Few recent topics in continuous (Bayesian) quantum measurement of solid-state qubits

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Support:

Outline

- **First solid-state experiment on continuous collapse**

  N. Katz, M. Ansmann, R.C. Bialczak, E. Lucero, R. McDermott, M. Neeley, M. Steffen, E.M. Weig, A.N. Cleland, J.M. Martinis, and A.N. Korotkov, Science (June 06)

- **Leggett-Garg inequalities (Bell inequalities in time) for continuous measurement of a solid-state qubit**

  R. Ruskov, A.N. Korotkov, and A. Mizel, PRL (May 06)
  A.N. Jordan, A.N. Korotkov, and M. Büttiker, PRL (July 06)

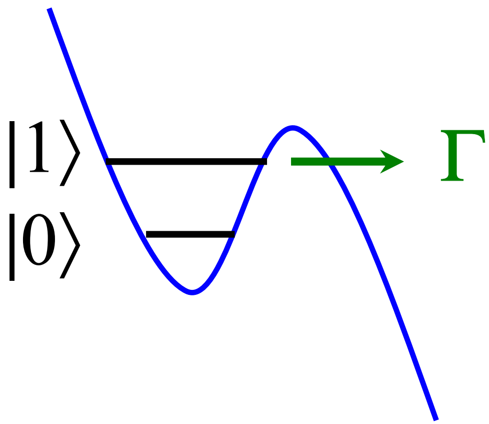
- **Undoing a weak measurement of a qubit**

  ("solid-state quantum eraser", "quantum un-demolition measurement")

  A.N. Jordan and A.N. Korotkov, cond-mat/0606713

Partial collapse of a phase qubit

N. Katz *et al.*, Science-06



How does a coherent state evolve in time before tunneling event?

Qubit "ages" in contrast to a radioactive atom!

Main idea:

$$\psi = \alpha\,|0\rangle + \beta\,|1\rangle \;\rightarrow\; \psi(t) = \begin{cases} |out\rangle, \text{ if tunneled} \\ \dfrac{\alpha\,|0\rangle + \beta\, e^{-\Gamma t/2}\,|1\rangle}{Norm}, \text{ if not tunneled} \end{cases}$$

$$Norm = \sqrt{|\alpha|^2 + |\beta|^2\, e^{-\Gamma t}}$$

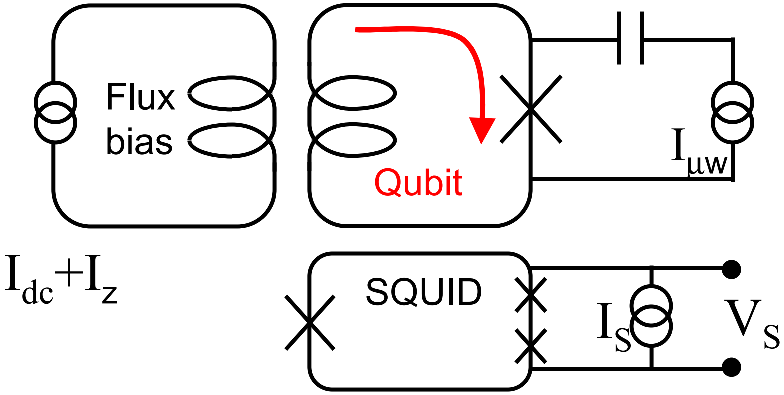
amplitude of state |0> grows without physical interaction

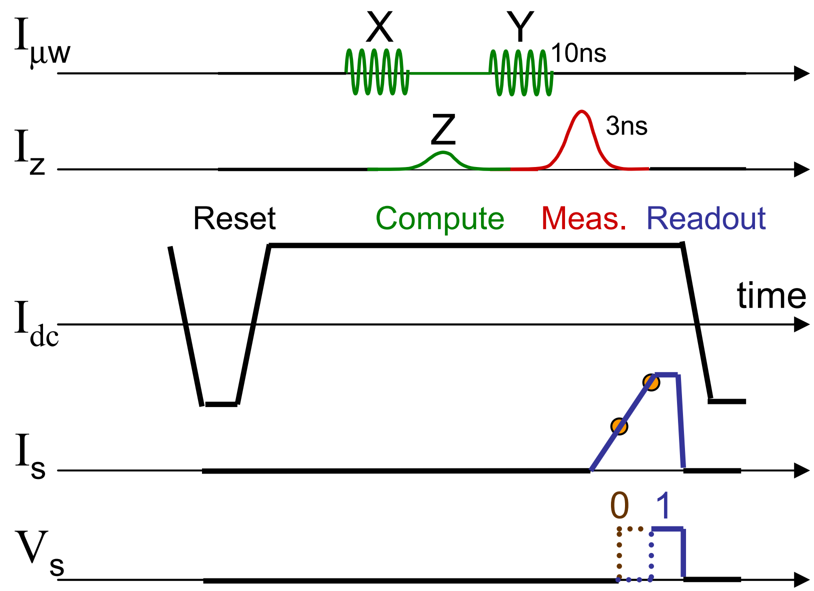
continuous null-result collapse

(similar to optics, Dalibard-Castin-Molmer, PRL-1992)

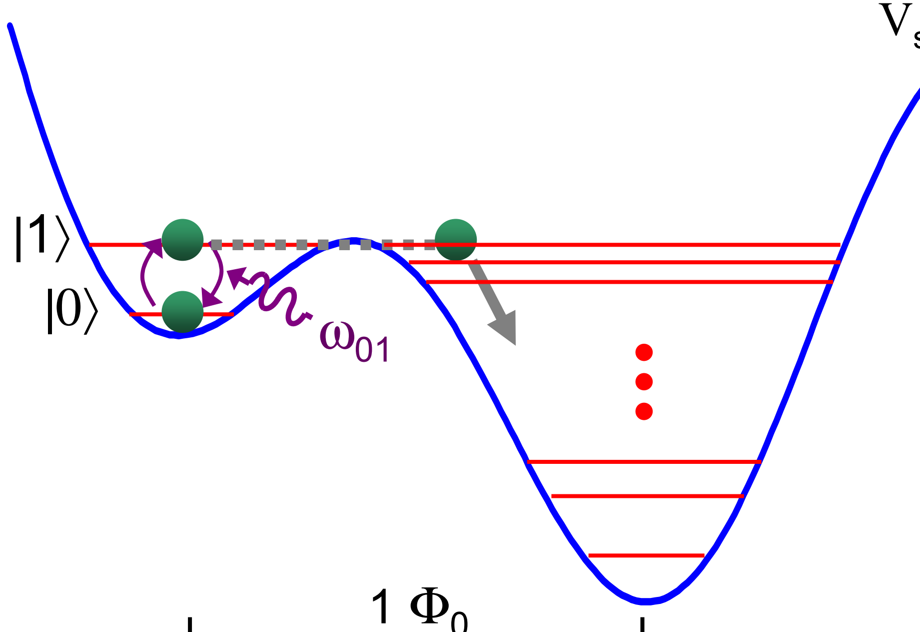
Phase qubit at UCSB

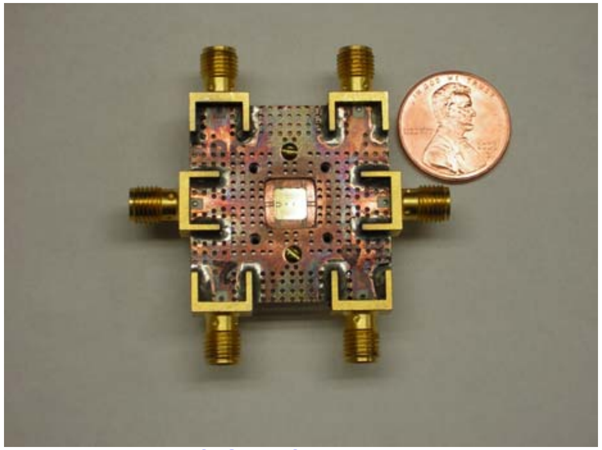
Courtesy of Nadav Katz (UCSB)



