Quantum feedback control in solid-state mesoscopics

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Outline:

- Introduction
  - *Quantum feedback in optics*
  - *Bayesian formalism for solid-state quantum measurements*
- **Non-decaying coherent (Rabi) oscillations in a qubit**
  - *Bayesian feedback*: **R.Ruskov-A.K., PRB 66, 041401(R) (2002)**
  - *Simple feedback*: **A.K., cond-mat/0404696**
- **QND squeezing of a nanomechanical resonator**

  R.Ruskov-K.Schwab-A.K., cond-mat/0406416, cond-mat/0411617

Support:

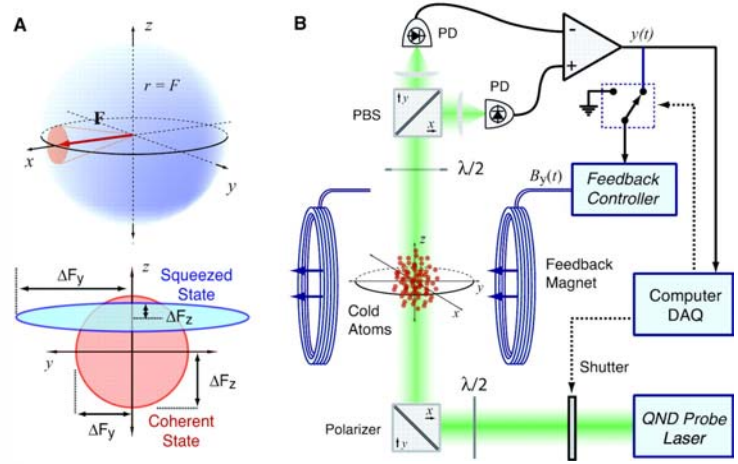
Quantum feedback in optics

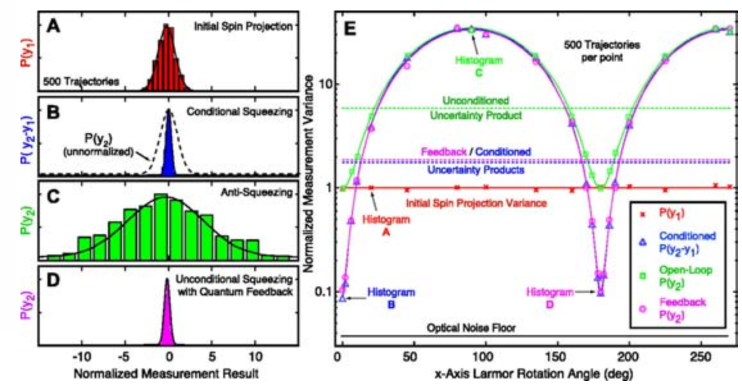
Recent experiment: **Science 304, 270 (2004)**
(Mabuchi's group, Caltech)

Real-Time Quantum Feedback Control of Atomic Spin-Squeezing

JM Geremia,* John K. Stockton, Hideo Mabuchi

Real-time feedback performed during a quantum nondemolition measurement of atomic spin-angular momentum allowed us to influence the quantum statistics of the measurement outcome. We showed that it is possible to harness measurement backaction as a form of actuation in quantum control, and thus we describe a valuable tool for quantum information science. Our feedback-mediated procedure generates spin-squeezing, for which the reduction in quantum uncertainty and resulting atomic entanglement are not conditioned on the measurement outcome.



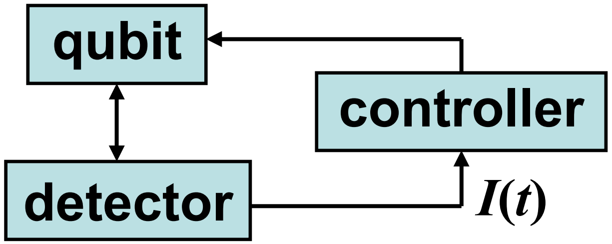


First detailed theory:

H.M. Wiseman and G.J. Milburn,
Phys. Rev. Lett. 70, 548 (**1993**)

Similar subject in solid state is delayed by 5-10 years, still theory only

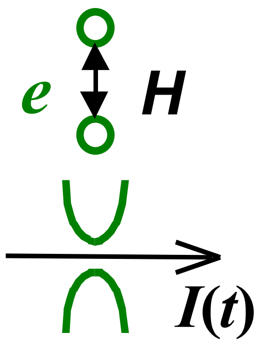
Quantum feedback of a solid-state qubit for non-decaying Rabi oscillations



Goal: maintain coherent (Rabi) oscillations in a qubit for arbitrarily long time

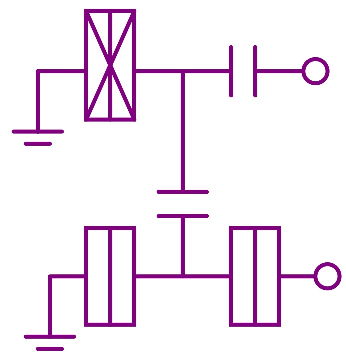
Result: yes, possible!

Potential implementations:



qubit: double quantum dot occupied by one electron

detector: quantum point contact (QPC)



qubit: Cooper-pair box

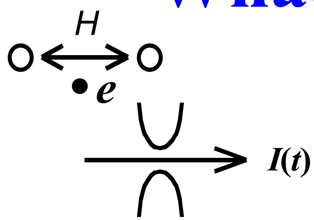
detector: single-electron transistor (SET)

qubit and detector have been demonstrated experimentally

Buks *et al*., Nature (1998)
Sprinzak *et al*., PRL (1999)
Hayashi *et al*., PRL (2003)

Nakamura *et al*., Nature (1999)
Vion *et al*., Science (2002)
Pashkin *et al*., Nature (2003)

What happens to qubit state during measurement?



For simplicity (for a moment) consider evolution due to measurement only (assume qubit with infinite barrier, $H = \varepsilon = 0$)

"Orthodox" answer

qubit density matrix: $\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \begin{matrix} \to \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \\ \to \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \end{matrix}$

|1> or |2>, depending on the result

"Conventional" (decoherence) answer (Leggett, Zurek)

$$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \to \begin{pmatrix} \frac{1}{2} & \frac{\exp(-\Gamma t)}{2} \\ \frac{\exp(-\Gamma t)}{2} & \frac{1}{2} \end{pmatrix} \to \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

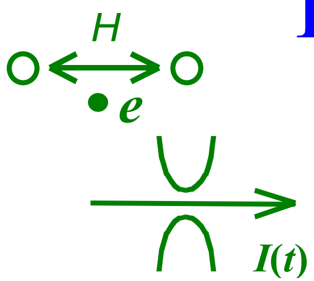
no measurement result! ensemble averaged

Orthodox and decoherence answers contradict each other!

applicable for:	Single quantum systems	Continuous measurements
Orthodox	yes	no
Conventional (ensemble)	no	yes
Bayesian	yes	yes

Bayesian formalism describes gradual collapse of single quantum systems
Noisy detector output $I(t)$ should be taken into account

Bayesian formalism for single qubit



$$\hat{H}_{QB} = \frac{\varepsilon}{2}(c_1^\dagger c_1 - c_2^\dagger c_2) + H(c_1^\dagger c_2 + c_2^\dagger c_1)$$

ρ – density matrix

$|1\rangle \to I_1$, $|2\rangle \to I_2$, $\Delta I = I_1 - I_2$, $I_0 = (I_1 + I_2)/2$, S_I **– detector noise**

