

Continuous quantum measurement of solid-state qubits and quantum feedback

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Outline:

- Introduction (quantum measurement)
- Bayesian formalism for continuous quantum measurement of a **single** quantum system
- Experimental predictions and proposals

General theme: Information and collapse in quantum mechanics

Acknowledgement: **Rusko Ruskov**

Support:



Niels Bohr:

“If you are not confused by quantum physics then you haven’t really understood it”

Richard Feynman:

“I think I can safely say that nobody understands quantum mechanics”



Quantum mechanics = Schroedinger equation + collapse postulate

1) Probability of measurement result $p_r = |\langle \psi | \psi_r \rangle|^2$

2) Wavefunction after measurement = ψ_r

- State collapse follows from common sense
- Does not follow from Schr. Eq. (contradicts; Schr. cat, random vs. deterministic)

What if measurement is continuous?
(as practically always in solid-state experiments)



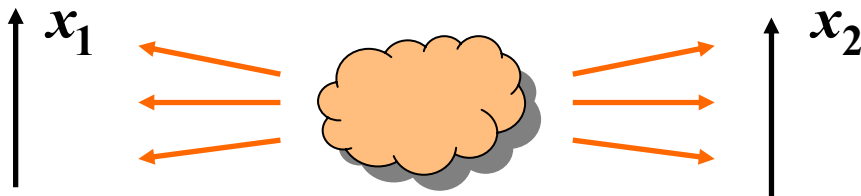
Einstein-Podolsky-Rosen (EPR) paradox

Phys. Rev., 1935

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system.

$$\psi(x_1, x_2) = \sum_n \psi_n(x_2) u_n(x_1) \quad (\text{nowadays we call it entangled state})$$

$$\psi(x_1, x_2) = \int_{-\infty}^{\infty} \exp[(i/\hbar)(x_1 - x_2)p] dp \sim \delta(x_1 - x_2)$$



**Measurement of particle 1
cannot affect particle 2,
while QM says it affects
(contradicts causality)**

=> Quantum mechanics is incomplete

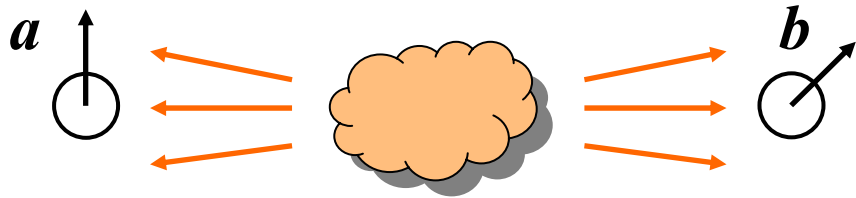
Bohr's reply (Phys. Rev., 1935) (seven pages, one formula: $\Delta p \Delta q \sim h$)

It is shown that a certain “criterion of physical reality” formulated ... by A. Einstein, B. Podolsky and N. Rosen contains an essential ambiguity when it is applied to quantum phenomena.

Crudely: No need to understand QM, just use the result



Bell's inequality (John Bell, 1964)



(setup due to David Bohm)

$$\psi = \frac{1}{\sqrt{2}} (\uparrow_1 \downarrow_2 - \downarrow_1 \uparrow_2)$$

Perfect anticorrelation of measurement results for the same measurement directions, $\vec{a} = \vec{b}$

Is it possible to explain the QM result assuming local realism and hidden variables **or** collapse “propagates” instantaneously (faster than light, “spooky action-at-a-distance”)?

Assume: $A(\vec{a}, \lambda) = \pm 1$, $B(\vec{b}, \lambda) = \pm 1$ (deterministic result with hidden variable λ)

Then: $|P(\vec{a}, \vec{b}) - P(\vec{a}, \vec{c})| \leq 1 + P(\vec{b}, \vec{c})$

where $P \equiv P(++) + P(--) - P(+-) - P(-+)$

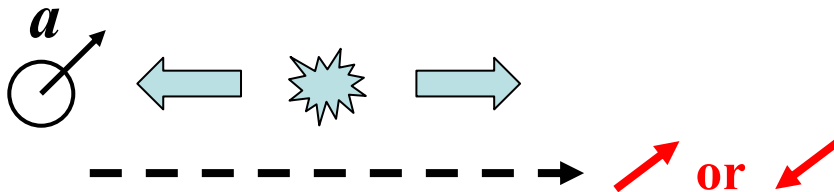
QM: $P(\vec{a}, \vec{b}) = -\vec{a} \cdot \vec{b}$ For 0° , 90° , and 45° : $0.71 \not\leq 1 - 0.71$ **violation!**

Experiment (Aspect et al., 1982; photons instead of spins, CHSH):
yes, “spooky action-at-a-distance”



What about causality?

Actually, not too bad: you cannot transmit your own information choosing a particular measurement direction a



Result of the other measurement does not depend on direction a

Randomness saves causality

Collapse is still instantaneous: OK, just our recipe, not an “objective reality”, not a “physical” process

Consequence of causality: **No-cloning theorem**

Wootters-Zurek, 1982; Dieks, 1982; Yurke

You cannot copy an unknown quantum state

Proof: Otherwise get information on direction a (and causality violated)

Application: quantum cryptography

Information is an important concept in quantum mechanics

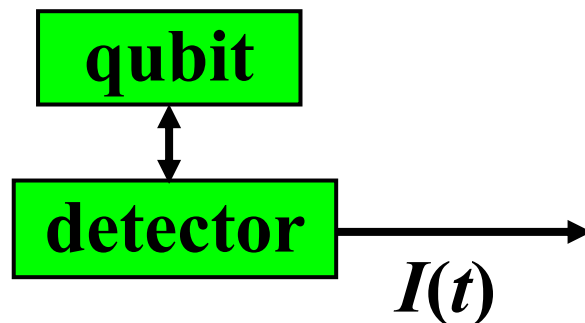


Quantum measurement in solid-state systems

No violation of locality – too small distances

**However, interesting informational aspects
of continuous measurement (gradual collapse)**

Starting point:



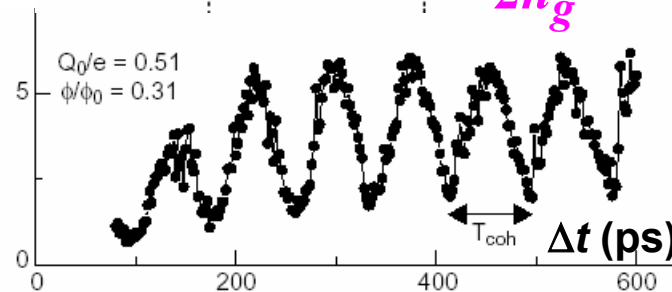
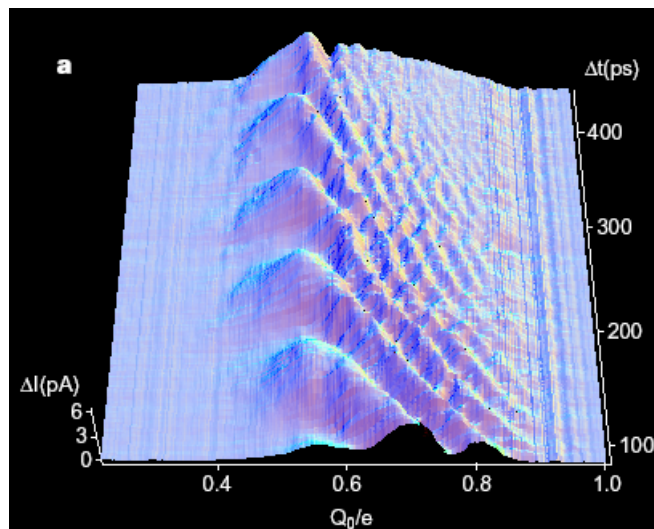
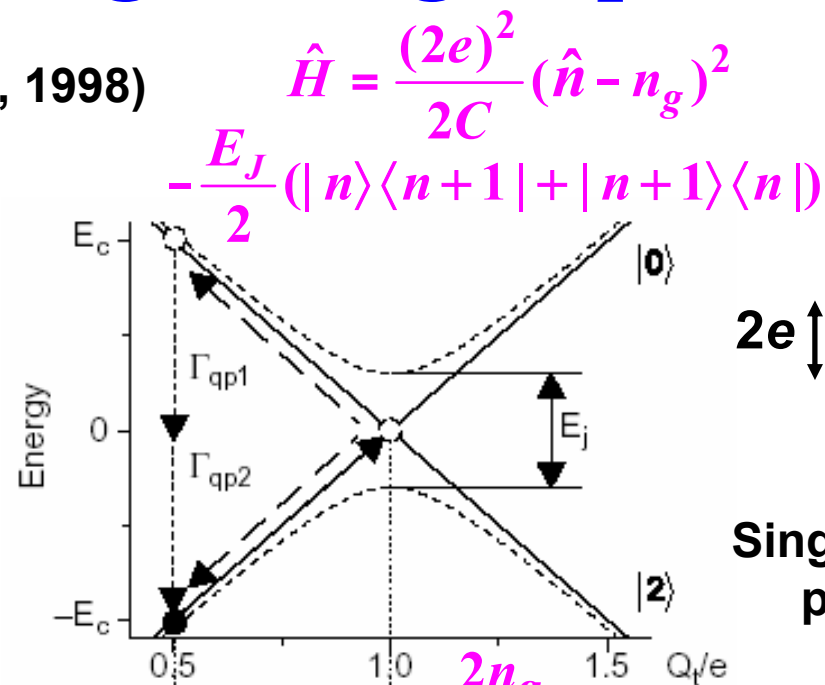
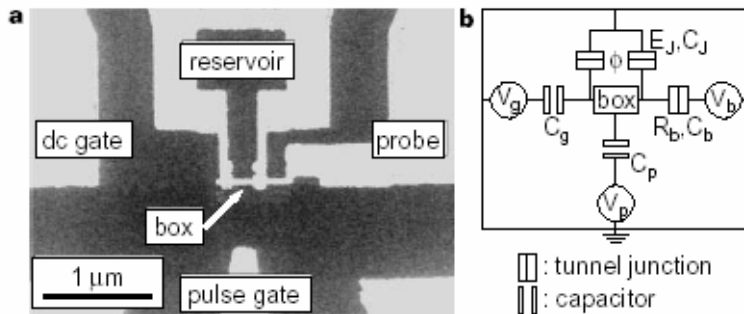
**What happens to a solid-state qubit (two-level system)
during its continuous measurement by a detector?**

**How qubit evolution is related to the noisy detector
output $I(t)$?**



Superconducting “charge” qubits

Nakamura, Pashkin, Tsai (Nature, 1998)



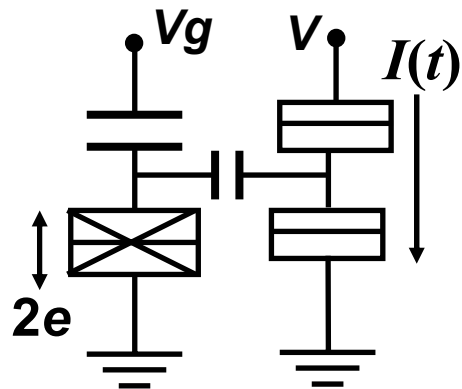
Quantum coherent (Rabi) oscillations

Vion et al. (Devoret's group); Science, 2002

Q-factor of coherent (Rabi) oscillations = 25,000



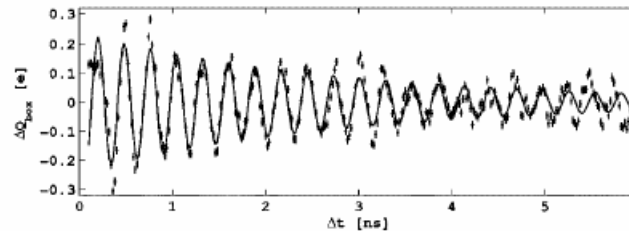
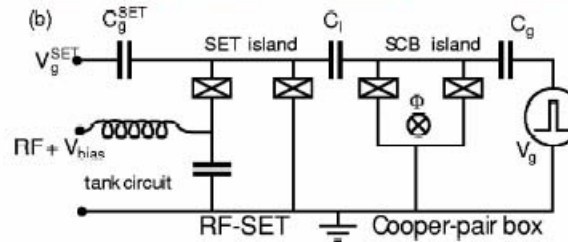
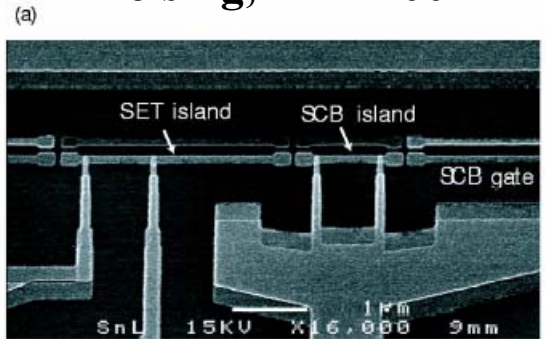
Superconducting “charge” qubits (2)



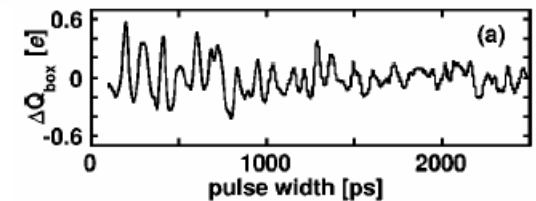
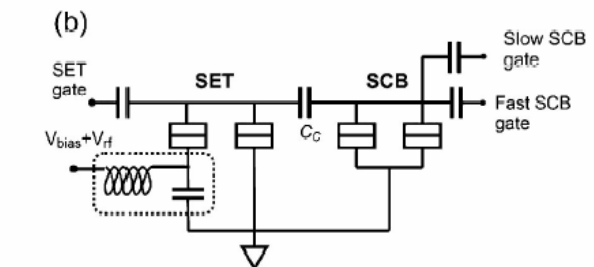
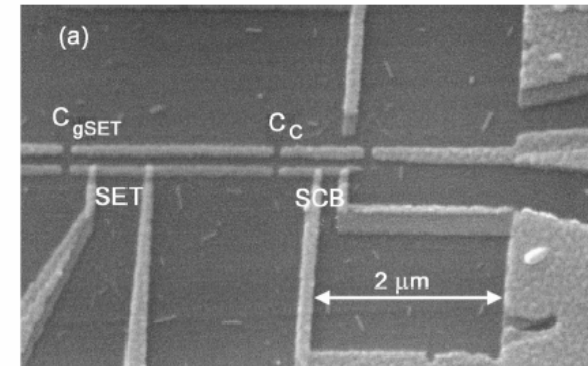
Cooper-pair box
measured by single-
electron transistor
(SET)
(actually, RF-SET)

