Measurement theory for phase qubits

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The team: 1) Qin Zhang, *graduate student*
2) Dr. Abraham Kofman, *researcher* (started 06/05)
3) Alexander Korotkov, *professor*

Milestones for Years 1 and 2:

- Develop a quantum theory for the classical measurement cross-coupling and use it to calculate the energy transfer from one phase Qbit to another.
- Calculate the theoretical fidelity of single phase-Qbit one-shot measurements
- Calculate the theoretical measurement fidelity for coupled phase Qbits
- Calculate the degree of quantum measurement back action in a phase Qbit

Progress: All Done

Sponsored publications during first 2 years of the project

Published: **11 journal papers and 2 proceedings**

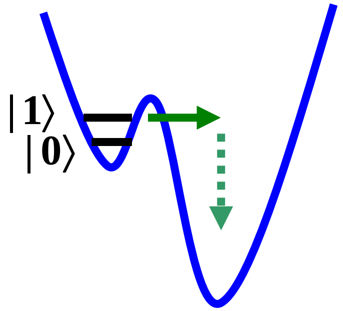
Submitted: **3 journal papers**

1. A.N. Korotkov, Phys. Rev. B 71, 201305(R) (2005)
2. R. Ruskov, K. Schwab, and A.N. Korotkov, IEEE Trans. Nanotech. 4, pp. 132-140 (2005)
3. R. Ruskov, K. Schwab, and A.N. Korotkov, Phys. Rev. B 71, 235407, pp. 1-19 (2005)
4. D.V. Averin and A.N. Korotkov, Phys. Rev. Lett. 94, 069701 (2005)
5. A.N. Korotkov, Microelectronics Journal 36, 253-255 (2005)
6. A.N. Korotkov, Proceedings of SPIE, v. 5846, pp. 46-56 (2005)
7. Q. Zhang, R. Ruskov, and A.N. Korotkov, Phys. Rev. B 72, 245322, pp. 1-11 (2005)
8. R. Ruskov, A.N. Korotkov, and A. Mizel, Phys. Rev. B 73, 085317, pp. 1-17 (2006)
9. R. Ruskov, A.N. Korotkov, and A. Mizel, Phys. Rev. Lett. 96, 200404 (2006)
10. N. Katz, M. Ansmann, R.C. Bialczak, E. Lucero, R. McDermott, M. Neeley, M. Steffen, E.M. Weig, A.N. Cleland, J.M. Martinis, A.N. Korotkov, Science 312, pp. 1498-1500 (2006)
11. A.N. Jordan, A.N. Korotkov, and M. Buttiker, Phys. Rev. Lett. 97, 026805 (2006)
12. A.N. Jordan and A.N. Korotkov, Phys. Rev. B 74, 085307, pp. 1-12 (2006)
13. Q. Zhang, R. Ruskov, A.N. Korotkov, J.Phys.: Conf. Series 38, pp. 163-166 (2006)

1. A.G. Kofman, Q. Zhang, J.M. Martinis, and A. N. Korotkov, cond-mat/0606078, subm. PRB
2. A. N. Korotkov and A. N. Jordan, cond-mat/0606713, subm. PRL
3. Q. Zhang, A.G. Kofman, J.M. Martinis, and A.N. Korotkov, cond-mat/0607385, subm. PRB

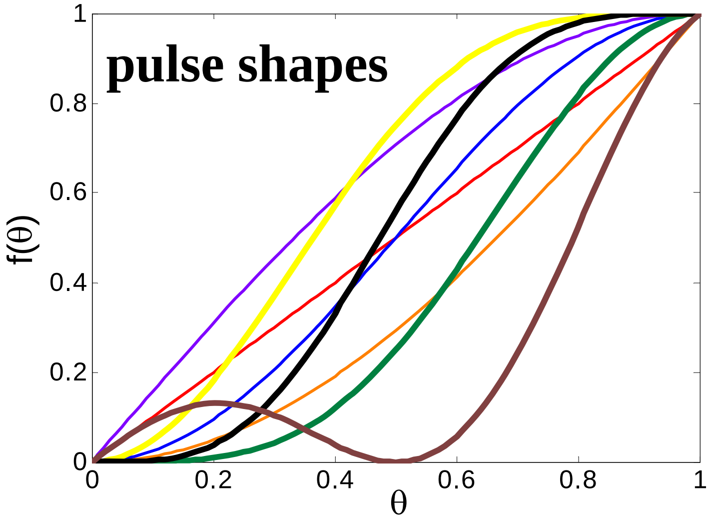
Milestone: theoretical one-qubit measurement fidelity

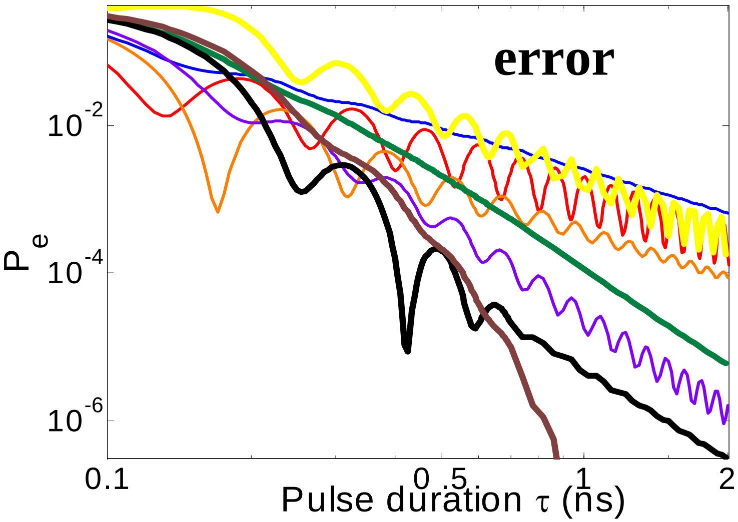
- Calculate the theoretical fidelity of single phase-Qbit one-shot measurements