$I(t)$ **– output**

$$\begin{cases} \dot{\rho}_{11} = -\dot{\rho}_{22} = -2H\,\mathrm{Im}\,\rho_{12} + \rho_{11}\rho_{22}(2\Delta I / S_I)[I(t) - I_0] \\ \dot{\rho}_{12} = i\varepsilon\rho_{12} + iH(\rho_{11} - \rho_{22}) + \rho_{12}(\rho_{11} - \rho_{22})(\Delta I / S_I)[I(t) - I_0] - \gamma\rho_{12} \end{cases}$$

A.K., 1998

$\gamma = \Gamma - (\Delta I)^2 / 4S_I$, Γ – ensemble decoherence

$\eta = 1 - \gamma / \Gamma = (\Delta I)^2 / 4S_I\Gamma$ – detector ideality (efficiency), $\eta \le 100\%$

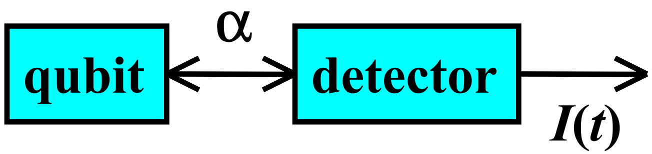
$I(t) - I_0 = (\rho_{22} - \rho_{11})\Delta I / 2 + \xi(t)$, $S_\xi = S_I$ **Averaging over ξ(t) ⇒ master equation**

Ideal detector (η=1) does not decohere a single qubit;
then random evolution of qubit *wavefunction* can be monitored

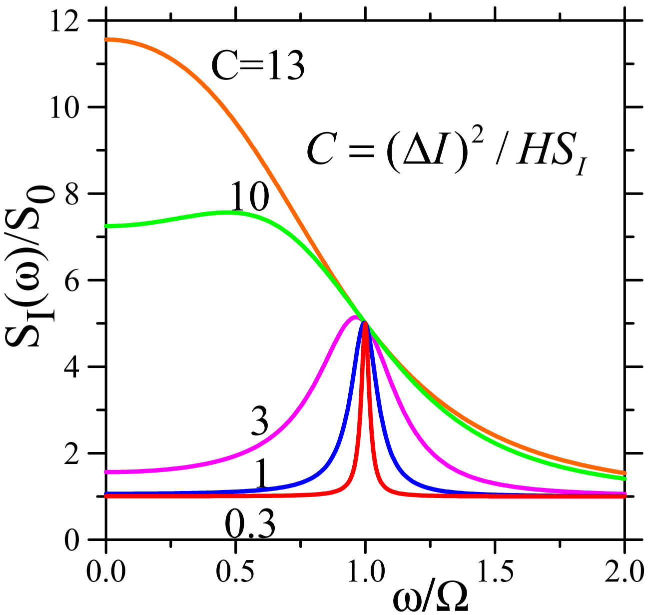
Theoretically, quantum point contact is an ideal detector (η=1),
experimentally, η ~ 0.8 demonstrated (Buks *et al*., 1998)

Similar formalisms developed earlier. Key words: Imprecise, weak, selective or conditional measurements, POVM, Quantum trajectories, Quantum jumps, Restricted path integral, etc.

Measured spectrum of qubit coherent oscillations



What is the spectral density $S_I(\omega)$ of detector current?



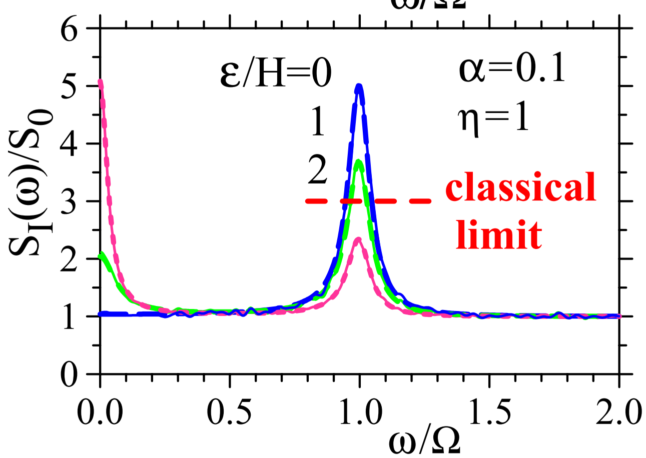
Assume classical output, eV » ħΩ

$$\varepsilon = 0, \quad \Gamma = \eta^{-1}(\Delta I)^2/4S_0$$

$$S_I(\omega) = S_0 + \frac{\Omega^2(\Delta I)^2\Gamma}{(\omega^2-\Omega^2)^2+\Gamma^2\omega^2}$$

Spectral peak can be seen, but peak-to-pedestal ratio ≤ 4η ≤ 4

(result can be obtained using various methods, not only Bayesian method)



Weak coupling, $\alpha = C/8 \ll 1$

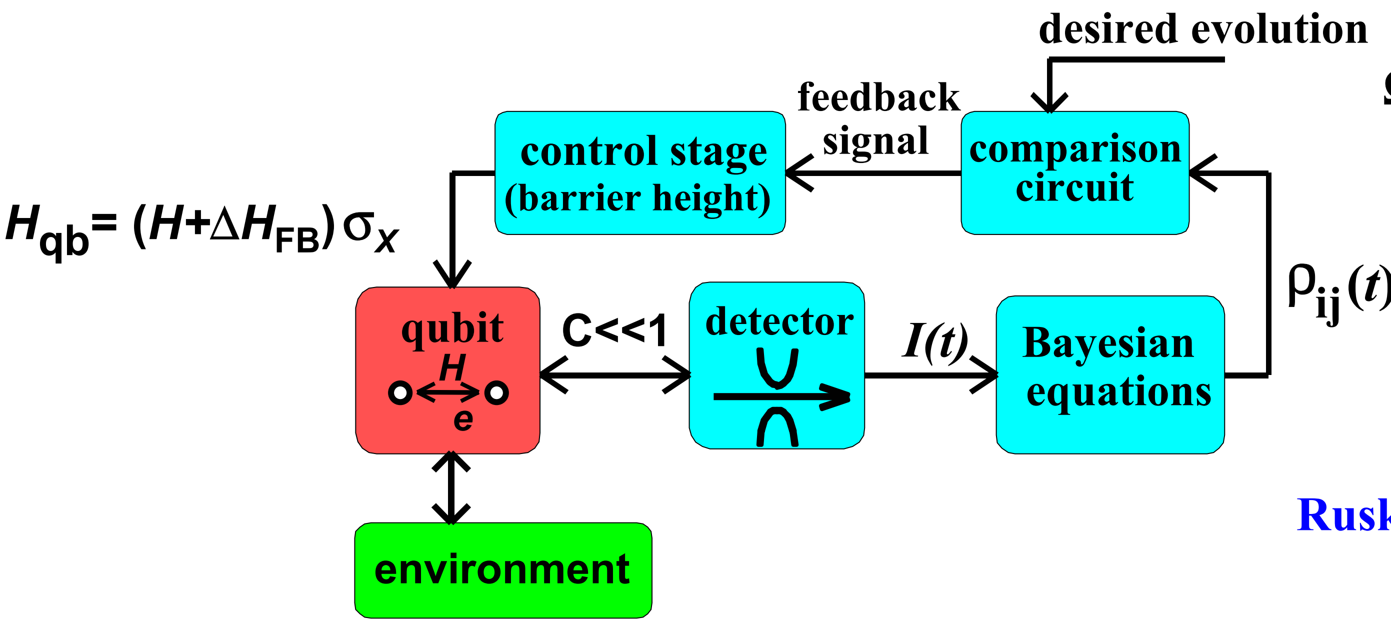
$$S_I(\omega) = S_0 + \frac{\eta S_0\varepsilon^2/H^2}{1+(\omega\hbar^2\Omega^2/4H^2\Gamma)^2} + \frac{4\eta S_0(1+\varepsilon^2/2H^2)^{-1}}{1+[(\omega-\Omega)\Gamma(1-2H^2/\hbar^2\Omega^2)]^2}$$

