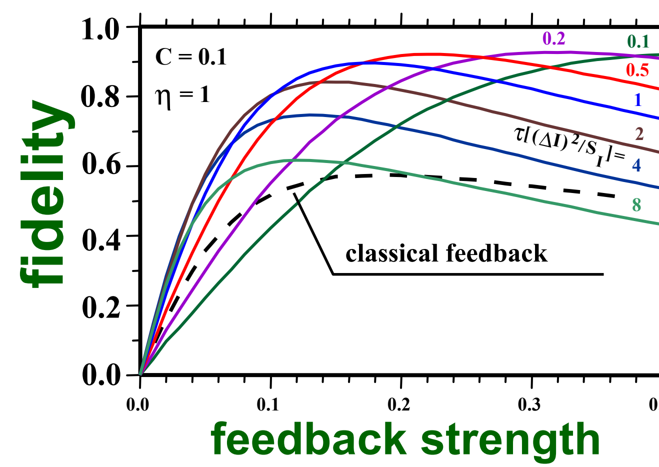


Goal: keep coherent oscillations forever





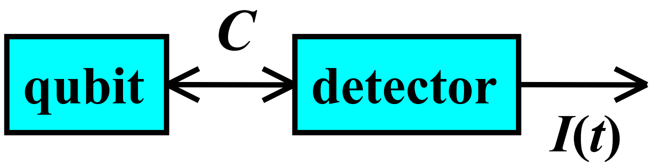
Outline:

- Introduction
- Bayesian quantum feedback
- Simple quantum feedback

Support:

cond-mat/0507011
PRB 71, 201305(R) (2005)

Measured spectrum of qubit coherent oscillations



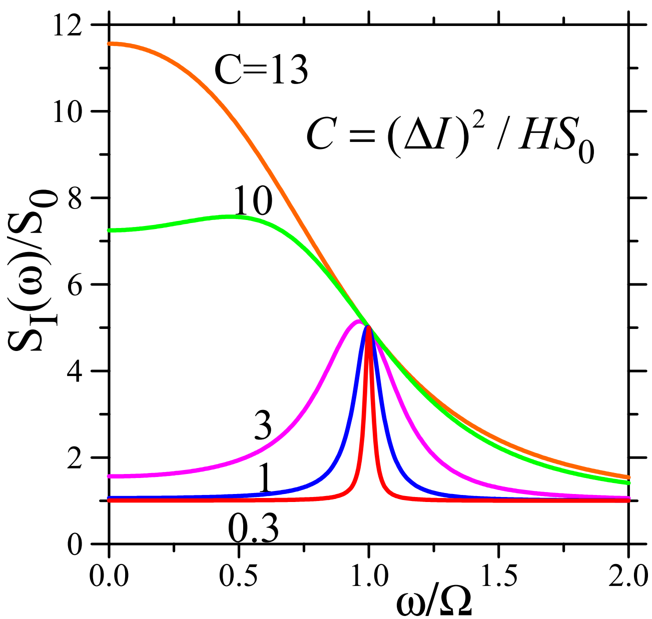
What is the spectral density $S_I(\omega)$ of detector current?

$$\hat{H}_{QB} = \frac{\varepsilon}{2}(c_1^\dagger c_1 - c_2^\dagger c_2) + H(c_1^\dagger c_2 + c_2^\dagger c_1)$$

|1ò Æ I_1, |2ò Æ I_2, $\Delta I = I_1 - I_2$, S_0 – detector noise

$\Omega = (4H^2+\varepsilon^2)^{1/2}$/Ñ – Rabi frequency

h – detector efficiency (h=1 for QPC)



Assume classical output, eV » $\hbar\Omega$

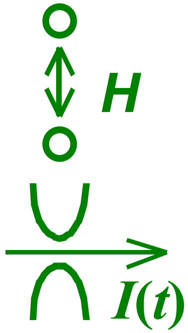
$$\varepsilon = 0, \quad \Gamma = \eta^{-1}(\Delta I)^2 / 4S_0$$

$$S_I(\omega) = S_0 + \frac{\Omega^2(\Delta I)^2\Gamma}{(\omega^2-\Omega^2)^2+\Gamma^2\omega^2}$$

Spectral peak at weak coupling, peak-to-pedestal ratio $\le 4\eta \le 4$

A.K., LT'99
Averin-A.K., 2000
A.K., 2000
Averin, 2000
Goan-Milburn, 2001
Makhlin et al., 2001
Balatsky-Martin, 2001
Ruskov-A.K., 2002
Mozyrsky et al., 2002
Balatsky et al., 2002
Bulaevskii et al., 2002
Shnirman et al., 2002
Bulaevskii-Ortiz, 2003
Shnirman et al., 2003

Contrary:
Stace-Barrett, 2003 (PRL 2004)



Bayesian formalism for a single qubit

$$\hat{H}_{QB} = \frac{\varepsilon}{2}(c_1^\dagger c_1 - c_2^\dagger c_2) + H(c_1^\dagger c_2 + c_2^\dagger c_1)$$

|1ò Æ I_1, |2ò Æ I_2 $\Delta I = I_1 - I_2$, $I_0 = (I_1+I_2)/2$, S_I – detector noise

$$\dot{\rho}_{11} = -\dot{\rho}_{22} = -2(H/\hbar)\,\mathrm{Im}\,\rho_{12} + \rho_{11}\rho_{22}(2\Delta I/S_I)[\underline{\underline{I(t)}} - I_0]$$

$$\dot{\rho}_{12} = i(\varepsilon/\hbar)\rho_{12} + i(H/\hbar)(\rho_{11} - \rho_{22}) + \rho_{12}(\rho_{11} - \rho_{22})(\Delta I/S_I)[\underline{\underline{I(t)}} - I_0] - \gamma\rho_{12}$$

(A.K., 1998)

$\gamma = \Gamma - (\Delta I)^2/4S_I$, Γ – ensemble decoherence

$\eta = 1 - \gamma/\Gamma = (\Delta I)^2/4S_I\Gamma$ – detector ideality (efficiency), $\eta \leq 100\%$

Ideal detector (η=1) does not decohere a single qubit;
then random evolution of qubit *wavefunction* can be monitored

For simulations: $I(t) - I_0 = (\rho_{22} - \rho_{11})\Delta I/2 + \xi(t)$, $S_\xi = S_I$