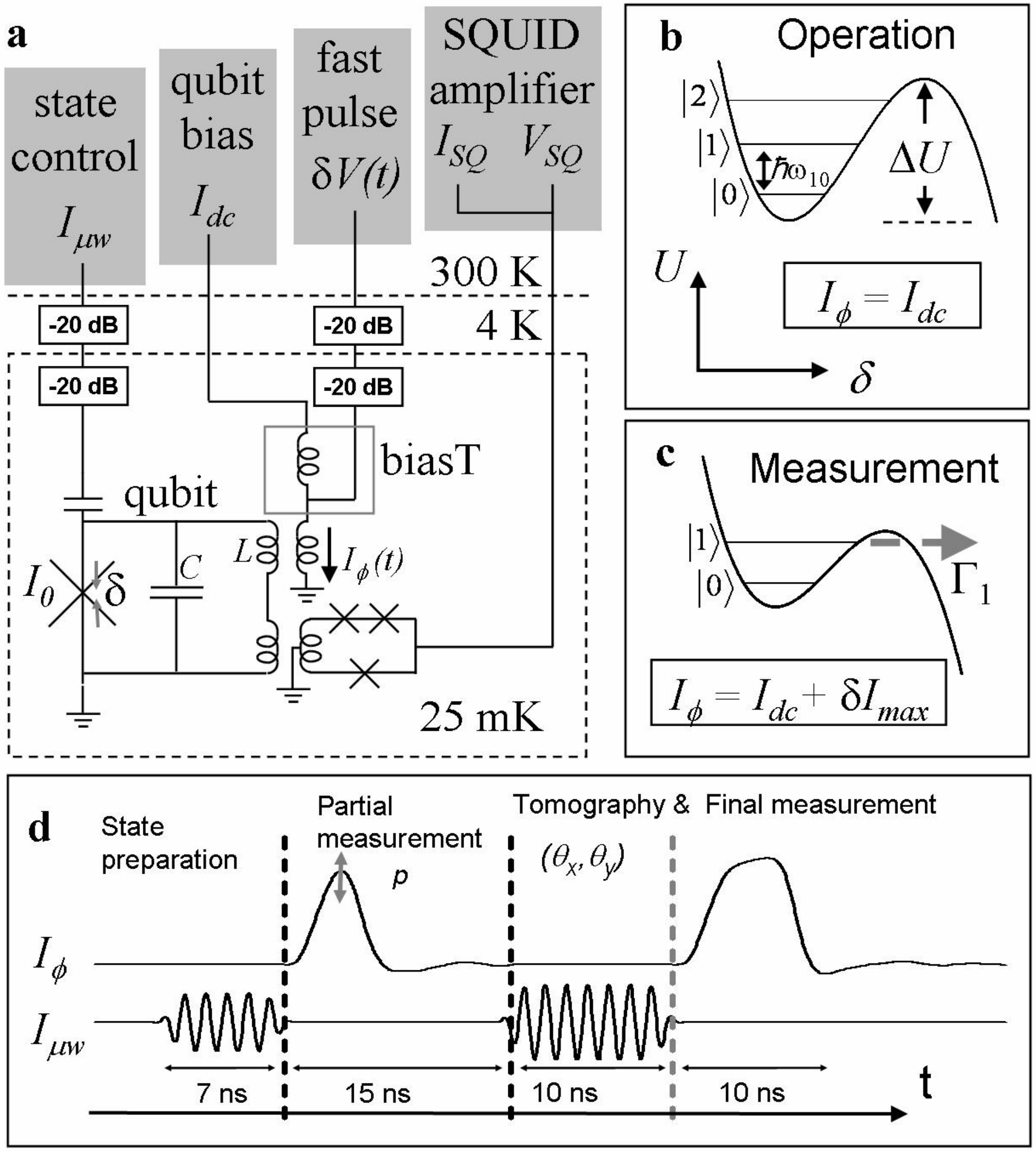


Repeat 1000x
prob. 0,1





Experimental technique for partial collapse



Nadav Katz *et al.*
(John Martinis' group)

Protocol:

1) State preparation by applying microwave pulse (via Rabi oscillations)

2) Partial measurement by lowering barrier for time *t*

3) State tomography (micro-wave + full measurement)

Measurement strength
$p = 1 - \exp(-\Gamma t)$
is actually controlled by Γ, not by t

p=0: no measurement
p=1: orthodox collapse

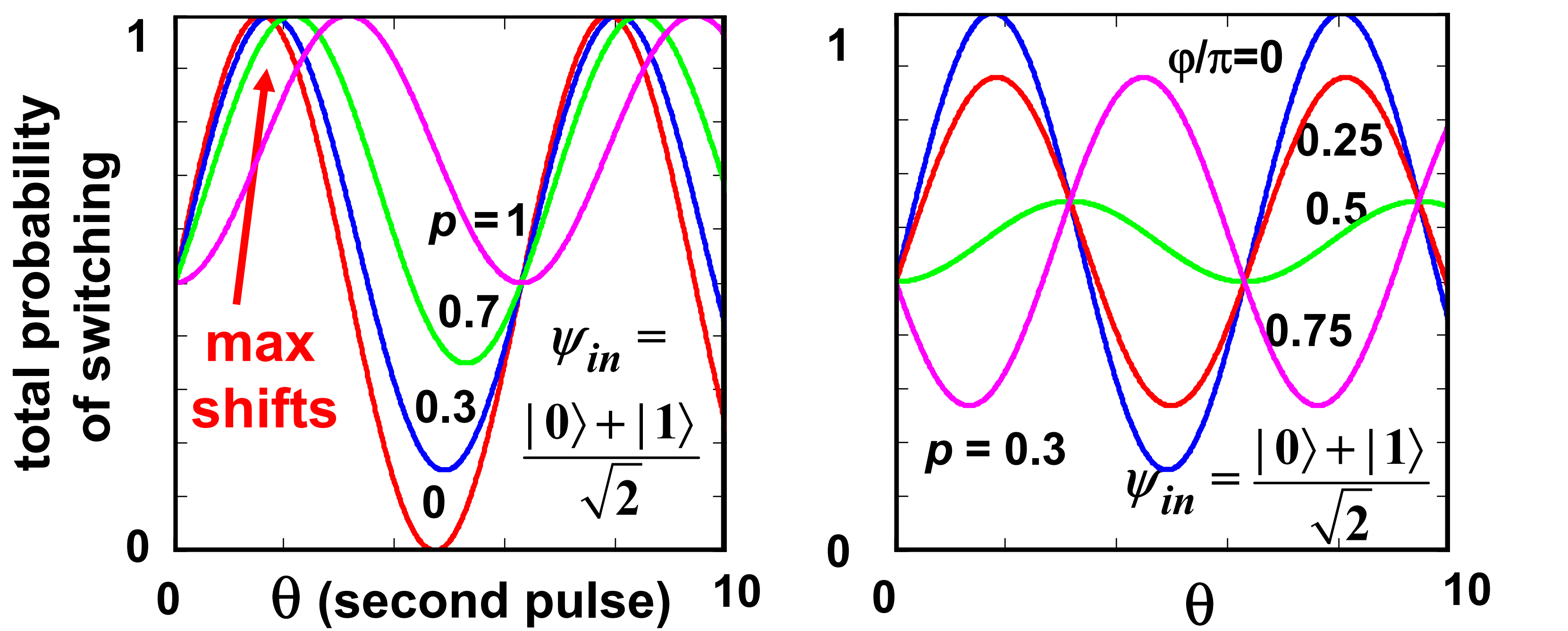
Two more technical details

- Extra phase shift φ (z-rotation) because of small energy change

$$\psi(t) = \frac{\alpha|0\rangle + \beta e^{-i\varphi} e^{-\Gamma t/2}|1\rangle}{\sqrt{|\alpha|^2 + |\beta|^2 e^{-\Gamma t}}}$$

(in rotating frame, so $\exp[-i(E_1 - E_0)t]$ is cancelled)

- Only total switching probability is measurable because SQUID is so far not fast enough (simple solution: subtract background)



Formulae for ideal case

Step 1. Rabi pulse θ_0 prepares state $\cos(\theta_0/2)|0\rangle + \sin(\theta_0/2)|1\rangle$

Step 2. Incomplete measurement with strength $p = 1 - \exp(-\Gamma\tau)$

switches qubit with probability $P_1 = p\sin^2(\theta_0)$. With probability $1 - P_1$
the state becomes $\cos(\theta_m/2)|0\rangle + \sin(\theta_m/2)e^{-i\varphi_m}|1\rangle$, where
φ_m – accumulated phase shift in rotating frame (levels change) and

$$\theta_m = 2\,\mathrm{atan}(\sqrt{1-p}\tan(\theta_0/2))$$

Step 3. Z-rotation φ and Rabi pulse θ.

