Possible experiments on continuous measurement and quantum feedback of solid-state qubits

Alexander Korotkov

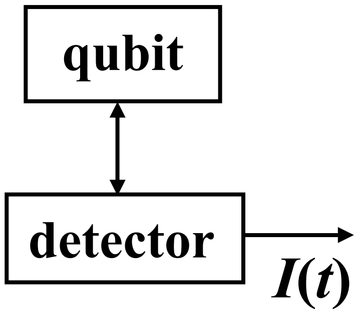
University of California, Riverside

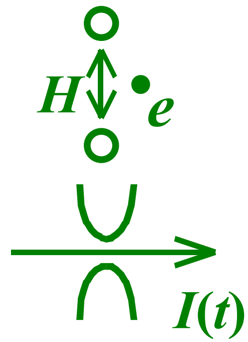
A number of experimental proposals have originated from the Bayesian approach to continuous quantum measurement. Here we present several of them, which are reasonably simple for a real experiment and therefore seem to be realizable in near future or have already been realized.

Support: ARDA

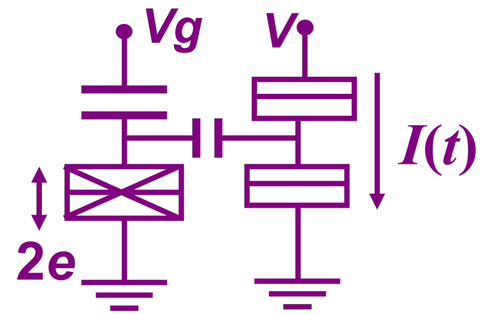
Continuous monitoring of oscillating qubit: spectral peak at Rabi frequency

Korotkov'1999, Korotkov-Averin'2000, …

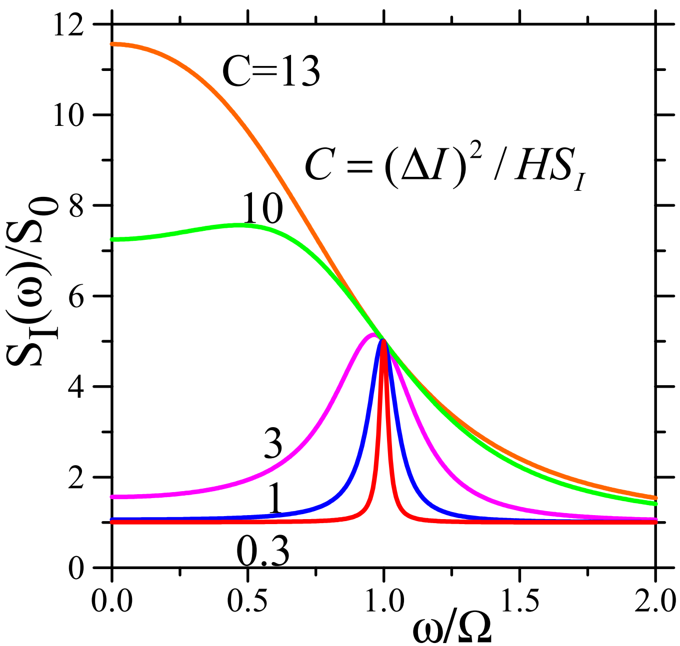




Double-quantum-dot (DQD) and quantum point contact (QPC)



Cooper-pair box (CPB) and single-electron transistor (SET)



eV » $\hbar\Omega$ (classical output), $\varepsilon = 0$ (symmetric qubit)

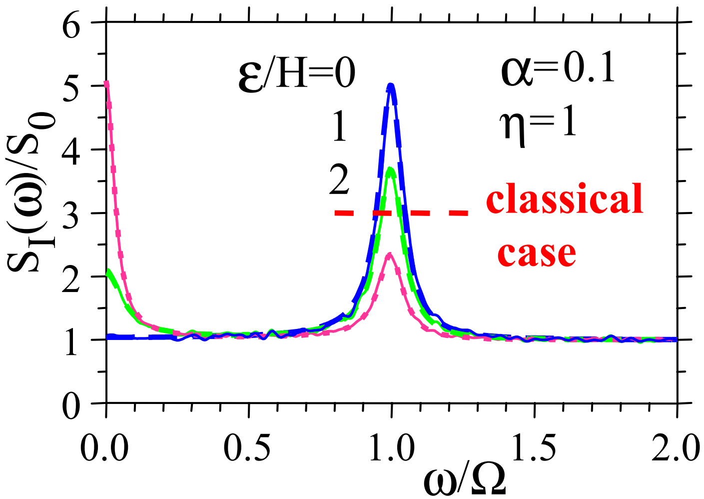
$\Gamma = \eta^{-1}(\Delta I)^2 / 4S_0$ (η - efficiency, S_0 – noise)

$$S_I(\omega) = S_0 + \frac{\Omega^2 (\Delta I)^2 \Gamma}{(\omega^2 - \Omega^2)^2 + \Gamma^2 \omega^2}$$

Spectral peak can be seen, but peak-to-pedestal ratio ≤ 4η ≤ 4

Spectral peak at Rabi frequency

(2)



Weak coupling, $\alpha = C/8 \ll 1$

$$S_I(\omega) = S_0 + \frac{\eta S_0 \varepsilon^2 / H^2}{1 + (\omega \hbar^2 \Omega^2 / 4H^2 \Gamma)^2} + \frac{4\eta S_0 (1 + \varepsilon^2 / 2H^2)^{-1}}{1 + [(\omega - \Omega)\Gamma(1 - 2H^2/\hbar^2\Omega^2)]^2}$$

What this experiment can demonstrate:

- Quantum coherent (Rabi) oscillations do not decay (!!!) in presence of continuous monitoring, just the oscillation phase is gradually changing (dephasing, not decay)
- Experiment can demonstrate violation of Leggett-Garg inequality, therefore incompatibility with realism (Ruskov-Korotkov-Mizel'2005, Jordan-Korotkov-Buttiker'2005)

Possible experimental confirmation?

