F&N (SPIE)'05, Austin, TX, 05/24/2005

Simple quadrature-based quantum feedback of a solid-state qubit

Alexander Korotkov

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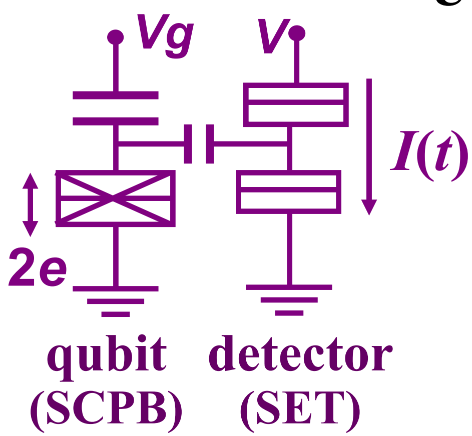
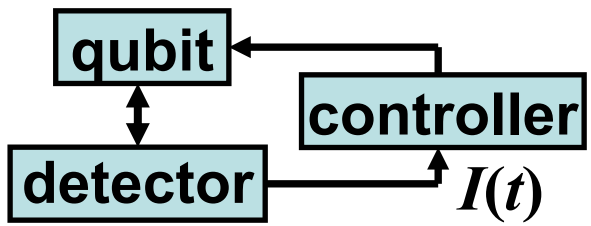
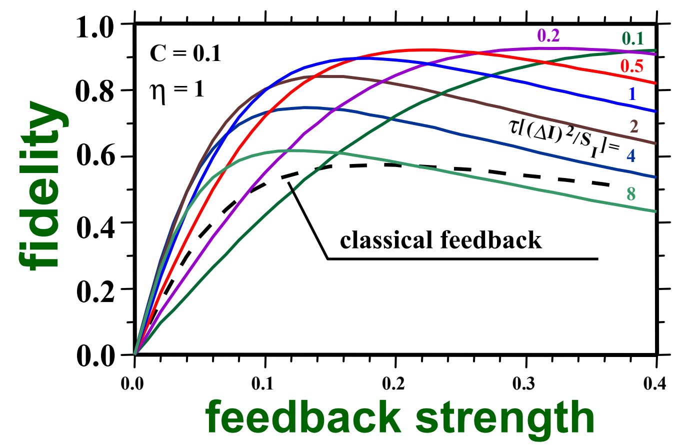


Goal: keep coherent oscillations forever



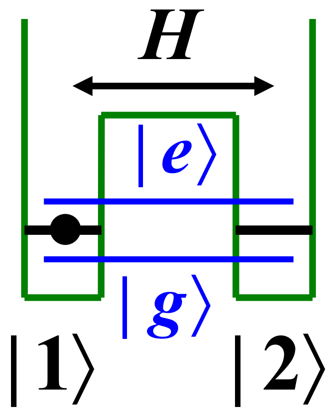


Outline:
- Problem statement and introduction
- Bayesian quantum feedback of a qubit
- Simple quantum feedback of a qubit

Support:

PRB 71, 201305(R) (2005)

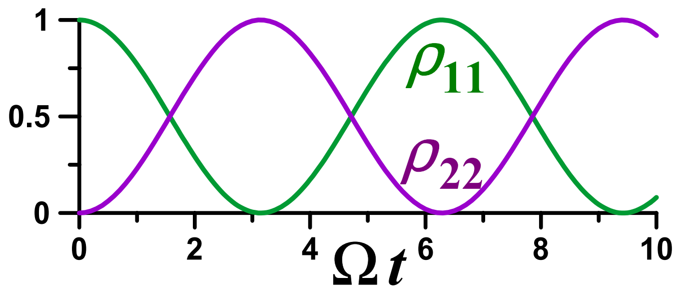
Quantum coherent ("Rabi") oscillations in a qubit (two-level system)



$$\hat{H}_{QB} = \frac{\varepsilon}{2}(|1\rangle\langle 1| - |2\rangle\langle 2|) + H(|1\rangle\langle 2| + |2\rangle\langle 1|)$$

ε - energy asymmetry, H - tunneling

$$\Omega = \frac{\sqrt{\varepsilon^2 + 4H^2}}{\hbar}$$ - oscillation frequency



If ε=0, and start with |1>, then

$$\psi(t) = |1\rangle \cos(\Omega t/2) + |2\rangle \sin(\Omega t/2)$$

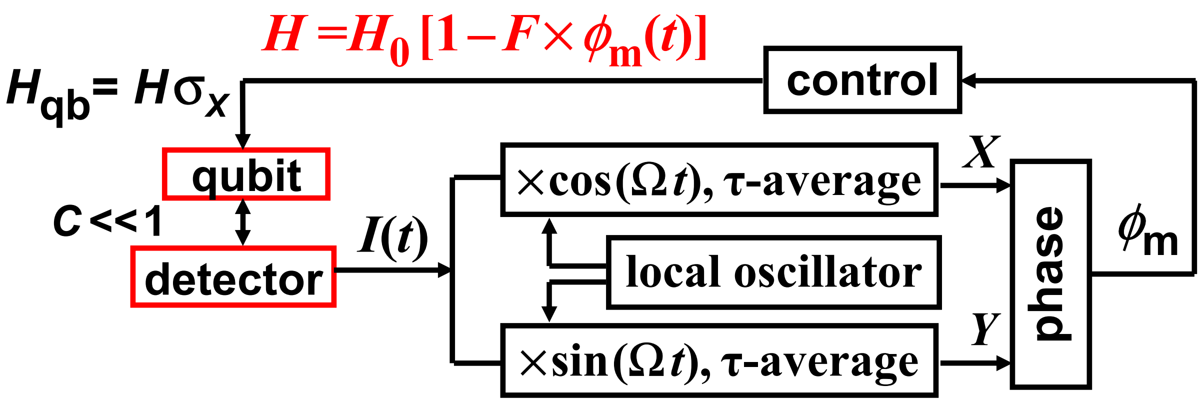
$$\rho_{11}(t) - \rho_{22}(t) = \cos \Omega t$$

Oscillations are fragile and decay quickly (decoherence).

Pure Hamiltonian control cannot help (shrinking of Liouville space).

How can we keep them forever? (Feedback?)

Proposal of quantum feedback setup

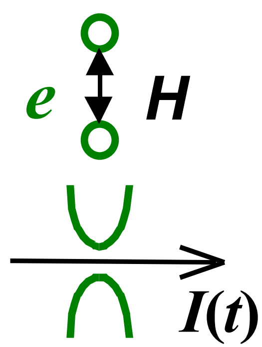


Goal: keep coherent (Rabi) oscillations in a qubit for arbitrarily long time

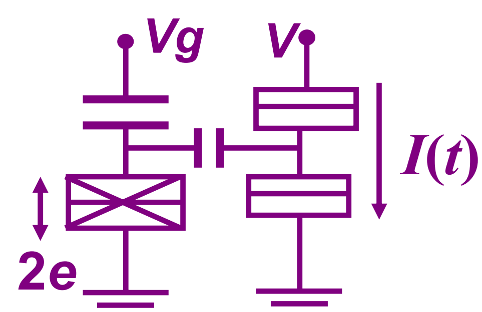
Idea: use *quadrature components* of the detector current $I(t)$ for monitoring the oscillation phase (as in a classical feedback!)

Result: **works surprisingly well** (fidelity up to 95%)

Implementations:



DQD and QPC



SCPB and SET

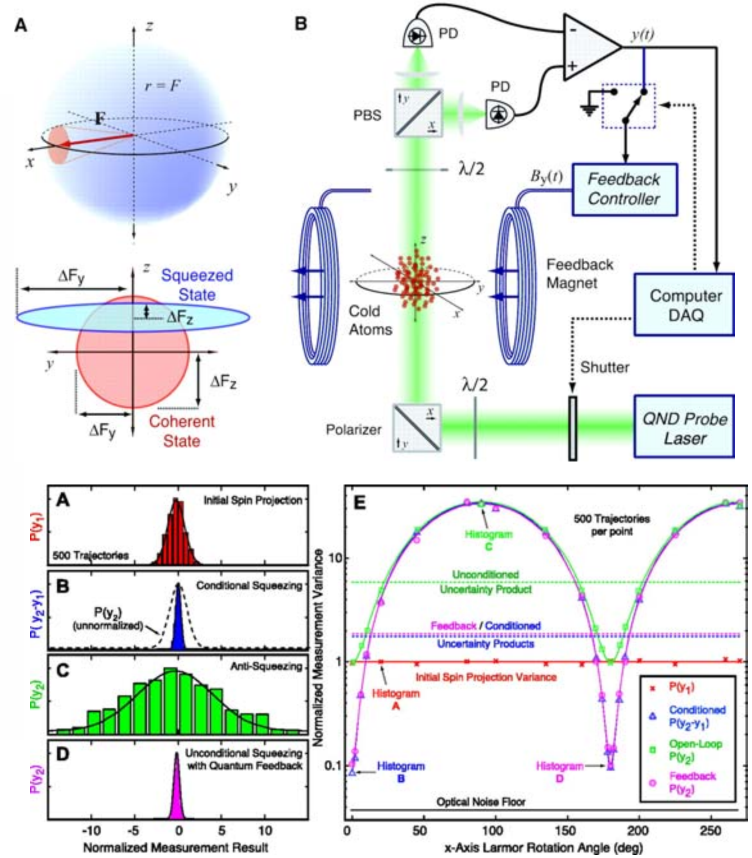
Quantum feedback in optics

Recent experiment: Science 304, 270 (2004)

Real-Time Quantum Feedback Control of Atomic Spin-Squeezing

JM Geremia,* John K. Stockton, Hideo Mabuchi

Real-time feedback performed during a quantum nondemolition measurement of atomic spin-angular momentum allowed us to influence the quantum statistics of the measurement outcome. We showed that it is possible to harness measurement backaction as a form of actuation in quantum control, and thus we describe a valuable tool for quantum information science. Our feedback-mediated procedure generates spin-squeezing, for which the reduction in quantum uncertainty and resulting atomic entanglement are not conditioned on the measurement outcome.