Step 4. Complete measurement, switching probability P_2.

Total switching probability $P_t = P_1 + P_2$

$$P_t = 1 - \frac{1}{2}[1 - p\sin^2(\frac{\theta_0}{2})][1 + \cos\theta_m\cos\theta - \sin\theta_m\sin\theta\cos(\varphi - \varphi_m)]$$

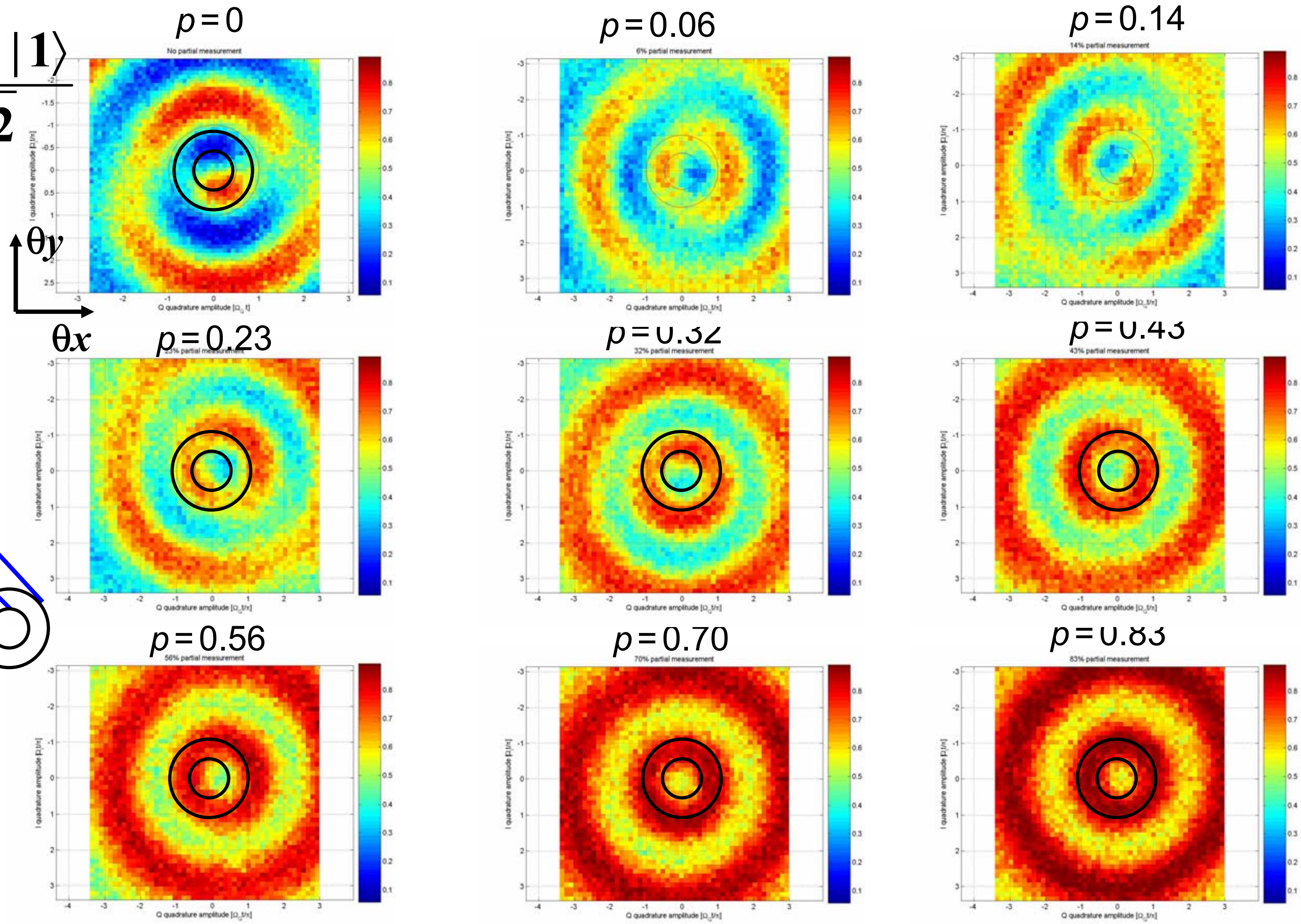
If φ_m is compensated ($\varphi = \varphi_m$) then maximum oscillation amplitude:

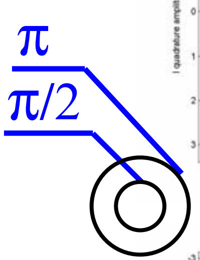
$$P_t = 1 - \frac{1}{2}[1 - p\sin^2(\frac{\theta_0}{2})][1 + \cos(\theta_m + \theta)]$$

Experimental tomography data

Nadav Katz *et al.* (UCSB)

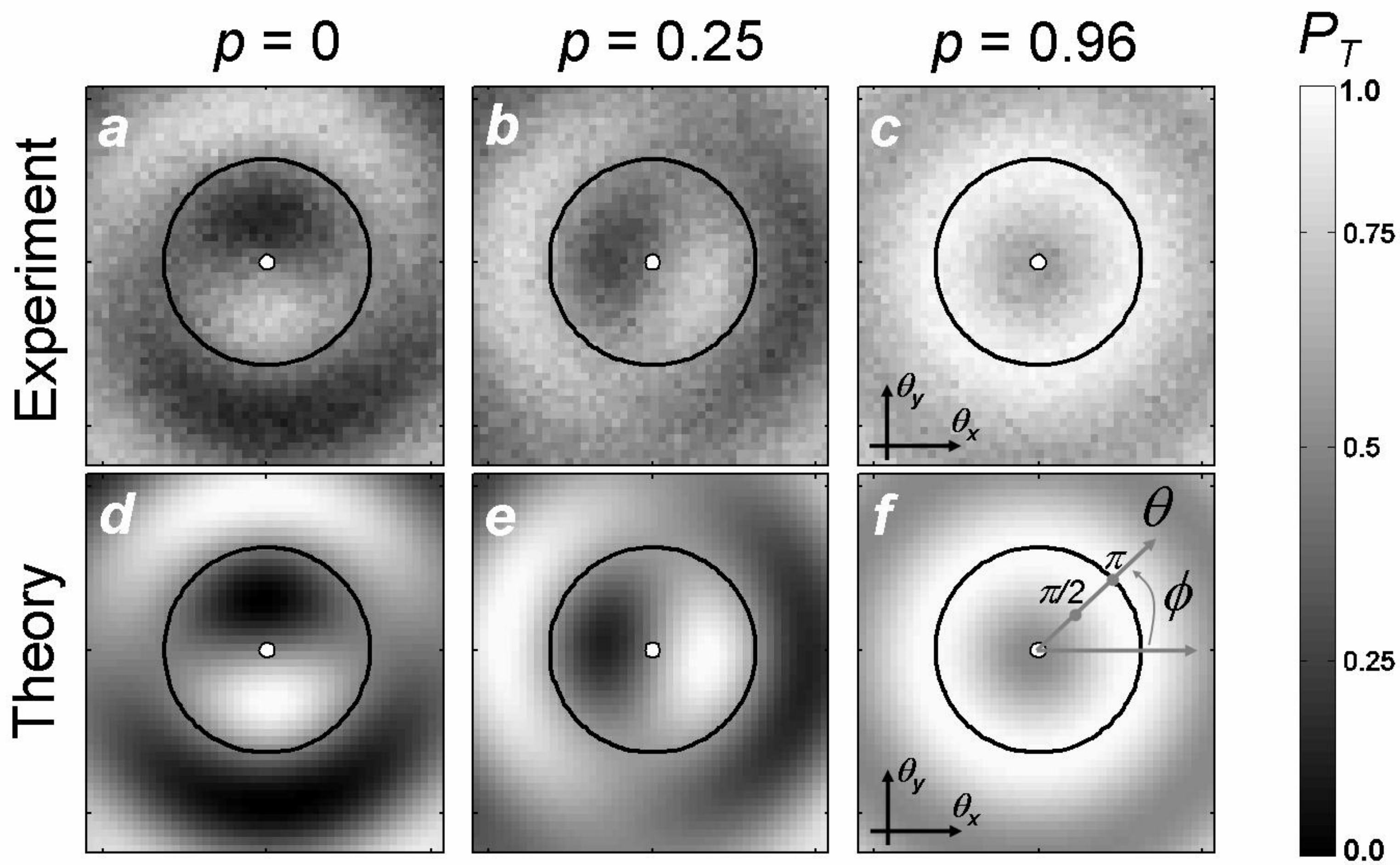
$$\psi_{in} = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$