Durkan and Welland, 2001 (STM-ESR experiment similar to Manassen-1989)

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Electronic spin detection in molecules using scanning-tunneling-microscopy-assisted electron-spin resonance

C. Durkan[a] and M. E. Welland
Nanoscale Science Laboratory, Department of Engineering, University of Cambridge, Trumpington Street, Cambridge CB2 1PZ, United Kingdom

(Received 8 May 2001; accepted for publication 8 November 2001)

By combining the spatial resolution of a scanning-tunneling microscope (STM) with the electronic spin sensitivity of electron-spin resonance, we show that it is possible to detect the presence of localized spins on surfaces. The principle is that a STM is operated in a magnetic field, and the resulting component of the tunnel current at the Larmor (precession) frequency is measured. This component is nonzero whenever there is tunneling into or out of a paramagnetic entity. We have

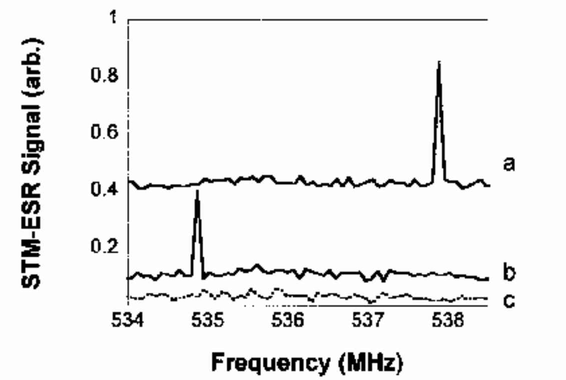


FIG. 3. STM-ESR spectra of (a), (b) two different areas (a few nm apart) of the molecule-covered sample and (c) bare HOPG. The graphs are shifted vertically for clarity.

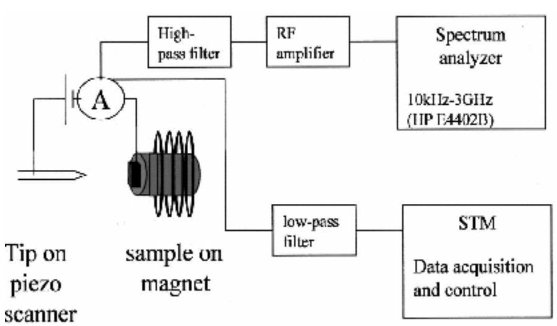


FIG. 1. Schematic of the electronics used in STM-ESR.

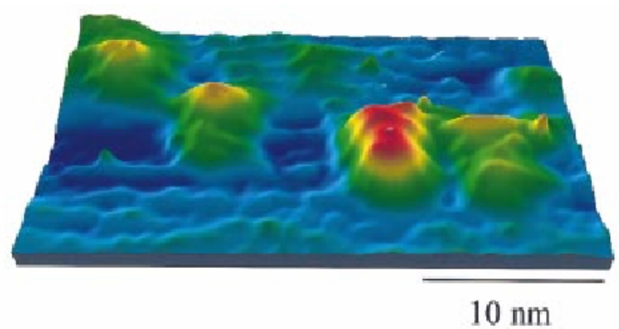


FIG. 2. (Color) STM image of a 250 Å×150 Å area of HOPG with four adsorbed BDPA molecules.

$$\frac{\text{peak}}{\text{noise}} \leq 3.5$$

(Colm Durkan, private comm.)

Somewhat similar experiment

"Continuous monitoring of Rabi oscillations in a Josephson flux qubit"

E. Il'ichev *et al.*, PRL, 2003

$$H = -\frac{1}{2}(\Delta\sigma_x + \varepsilon\sigma_z) - W\sigma_z \cos\omega_{HF} t$$

$$(\omega_{HF} \approx \sqrt{\Delta^2 + \varepsilon^2};\quad \varepsilon \neq 0)$$