Setup can be used
for continuous
measurements

Duty, Gunnarsson, Bladh,
Delsing, PRB 2004



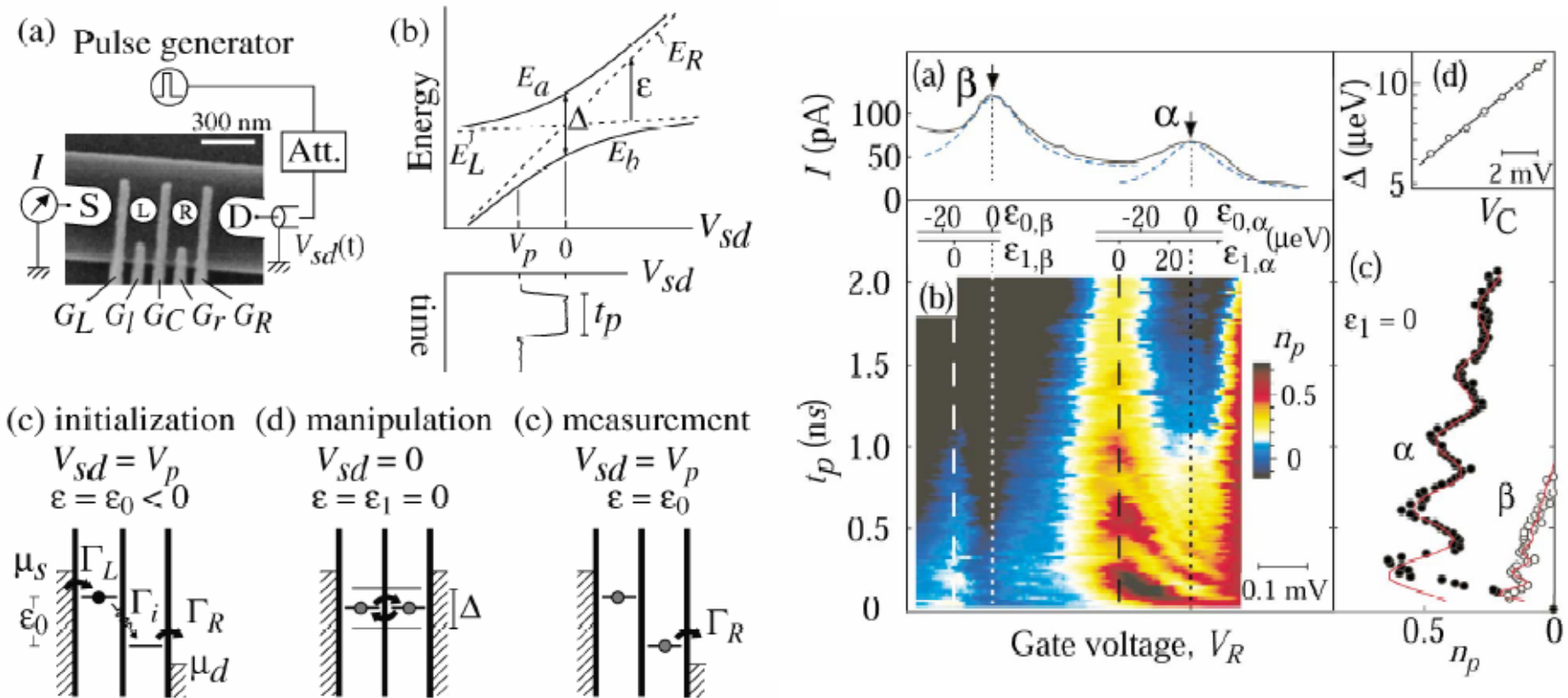
Guillaume et al. (Echternach's
group), PRB 2004



All results are averaged over many measurements (not “single-shot”)

Semiconductor (double-dot) qubit

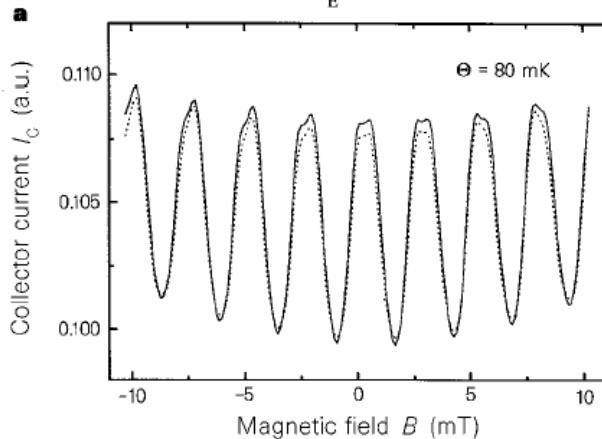
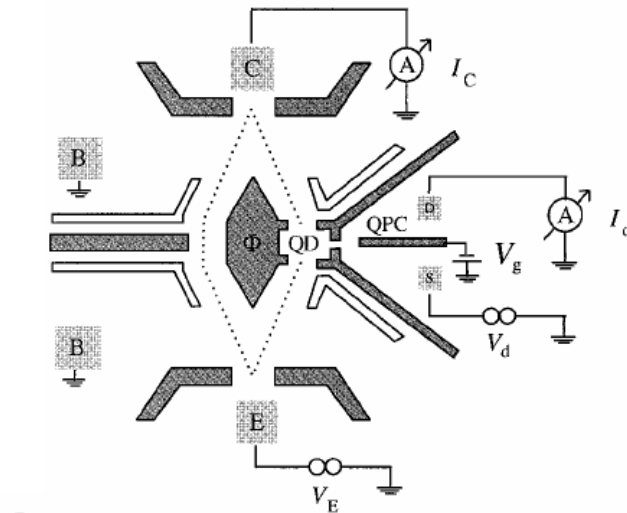
T. Hayashi et al., PRL 2003



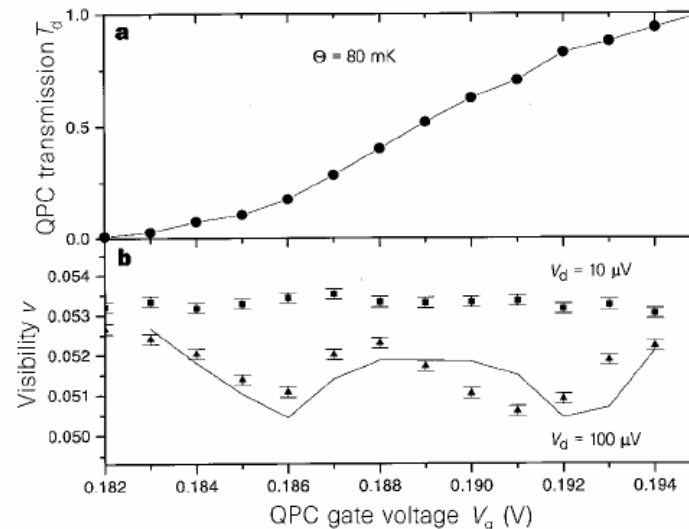
Detector is not separated similar to Nakamura-98,
also possible to use a separate detector

“Which-path detector” experiment

Buks, Schuster, Heiblum, Mahalu,
and Umansky, Nature 1998



Theory: Aleiner, Wingreen,
and Meir, PRL 1997



Dephasing rate:
$$\Gamma = \frac{eV}{h} \frac{(\Delta T)^2}{T(1-T)} = \frac{(\Delta I)^2}{4S_I}$$

ΔI – detector response, S_I – shot noise

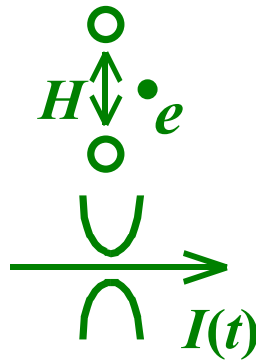
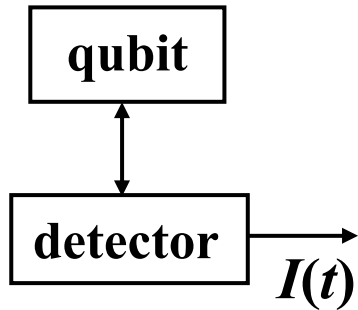
The larger noise, the smaller dephasing!!!

$(\Delta I)^2/4S_I \sim$ rate of “information flow”

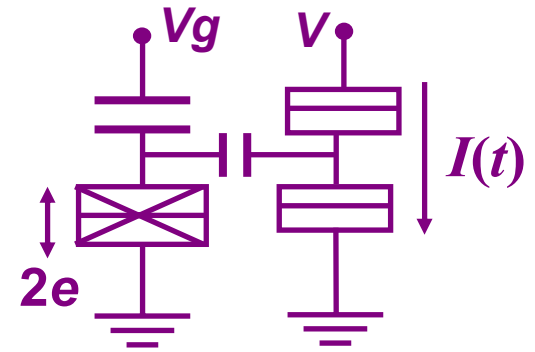
$\tau_m = 2S_I/(\Delta I)^2$ – “measurement time”
(Shnirman-Schon, 1998)



The system we consider: qubit + detector



Double-quantum-dot (DQD) and quantum point contact (QPC)



Cooper-pair box (CPB) and single-electron transistor (SET)

$$H = H_{\text{QB}} + H_{\text{DET}} + H_{\text{INT}}$$

$$H_{\text{QB}} = (\epsilon/2)(c_1^\dagger c_1 - c_2^\dagger c_2) + H(c_1^\dagger c_2 + c_2^\dagger c_1) \quad \epsilon - \text{asymmetry, } H - \text{tunneling}$$

$$\Omega = (4H^2 + \epsilon^2)^{1/2} / \tilde{N} - \text{frequency of quantum coherent (Rabi) oscillations}$$

Two levels of average detector current: I_1 for qubit state $|1\rangle$, I_2 for $|2\rangle$

Response: $\Delta I = I_1 - I_2$ Detector noise: white, spectral density S_I

DQD and QPC

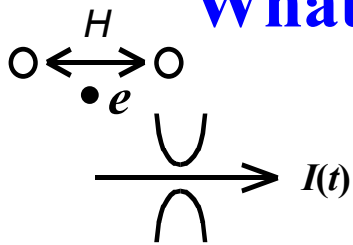
(setup due to Gurvitz, 1997)

$$H_{\text{DET}} = \sum_l E_l a_l^\dagger a_l + \sum_r E_r a_r^\dagger a_r + \sum_{l,r} T(a_r^\dagger a_l + a_l^\dagger a_r)$$

$$H_{\text{INT}} = \sum_{l,r} \Delta T (c_1^\dagger c_1 - c_2^\dagger c_2)(a_r^\dagger a_l + a_l^\dagger a_r) \quad S_I = 2eI$$



What happens to a qubit state during measurement?



For simplicity (for a moment) $H = \epsilon = 0$ (infinite barrier), evolution due to measurement only

“Orthodox” answer

$$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \begin{matrix} \nearrow \\ \searrow \end{matrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$|1\rangle$ or $|2\rangle$, depending on the result

“Conventional” (decoherence) answer (Leggett, Zurek)

$$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \rightarrow \begin{pmatrix} \frac{1}{2} & \frac{\exp(-\Gamma t)}{2} \\ \frac{\exp(-\Gamma t)}{2} & \frac{1}{2} \end{pmatrix} \rightarrow \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

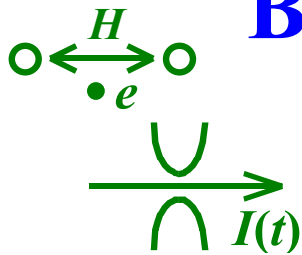
no measurement result! ensemble averaged

Orthodox and decoherence answers contradict each other!

applicable for:	Single quantum systems	Continuous measurements
Orthodox	yes	no
Conventional (ensemble)	no	yes
Bayesian, 1998	yes	yes

Bayesian formalism describes gradual collapse of a single quantum system
Noisy detector output $I(t)$ should be taken into account





Bayesian formalism for a single qubit

$$\hat{H}_{QB} = \frac{\varepsilon}{2}(c_1^\dagger c_1 - c_2^\dagger c_2) + H(c_1^\dagger c_2 + c_2^\dagger c_1)$$

$$|1\rangle \leftrightarrow \Delta I_1, |2\rangle \leftrightarrow \Delta I_2, \Delta I = I_1 - I_2, I_0 = (I_1 + I_2)/2, S_I - \text{detector noise}$$

$$\dot{\rho}_{11} = -\dot{\rho}_{22} = -2(H/\hbar) \text{Im} \rho_{12} + \rho_{11}\rho_{22} (2\Delta I / S_I) [\underline{I(t)} - I_0]$$

$$\dot{\rho}_{12} = i(\varepsilon/\hbar)\rho_{12} + i(H/\hbar)(\rho_{11} - \rho_{22}) + \rho_{12}(\rho_{11} - \rho_{22})(\Delta I / S_I) [\underline{I(t)} - I_0] - \gamma\rho_{12}$$

(A.K., 1998)

$$\gamma = \Gamma - (\Delta I)^2 / 4S_I, \quad \Gamma - \text{ensemble decoherence}$$

$$\eta = 1 - \gamma / \Gamma = (\Delta I)^2 / 4S_I \Gamma - \text{detector ideality (efficiency), } \eta \leq 100\%$$

Ideal detector ($\eta=1$) does not decohere a single qubit;
then random evolution of qubit *wavefunction* can be monitored

$$\text{For simulations: } I(t) - I_0 = (\rho_{22} - \rho_{11})\Delta I / 2 + \xi(t), \quad S_\xi = S_I$$

Averaging over $\xi(t)$ i conventional master equation

Similar formalisms developed earlier. Key words: **Imprecise, weak, selective, or conditional measurements, POVM, Quantum trajectories, Quantum jumps, Restricted path integral, etc.**

Names: Davies, Kraus, Holevo, Mensky, Caves, Gardiner, Carmichael, Plenio, Knight, Walls, Gisin, Percival, **Milburn, Wiseman**, Onofrio, Habib, Doherty, etc. (incomplete list)



Assumptions needed for the Bayesian formalism

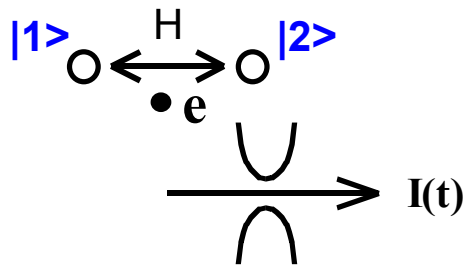
- Detector voltage is much larger than the qubit energies involved
 $eV \gg \tilde{N}\Omega, eV \gg \tilde{N}\Gamma$ (no coherence in the detector,
 $\tilde{N}eV \ll (1/\Omega, 1/\Gamma)$; Markovian approximation)
- Small detector response, $|\Delta I| \ll I_0$, $\Delta I = I_1 - I_2$, $I_0 = (I_1 + I_2)/2$
 Many electrons pass through detector before qubit evolves noticeably.
 (Not a really important condition, but simplifies formalism.)

