Continuous quantum measurement of solid-state systems

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Outline:

- What is "inside" wavefunction collapse (quantum Bayesian formalism)
- Experimental consequences (predictions)
- Recent experiments on partial collapse and "wavefunction uncollapsing"

Support: DTO

Motivation and relevance

- quantum measurement: necessary step in any QIP; also fundamentally interesting (still controversial after 80 years(!), becoming possible to check experimentally)
- solid-state: big potential impact on practical electronics

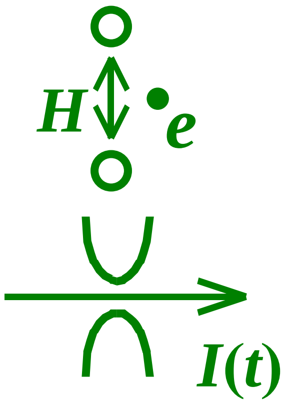
Meanings of "quantum measurement"

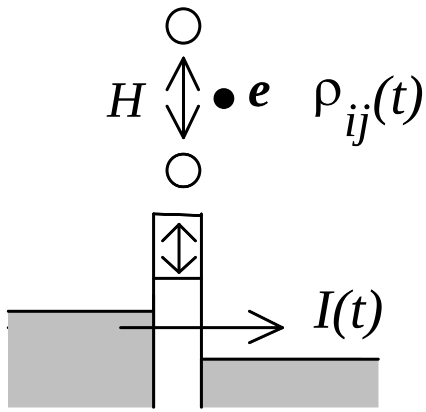
- any measurement of a quantum system
- "orthodox" projective measurement (not easy!)
- beyond-orthodox measurement (generalized, POVM, partial, continuous, weak, etc.)

Problem: How does collapse develop in time? (What is "inside" collapse?)

In solid state collapse is typically gradual $\Rightarrow$ important to know

Simple model





Double-quantum-dot (DQD) as a qubit and quantum point contact (QPC) as a detector

QPC barrier height depends on the qubit state

Coupling is weak

Detector shot noise S_I=2eI(1-T)

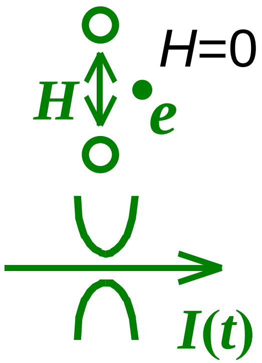
Gradual acquisition of information

⇒ continuous (weak) measurement

What is the evolution of the DQD qubit state ρ_{ij}?

- **Orthodox projection? No.** (Weak coupling, should be gradual evolution.)
- **Decoherence? No.** (Surprisingly, single qubit retains purity.)
- **Evolution is gradual, coherent, and information-related!** (A.K.,1998)

Quantum Bayesian formalism for DQD-QPC (qubit-detector) system



Qubit evolution due to continuous measurement:

1) Diagonal matrix elements of the qubit density matrix evolve as classical probabilities (i.e. according to the classical Bayes rule)

2) Non-diagonal matrix elements evolve so that the degree of purity $\rho_{ij}/[\rho_{ii}\,\rho_{jj}]^{1/2}$ is conserved

(A.K., 1998)

Bayes rule:

$$P(A_i\,|\,R) = \frac{P(A_i)\,P(R\,|\,A_i)}{\sum_k P(A_k)\,P(R\,|\,A_k)}$$

So simple because:

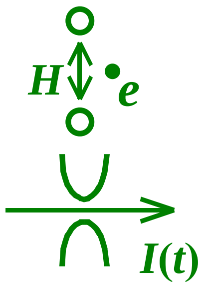
1) QPC happens to be an ideal detector
2) no Hamiltonian evolution of the qubit

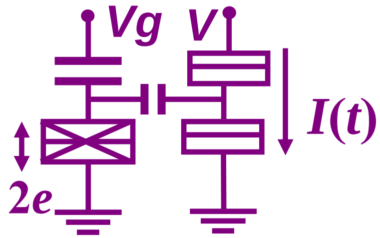
Cannot be derived from Schrödinger equation only

Similar formalisms developed earlier. Key words: Imprecise, weak, selective, or conditional measurements, POVM, Quantum trajectories, Quantum jumps, Restricted path integral, etc.

