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Continuous quantum measurement and feedback control of solid-state qubits

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Outline:

- Introduction (quantum measurement)
- Bayesian formalism for continuous quantum measurement of a **single** quantum system
- Experimental predictions and proposals
- Recent experiment on partial collapse

General theme: Information and collapse in quantum mechanics

Acknowledgement: Rusko Ruskov

Support: DTO

Niels Bohr:

“If you are not confused by quantum physics then you haven’t really understood it”

Richard Feynman:

“I think I can safely say that nobody understands quantum mechanics”

Quantum mechanics =

Schroedinger equation
+
collapse postulate

1) Probability of measurement result $p_r = |\langle \psi | \psi_r \rangle|^2$

2) Wavefunction after measurement = ψ_r

- State collapse follows from common sense
- Does not follow from Schr. Eq. (contradicts; Schr. cat, random vs. deterministic)

What if measurement is continuous?

(as practically always in solid-state experiments)

Einstein-Podolsky-Rosen (EPR) paradox

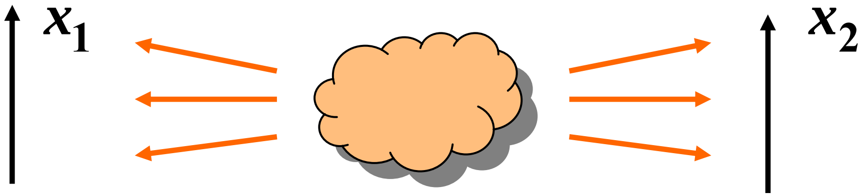
Phys. Rev., 1935

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system.

$$\psi(x_1,x_2)=\sum\nolimits_n \psi_n(x_2)\,u_n(x_1)$$

(nowadays we call it entangled state)

$$\psi(x_1,x_2)=\int_{-\infty}^{\infty}\exp[(i/\hbar)(x_1-x_2)p]dp \sim \delta(x_1-x_2)$$



Measurement of particle 1 cannot affect particle 2, while QM says it affects (contradicts causality)

=> Quantum mechanics is incomplete

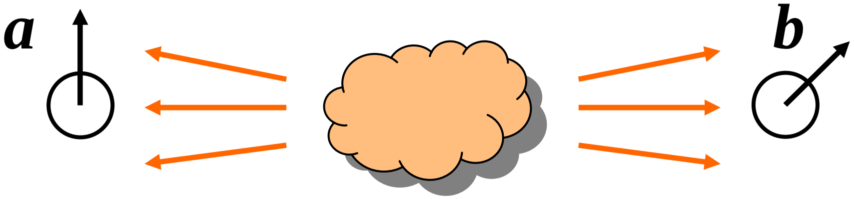
Bohr's reply

(Phys. Rev., 1935) (seven pages, one formula: $\Delta p\,\Delta q \sim h$)

It is shown that a certain “criterion of physical reality” formulated … by A. Einstein, B. Podolsky and N. Rosen contains an essential ambiguity when it is applied to quantum phenomena.

Crudely: No need to understand QM, just use the result

Bell's inequality (John Bell, 1964)



(setup due to David Bohm)

$$\psi=\frac{1}{\sqrt{2}}(\uparrow_1\downarrow_2-\downarrow_1\uparrow_2)$$

Perfect anticorrelation of measurement results for the same measurement directions, $\vec{a}=\vec{b}$

Is it possible to explain the QM result assuming local realism and hidden variables *or* collapse "propagates" instantaneously (faster than light, "spooky action-at-a-distance")?

Assume: $A(\vec{a},\lambda)=\pm1,\quad B(\vec{b},\lambda)=\pm1$ (deterministic result with hidden variable λ)

Then: $|P(\vec{a},\vec{b})-P(\vec{a},\vec{c})|\leq 1+P(\vec{b},\vec{c})$

where $P\equiv P(++)+P(--)-P(+-)-P(-+)$

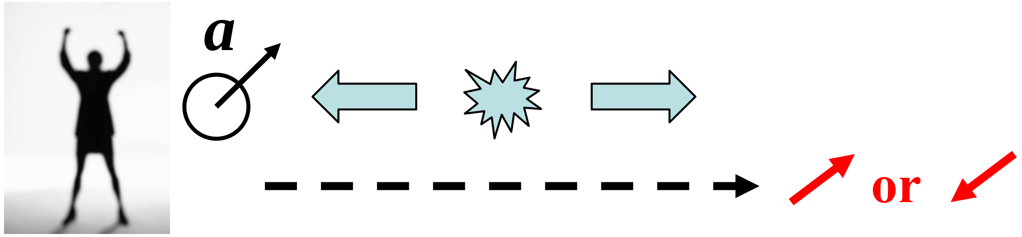
QM: $P(\vec{a},\vec{b})=-\vec{a}\bullet\vec{b}$ For 0°, 90°, and 45°: $0.71\nleq 1-0.71$ **violation!**

Experiment (Aspect et al., 1982; photons instead of spins, CHSH):

yes, "spooky action-at-a-distance"

What about causality?

Actually, not too bad: you cannot transmit your own information choosing a particular measurement direction $\boldsymbol{a}$



Result of the other measurement does not depend on direction $\boldsymbol{a}$

Randomness saves causality

Collapse is still instantaneous: OK, just our recipe, not an “objective reality”, not a “physical” process

Consequence of causality: No-cloning theorem

Wootters-Zurek, 1982; Dieks, 1982; Yurke

You cannot copy an unknown quantum state

Proof: Otherwise get information on direction $\boldsymbol{a}$ (and causality violated)

Application: quantum cryptography

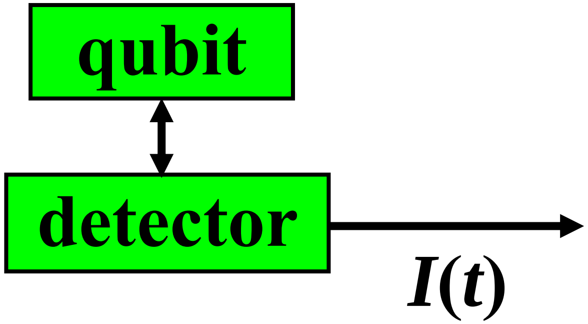
Information is an important concept in quantum mechanics

Quantum measurement in solid-state systems

No violation of locality – too small distances

However, interesting informational aspects of continuous quantum measurement (weak coupling, noise ⇒ gradual collapse)

Starting point:

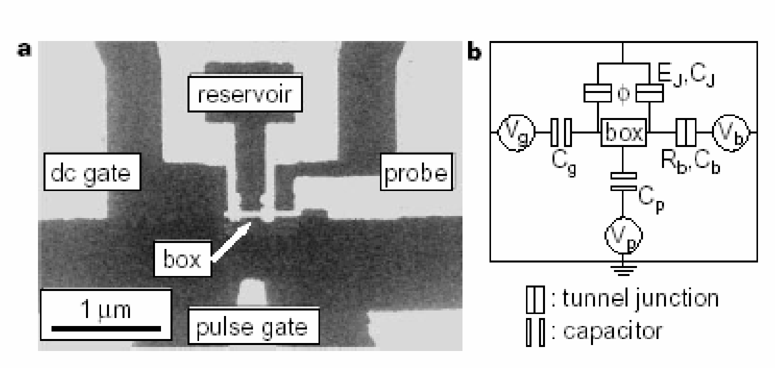


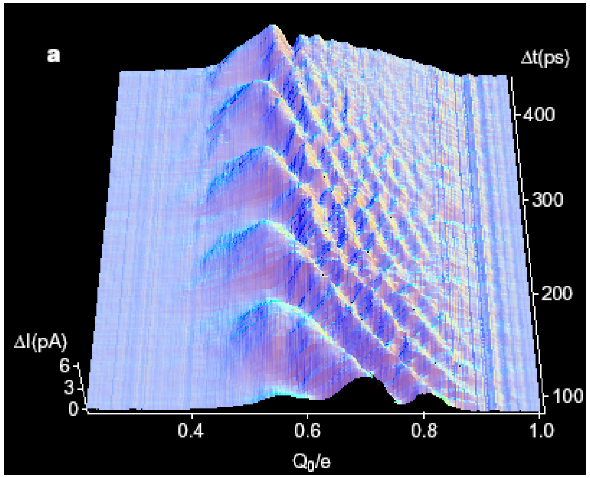
What happens to a solid-state qubit (two-level system) during its continuous measurement by a detector?

How qubit evolution is related to detector output $I(t)$? (output noise is important!)

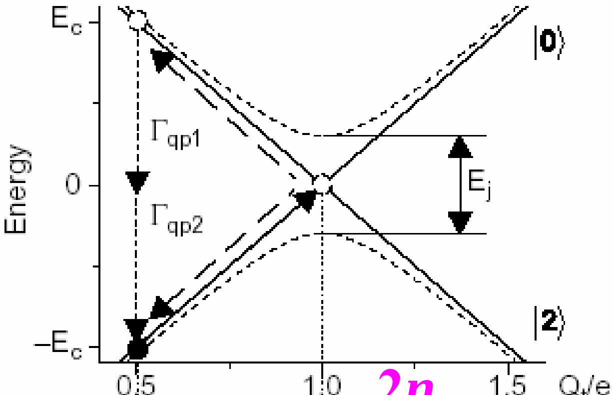
Superconducting “charge” qubit

Y. Nakamura, Yu. Pashkin, and J.S. Tsai (Nature, 1998)

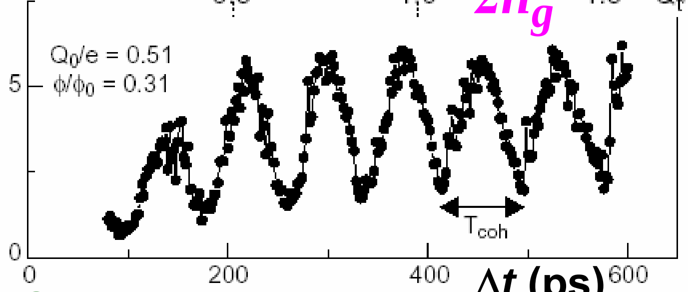




$$\hat{H} = \frac{(2e)^2}{2C}(\hat{n} - n_g)^2 - \frac{E_J}{2}(|n\rangle\langle n+1| + |n+1\rangle\langle n|)$$



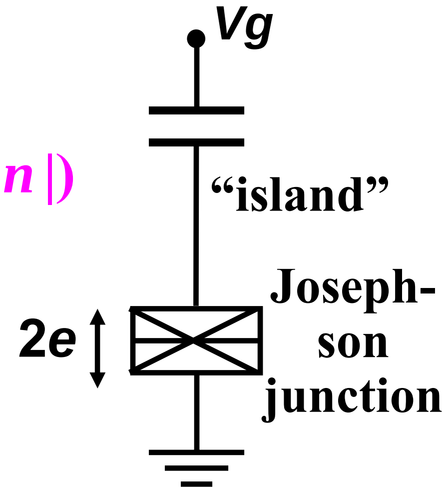
$2n_g$



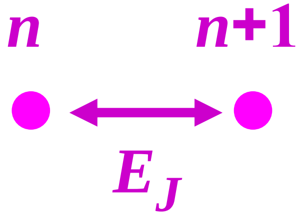
Quantum coherent (Rabi) oscillations

Vion et al. (Devoret’s group); Science, 2002

Q-factor of coherent (Rabi) oscillations = 25,000

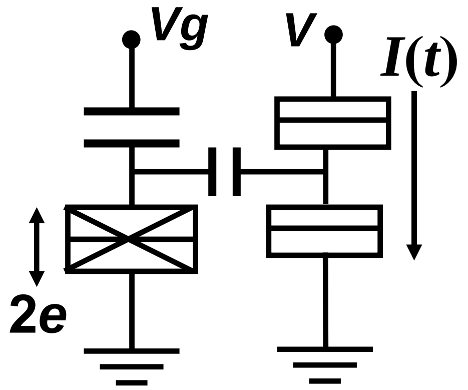


Single Cooper pair box



n: number of Cooper pairs on the island

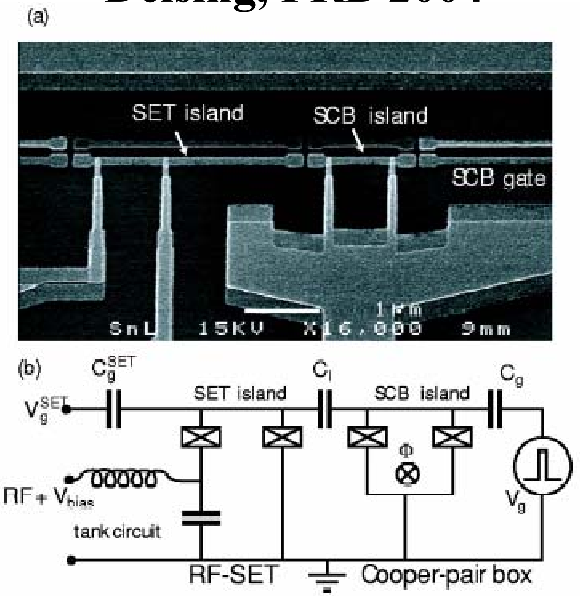
More of superconducting charge qubits

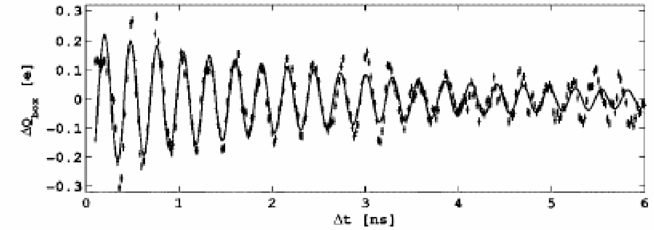


Cooper-pair box measured by single-electron transistor (SET)
(actually, RF-SET)

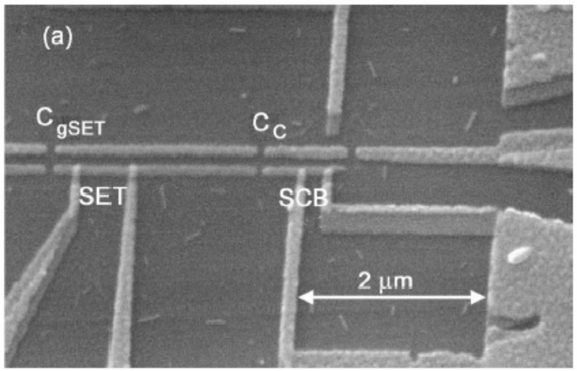
Setup can be used for continuous measurements

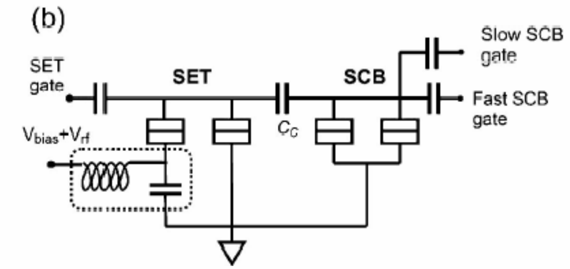
Duty, Gunnarsson, Bladh, Delsing, PRB 2004