Studied three error mechanisms: 1) non-adiabatic error, 2) incomplete level discrimination, 3) left well repopulation

non-adiabatic error





very small errors for experimental pulse duration (~2 ns)

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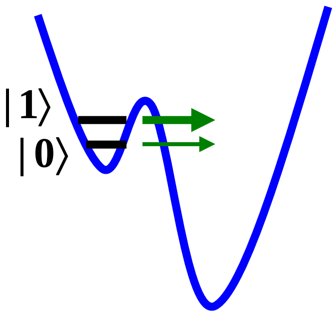
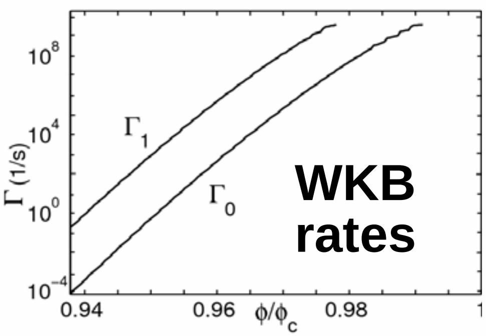
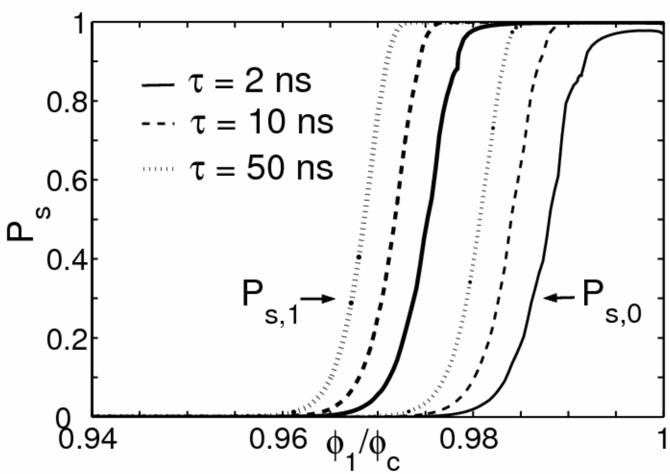
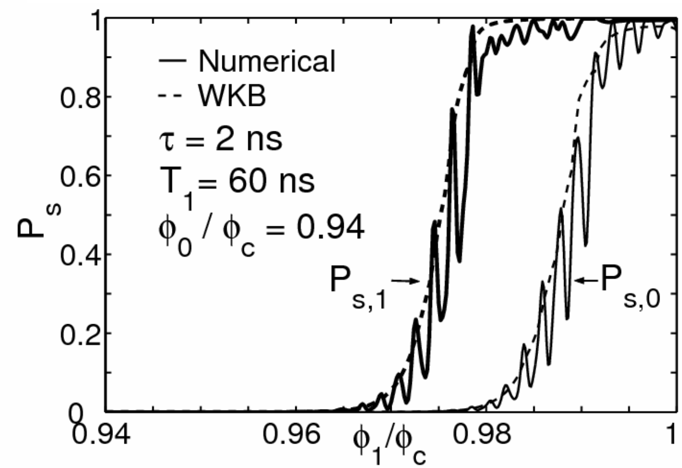
Error due to incomplete level discrimination





Switching probability P_s vs. flux ("S-curves")

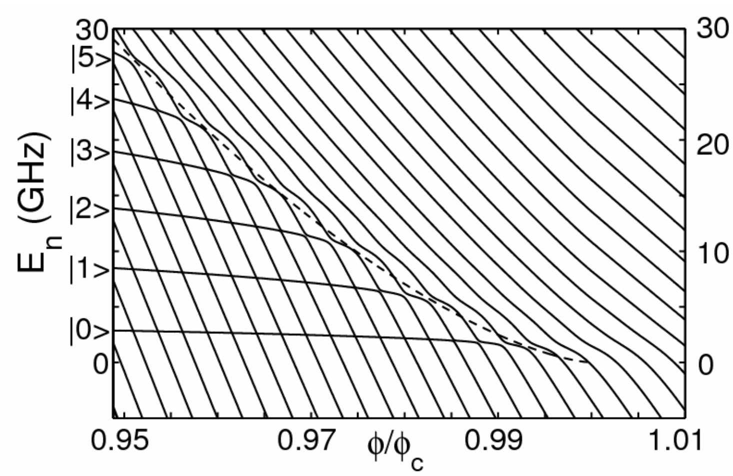


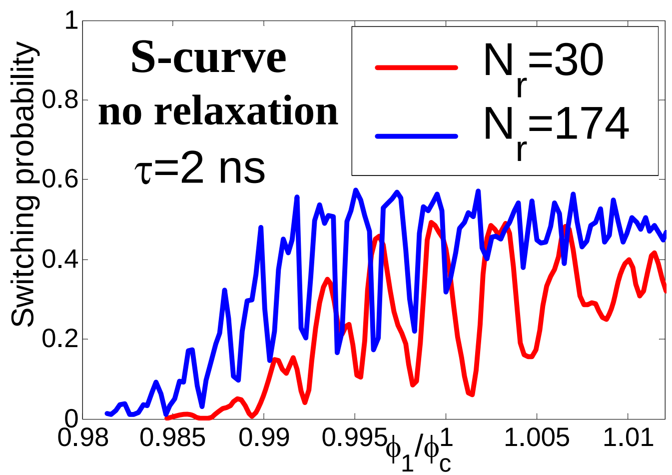


- error ~1% at the optimal point
- error decreases for longer pulses
- error well below 1% requires qubit redesign
  (good news: exponential sensitivity on parameters)