A.K., LT'99
Averin-A.K., 2000
A.K., 2000
Averin, 2000
Goan-Milburn, 2001
Makhlin et al., 2001
Balatsky-Martin, 2001
Ruskov-A.K., 2002
Mozyrsky et al., 2002
Balatsky et al., 2002
Bulaevskii et al., 2002
Shnirman et al., 2002
Bulaevskii-Ortiz, 2003
Shnirman et al., 2003

Contrary:
Stace-Barrett, 2003 (PRL 2004)

Bayesian quantum feedback of a qubit

Since qubit state can be monitored, the feedback is possible!



$$\Omega = \sqrt{4H^2 + \varepsilon^2}/\hbar$$
$$= 2H/\hbar$$

Ruskov & A.K., 2001

Goal: maintain desired phase of coherent (Rabi) oscillations in spite of environmental dephasing (keep qubit “fresh”)

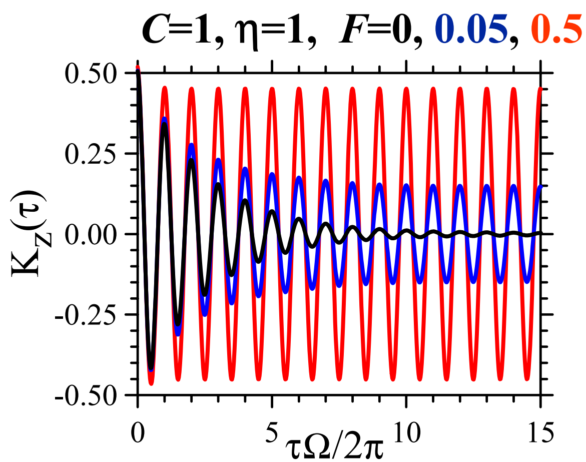
Idea: monitor the Rabi phase ϕ by continuous measurement and apply feedback control of the qubit barrier height, $\Delta H_{FB}/H = -F \times \Delta\phi$

To monitor phase ϕ we plug detector output $I(t)$ into Bayesian equations

Performance of quantum feedback

(no extra environment)

Qubit correlation function



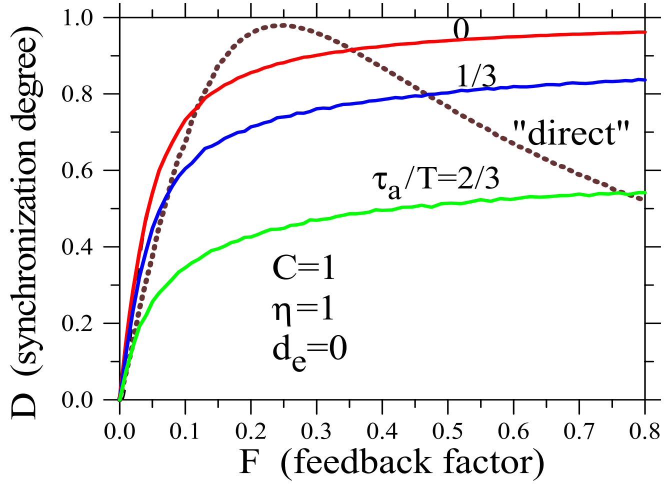
$$K_z(\tau)=\frac{\cos\Omega t}{2}\ \exp\left[\frac{C}{16F}(e^{-2FH\tau/\hbar}-1)\right]$$

(for weak coupling and good fidelity)

Detector current correlation function

$$K_I(\tau)=\frac{(\Delta I)^2}{4}\frac{\cos\Omega t}{2}\ (1+e^{-2FH\tau/\hbar})$$
$$\times\exp\left[\frac{C}{16F}(e^{-2FH\tau/\hbar}-1)\right]+\frac{S_I}{2}\delta(\tau)$$

Fidelity (synchronization degree)



$C=\hbar(\Delta I)^2/S_I H$ – coupling

τ_a^{-1} – available bandwidth

F – feedback strength

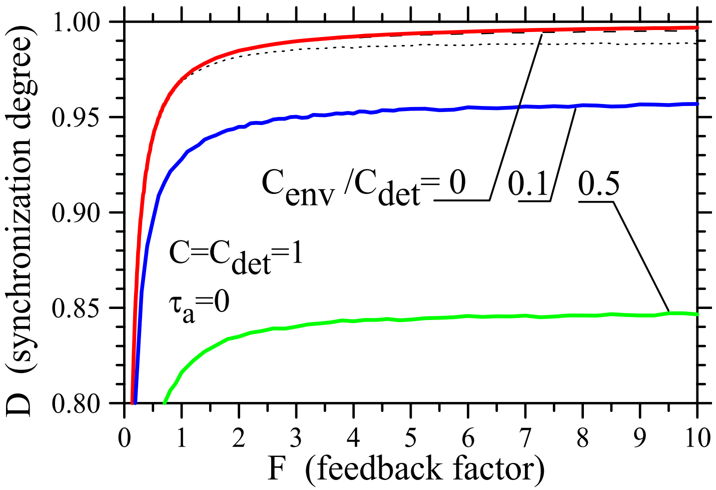
$$D=2\langle \mathrm{Tr}\rho\rho_{desir}\rangle-1$$

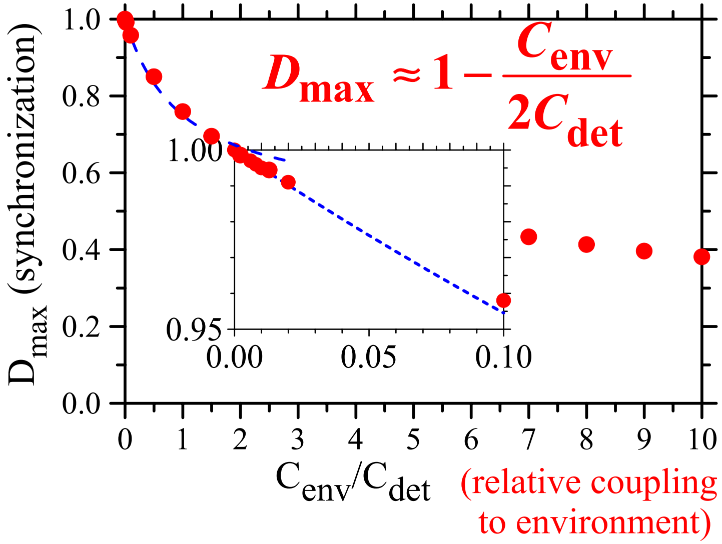
For ideal detector and wide bandwidth, fidelity can be arbitrary close to 100%

$$D=\exp(-C/32F)$$

Ruskov & Korotkov, PRB 66, 041401(R) (2002)