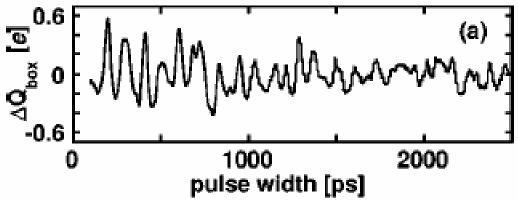




Guillaume et al. (Echternach's group), PRB 2004



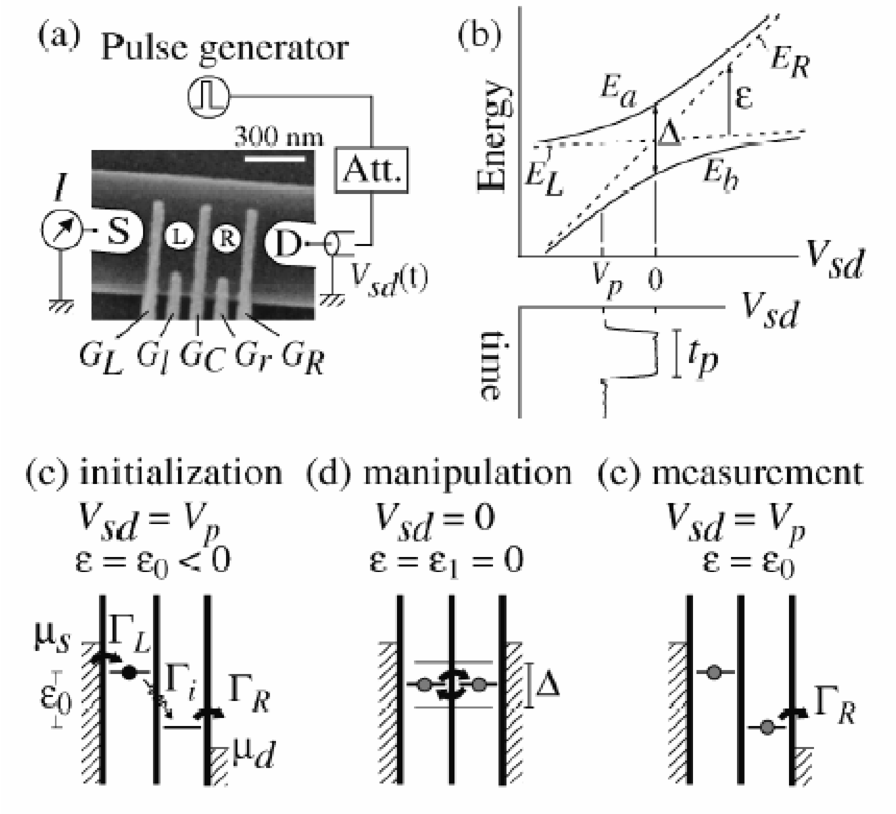


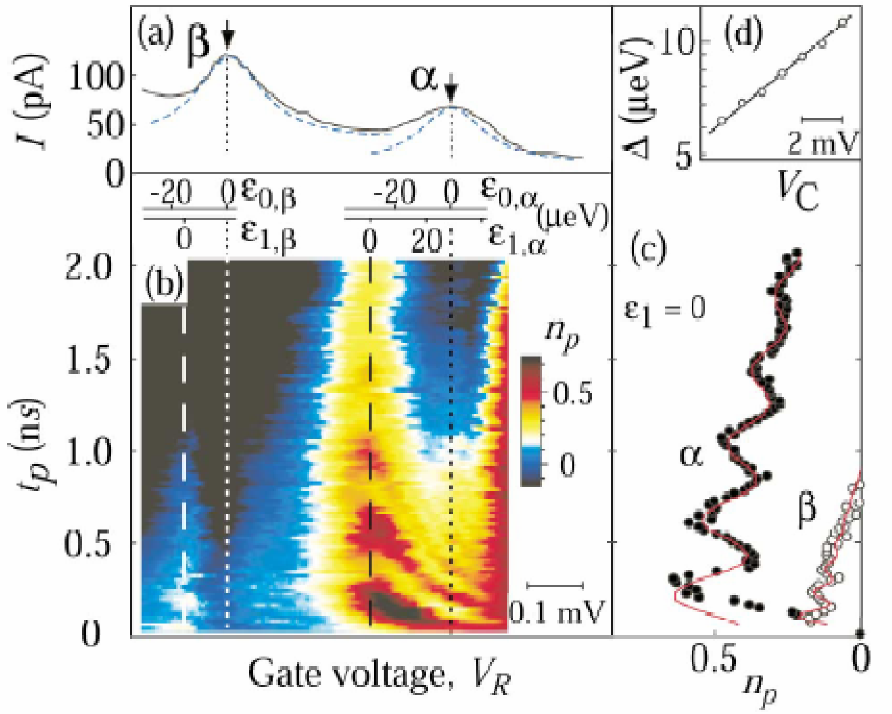


All results are averaged over many measurements (not "single-shot")

Semiconductor (double-dot) qubit

T. Hayashi et al., PRL 2003





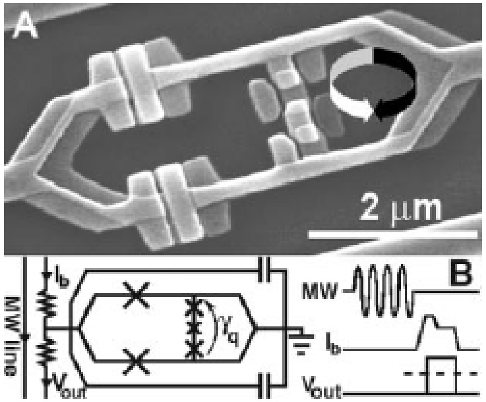
Rabi oscillations

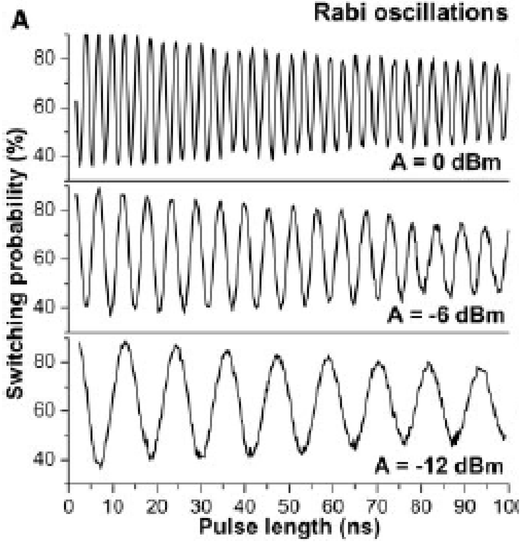
Detector is not separated from qubit,
also possible to use a separate detector

Some other solid-state qubits

Flux qubit

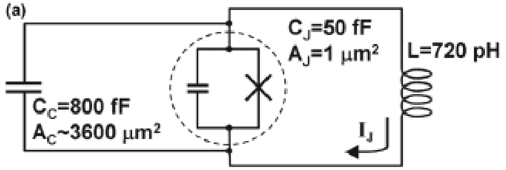
Mooij et al. (Delft)

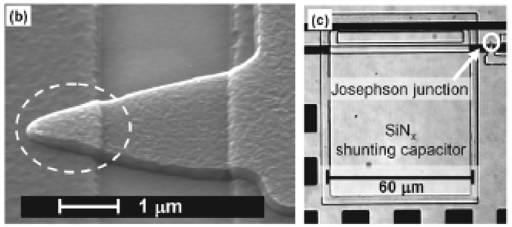


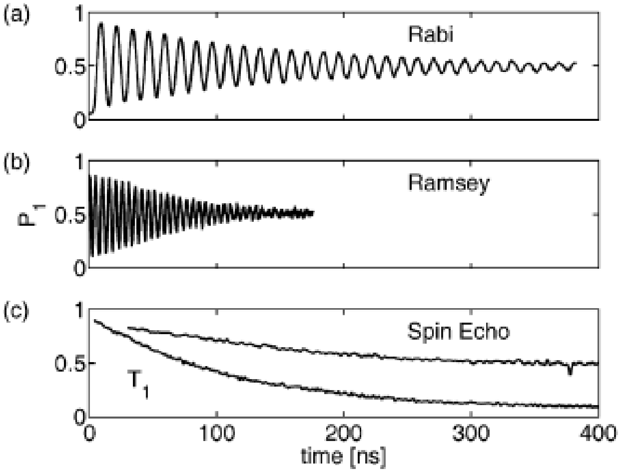


Phase qubit

J. Martinis et al.
(UCSB and NIST)

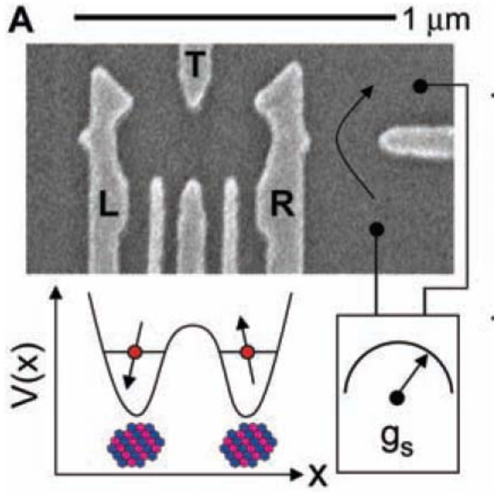


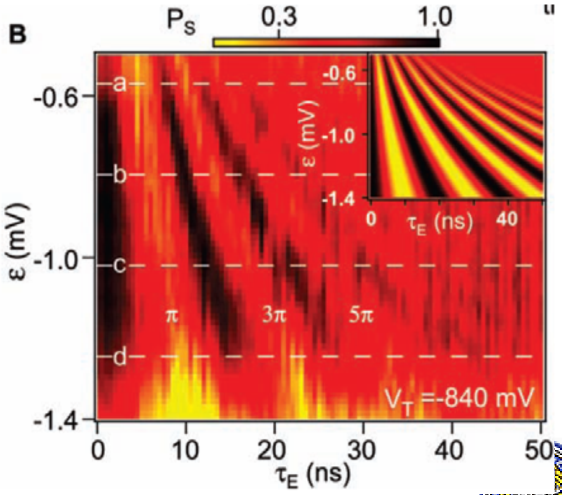




Spin qubit

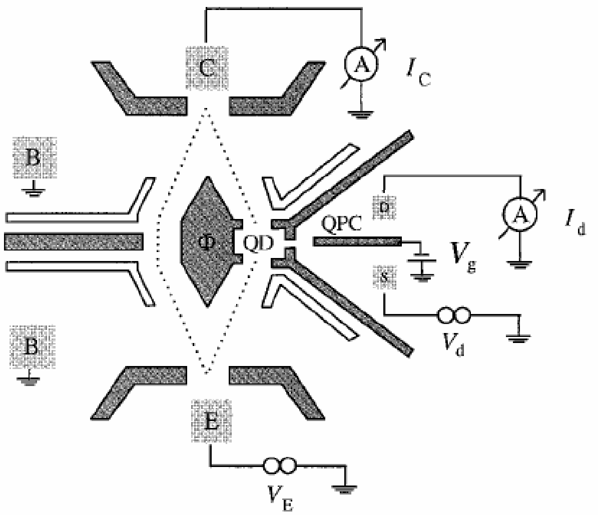
C. Marcus et al. (Harvard)

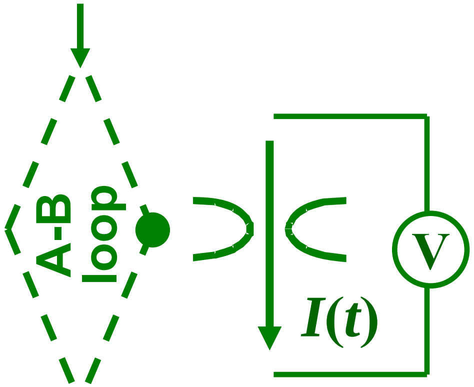




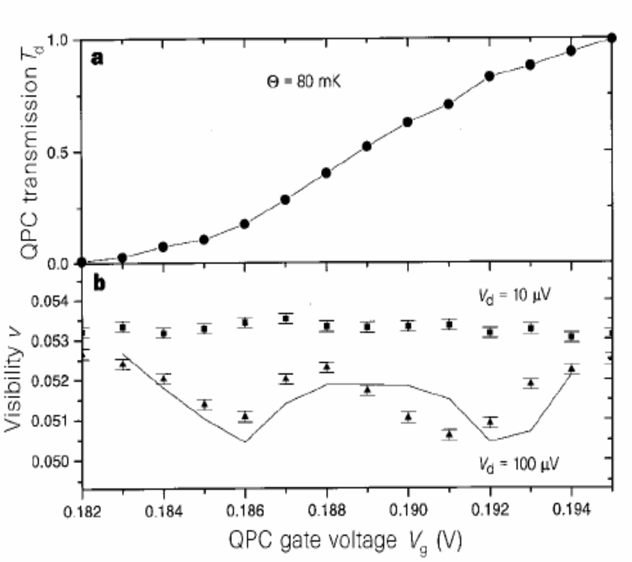
“Which-path detector” experiment

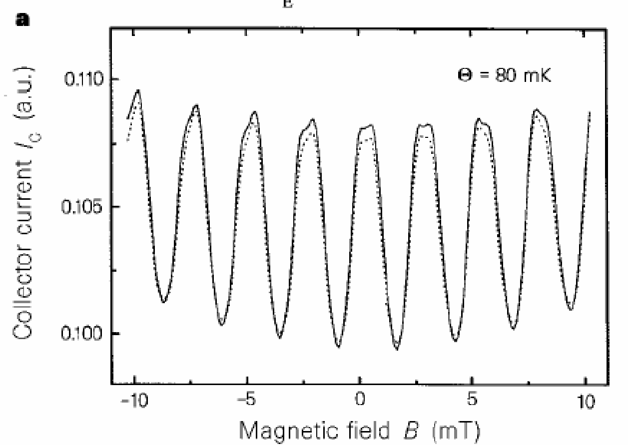
Buks, Schuster, Heiblum, Mahalu, and Umansky, Nature 1998





QPC detector





Dephasing rate:

$$\Gamma = \frac{eV}{h}\frac{(\Delta T)^2}{T(1-T)} = \frac{(\Delta I)^2}{4S_I}$$

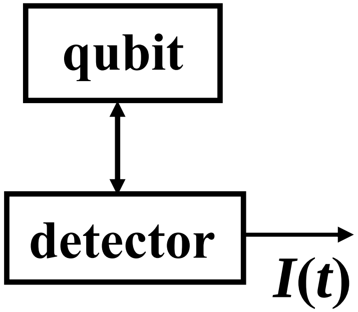
ΔI – detector response, S_I – shot noise

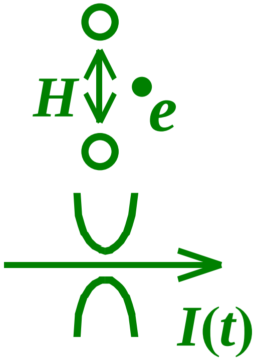
The larger noise, the smaller dephasing!!!