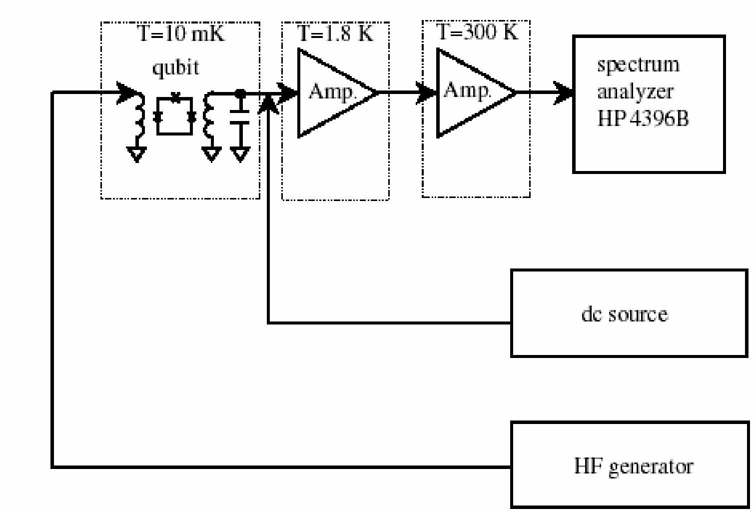


FIG. 1. Measurement setup. The flux qubit is inductively coupled to a tank circuit. The dc source applies a constant flux $\Phi_e \approx \frac{1}{2}\Phi_0$. The HF generator drives the qubit through a separate coil at a frequency close to the level separation $\Delta/h =$ 868 MHz. The output voltage at the resonant frequency of the tank is measured as a function of HF power.

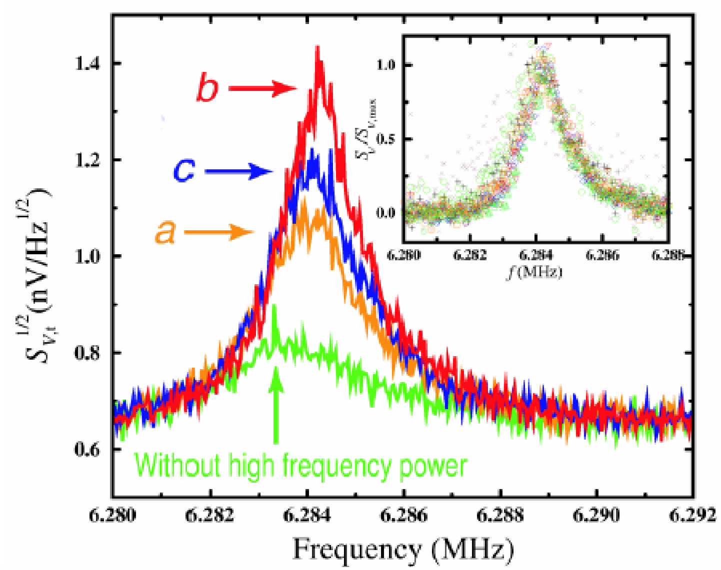
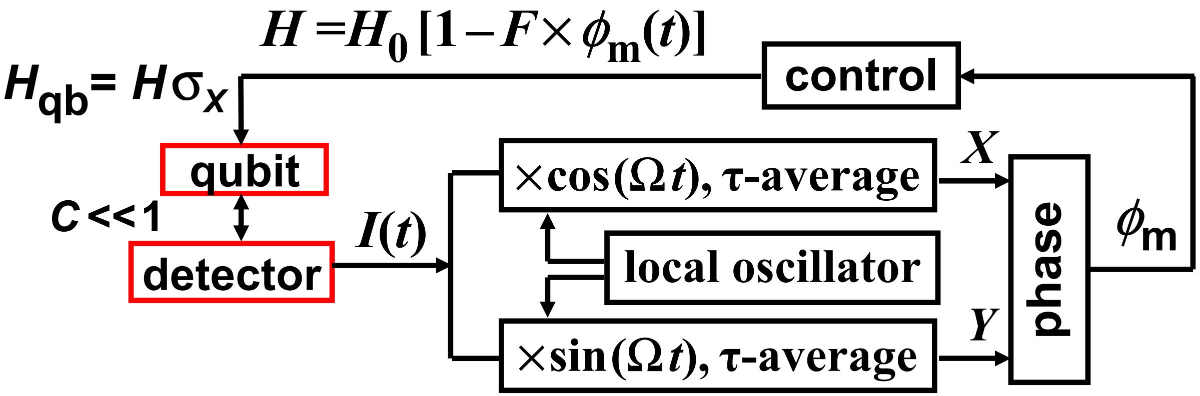


FIG. 3 (color online). The spectral amplitude of the tank voltage for HF powers $P_a < P_b < P_c$ at 868 MHz, detected using the setup of Fig. 1. The bottom curve corresponds to the background noise without an HF signal. The inset shows normalized voltage spectra for seven values of HF power, with background subtracted. The shape of the resonance, being determined by the tank circuit, is essentially the same in each case. Remaining tiny variations visible in the main figure are due to the irradiated qubit modifying the tank's inductance and

Simple quantum feedback of a solid-state qubit

Korotkov'2005



Goal: maintain coherent (Rabi) oscillations for arbitrary long time

Idea: use two quadrature components of the detector current $I(t)$ to monitor approximately the phase of qubit oscillations

(a very natural way for usual classical feedback!)

$$X(t) = \int_{-\infty}^{t} [I(t') - I_0] \cos(\Omega t') \exp[-(t-t')/\tau]\, dt$$

$$Y(t) = \int_{-\infty}^{t} [I(t') - I_0] \sin(\Omega t') \exp[-(t-t')/\tau]\, dt$$

$$\phi_m = -\arctan(Y/X)$$

(similar formulas for a tank circuit instead of mixing with local oscillator)

Advantage: simplicity and relatively narrow bandwidth $(1/\tau \sim \Gamma_d << \Omega)$