Coupling $C \sim \Gamma/\Omega$ is arbitrary [we define $C = \tilde{N}(\Delta I)^2/S_I H$]

$$\begin{aligned} \frac{d}{dt} \rho_{11} &= -\frac{d}{dt} \rho_{22} = -2 \frac{H}{\hbar} \text{Im} \rho_{12} + \rho_{11} \rho_{22} \frac{2\Delta I}{S_I} [I(t) - I_0] \\ \frac{d}{dt} \rho_{12} &= i \frac{\varepsilon}{\hbar} \rho_{12} + i \frac{H}{\hbar} (\rho_{11} - \rho_{22}) + \rho_{12} (\rho_{11} - \rho_{22}) \frac{\Delta I}{S_I} [I(t) - I_0] - \gamma \rho_{12} \end{aligned}$$



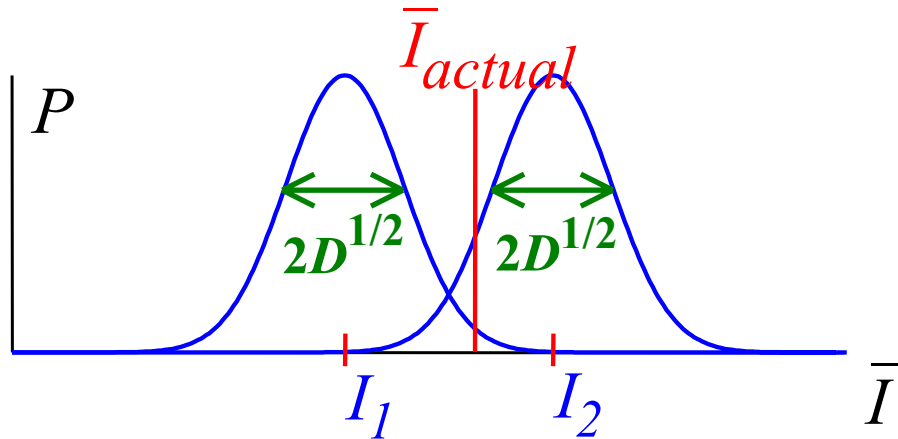
“Quantum Bayes theorem” (ideal detector assumed)



$H = \varepsilon = 0$
 (“frozen” qubit)

Initial state: $\begin{pmatrix} \rho_{11}(0) & \rho_{12}(0) \\ \rho_{21}(0) & \rho_{22}(0) \end{pmatrix}$

Measurement (during time τ):



$$\bar{I} \equiv \frac{1}{\tau} \int_0^\tau I(t) dt$$

$$P(\bar{I}, \tau) = \rho_{11}(0) P_1(\bar{I}, \tau) + \rho_{22}(0) P_2(\bar{I}, \tau)$$

$$P_i(\bar{I}, \tau) = \frac{1}{\sqrt{2\pi D}} \exp[-(\bar{I} - I_i)^2 / 2D],$$

$$D = S_I / 2\tau, \quad |I_1 - I_2| \ll I_i, \quad \tau \gg S_I / I_i^2$$

After the measurement during time τ , the probabilities should be updated using the **standard Bayes formula**:

$$P(B_i | A) = \frac{P(B_i)P(A | B_i)}{\sum_k P(B_k)P(A | B_k)}$$

Quantum Bayes formulas:

$$\rho_{11}(\tau) = \frac{\rho_{11}(0) \exp[-(\bar{I} - I_1)^2 / 2D]}{\rho_{11}(0) \exp[-(\bar{I} - I_1)^2 / 2D] + \rho_{22}(0) \exp[-(\bar{I} - I_2)^2 / 2D]}$$

$$\frac{\rho_{12}(\tau)}{[\rho_{12}(\tau) \rho_{22}(\tau)]^{1/2}} = \frac{\rho_{12}(0)}{[\rho_{12}(0) \rho_{22}(0)]^{1/2}}, \quad \rho_{22}(\tau) = 1 - \rho_{11}(\tau)$$



“Informational” derivation of the Bayesian formalism

Step 1. Assume $H = \varepsilon = 0$, “frozen” qubit

Since ρ_{12} is not involved, **evolution of ρ_{11} and ρ_{22} should be the same as in the classical case**, i.e. Bayes formula (correspondence principle).

Step 2. Assume $H = \varepsilon = 0$ and pure initial state, $\rho_{12}(0) = [\rho_{11}(0) \rho_{22}(0)]^{1/2}$

For any realization **$|\rho_{12}(t)| \leq [\rho_{11}(t) \rho_{22}(t)]^{1/2}$** . Hence, averaging over

ensemble of realizations gives **$|\rho_{12}^{\text{av}}(t)| \leq \rho_{12}^{\text{av}}(0) \exp[\alpha \Delta I^2 / 4 S_I] t$**

However, **conventional (ensemble) result** (Gurvitz-1997, Aleiner *et al.*-1997)

for QPC **is exactly the upper bound**: **$\rho_{12}^{\text{av}}(t) = \rho_{12}^{\text{av}}(0) \exp[\alpha \Delta I^2 / 4 S_I] t$** .

Therefore, pure state remains pure: **$\rho_{12}(t) = [\rho_{11}(t) \rho_{22}(t)]^{1/2}$** .

Step 3. Account of a mixed initial state

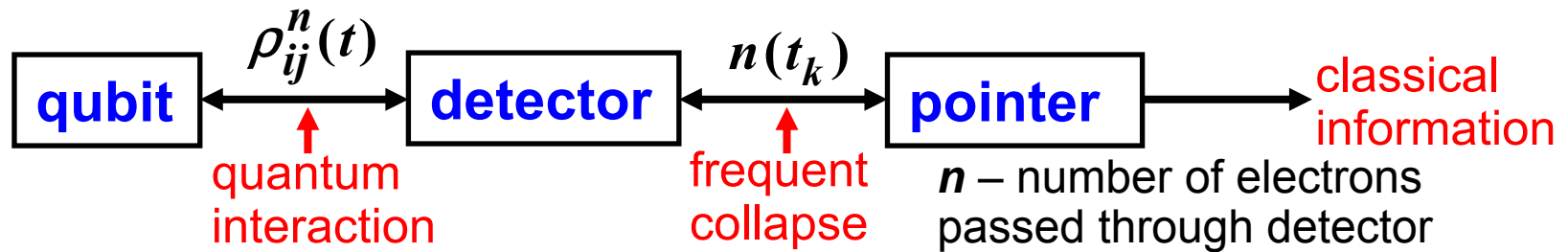
Result: the degree of purity $\rho_{12}(t) / [\rho_{11}(t) \rho_{22}(t)]^{1/2}$ is conserved.

Step 4. Add qubit evolution due to H and ε .

Step 5. Add extra dephasing due to detector nonideality (i.e., for SET).



“Microscopic” derivation of the Bayesian formalism



Schrödinger evolution of “qubit + detector”
for a low- T QPC as a detector (Gurvitz, 1997)

$$\frac{d}{dt} \rho_{11}^n = -\frac{I_1}{e} \rho_{11}^n + \frac{I_1}{e} \rho_{11}^{n-1} - 2 \frac{H}{\hbar} \text{Im} \rho_{12}^n$$

$$\frac{d}{dt} \rho_{22}^n = -\frac{I_2}{e} \rho_{22}^n + \frac{I_2}{e} \rho_{22}^{n-1} + 2 \frac{H}{\hbar} \text{Im} \rho_{12}^n$$

$$\frac{d}{dt} \rho_{12}^n = i \frac{\varepsilon}{\hbar} \rho_{12}^n + i \frac{H}{\hbar} (\rho_{11}^n - \rho_{22}^n) - \frac{I_1 + I_2}{2e} \rho_{12}^n + \frac{\sqrt{I_1 I_2}}{e} \rho_{12}^{n-1}$$

If $H = \varepsilon = 0$,
this leads to

$$\rho_{11}(t) = \frac{\rho_{11}(0) P_1(n)}{\rho_{11}(0) P_1(n) + \rho_{22}(0) P_2(n)},$$

$$\rho_{12}(t) = \rho_{12}(0) \frac{[\rho_{11}(t) \rho_{22}(t)]^{1/2}}{[\rho_{11}(0) \rho_{22}(0)]^{1/2}},$$

$$\rho_{22}(t) = \frac{\rho_{22}(0) P_2(n)}{\rho_{11}(0) P_1(n) + \rho_{22}(0) P_2(n)}$$

$$P_i(n) = \frac{(I_i t / e)^n}{n!} \exp(-I_i t / e),$$

Detector collapse at $t = t_k$
Particular n_k is chosen at t_k

$$P(n) = \rho_{11}^n(t_k) + \rho_{22}^n(t_k)$$

$$\rho_{ij}^n(t_k + 0) = \delta_{n, n_k} \rho_{ij}^n(t_k + 0)$$

$$\rho_{ij}(t_k + 0) = \frac{\rho_{ij}^{n_k}(t_k)}{\rho_{11}^{n_k}(t_k) + \rho_{22}^{n_k}(t_k)}$$

which are exactly quantum Bayes formulas



One more derivation

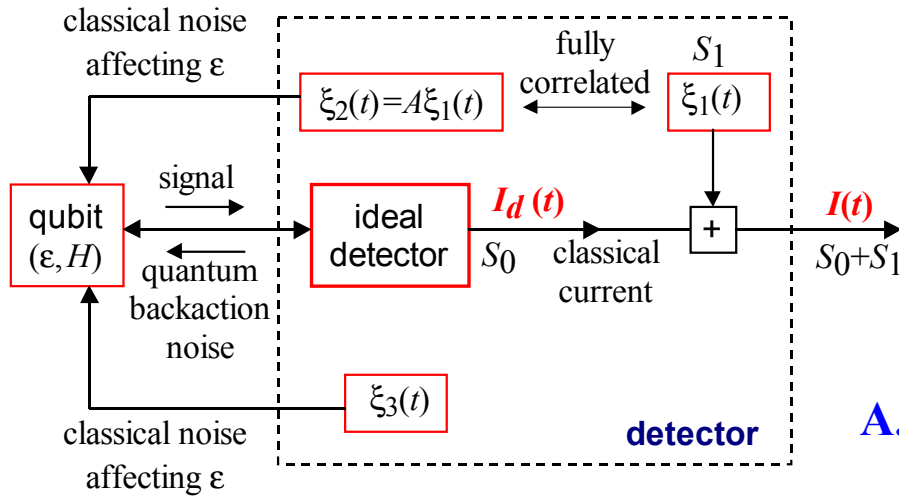
Translating well-developed “quantum trajectory” formalism
from quantum optics into solid-state language
(equivalent though looks very different)

Goan and Milburn, 2001

Also: Wiseman, Sun, Oxtoby, Warszawsky,
Polkinghorne, etc.



Nonideal detectors with input-output noise correlation



$$K = \frac{AS_1 + \theta S_0}{\hbar S_I}, \quad S_I = S_0 + S_1$$

K – correlation between output and ε –backaction noises

A.K., 2002

$$\frac{d}{dt} \rho_{11} = -\frac{d}{dt} \rho_{22} = -2H \operatorname{Im} \rho_{12} + \rho_{11} \rho_{22} \frac{2\Delta I}{S_I} [I(t) - I_0]$$

$$\frac{d}{dt} \rho_{12} = i\varepsilon \rho_{12} + iH(\rho_{11} - \rho_{22}) + \rho_{12}(\rho_{11} - \rho_{22}) \frac{\Delta I}{S_I} [I(t) - I_0] + \underline{\underline{iK[I(t) - I_0] - \tilde{\gamma} \rho_{12}}}$$

Fundamental limits for ensemble decoherence

$$\Gamma = \gamma + (\Delta I)^2 / 4S_I, \quad \gamma \geq 0 \Rightarrow \Gamma \geq (\Delta I)^2 / 4S_I$$

$$\Gamma = \gamma + (\Delta I)^2 / 4S_I + K^2 S_I / 4, \quad \gamma \geq 0 \Rightarrow \Gamma \geq (\Delta I)^2 / 4S_I + K^2 S_I / 4$$

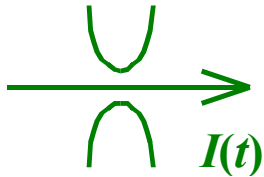
Translated into energy sensitivity: $(\epsilon_I \epsilon_{BA})^{1/2} \geq \hbar/2$ or $(\epsilon_I \epsilon_{BA} - \epsilon_{I,BA}^2)^{1/2} \geq \hbar/2$
(known since 1980s)



Ideality of solid-state detectors

(ideal detector does not cause single qubit decoherence)

1. Quantum point contact



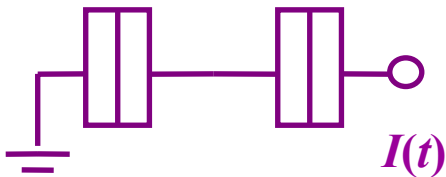
Theoretically, **ideal quantum detector**, $\eta = 1$

A.K., 1998 (Gurvitz, 1997; Aleiner *et al.*, 1997)

Experimentally, $\eta > 80\%$

(using Buks *et al.*, 1998)

2. SET-transistor



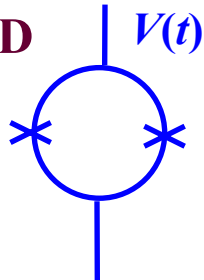
Very non-ideal in usual operation regime, $\eta \ll 1$

Shnirman-Schön, 1998; A.K., 2000, Devoret-Schoelkopf, 2000

However, reaches ideality, $\eta = 1$ if:

- in deep cotunneling regime (Averin, van den Brink, 2000)
- S-SET, using supercurrent (Zorin, 1996)
- S-SET, double-JQP peak ($\eta \sim 1$) (Clerk *et al.*, 2002)
- resonant-tunneling SET, low bias (Averin, 2000)

3. SQUID



Can reach ideality, $\eta = 1$

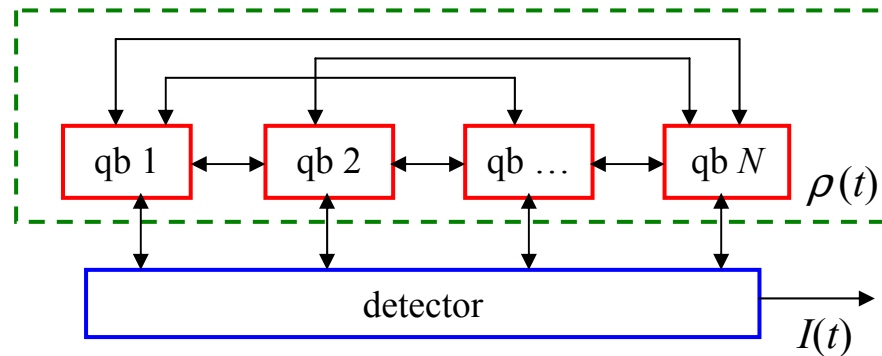
(Danilov-Likharev-Zorin, 1983;
Averin, 2000)

4. FET ?? HEMT ??

ballistic FET/HEMT ??