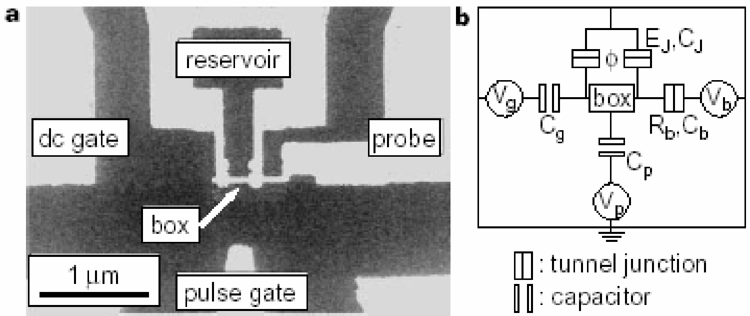
First detailed theory:

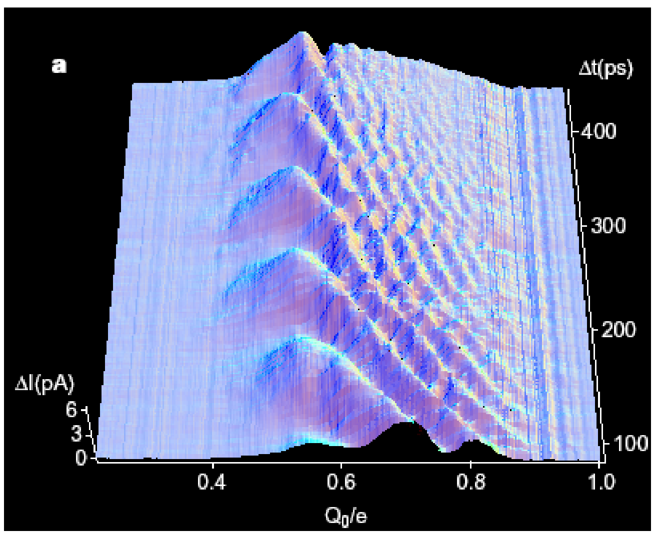
H.M. Wiseman and G. J. Milburn, Phys. Rev. Lett. 70, 548 (**1993**)

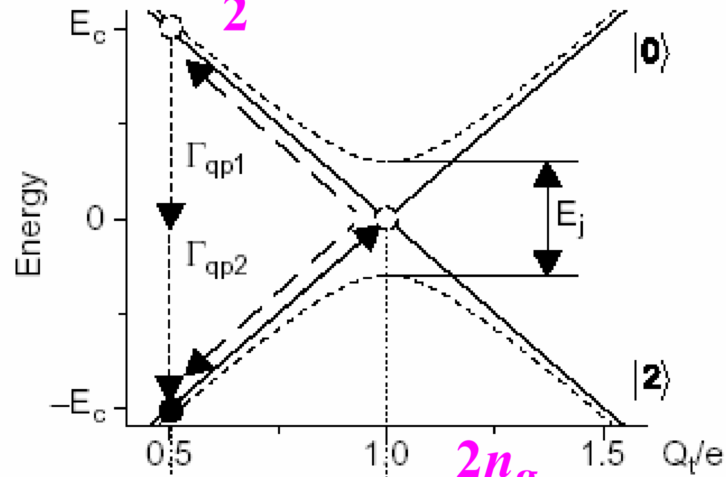
Superconducting “charge” qubits

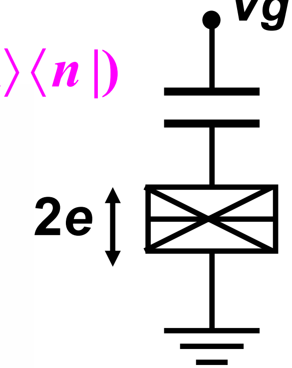
Nakamura, Pashkin, Tsai (Nature, 1998)

$$\hat{H} = \frac{(2e)^2}{2C}(\hat{n} - n_g)^2 - \frac{E_J}{2}(|n\rangle\langle n+1| + |n+1\rangle\langle n|)$$

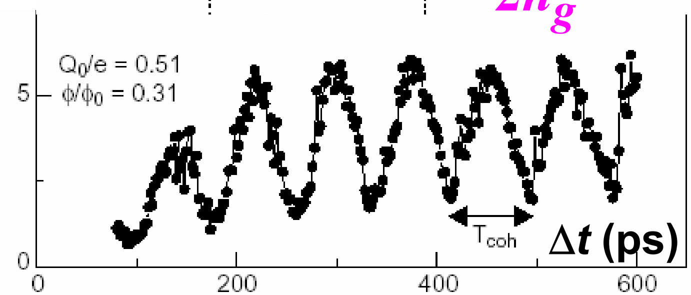


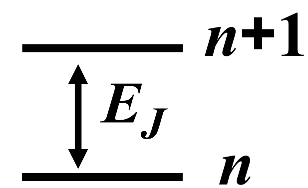






Single Cooper pair box



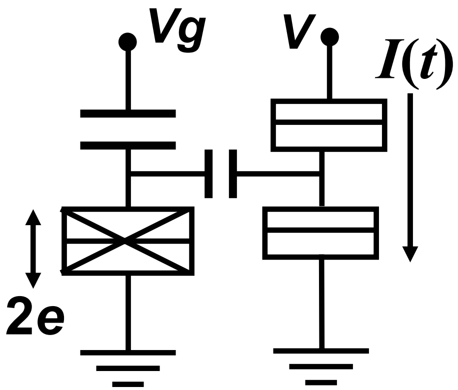


Quantum coherent (Rabi) oscillations

Vion et al. (Devoret’s group); Science, 2002

Q-factor of coherent (Rabi) oscillations = 25,000

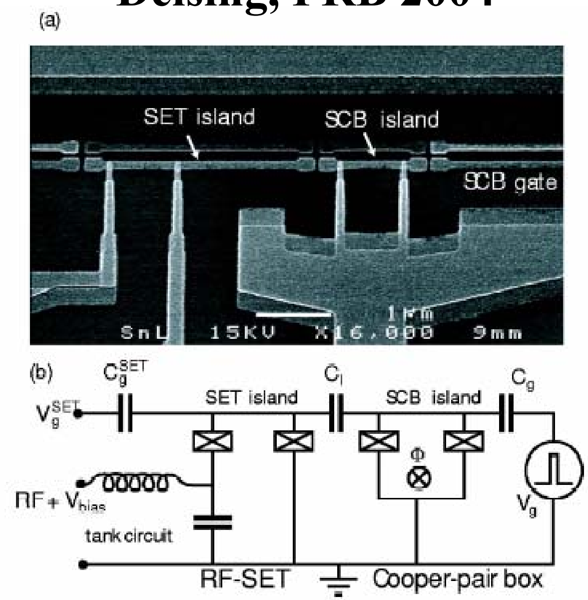
Superconducting “charge” qubits (2)

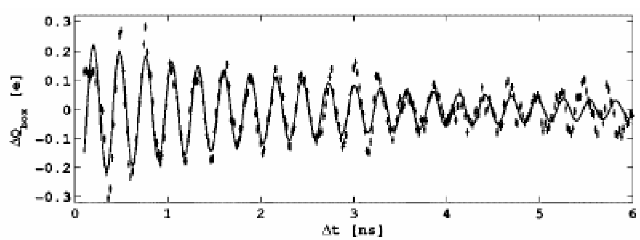


Cooper-pair box measured by single-electron transistor (SET)
(actually, RF-SET)

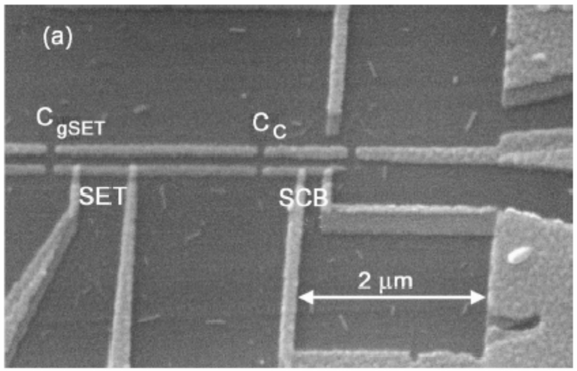
Setup can be used for continuous measurements

