Measurement theory for phase qubits

Co-P.I.: Alexander Korotkov, UC Riverside

(Lead P.I.: John Martinis, UCSB)

The team: 1) Qin Zhang, *graduate student*
2) Dr. Abraham Kofman, *researcher*
3) Alexander Korotkov, *associate prof.*

Since last review (August 2005)

Published/accepted : 4 journal papers (incl. PRL and Science) and 1 proceedings

Submitted (not yet accepted): 2 journal papers

Research accomplishments since last review

- Developed quantum theory of the classical measurement cross-talk for phase qubits; obtained limitations for two-qubit coupling capacitance
- Analyzed one-qubit measurement fidelity limitations due to non-adiabatic measurement pulse and too slow energy dissipation
- Analyzed the phase qubit evolution in the process of measurement due to quantum back-action
- Proposed quantum eraser based on phase qubit
- Analyzed QND measurement of charge qubit
- Started analysis of Bell inequalities for phase qubits

Research topics for the next year

- Requirements for phase qubit entanglement demonstration via violation of Bell inequalities
- Detailed theory of the quantum eraser based on phase or charge qubit
- Role of energy dissipation in phase qubit measurement (repopulation of initial state, level discreteness in “continuum” affecting fidelity, etc.)
- Related problems of quantum measurement

Quantum eraser based on phase qubit

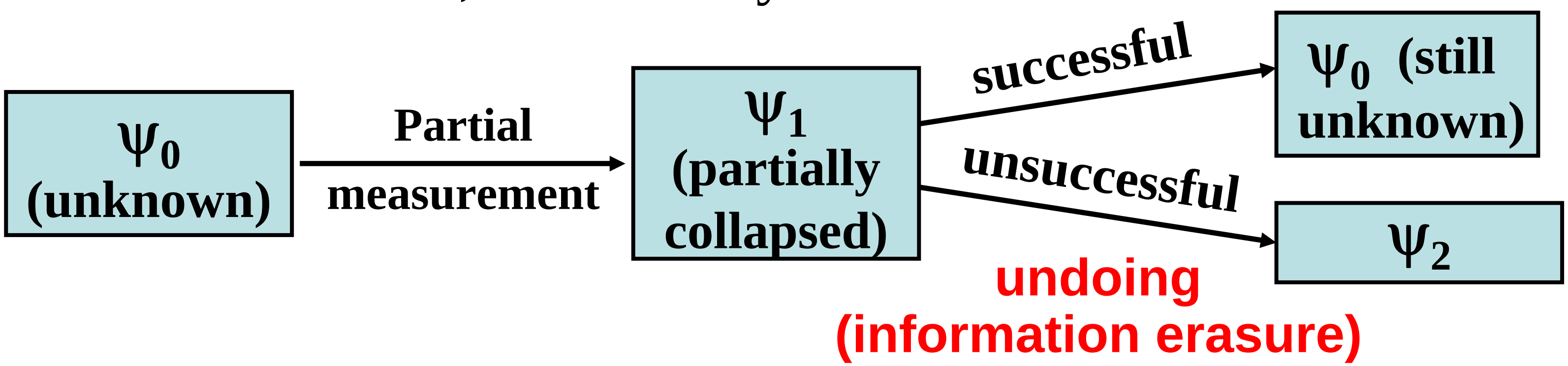
(A. Korotkov and A. Jordan, to be submitted)

It is impossible to undo "orthodox" quantum measurement (for an unknown initial state)

Is it possible to undo partial quantum measurement?
(To restore a "precious" qubit accidentally measured)

Yes! (but with a finite probability)

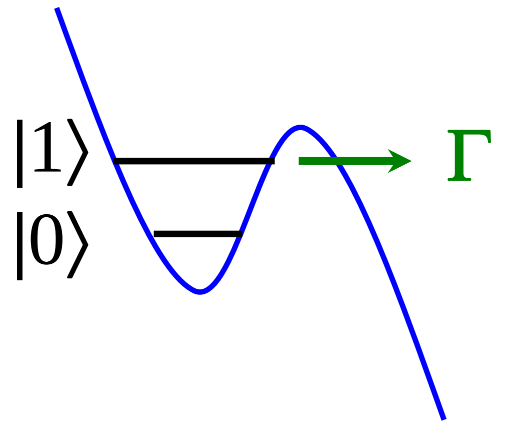
If undoing is successful, an unknown state is **fully** restored, if unsuccessful, then destroyed



Simple experimental procedure: similar to N. Katz et al. (2006)