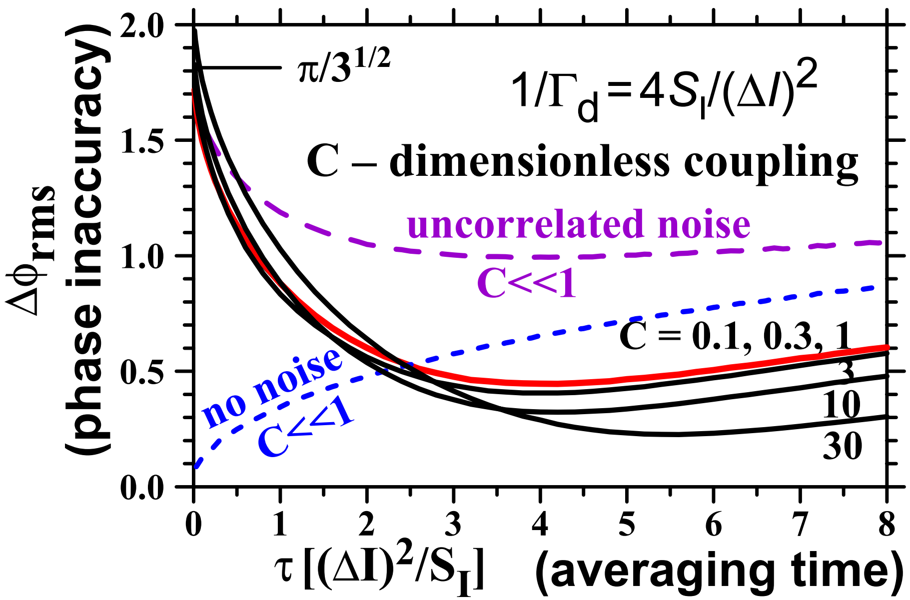
Anticipated problem: without feedback the spectral peak-to-pedestal ratio <4, therefore not much information in quadratures

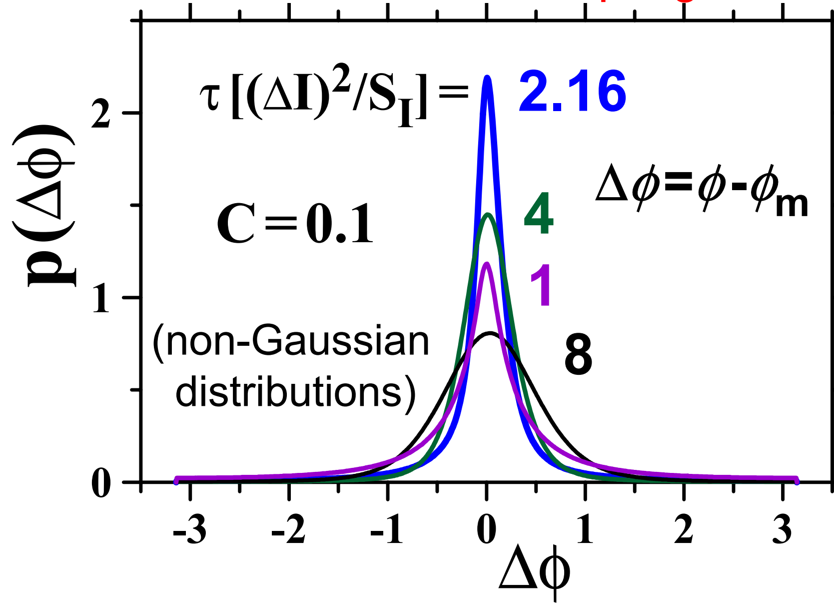
(surprisingly, situation is much better than anticipated!)

Accuracy of phase monitoring via quadratures (2)

(no feedback yet)

weak coupling C<<1





Noise improves the monitoring accuracy!

(purely quantum effect, “reality follows observations”)

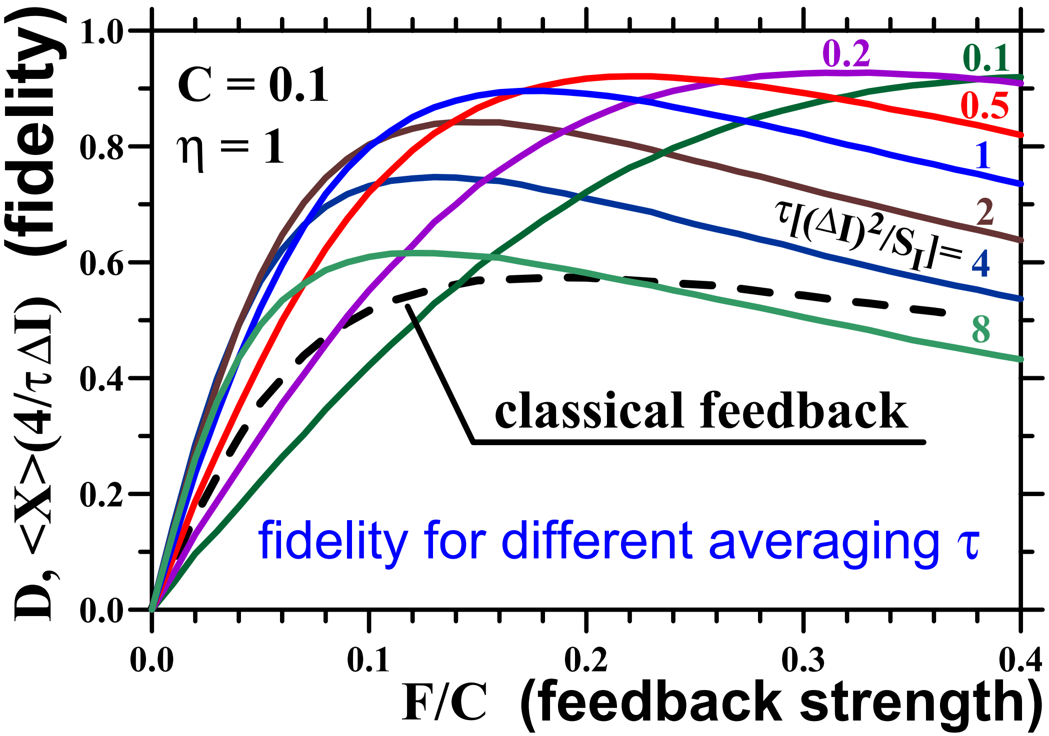
$$d\phi/dt = -[I(t)-I_0]\sin(\Omega t+\phi)(\Delta I/S_I)$$ (actual phase shift, ideal detector)