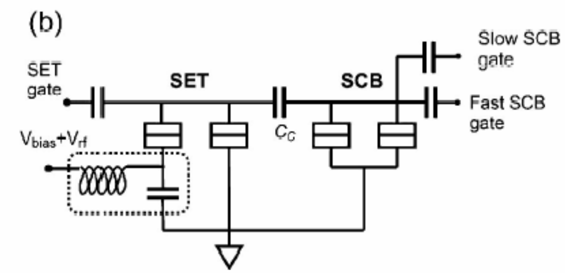
Duty, Gunnarsson, Bladh, Delsing, PRB 2004

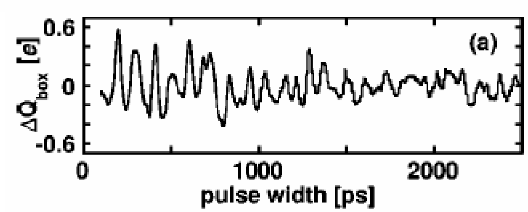




Guillaume et al. (Echternach’s group), PRB 2004



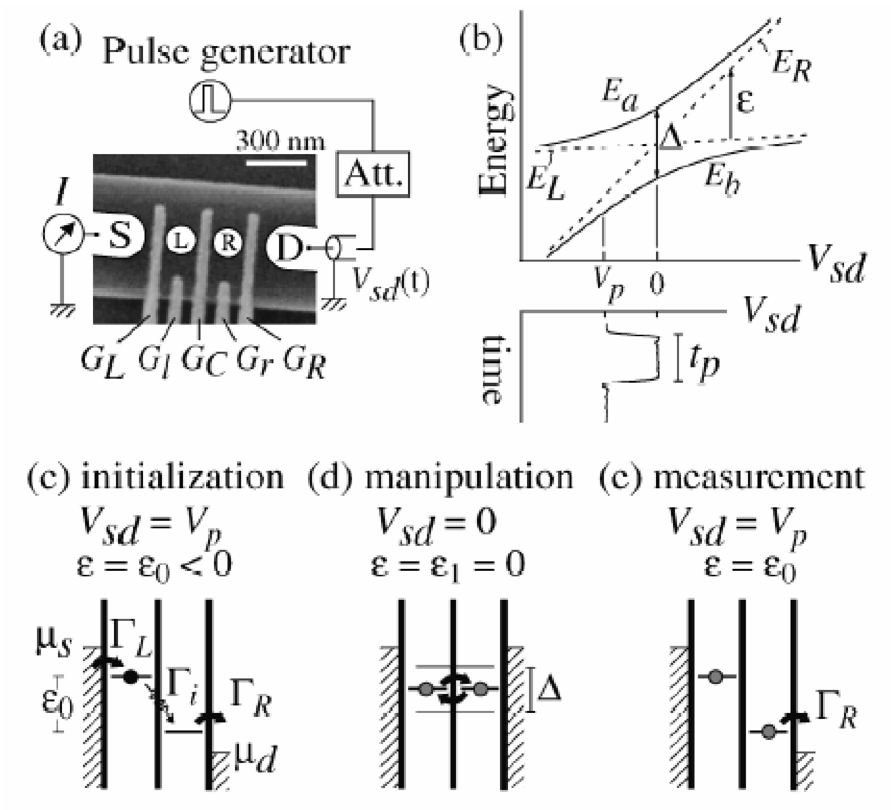


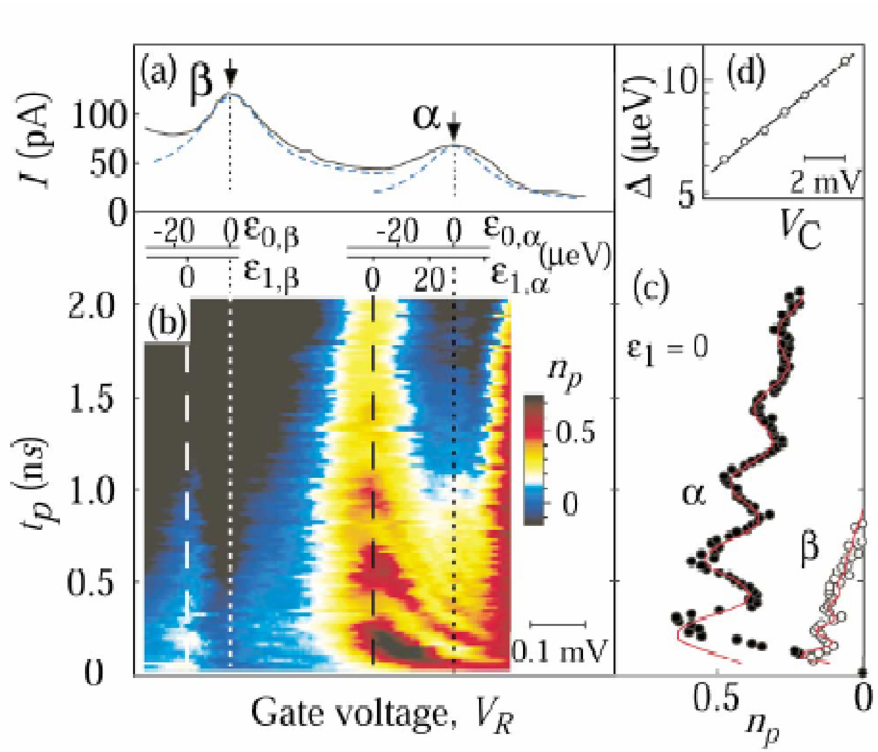


All results are averaged over many measurements (not “single-shot”)

Semiconductor (double-dot) qubit

T. Hayashi et al., PRL 2003

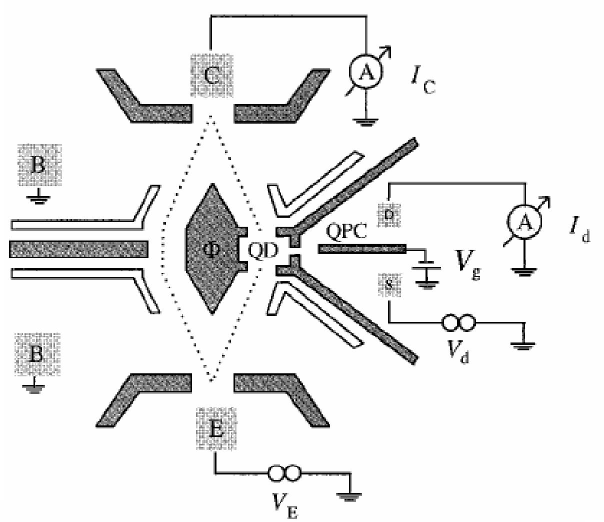


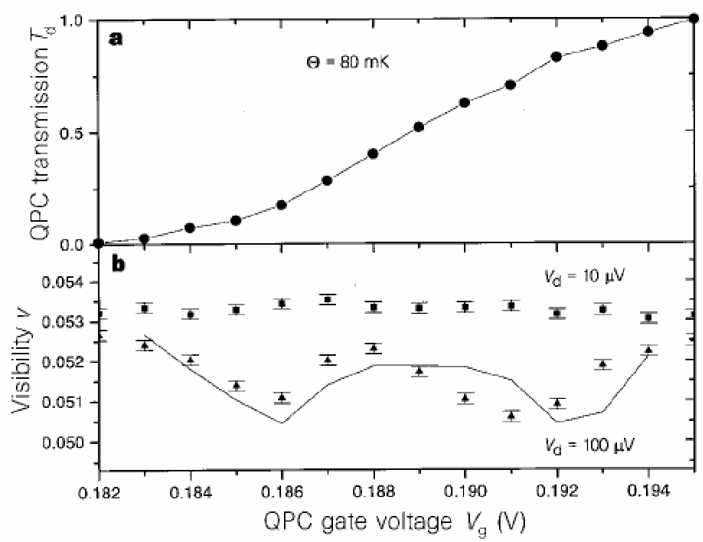


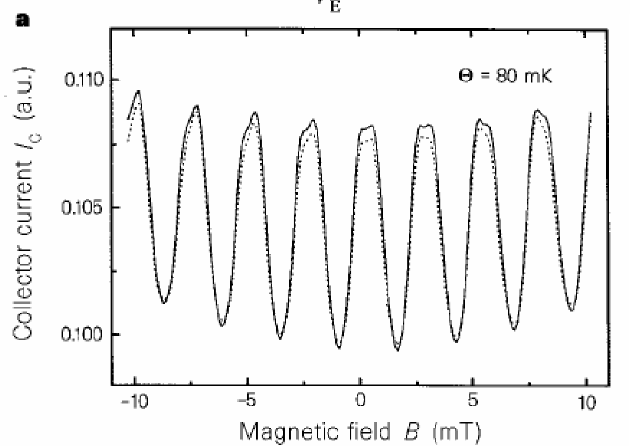
Detector is not separated similar to Nakamura-98,
also possible to use a separate detector

"Which-path detector" experiment

Buks, Schuster, Heiblum, Mahalu, and Umansky, Nature 1998







Dephasing rate: $$\Gamma = \frac{eV}{h}\frac{(\Delta T)^2}{T(1-T)} = \frac{(\Delta I)^2}{4S_I}$$

ΔI – detector response, S_I – shot noise

The larger noise, the smaller dephasing!!!