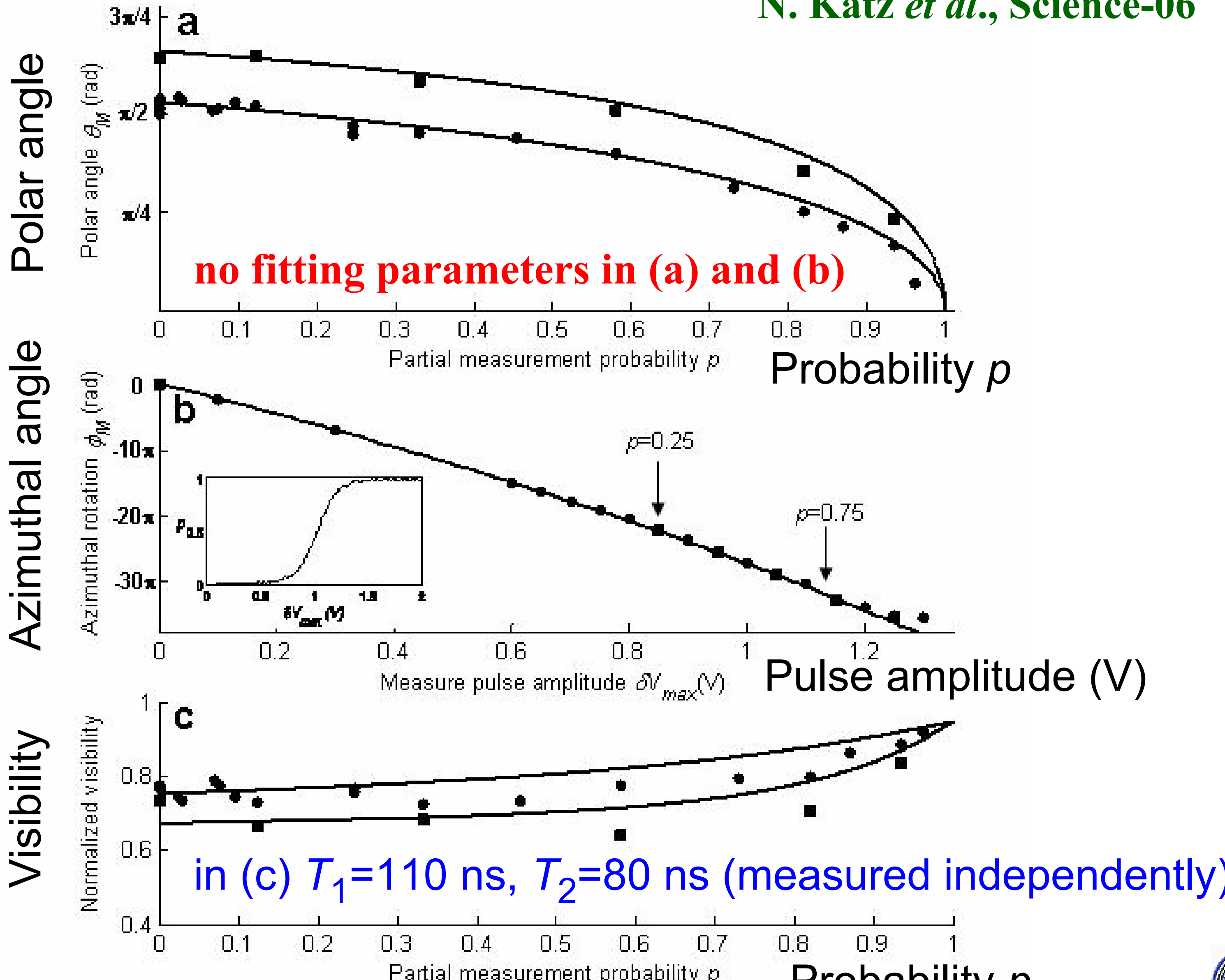
Tomography: experiment vs. theory

N. Katz *et al.*, Science-06

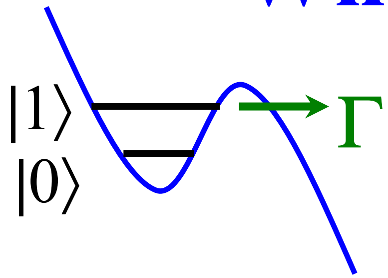


Main results

N. Katz *et al.*, Science-06



What this experiment has shown



N. Katz, M. Ansmann, R.C. Bialczak, E. Lucero, R. McDermott, M. Neeley, M. Steffen, E.M. Weig, A.N. Cleland, J.M. Martinis, and A.N. Korotkov, Science-06

- **In case of no tunneling (null-result measurement) phase qubit evolves**
- **This evolution is well-described by a very simple (Bayesian) theory, without fitting parameters**
- **Phase qubit remains fully coherent in the process of continuous collapse (experimentally ~80% raw data, ~96% after excluding effect of T1 and T2)**

Still a little loophole:

Tunneling and no-tunneling cases are mixed together, so need to subtract tunneling cases instead of selecting only no-tunneling cases; possibly will be solved soon by a faster SQUID

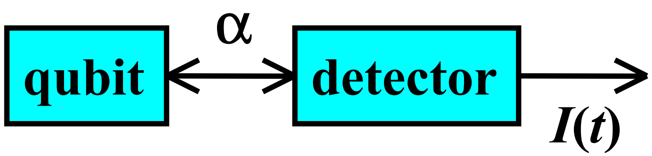
Next topic:

Leggett-Garg inequalities (Bell inequalities in time) for continuous measurement of a solid-state qubit

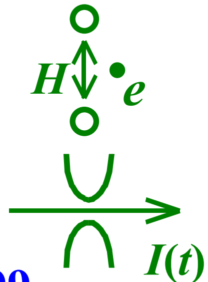
R. Ruskov, A.N. Korotkov, and A. Mizel, PRL (May 06)

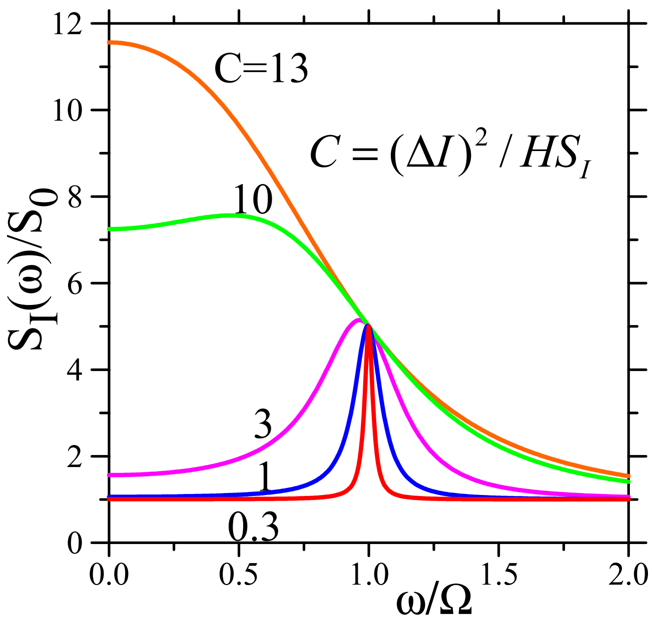
A.N. Jordan, A.N. Korotkov, and M. Büttiker, PRL (July 06)

Measured spectrum of qubit coherent oscillations



What is the spectral density $S_I(\omega)$ of detector current?