Names: Davies, Kraus, Holevo, Mensky, Caves, Gardiner, Carmichael, Plenio, Knight, Walls, Gisin, Percival, Milburn, Wiseman, Habib, etc. (very incomplete list)

Bayesian formalism for a single qubit





$$\hat{H}_{QB} = \frac{\varepsilon}{2}(c_1^\dagger c_1 - c_2^\dagger c_2) + H(c_1^\dagger c_2 + c_2^\dagger c_1)$$

$|1\rangle \to I_1$, $|2\rangle \to I_2$, $\Delta I = I_1 - I_2$, $I_0 = (I_1+I_2)/2$

S_I – detector noise

$$\dot{\rho}_{11} = -\dot{\rho}_{22} = -2(H/\hbar)\,\mathrm{Im}\,\rho_{12} + \rho_{11}\rho_{22}(2\Delta I/S_I)[\underline{\underline{I(t)}} - I_0]$$

$$\dot{\rho}_{12} = i(\varepsilon/\hbar)\rho_{12} + i(H/\hbar)(\rho_{11} - \rho_{22}) + \rho_{12}(\rho_{11} - \rho_{22})(\Delta I/S_I)[\underline{\underline{I(t)}} - I_0] - \gamma\rho_{12}$$

(A.K., 1998)

$\gamma = \Gamma - (\Delta I)^2/4S_I$, Γ – ensemble decoherence

$\eta = 1 - \gamma/\Gamma = (\Delta I)^2/4S_I\Gamma$ – detector ideality (efficiency), $\eta \le 100\%$

Ideal detector (η=1, as QPC) does not decohere a qubit, then random evolution of qubit *wavefunction* can be monitored

Averaging over result $I(t)$ leads to conventional master equation:

$$d\rho_{11}/dt = -d\rho_{22}/dt = -2(H/\hbar)\,\mathrm{Im}\,\rho_{12}$$

$$d\rho_{12}/dt = i(\varepsilon/\hbar)\rho_{12} + i(H/\hbar)(\rho_{11} - \rho_{22}) - \Gamma\rho_{12}$$

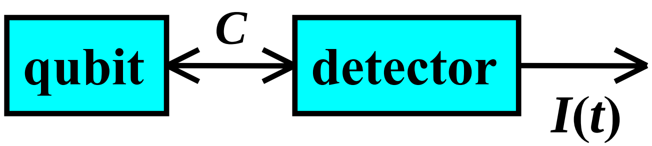
Fundamental limit: $\Gamma \geq (\Delta I)^2/4S_I$ — ensemble decoherence always faster than rate of information acquisition

Simple generalizations to entangled qubits and other systems

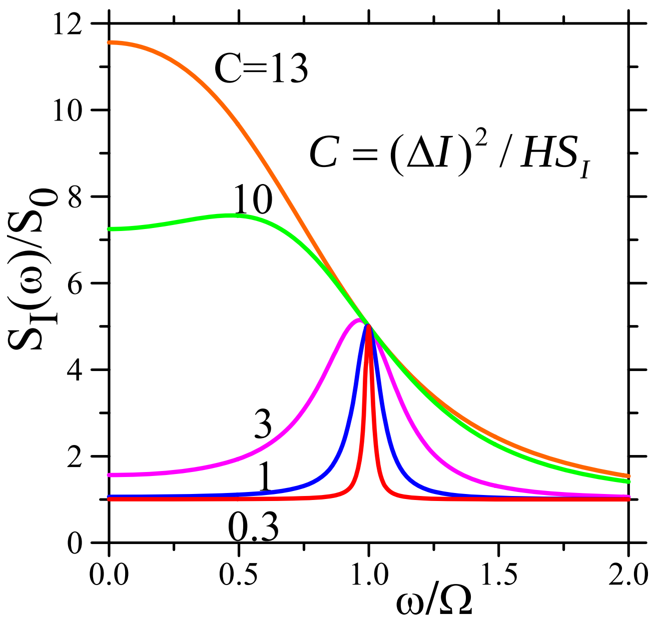
Experimental predictions and proposals from Bayesian formalism