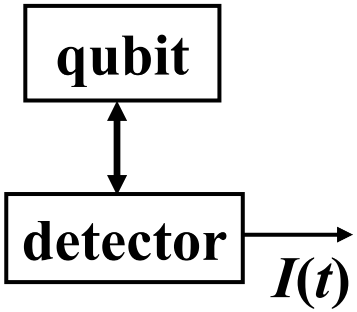
$(\Delta I)^2/4S_I$ ~ rate of "information flow"

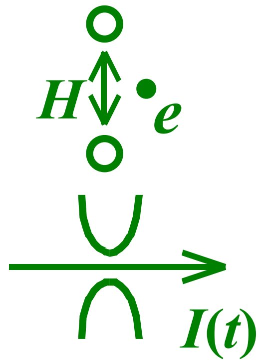
$\tau_m = 2S_I/(\Delta I)^2$ – "measurement time"

(Shnirman-Schon, 1998)

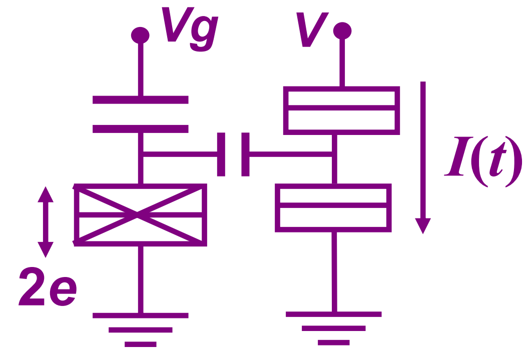
Theory: Aleiner, Wingreen, and Meir, PRL 1997

The system we consider: qubit + detector





Double-quantum-qot (DQD) and quantum point contact (QPC)



Cooper-pair box (CPB) and single-electron transistor (SET)

$H = H_{\mathrm{QB}} + H_{\mathrm{DET}} + H_{\mathrm{INT}}$

$H_{\mathrm{QB}} = (\varepsilon/2)(c_1^+c_1 - c_2^+c_2) + H(c_1^+c_2 + c_2^+c_1)$ ε – asymmetry, H – tunneling

$\Omega = (4H^2+\varepsilon^2)^{1/2}/\hbar$ – frequency of quantum coherent (Rabi) oscillations

Two levels of average detector current: I_1 for qubit state $|1\rangle$, I_2 for $|2\rangle$

Response: $\Delta I = I_1 - I_2$ Detector noise: white, spectral density S_I

DQD and QPC
(setup due to Gurvitz, 1997)

$$H_{DET} = \sum_l E_l a_l^\dagger a_l + \sum_r E_r a_r^\dagger a_r + \sum_{l,r} T(a_r^\dagger a_l + a_l^\dagger a_r)$$

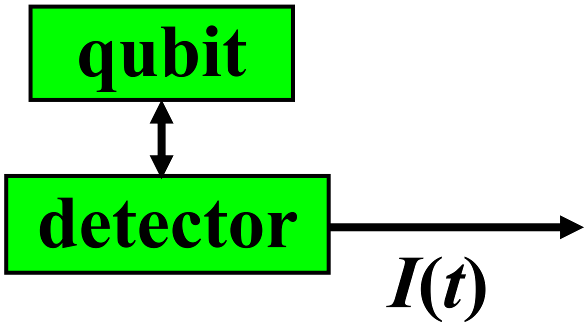
$$H_{INT} = \sum_{l,r} \Delta T\,(c_1^\dagger c_1 - c_2^\dagger c_2)(a_r^\dagger a_l + a_l^\dagger a_r)$$

$$S_I = 2eI$$

Continuous quantum measurement of a solid-state qubit

continuous measurement ⇔ gradual collapse

(Shot noise = quantum noise!)

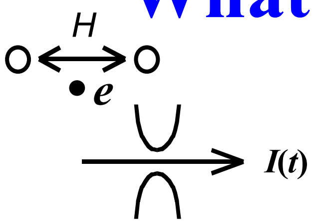


Starting point:

What is the qubit evolution due to continuous measurement by a detector?

How qubit evolution is related to the noisy detector output $I(t)$?

What happens to qubit state during measurement?



For simplicity (for a moment) $H=\varepsilon=0$, infinite barrier, evolution due to measurement only

"Orthodox" answer

$$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \begin{matrix} \to \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \\ \to \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \end{matrix}$$

|1> or |2>, depending on the result

"Conventional" (decoherence) answer (Leggett, Zurek)

$$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \to \begin{pmatrix} \frac{1}{2} & \frac{\exp(-\Gamma t)}{2} \\ \frac{\exp(-\Gamma t)}{2} & \frac{1}{2} \end{pmatrix} \to \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

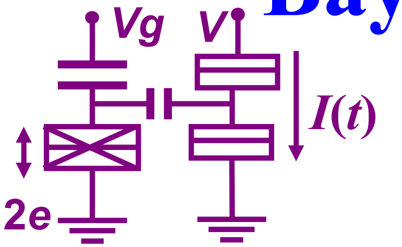
no measurement result! ensemble averaged

Orthodox and decoherence answers contradict each other!

applicable for:	Single quantum systems	Continuous measurements
Orthodox	yes	no
Conventional (ensemble)	no	yes
Bayesian	yes	yes

Bayesian formalism describes gradual collapse of single quantum systems
Noisy detector output $I(t)$ should be taken into account

Bayesian formalism for a single qubit



$$\hat{H}_{QB} = \frac{\varepsilon}{2}(c_1^\dagger c_1 - c_2^\dagger c_2) + H(c_1^\dagger c_2 + c_2^\dagger c_1)$$

|1ò Æ I_1, |2ò Æ I_2 $\Delta I = I_1 - I_2$, $I_0 = (I_1 + I_2)/2$, S_I – detector noise

$$\dot{\rho}_{11} = -\dot{\rho}_{22} = -2(H/\hbar)\,\mathrm{Im}\,\rho_{12} + \rho_{11}\rho_{22}(2\Delta I/S_I)[\underline{\underline{I(t)}} - I_0]$$

$$\dot{\rho}_{12} = i(\varepsilon/\hbar)\rho_{12} + i(H/\hbar)(\rho_{11} - \rho_{22}) + \rho_{12}(\rho_{11} - \rho_{22})(\Delta I/S_I)[\underline{\underline{I(t)}} - I_0] - \gamma\rho_{12}$$

(A.K., 1998)

$\gamma = \Gamma - (\Delta I)^2/4S_I$, Γ – ensemble decoherence

$\eta = 1 - \gamma/\Gamma = (\Delta I)^2/4S_I\Gamma$ – detector ideality (efficiency), $\eta \le 100\%$

Ideal detector (η=1, e.g., QPC) does not decohere a single qubit; then random evolution of qubit *wavefunction* can be monitored

For simulations: $I(t) - I_0 = (\rho_{22} - \rho_{11})\Delta I/2 + \xi(t)$, $S_\xi = S_I$

Averaging over ξ(t) ï conventional master equation

Similar formalisms developed earlier. Key words: Imprecise, weak, selective, or conditional measurements, POVM, Quantum trajectories, Quantum jumps, Restricted path integral, etc.

Names: Davies, Kraus, Holevo, Mensky, Caves, Gardiner, Carmichael, Plenio, Knight, Walls, Gisin, Percival, **Milburn**, **Wiseman,** Onofrio, Habib, Doherty, etc. (incomplete list)

Measurement vs. decoherence

Widely accepted point of view:

measurement = decoherence (environment)

Is it true?