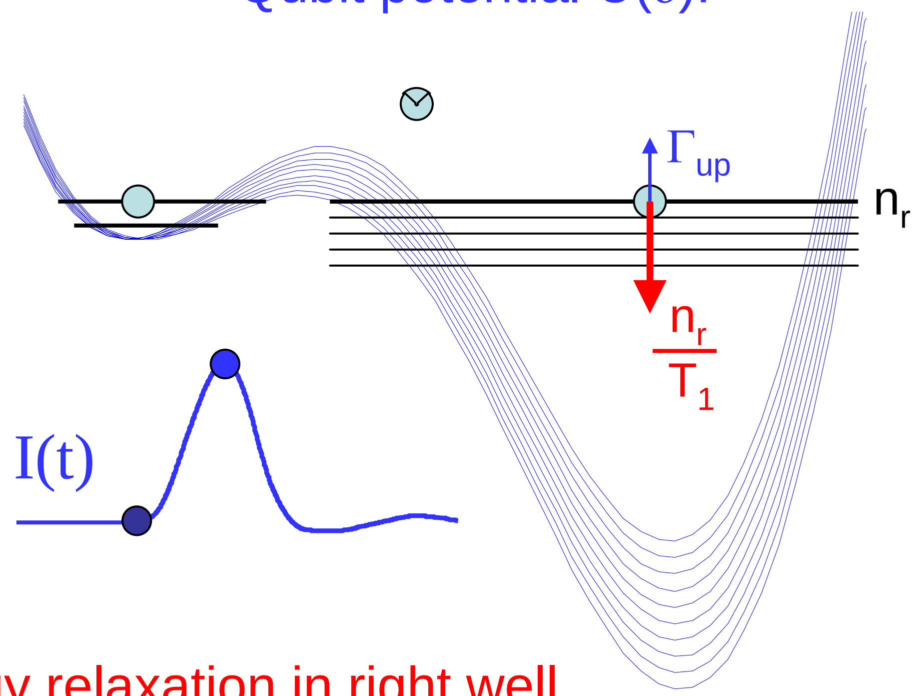
Error due to left well repopulation

Fully quantum simulation





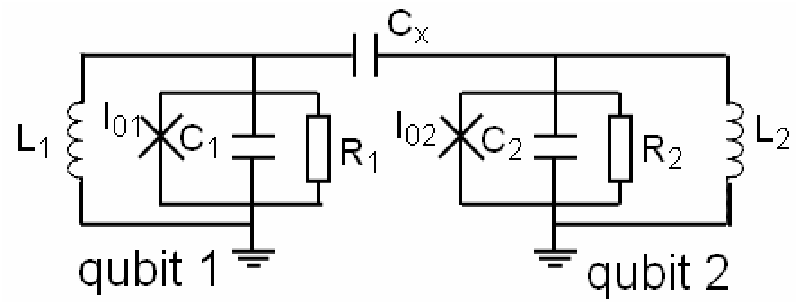
Qubit potential U(δ):

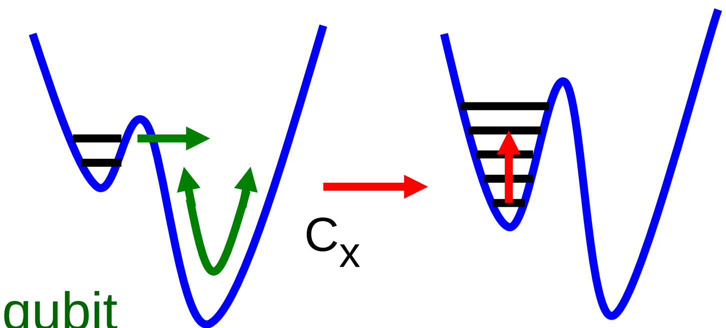


- energy relaxation in right well is very important for irreversibility
- requires slow resetting of measurement pulse (or turning damping on/off)
- special experimental study (N. Katz et al.)

Milestones: theory of cross-talk and theoretical fidelity for coupled qubits

- Develop a quantum theory for the classical measurement cross-coupling and use it to calculate the energy transfer from one phase Qbit to another
- Calculate the theoretical measurement fidelity for coupled phase Qbits