$$d\phi_m/dt = -[I(t)-I_0]\sin(\Omega t+\phi_m)/(X^2+Y^2)^{1/2}$$ (observed phase shift)

Noise enters the actual and observed phase evolution in a similar way

Quite accurate monitoring! cos(0.44)≈0.9

Simple quantum feedback



weak coupling *C*

D – feedback efficiency

$$D \equiv 2F_Q - 1$$

$$F_Q \equiv \langle \mathrm{Tr}\, \rho(t)\, \rho_{des}(t) \rangle$$

$D_{max} \approx 90\%$

($F_Q \approx 95\%$)

How to verify feedback operation experimentally?

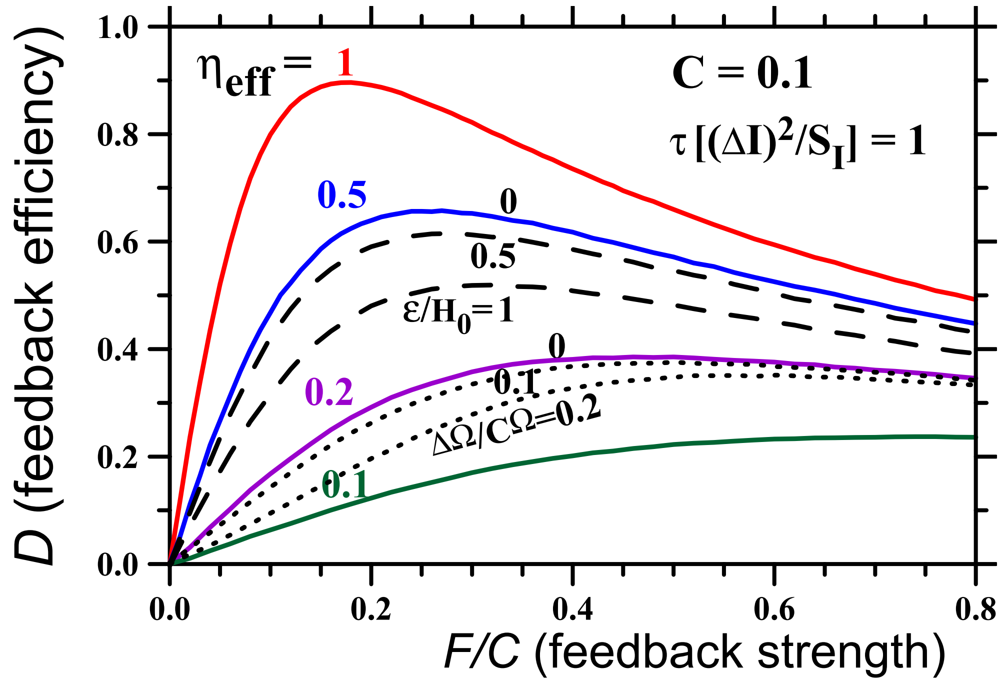
Simple: just check that in-phase quadrature $\langle X \rangle$ of the detector current is positive $D = \langle X \rangle (4/\tau\Delta I)$

$\langle X \rangle$=0 for *any* non-feedback Hamiltonian control of the qubit

Effect of nonidealities

- nonideal detectors (finite quantum efficiency η) and environment
- qubit energy asymmetry ε
- frequency mismatch ΔΩ

Quantum feedback still works quite well



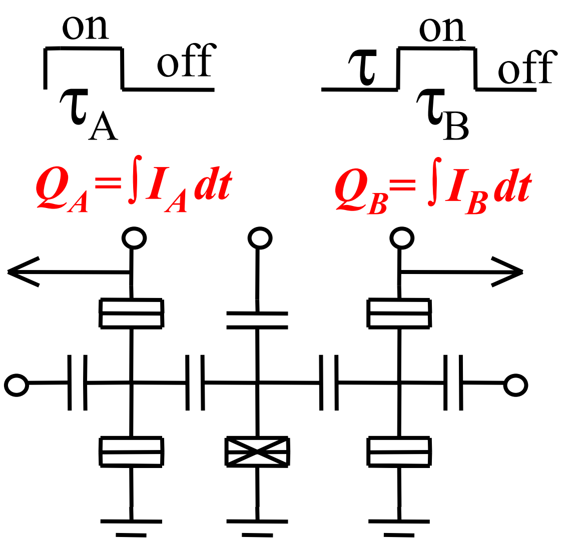
Main features:

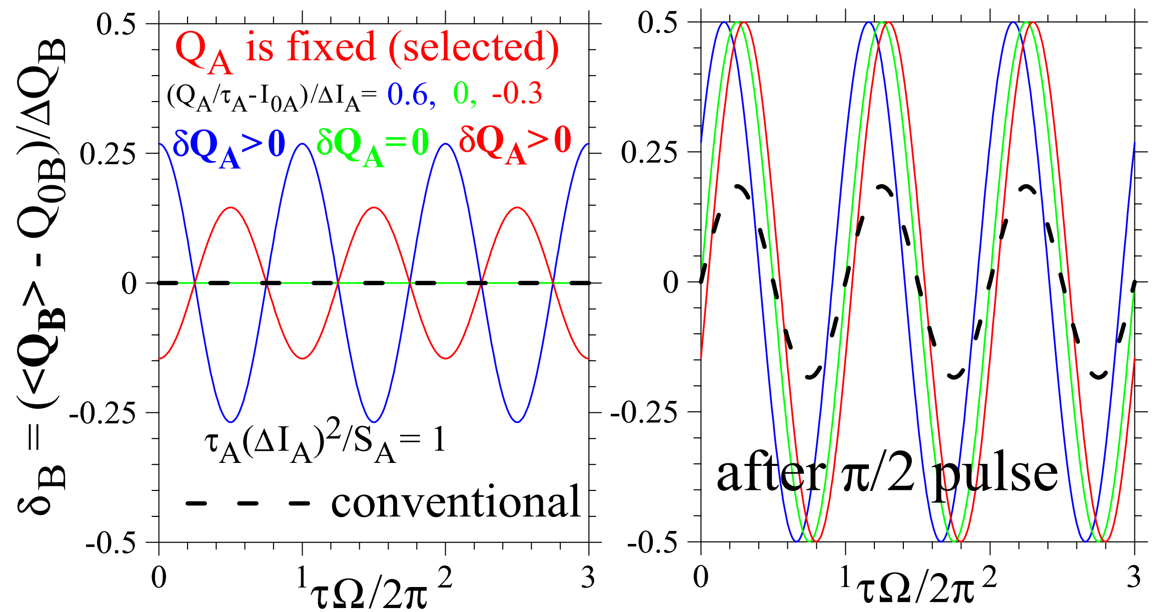
- **Fidelity F_Q up to ~95% achievable (D~90%)**
- **Natural, practically classical feedback setup**
- **Averaging τ~1/Γ>>1/Ω (narrow bandwidth!)**
- **Detector efficiency (ideality) η~0.1 still OK**
- **Robust to asymmetry ε and frequency shift ΔΩ**
- **Simple verification: positive in-phase quadrature $\langle X \rangle$**

Simple enough experiment?!

Bell-type measurement correlation

Korotkov'2000





Idea: two consecutive finite-time (imprecise) measurements of a qubit by two detectors; probability distribution **P(Q_A, Q_B, τ)** shows the effect of the first measurement on the qubit state.

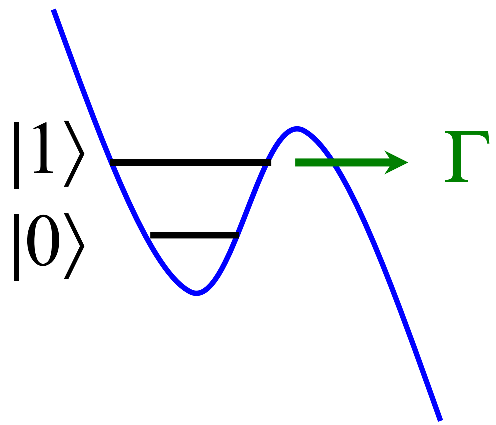
Proves that qubit remains in a pure state during measurement (for η=1)

Advantage: no need to record noisy detector output with GHz bandwidth; instead, we use two detectors and fast ON/OFF switching.

Modification: correlation of low-frequency noises (not considered yet)

“Aging” of a phase qubit due to measurement

Recently realized experimentally by
Nadav Katz *et al*. (John Martinis’ group)



How does a coherent state evolve in time before tunneling event?

Main idea:

$$\psi = \alpha\,|0\rangle + \beta\,|1\rangle \;\rightarrow\; \psi(t) = \begin{cases} |out\rangle, \text{ if tunneled} \\ \dfrac{\alpha\,|0\rangle + \beta\, e^{-\Gamma t/2}\,|1\rangle}{Norm}, \text{ if not tunneled} \end{cases}$$

$$Norm = \sqrt{|\alpha|^2 + |\beta|^2\, e^{-\Gamma t}}$$

continuous null-result collapse

(similar to optics, Dalibard *et al*., PRL-1992)