Bayesian formalism for N entangled qubits measured by one detector



Up to 2^N levels of current

$$\frac{d}{dt} \rho_{ij} = \frac{-i}{\hbar} [\hat{H}_{qb}, \rho]_{ij} + \rho_{ij} \frac{1}{S} \sum_k \rho_{kk} \left[\left(I(t) - \frac{I_k + I_i}{2} \right) (I_i - I_k) + \left(I(t) - \frac{I_k + I_j}{2} \right) (I_j - I_k) \right] - \gamma_{ij} \rho_{ij} \quad (\text{Stratonovich form})$$

$$\gamma_{ij} = (\eta^{-1} - 1)(I_i - I_j)^2 / 4S_I \quad I(t) = \sum_i \rho_{ii}(t) I_i + \xi(t)$$

Averaging over $\xi(t)$ $\hat{=}$ master equation

No measurement-induced dephasing between states $|i\rangle$ and $|j\rangle$ if $I_i = I_j$!

A.K., PRA 65 (2002),
PRB 67 (2003)



Measurement vs. decoherence

Widely accepted point of view:

measurement = decoherence (environment)

Is it true?

- **Yes**, if not interested in information from detector (ensemble-averaged evolution)
- **No**, if take into account measurement result (single quantum system)

Measurement result obviously gives us more information about the measured system, so we know its quantum state better (ideally, a pure state instead of a mixed state)



Some experimental predictions and proposals

- **Direct experimental verification (1998)**
- **Measured spectral density of Rabi oscillations (1999, 2000, 2002)**
- **Bell-type correlation experiment (2000)**
- **Quantum feedback control of a qubit (2001)**
- **Entanglement by measurement (2002)**
- **Measurement by a quadratic detector (2003)**
- **Simple quantum feedback of a qubit (2004)**
- **Squeezing of a nanomechanical resonator (2004)**

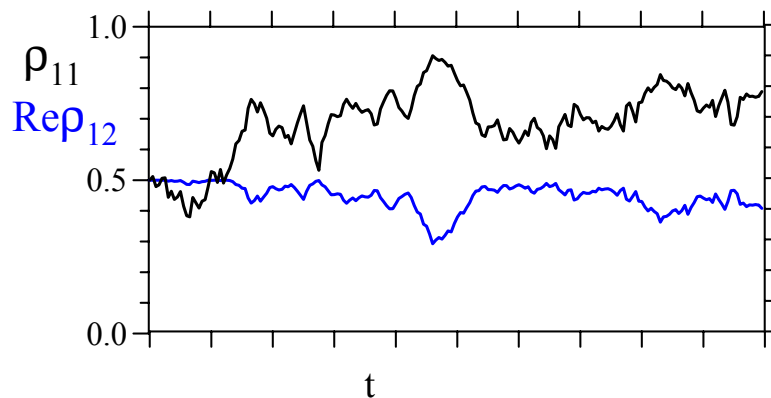


Direct verification of the Bayesian evolution

(A.K., 1998)

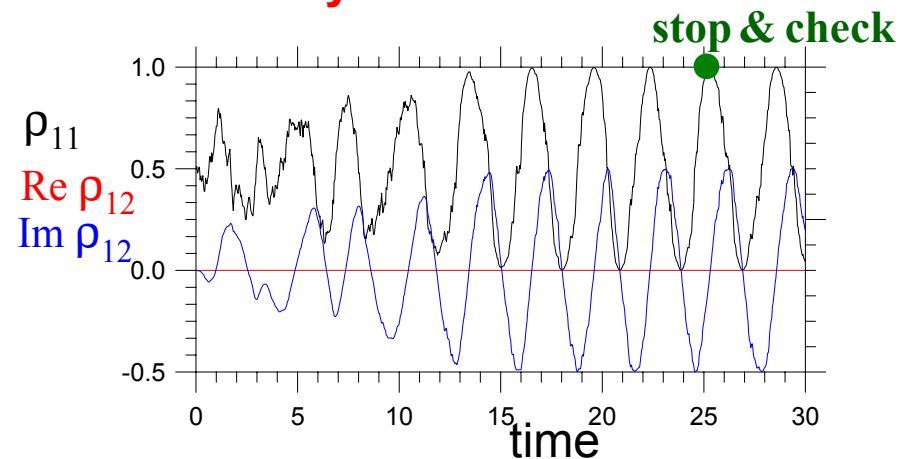
Idea: check the predicted evolution of an almost pure qubit state

Evolution from 1/2-alive to
1/3-alive Schrödinger cat



1. Prepare coherent state and make $H=0$.
2. Measure for a finite time t .
3. Check the predicted wavefunction (using evolution with $H \neq 0$ to get the state $|1\rangle$).

Density matrix purification
by measurement

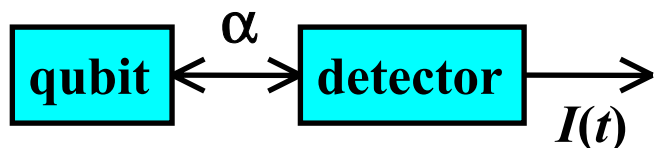


1. Start with completely mixed state.
2. Measure and monitor the Rabi phase.
3. Stop evolution (make $H=0$) at state $|1\rangle$.
4. Measure and check.

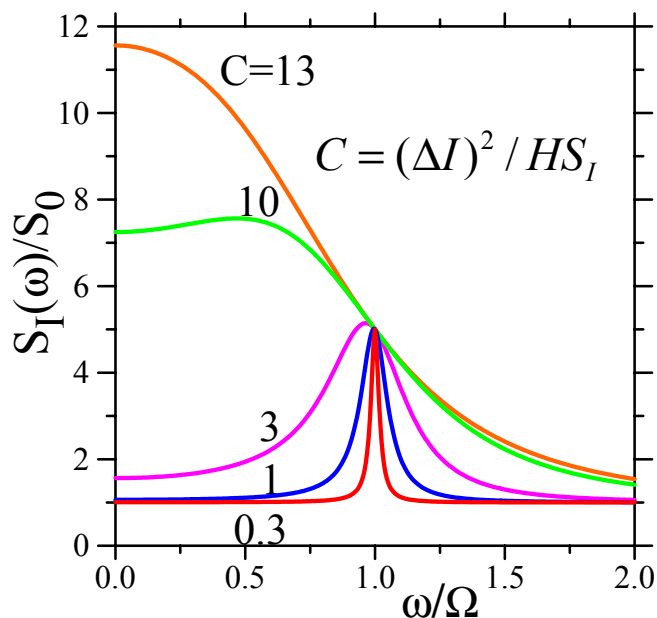
Difficulty: need to record noisy detector current $I(t)$ and solve Bayesian equations in real time; typical required bandwidth: 1-10 GHz.



Measured spectrum of qubit coherent oscillations



What is the spectral density $S_I(\omega)$ of detector current?



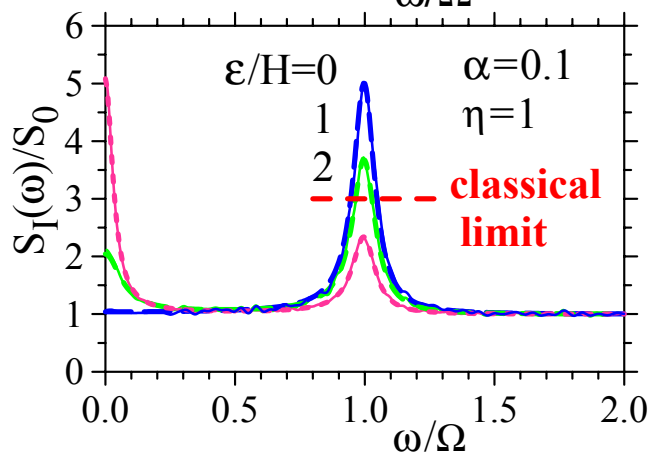
Assume classical output, $\text{eV} \gg \hbar\Omega$

$$\varepsilon = 0, \quad \Gamma = \eta^{-1}(\Delta I)^2 / 4S_0$$

$$S_I(\omega) = S_0 + \frac{\Omega^2(\Delta I)^2\Gamma}{(\omega^2 - \Omega^2)^2 + \Gamma^2\omega^2}$$

Spectral peak can be seen, but
peak-to-pedestal ratio $\leq 4\eta \leq 4$

(result can be obtained using various
methods, not only Bayesian method)



Weak coupling, $\alpha = C/8 \ll 1$

$$S_I(\omega) = S_0 + \frac{\eta S_0 \varepsilon^2 / H^2}{1 + (\omega \hbar^2 \Omega^2 / 4 H^2 \Gamma)^2} + \frac{4\eta S_0 (1 + \varepsilon^2 / 2 H^2)^{-1}}{1 + [(\omega - \Omega)\Gamma(1 - 2 H^2 / \hbar^2 \Omega^2)]^2}$$

A.K., LT'99
Averin-A.K., 2000
A.K., 2000
Averin, 2000
Goan-Milburn, 2001
Makhlin et al., 2001
Balatsky-Martin, 2001
Ruskov-A.K., 2002
Mozyrsky et al., 2002
Balatsky et al., 2002
Bulaevskii et al., 2002
Shnirman et al., 2002
Bulaevskii-Ortiz, 2003
Shnirman et al., 2003

Contrary:
Stace-Barrett, 2003
(PRL 2004)



Possible experimental confirmation?

Durkan and Welland, 2001 (STM-ESR experiment similar to Manassen-1989)

APPLIED PHYSICS LETTERS

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Electronic spin detection in molecules using scanning-tunneling-microscopy-assisted electron-spin resonance

C. Durkan^{a)} and M. E. Welland

Nanoscale Science Laboratory, Department of Engineering, University of Cambridge, Trumpington Street, Cambridge CB2 1PZ, United Kingdom

(Received 8 May 2001; accepted for publication 8 November 2001)

By combining the spatial resolution of a scanning-tunneling microscope (STM) with the electronic spin sensitivity of electron-spin resonance, we show that it is possible to detect the presence of localized spins on surfaces. The principle is that a STM is operated in a magnetic field, and the resulting component of the tunnel current at the Larmor (precession) frequency is measured. This component is nonzero whenever there is tunneling into or out of a paramagnetic entity. We have

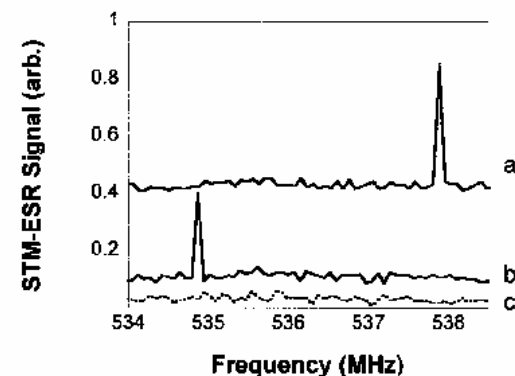


FIG. 3. STM-ESR spectra of (a), (b) two different areas (a few nm apart) of the molecule-covered sample and (c) bare HOPG. The graphs are shifted vertically for clarity.

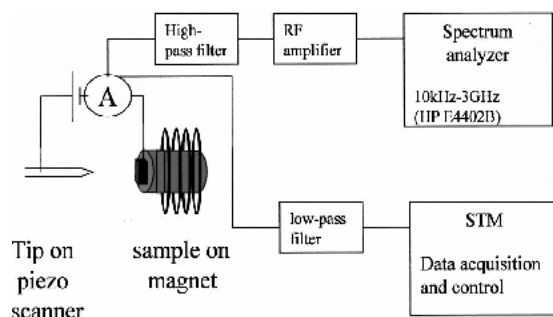


FIG. 1. Schematic of the electronics used in STM-ESR.

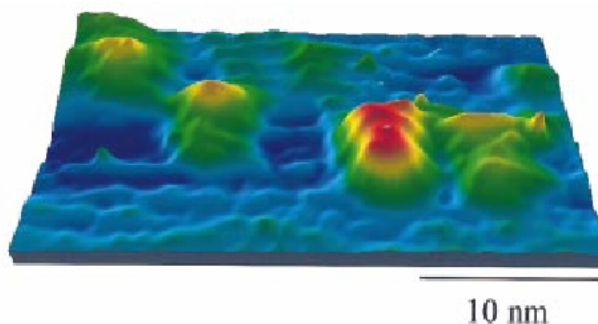


FIG. 2. (Color) STM image of a 250 Å x 150 Å area of HOPG with four adsorbed BDPA molecules.

$$\frac{\text{peak}}{\text{noise}} \leq 3.5$$

(Colm Durkan,
private comm.)



Somewhat similar experiment

“Continuous monitoring of Rabi oscillations in a Josephson flux qubit”

E. Il'ichev *et al.*, PRL, 2003

$$H = -\frac{1}{2}(\Delta \sigma_x + \varepsilon \sigma_z) - W \sigma_z \cos \omega_{HF} t$$

$$(\omega_{HF} \approx \sqrt{\Delta^2 + \varepsilon^2}; \varepsilon \neq 0)$$

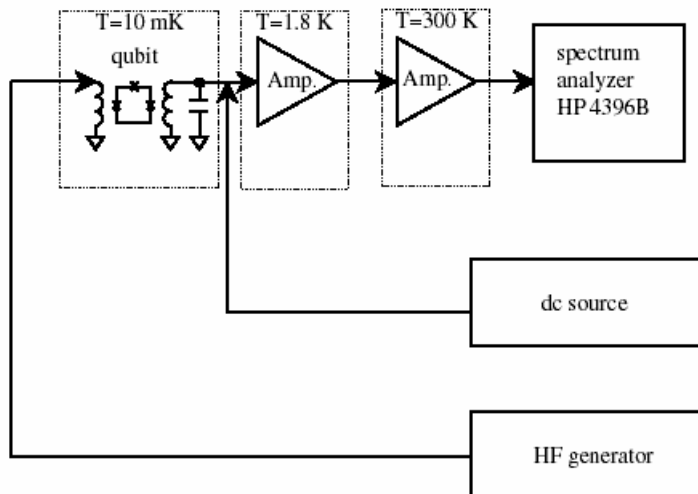


FIG. 1. Measurement setup. The flux qubit is inductively coupled to a tank circuit. The dc source applies a constant flux $\Phi_e \approx \frac{1}{2} \Phi_0$. The HF generator drives the qubit through a separate coil at a frequency close to the level separation $\Delta/h = 868$ MHz. The output voltage at the resonant frequency of the tank is measured as a function of HF power.