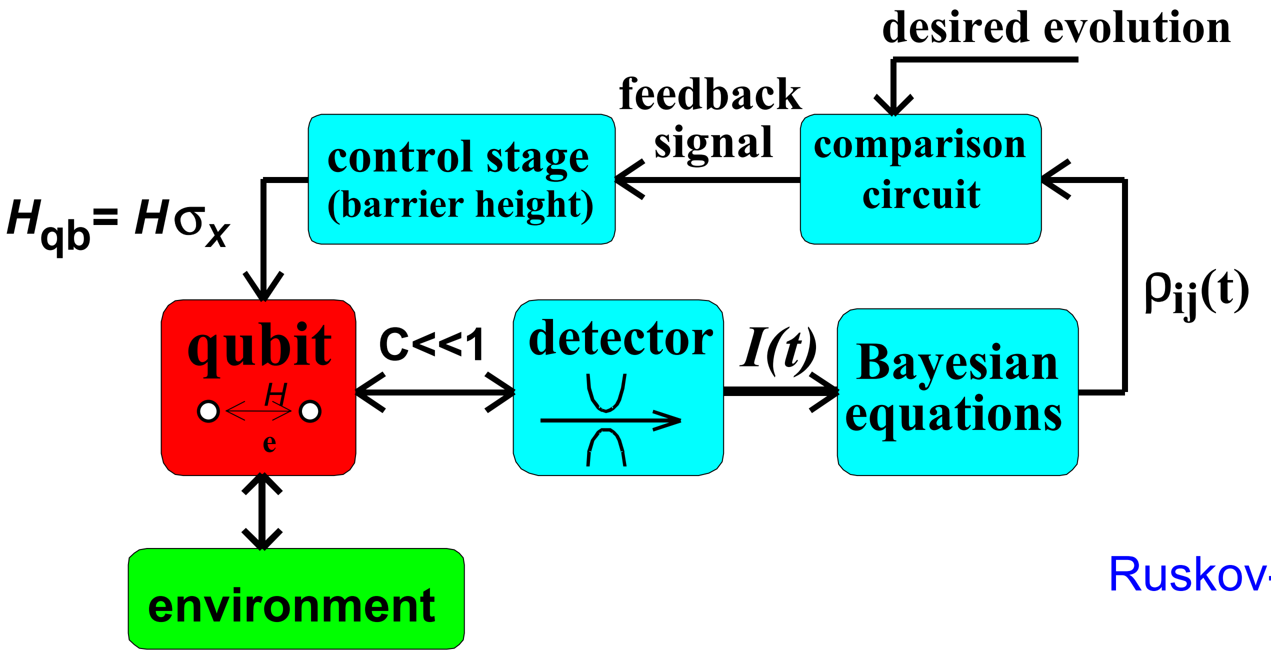
Averaging over ξ(t) ï conventional master equation

Similar formalisms developed earlier. **Key words**: **Imprecise, weak, selective, or conditional measurements, POVM, Quantum trajectories, Quantum jumps, Restricted path integral, etc.**

Names: Davies, Kraus, Holevo, Mensky, Caves, Gardiner, Carmichael, Plenio, Knight, Walls, Gisin, Percival, **Milburn**, **Wiseman,** Onofrio, Habib, Doherty, etc. (incomplete list)

Quantum feedback control of a qubit

Since qubit state can be monitored, the feedback is possible!



Ruskov-Korotkov, 2001

Goal: to maintain desired phase of coherent (Rabi) oscillations in spite of environmental dephasing (keep qubit "fresh")

Idea: monitor the Rabi phase ϕ by continuous measurement and apply feedback control of the qubit barrier height, $\Delta H_{FB}/H = -F\times\Delta\phi$

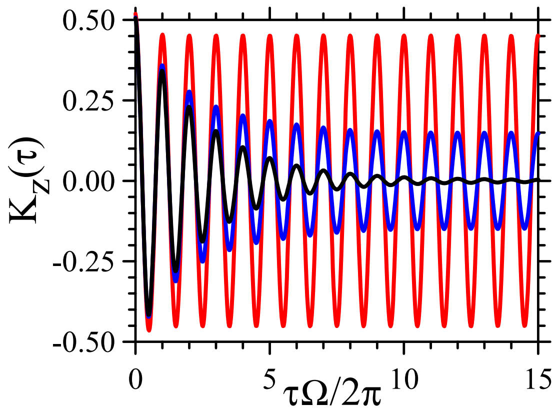
To monitor phase ϕ we plug detector output $I(t)$ into Bayesian equations

Performance of Bayesian quantum feedback

(no extra environment)

Qubit correlation function

C=1, η=1, F=0, 0.05, 0.5



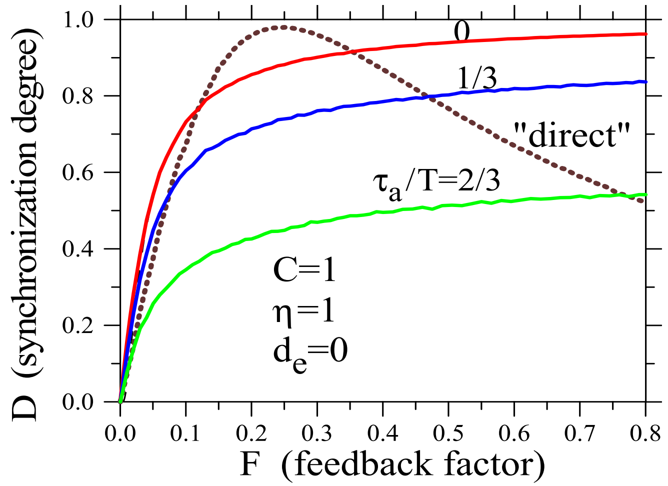
$$K_z(\tau) = \frac{\cos \Omega t}{2} \exp\left[\frac{C}{16F}(e^{-2FH\tau/\hbar} - 1)\right]$$

(for weak coupling and good fidelity)

Detector current correlation function

$$K_I(\tau) = \frac{(\Delta I)^2}{4} \frac{\cos \Omega t}{2} (1 + e^{-2FH\tau/\hbar}) \times \exp\left[\frac{C}{16F}(e^{-2FH\tau/\hbar} - 1)\right] + \frac{S_I}{2}\delta(\tau)$$

Fidelity (synchronization degree)