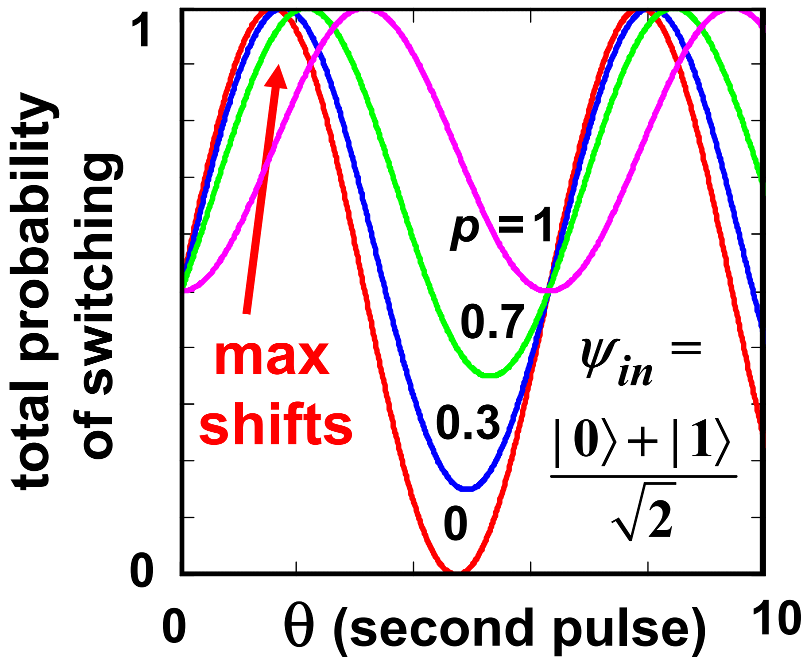
Effect of remaining coherence after incomplete (too short) measurement

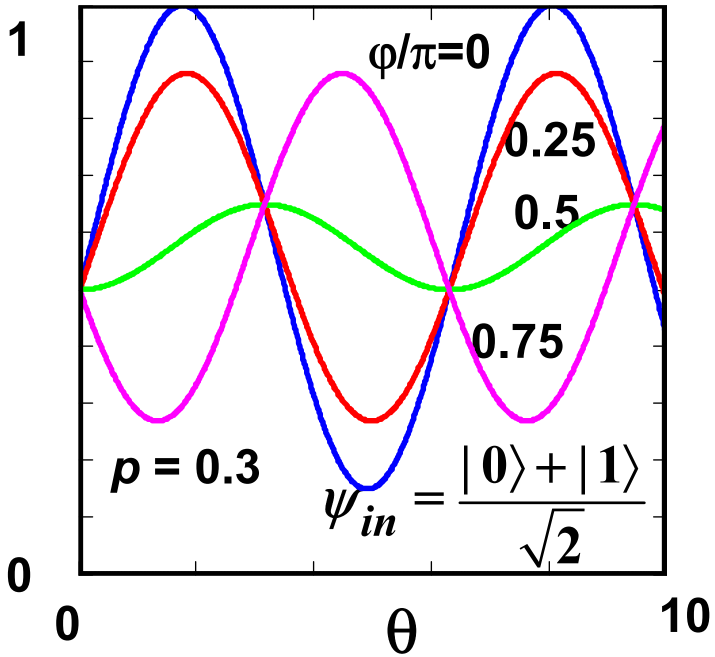
Protocol:

0) state preparation by rf pulse
1) incomplete measurement
2) additional rf pulse (θ-pulse)
3) measurement again (complete)

p = 1-exp($-\Gamma t$) – probability of state $|1\rangle$ switching after incomplete measurement

φ – extra phase (z-rotation)





Formulas for ideal case

Step 1. Rabi pulse θ_0 prepares state $\cos(\theta_0/2)|0\rangle + \sin(\theta_0/2)|1\rangle$

Step 2. Incomplete measurement with strength $p = 1 - \exp(-\Gamma\tau)$

switches qubit with probability $P_1 = p\sin^2(\theta_0)$. With probability $1 - P_1$

the state becomes $\cos(\theta_m/2)|0\rangle + \sin(\theta_m/2)\,e^{-i\varphi_m}|1\rangle$, where

φ_m – accumulated phase shift in rotating frame (levels change) and

$$\theta_m = 2\,\mathrm{atan}(\sqrt{1-p}\,\tan(\theta_0/2))$$

Step 3. Z-rotation φ and Rabi pulse θ.

Step 4. Complete measurement, switching probability P_2.

Total switching probability $P_t = P_1 + P_2$

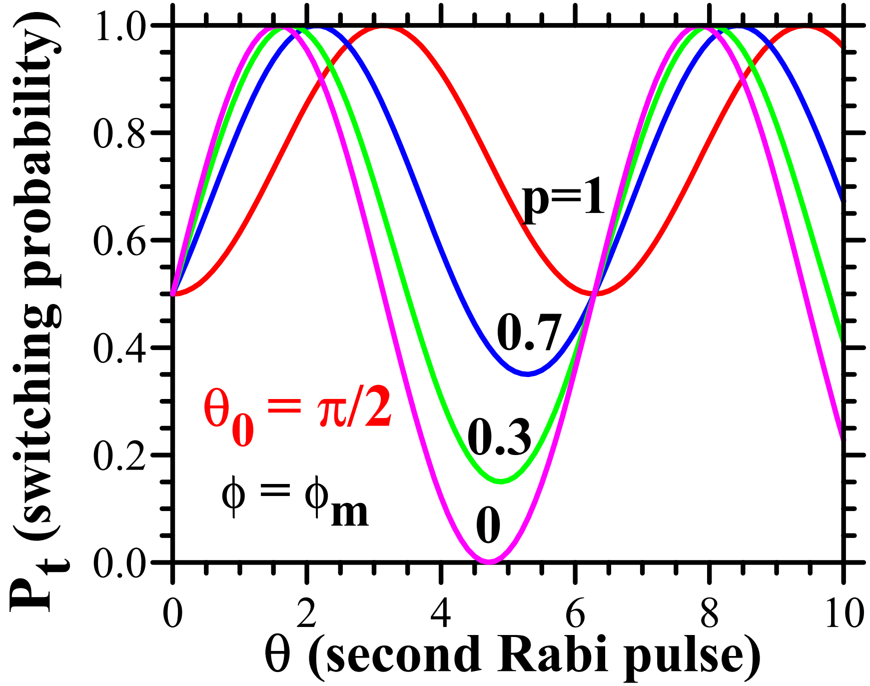
$$P_t = 1 - \frac{1}{2}[1 - p\sin^2(\frac{\theta_0}{2})][1 + \cos\theta_m\cos\theta - \sin\theta_m\sin\theta\cos(\varphi - \varphi_m)]$$