Suppression of environment-induced decoherence by quantum feedback



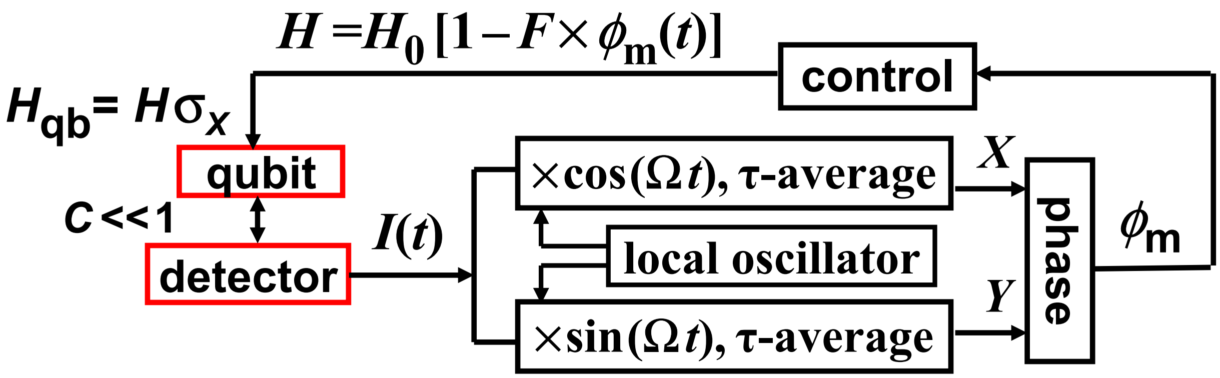


Big experimental problems:

- necessity of very fast real-time solution of the Bayesian equations
- wide bandwidth (>>Ω, GHz-range) of the line delivering noisy signal $I(t)$ to the "processor"

Simple quantum feedback of a solid-state qubit

(A.K., cond-mat/0404696)



Goal: maintain coherent (Rabi) oscillations for arbitrary long time

$\rho_{11}-\rho_{22}=\cos(\Omega t)$, $\rho_{12}=i\sin(\Omega t)/2$

Idea: **use two quadrature components of the detector current $I(t)$ to monitor approximately the phase of qubit oscillations**

(a very natural way for usual classical feedback!)

$$X(t) = \int_{-\infty}^{t} [I(t') - I_0] \cos(\Omega t') \exp[-(t-t')/\tau]\, dt$$

$$Y(t) = \int_{-\infty}^{t} [I(t') - I_0] \sin(\Omega t') \exp[-(t-t')/\tau]\, dt$$

$$\phi_m = -\arctan(Y/X)$$

(similar formulas for a tank circuit instead of mixing with local oscillator)

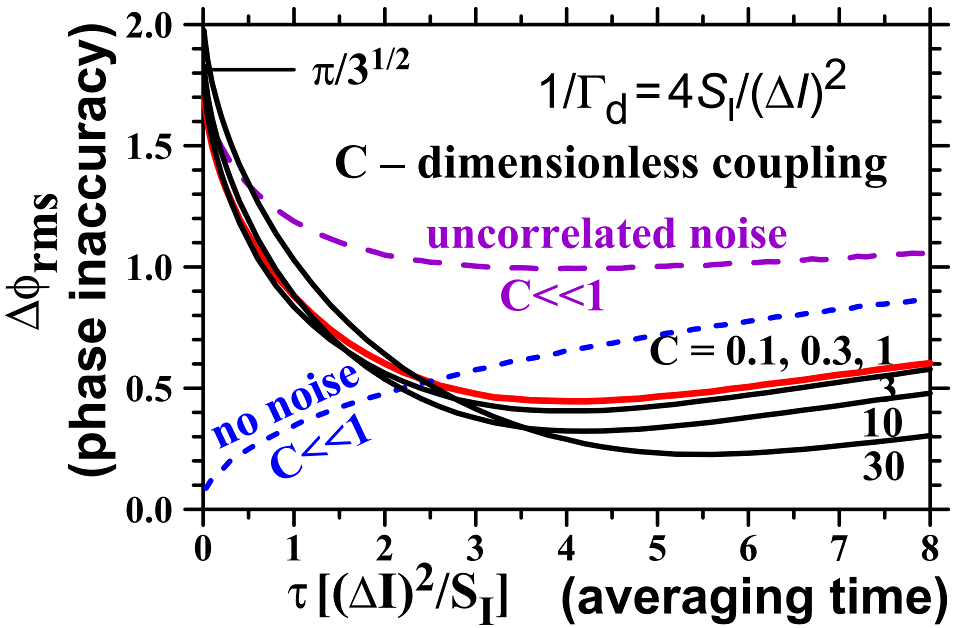
Advantage: **simplicity and relatively narrow bandwidth** $(1/\tau \sim \Gamma_d \ll \Omega)$

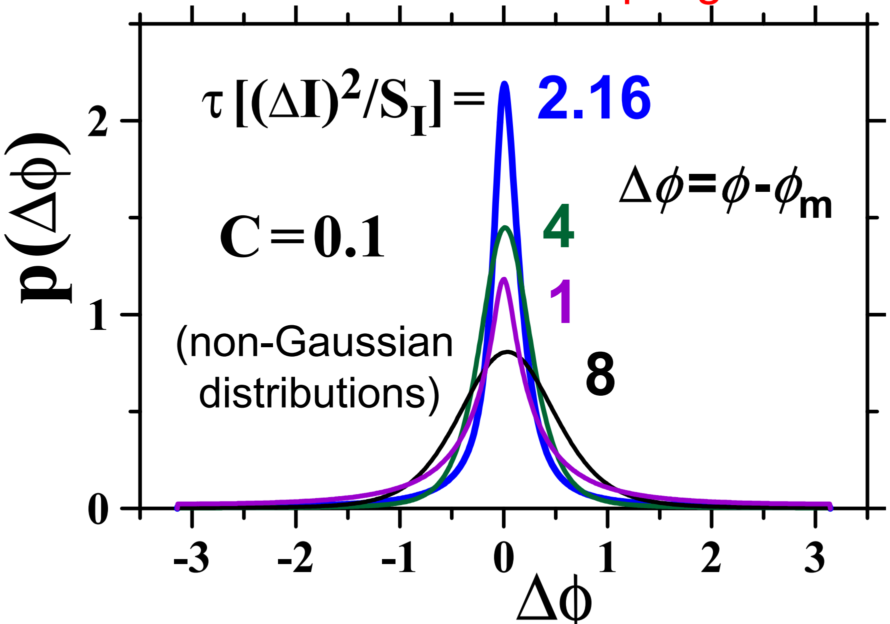
Anticipated problem: without feedback the spectral peak-to-pedestal ratio <4, therefore not much information in quadratures

(surprisingly, situation is much better than anticipated!)

Accuracy of phase monitoring via quadratures
(no feedback yet)

weak coupling $C \ll 1$





Best approximation
$\langle X^2+Y^2\rangle = (S_I/\Delta I)^2$
$(2/5)(41^{1/2}-1) \approx 2.16$

Noise improves the monitoring accuracy!
(purely quantum effect, "reality follows observations")