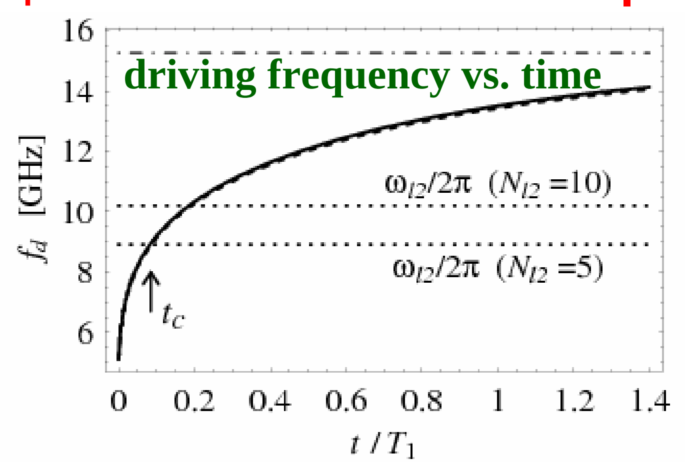




Measurement of state |1⟩ drives second qubit

⇒ possible errors for states |10⟩ and |01⟩

McDermott et al., Science-2005

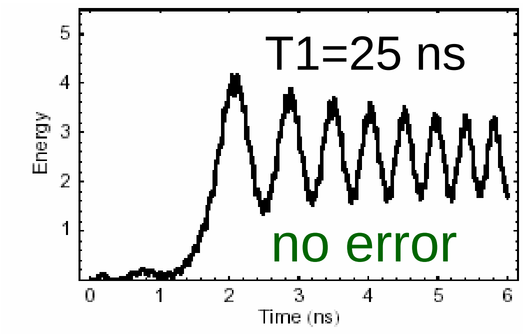


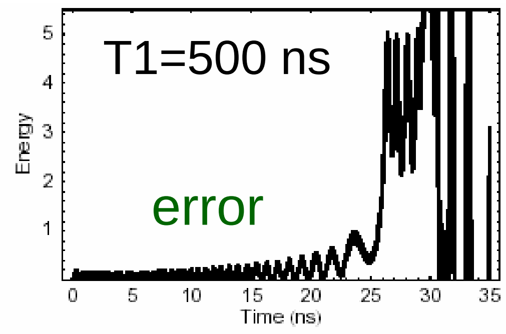
Approaches for the driven qubit:

- classical
- quantum

Classical approach

Qubit energy vs. time

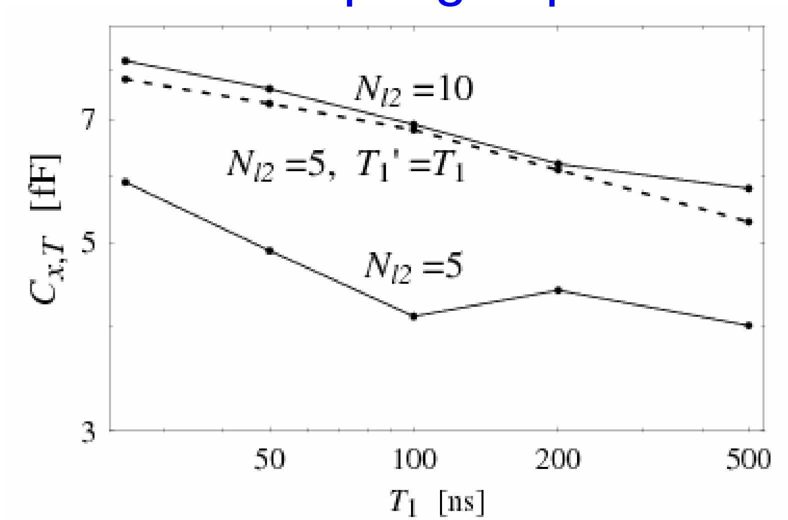




Measurement error: excitation over barrier

No-error requirement limits coupling capacitance

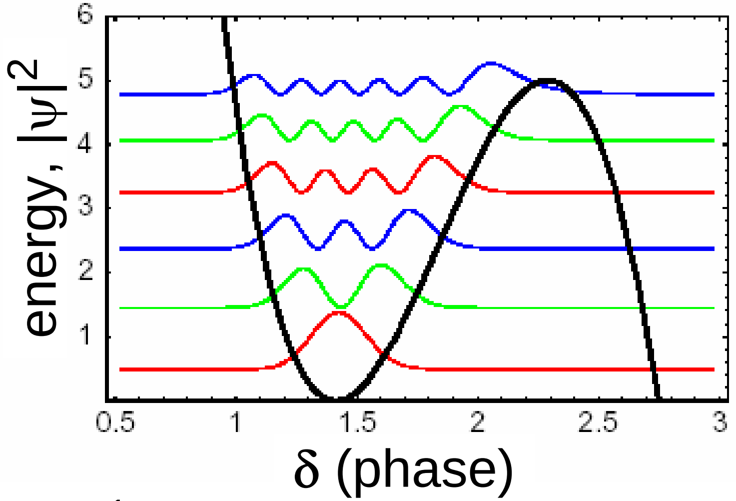
Maximum coupling capacitance vs. T1

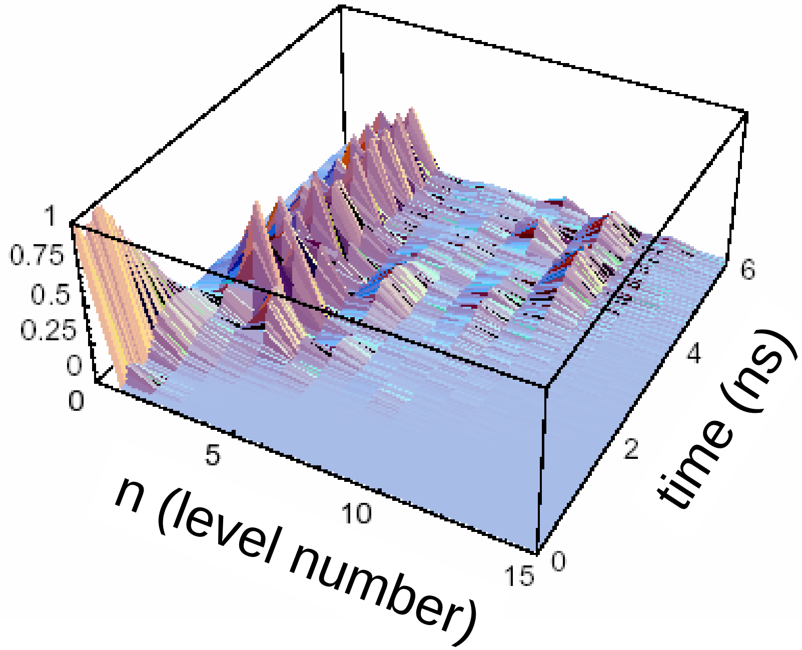


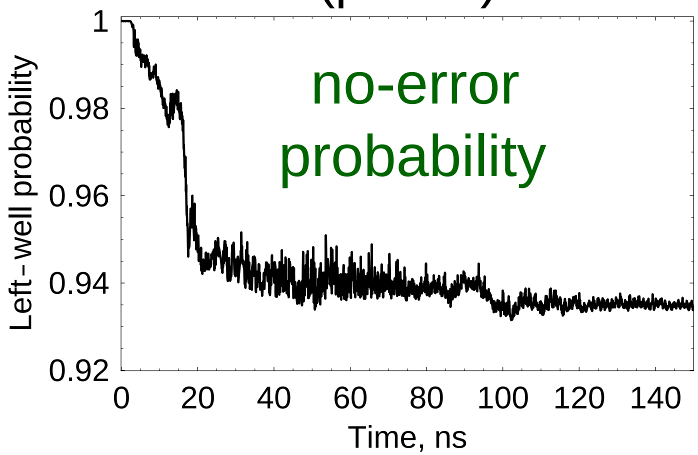
qubit parameters taken from McDermott et al., Science-05

Larger T_1 requires smaller coupling

Quantum approach
(numerical solution of Schrödinger equation)



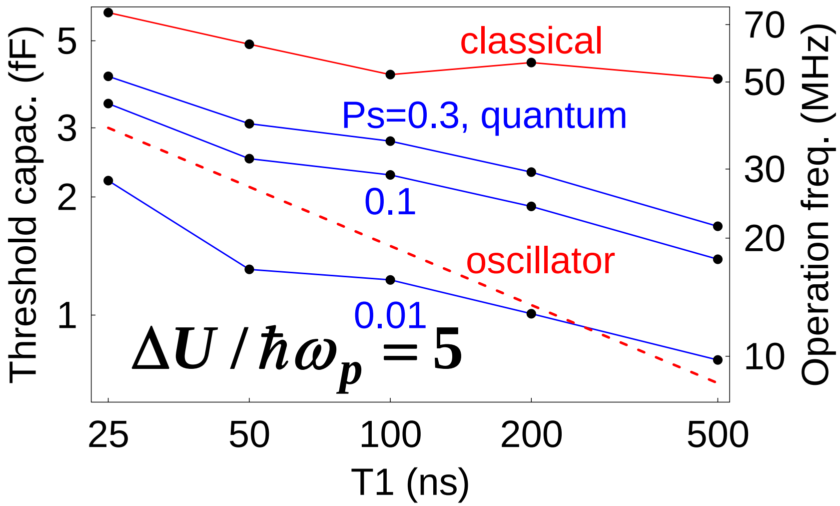


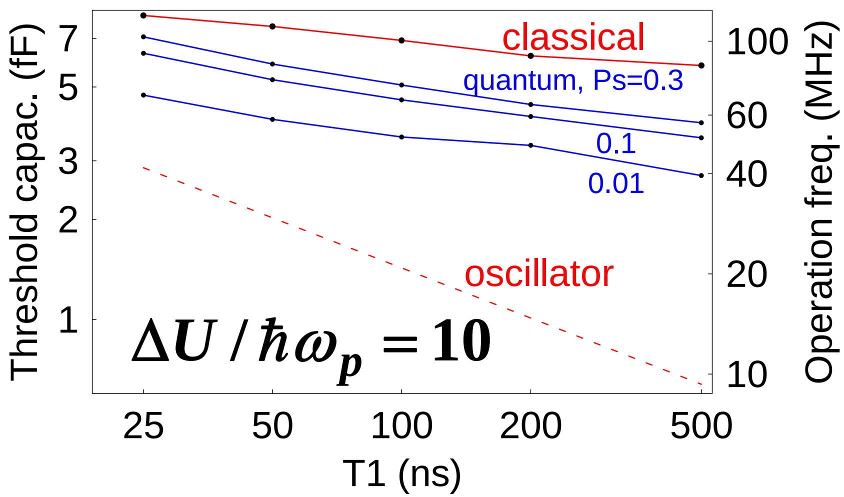


In contrast to classical case, quantum measurement error is probabilistic

Measurement fidelity and coupling limitation for coupled phase qubits

Maximum coupling capacitance (and gate frequency) vs. T1



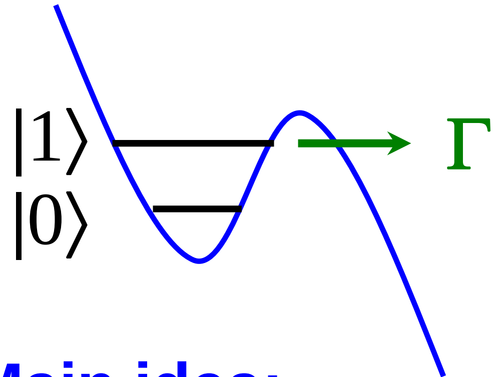


- Cross-talk is small for 2-qubit gate frequency < 10-40 MHz
- Increasing qubit barrier helps, but not much
- Larger T1 requires smaller frequency, but weak dependence

For proof-of-principle experiments cross-talk is not a big problem, but for future high-fidelity experiments variable coupling is desirable

Milestone: quantum measurement back-action for phase qubit

- Calculate the degree of quantum measurement back action in a phase Qbit



How does a coherent state evolve in time before tunneling event?