$C = \hbar(\Delta I)^2 / S_I H$ – coupling

τ_a^{-1} – available bandwidth

F – feedback strength

$$D = 2\langle \mathrm{Tr}\rho\rho_{desir}\rangle - 1$$

For ideal detector and wide bandwidth, fidelity can be arbitrarily close to 100%

$$D = \exp(-C/32F)$$

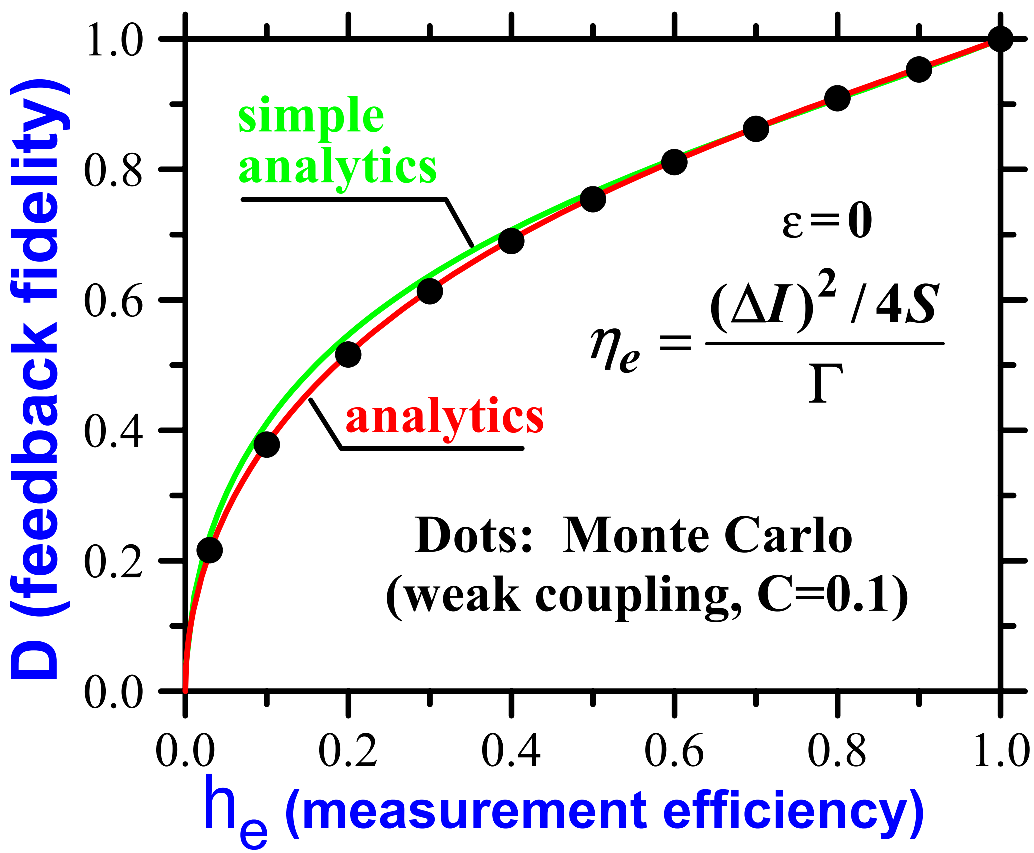
Ruskov-Korotkov, PRB 66, 041401(R) (2002)

Effect of non-ideal detector and extra environment

Zhang-Ruskov-Korotkov, 2005

Effective ideality of measurement h_e

$$\frac{1}{\eta_e} - 1 = \frac{\text{decoherence coupling}}{\text{measurement coupling}}$$



Simple analytics:

$$D = \sqrt{1 + \frac{1}{2\eta_e} - \sqrt{\left(1 + \frac{1}{2\eta_e}\right)^2 - 2}}$$

Full analytics:

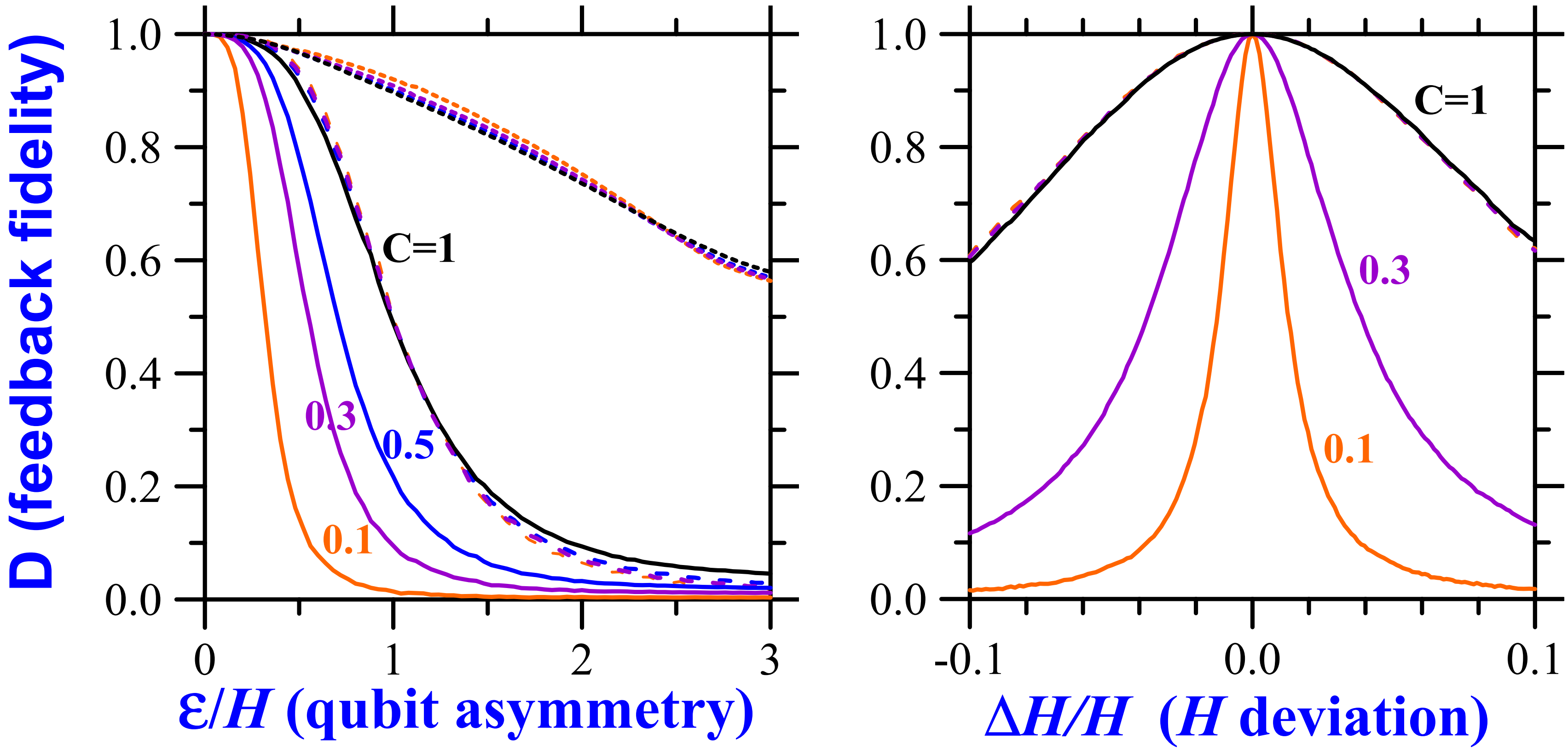
$$D = \frac{\int_0^1 P^2 G(P^2)\, dP}{\int_0^1 P\, G(P^2)\, dP}$$