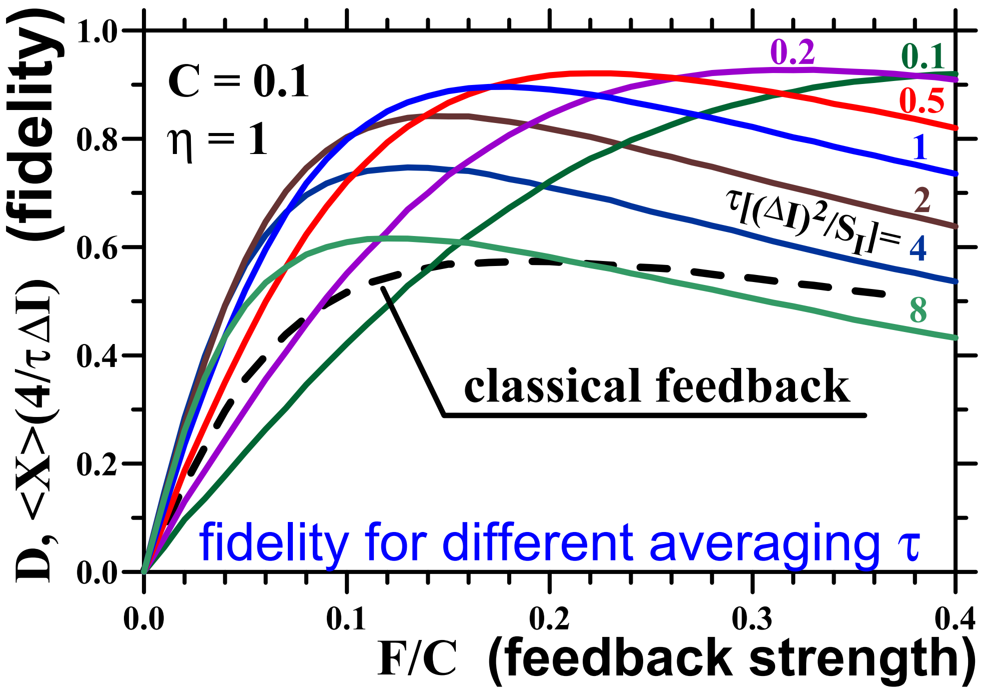
$$d\phi/dt = -[I(t) - I_0]\sin(\Omega t + \phi)(\Delta I/S_I)$$ (actual phase shift, ideal detector)

$$d\phi_m/dt = -[I(t) - I_0]\sin(\Omega t + \phi_m)/(X^2 + Y^2)^{1/2}$$ (observed phase shift)

Noise enters the actual and observed phase evolution in a similar way

Quite accurate monitoring! $\cos(0.44) \approx 0.9$

Simple quantum feedback



weak coupling C

D – feedback efficiency

$$D \equiv 2F_Q - 1$$

$$F_Q \equiv \langle \mathrm{Tr}\, \rho(t)\, \rho_{des}(t) \rangle$$

$D_{max} \approx 90\%$

$(F_Q \approx 95\%)$

How to verify feedback operation experimentally?

Simple: just check that in-phase quadrature $\langle X \rangle$ of the detector current is positive

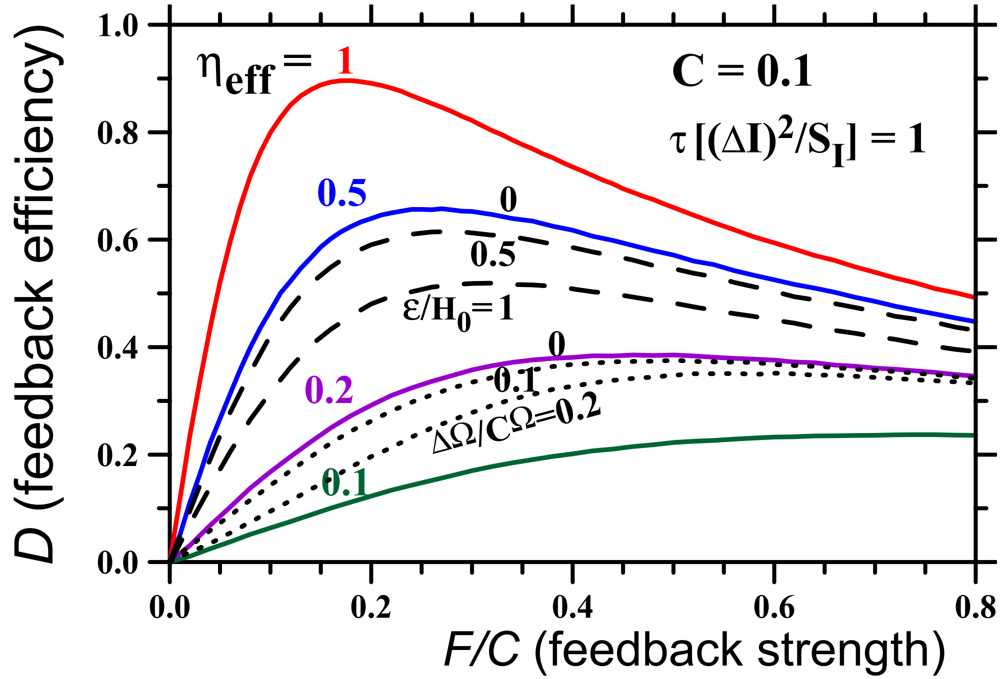
$$D = \langle X \rangle (4/\tau\Delta I)$$

$\langle X \rangle$=0 for *any* non-feedback Hamiltonian control of the qubit

Effect of nonidealities

- nonideal detectors (finite quantum efficiency η) and environment
- qubit energy asymmetry ε
- frequency mismatch ΔΩ

Quantum feedback still works quite well

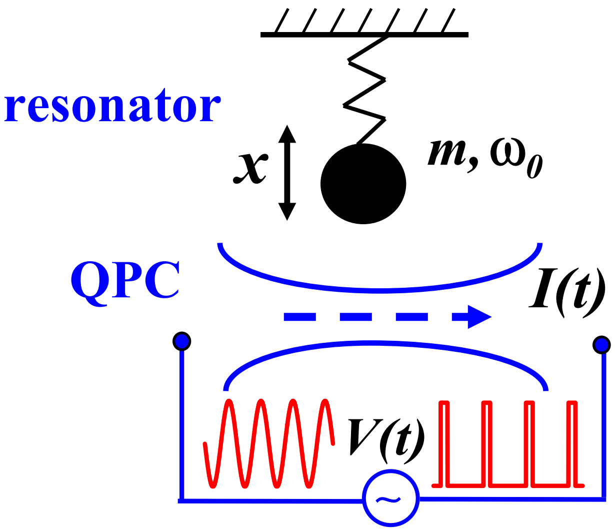


Main features:

- Natural, practically classical feedback setup
- Narrow bandwidth ($1/\tau \sim \Gamma << \Omega$)
- Fidelity F_Q up to ~95% achievable (D~90%)
- Robust to asymmetry ε and frequency shift ΔΩ
- Small detector efficiency (ideality) η~0.1 still OK
- Simple verification: positive in-phase quadrature $\langle X \rangle$

Simple enough for real experiment!

QND squeezing of a nanomechanical resonator



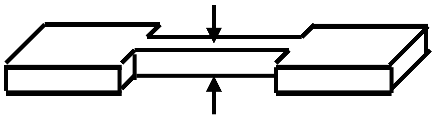
Ruskov, Schwab, Korotkov, cond-mat/0406416, cond-mat/0411617

$$\hat{H}_0 = \hat{p}^2/2m + m\omega_0^2\hat{x}^2/2$$

$$\hat{H}_{DET} = \sum_l E_l a_l^\dagger a_l + \sum_r E_r a_r^\dagger a_r + \sum_{l,r}(M a_l^\dagger a_r + H.c.)$$

$$\hat{H}_{INT} = \sum_{l,r}(\Delta M\, \hat{x}\, a_l^\dagger a_r + H.c.)$$

Experimental status:



$\omega_0/2\pi \sim 1$ GHz ($\hbar\omega_0 \sim 80$ mK), Roukes' group, 2003

$\Delta x/\Delta x_0 \sim 5$ [SQL $\Delta x_0 = (\hbar/2m\omega_0)^{1/2}$], Schwab's group, 2004

Continuous monitoring and quantum feedback can cool nanoresonator down to the ground state (Hopkins, Jacobs, Habib, Schwab, PRB 2003)

New feature: Braginsky's stroboscopic QND measurement using modulation of detector voltage ⇒ **squeezing becomes possible**

Potential application: ultrasensitive force measurements

Other most important papers:

Doherty, Jacobs, PRA 1999 (formalism for Gaussian states)