Recent partial-collapse experiment

(N. Katz, M. Ansmann, R.C. Bialczak, E. Lucero, R. McDermott, M. Neeley, M. Steffen, E.M. Weig, A.N. Cleland, J.M. Martinis, A.N. Korotkov, Science, 2006)



Radioactive atom remains as new until it decays. In contrast, qubit with decaying state $|1\rangle$ evolves during no-decay stage

$$\psi = \alpha|0\rangle + \beta|1\rangle \;\rightarrow\; \psi(t) = \begin{cases} |out\rangle, \text{ if tunneled} \\ \dfrac{\alpha|0\rangle + \beta e^{i\phi} e^{-\Gamma t/2}|1\rangle}{\sqrt{|\alpha|^2 + |\beta|^2 e^{-\Gamma t}}}, \text{ if not tunneled} \end{cases}$$

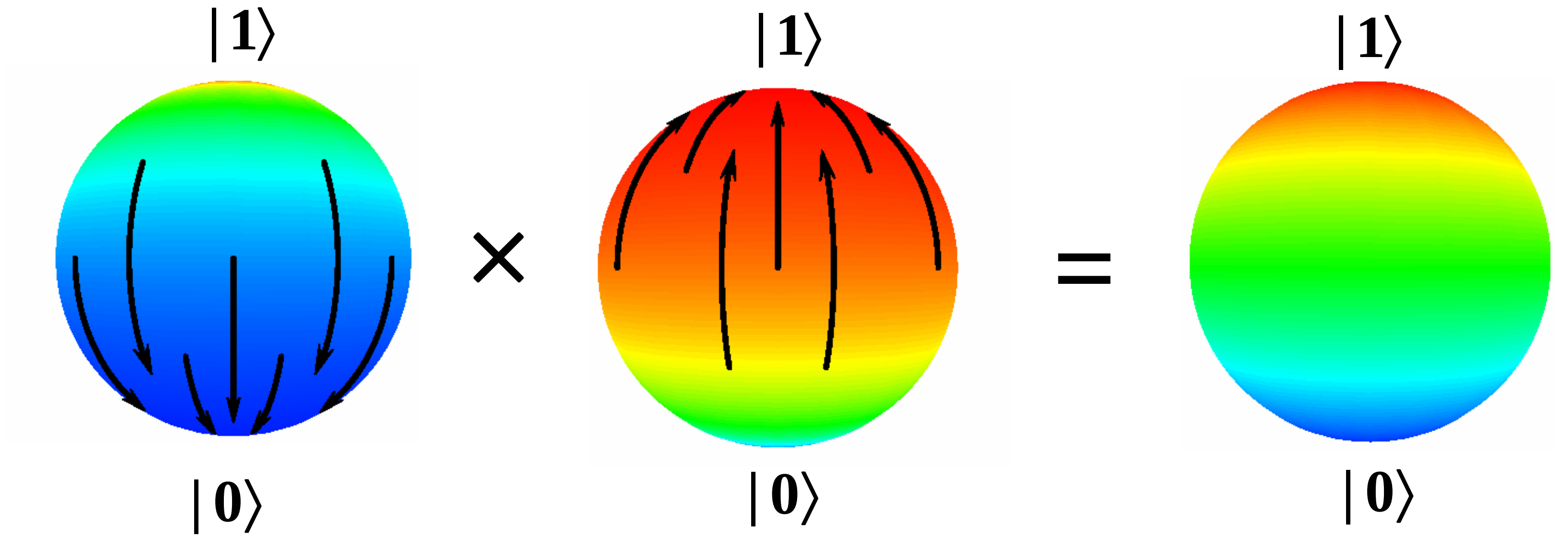
Phase qubit undergoes non-unitary evolution (while remaining in a pure state) due to continuous null-result measurement

In the experiment the measurement strength p=1-e$^{-\Gamma t}$ is varied by varying Γ, while keeping t constant.
For experimental results see John Martinis' poster.

Measurement undoing

Evolution due to partial measurement is **non-unitary**, therefore impossible to undo it by Hamiltonian dynamics.