$$G(P^2) = (1 - P^2)^{-5/2} \times$$
$$\times \exp[-(\eta_e^{-1} - 1)/2(1 - P^2)]$$

Example:

h_e=0.1, then fidelity 0.4
(still quite good!)

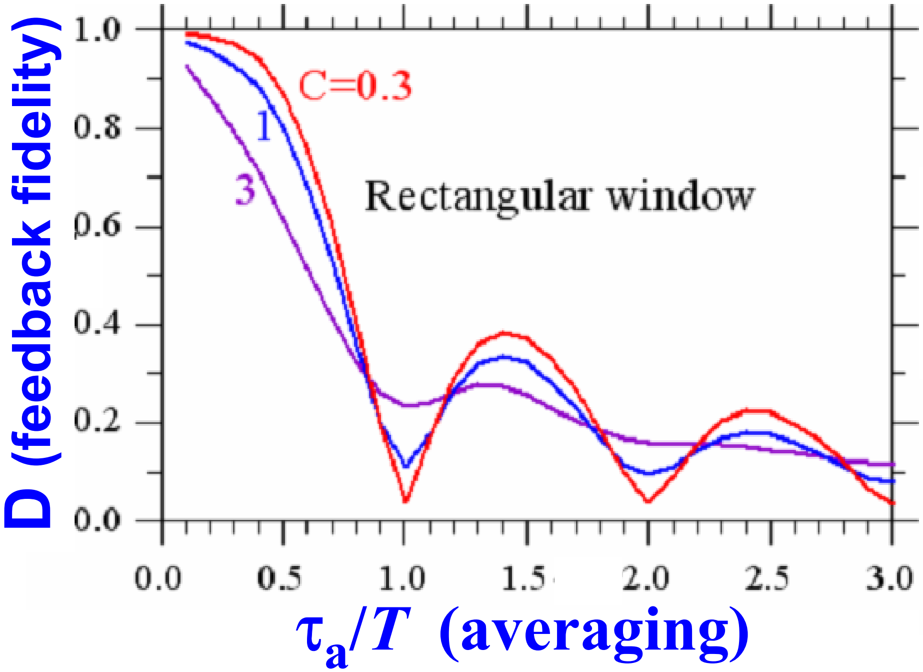
Effect of qubit parameter deviations (ε, *H*)

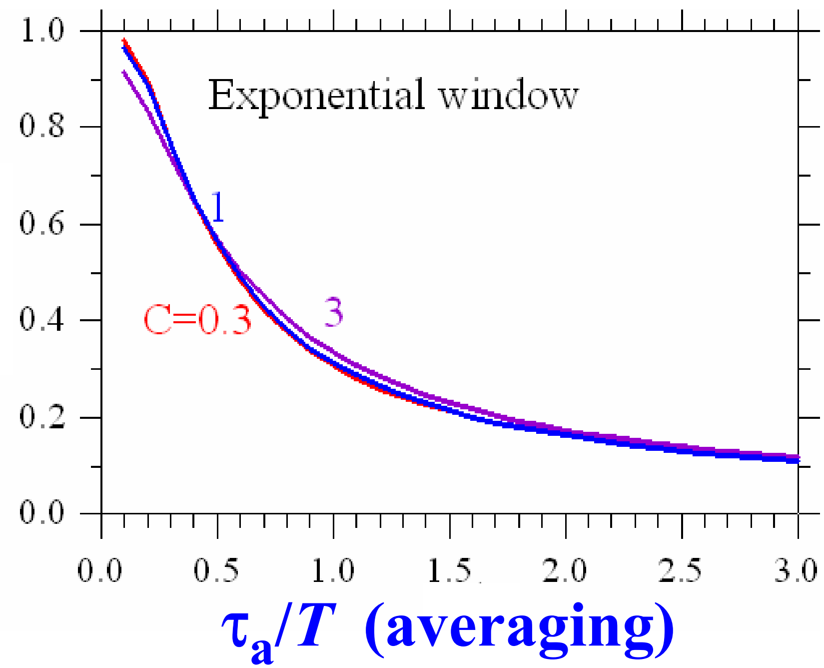


Feedback operation is robust against small unknown deviations of qubit parameters

Effect of finite bandwidth

Averaging of the detector signal over time τ_a (using rectangular or exponential window) leads to information loss and therefore to the decrease of feedback fidelity D.