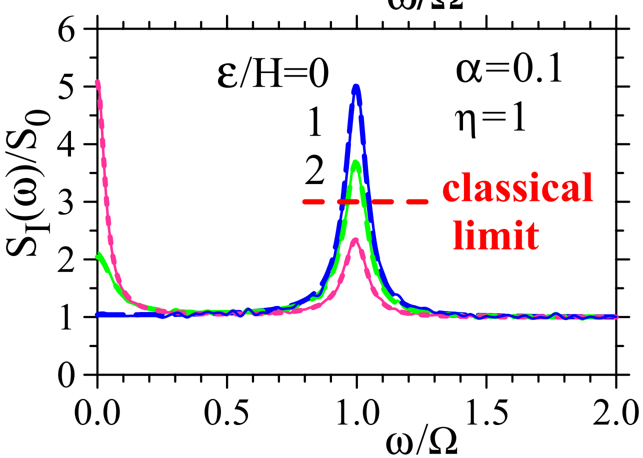
Assume classical output, eV » $\hbar\Omega$

$$\varepsilon = 0, \quad \Gamma = \eta^{-1} (\Delta I)^2 / 4 S_0$$

$$S_I(\omega) = S_0 + \frac{\Omega^2 (\Delta I)^2 \Gamma}{(\omega^2 - \Omega^2)^2 + \Gamma^2 \omega^2}$$

Spectral peak can be seen, but peak-to-pedestal ratio ≤ 4η ≤ 4

(result can be obtained using various methods, not only Bayesian method)



4 = 2 + 2

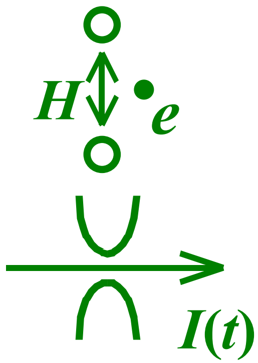
classical **quantum back-action**

A.K., LT'99
Averin-A.K., 2000
A.K., 2000
Averin, 2000
Goan-Milburn, 2001
Makhlin et al., 2001
Balatsky-Martin, 2001
Ruskov-A.K., 2002
Mozyrsky et al., 2002
Balatsky et al., 2002
Bulaevskii et al., 2002
Shnirman et al., 2002
Bulaevskii-Ortiz, 2003
Shnirman et al., 2003

Contrary:
Stace-Barrett, 2004

Interpretation in the Bayesian formalism

(A.K., 2000)



4 = 2 + 2

$$I(t) = I_0 + \frac{\Delta I}{2} z(t) + \xi(t)$$

$\xi(t)$ – white noise (S_0)

$$z(t) = \cos[\Omega t + \varphi(t)]$$

– signal: nondecaying oscillations with slowly diffusing phase $\varphi(t)$

$$\langle z(t)\, z(t+\tau)\rangle = \cos(\Omega t)\exp[-(\Delta I)^2 \tau / 8S_0]/2$$

("classical" contribution, gives factor 2)

$$\langle \xi(t)\, z(t+\tau)\rangle = \Delta I \cos(\Omega t)\exp[-(\Delta I)^2 \tau / 8S_0]/4$$

("quantum" contribution, also gives factor 2)

Quantum back-action: qubit evolves in the same direction as we see it due to noise ("reality follows observation")

However, this is only the Bayesian interpretation. May be some peculiar $z(t)$ can explain the factor of 4 without this strange back-action? What is the ultimate classical limitation (like Bell inequality) for the spectral peak?

Leggett-Garg inequalities (1985)

Assumptions of macrorealism:

1) $z(t)$ is well-defined at all times
2) noninvasive measurability of $z(t)$

(instantaneous "strong" measurement)

$$z(t) = \pm 1, \quad K_{ij} = \langle z_i z_j \rangle$$

$$1 + K_{12} + K_{23} + K_{13} \geq 0$$

$$K_{12} + K_{23} + K_{34} - K_{14} \leq 2$$

Similar inequality for continuous measurement

Ruskov et al., PRL-2006

Assumptions:

1) $I(t) = I_0 + \frac{\Delta I}{2} z(t) + \xi(t)$

2) $|z(t)| \leq 1$

3) $\langle \xi(t) z(t+\tau) \rangle = 0$

$$K_I(\tau) = \langle I(t) I(t+\tau) \rangle - I_0^2$$

Then classical bound

$$K_I(\tau_1) + K_I(\tau_2) - K_I(\tau_1 + \tau_2) \leq \frac{\Delta I^2}{4}$$

Quantum result (Bayesian, etc.) for

$\tau_1 = \tau_2 = \tau = \pi / 3\Omega$

$$K_I(\tau) + K_I(\tau) - K_I(2\tau) = \frac{3}{2} \frac{\Delta I^2}{4}$$

Violation of the classical bound by factor 3/2

Inequalities for measured spectrum

Ruskov et al., PRL-2006

Again assume: $I(t) = I_0 + (\Delta I/2)\, z(t) + \xi(t),\ |z(t)| \le 1,\ \langle \xi(t)\, z(t+\tau)\rangle = 0$

Then

$$\int_{-\infty}^{\infty} [S_I(\Omega+\omega) - S_0]\, f(\omega) \frac{d\omega}{2\pi} < \frac{8}{\pi^2} \frac{\Delta I^2}{4} [1 + o(\Delta/\Omega)]$$

where $f(\omega) = \exp(-\omega^2/2\Delta^2)$; $f(\omega) \approx 1$ if $W \ll \Delta \ll \Omega$ (W - peak width)

So, integral under the spectral peak is limited (crudely) by $\frac{8}{\pi^2}\frac{\Delta I^2}{4}$

Quantum result for this integral: $\Delta I^2/4$

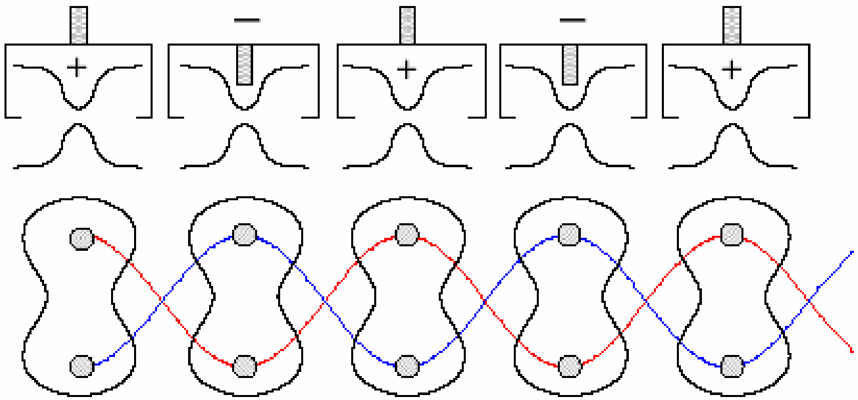
Violation of the classical bound by factor $\pi^2/8=1.23$

Assuming single Lorentzian peak, the classical inequality can be made stronger: the integral under the peak is limited (crudely) by $(2/3)(\Delta I^2/4)$. Then **violation is by factor 3/2** (as for correlation function).

Notice that the inequalities are for the peak area, not for the peak height (these quantities are obviously related)

Inequality for kicked (QND) measurement

Jordan-Korotkov-Büttiker, PRL-06



Pulsed QPC voltage: QND measurement

Alternating sign of pulses: quantum pump (average current depends on phase of the qubit Rabi oscillations)

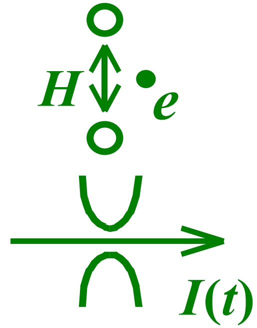
Leggett-Garg inequality: $B \leq 1, \;\; B = \langle I_1 I_2 \rangle + \langle I_2 I_3 \rangle - \langle I_1 I_3 \rangle$

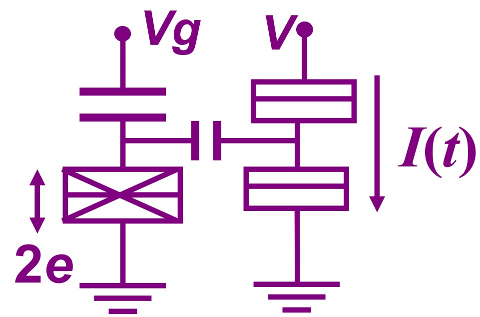
(currents I_i are normalized to be between -1 and +1)

For phase shifts $\phi_1 = \phi_2 = \pi/3$ between pulses and weak measurement (strength << 1), the quantum (Bayesian) result is **B = 3/2**

Violation by factor 3/2

What violation of the inequalities shows





If we think that $I(t) = I_0 + (\Delta I/2)\, z(t) + \xi(t)$

and qubit signal is bounded $|z(t)| \leq 1$ **,**

then we have to accept that the <u>output</u> detector noise $\xi(t)$ **necessarily (even in principle) causes a change of** $z(t)$ **evolution (quantum back-action)**