$(\Delta I)^2/4S_I$ ~ rate of “information flow”

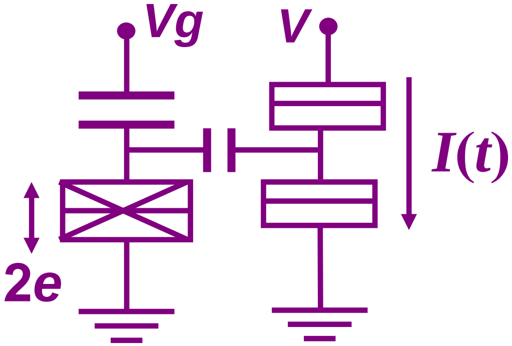
Theory: Aleiner, Wingreen, and Meir, PRL 1997

The system we consider: qubit + detector





Double-quantum-qot (DQD) and quantum point contact (QPC)



Cooper-pair box (CPB) and single-electron transistor (SET)

$$H = H_{\mathrm{QB}} + H_{\mathrm{DET}} + H_{\mathrm{INT}}$$

$$H_{\mathrm{QB}} = (\varepsilon/2)(c_1^+c_1 - c_2^+c_2) + H(c_1^+c_2 + c_2^+c_1)$$ ε – asymmetry, H – tunneling

$\Omega = (4H^2+\varepsilon^2)^{1/2}/\hbar$ – frequency of quantum coherent (Rabi) oscillations

Two levels of average detector current: I_1 for qubit state $|1\rangle$, I_2 for $|2\rangle$

Response: $\Delta I = I_1 - I_2$ Detector noise: white, spectral density S_I

DQD and QPC
(setup due to Gurvitz, 1997)

$$H_{DET} = \sum_l E_l a_l^\dagger a_l + \sum_r E_r a_r^\dagger a_r + \sum_{l,r} T(a_r^\dagger a_l + a_l^\dagger a_r)$$

$$H_{INT} = \sum_{l,r} \Delta T\,(c_1^\dagger c_1 - c_2^\dagger c_2)(a_r^\dagger a_l + a_l^\dagger a_r)$$

$$S_I = 2eI$$

What happens to a qubit state during measurement?

Start with density matrix evolution due to measurement only ($H=\varepsilon=0$)

"Orthodox" answer

$$\begin{pmatrix}\frac{1}{2} & \frac{1}{2}\\ \frac{1}{2} & \frac{1}{2}\end{pmatrix}\begin{matrix}\nearrow\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}\\ \searrow\begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix}\end{matrix}$$

|1> or |2>, depending on the result

"Conventional" (decoherence) answer (Leggett, Zurek)

$$\begin{pmatrix}\frac{1}{2} & \frac{1}{2}\\ \frac{1}{2} & \frac{1}{2}\end{pmatrix}\rightarrow\begin{pmatrix}\frac{1}{2} & \frac{\exp(-\Gamma t)}{2}\\ \frac{\exp(-\Gamma t)}{2} & \frac{1}{2}\end{pmatrix}\rightarrow\begin{pmatrix}\frac{1}{2} & 0\\ 0 & \frac{1}{2}\end{pmatrix}$$

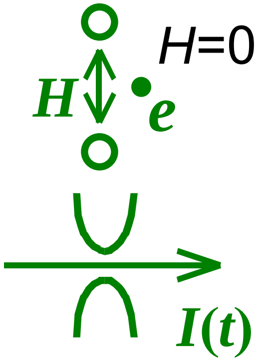
no measurement result! (ensemble averaged)

Orthodox and decoherence answers contradict each other!

applicable for:	Single quantum systems	Continuous measurements
Orthodox	yes	no
Conventional (ensemble)	no	yes
Bayesian	yes	yes

Bayesian formalism describes gradual collapse of a single quantum system, taking into account noisy detector output $I(t)$

Bayesian formalism for DQD-QPC (qubit-detector) system



Qubit evolution due to continuous measurement:

1) Diagonal matrix elements of the qubit density matrix evolve as classical probabilities (i.e. according to the classical Bayes rule)

2) Non-diagonal matrix elements evolve so that the degree of purity $\rho_{ij}/[\rho_{ii}\,\rho_{jj}]^{1/2}$ is conserved

(A.K., 1998)

Bayes rule:

$$P(A_i\,|\,R)=\frac{P(A_i)\,P(R\,|\,A_i)}{\sum_k P(A_k)\,P(R\,|\,A_k)}$$

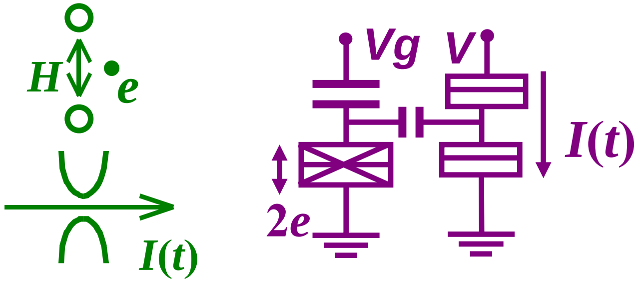
So simple because:

1) QPC happens to be an ideal detector
2) no Hamiltonian evolution of the qubit

Similar formalisms developed earlier. Key words: Imprecise, weak, selective, or conditional measurements, POVM, Quantum trajectories, Quantum jumps, Restricted path integral, etc.

Names: Davies, Kraus, Holevo, Belavkin, Mensky, Caves, Gardiner, Carmichael, Knight, Walls, Gisin, Percival, Milburn, Wiseman, Habib, etc. (very incomplete list)

Bayesian formalism for a single qubit



$$\hat{H}_{QB} = \frac{\varepsilon}{2}(c_1^\dagger c_1 - c_2^\dagger c_2) + H(c_1^\dagger c_2 + c_2^\dagger c_1)$$

$|1\rangle \to I_1$, $|2\rangle \to I_2$, $\Delta I = I_1 - I_2$, $I_0 = (I_1 + I_2)/2$

S_I – detector noise

$$\boxed{\begin{aligned} &\dot{\rho}_{11} = -\dot{\rho}_{22} = -2(H/\hbar)\,\mathrm{Im}\,\rho_{12} + \rho_{11}\rho_{22}(2\Delta I/S_I)[\underline{\underline{I(t)}} - I_0] \\ &\dot{\rho}_{12} = i(\varepsilon/\hbar)\rho_{12} + i(H/\hbar)(\rho_{11} - \rho_{22}) + \rho_{12}(\rho_{11} - \rho_{22})(\Delta I/S_I)[\underline{\underline{I(t)}} - I_0] - \gamma\rho_{12} \end{aligned}}$$

(A.K., 1998)

$\gamma = \Gamma - (\Delta I)^2/4S_I$, Γ – ensemble decoherence

$\eta = 1 - \gamma/\Gamma = (\Delta I)^2/4S_I\Gamma$ – detector ideality (efficiency), $\eta \le 100\%$

Ideal detector (η=1, as QPC) does not decohere a qubit, then random evolution of qubit *wavefunction* can be monitored

Averaging over result $I(t)$ leads to conventional master equation:

$$d\rho_{11}/dt = -d\rho_{22}/dt = -2(H/\hbar)\,\mathrm{Im}\,\rho_{12}$$

$$d\rho_{12}/dt = i(\varepsilon/\hbar)\rho_{12} + i(H/\hbar)(\rho_{11} - \rho_{22}) - \Gamma\rho_{12}$$

Main assumption needed for the Bayesian formalism:

Detector voltage is much larger than the qubit energies involved

$eV \gg \hbar\Omega,\ eV \gg \hbar\Gamma,\ \ \hbar/eV \ll (1/\Omega, 1/\Gamma)$

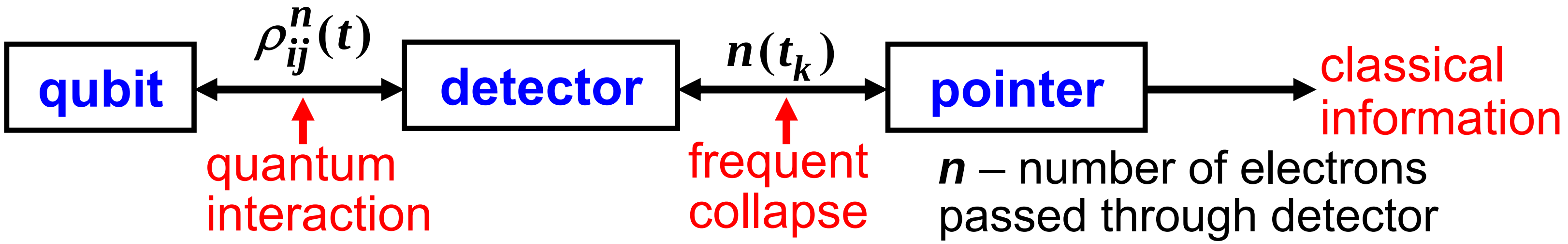
(no coherence in the detector, classical output, Markovian approximation)

(Coupling $C \sim \Gamma/\Omega$ is arbitrary)

Derivations:

1) “logical”: via correspondence principle and comparison with decoherence approach (A.K., 1998)

2) “microscopic”: Schr. eq. + collapse of the detector (A.K., 2000)



qubit $\xleftrightarrow{\rho_{ij}^{n}(t)}$ **detector** $\xleftrightarrow{n(t_k)}$ **pointer** → classical information

quantum interaction

frequent collapse

n – number of electrons passed through detector

3) from “quantum trajectory” formalism developed for quantum optics (Goan-Milburn, 2001; also: Wiseman, Sun, Oxtoby, etc.)

4) from POVM formalism (Jordan-A.K., 2006)

Fundamental limit for ensemble decoherence

$$\Gamma = (\Delta I)^2/4S_I + \gamma$$

ensemble decoherence rate

rate of information acquisition [bit/s]

single-qubit decoherence

$$\gamma \geq 0 \;\Rightarrow\; \boxed{\Gamma \geq (\Delta I)^2/4S_I}$$

$\eta = 1 - \gamma / \Gamma$ – detector ideality (quantum efficiency), $\eta \leq 100\%$

Translated into energy sensitivity: $(\epsilon_O \, \epsilon_{BA})^{1/2} \geq \hbar/2$

where ϵ_O is output-noise-limited sensitivity [J/Hz]

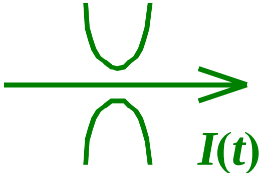
and ϵ_{BA} is back-action-limited sensitivity [J/Hz]

Sensitivity limitation is known since 1980s (Clarke, Tesche, Likharev, etc.); also Averin-2000, Clerk et al.-2002, Pilgram et al.-2002, etc.

Quantum efficiency of solid-state detectors

(ideal detector does not cause single qubit decoherence)

1. Quantum point contact



Theoretically, ideal quantum detector, $\eta = 1$

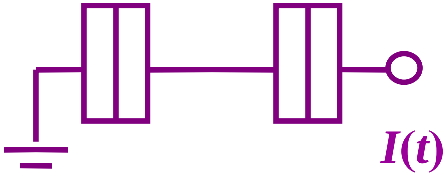
A.K., 1998 (Gurvitz, 1997; Aleiner *et al*., 1997)

Averin, 2000; Pilgram et al., 2002, Clerk et al., 2002

Experimentally, $\eta > 80\%$

(using Buks *et al*., 1998)

2. SET-transistor



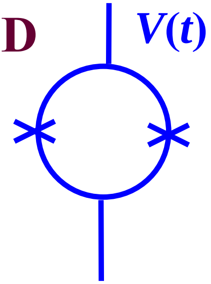
Very non-ideal in usual operation regime, $\eta \ll 1$

Shnirman-Schön, 1998; A.K., 2000, Devoret-Schoelkopf, 2000

However, reaches ideality, $\eta = 1$ if:

- in deep cotunneling regime (Averin, van den Brink, 2000)
- S-SET, using supercurrent (Zorin, 1996)
- S-SET, double-JQP peak ($\eta \sim 1$) (Clerk *et al*., 2002)
- resonant-tunneling SET, low bias (Averin, 2000)

3. SQUID

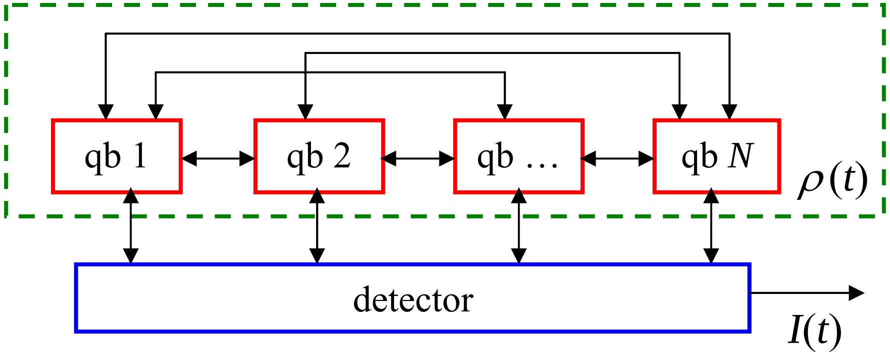


Can reach ideality, $\eta = 1$

(Danilov-Likharev-Zorin, 1983;
Averin, 2000)

4. FET ?? HEMT ??
ballistic FET/HEMT ??

Bayesian formalism for *N* entangled qubits measured by one detector



Up to 2^N levels of current

$$\frac{d}{dt}\rho_{ij} = \frac{-i}{\hbar}[\hat{H}_{qb},\rho]_{ij} + \rho_{ij}\frac{1}{S}\sum_k \rho_{kk}\Big[\Big(I(t)-\frac{I_k+I_i}{2}\Big)(I_i-I_k)+$$

$$+\Big(I(t)-\frac{I_k+I_j}{2}\Big)(I_j-I_k)\Big]-\gamma_{ij}\rho_{ij}$$

(Stratonovich form)

$$\gamma_{ij} = (\eta^{-1}-1)(I_i-I_j)^2/4S_I \qquad I(t)=\sum_i \rho_{ii}(t)I_i+\xi(t)$$

Averaging over ξ(t) ⟹ master equation

No measurement-induced dephasing between states $|i\rangle$ and $|j\rangle$ if $I_i = I_j$!

**A.K., PRA 65 (2002),
PRB 67 (2003)**

Measurement vs. decoherence

Widely accepted point of view:

measurement = decoherence (environment)

Is it true?