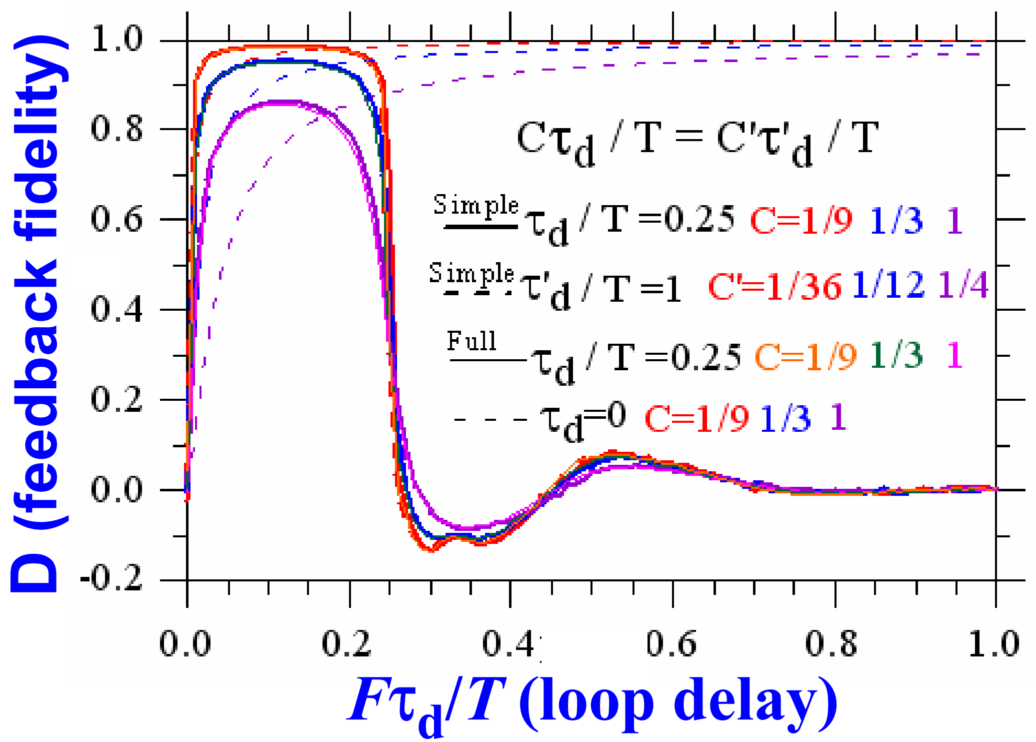




For good feedback performance the averaging time τ_a should be much smaller than Rabi period $T=2\pi/\Omega$

(**signal bandwidth >> Rabi frequency**)

Effect of feedback loop delay



F **– feedback strength**

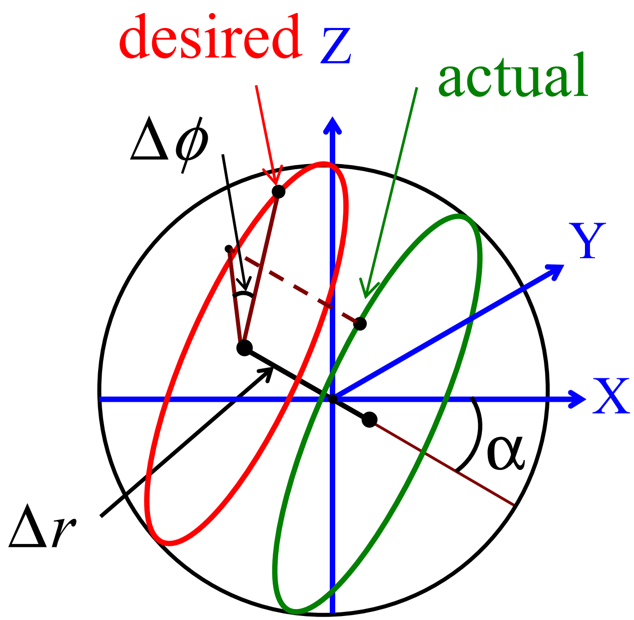
τ_d **– loop delay**

$T = 2\pi/\Omega$ **– Rabi period**

C – detector coupling

Feedback loop becomes unstable ("oversteering") at $F\tau_d/T > 1/4$

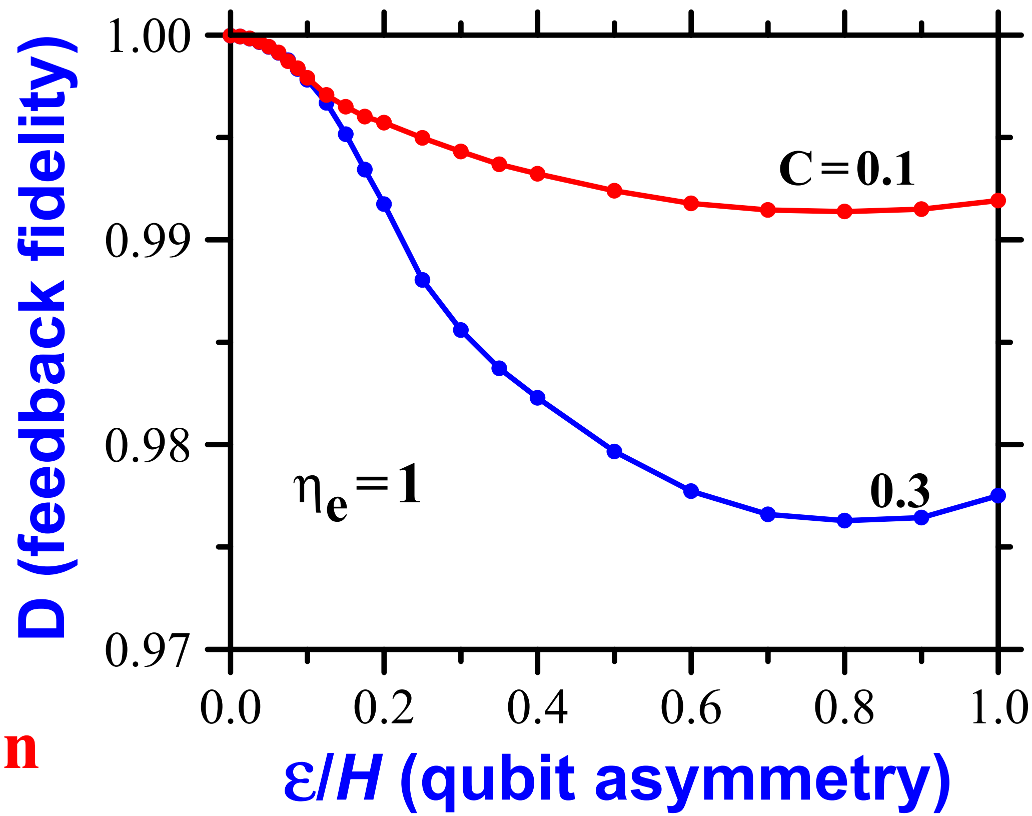
Control of energy-asymmetric qubit (ε≠0)



Now two degrees of freedom
for deviation: $\Delta\phi$ and Δr

New controller:

$$\Delta H_{FB} = -F\, H\, \Delta\phi - F_r\, H \sin\phi\, \Delta r$$



Even an asymmetric qubit can be efficiently feedback-controlled using only *H*-modulation