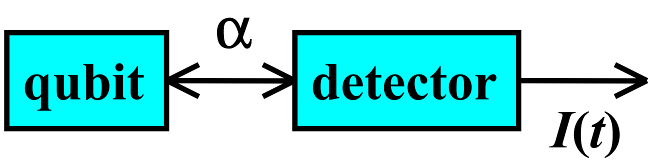
- **Yes**, if not interested in information from detector (ensemble-averaged evolution)

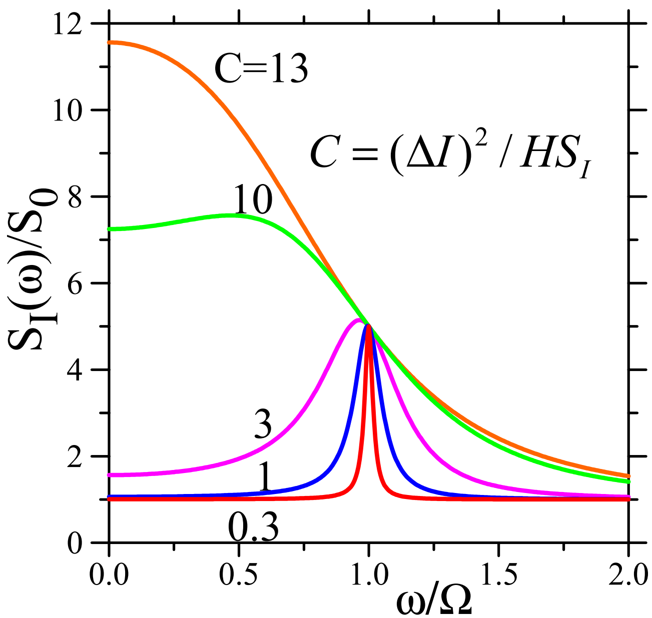
- **No**, if take into account measurement result (single quantum system)

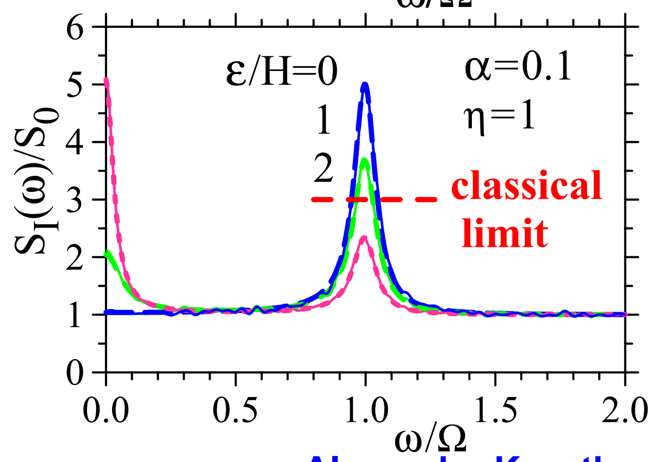
Measurement result obviously gives us more information about the measured system, so we know its quantum state better (ideally, a pure state instead of a mixed state)

measurement = purification!

Measured spectrum of qubit coherent oscillations







What is the spectral density $S_I(\omega)$ of detector current?

Assume classical output, eV » $\hbar\Omega$

$$\varepsilon = 0, \quad \Gamma = \eta^{-1}(\Delta I)^2 / 4S_0$$

$$S_I(\omega) = S_0 + \frac{\Omega^2 (\Delta I)^2 \Gamma}{(\omega^2 - \Omega^2)^2 + \Gamma^2 \omega^2}$$

Spectral peak can be seen, but peak-to-pedestal ratio ≤ 4η ≤ 4

(result can be obtained using various methods, not only Bayesian method)

Weak coupling, $\alpha = C/8 \ll 1$

$$S_I(\omega) = S_0 + \frac{\eta S_0 \varepsilon^2 / H^2}{1 + (\omega\hbar^2\Omega^2 / 4H^2\Gamma)^2} + \frac{4\eta S_0 (1 + \varepsilon^2 / 2H^2)^{-1}}{1 + [(\omega - \Omega)\Gamma(1 - 2H^2/\hbar^2\Omega^2)]^2}$$

A.K., LT'99
Averin-A.K., 2000
A.K., 2000
Averin, 2000
Goan-Milburn, 2001
Makhlin et al., 2001
Balatsky-Martin, 2001
Ruskov-A.K., 2002
Mozyrsky et al., 2002
Balatsky et al., 2002
Bulaevskii et al., 2002
Shnirman et al., 2002
Bulaevskii-Ortiz, 2003
Shnirman et al., 2003

Contrary:
Stace-Barrett, PRL'04

Possible experimental confirmation?

Durkan and Welland, 2001 (STM-ESR experiment similar to Manassen-1989)

APPLIED PHYSICS LETTERS VOLUME 80, NUMBER 3 21 JANUARY 2002

Electronic spin detection in molecules using scanning-tunneling-microscopy-assisted electron-spin resonance

C. Durkan[a] and M. E. Welland
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(Received 8 May 2001; accepted for publication 8 November 2001)

By combining the spatial resolution of a scanning-tunneling microscope (STM) with the electronic spin sensitivity of electron-spin resonance, we show that it is possible to detect the presence of localized spins on surfaces. The principle is that a STM is operated in a magnetic field, and the resulting component of the tunnel current at the Larmor (precession) frequency is measured. This component is nonzero whenever there is tunneling into or out of a paramagnetic entity. We have

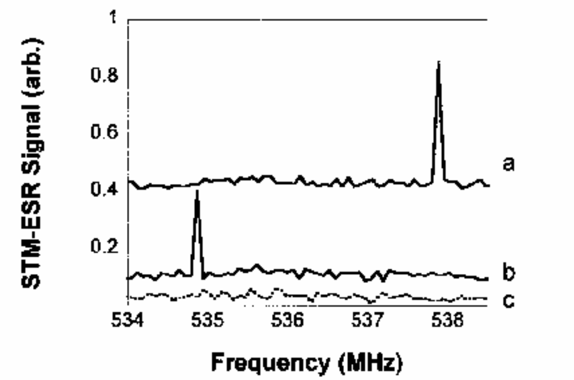


FIG. 3. STM-ESR spectra of (a), (b) two different areas (a few nm apart) of the molecule-covered sample and (c) bare HOPG. The graphs are shifted vertically for clarity.

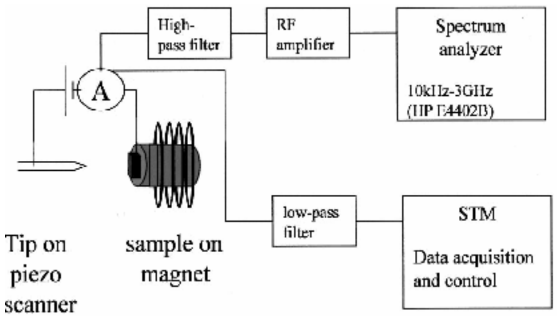


FIG. 1. Schematic of the electronics used in STM-ESR.

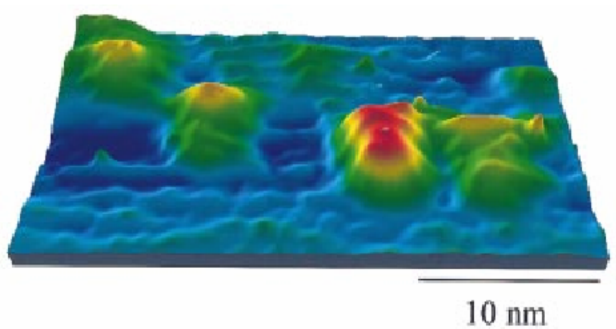


FIG. 2. (Color) STM image of a 250 Å×150 Å area of HOPG with four adsorbed BDPA molecules.

$$\frac{\text{peak}}{\text{noise}} \le 3.5$$

(Colm Durkan, private comm.)

Somewhat similar experiment

"Continuous monitoring of Rabi oscillations in a Josephson flux qubit"

E. Il'ichev *et al.*, PRL, 2003

$$H = -\frac{1}{2}(\Delta\sigma_x + \varepsilon\sigma_z) - W\sigma_z \cos\omega_{HF} t$$

$$(\omega_{HF} \approx \sqrt{\Delta^2 + \varepsilon^2};\ \ \varepsilon \neq 0)$$

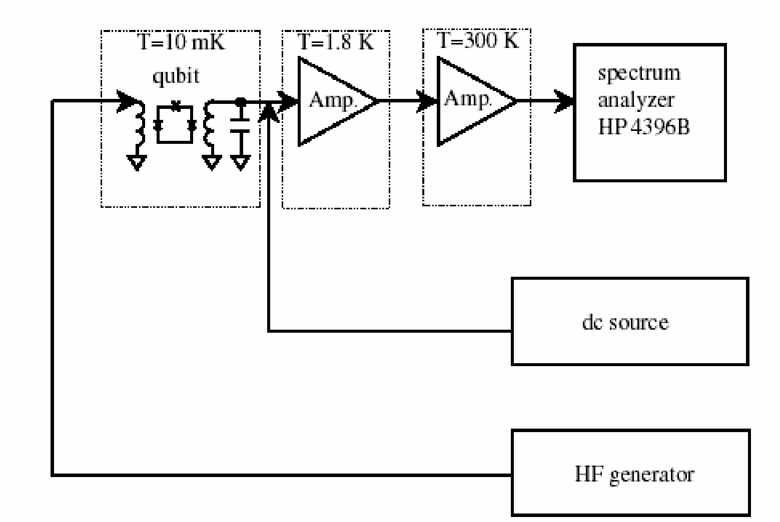


FIG. 1. Measurement setup. The flux qubit is inductively coupled to a tank circuit. The dc source applies a constant flux $\Phi_e \approx \frac{1}{2}\Phi_0$. The HF generator drives the qubit through a separate coil at a frequency close to the level separation $\Delta/h =$ 868 MHz. The output voltage at the resonant frequency of the tank is measured as a function of HF power.