- **Yes**, if not interested in information from detector (ensemble-averaged evolution)

- **No**, if take into account measurement result (single quantum system)

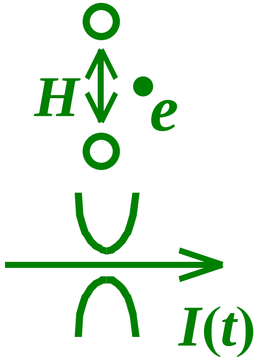
Measurement result obviously gives us more information about the measured system, so we know its quantum state better (ideally, a pure state instead of a mixed state)

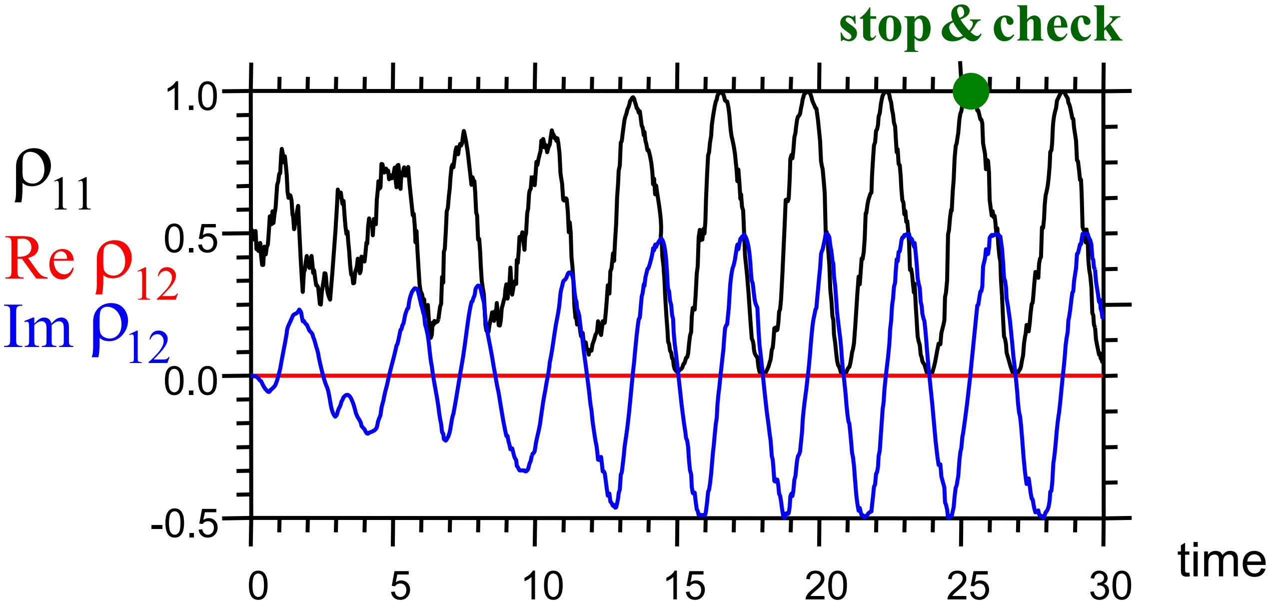
Experimental predictions and proposals from Bayesian formalism

- Direct experimental verification (1998)
- Measured spectral density of Rabi oscillations (1999, 2000, 2002)
- Bell-type correlation experiment (2000)
- Quantum feedback control of a qubit (2001)
- Entanglement by measurement (2002)
- Measurement by a quadratic detector (2003)
- Simple quantum feedback of a qubit (2004)
- Squeezing of a nanomechanical resonator (2004)
- Violation of Leggett-Garg inequality (2005)
- Partial collapse of a phase qubit (2005)
- Undoing of a weak measurement (2006)

Density matrix purification by measurement

(A.K., 1998)

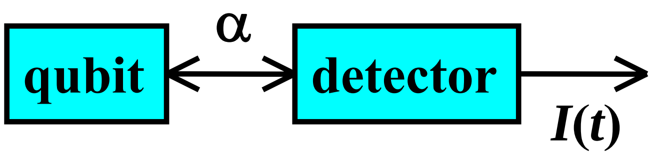




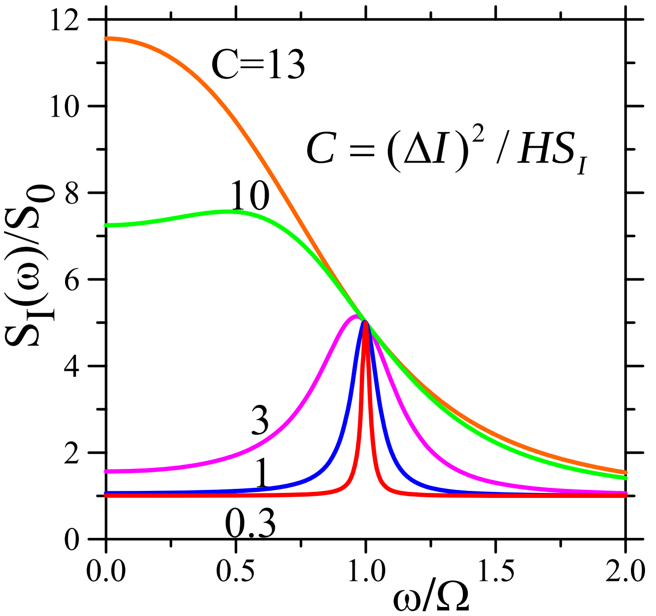
1. Start with completely mixed state.
2. Measure and monitor the Rabi phase.
3. Stop evolution (make *H*=0) at state |1›.
4. Measure and check.

Difficulty: need to record noisy detector current *I*(*t*) and solve Bayesian equations in real time; typical required bandwidth: 1-10 GHz.

Measured spectrum of coherent (Rabi) oscillations



What is the spectral density $S_I(\omega)$ of detector current?



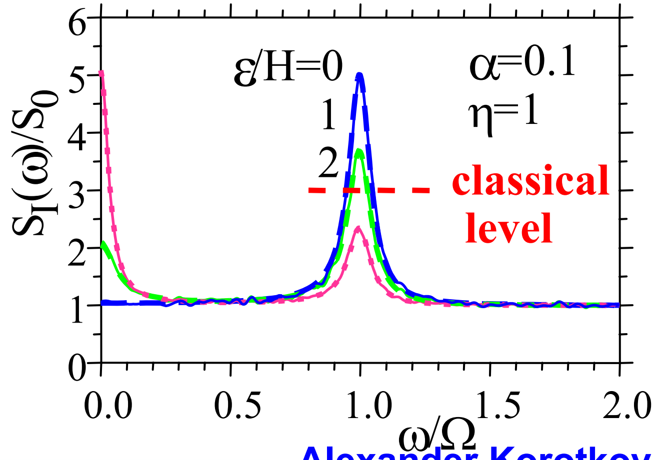
Assume classical output, eV » ℏΩ

$$\varepsilon = 0, \quad \Gamma = \eta^{-1}(\Delta I)^2 / 4S_0$$

$$S_I(\omega) = S_0 + \frac{\Omega^2(\Delta I)^2\Gamma}{(\omega^2-\Omega^2)^2+\Gamma^2\omega^2}$$

Spectral peak can be seen, but peak-to-pedestal ratio ≤ 4η ≤ 4

(result can be obtained using various methods, not only Bayesian method)



A.K., LT'99
A.K.-Averin, 2000
A.K., 2000
Averin, 2000
Goan-Milburn, 2001
Makhlin et al., 2001
Balatsky-Martin, 2001
Ruskov-A.K., 2002
Mozyrsky et al., 2002
Balatsky et al., 2002
Bulaevskii et al., 2002
Shnirman et al., 2002
Bulaevskii-Ortiz, 2003
Shnirman et al., 2003

Contrary:
Stace-Barrett,
PRL-2004

Possible experimental confirmation?

Durkan and Welland, 2001 (STM-ESR experiment similar to Manassen-1989)

APPLIED PHYSICS LETTERS VOLUME 80, NUMBER 3 21 JANUARY 2002

Electronic spin detection in molecules using scanning-tunneling-microscopy-assisted electron-spin resonance

C. Durkan[a] and M. E. Welland
Nanoscale Science Laboratory, Department of Engineering, University of Cambridge, Trumpington Street, Cambridge CB2 1PZ, United Kingdom

(Received 8 May 2001; accepted for publication 8 November 2001)

By combining the spatial resolution of a scanning-tunneling microscope (STM) with the electronic spin sensitivity of electron-spin resonance, we show that it is possible to detect the presence of localized spins on surfaces. The principle is that a STM is operated in a magnetic field, and the resulting component of the tunnel current at the Larmor (precession) frequency is measured. This component is nonzero whenever there is tunneling into or out of a paramagnetic entity. We have

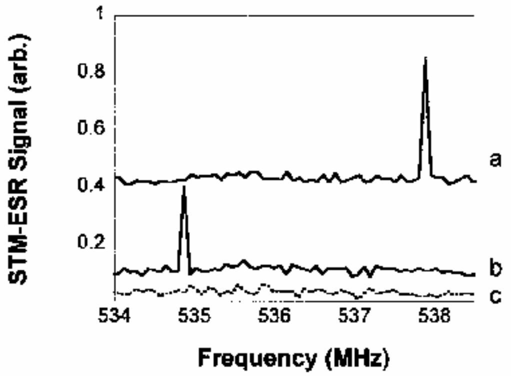


FIG. 3. STM-ESR spectra of (a), (b) two different areas (a few nm apart) of the molecule-covered sample and (c) bare HOPG. The graphs are shifted vertically for clarity.

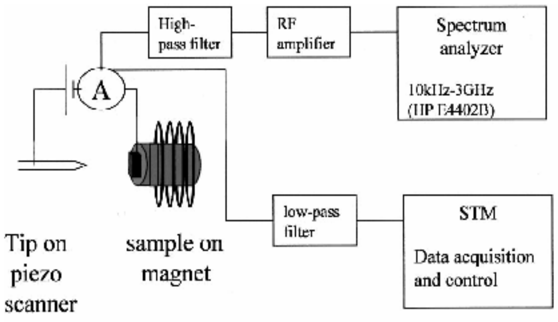


FIG. 1. Schematic of the electronics used in STM-ESR.

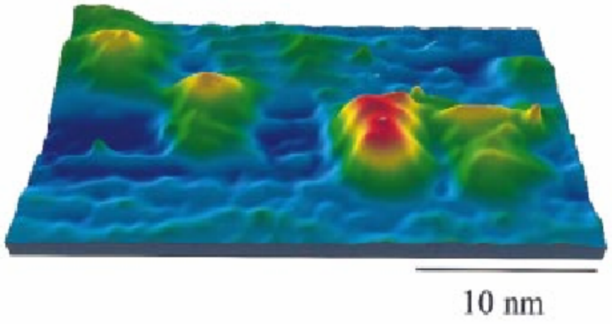


FIG. 2. (Color) STM image of a 250 Å × 150 Å area of HOPG with four adsorbed BDPA molecules.

$$\frac{\text{peak}}{\text{noise}} \leq 3.5$$

(Colm Durkan, private comm.)

Somewhat similar experiment

“Continuous monitoring of Rabi oscillations in a Josephson flux qubit”

E. Il’ichev *et al*., PRL, 2003

$$H = -\frac{1}{2}(\Delta\sigma_x + \varepsilon\sigma_z) - W\sigma_z\cos\omega_{HF}t$$

$$(\omega_{HF} \approx \sqrt{\Delta^2 + \varepsilon^2};\quad \varepsilon \neq 0)$$

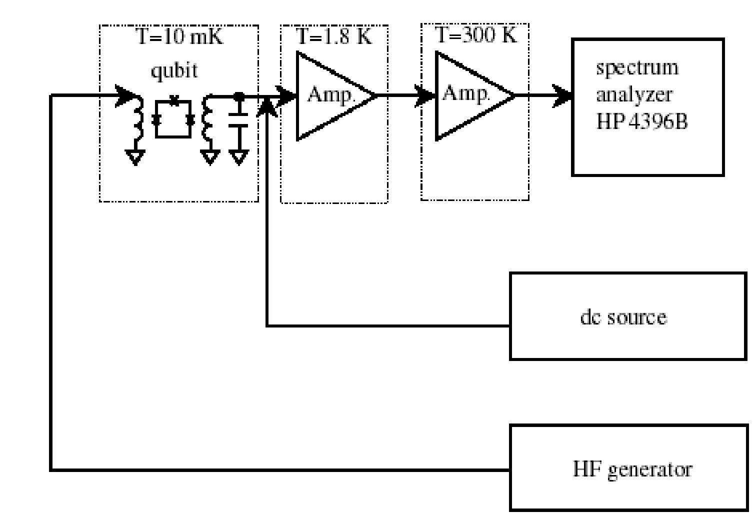


FIG. 1. Measurement setup. The flux qubit is inductively coupled to a tank circuit. The dc source applies a constant flux $\Phi_e \approx \frac{1}{2}\Phi_0$. The HF generator drives the qubit through a separate coil at a frequency close to the level separation $\Delta/h = 868$ MHz. The output voltage at the resonant frequency of the tank is measured as a function of HF power.

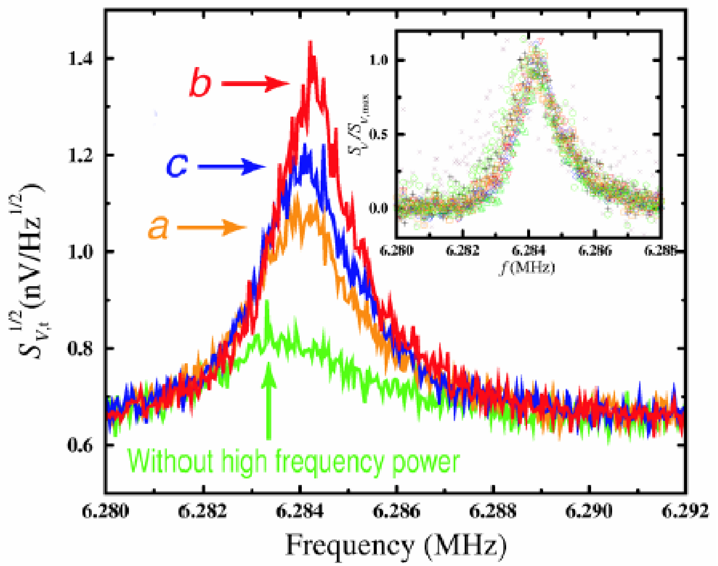
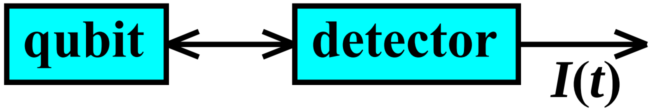


FIG. 3 (color online). The spectral amplitude of the tank voltage for HF powers $P_a < P_b < P_c$ at 868 MHz, detected using the setup of Fig. 1. The bottom curve corresponds to the background noise without an HF signal. The inset shows normalized voltage spectra for seven values of HF power, with background subtracted. The shape of the resonance, being determined by the tank circuit, is essentially the same in each case. Remaining tiny variations visible in the main figure are due to the irradiated qubit modifying the tank’s inductance and