- Direct experimental verification (1998)
- Measured spectral density of Rabi oscillations (1999, 2000, 2002)
- Bell-type correlation experiment (2000)
- Quantum feedback control of a qubit (2001)
- Entanglement by measurement (2002)
- Measurement by a quadratic detector (2003)
- Simple quantum feedback of a qubit (2004)
- Squeezing of a nanomechanical resonator (2004)
- Violation of Leggett-Garg inequality (2005)
- Partial collapse of a phase qubit (2005)
- Undoing of a weak measurement (2006)

Measured spectrum of Rabi oscillations



What is the spectral density $S_I(\omega)$ of detector current?



Assume classical output, eV » $\hbar\Omega$

$$\varepsilon = 0, \quad \Gamma = \eta^{-1}(\Delta I)^2 / 4S_0$$

$$S_I(\omega) = S_0 + \frac{\Omega^2 (\Delta I)^2 \Gamma}{(\omega^2 - \Omega^2)^2 + \Gamma^2 \omega^2}$$

Spectral peak can be seen, but peak-to-pedestal ratio ≤ 4η ≤ 4

(result can be obtained using various methods, not only Bayesian method)

A.K., LT'99
A.K.-Averin, 2000
A.K., 2000
Averin, 2000
Goan-Milburn, 2001
Makhlin et al., 2001
Balatsky-Martin, 2001
Ruskov-A.K., 2002
Mozyrsky et al., 2002
Balatsky et al., 2002
Bulaevskii et al., 2002
Shnirman et al., 2002
Bulaevskii-Ortiz, 2003
Shnirman et al., 2003
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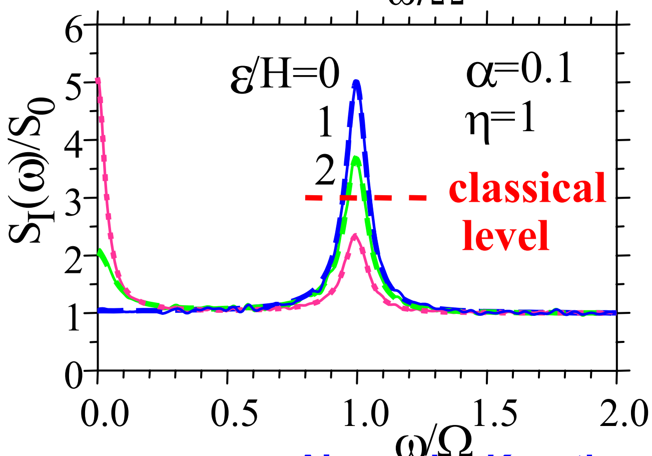
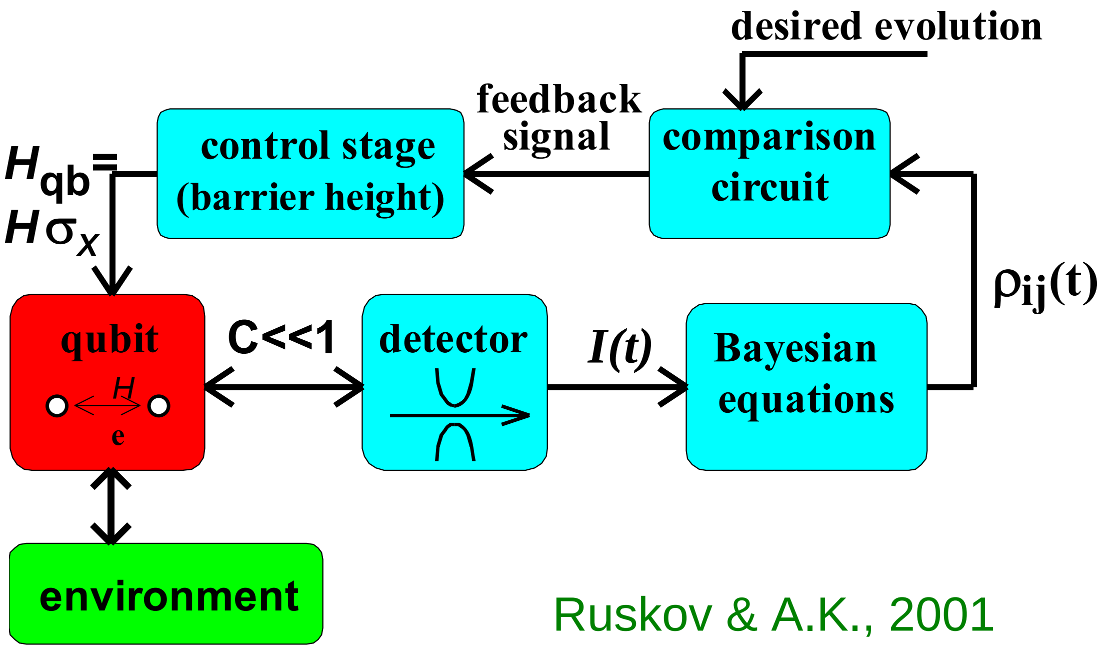