Remark 1: violation of the inequalities holds for non-ideal detectors (makes it easier experimentally: can use SET)

Remark 2: assumption of the classical back-action affecting qubit parameters (ε, H) cannot explain the quantum result

Next topic:

Undoing a weak measurement of a qubit

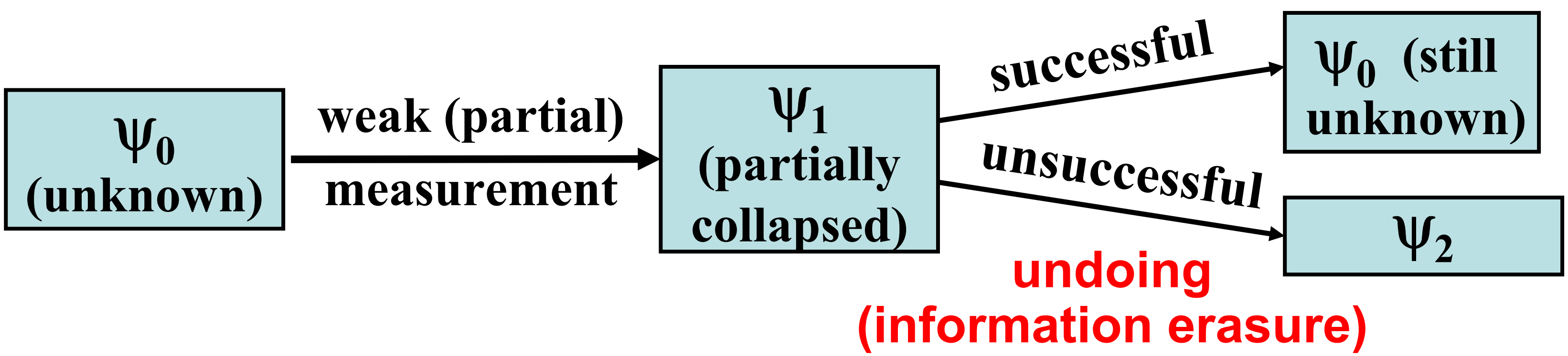
Jordan and Korotkov, cond-mat/0606713

It is impossible to undo “orthodox” quantum measurement (for an unknown initial state)

Is it possible to undo partial quantum measurement?
(To restore a “precious” qubit accidentally measured)

Yes! (but with a finite probability)

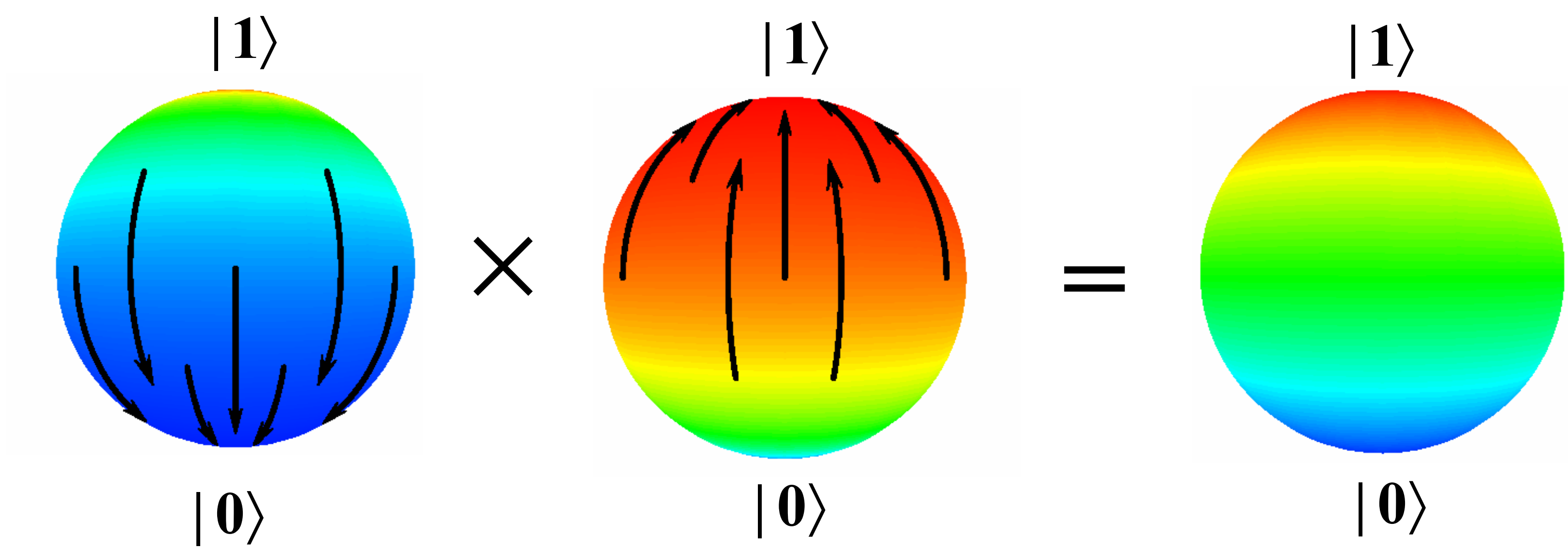
If undoing is successful, an unknown state is **fully** restored



Measurement undoing

Evolution due to partial (weak, continuous, etc.) measurement is **non-unitary** (though coherent if detector is good!), therefore it is impossible to undo it by Hamiltonian dynamics.

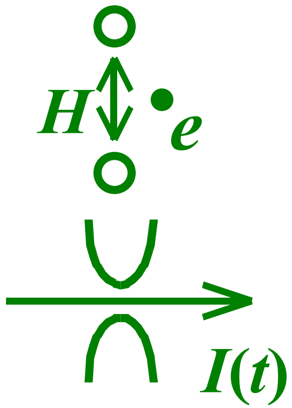
How to undo? One more measurement!



(similar to Koashi-Ueda, PRL-1999)

(Figure partially adopted from A. Jordan, A. Korotkov, and M. Büttiker, PRL-2006

First example: DQD qubit with no tunneling, measured by QPC



$$\hat{H}_{QB} = (\varepsilon/2)(c_1^\dagger c_1 - c_2^\dagger c_2) + H(c_1^\dagger c_2 + c_2^\dagger c_1)$$

Assume “frozen” qubit: $\varepsilon = H = 0$

Bayesian evolution due to measurement (Korotkov-1998)

1) Diagonal matrix elements of the density matrix evolve according to the classical Bayes rule

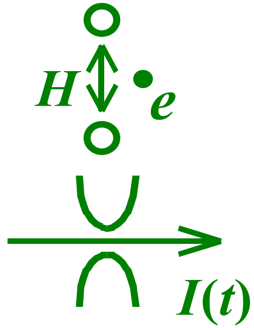
2) Non-diagonal matrix elements evolve so that the degree of purity $\rho_{ij}/[\rho_{ii}\,\rho_{jj}]^{1/2}$ is conserved

$$\rho_{11}(\tau) = \frac{\rho_{11}(0)\exp[-(\bar{I}-I_1)^2/2D]}{\rho_{11}(0)\exp[-(\bar{I}-I_1)^2/2D] + \rho_{22}(0)\exp[-(\bar{I}-I_2)^2/2D]}$$

$$\frac{\rho_{12}(\tau)}{[\rho_{12}(\tau)\,\rho_{22}(\tau)]^{1/2}} = \frac{\rho_{12}(0)}{[\rho_{12}(0)\,\rho_{22}(0)]^{1/2}}, \qquad \rho_{22}(\tau) = 1 - \rho_{11}(\tau)$$