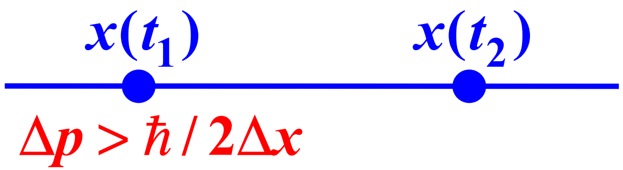
Mozyrsky, Martin, PRL 2002 (ensemble-averaged evolution)

Stroboscopic QND measurements

Quantum nondemolition (QND) measurements (Braginsky-Khalili book)
(a way to suppress measurement backaction and overcome standard quantum limit)

Idea: to avoid measuring the magnitude conjugated to the magnitude of interest

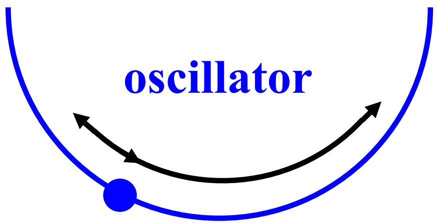
Standard quantum limit



Example: measurement of $x(t_2)$-$x(t_1)$

First measurement: $\Delta p(t_1) > \hbar/2\Delta x(t_1)$, then even for accurate second measurement inaccuracy of position difference is $\Delta x(t_1) + (t_2 - t_1)\hbar/2m\Delta x(t_1) > (t_2 - t_1)\hbar/2^{1/2}m$

Stroboscopic QND measurements (Braginsky *et al*., 1978; Thorne *et al*., 1978)

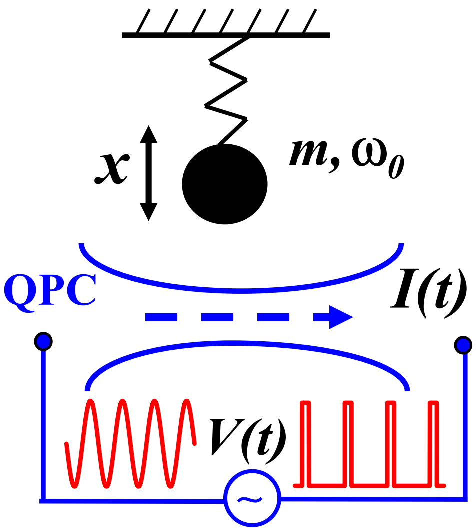


Idea: second measurement exactly one oscillation period later is insensitive to Δp
(or $\Delta t = nT/2$, $T=2\pi/\omega_0$)

Difference in our case:

- **continuous measurement**
- **weak coupling with detector**
- **quantum feedback to suppress "heating"**

Bayesian formalism for continuous measurement of a nanoresonator



$$\hat{H}_0 = \hat{p}^2/2m + m\omega_0^2\hat{x}^2/2$$

$$\hat{H}_{DET} = \sum_l E_l a_l^\dagger a_l + \sum_r E_r a_r^\dagger a_r + \sum_{l,r}(M a_l^\dagger a_r + H.c.)$$

$$\hat{H}_{INT} = \sum_{l,r}(\Delta M\, \hat{x}\, a_l^\dagger a_r + H.c.)$$

Current $I_x = 2\pi\left(M + \Delta M\, x\right)^2 \rho_l \rho_r e^2 V/\hbar = I_0 + k\,x$

Detector noise $S_x = S_0 \equiv 2eI_0$

Recipe: quantum Bayes procedure

Nanoresonator evolution (Stratonovich form), same equation as for qubits:

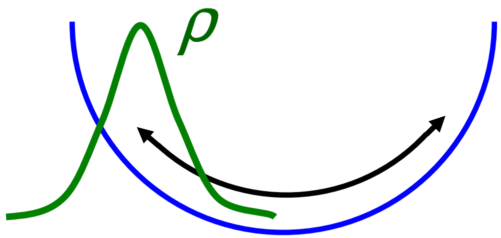
$$\frac{d\rho(x,x')}{dt} = \frac{-i}{\hbar}[\hat{H}_0, \rho] + \frac{\rho(x,x')}{S_0}\left\{ I(t)(I_x + I_{x'} - 2\langle I\rangle) - \frac{1}{2}\left(I_x^2 + I_{x'}^2 - 2\langle I^2\rangle\right)\right\}$$

$$\langle I\rangle = \sum I_x \rho(x,x), \quad I(t) = I_x + \xi(t), \quad S_\xi = S_0$$

Ito form (same as in many papers on conditional measurement of oscillators):

$$\frac{d\rho(x,x')}{dt} = \frac{-i}{\hbar}[\hat{H}_0, \rho] - \frac{k^2}{4S_0\eta}(x-x')^2\rho(x,x') + \frac{k}{S_0}(x + x' - 2\langle x\rangle)\,\rho(x,x')\,\xi(t)$$

Evolution of Gaussian states



Assume Gaussian states (following Doherty-Jacobs and Hopkins-Jacobs-Habib-Schwab), then ρ(x,x') is described by only 5 magnitudes:

$\langle \boldsymbol{x} \rangle$, $\langle p \rangle$ - average position and momentum (packet center),

$\boldsymbol{D_x}$, $\boldsymbol{D_p}$, $\boldsymbol{D_{xp}}$ – variances (packet width)

Assume large Q-factor (then no temperature)

Voltage modulation $f(t)V_0$: $\quad k = f(t)k_0, \quad I_x = f(t)(I_{00} + k_0 x), \quad S_I = |f(t)|\, S_0$

Then coupling (measurement strength) is also modulated in time:

$$C = |f(t)|\, C_0, \quad C = \hbar k^2 / S_I m\omega_0^2 = 4/\omega_0 \tau_{meas}$$

Packet center evolves randomly and needs feedback (force *F*) to cool down

$$d\langle x \rangle / dt = \langle p \rangle / m + (2k_0 / S_0)\, \mathrm{sgn}[f(t)]\, D_x\, \xi(t)$$

$$d\langle p \rangle / dt = -m\omega_0^2 \langle x \rangle + (2k_0 / S_0)\, \mathrm{sgn}[f(t)]\, D_{xp}\, \xi(t) + F(t)$$

Packet width evolves deterministically and is QND squeezed by periodic *f*(*t*)

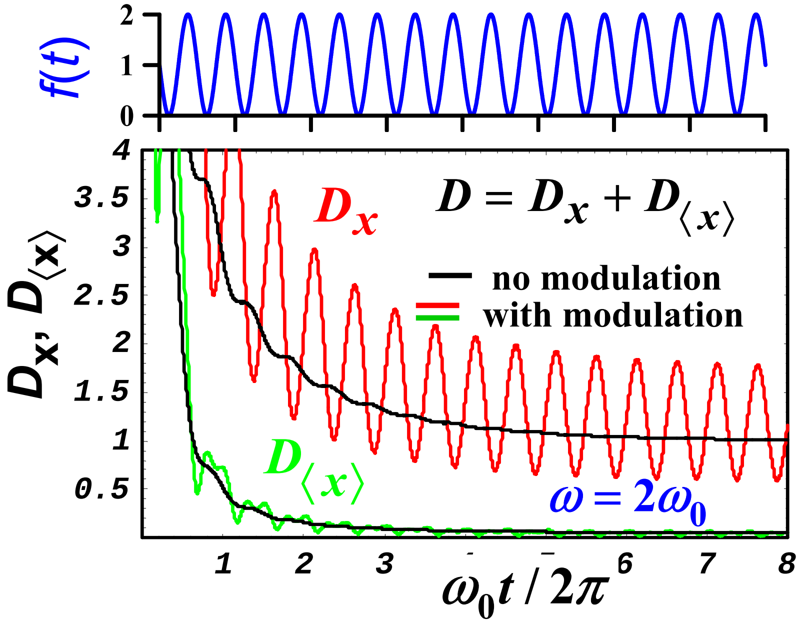
$$d\langle D_x \rangle / dt = (2/m) D_{xp} - (2k_0^2 / S_0)\, |f(t)|\, D_x^2$$