(What happens when nothing happens?)

Qubit "ages" in contrast to a radioactive atom!

Main idea:

$$\psi = \alpha\,|0\rangle + \beta\,|1\rangle \;\rightarrow\; \psi(t) = \begin{cases} |out\rangle, \text{ if tunneled} \\ \dfrac{\alpha\,|0\rangle + \beta\, e^{-\Gamma t/2}\, e^{i\varphi}\,|1\rangle}{Norm}, \text{ if not tunneled} \end{cases}$$

$$Norm = \sqrt{|\alpha|^2 + |\beta|^2\, e^{-\Gamma t}}$$

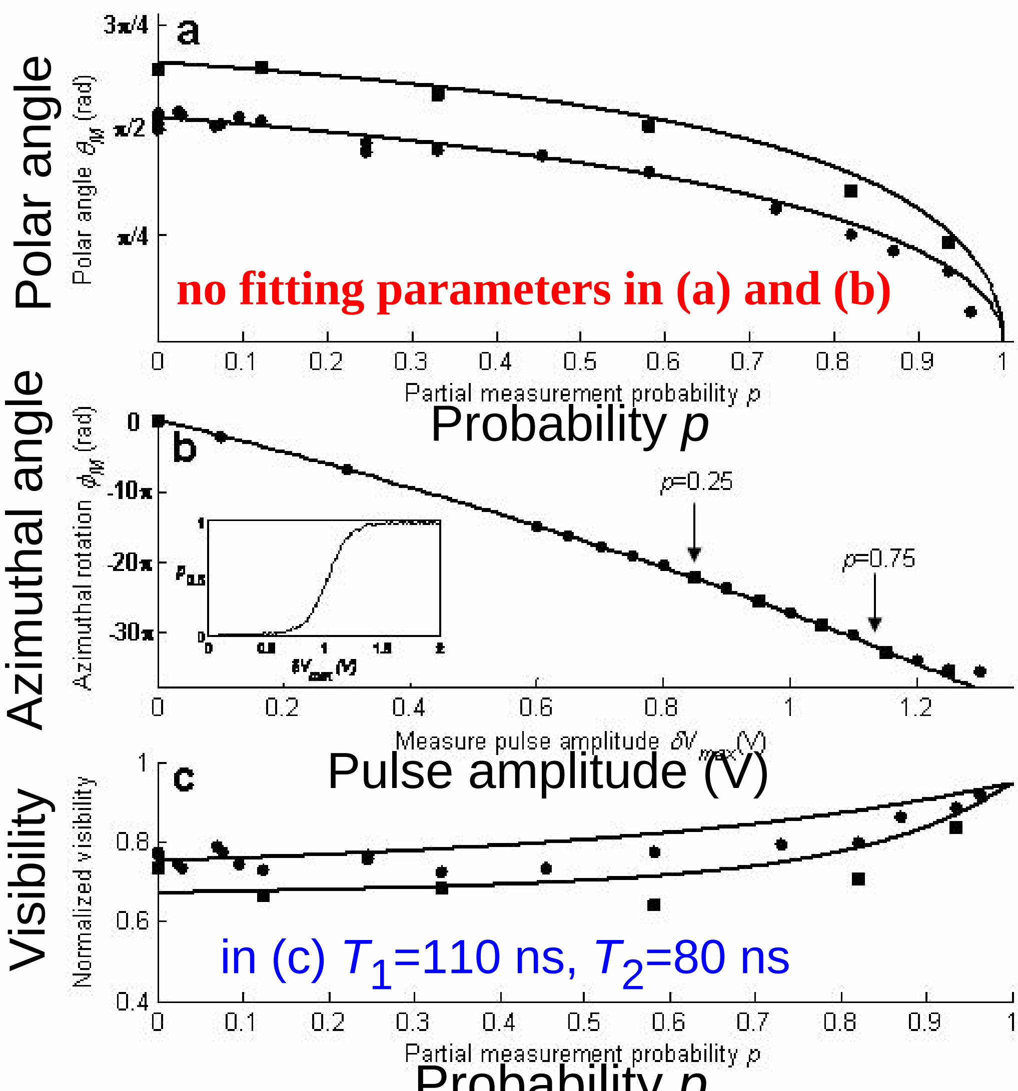
amplitude of state |0> grows without physical interaction

continuous null-result collapse

(similar to optics, Dalibard-Castin-Molmer, PRL-1992)

Partial collapse of a phase qubit

N. Katz *et al.*, Science-2006



Measurement strength

$p = 1 - \exp(-\Gamma t)$

is actually controlled by Γ, not by t

$p = 0$: no measurement

$p = 1$: orthodox collapse

- **First solid-state experiment on non-trivial collapse (quantum back-action)**
- **Showed Bayesian nature of collapse (state remains pure!)**
- **Non-unitary quantum gate for a quantum computer**

Quantum eraser based on phase qubit
(exact undoing of a quantum measurement)

ψ_0 (unknown) → Partial measurement → ψ_1 (partially collapsed)