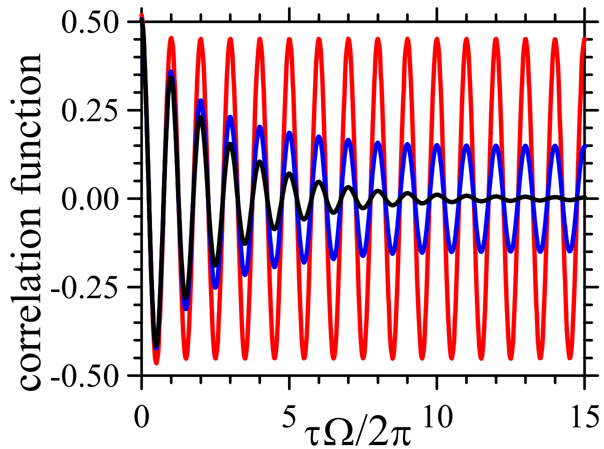
Spectral peak ⇒ Rabi oscillations persist forever if measured (but phase fluctuates)

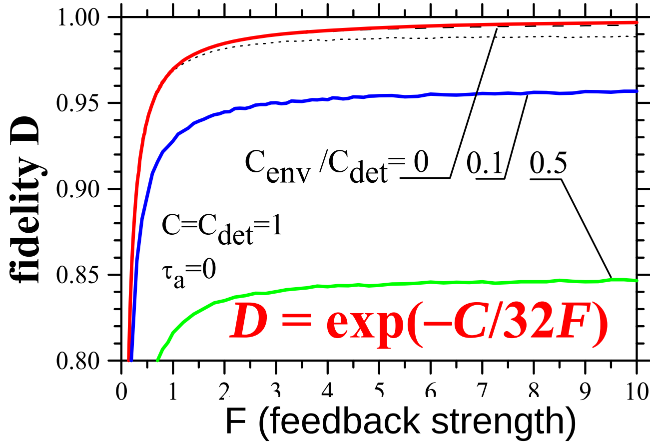
Quantum feedback control of a qubit

Since qubit state can be monitored, the feedback is possible!