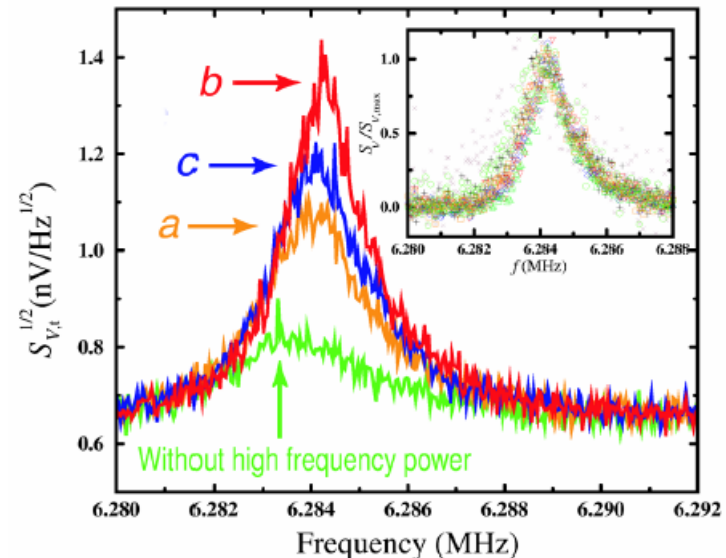
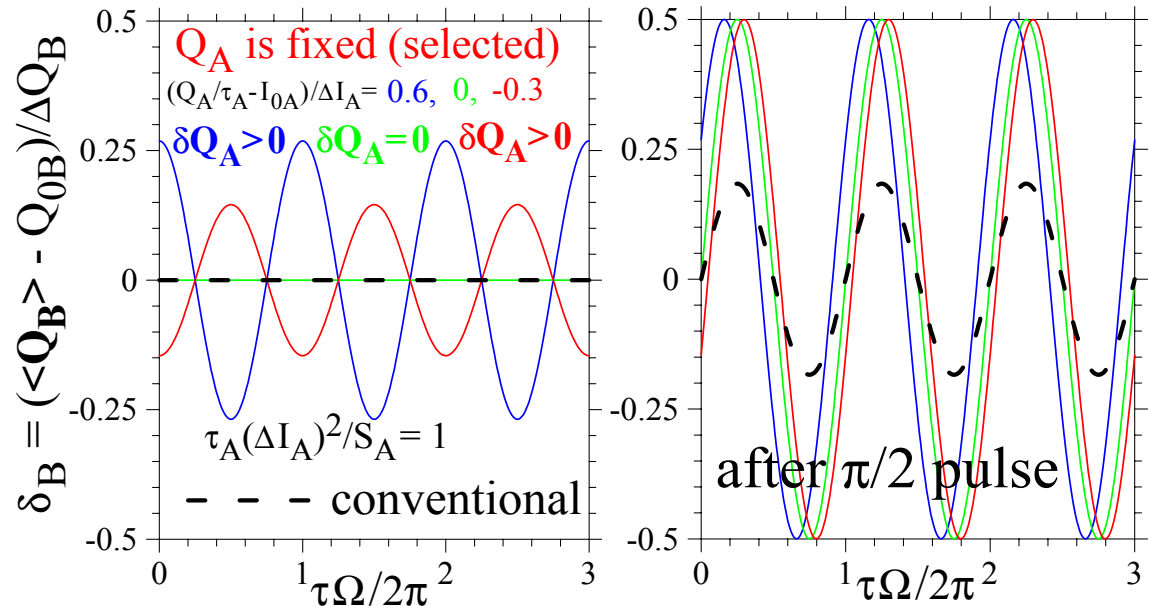
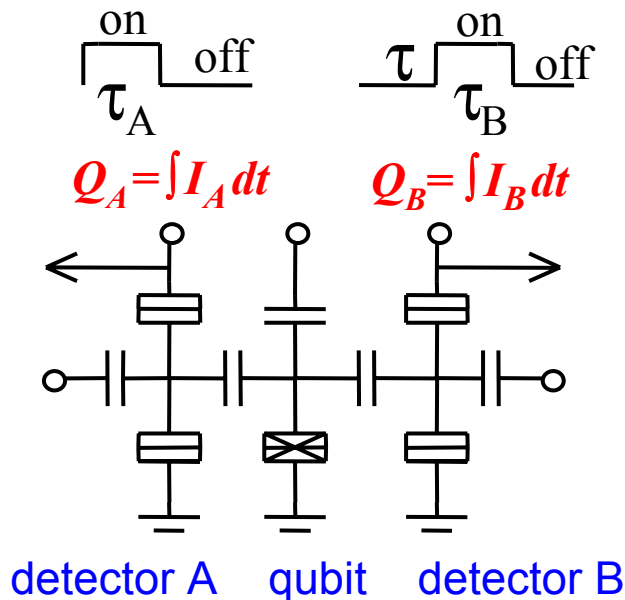


FIG. 3 (color online). The spectral amplitude of the tank voltage for HF powers $P_a < P_b < P_c$ at 868 MHz, detected using the setup of Fig. 1. The bottom curve corresponds to the background noise without an HF signal. The inset shows normalized voltage spectra for seven values of HF power, with background subtracted. The shape of the resonance, being determined by the tank circuit, is essentially the same in each case. Remaining tiny variations visible in the main figure are due to the irradiated qubit modifying the tank's inductance and



Bell-type measurement correlation

(A.K., 2000)



Idea: two consecutive finite-time (imprecise) measurements of a qubit by two detectors; probability distribution $\mathbf{P}(Q_A, Q_B, \tau)$ shows the effect of the first measurement on the qubit state.

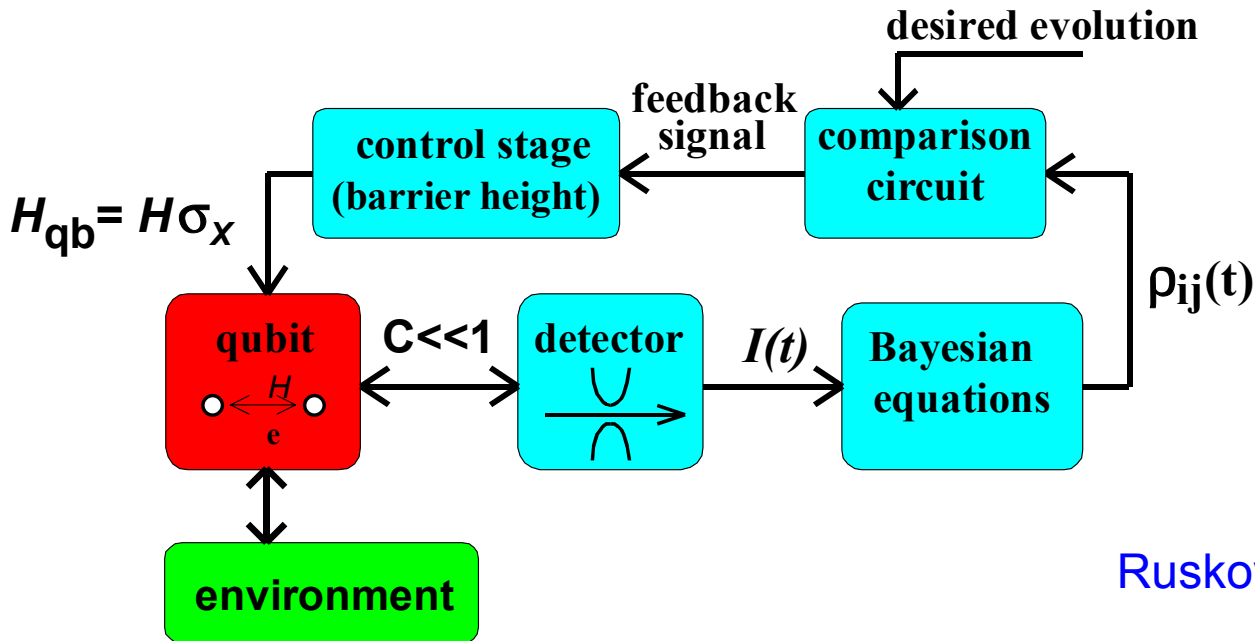
Proves that qubit remains in a pure state during measurement (for $\eta=1$)

Advantage: no need to record noisy detector output with GHz bandwidth; instead, we use two detectors and fast ON/OFF switching.



Quantum feedback control of a qubit

Since qubit state can be monitored, the feedback is possible!



Ruskov & A.K., 2001

Goal: maintain desired phase of coherent (Rabi) oscillations in spite of environmental dephasing (keep qubit “fresh”)

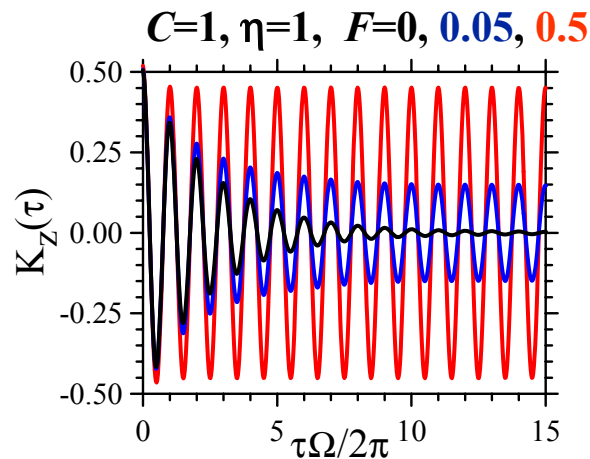
Idea: monitor the Rabi phase ϕ by continuous measurement and apply feedback control of the qubit barrier height, $\Delta H_{FB}/H = -F \times \Delta \phi$

To monitor phase ϕ we plug detector output $I(t)$ into Bayesian equations



Performance of quantum feedback (no extra environment)

Qubit correlation function



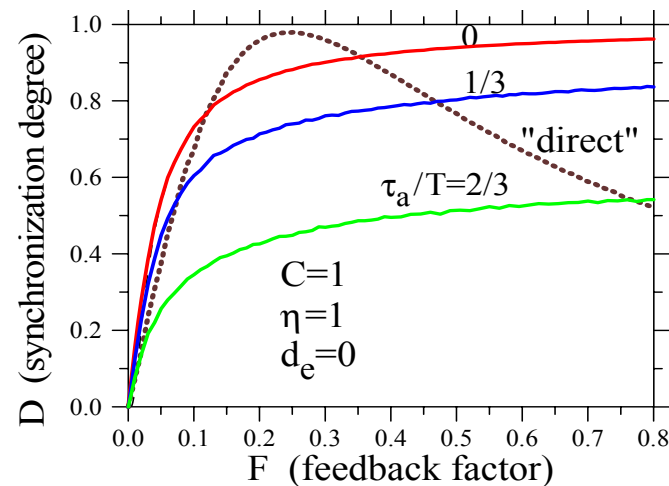
$$K_z(\tau) = \frac{\cos \Omega \tau}{2} \exp \left[\frac{C}{16F} (e^{-2FH\tau/\hbar} - 1) \right]$$

(for weak coupling and good fidelity)

Detector current correlation function

$$K_I(\tau) = \frac{(\Delta I)^2}{4} \frac{\cos \Omega \tau}{2} (1 + e^{-2FH\tau/\hbar}) \\ \times \exp \left[\frac{C}{16F} (e^{-2FH\tau/\hbar} - 1) \right] + \frac{S_I}{2} \delta(\tau)$$

Fidelity (synchronization degree)



$C = \hbar(\Delta I)^2 / S_I H$ – coupling

τ_a^{-1} – available bandwidth

F – feedback strength

$$D = 2 \langle \text{Tr} \rho \rho_{\text{desir}} \rangle - 1$$

**For ideal detector and wide bandwidth,
fidelity can be arbitrarily close to 100%**

$$D = \exp(-C/32F)$$

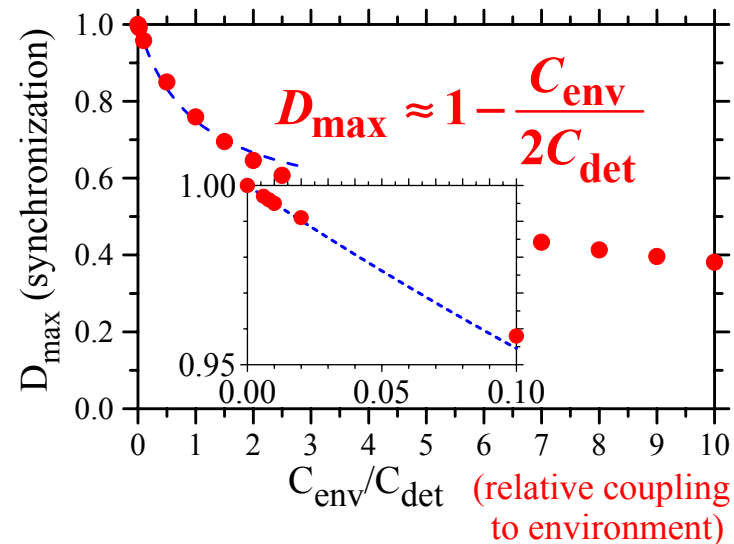
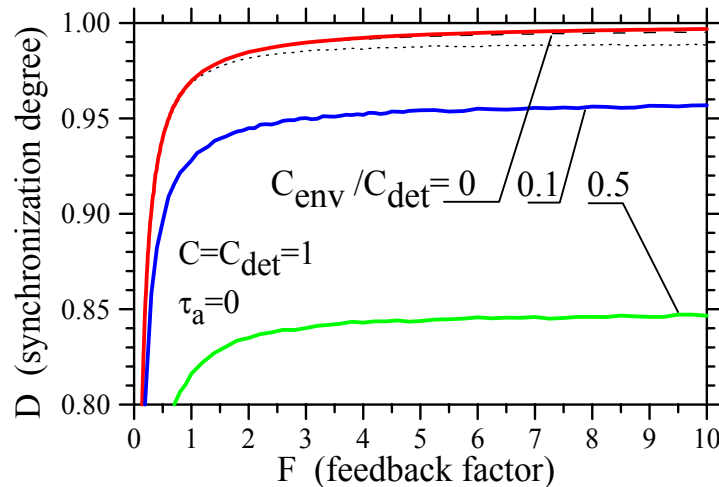
Ruskov & Korotkov, PRB 66, 041401(R) (2002)

University of California, Riverside

Alexander Korotkov



Suppression of environment-induced decoherence by quantum feedback



Example: if qubit coupling to environment is 10 times weaker than to detector, then $D_{\text{max}} = 95\%$ and qubit fidelity 97.5%. ($D = 0$ without feedback.)