Bell-type (Leggett-Garg-type) inequalities for continuous measurement of a qubit

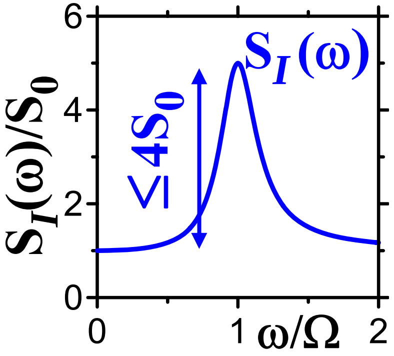


Ruskov-A.K.-Mizel, PRL-2006
Jordan-A.K.-Büttiker, PRL-2006

Assumptions of macrorealism
(similar to Leggett-Garg'85):

$$I(t) = I_0 + (\Delta I/2)\,Q(t) + \xi(t)$$

$$|Q(t)| \leq 1, \quad \langle \xi(t)\, Q(t+\tau)\rangle = 0$$



Then for correlation function

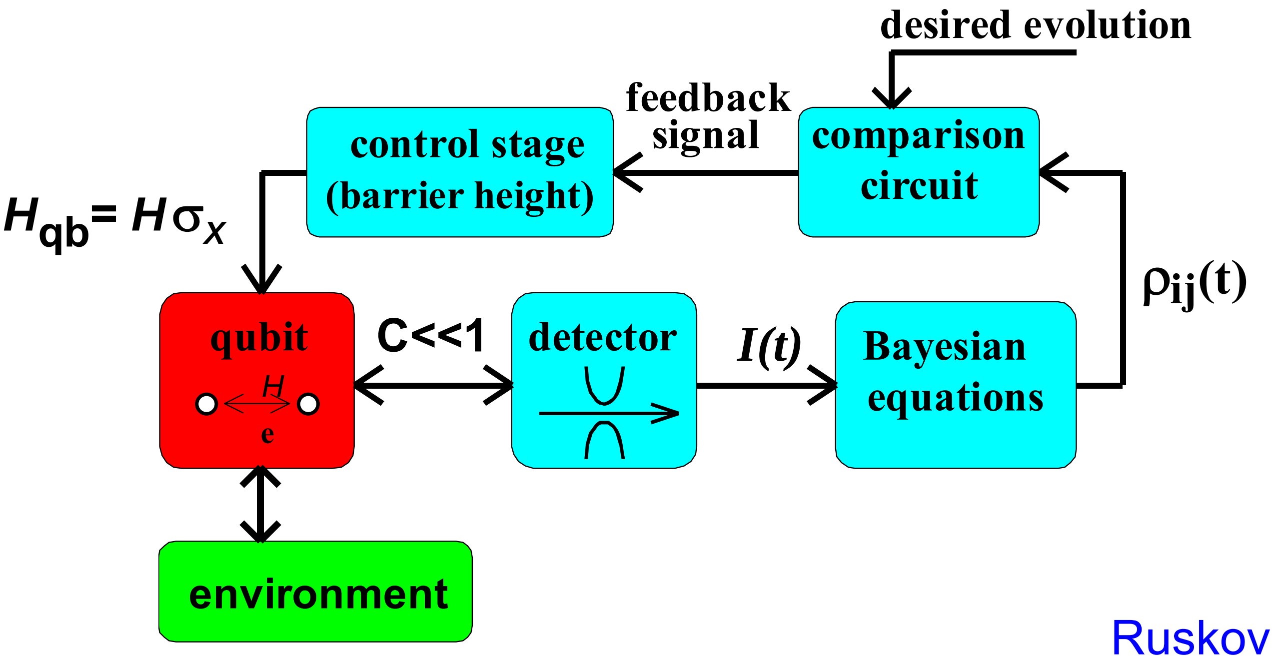
$$K(\tau) = \langle I(t)\, I(t+\tau)\rangle$$

	quantum result	violation
$K(\tau_1) + K(\tau_2) - K(\tau_1+\tau_2) \leq (\Delta I/2)^2$	$\frac{3}{2}(\Delta I/2)^2$	$\times\frac{3}{2}$
and for area under spectral peak $\int [S_I(f) - S_0]\,df \leq (8/\pi^2)(\Delta I/2)^2$	$(\Delta I/2)^2$	$\times\frac{8}{\pi^2}$

Experimentally measurable violation of classical bound

Quantum feedback control of a qubit

Since qubit state can be monitored, the feedback is possible!



Ruskov & A.K., 2001

Goal: maintain perfect Rabi oscillations forever

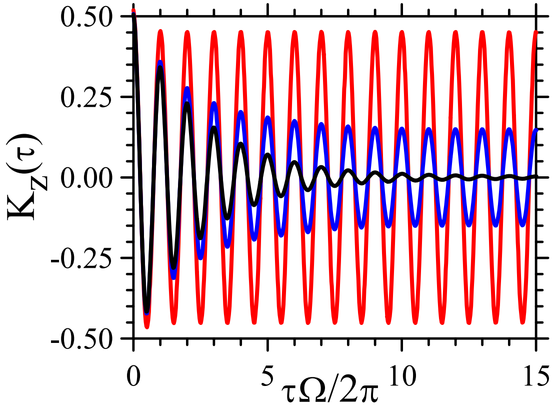
Idea: monitor the Rabi phase ϕ by continuous measurement and apply feedback control of the qubit barrier height, $\Delta H_{FB}/H = -F\times\Delta\phi$

To monitor phase ϕ we plug detector output $I(t)$ into Bayesian equations

Performance of quantum feedback

Qubit correlation function

C=1, η=1, F=0, 0.05, 0.5



$$K_z(\tau) = \frac{\cos\Omega t}{2}\ \exp\left[\frac{C}{16F}(e^{-2FH\tau/\hbar} - 1)\right]$$

$C = \hbar(\Delta I)^2 / S_I H$ – coupling

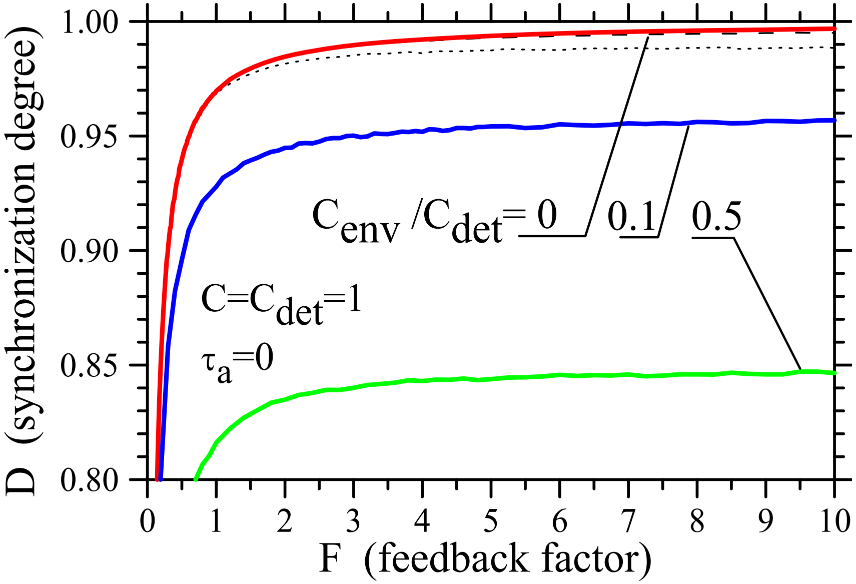
F – feedback strength

$D = 2\langle \mathrm{Tr}\rho\rho_{\mathrm{desir}}\rangle - 1$

For ideal detector and wide bandwidth, fidelity can be arbitrarily close to 100%

$D = \exp(-C/32F)$

Fidelity (synchronization degree)



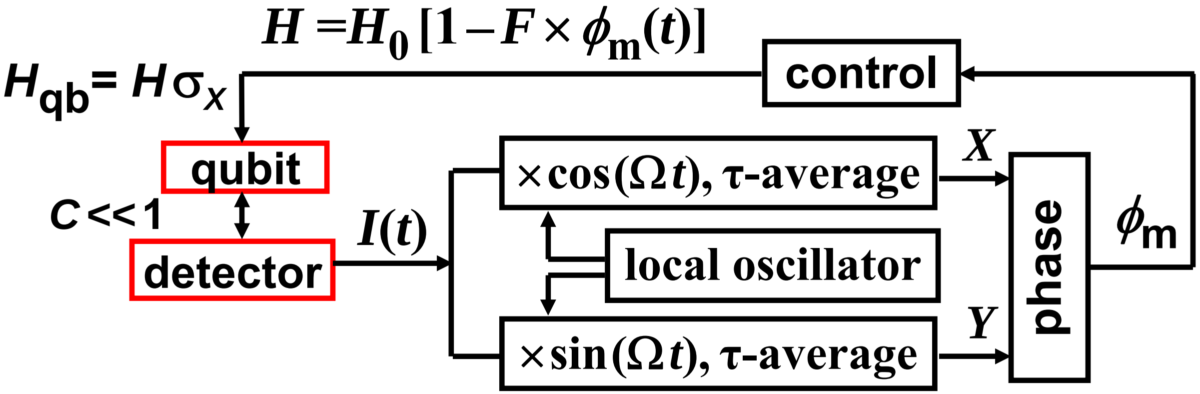
Experimental difficulties:

- necessity of **very fast** real-time solution of Bayesian equations
- **wide bandwidth** (>>Ω, GHz-range) of the line delivering noisy signal $I(t)$ to the "processor"

Ruskov & A.K., PRB-2002

Simple quantum feedback of a solid-state qubit

(A.K., 2005)



Goal: maintain coherent (Rabi) oscillations for arbitrarily long time

Idea: **use two quadrature components of the detector current *I*(*t*) to monitor approximately the phase of qubit oscillations**
(a very natural way for usual classical feedback!)

$$X(t) = \int_{-\infty}^{t} [I(t') - I_0] \cos(\Omega t') \exp[-(t-t')/\tau] \, dt$$
$$Y(t) = \int_{-\infty}^{t} [I(t') - I_0] \sin(\Omega t') \exp[-(t-t')/\tau] \, dt$$

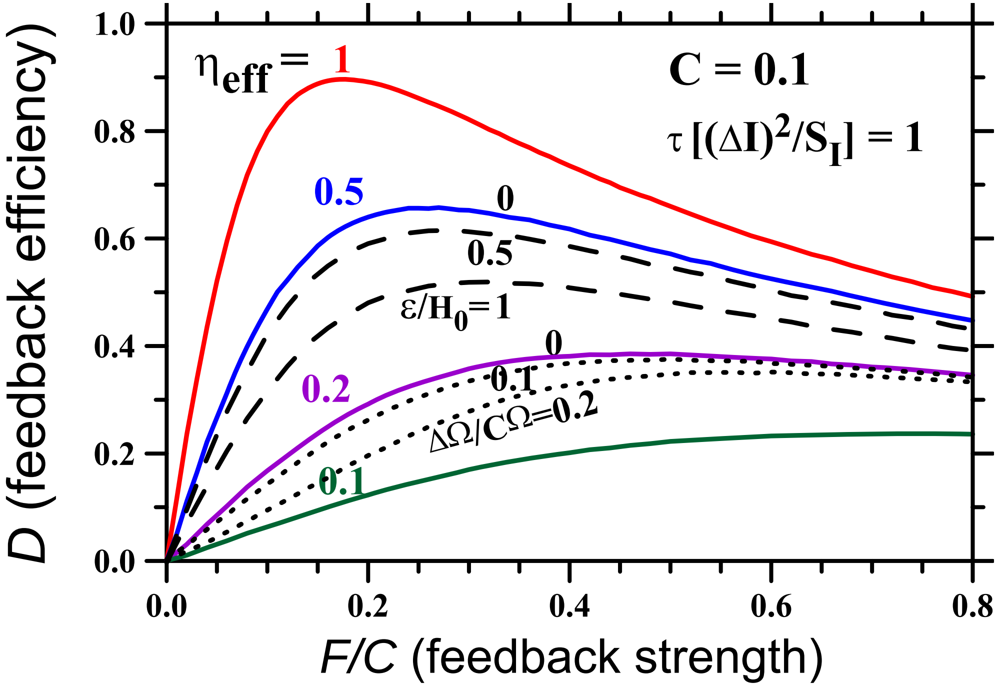
$$\phi_m = -\arctan(Y/X)$$

(similar formulas for a tank circuit instead of mixing with local oscillator)

Advantage: **simplicity and relatively narrow bandwidth** $(1/\tau \sim \Gamma_d << \Omega)$

Essentially classical feedback. Does it really work?

Fidelity of simple quantum feedback



$D_{max} \approx 90\%$

$$D \equiv 2F_Q - 1$$

$$F_Q \equiv \langle \mathrm{Tr}\, \rho(t)\, \rho_{des}(t) \rangle$$

Robust to imperfections
(inefficient detector, frequency
mismatch, qubit asymmetry)

How to verify feedback operation experimentally?

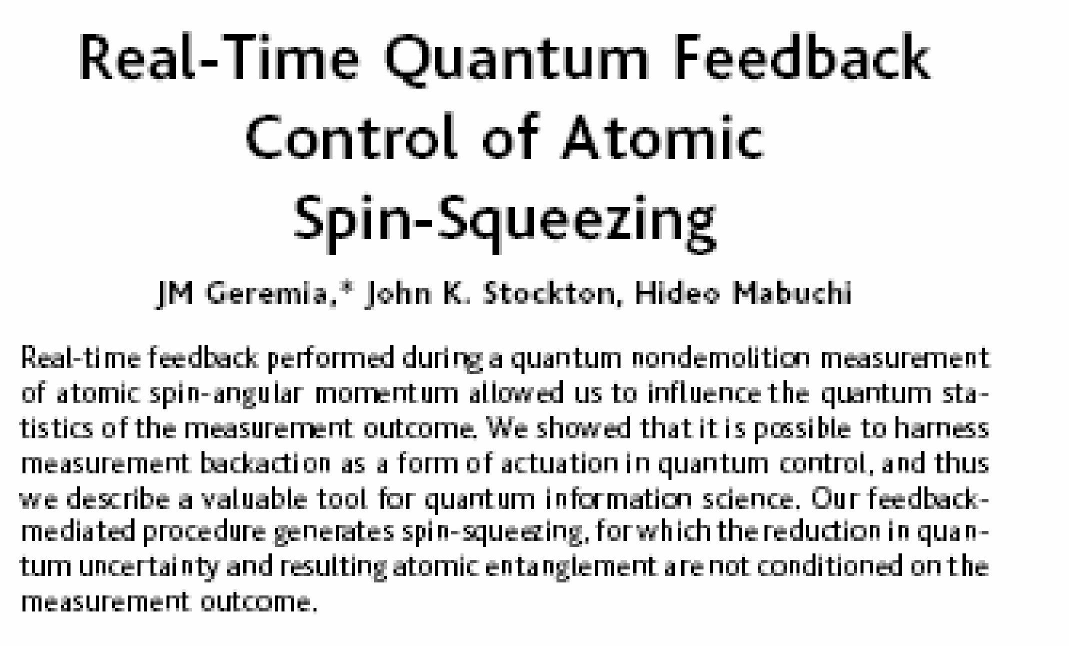
Simple: just check that in-phase quadrature $\langle X \rangle$
of the detector current is positive $D = \langle X \rangle (4/\tau\Delta I)$

$\langle X \rangle$=0 for *any* non-feedback Hamiltonian control of the qubit

Simple enough for real experiment!