Ruskov & A.K., 2001





Goal: maintain perfect Rabi oscillations

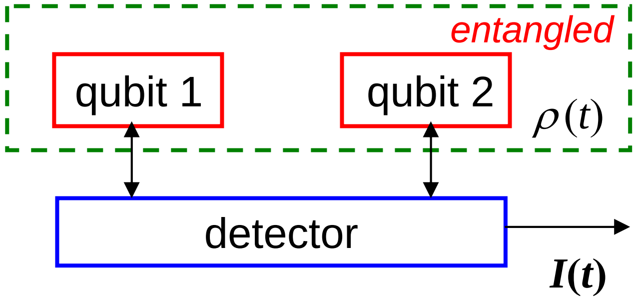
Idea: monitor the Rabi phase ϕ by continuous measurement and apply feedback control of the qubit barrier height, $\Delta H_{FB}/H = -F\times\Delta\phi$

For ideal detector and wide bandwidth, fidelity can approach 100%

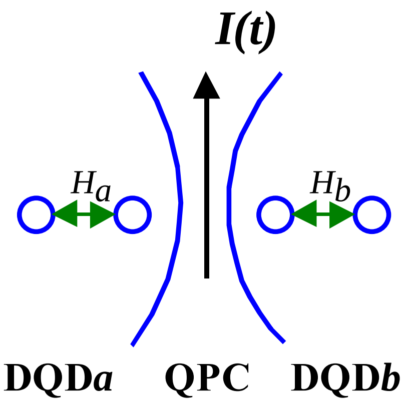
First experimental quantum feedback in optics: JM Geremia et al., Science 304, 270 (2004).