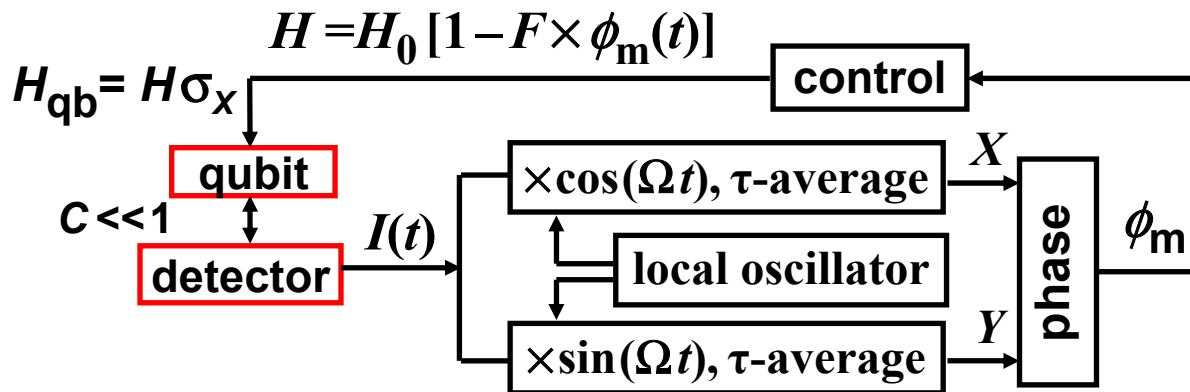
Experimental problems:

- necessity of very fast real-time solution of the Bayesian equations
- wide bandwidth ($\gg \Omega$, GHz-range) of the line delivering noisy signal $I(t)$ to the “processor”



Simple quantum feedback of a solid-state qubit

(A.K., cond-mat/0404696)



Goal: maintain coherent (Rabi) oscillations for arbitrary long time

Idea: use two quadrature components of the detector current $I(t)$ to monitor approximately the phase of qubit oscillations (a very natural way for usual classical feedback!)

$$X(t) = \int_{-\infty}^t [I(t') - I_0] \cos(\Omega t') \exp[-(t - t') / \tau] dt$$

$$Y(t) = \int_{-\infty}^t [I(t') - I_0] \sin(\Omega t') \exp[-(t - t') / \tau] dt$$

$$\phi_m = -\arctan(Y / X)$$

(similar formulas for a tank circuit instead of mixing with local oscillator)

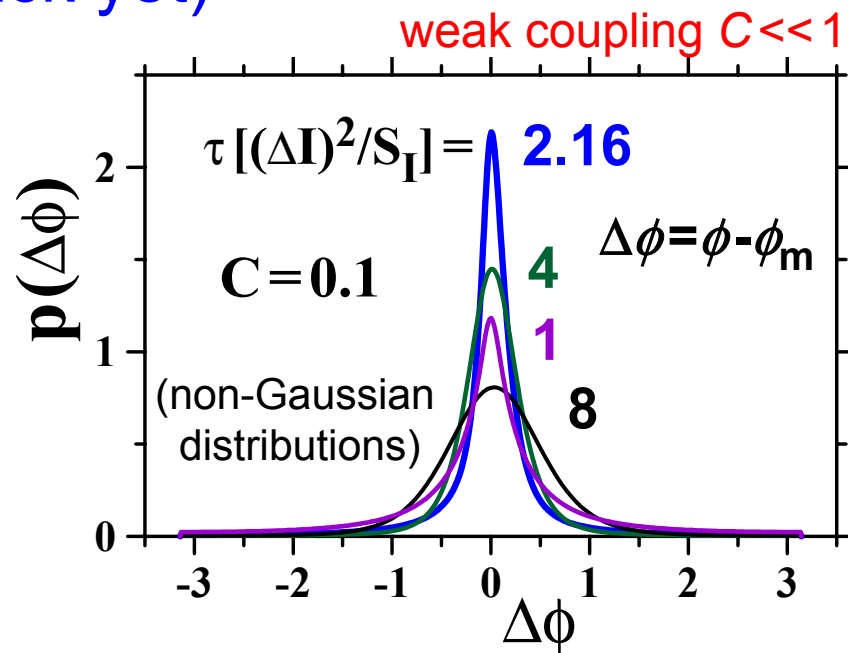
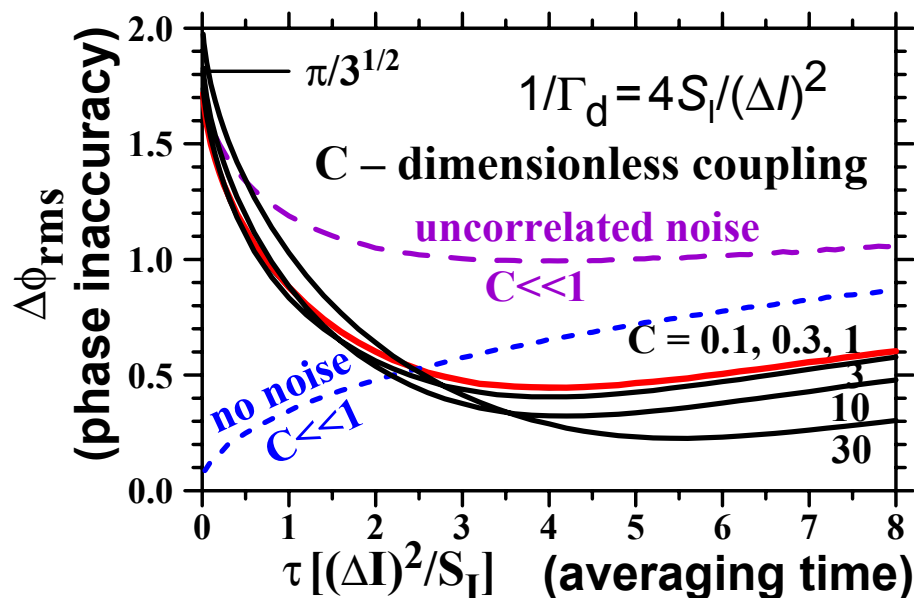
Advantage: simplicity and relatively narrow bandwidth ($1/\tau \sim \Gamma_d \ll \Omega$)

Anticipated problem: without feedback the spectral peak-to-pedestal ratio < 4 , therefore not much information in quadratures

(surprisingly, situation is much better than anticipated!)



Accuracy of phase monitoring via quadratures (no feedback yet)



Noise improves the monitoring accuracy!
(purely quantum effect, “reality follows observations”)

$$d\phi / dt = -[I(t) - I_0] \sin(\Omega t + \phi) (\Delta I / S_I) \quad (\text{actual phase shift, ideal detector})$$

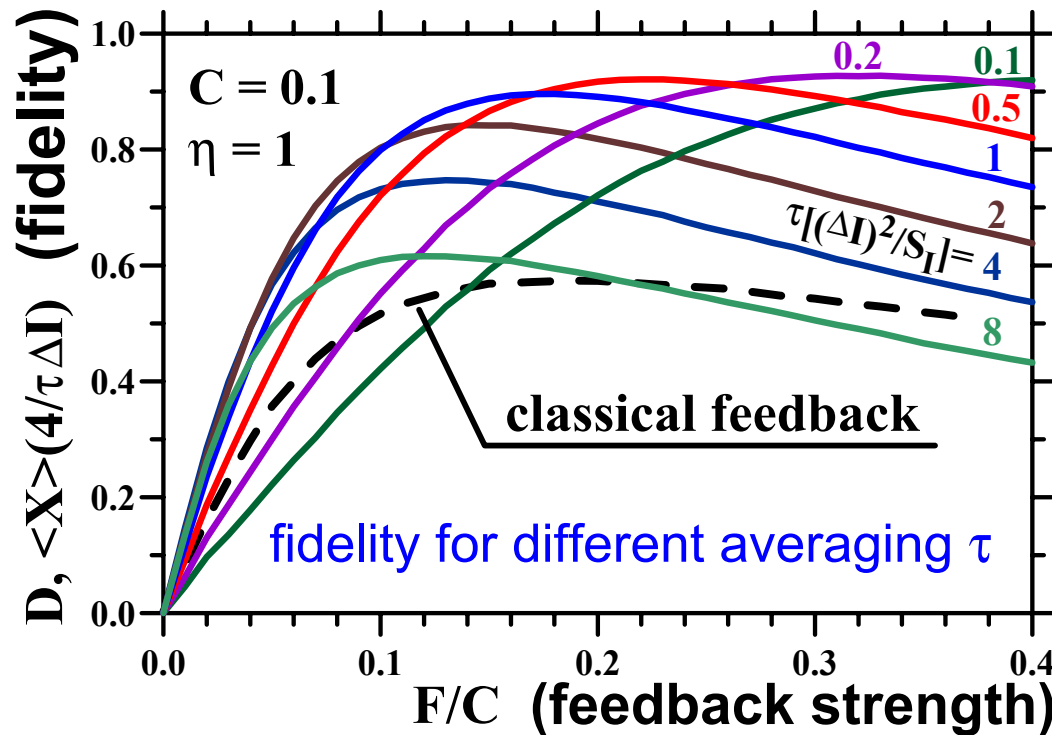
$$d\phi_m / dt = -[I(t) - I_0] \sin(\Omega t + \phi_m) / (X^2 + Y^2)^{1/2} \quad (\text{observed phase shift})$$

Noise enters the actual and observed phase evolution in a similar way

Quite accurate monitoring! $\cos(0.44) \approx 0.9$



Simple quantum feedback



weak coupling C

D – feedback efficiency

$$D \equiv 2F_Q - 1$$

$$F_Q \equiv \langle \text{Tr } \rho(t) \rho_{des}(t) \rangle$$

$$D_{\max} \approx 90\%$$

$$(F_Q \approx 95\%)$$

How to verify feedback operation experimentally?

Simple: just check that in-phase quadrature $\langle X \rangle$

of the detector current is positive $D = \langle X \rangle (4/\tau \Delta I)$

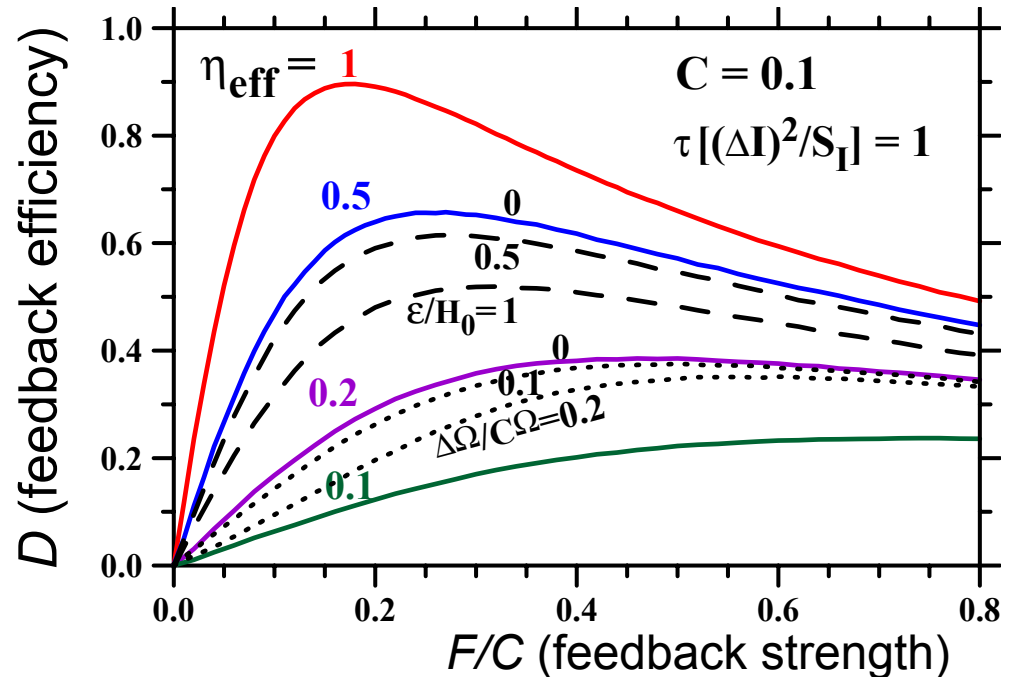
$\langle X \rangle = 0$ for any non-feedback Hamiltonian control of the qubit



Effect of nonidealities

- nonideal detectors (finite quantum efficiency η) and environment
- qubit energy asymmetry ε
- frequency mismatch $\Delta\Omega$

**Quantum feedback
still works quite well**



Main features:

- Fidelity F_Q up to $\sim 95\%$ achievable ($D \sim 90\%$)
- Natural, practically classical feedback setup
- Averaging $\tau \sim 1/\Gamma \gg 1/\Omega$ (narrow bandwidth!)
- Detector efficiency (ideality) $\eta \sim 0.1$ still OK
- Robust to asymmetry ε and frequency shift $\Delta\Omega$
- Simple verification: positive in-phase quadrature $\langle X \rangle$

**Simple enough
experiment?!**



Quantum feedback in optics

Recent experiment: Science 304, 270 (2004)

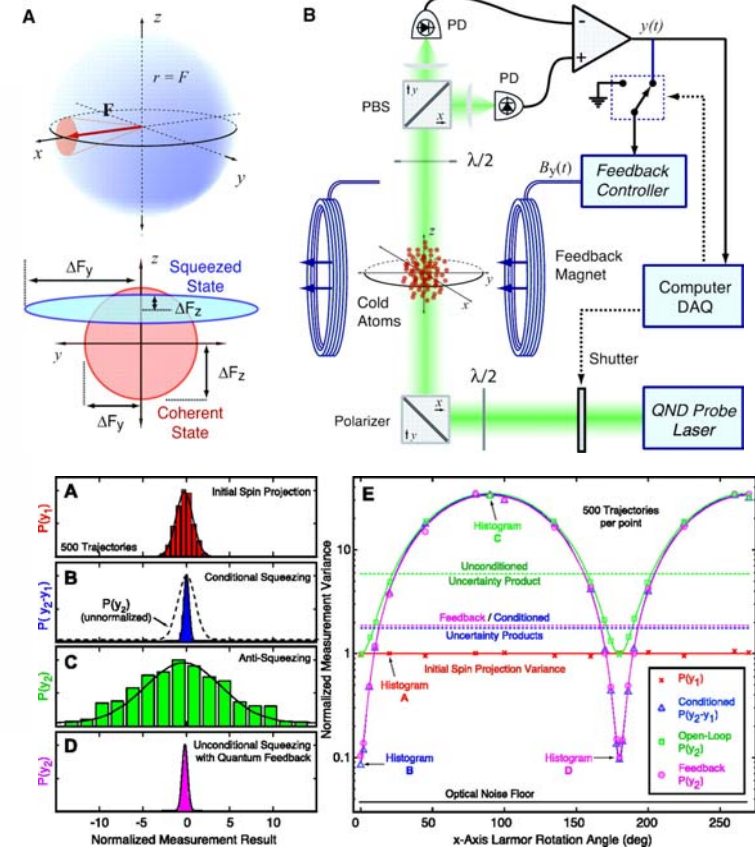
Real-Time Quantum Feedback Control of Atomic Spin-Squeezing

JM Geremia,* John K. Stockton, Hideo Mabuchi

Real-time feedback performed during a quantum nondemolition measurement of atomic spin-angular momentum allowed us to influence the quantum statistics of the measurement outcome. We showed that it is possible to harness measurement backaction as a form of actuation in quantum control, and thus we describe a valuable tool for quantum information science. Our feedback-mediated procedure generates spin-squeezing, for which the reduction in quantum uncertainty and resulting atomic entanglement are not conditioned on the measurement outcome.