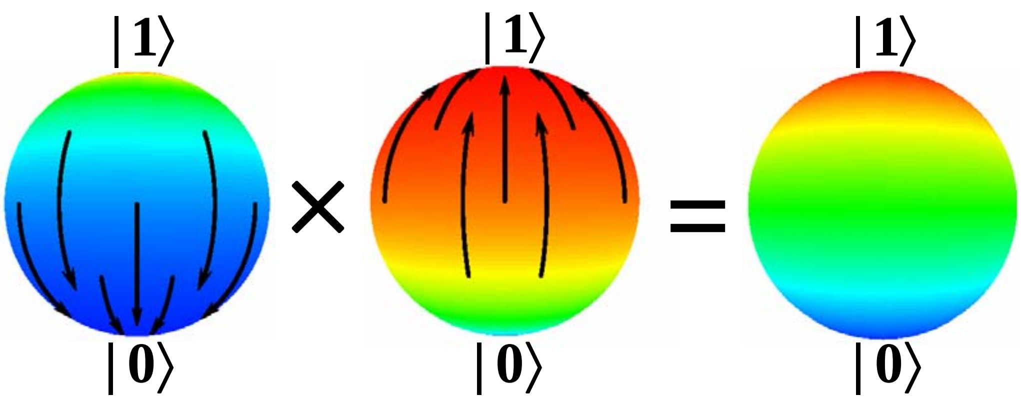
ψ_1 → successful → ψ_0 (still unknown)

ψ_1 → unsuccessful → ψ_2

undoing (information erasure)

Evolution due to partial measurement is **non-unitary**, therefore impossible to undo it by Hamiltonian dynamics.

How to undo? One more measurement!



Korotkov-Jordan, 2006

Similar to experiment by N. Katz et al., just one more π-pulse and one more partial measurement. If no tunneling twice, then an unknown state is fully restored

Probability of success:

$$P_S = \frac{1-p}{\rho_{00}(0)+(1-p)\rho_{11}(0)}$$

Related topics (quantum measurement back-action)

Besides completing the planned milestones, we have also published papers on the following related topics:

- **Quantum feedback control of a charge qubit**
- **QND measurement and squeezing of a nanoresonator**
- **QND measurement of a charge qubit**
- **Leggett-Garg inequalities ("Bell inequalities in time") for continuous measurement**

Proposed milestones for Years 3 and 4 (theory, UCR)

Milestones for years 3 and 4 planned in 2004:

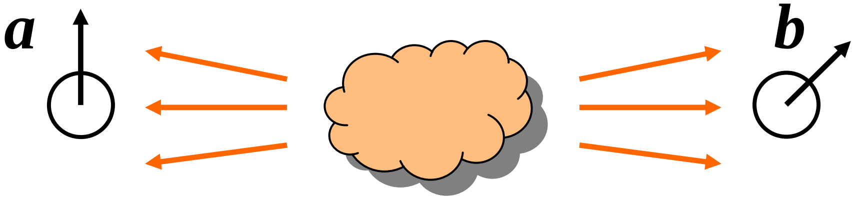
Year 3: Compute fidelity threshold for entanglement demonstration
Year 4: Relate analysis of state tomography to models of decoherence

Proposed revised (detailed) milestones for years 3 and 4:

- Compute fidelity threshold and decoherence limitations for entanglement demonstration. Analyze applicability of the Clauser-Horne-Shimony-Holt version of the Bell inequality for entangled phase qubits; study potential improvement using Eberhard-like inequality.
- Develop comprehensive theory of coherent partial collapse for single and entangled phase qubits due to null-result measurement (taking into account imperfections and simultaneous Hamiltonian evolution). Develop experiment-oriented theory and compute fidelity of undoing of a partial measurement (restoration of an unknown state) for a phase qubit.
- Calculate gate fidelities for coupled qubits. Develop procedure of process tomography characterization suitable for quantum gates based on phase qubits.

Milestone: Threshold for entanglement demonstration

Original Bell's inequality (1964)



$$\psi = \frac{1}{\sqrt{2}}(\uparrow_1\downarrow_2 - \downarrow_1\uparrow_2)$$

QM: $P(\vec{a},\vec{b}) = -\vec{a}\bullet\vec{b}$

$$|P(\vec{a},\vec{b}) - P(\vec{a},\vec{c})| \le 1 + P(\vec{b},\vec{c})$$

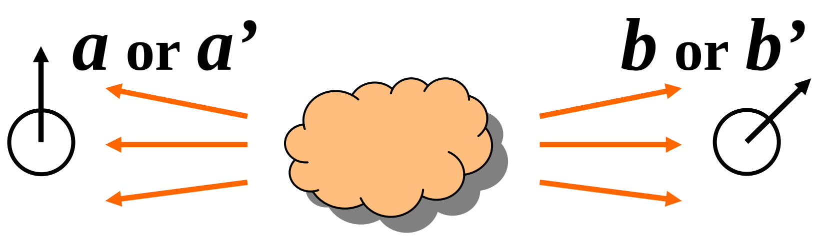
For 0°, 90°, and 45°: $0.71 \le 1 - 0.71$

violation

$$P \equiv p(++) + p(--) - p(+-) - p(-+)$$

For the Bell's inequality we need to assume <u>perfect</u> anticorrelation for the same measurement direction $\Rightarrow$ not practical!