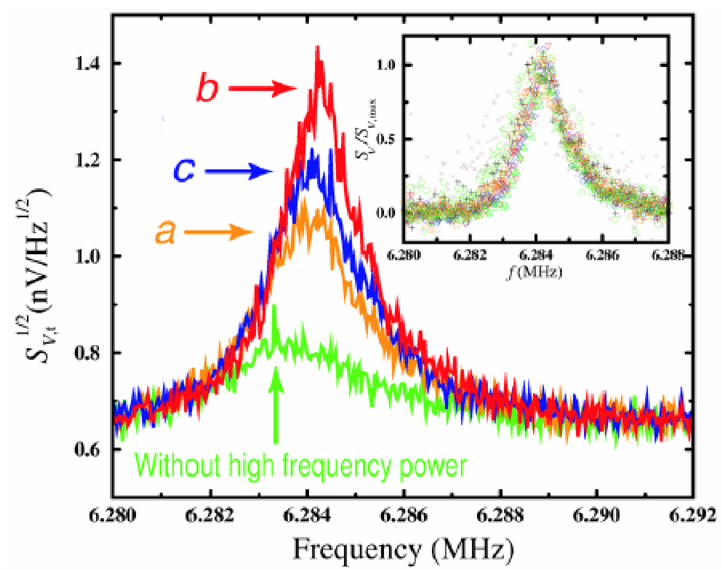
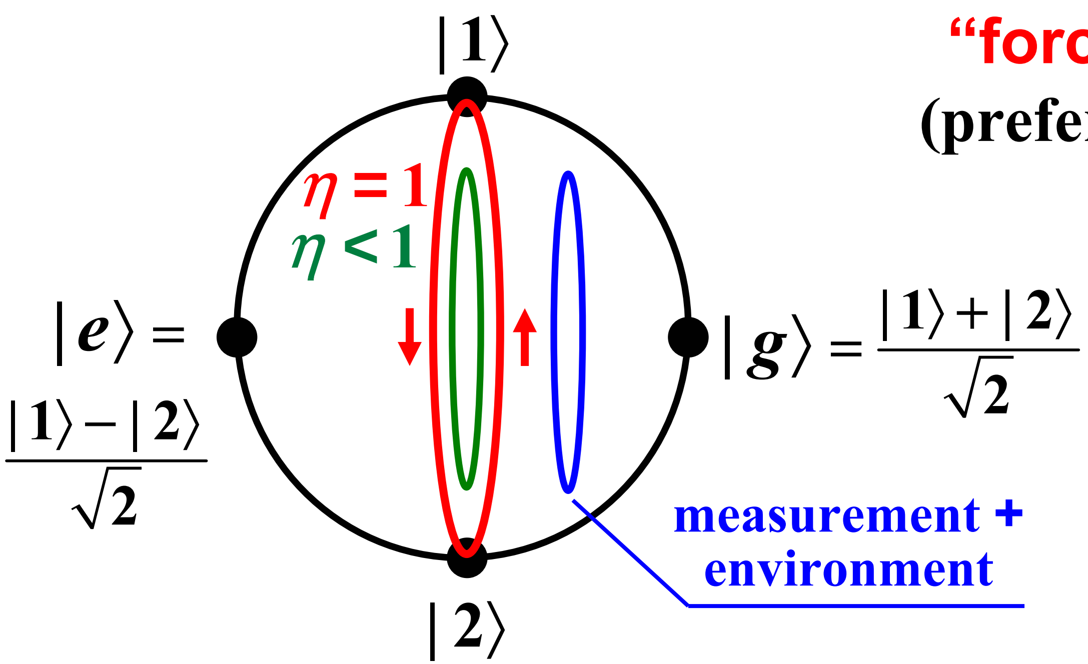


FIG. 3 (color online). The spectral amplitude of the tank voltage for HF powers $P_a < P_b < P_c$ at 868 MHz, detected using the setup of Fig. 1. The bottom curve corresponds to the background noise without an HF signal. The inset shows normalized voltage spectra for seven values of HF power, with background subtracted. The shape of the resonance, being determined by the tank circuit, is essentially the same in each case. Remaining tiny variations visible in the main figure are due to the irradiated qubit modifying the tank's inductance and

Oscillations in a measured qubit

Bloch sphere



Continuous measurement "forces" qubit to oscillate

(prefers to localize in $|1\rangle$ or $|2\rangle$)

$$\rho_{11} - \rho_{22} = \cos[\Omega t + \phi(t)]$$

(ideal, η=1)

Quantum back-action: random evolution of $\phi(t)$

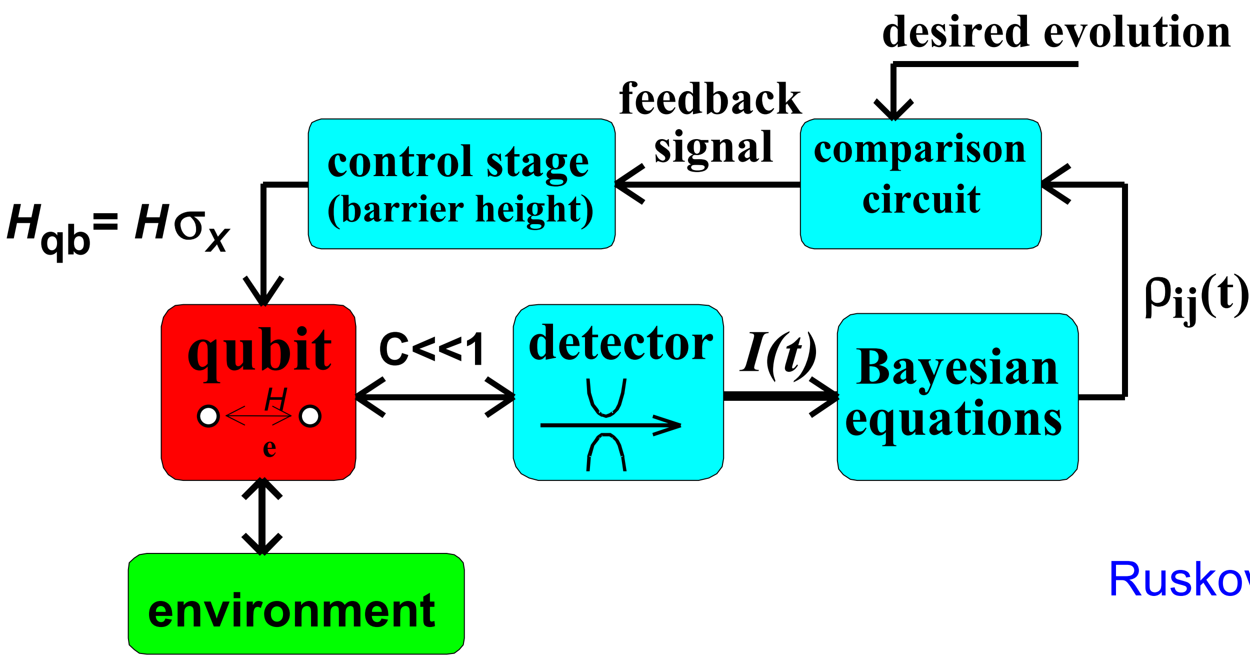
Bad detector (environment): averaging over $\phi(t)$

Does the qubit "really" oscillate in the case of a bad detector? Does not matter (just philosophy), density matrix takes care.

Goal: perfect oscillations, $\phi(t)$=0

Quantum feedback control of a qubit

Since qubit state can be monitored, the feedback is possible!



Ruskov-Korotkov, 2001

Goal: maintain desired phase of coherent (Rabi) oscillations in spite of environmental dephasing (keep qubit “fresh”)

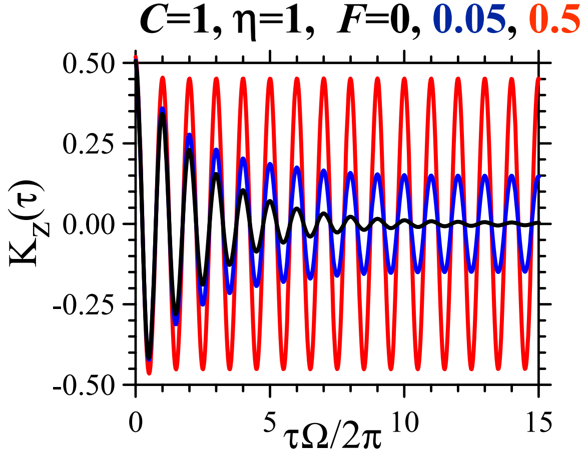
Idea: monitor the Rabi phase ϕ by continuous measurement and apply feedback control of the qubit barrier height, $\Delta H_{FB}/H = -F\times\Delta\phi$

To monitor phase ϕ we plug detector output $I(t)$ into Bayesian equations

Performance of quantum feedback

(no extra environment)