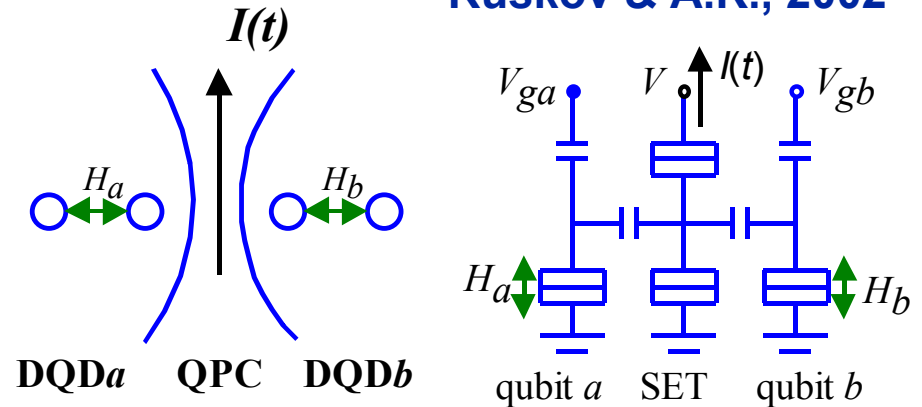
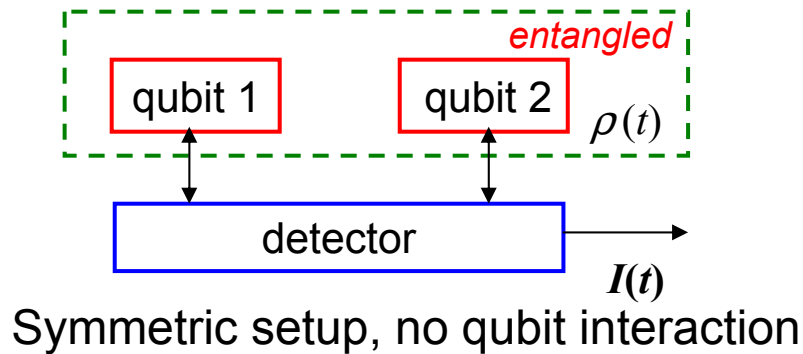
First detailed theory:

H.M. Wiseman and G. J. Milburn,
Phys. Rev. Lett. 70, 548 (1993)

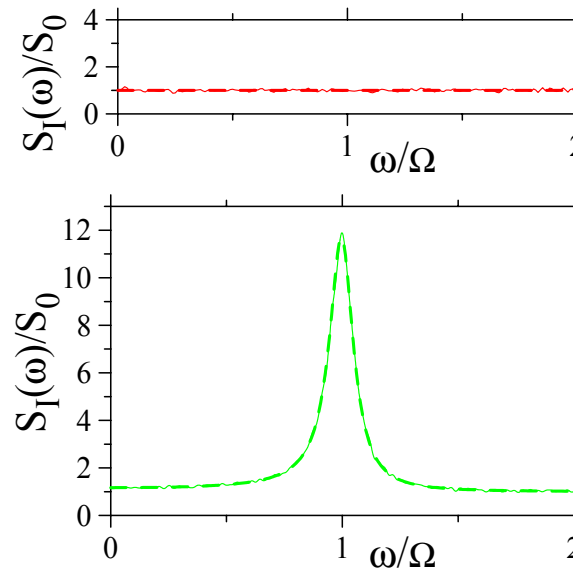
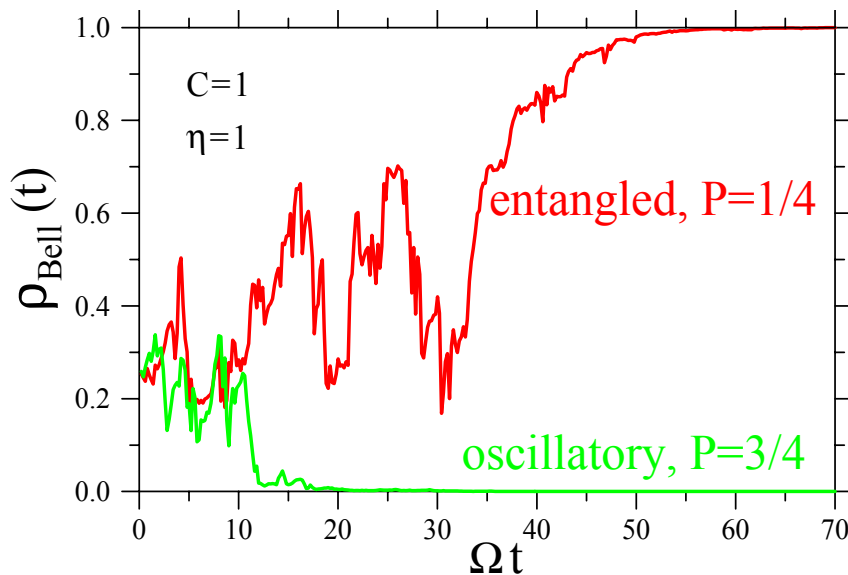


Two-qubit entanglement by measurement

Ruskov & A.K., 2002



Two evolution scenarios:



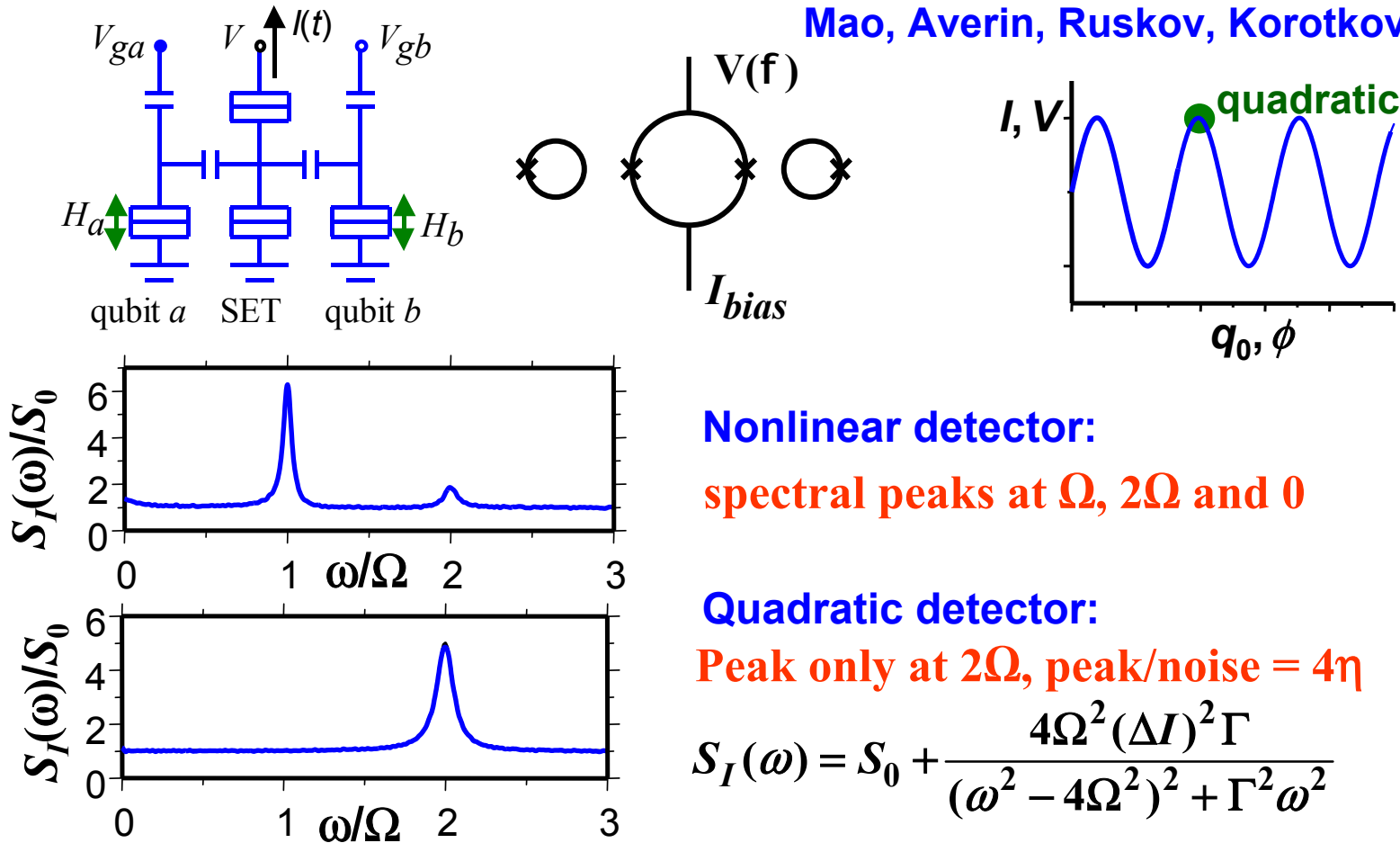
Peak/noise
= $(32/3)\eta$

Collapse into $|\text{Bell}\rangle$ state (spontaneous entanglement)
with probability 1/4 starting from fully mixed state



Quadratic quantum detection

Mao, Averin, Ruskov, Korotkov, PRL-2004



Nonlinear detector:

spectral peaks at Ω , 2Ω and 0

Quadratic detector:

Peak only at 2Ω , peak/noise = 4η

$$S_I(\omega) = S_0 + \frac{4\Omega^2 (\Delta I)^2 \Gamma}{(\omega^2 - 4\Omega^2)^2 + \Gamma^2 \omega^2}$$

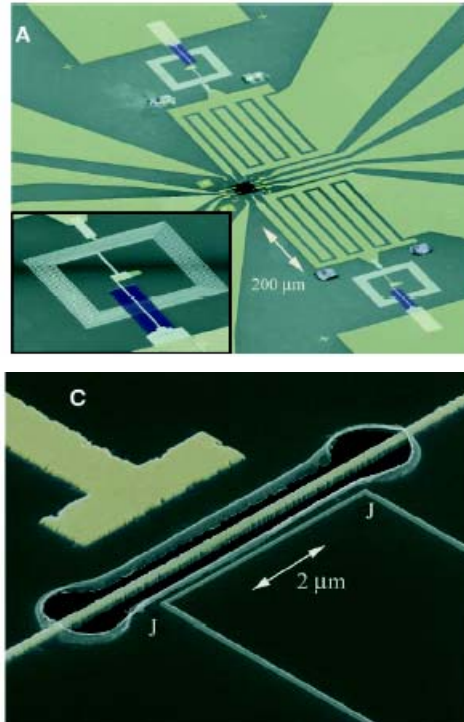
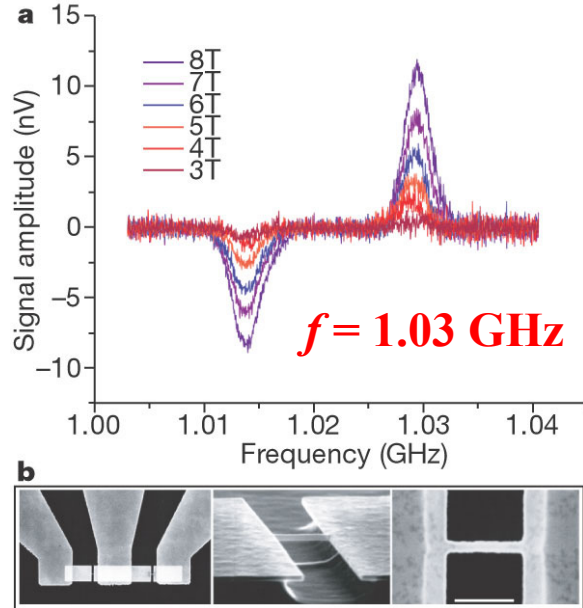
Three evolution scenarios: 1) collapse into $|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$ current I_{AE} , flat spectrum
 2) collapse into $|\uparrow\uparrow\rangle - |\downarrow\downarrow\rangle$ current $I_{A\bar{A}\bar{E}}$ flat spectrum; 3) collapse into remaining subspace, current $(I_{AE} + I_{A\bar{A}\bar{E}})/2$, spectral peak at 2Ω

Entangled states distinguished by average detector current

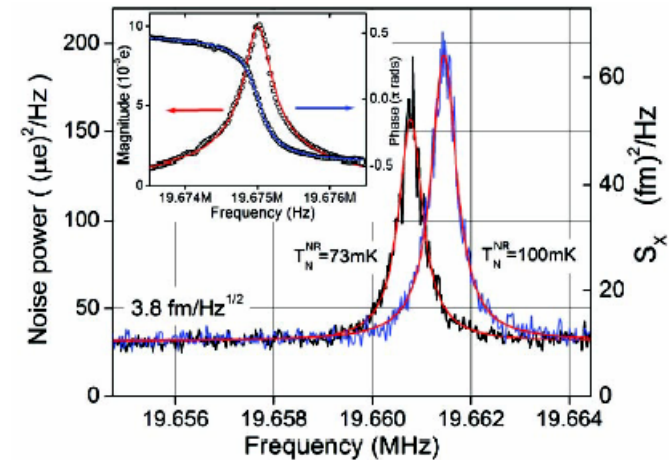


Some experiments on nanoresonators

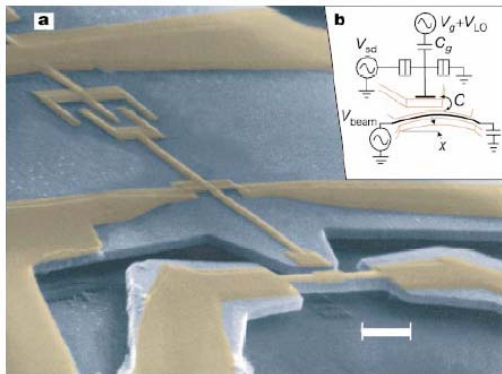
Ming *et al.* (Roukes' group),
Nature-2003