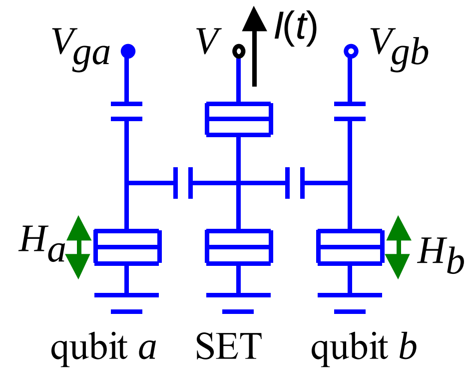
Two-qubit entanglement by measurement

Ruskov & A.K., 2002

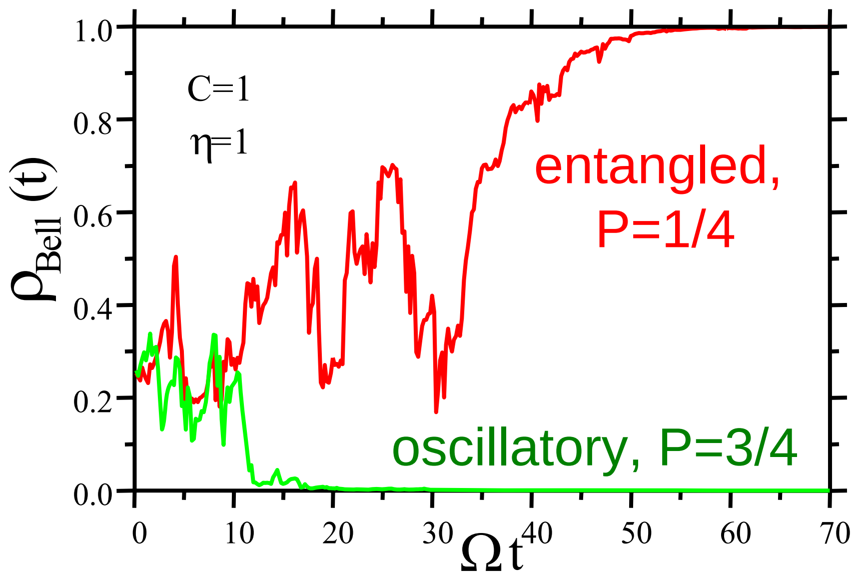


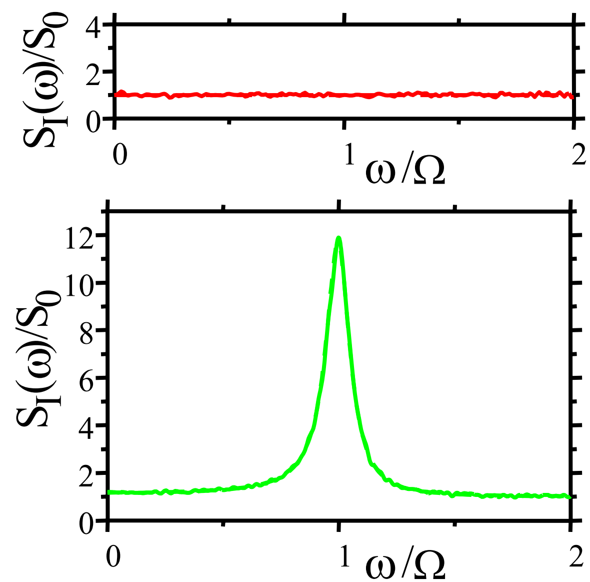
Symmetric setup, no qubit interaction





Two evolution scenarios:



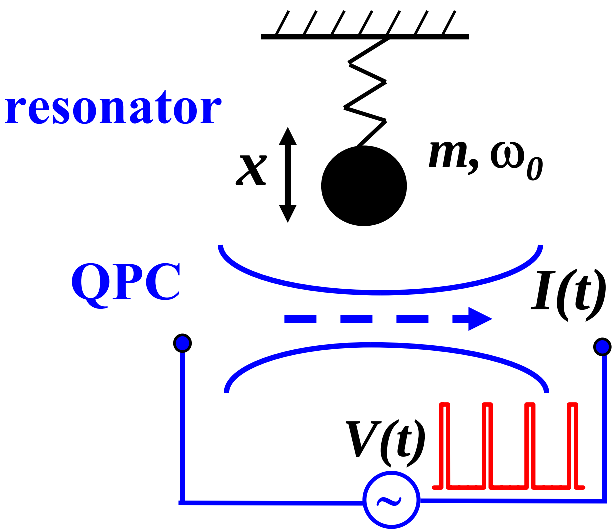


Peak/noise = (32/3) η

Collapse into |Bell⟩ state (spontaneous entanglement) with probability 1/4 starting from fully mixed state

QND squeezing of a nanomechanical resonator

Ruskov, Schwab, Korotkov, PRB-2005



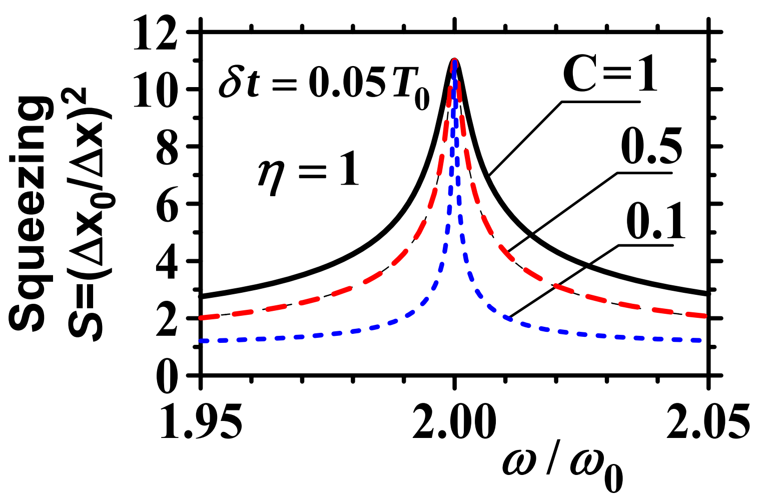
Experimental status:

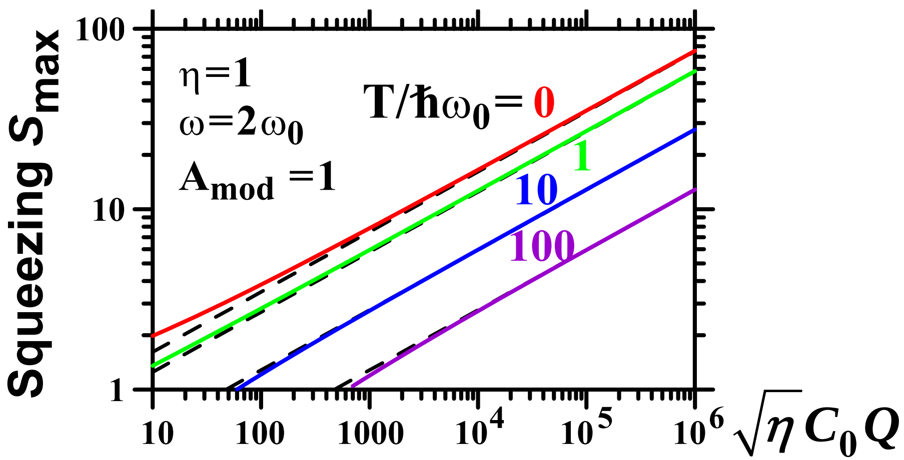
$\omega_0/2\pi \sim 1$ GHz ($\hbar\omega_0 \sim 80$ mK), Roukes' group, 2003

$\Delta x/\Delta x_0 \sim 5$ [SQL $\Delta x_0=(\hbar/2m\omega_0)^{1/2}$], Schwab's group, 2004

$$S_{max} = \frac{3}{4}\left[\frac{\sqrt{\eta}\,C_0 Q}{\coth(\hbar\omega_0/2T)}\right]^{1/3}$$

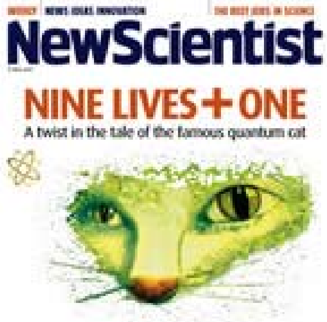
C_0 – coupling with detector, η – detector efficiency,
T – temperature, Q – resonator Q-factor





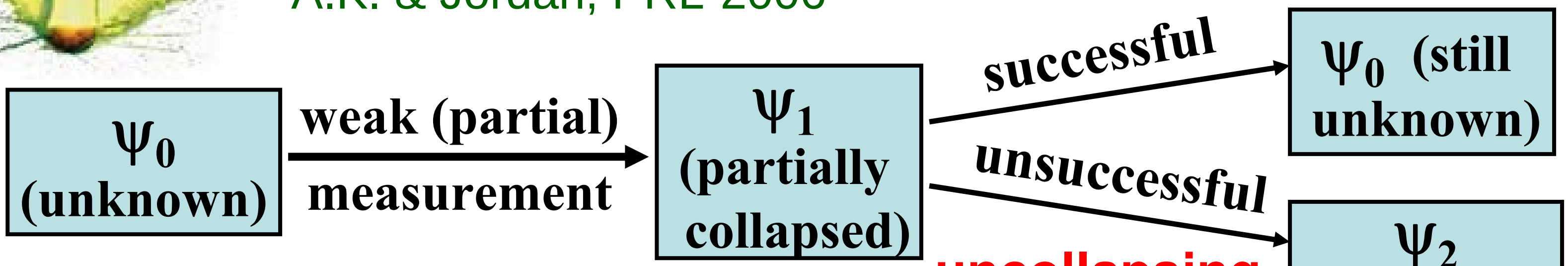
(So far in experiment $\eta^{1/2}C_0Q \sim 0.1$)