Qubit correlation function



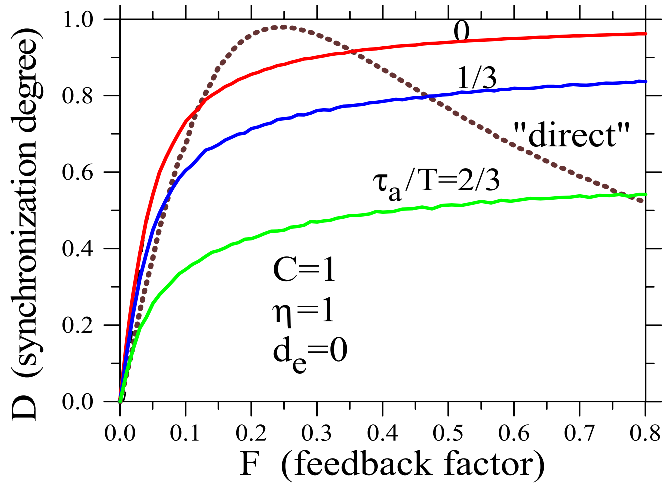
$$K_z(\tau) = \frac{\cos \Omega t}{2} \exp\left[\frac{C}{16F}(e^{-2FH\tau/\hbar} - 1)\right]$$

(for weak coupling and good fidelity)

Detector current correlation function

$$K_I(\tau) = \frac{(\Delta I)^2}{4} \frac{\cos \Omega t}{2} (1 + e^{-2FH\tau/\hbar}) \times \exp\left[\frac{C}{16F}(e^{-2FH\tau/\hbar} - 1)\right] + \frac{S_I}{2}\delta(\tau)$$

Fidelity (synchronization degree)



$C = \hbar(\Delta I)^2 / S_I H$ – coupling

τ_a^{-1} – available bandwidth

F – feedback strength

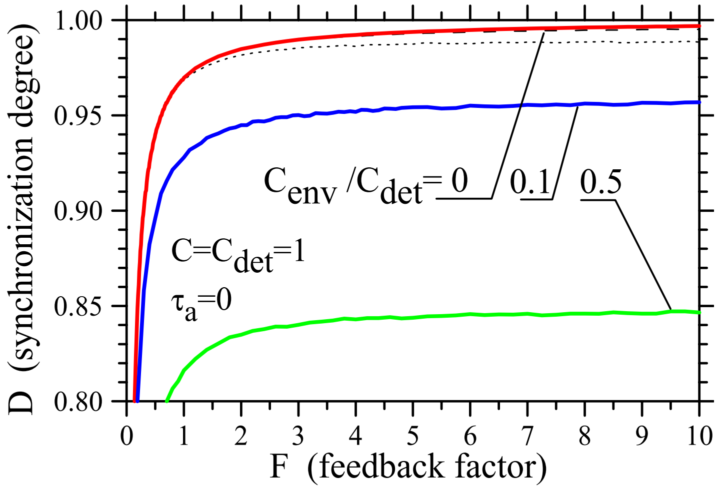
$$D = 2\langle \mathrm{Tr}\rho\rho_{desir}\rangle - 1$$

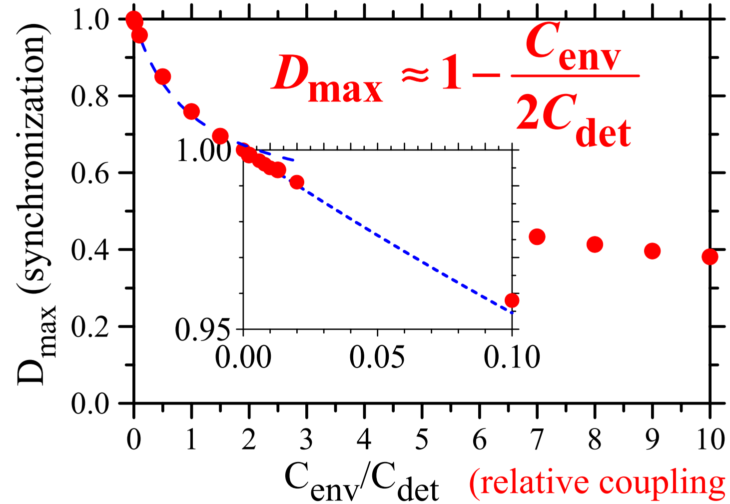
For ideal detector and wide bandwidth, fidelity can be arbitrary close to 100%

$$D = \exp(-C/32F)$$

Ruskov & Korotkov, PRB 66, 041401(R) (2002)

Quantum feedback in presence of decoherence by environment



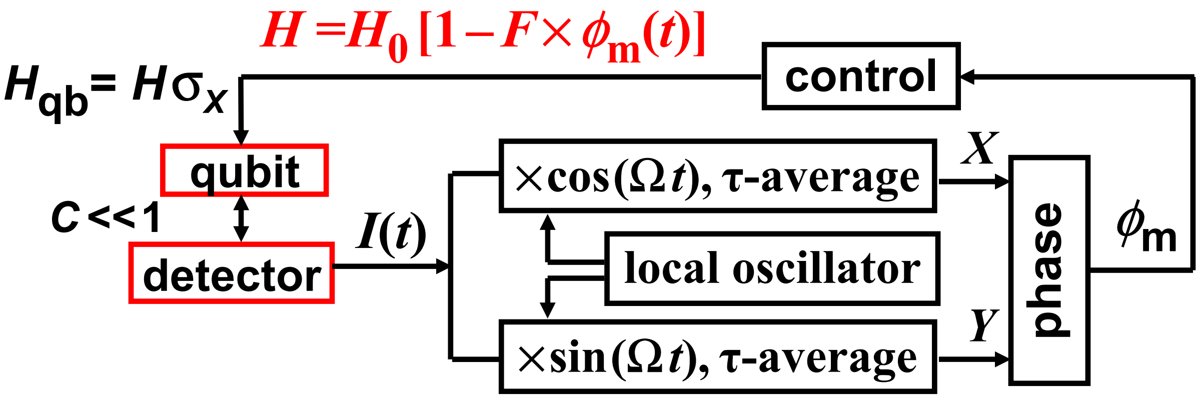


Big experimental problems:

- necessity of very fast real-time solution of the Bayesian equations
- wide bandwidth (>>Ω, GHz-range) of the line delivering noisy signal $I(t)$ to the "processor"

Simple quantum feedback of a solid-state qubit

(A.K., PRB'05)



Goal: maintain coherent (Rabi) oscillations for arbitrary long time

Idea: use two quadrature components of the detector current $I(t)$ to monitor approximately the phase of qubit oscillations
(a very natural way for usual classical feedback!)

$$X(t) = \int_{-\infty}^{t} [I(t') - I_0] \cos(\Omega t') \exp[-(t-t')/\tau]\, dt$$

$$Y(t) = \int_{-\infty}^{t} [I(t') - I_0] \sin(\Omega t') \exp[-(t-t')/\tau]\, dt$$

$$\phi_m = -\arctan(Y/X)$$

(similar formulas for a tank circuit instead of mixing with local oscillator)

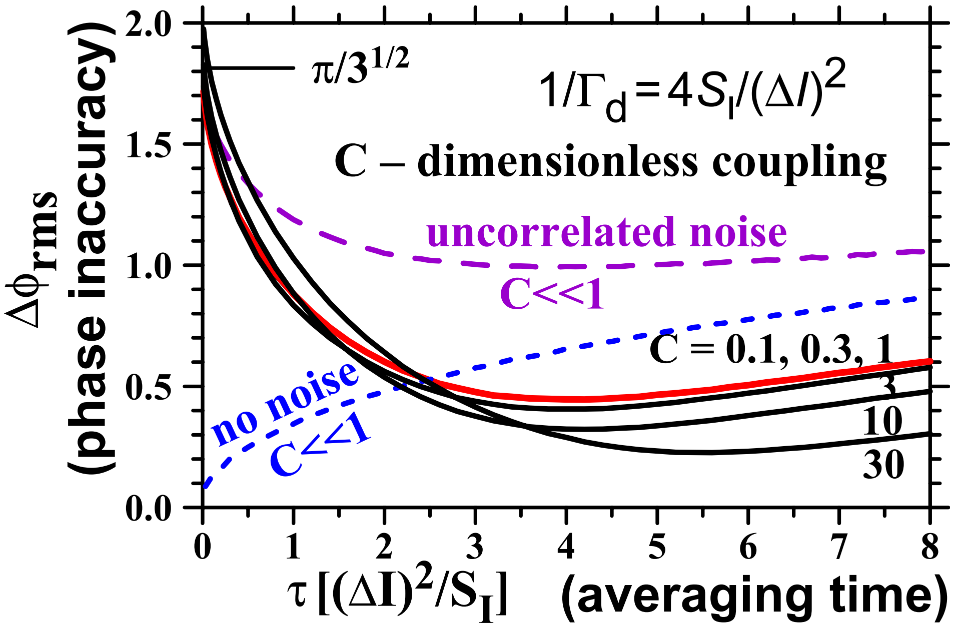
Advantage: simplicity and relatively narrow bandwidth $(1/\tau \sim \Gamma_d << \Omega)$