where $\bar{I} = \frac{1}{\tau}\int_0^\tau I(t)\,dt$

Graphical representation of the evolution

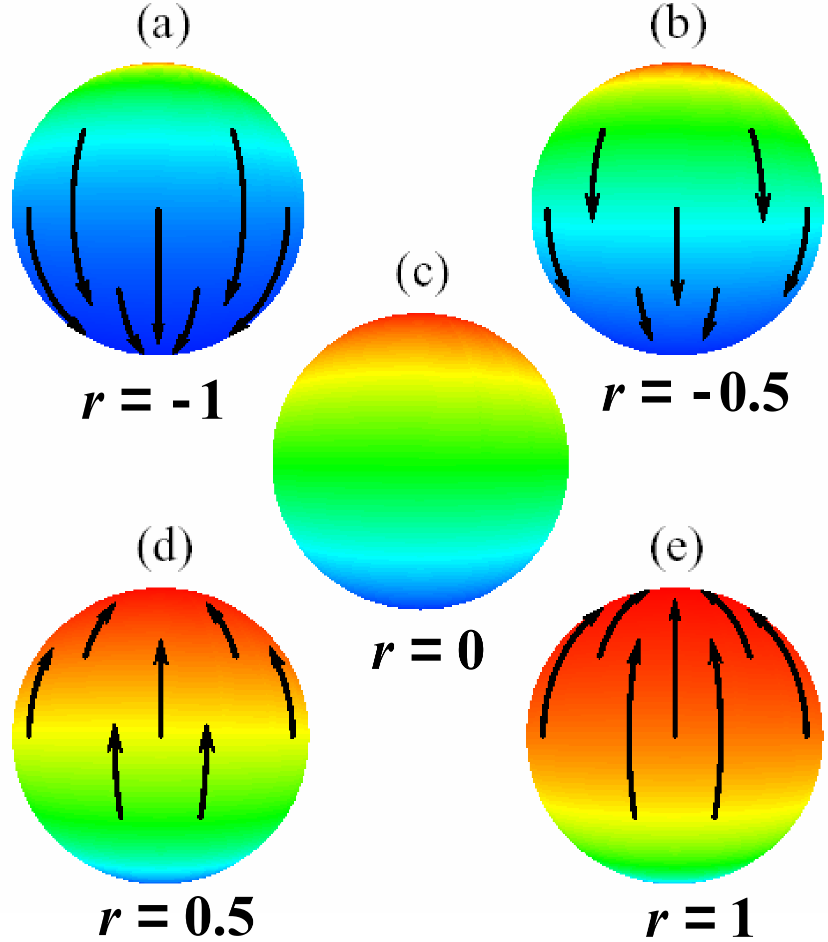


$$\frac{\rho_{11}(t)}{\rho_{22}(t)} = \frac{\rho_{11}(0)}{\rho_{22}(0)} \exp[2r(t)]$$

$$\frac{\rho_{12}(t)}{\sqrt{\rho_{11}(t)\rho_{22}(t)}} = \text{const}$$

where measurement result $r(t)$ is

$$r(t) = \frac{\Delta I}{S_I}\left[\int_0^t I(t')\,dt' - I_0 t\right]$$

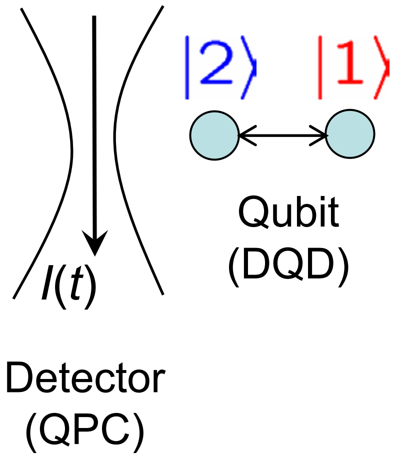


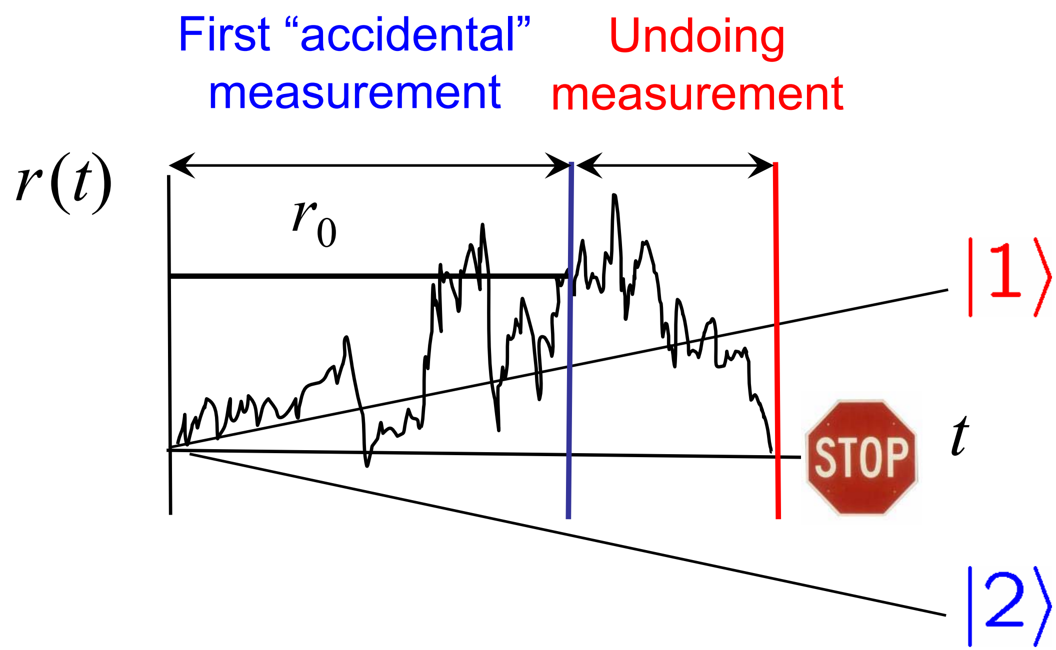
Jordan-Korotkov-Büttiker, PRL-06

If $r = 0$, then no information and no evolution!

Measurement undoing for DQD-QPC system

Jordan and Korotkov, 2006





Simple strategy: continue measuring until $r(t)$ becomes zero!

Then any unknown initial state is fully restored.

(same for an entangled qubit)

It may happen though that $r = 0$ never happens;
then undoing procedure is unsuccessful.

Probability of success

Trick: since non-diagonal matrix elements are not directly involved, we can analyze classical probabilities (as if qubit is in some certain, but unknown state); then simple diffusion with drift

Results:

Probability of successful undoing

$$P_S = \frac{e^{-|r_0|}}{e^{|r_0|}\rho_{11}(0) + e^{-|r_0|}\rho_{22}(0)}$$

where r_0 is the result of the measurement to be undone, and $\rho(0)$ is our knowledge about an unknown initial state; in case of an entangled qubit $\rho(0)$ is traced over other qubits

Average time to wait $T_{\text{undo}} = T_m\,|r_0|$ where $T_m = 2S_I/(\Delta I)^2$ ("measurement time")

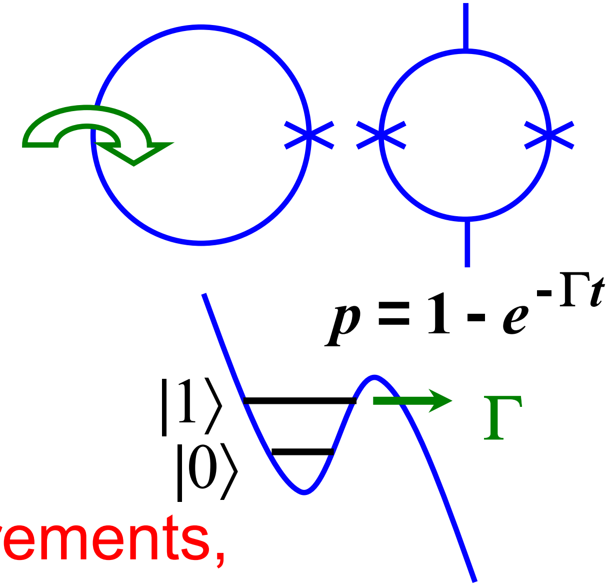
Averaged probability of success (over result r_0)

$$P_{\text{av}} = 1 - \text{erf}[\sqrt{t/2T_m}\,]$$

(does not depend on initial state!)

Second example: Undoing partial measurement of a phase qubit

1) Start with an unknown state
2) Partial measurement of strength *p*
3) π-pulse (exchange |0> ↔ |1>)
4) One more measurement with the **same strength *p***
5) π-pulse



If no tunneling for both measurements, then initial state is fully restored!

$$\alpha|0\rangle + \beta|1\rangle \rightarrow \frac{\alpha|0\rangle + e^{i\phi}\beta\sqrt{1-p}\,|1\rangle}{\text{Norm}} \rightarrow$$

$$\frac{e^{i\phi}\alpha\sqrt{1-p}\,|0\rangle + e^{i\phi}\beta\sqrt{1-p}\,|1\rangle}{\text{Norm}} = e^{i\phi}(\alpha|0\rangle + \beta|1\rangle)$$

Probability of successful measurement undoing for phase qubit

Success probability if no tunneling during first measurement:

$$P_S = \frac{e^{-\Gamma t}}{\rho_{00}(0) + e^{-\Gamma t}\rho_{11}(0)} = \frac{1-p}{\rho_{00}(0) + (1-p)\rho_{11}(0)}$$

where ρ(0) is the density matrix of the initial state (either averaged unknown state or an entangled state traced over all other qubits)

Total (averaged) success probability: $P_{av} = 1 - p$

For measurement strength *p* increasing to 1, success probability decreases to zero (orthodox collapse), but still exact undoing

Such an experiment is only slightly more difficult than recent experiment on partial collapse (N. Katz et al., 2006).