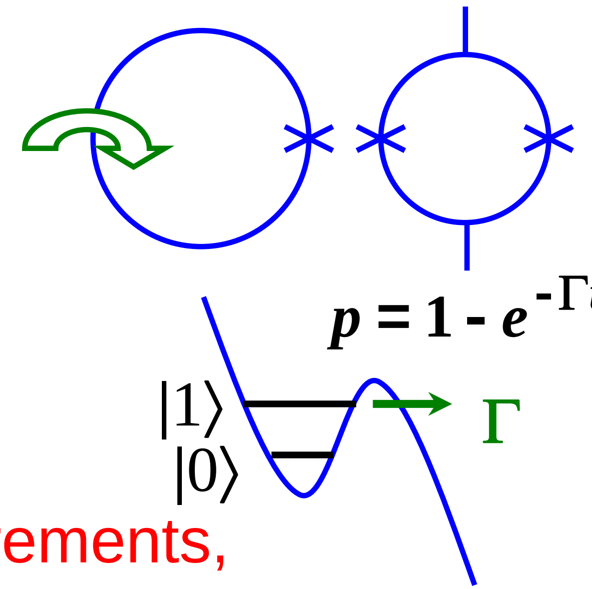
How to undo? One more measurement!



(Figure partially adopted from A. Jordan, A. Korotkov, and M. Büttiker, cond-mat/0510782)

Undoing protocol (quantum eraser)

1) Start with an unknown state
2) Partial measurement of strength *p*
3) π-pulse (exchange |0> ↔ |1>)
4) One more measurement with the **same strength** *p*
5) π-pulse



If no tunneling for both measurements, then initial state is fully restored!

$$\alpha|0\rangle + \beta|1\rangle \to \frac{\alpha|0\rangle + e^{i\phi}\beta\sqrt{1-p}\,|1\rangle}{\text{Norm}} \to$$

$$\frac{e^{i\phi}\alpha\sqrt{1-p}\,|0\rangle + e^{i\phi}\beta\sqrt{1-p}\,|1\rangle}{\text{Norm}} = e^{i\phi}(\alpha|0\rangle + \beta|1\rangle)$$

Probability of successful measurement undoing for phase qubit

Success probability for measurement undoing
(assuming no tunneling during first measurement):

$$s = \frac{e^{-\Gamma t}}{\rho_{00}(0) + e^{-\Gamma t}\rho_{11}(0)} = \frac{1-p}{\rho_{00}(0) + (1-p)\rho_{11}(0)}$$

where ρ(0) is the density matrix of the initial state (either averaged unknown state or an entangled state traced over all other qubits)

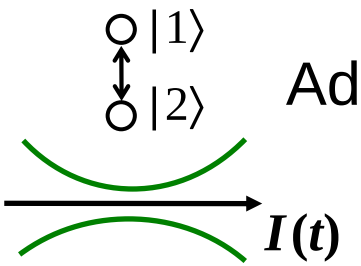
For p increasing to 1, success probability decreases to zero (orthodox collapse), but still exact undoing

Success probability coincides with the upper bound allowed by quantum mechanics $\Rightarrow$ maximally efficient undoing

Undoing partial measurement of a charge qubit

qubit (DQD)

detector (QPC)



Advantage: measurement does not destroy the qubit, just evolution due to measurement

Disadvantages: need QPC (SET is a non-ideal detector), so far nobody demonstrated good measurement

Similar idea of undoing: just continue to measure until acquired information is (hopefully) erased

$$r(t) = 0, \text{ where } r(t) = (\Delta I / S_I)\,[\int_0^t I(t')\,dt' - I_0 t]$$

Success probability:

$$s = \frac{e^{-|r|}}{e^{|r|}\rho_{11}(0) + e^{-|r|}\rho_{22}(0)}$$

(also maximally efficient)

General theory of quantum measurement undoing

Measurement operator M_m: $\quad \rho \rightarrow \dfrac{M_m \rho M_m^\dagger}{\mathrm{Tr}(M_m \rho M_m^\dagger)}$

Undoing measurement operator: $\quad C \times M_m^{-1}$

$$\max(C) = \min_i \sqrt{p_i}\,, \quad p_i = \mathrm{Tr}(M_m^\dagger M_m |i\rangle \langle i|)$$

p_i – probability of the measurement result m for initial state $|i\rangle$

Probability of success:

$$s \le \frac{\min_i p_i}{\Sigma_i p_i \rho_{ii}(0)} = \frac{\min P_m}{P_m(\rho(0))}$$

$P_m(\rho(0))$ – probability of result m for initial state $\rho(0)$,
$\min(P_m)$ – probability of result m minimized over all possible initial states

(similar to Koashi-Ueda, PRL, 1999)

Measurement undoing (quantum eraser) in various solid-state systems

(different from quantum erasers in optics!)