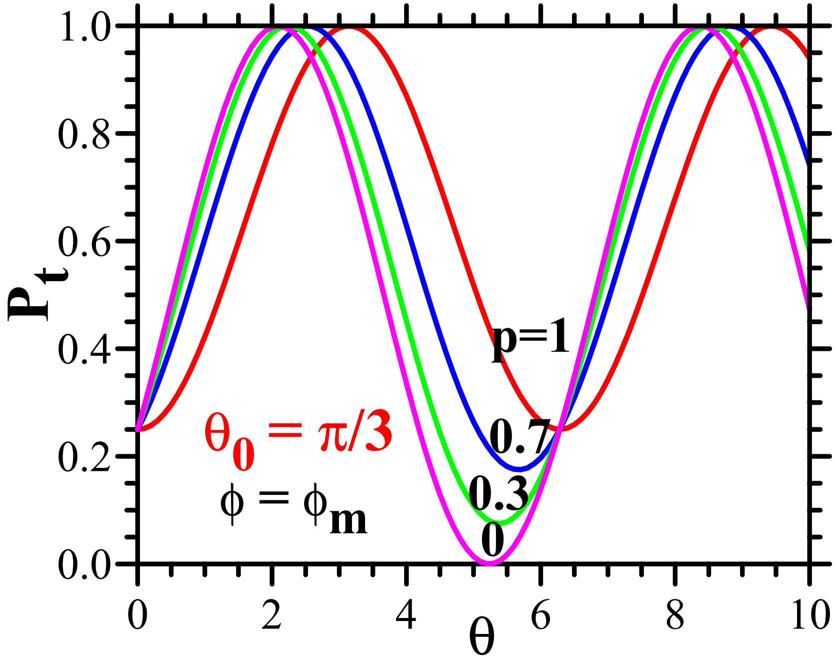
If φ_m is compensated ($\varphi = \varphi_m$) then maximum oscillation amplitude:

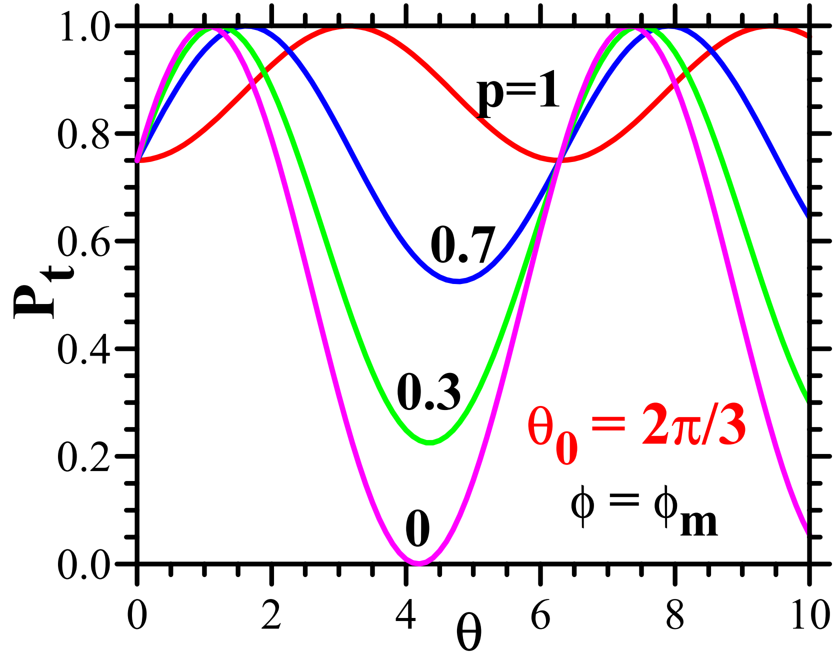
$$P_t = 1 - \frac{1}{2}[1 - p\sin^2(\frac{\theta_0}{2})][1 + \cos(\theta_m + \theta)]$$

(4)

Dependences $P_t(\theta)$ for several p and θ_0 (φ_m is compensated)







Conclusion

- Interesting ("ideologically" nontrivial) experiments on continuous quantum measurement and quantum feedback can be performed with solid-state qubits

- One such experiment has been already realized; one more partially realized

- Few more experimental proposals seem to be realizable now (or in near future)