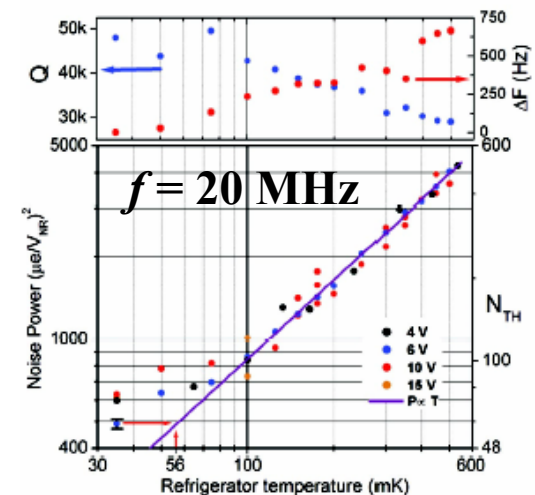
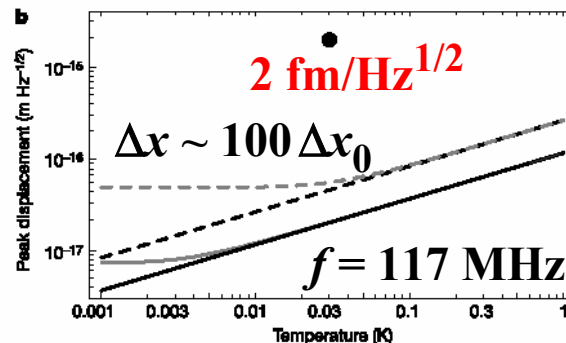
LaHaye, Buu, Camarota,
and Schwab, Science-2004



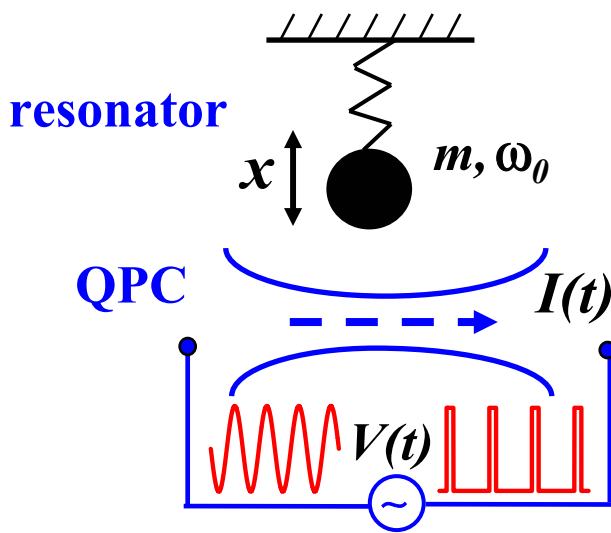
$$\Delta x = 5.8 \Delta x_0 \quad 3.8 \text{ fm}/\text{Hz}^{1/2}$$



Knobel, Cleland, Nature-2003



QND squeezing of a nanomechanical resonator



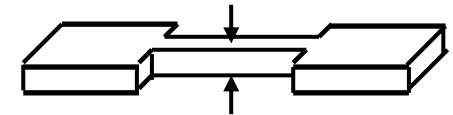
Ruskov, Schwab, Korotkov, cond-mat/0406416,
cond-mat/0411617

$$\hat{H}_0 = \hat{p}^2 / 2m + m\omega_0^2 \hat{x}^2 / 2$$

$$\hat{H}_{DET} = \sum_l E_l a_l^\dagger a_l + \sum_r E_r a_r^\dagger a_r + \sum_{l,r} (M a_l^\dagger a_r + H.c.)$$

$$\hat{H}_{INT} = \sum_{l,r} (\Delta M \hat{x} a_l^\dagger a_r + H.c.)$$

Experimental status:



$\omega_0/2\pi \sim 1$ GHz ($\hbar\omega_0 \sim 80$ mK), Roukes' group, 2003
 $\Delta x/\Delta x_0 \sim 5$ [SQL $\Delta x_0 = (\hbar/2m\omega_0)^{1/2}$], Schwab's group, 2004

Continuous monitoring and quantum feedback can cool nanoresonator down to the ground state (Hopkins, Jacobs, Habib, Schwab, PRB 2003)

Our paper: Braginsky's stroboscopic QND measurement using modulation of detector voltage \Rightarrow **squeezing becomes possible**

Potential application: ultrasensitive force measurements

Other most important papers:

Doherty, Jacobs, PRA 1999 (formalism for Gaussian states)

Mozyrsky, Martin, PRL 2002 (ensemble-averaged evolution)



Stroboscopic QND measurements

Quantum nondemolition (QND) measurements (Braginsky-Khalili book)
(a way to suppress measurement backaction and overcome standard quantum limit)

Idea: to avoid measuring the magnitude conjugated to the magnitude of interest

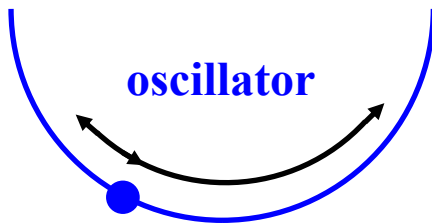
Standard quantum limit

Example: measurement of $x(t_2) - x(t_1)$



First measurement: $\Delta p(t_1) > \hbar/2\Delta x(t_1)$, then even for accurate second measurement inaccuracy of position difference is $\Delta x(t_1) + (t_2 - t_1)\hbar/2m\Delta x(t_1) > (t_2 - t_1)\hbar/2^{1/2}m$

Stroboscopic QND measurements (Braginsky *et al.*, 1978; Thorne *et al.*, 1978)



Idea: second measurement exactly one oscillation period later is insensitive to Δp
(or $\Delta t = nT/2$, $T = 2\pi/\omega_0$)

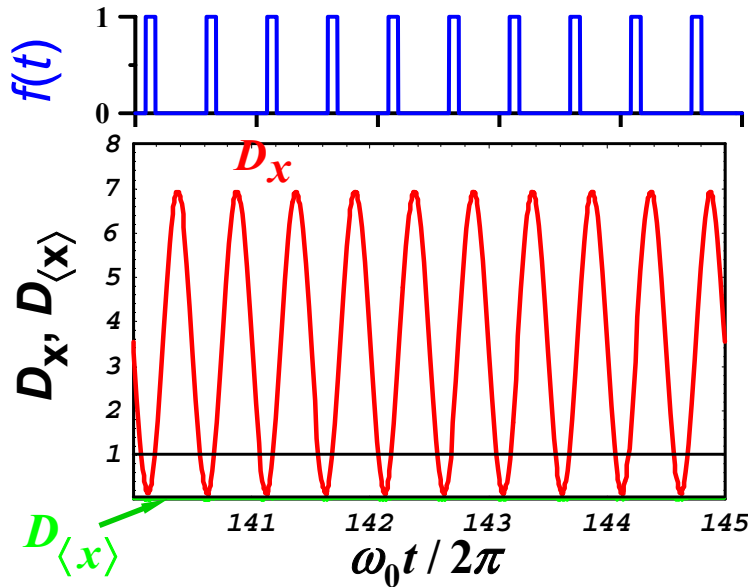
Difference in our case:

- continuous measurement
- weak coupling with detector
- quantum feedback to suppress “heating”

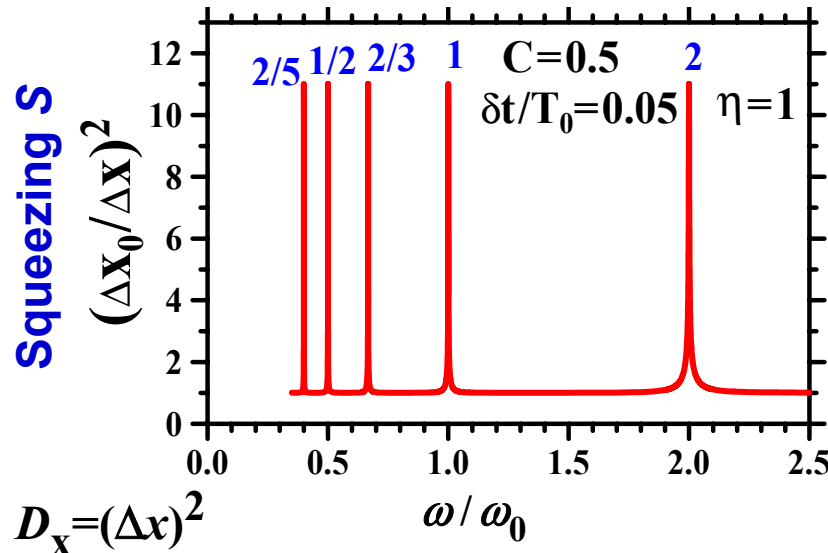


Squeezing by stroboscopic (pulse) modulation

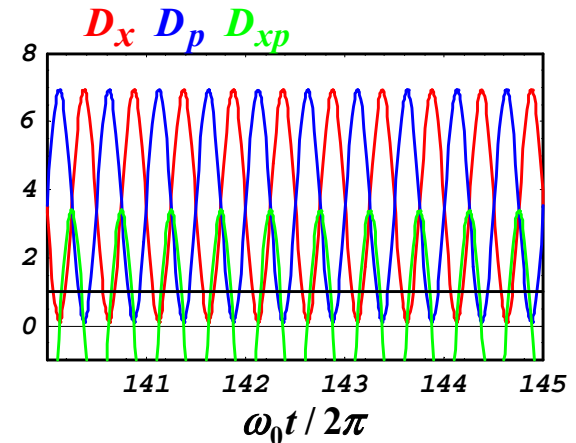
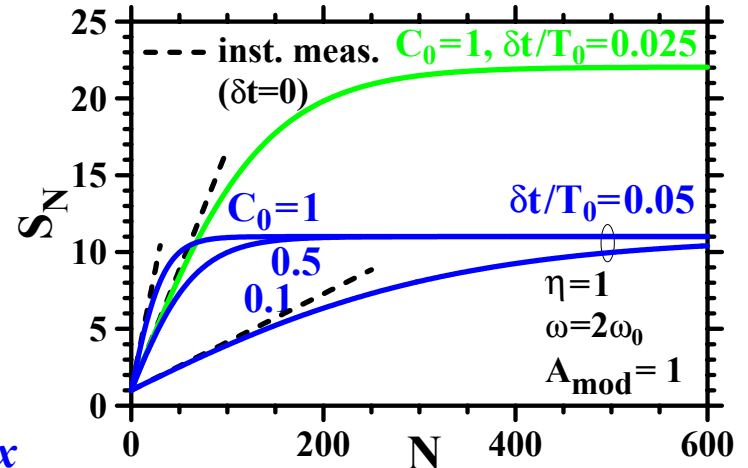
pulse modulation $\omega = 2\omega_0 / n$



$D_{\langle x \rangle} \ll D_x$
using feedback



Squeezing buildup (in time)



Sá 1

**Efficient squeezing at $\omega = 2\omega_0 / n$
(natural QND condition)**

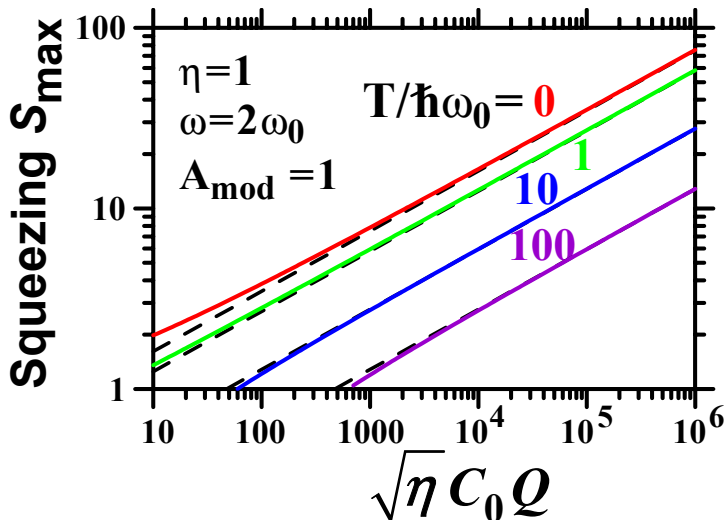
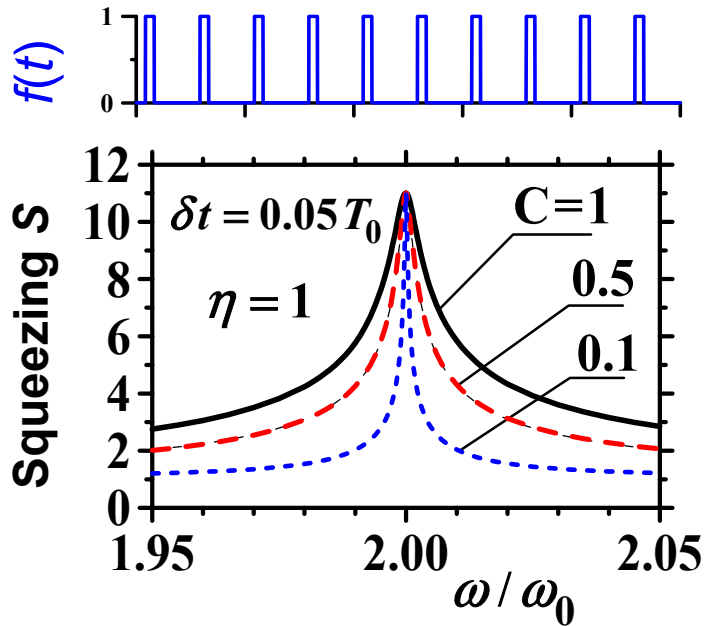
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Squeezing by stroboscopic modulation



Analytics (weak coupling, short pulses)

Maximum squeezing

$$S(2\omega_0 / n) = \frac{2\sqrt{3}\eta}{\omega_0 \delta t}$$

Linewidth

$$\Delta\omega = \frac{4C_0(\delta t)^3 \omega_0^4}{\pi n^2 \sqrt{3}\eta}$$

C_0 – dimensionless coupling with detector

δt – pulse duration, $T_0 = 2\pi/\omega_0$

η – quantum efficiency of detector

Squeezing requires $\sim \sqrt{3\eta} / C_0(\omega_0 \delta t)^2$ pulses

Finite Q-factor and finite temperature limit

maximum squeezing S_{\max}

$$S_{\max} = \frac{3}{4} \left[\frac{\sqrt{\eta} C_0 Q}{\coth(\hbar\omega_0 / 2T)} \right]^{1/3}$$

(So far in experiment $\eta^{1/2} C_0 Q \sim 0.1$)



Conclusions

- Continuous quantum measurement is *not* equivalent to decoherence (environment) if detector output (information) is taken into account, in contrast to ensemble-averaged case
- Bayesian approach to continuous quantum measurement is a simple, but new and interesting subject in solid-state mesoscopics
- Several experimental predictions have been already made; however, many problems not studied yet
- No direct experiments yet (few indirect ones); hopefully, coming soon