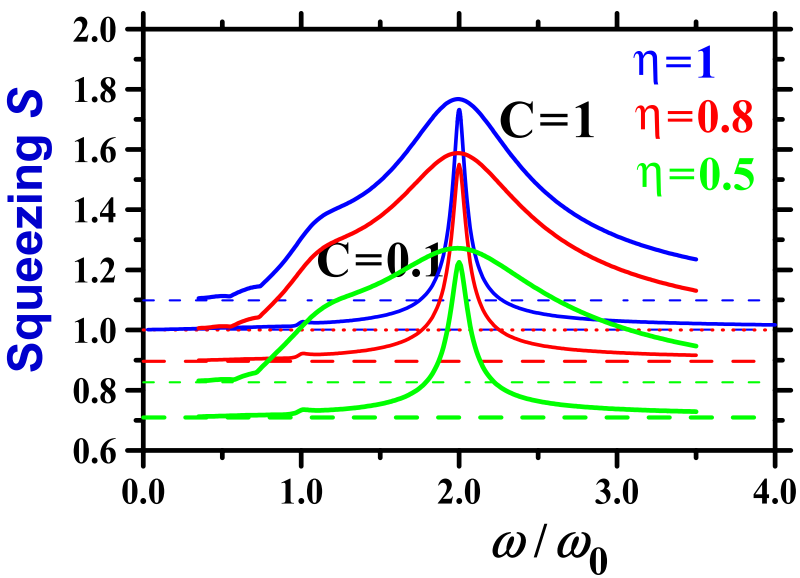
$$d\langle D_p \rangle / dt = -2m\omega_0^2 D_{xp} + (k_0^2 \hbar^2 / 2S_0 \eta)\, |f(t)| - (2k_0^2 / S_0)\, |f(t)|\, D_{xp}^2$$

$$d\langle D_{xp} \rangle / dt = (1/m) D_p - m\omega_0^2 D_x - (2k_0^2 / S_0)\, |f(t)|\, D_x D_{xp}$$

Squeezing by sine-modulation, $V(t)=V_0\sin(\omega t)$

Ruskov-Schwab-Korotkov





Squeezing obviously oscillates in time, maximum squeezing at maximum voltage, momentum squeezing shifted in phase by π/2.

$$S \equiv \max_t (\Delta x_0)^2 / D_x$$

Analytics (weak coupling):

$$S(2\omega_0) = \sqrt{3\eta}, \quad \Delta\omega = 0.36\omega_0 C_0 / \sqrt{\eta}$$

η - detector efficiency, C_0 – coupling

$\Delta x_0 = (\hbar/2m\omega_0)^{1/2}$ – ground state width

$D_x = (\Delta x)^2$, $D_{\langle x\rangle} = \langle\langle x\rangle^2\rangle - \langle\langle x\rangle\rangle^2$

Quantum feedback:

$$F = -m\omega_0\gamma_x\langle x\rangle - \gamma_p\langle p\rangle$$

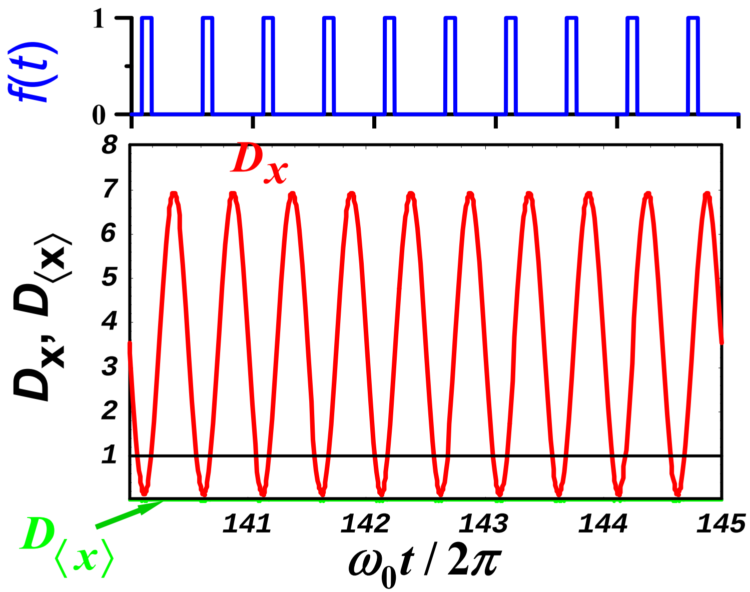
(same as in Hopkins *et al.*; without modulation it cools the state down to the ground state)

Feedback is sufficiently efficient, $D_{\langle x\rangle}$ Ü D_x

Squeezing up to 1.73 at $\omega = 2\omega_0$

Squeezing by stroboscopic (pulse) modulation

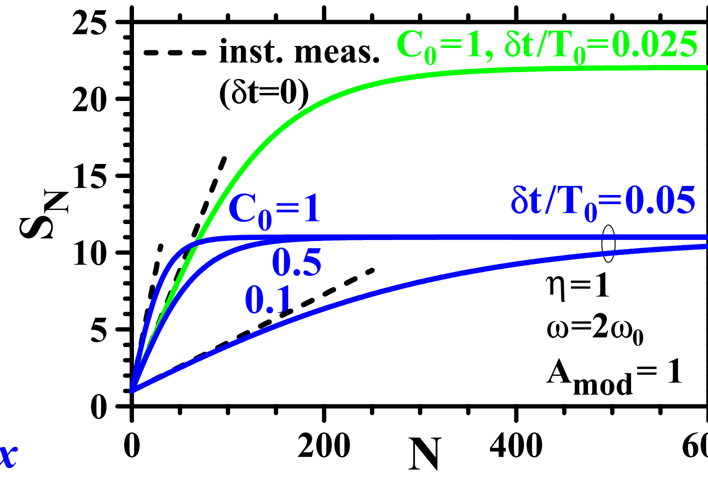
pulse modulation $\omega = 2\omega_0 / n$