Quantum feedback in optics

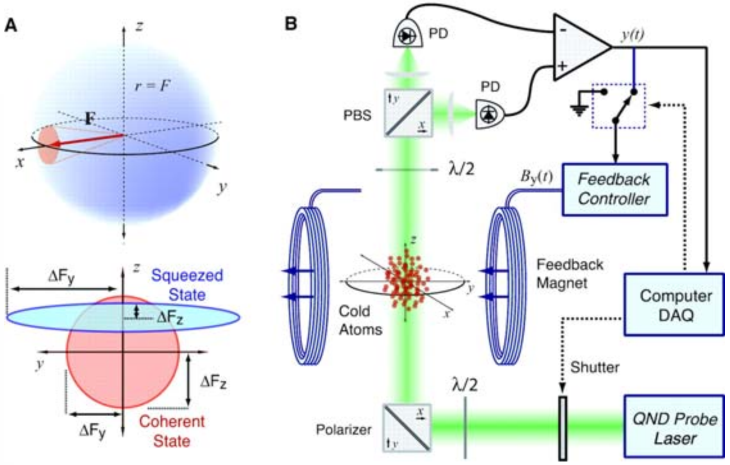
First experiment: Science 304, 270 (2004)

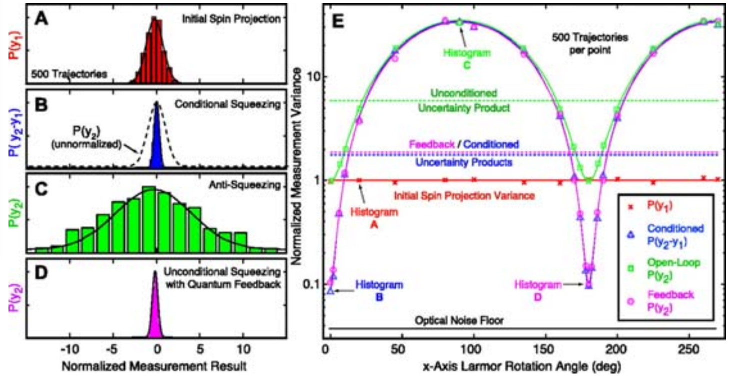


Real-Time Quantum Feedback Control of Atomic Spin-Squeezing

JM Geremia,* John K. Stockton, Hideo Mabuchi

Real-time feedback performed during a quantum nondemolition measurement of atomic spin-angular momentum allowed us to influence the quantum statistics of the measurement outcome. We showed that it is possible to harness measurement backaction as a form of actuation in quantum control, and thus we describe a valuable tool for quantum information science. Our feedback-mediated procedure generates spin-squeezing, for which the reduction in quantum uncertainty and resulting atomic entanglement are not conditioned on the measurement outcome.



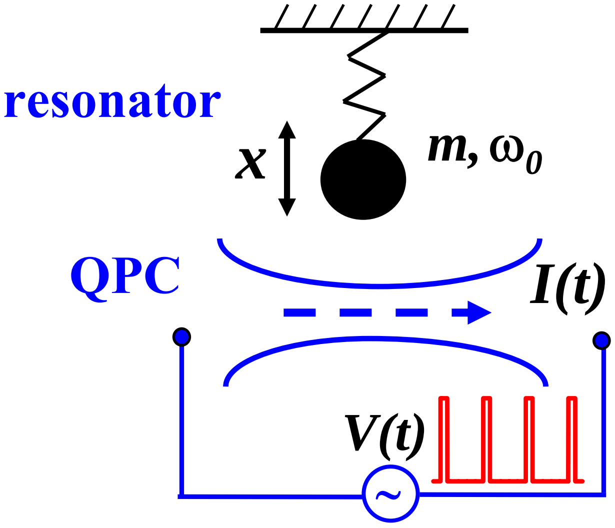


First detailed theory:

H.M. Wiseman and G. J. Milburn,
Phys. Rev. Lett. 70, 548 (**1993**)

QND squeezing of a nanomechanical resonator

Ruskov, Schwab, Korotkov, PRB-2005



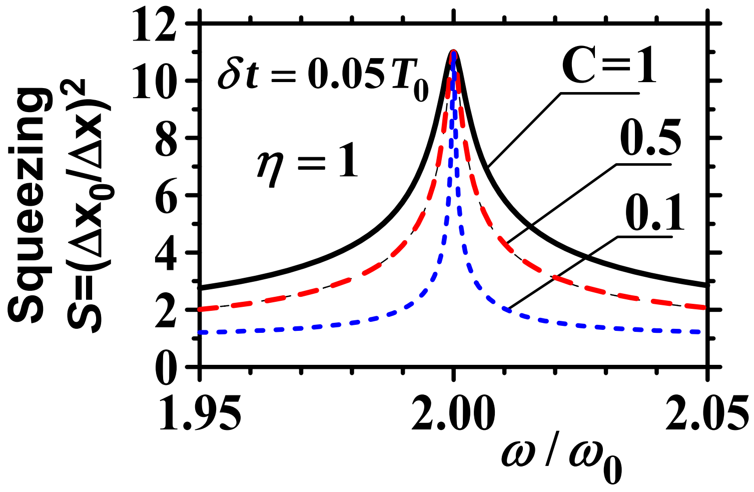
Experimental status:

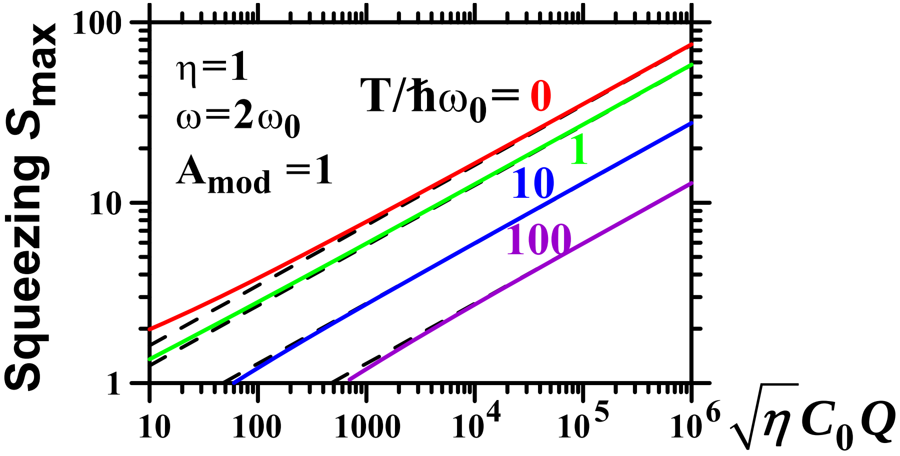
$\omega_0/2\pi \sim 1$ GHz ($\hbar\omega_0 \sim 80$ mK), Roukes' group, 2003

$\Delta x/\Delta x_0 \sim 5$ [SQL $\Delta x_0=(\hbar/2m\omega_0)^{1/2}$], Schwab's group, 2004

$$S_{\max} = \frac{3}{4}\left[\frac{\sqrt{\eta}\,C_0 Q}{\coth(\hbar\omega_0/2T)}\right]^{1/3}$$

C_0 – coupling with detector, η – detector efficiency, T – temperature, Q – resonator Q-factor

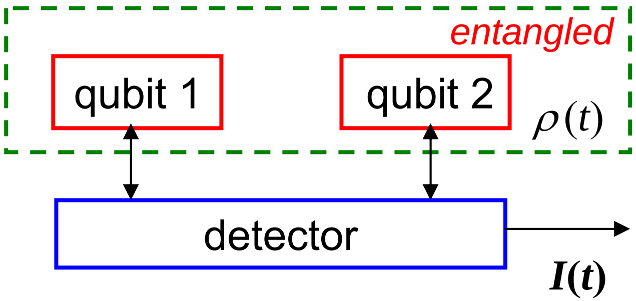




(So far in experiment $\eta^{1/2}C_0Q \sim 0.1$)

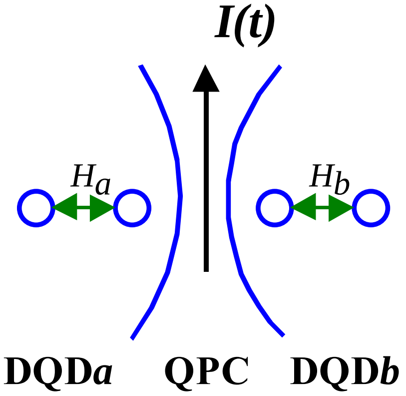
Potential application: ultrasensitive force measurements

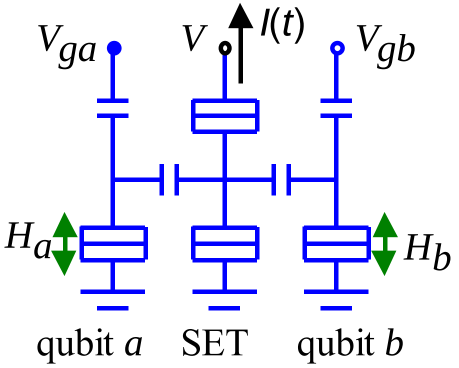
Two-qubit entanglement by measurement



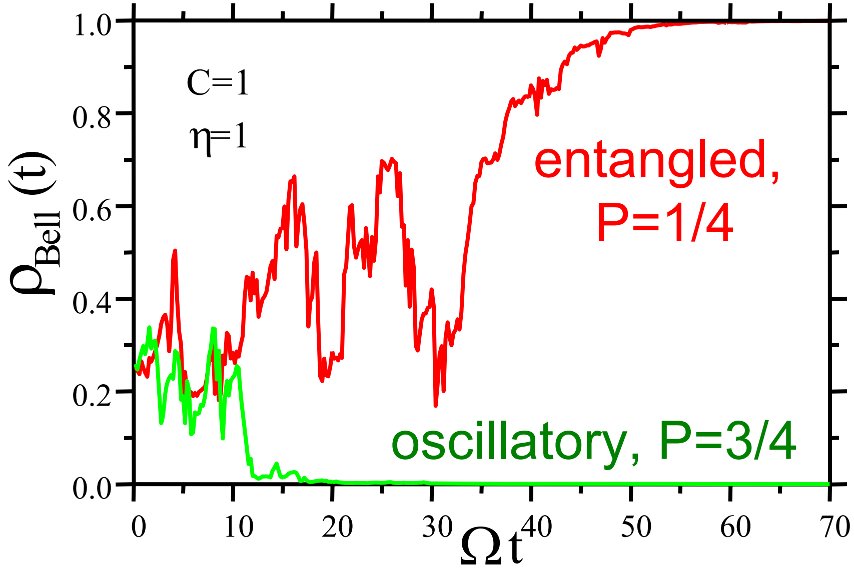
Symmetric setup, no qubit interaction

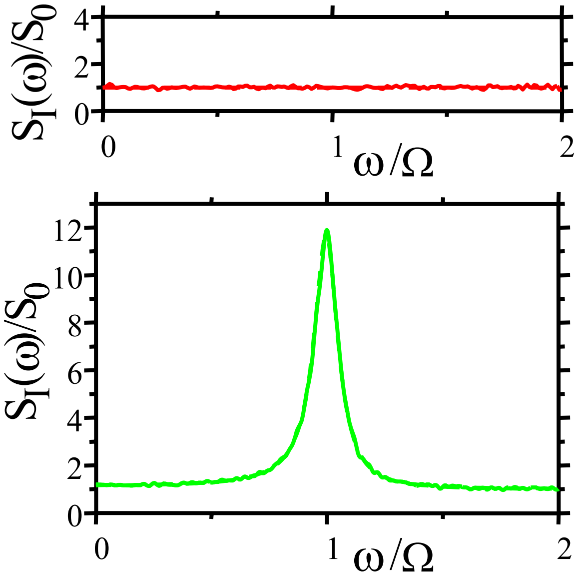
Ruskov & A.K., 2002





Two evolution scenarios:



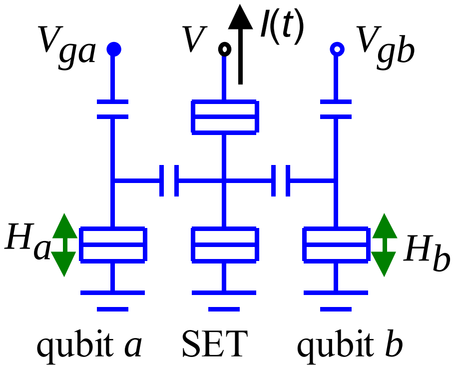


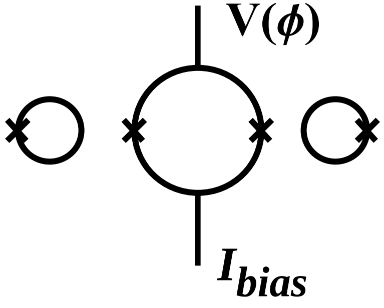
Peak/noise = (32/3)η

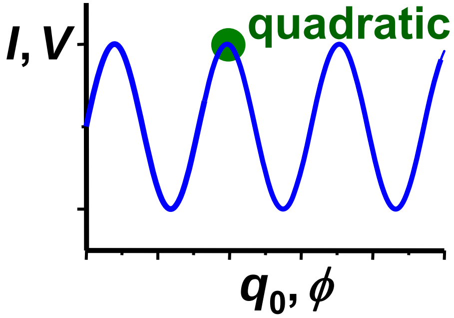
Collapse into |Bell⟩ state (spontaneous entanglement) with probability 1/4 starting from fully mixed state

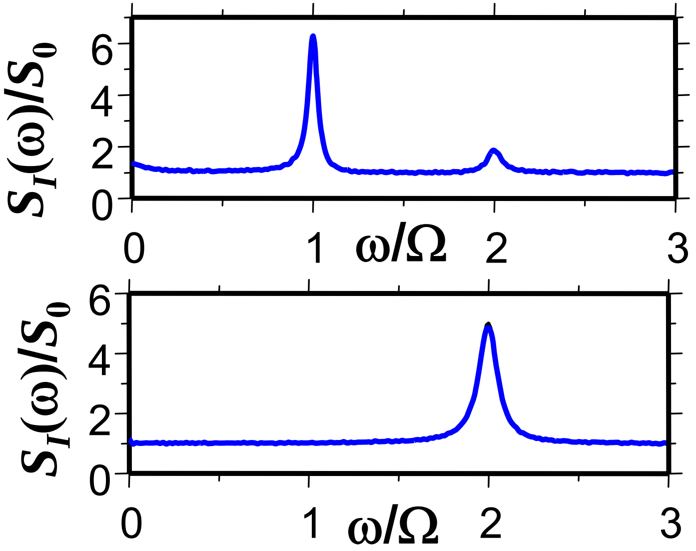
Quadratic quantum detection

Mao, Averin, Ruskov, Korotkov, PRL-2004









Nonlinear detector:

spectral peaks at Ω, 2Ω and 0

Quadratic detector:

Peak only at 2Ω, peak/noise = 4η

$$S_I(\omega) = S_0 + \frac{4\Omega^2(\Delta I)^2\Gamma}{(\omega^2-4\Omega^2)^2+\Gamma^2\omega^2}$$

Three evolution scenarios: 1) collapse into |↑↓ – ↓↑⟩, current $I_{\uparrow\downarrow}$, flat spectrum 2) collapse into |↑↑ – ↓↓⟩, current $I_{\uparrow\uparrow}$, flat spectrum; 3) collapse into remaining subspace, current $(I_{\uparrow\downarrow}+I_{\uparrow\uparrow})/2$, spectral peak at 2Ω

Entangled states distinguished by average detector current

Undoing a weak measurement of a qubit

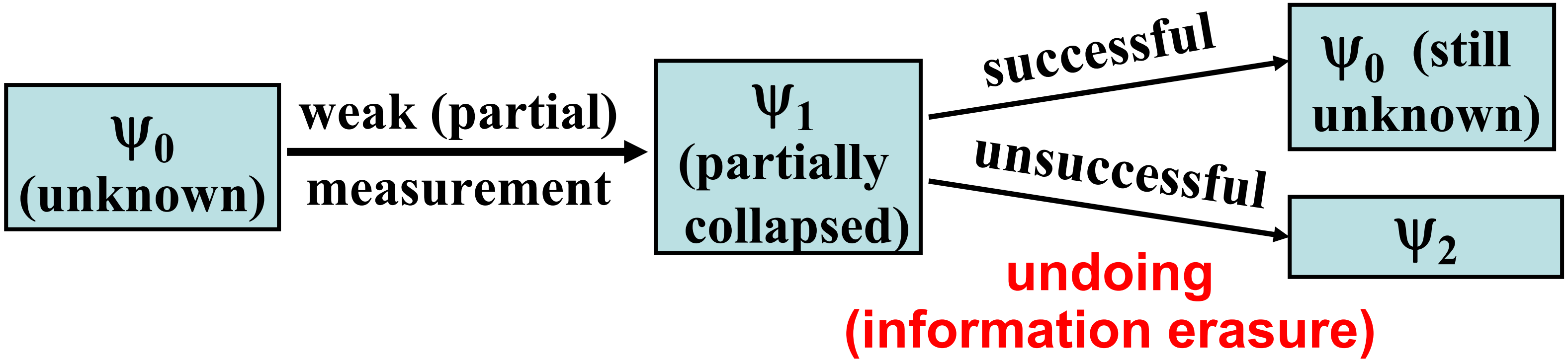
A.K. & Jordan, PRL-2006

It is impossible to undo “orthodox” quantum measurement (for an unknown initial state)

Is it possible to undo partial quantum measurement?
(To restore a “precious” qubit accidentally measured)

Yes! (but with a finite probability)

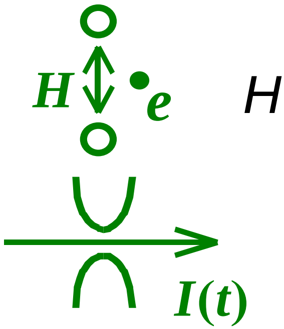
If undoing is successful, an unknown state is **fully** restored



undoing (information erasure)

“Quantum Un-Demolition (QUD) measurement”

Evolution of a charge qubit



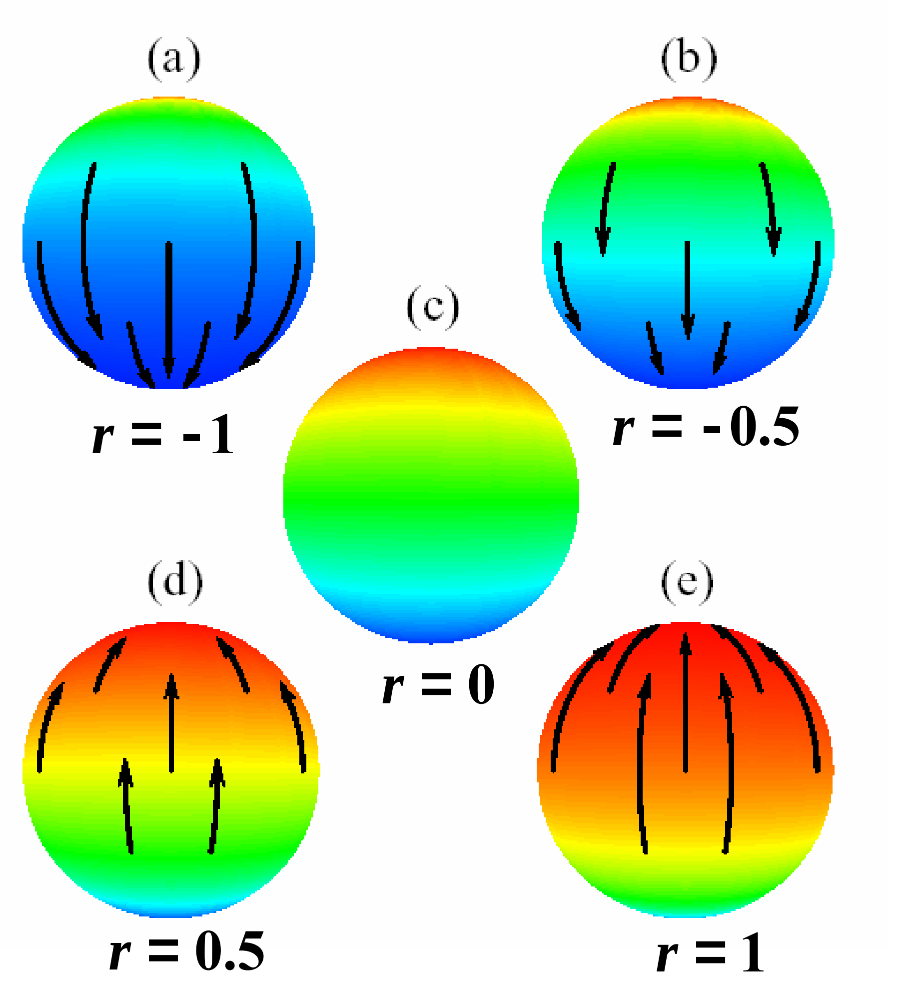
$H=0$

$$\frac{\rho_{11}(t)}{\rho_{22}(t)} = \frac{\rho_{11}(0)}{\rho_{22}(0)}\exp[2r(t)]$$

$$\frac{\rho_{12}(t)}{\sqrt{\rho_{11}(t)\rho_{22}(t)}} = \text{const}$$

where measurement result $r(t)$ is

$$r(t) = \frac{\Delta I}{S_I}[\int_0^t I(t')\,dt' - I_0 t]$$



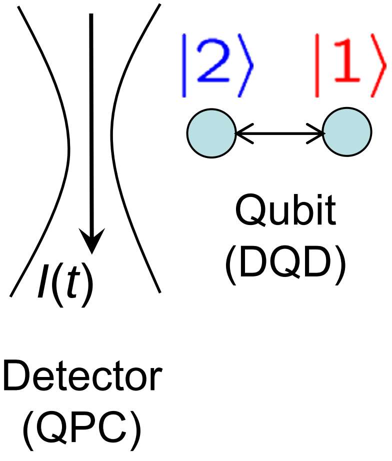
$r = -1$ $r = -0.5$ $r = 0$ $r = 0.5$ $r = 1$

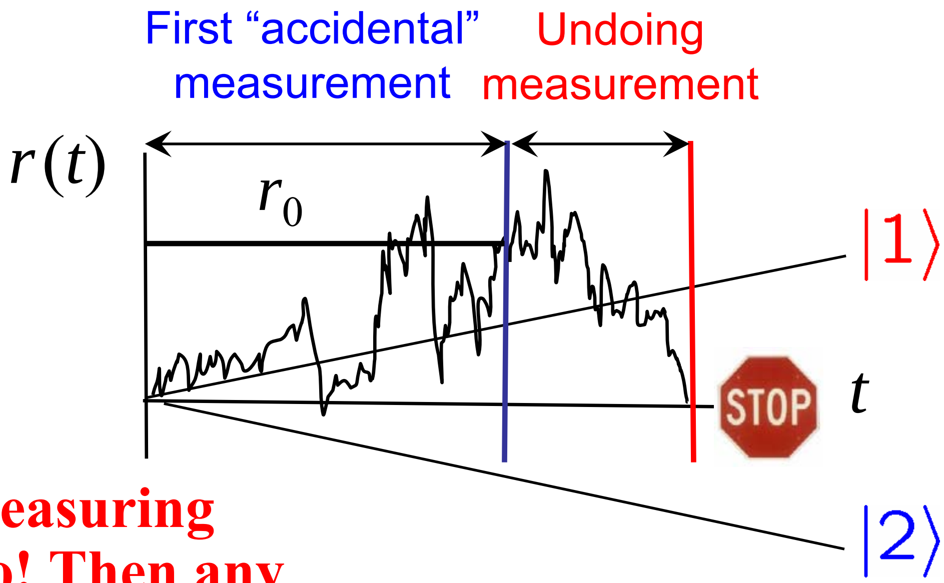
Jordan-Korotkov-Büttiker, PRL-06

If $r = 0$, then no information and no evolution!

Measurement undoing for DQD-QPC system

A.K. & Jordan, PRL-2006





Simple strategy: continue measuring until result *r*(*t*) becomes zero! Then any unknown initial state is fully restored.

$$r(t) = \frac{\Delta I}{S_I}\left[\int_0^t I(t')\,dt' - I_0 t\right]$$

(same for an entangled qubit)

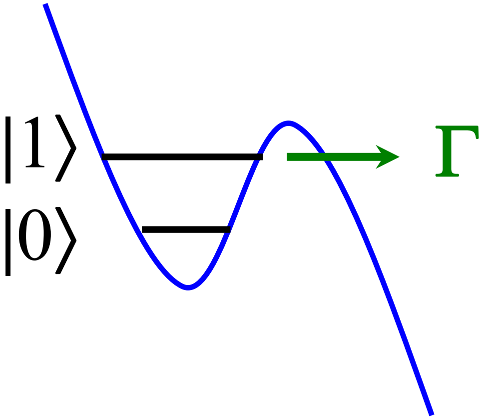
It may happen though that $r = 0$ never happens; then undoing procedure is unsuccessful.

Probability of success:

$$P_S = \frac{e^{-|r_0|}}{e^{|r_0|}\rho_{11}(0) + e^{-|r_0|}\rho_{22}(0)}$$

Partial collapse of a "phase" qubit

N. Katz, M. Ansmann, R. Bialczak, E. Lucero,
R. McDermott, M. Neeley, M. Steffen, E. Weig,
A. Cleland, J. Martinis, A. Korotkov, Science-06



How does a coherent state evolve in time before tunneling event?

(What happens when nothing happens?)

Qubit "ages" in contrast to a radioactive atom!

Main idea:

$$\psi = \alpha|0\rangle + \beta|1\rangle \;\rightarrow\; \psi(t) = \begin{cases} |out\rangle, \text{ if tunneled} \\ \dfrac{\alpha|0\rangle + \beta e^{-\Gamma t/2} e^{i\varphi}|1\rangle}{Norm}, \text{ if not tunneled} \end{cases}$$

$$Norm = \sqrt{|\alpha|^2 + |\beta|^2 e^{-\Gamma t}}$$

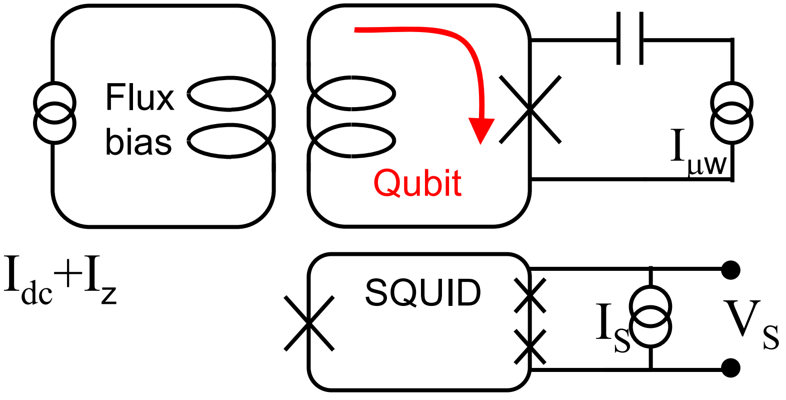
amplitude of state |0> grows without physical interaction

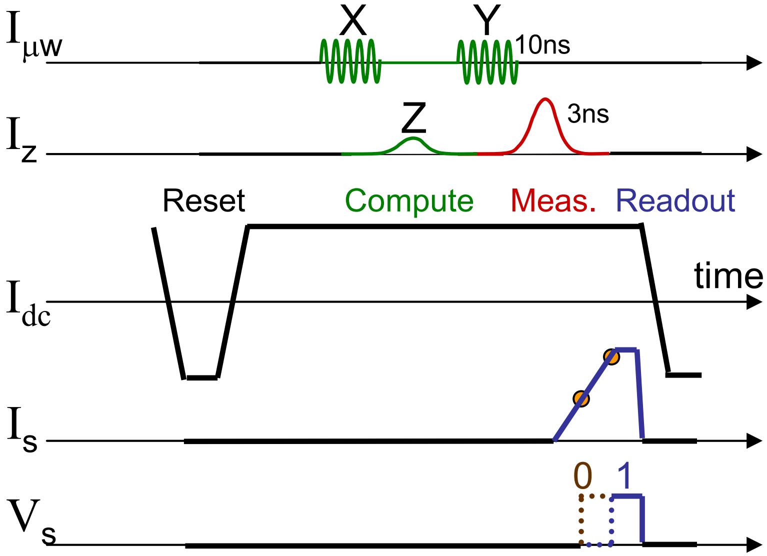
continuous null-result collapse

(similar to optics, Dalibard-Castin-Molmer, PRL-1992)