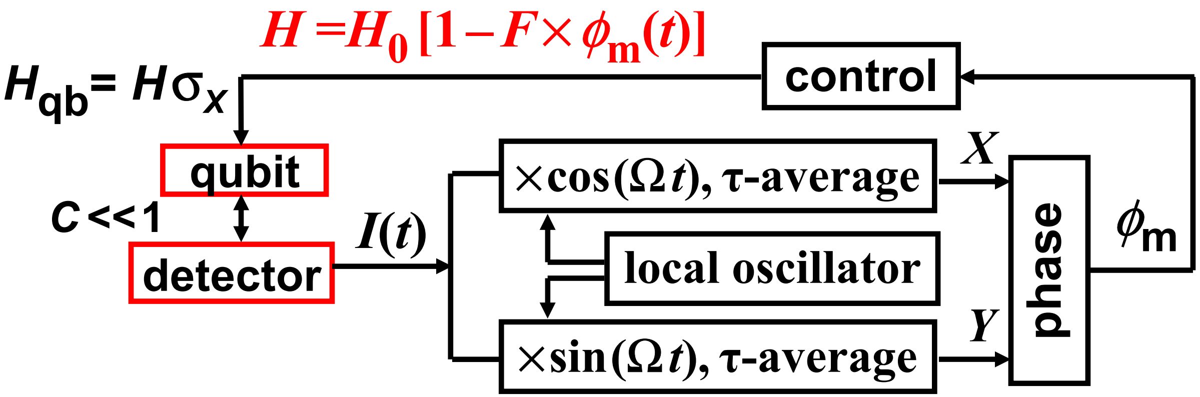
So, Bayesian quantum feedback of a qubit works very well, but …

Difficult experimental problems:

- necessity of very fast real-time solution of the Bayesian equations
- wide bandwidth (>>Ω, GHz-range) of the line delivering noisy signal *I*(*t*) to the "processor"

Simple quantum feedback of a solid-state qubit

(A.K., PRB-2005)



Goal: maintain coherent (Rabi) oscillations for arbitrary long time

Idea: use two quadratures of the detector current $I(t)$ to monitor the phase of qubit oscillations

(just as in usual classical feedback!)

$$X(t) = \int_{-\infty}^{t} [I(t') - I_0] \cos(\Omega t') \exp[-(t-t')/\tau]\, dt$$

$$Y(t) = \int_{-\infty}^{t} [I(t') - I_0] \sin(\Omega t') \exp[-(t-t')/\tau]\, dt$$

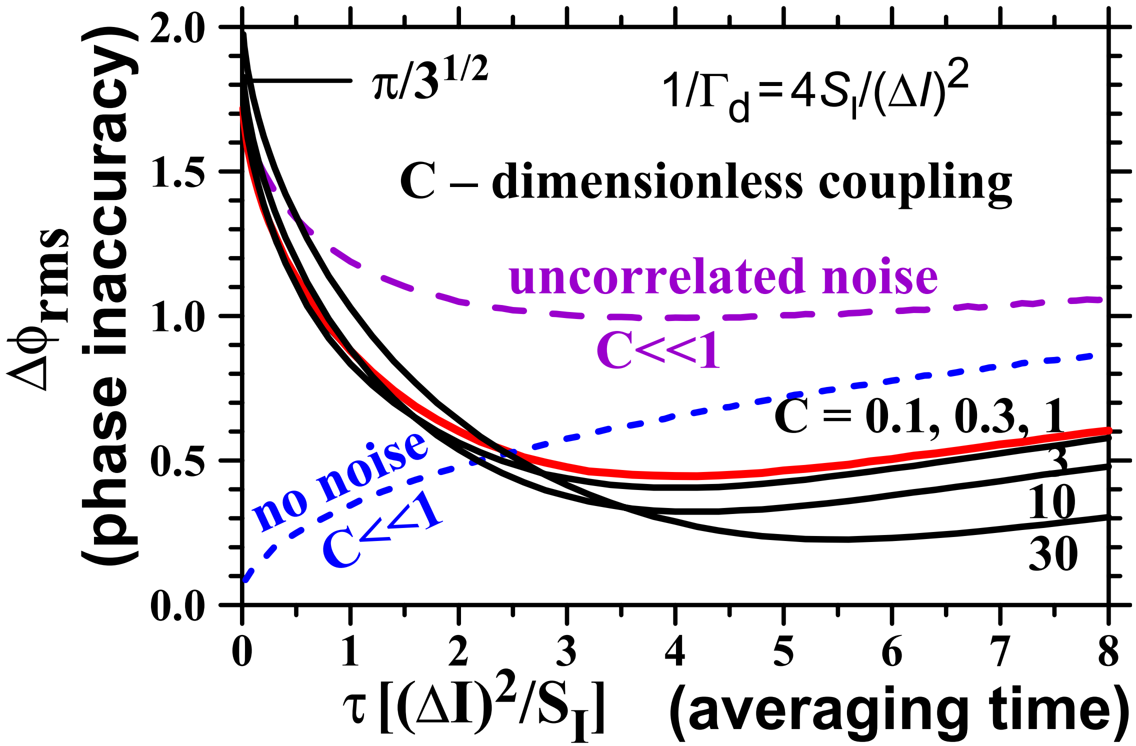
$$\phi_m = -\arctan(Y/X)$$

(similar formulas for a tank circuit instead of mixing with local oscillator)

Advantage: simplicity and relatively narrow bandwidth $(1/\tau \sim \Gamma_d << \Omega)$

Surprisingly, works well in spite of being so simple!