CHSH inequality (Clauser, Horne, Shimony, Holt, 1969)



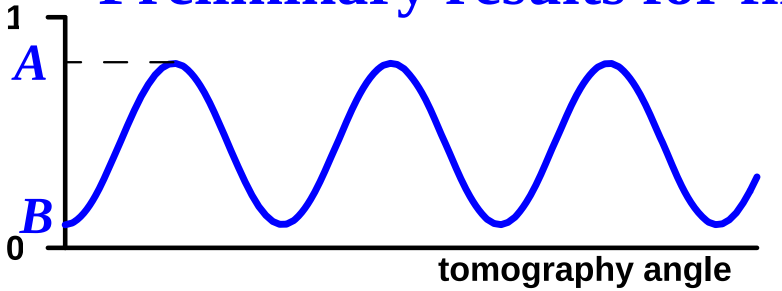
$|S| \le 2,$ **where**

$$S = P(a,b) - P(a,b') + P(a',b) + P(a',b')$$

Maximum violation: $S = \pm 2\sqrt{2}$

CHSH: Translation into phase qubits

Preliminary results for finite visibility



Models: 1) "orthodox" collapse and classical errors
2) Partial measurement, Γ and γ (not yet studied)

$$\max(S) = 2(A_1 + B_1 - 1)(A_2 + B_2 - 1) + 2\sqrt{2}(A_1 - B_1)(A_2 - B_2)$$

Assume $A_1=A_2=A$, $B_1=B_2=B$, then violation (S>2) requires:

$$A - B > 0.83$$ (same as in CHSH paper)

Unanswered questions

- Visibility limitation for the second and other models
- Limitation for decoherence
- Finite visibility together with decoherence
- Required accuracy of control

Other inequalities

Our preliminary proposal (similar to CHSH)

$$-1 \le T \le 0, \text{ where } T = R(a,b) - R(a,b') + R(a',b) + R(a',b') - R_1(a') - R_2(b)$$

$$R = p(--), \;\; R_1 = p(-,\text{any}), \;\; R_2 = p(\text{any},-)$$

Max violation: $T = -\frac{1}{2} \pm \frac{\sqrt{2}}{2}$

There is a direct relation between *S* and *T*; however, only negative-result cases are counted in *T*, so **cross-talk is not important!**

One more inequality (Eberhard, 1993)

$$J \ge 0 \qquad J = p(+-|a,b') + p(+u|a,b') + p(-+|a',b) + p(u+|a',b) + p(++|a',b') - p(++|a,b)$$

(u – unobserved)

Efficiency η >2/3 (instead of >83%) is sufficient to observe violation, initial state is not the Bell state

Can something similar be used for phase qubits to relax requirements for visibility and decoherence?

Milestone: Partial collapse of phase qubits (further developments)

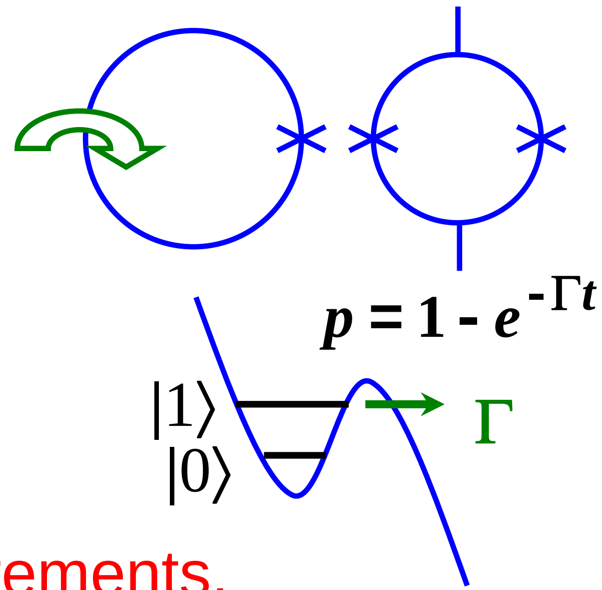
Problems to be studied:

- Rigorous theory of partial collapse; account of decoherence, microwave, etc., theory for more advanced experiments (collaboration with Prof. Pryadko, Dr. Ruskov, and Prof. Mizel)
- Theoretical support of experiment (hopefully!) on quantum eraser: account of imperfections (collab. with Prof. Jordan)
- Partial collapse of entangled qubits; gradual violation of Bell (CHSH) inequalities (not possible in optics!)
- Non-unitary probabilistic gates for entangled qubits based on partial collapse: new operations for QC library

Really interesting ("ideologically" non-trivial) experiments on quantum information processing are within reach. Quantum optics no longer holds monopoly!

Proposed experiment on undoing partial collapse of a phase qubit

1) start with an "unknown" state
2) partial measurement of strength *p*
3) π-pulse (exchange |0> ↔ |1>)
4) one more measurement with the **same strength** *p*
5) π-pulse *(not really needed)*
6) tomography



If no tunneling for both measurements, then initial state is fully restored!

$$\alpha|0\rangle + \beta|1\rangle \rightarrow \frac{\alpha|0\rangle + e^{i\phi}\beta\sqrt{1-p}\,|1\rangle}{\text{Norm}} \rightarrow$$

$$\frac{e^{i\phi}\alpha\sqrt{1-p}\,|0\rangle + e^{i\phi}\beta\sqrt{1-p}\,|1\rangle}{\text{Norm}} = e^{i\phi}(\alpha|0\rangle + \beta|1\rangle)$$

Milestone: Gate fidelities and process tomography

Gate fidelities

We have learned a lot about one-qubit and coupled-qubit measurement fidelities, now can apply it to quantum gates

Simple approach: simplest model of gate operation + T1 and T2 for each qubit

Advanced approach: 1) various models of decoherence (intra- and inter-qubit); 2) account of cross-talk; 3) account of measurement fidelity

Process tomography

- Develop superoperator formalism for evolution of phase qubits
- Adapt optical language for process tomography to phase qubits
- Extract process information (types of decoherence, measurement disturbance, cross-talk, etc.) from experimental tomography data

Proposed milestones for Years 3 and 4 (theory, UCR)

- Compute fidelity threshold and decoherence limitations for entanglement demonstration. Analyze applicability of the Clauser-Horne-Shimony-Holt version of the Bell inequality for entangled phase qubits; study potential improvement using Eberhard-like inequality.

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