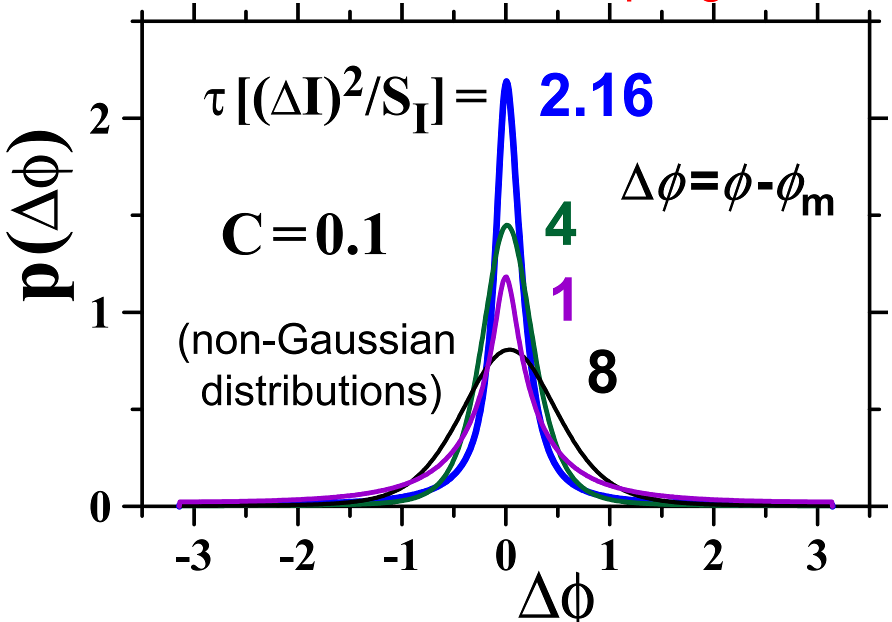
Anticipated problem: without feedback the spectral peak-to-pedestal ratio <4, therefore not much information in quadratures

(surprisingly, situation is much better than anticipated!)

Accuracy of phase monitoring via quadratures
(no feedback yet)

weak coupling $C\ll 1$





Best approximation
$\langle X^2+Y^2\rangle = (S_I/\Delta I)^2$
$(2/5)(41^{1/2}-1)\approx 2.16$

Noise improves the monitoring accuracy!
(purely quantum effect, "reality follows observations")

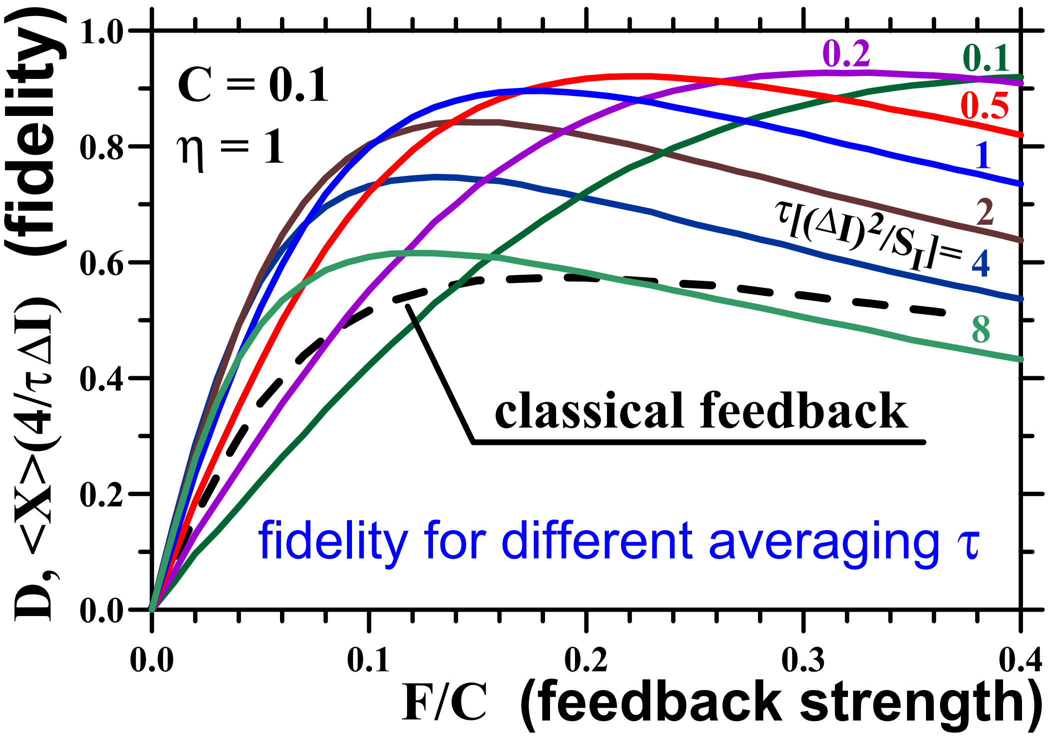
$$d\phi/dt = -[I(t)-I_0]\sin(\Omega t+\phi)(\Delta I/S_I)$$ (actual phase shift, ideal detector)

$$d\phi_m/dt = -[I(t)-I_0]\sin(\Omega t+\phi_m)/(X^2+Y^2)^{1/2}$$ (observed phase shift)

Noise enters the actual and observed phase evolution in a similar way

Quite accurate monitoring! $\cos(0.44)\approx 0.9$

Simple quantum feedback



weak coupling C

D – feedback efficiency

$$D \equiv 2F_Q - 1$$

$$F_Q \equiv \langle \mathrm{Tr}\,\rho(t)\,\rho_{des}(t)\rangle$$

$D_{max} \approx 90\%$

$(F_Q \approx 95\%)$

How to verify feedback operation experimentally?

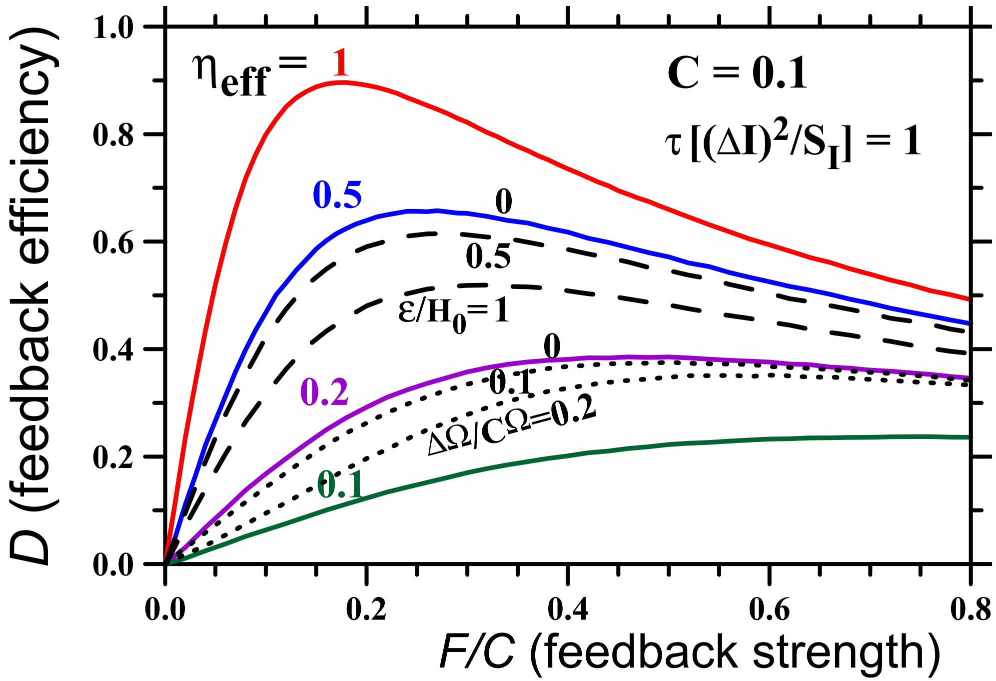
Simple: just check that in-phase quadrature $\langle X \rangle$ of the detector current is positive $D = \langle X \rangle (4/\tau\Delta I)$

$\langle X \rangle$=0 for *any* non-feedback Hamiltonian control of the qubit

Effect of nonidealities

- nonideal detectors (finite quantum efficiency η) and environment
- qubit energy asymmetry ε
- frequency mismatch ΔΩ

Quantum feedback still works quite well



Main features:

- **Natural, practically classical feedback setup**
- **Fidelity F_Q up to ~95% achievable ($D\sim$90%)**
- **Averaging τ~1/Γ>>1/Ω (narrow bandwidth!)**
- **Detector efficiency (ideality) η ~0.1 still OK**
- **Robust to asymmetry ε and frequency shift ΔΩ**
- **Simple verification: positive in-phase quadrature $\langle X\rangle$**

Simple enough experiment?!

Conclusion

- Very straightforward, practically classical feedback idea (monitoring the phase of oscillations via quadratures) works well for the qubit coherent oscillations

- Price for simplicity is a less-then-ideal operation (fidelity is limited by ~95%)

- Feedback operation is much better than expected

- Relatively simple experiment (simple setup, narrow bandwidth, inefficient detectors OK, simple verification)

- Possible realizations: superconducting “charge” or “flux” qubits, GaAs quantum dots and QPC, STM-ESR