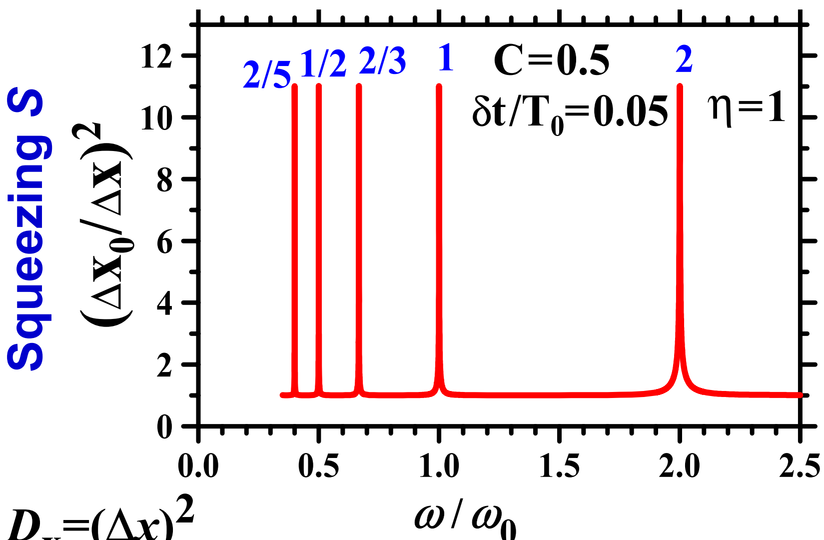


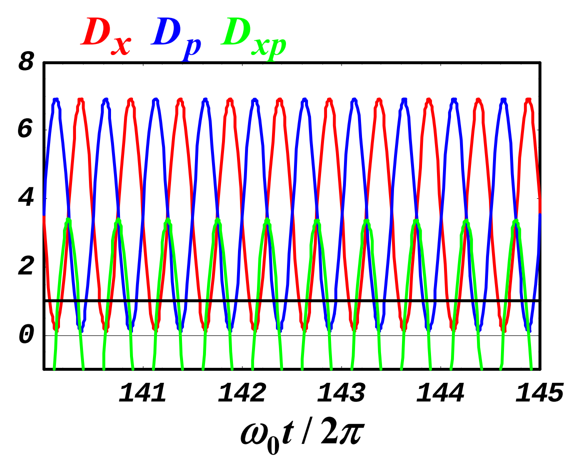
$D_{\langle x \rangle} \ll D_x$

using feedback

Squeezing buildup (in time)







$D_x = (\Delta x)^2$

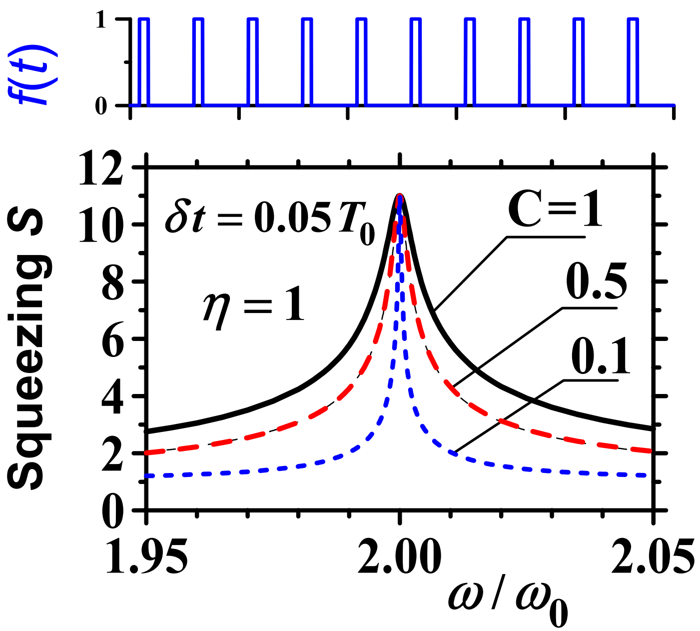
***S* á 1**

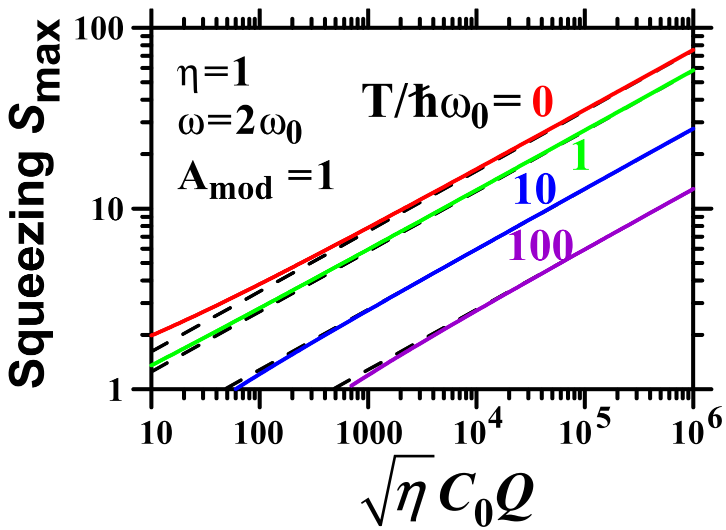
Efficient squeezing at $\omega = 2\omega_0/n$

(natural QND condition)

Ruskov-Schwab-Korotkov

Squeezing by stroboscopic modulation





Analytics (weak coupling, short pulses)

Maximum squeezing

$$S(2\omega_0/n) = \frac{2\sqrt{3\eta}}{\omega_0\delta t}$$

Linewidth

$$\Delta\omega = \frac{4C_0(\delta t)^3\omega_0^4}{\pi n^2\sqrt{3\eta}}$$

C_0 – dimensionless coupling with detector
δt – pulse duration, $T_0 = 2\pi/\omega_0$
η – quantum efficiency of detector

Squeezing requires $\sim \sqrt{3\eta}/C_0(\omega_0\delta t)^2$ pulses

Finite *Q*-factor and finite temperature limit

maximum squeezing S_{max}

$$S_{max} = \frac{3}{4}\left[\frac{\sqrt{\eta}\,C_0 Q}{\coth(\hbar\omega_0/2T)}\right]^{1/3}$$

(So far in experiment $\eta^{1/2}C_0Q \sim 0.1$)

Verification of nanoresonator squeezing

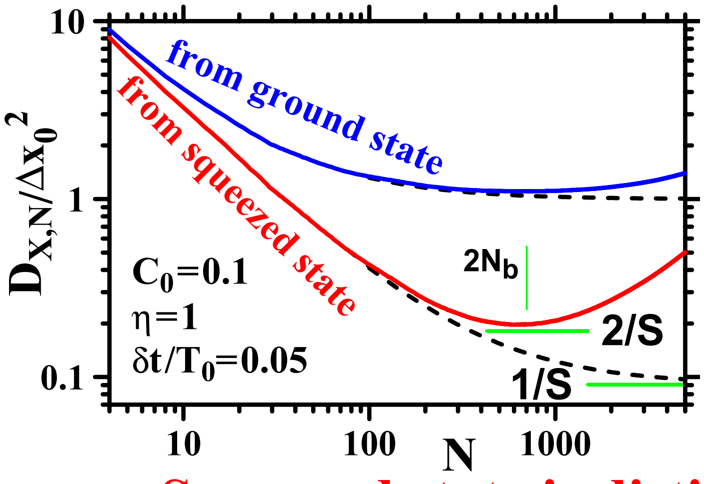
Procedure: 1) prepare squeezed state by stroboscopic measurement,
2) switch off quantum feedback
3) measure in the stroboscopic way $X_N = \frac{1}{N}\sum_{j=1}^{N} x_j$

For instantaneous measurements ($\delta t \to 0$) the variance of X_N is

$$D_{X,N} = \frac{\hbar}{2m\omega_0}\left(\frac{1}{S} + \frac{1}{NC_0\omega_0\delta t}\right) \to \frac{1}{S}(\Delta x_0)^2 \quad \text{at } N \to \infty$$

S – squeezing,
Δx_0 – ground state width

Distinguishable from ground state (S=1) in one run for S à 1
(error probability ~$S^{-1/2}$)



About twice worse for continuous measurements because of extra "heating"

$$\min_N D_{X,N} \sim 2(\Delta x_0)^2 / S$$

Squeezed state is distinguishable in one run (with small error probability), therefore suitable for ultrasensitive force measurement beyond standard quantum limit

Conclusion

- Quantum feedback in solid state is a relatively new subject, while in optics already demonstrated
- Quantum feedback can maintain non-decaying Rabi oscillations in a solid-state qubit with 100% fidelity
- A relatively simple experiment for non-decaying Rabi oscillations in a qubit is proposed (fidelity is limited to ~95%)
- Stroboscopic QND measurement and quantum feedback of a nanomechanical resonator produces squeezed state and is useful for ultrasensitive force detection