Can be realized experimentally pretty soon!!!

General theory of quantum measurement undoing

Measurement operator M_r: $\rho \to \dfrac{M_r \rho M_r^\dagger}{\mathrm{Tr}(M_r \rho M_r^\dagger)}$ (POVM formalism)

Undoing measurement operator: $C \times M_r^{-1}$ (to satisfy completeness, eigenvalues cannot be >1)

$$\max(C) = \min_i \sqrt{p_i}\,, \quad p_i = \mathrm{Tr}(M_r^\dagger M_r \,|i\rangle\langle i|)$$

p_i – probability of the measurement result r for initial state $|i\rangle$

Probability of success: $P_S \le \dfrac{\min_i p_i}{\Sigma_i p_i \rho_{ii}(0)} = \dfrac{\min P_r}{P_r(\rho(0))}$

$P_r(\rho(0))$ – probability of result r for initial state $\rho(0)$,
$\min(P_r)$ – probability of result r minimized over all possible initial states

Averaged (over r) probability of success: $P_{av} \le \sum_r \min P_r$

(similar to Koashi-Ueda, PRL, 1999)

Comparison of the general bound for undoing success with examples

General bound: $$P_S \le \frac{\min P_r}{P_r(\rho(0))}$$

First example (DQD+QPC) $$P_S \le \frac{\min(p_1, p_2)}{p_1\rho_{11}(0) + p_2\rho_{22}(0)}$$

where $p_i = (\pi S_I / t)^{-1/2} \exp[-(\bar{I} - I_i)^2 t / S_I]\, d\bar{I}$

Coincides with the pervious result, so the upper bound is reached, **therefore undoing strategy is optimal**

Second example (phase qubit) Probabilities of no-tunneling are 1 and exp(-Γt)=1-p

$$P_S \le \frac{1-p}{\rho_{00}(0) + (1-p)\rho_{11}(0)}$$

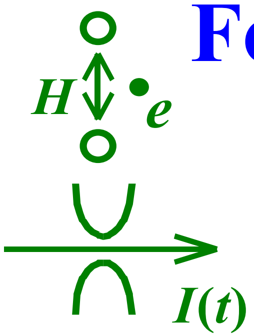
Again same as before, so measurement **undoing for phase qubit is also optimal**

Third example: General undoing procedure for entangled charge qubits

1) unitary transformation of N qubits
2) null-result measurement of a certain strength by a strongly nonlinear QPC (tunneling only for state $|11..1\rangle$)
3) repeat 2^N times, sequentially transforming the basis vectors of the measurement operator into $|11..1\rangle$

(also reaches the upper bound for success probability)

Fourth example: Evolving charge qubit



$$\hat{H}_{QB} = (\varepsilon/2)(c_1^\dagger c_1 - c_2^\dagger c_2) + H(c_1^\dagger c_2 + c_2^\dagger c_1)$$

(now non-zero *H* and ε, qubit evolves during measurement)

1) Bayesian equations to calculate measurement operator
2) unitary operation, measurement by QPC, unitary operation

Quantum erasers in optics

Quantum eraser proposal by Scully and Drühl, PRA (1982)

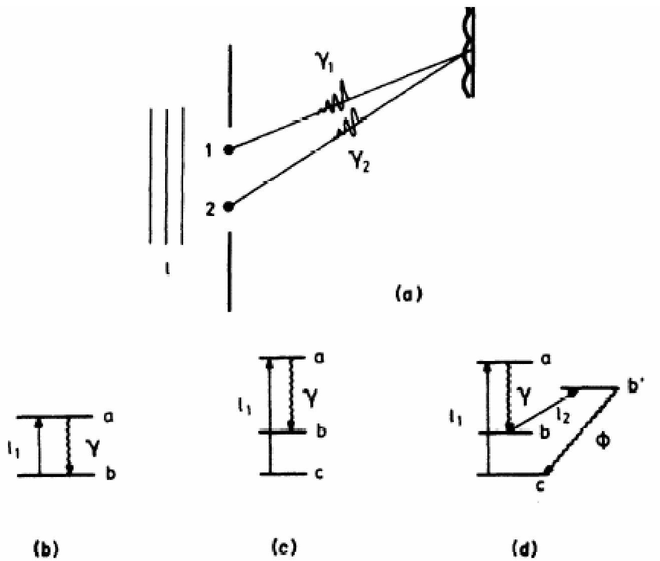


FIG. 1. (a) Figure depicting light impinging from left on atoms at sites 1 and 2. Scattered photons γ_1 and γ_2 produce interference pattern on screen. (b) Two-level atoms excited by laser pulse l_1, and emit γ photons in $a \rightarrow b$ transition. (c) Three-level atoms excited by pulse l_1 from $c \rightarrow a$ and emit photons in $a \rightarrow b$ transition. (d) Four-level system excited by pulse l_1 from $c \rightarrow a$ followed by emission of γ photons in $a \rightarrow b$ transition. Second pulse l_2 takes atoms from $b \rightarrow b'$. Decay from $b' \rightarrow c$ results in emission of ϕ photons.

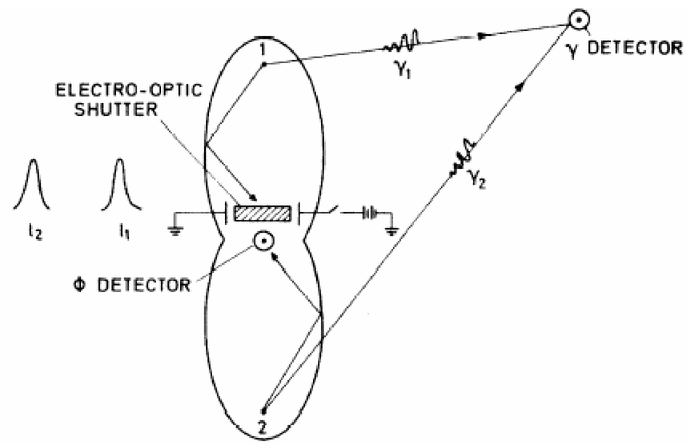


FIG. 2. Laser pulses l_1 and l_2 incident on atoms at sites 1 and 2. Scattered photons γ_1 and γ_2 result from $a \rightarrow b$ transition. Decay of atoms from $b' \rightarrow c$ results in ϕ photon emission. Elliptical cavities reflect ϕ photons onto common photodetector. Electro-optic shutter transmits ϕ photons only when switch is open. Choice of switch position determines whether we emphasize particle or wave nature of γ photons.

Interference fringes restored for two-detector correlations (since "which-path" information is erased)

Our idea of measurement undoing is quite different: we really extract quantum information and then erase it

Summary for measurement undoing

Jordan-Korotkov, 2006

- **Partial (incomplete, weak, etc.) quantum measurement can be undone, though with a finite probability P_s, decreasing with increasing strength of the measurement (P_s=0 for orthodox case)**

- **Though somewhat similar to the quantum eraser, undoing idea is actually quite different;**
  Quantum Un-Demolition measurement (suggested by J. Dowling)

- **Measurement undoing for single phase qubit is realizable now, experiment with a charge qubit will hopefully be possible soon (difficulty to use SET: need an ideal quantum detector)**

In conclusion

- **Optics (AMO) no longer holds monopoly on "ideologically" nontrivial experiments on quantum measurement (collapse); first solid-state experiment has been realized**

- **Phase qubit happened to be a good system for experiments on collapse, even though theoretically this is not a good system ("half-destructive" measurement)**

- **Much more interesting games are possible for charge qubits; unfortunately, no experiments so far; it is easier to start with experiments, which do not require an ideal detector (then OK to use SET): Leggett-Garg, simple quantum feedback, etc.**