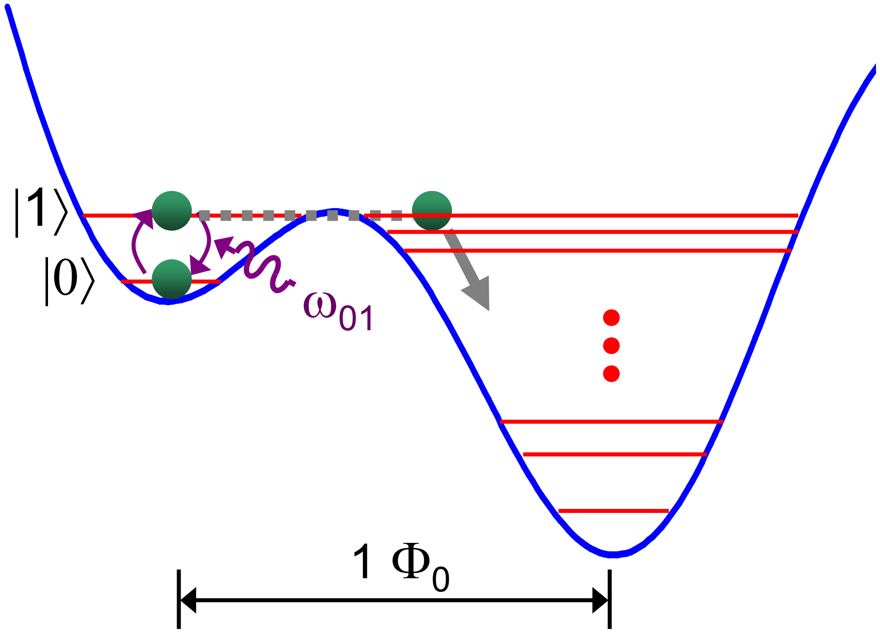
Superconducting phase qubit at UCSB

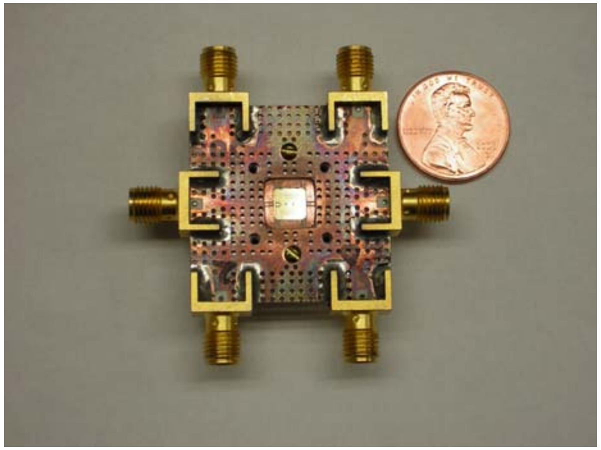
Courtesy of Nadav Katz (UCSB)



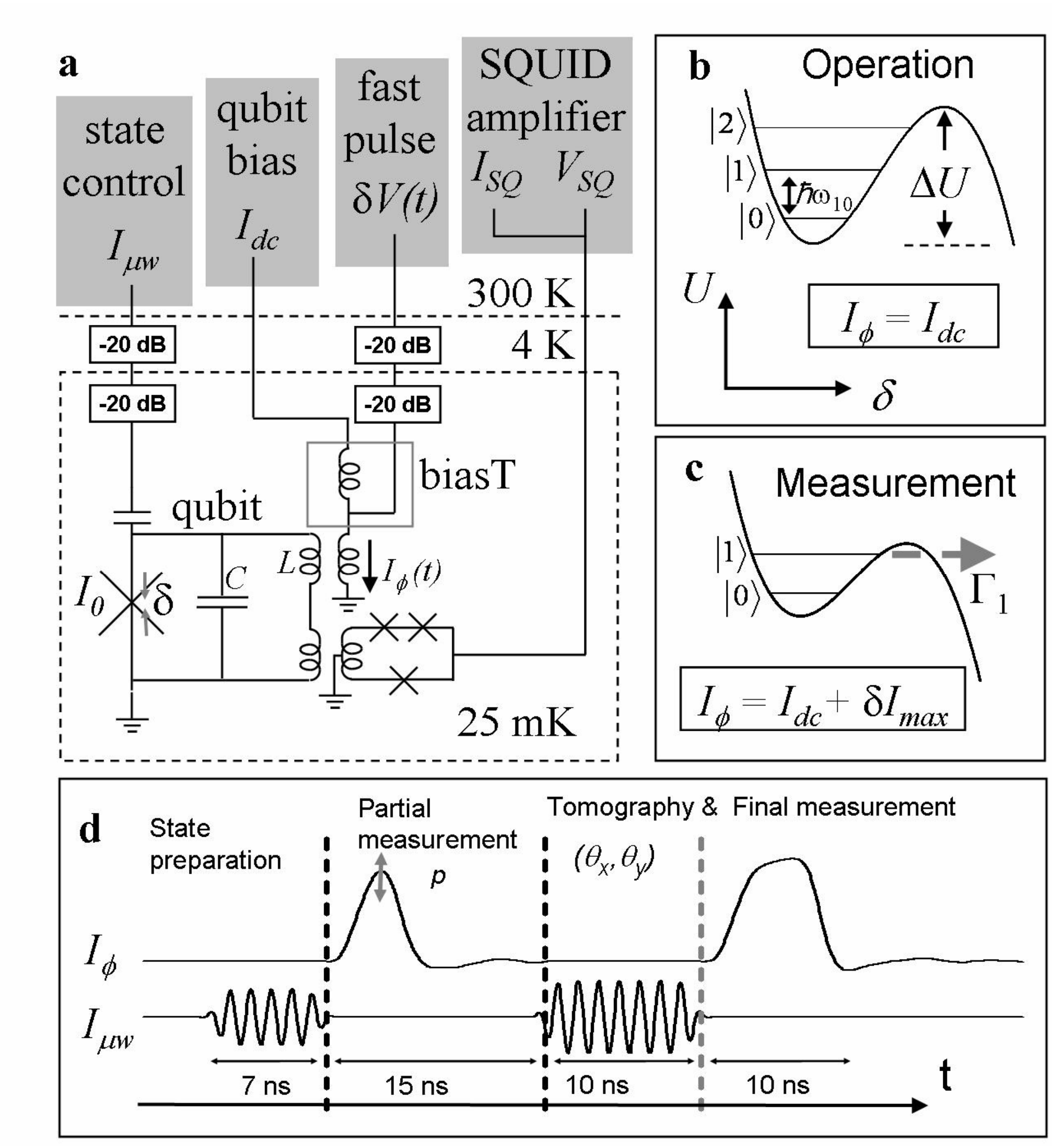


Repeat 1000x
prob. 0,1





Experimental technique for partial collapse



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(John Martinis' group)

Protocol:

1) State preparation by applying microwave pulse (via Rabi oscillations)

2) Partial measurement by lowering barrier for time *t*

3) State tomography (microwave + full measurement)

Measurement strength
$p = 1 - \exp(-\Gamma t)$
is actually controlled by Γ, not by *t*

$p=0$: no measurement
$p=1$: orthodox collapse

Experimental tomography data

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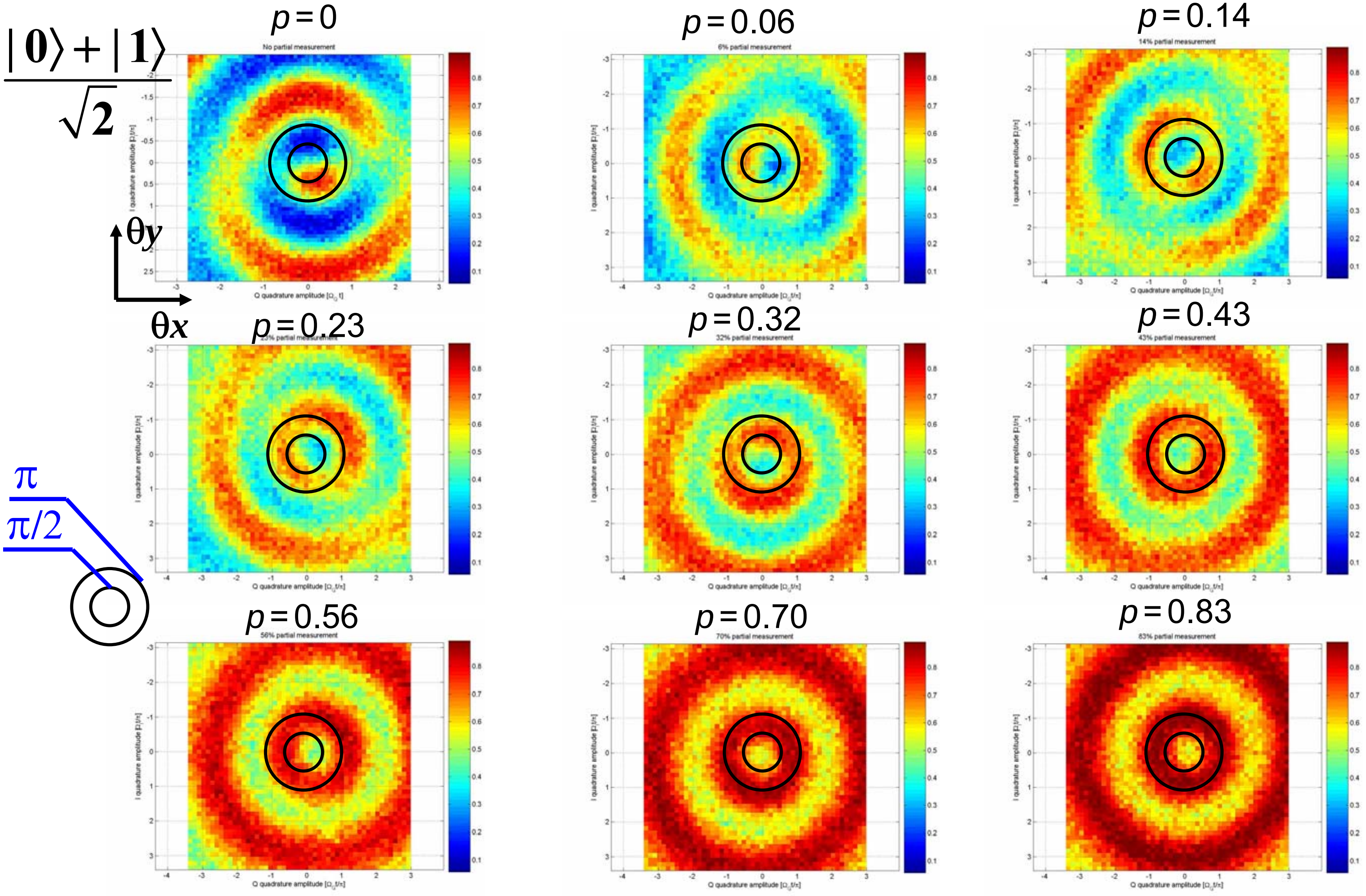
$$\psi_{in} = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$



$p=0$ No partial measurement

$p=0.06$ 6% partial measurement

$p=0.14$ 14% partial measurement

$p=0.23$ 23% partial measurement

$p=0.32$ 32% partial measurement

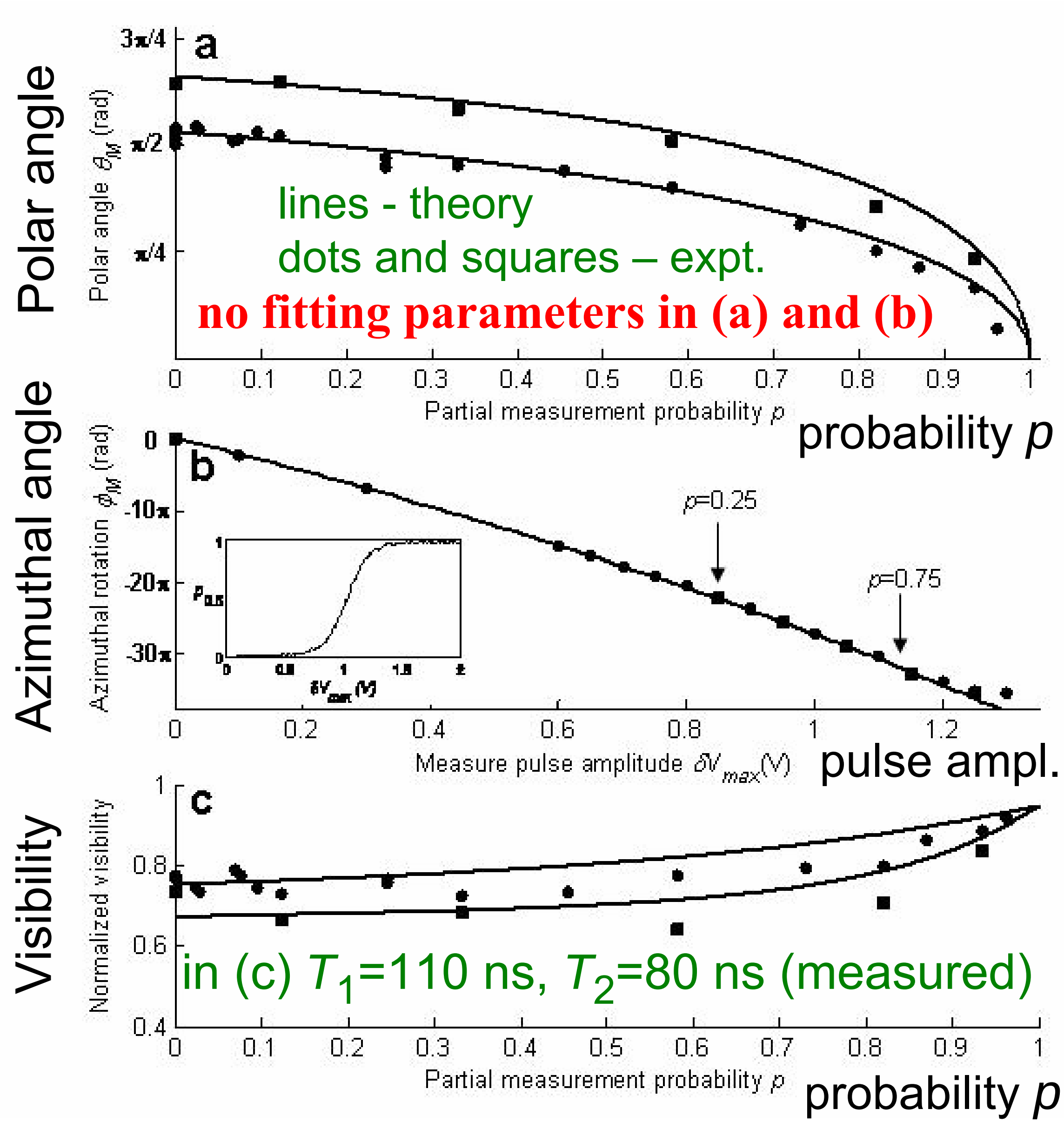
$p=0.43$ 43% partial measurement

$p=0.56$ 56% partial measurement

$p=0.70$ 70% partial measurement

$p=0.83$ 83% partial measurement

Partial collapse: experimental results



lines - theory

dots and squares – expt.

no fitting parameters in (a) and (b)

probability p

pulse ampl.

in (c) T_1=110 ns, T_2=80 ns (measured)

probability p

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- **In case of no tunneling (null-result measurement) phase qubit evolves**
- **This evolution is well described by a simple Bayesian theory, without fitting parameters**
- **Phase qubit remains fully coherent in the process of continuous collapse (experimentally ~80% raw data, ~96% after account for T1 and T2)**

Conclusions

- Bayesian approach to continuous quantum measurement is a simple, but new and interesting subject in solid-state mesoscopics
- A number of experimental predictions have been made
- Bayesian formalism can be used for the analysis of quantum feedback of solid-state qubits
- Somewhat surprisingly, a very simple, essentially classical feedback works well for Rabi oscillations of a qubit
- First direct experiment is realized (+ few indirect ones); hopefully, more experiments are coming soon