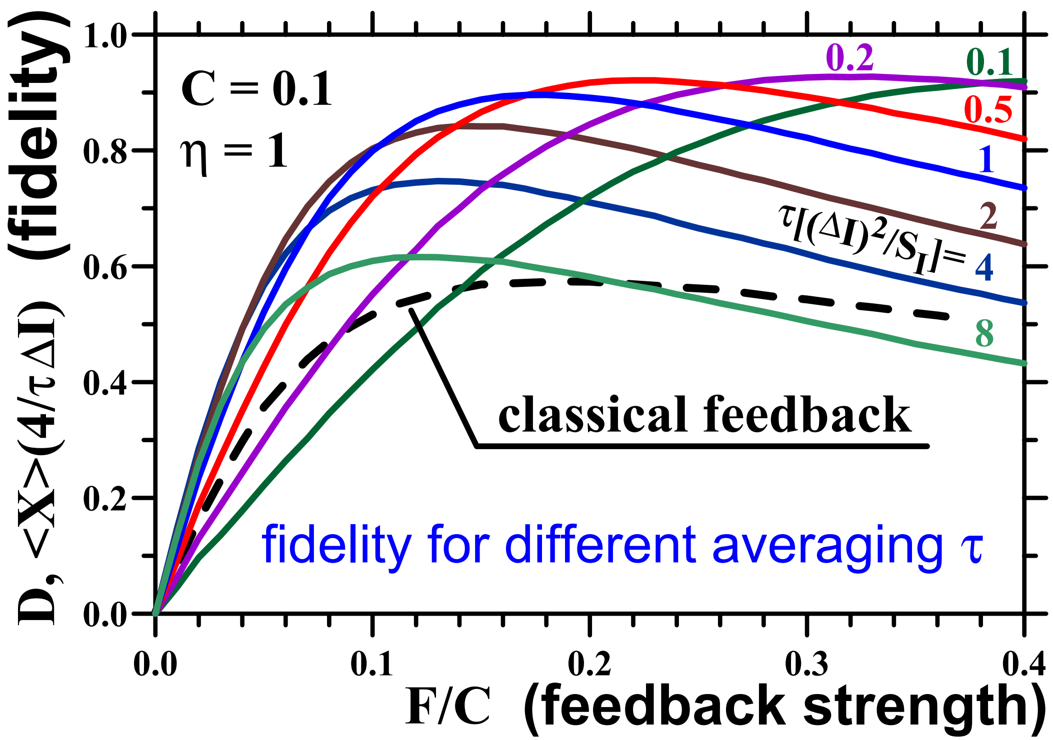
Accuracy of phase monitoring via quadratures
(no feedback yet)



Noise improves the monitoring accuracy!
(purely quantum effect, “reality follows observations”)

Quite accurate monitoring! $\cos(0.44) \approx 0.9$

Simple quantum feedback



weak coupling *C*

D – feedback efficiency

$$D \equiv 2F_Q - 1$$

$$F_Q \equiv \langle \mathrm{Tr}\, \rho(t)\, \rho_{des}(t) \rangle$$

$D_{max} \approx 90\%$

$(F_Q \approx 95\%)$

How to verify feedback operation experimentally?

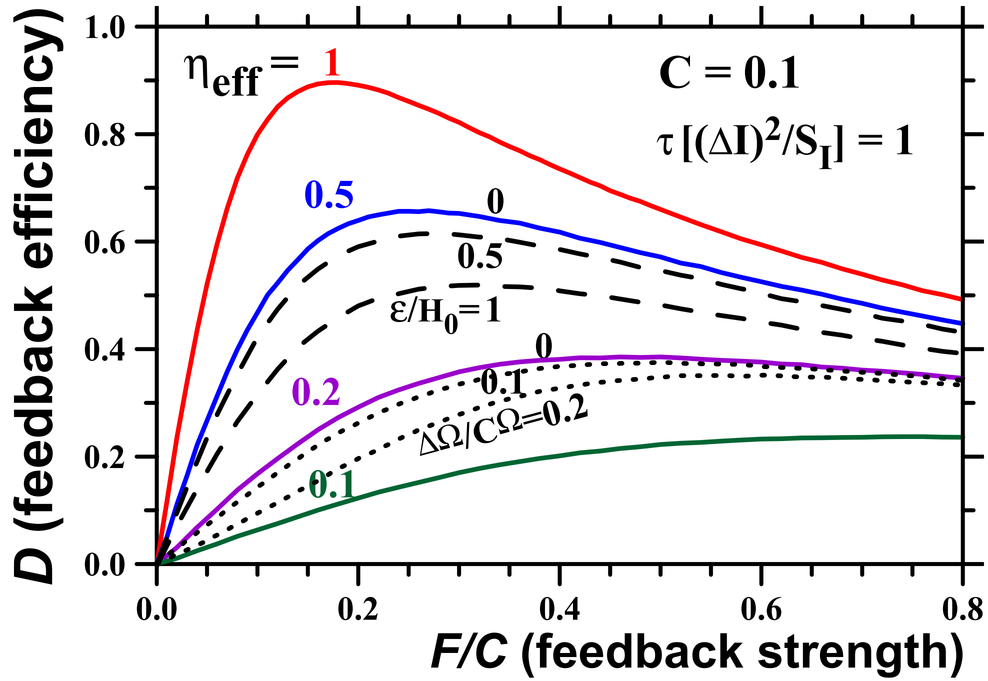
Simple: just check that in-phase quadrature $\langle X \rangle$ of the detector current is positive $D = \langle X \rangle (4/\tau\Delta I)$

$\langle X \rangle$=0 for *any* non-feedback Hamiltonian control of the qubit

Effect of nonidealities

- nonideal detectors (finite quantum efficiency η) and environment
- qubit energy asymmetry ε
- frequency mismatch ΔΩ

Quantum feedback still works quite well



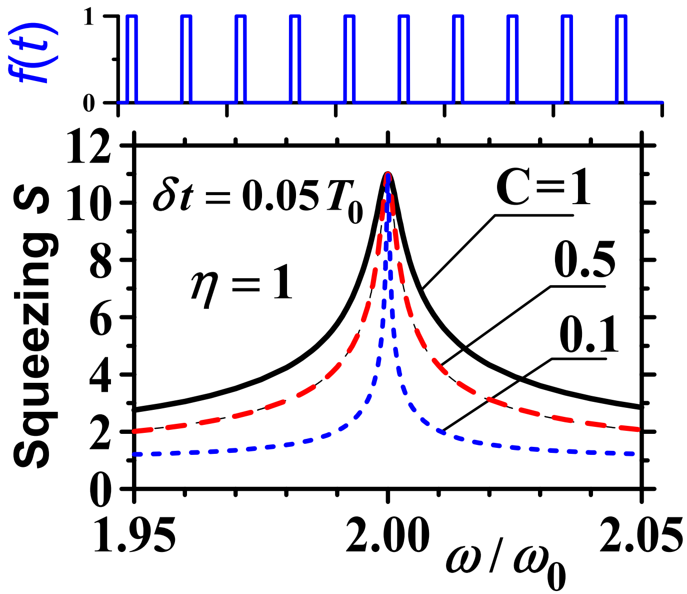
Main features:

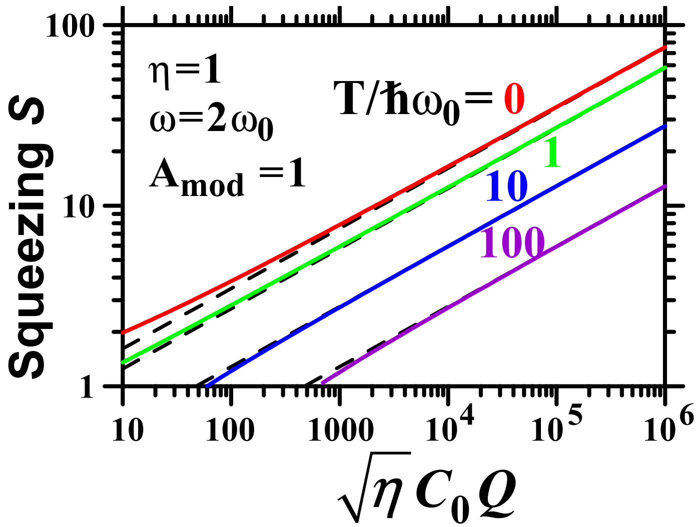
- **Fidelity F_Q up to ~95% achievable (D~90%)**
- **Natural, practically classical feedback setup**
- **Averaging τ~1/Γ>>1/Ω (narrow bandwidth!)**
- **Detector efficiency (ideality) η~0.1 still OK**
- **Robust to asymmetry ε and frequency shift ΔΩ**
- **Simple verification: positive in-phase quadrature ⟨*X*⟩**

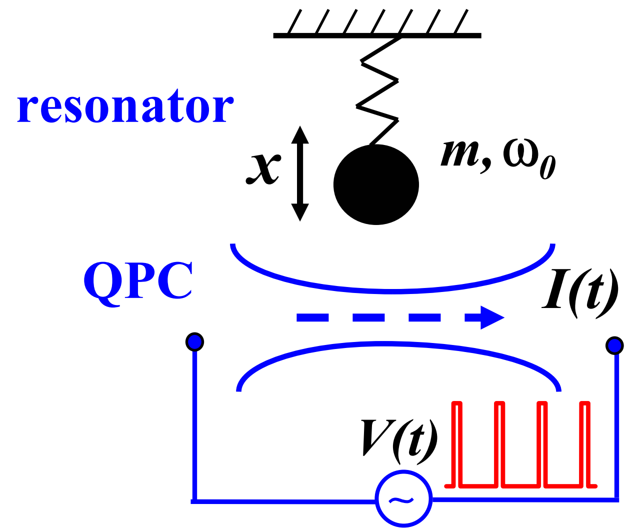
Simple enough experiment?!

QND measurement and quantum feedback for nanoresonator squeezing

Ruskov-Schwab-A.K., PRB-2005







Analytics:

$$S_{\max} = \frac{3}{4}\left[\frac{\sqrt{\eta}\, C_0 Q}{\coth(\hbar\omega_0 / 2T)}\right]^{1/3}$$

(So far in experiment $\eta^{1/2} C_0 Q \sim 0.1$)

Potential application:

ultrasensitive force measurement beyond standard quantum limit

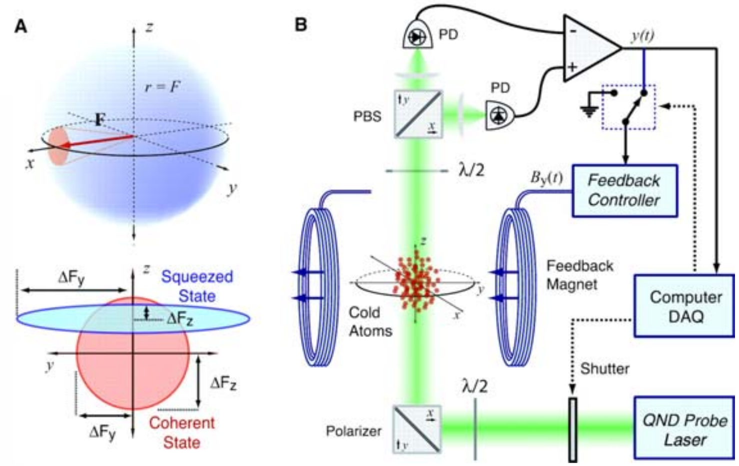
Quantum feedback in optics

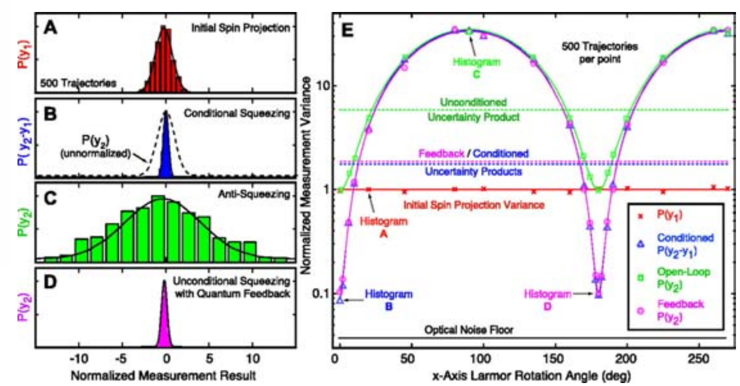
Recent experiment: Science 304, 270 (2004)

Real-Time Quantum Feedback Control of Atomic Spin-Squeezing

JM Geremia,* John K. Stockton, Hideo Mabuchi

Real-time feedback performed during a quantum nondemolition measurement of atomic spin-angular momentum allowed us to influence the quantum statistics of the measurement outcome. We showed that it is possible to harness measurement backaction as a form of actuation in quantum control, and thus we describe a valuable tool for quantum information science. Our feedback-mediated procedure generates spin-squeezing, for which the reduction in quantum uncertainty and resulting atomic entanglement are not conditioned on the measurement outcome.





First detailed theory:

H.M. Wiseman and G. J. Milburn,
Phys. Rev. Lett. 70, 548 (**1993**)

Conclusion

- Quantum evolution can be continuously controlled by a feedback loop
- Bayesian feedback of a qubit can maintain Rabi oscillations in a qubit with nearly 100% fidelity
- Simple (practically classical) quadrature-based feedback works well for a qubit; relatively simple experiment