Phase qubit: π-pulse and another measurement is optimal undoing (reaches the upper bound for success probability)

Non-evolving charge qubit: simple waiting strategy is also optimal undoing

General procedure for entangled charge qubits (also optimal):

1) unitary transformation of N qubits
2) null-result measurement of a certain strength by a strongly nonlinear QPC (tunneling only for state $|11..1\rangle$)
3) repeat 2^N times, sequentially transforming the basis vectors of the measurement operator into $|11..1\rangle$

Evolving charge qubit:

1) Bayesian equations to calculate measurement operator
2) unitary operation, measurement by QPC, unitary operation

Measurement undoing for single phase qubit is possible now, experiment with a charge qubit will hopefully be possible soon

Quantum erasers in optics

Quantum eraser proposal by Scully and Drühl, PRA (1982)

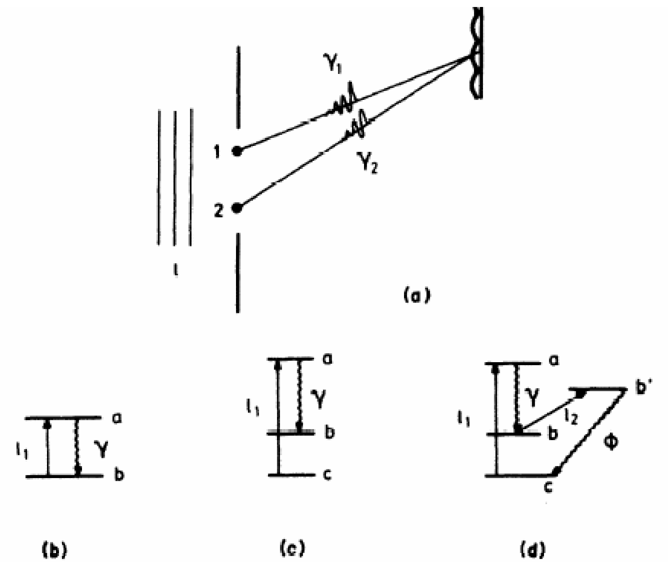


FIG. 1. (a) Figure depicting light impinging from left on atoms at sites 1 and 2. Scattered photons γ_1 and γ_2 produce interference pattern on screen. (b) Two-level atoms excited by laser pulse l_1, and emit γ photons in $a \to b$ transition. (c) Three-level atoms excited by pulse l_1 from $c \to a$ and emit photons in $a \to b$ transition. (d) Four-level system excited by pulse l_1 from $c \to a$ followed by emission of γ photons in $a \to b$ transition. Second pulse l_2 takes atoms from $b \to b'$. Decay from $b' \to c$ results in emission of ϕ photons.

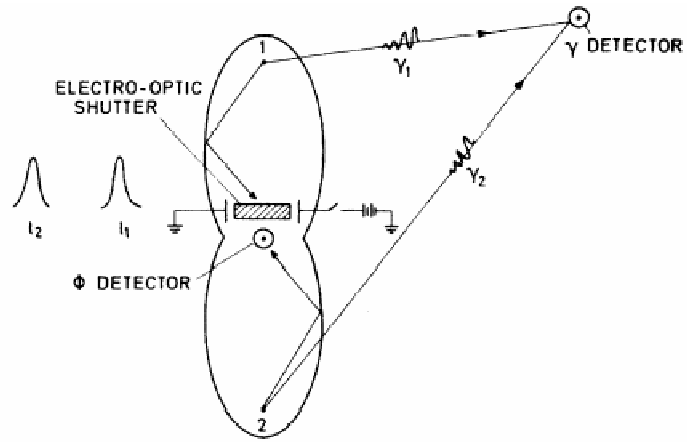


FIG. 2. Laser pulses l_1 and l_2 incident on atoms at sites 1 and 2. Scattered photons γ_1 and γ_2 result from $a \to b$ transition. Decay of atoms from $b' \to c$ results in ϕ photon emission. Elliptical cavities reflect ϕ photons onto common photodetector. Electro-optic shutter transmits ϕ photons only when switch is open. Choice of switch position determines whether we emphasize particle or wave nature of γ photons.

Interference fringes restored for two-detector correlations (since “which-path” information is erased)

Our idea of measurement undoing is quite different: we really extract quantum information and then erase it

Conclusion

- It is possible to fully undo partial quantum measurement. Undoing procedure has finite probability of success and clear experimental indication if undoing has been successful or not. Success probability decreases with the strength of measurement, going to zero in "orthodox" case.

- For phase qubit measurement undoing (quantum eraser) is a simple procedure and is only slightly more complex than recent experiment by N. Katz et al. (Science, 2006). Quantum eraser based on phase qubit is realizable today.

- Measurement undoing for a charge qubit is also simple, and hopefully can be demonstrated soon.

- Proposed measurement undoing for phase and charge qubits is optimal: the success probability coincides with the upper bound allowed by quantum mechanics.