Potential application: ultrasensitive force measurements

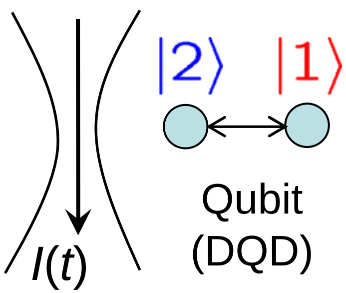


Undoing a weak measurement of a qubit
(quantum undemolition, "uncollapsing")

A.K. & Jordan, PRL-2006



uncollapsing (information erasure)

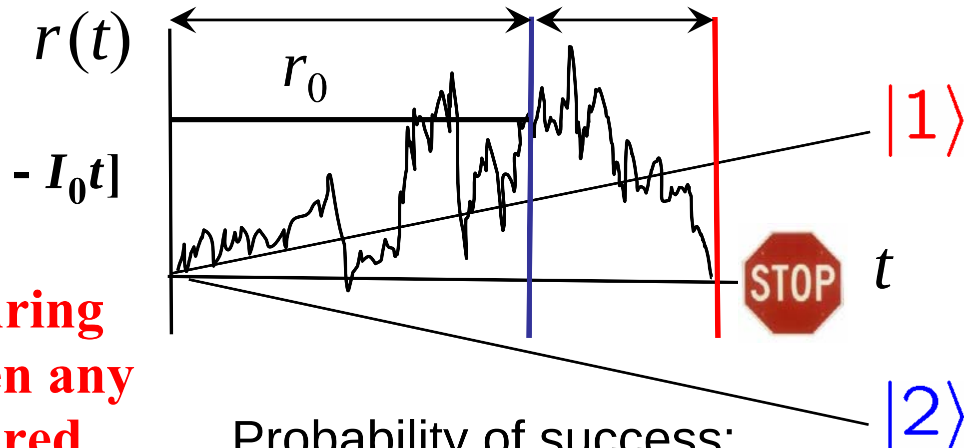


no information if r=0

$$r(t) = \frac{\Delta I}{S_I}\left[\int_0^t I(t')\,dt' - I_0 t\right]$$

First (collapsing) measurement

Uncollapsing measurement



Simple strategy: continue measuring until result $r(t)$ becomes zero! Then any unknown initial state is fully restored.

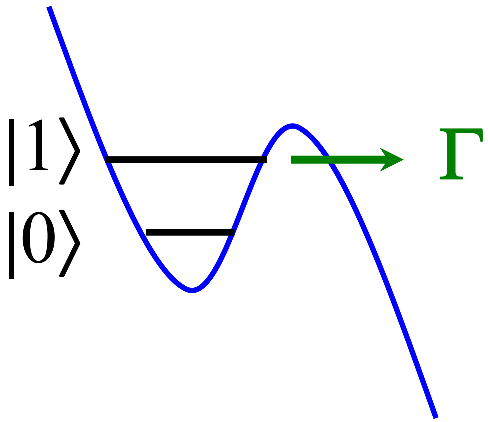
Probability of success:

$$P_S = e^{-|r_0|}/(e^{|r_0|}\rho_{11} + e^{-|r_0|}\rho_{22})$$

Experimentally realized for phase qubit

Partial collapse of a superconducting phase qubit

Katz, Ansmann, Bialczak, Lucero, McDermott, Neeley, Steffen, Weig, Cleland, Martinis, Korotkov, *Science-06*



How does a wavefunction evolve in time before tunneling event?

(What happens when nothing happens?)

Qubit "ages" in contrast to a radioactive atom!

Main idea:

$$\psi = \alpha|0\rangle + \beta|1\rangle \;\rightarrow\; \psi(t) = \begin{cases} |out\rangle, \text{ if tunneled} \\ \dfrac{\alpha|0\rangle + \beta e^{-\Gamma t/2} e^{i\varphi}|1\rangle}{\sqrt{|\alpha|^2 + |\beta|^2 e^{-\Gamma t}}}, \text{ if not tunneled} \end{cases}$$

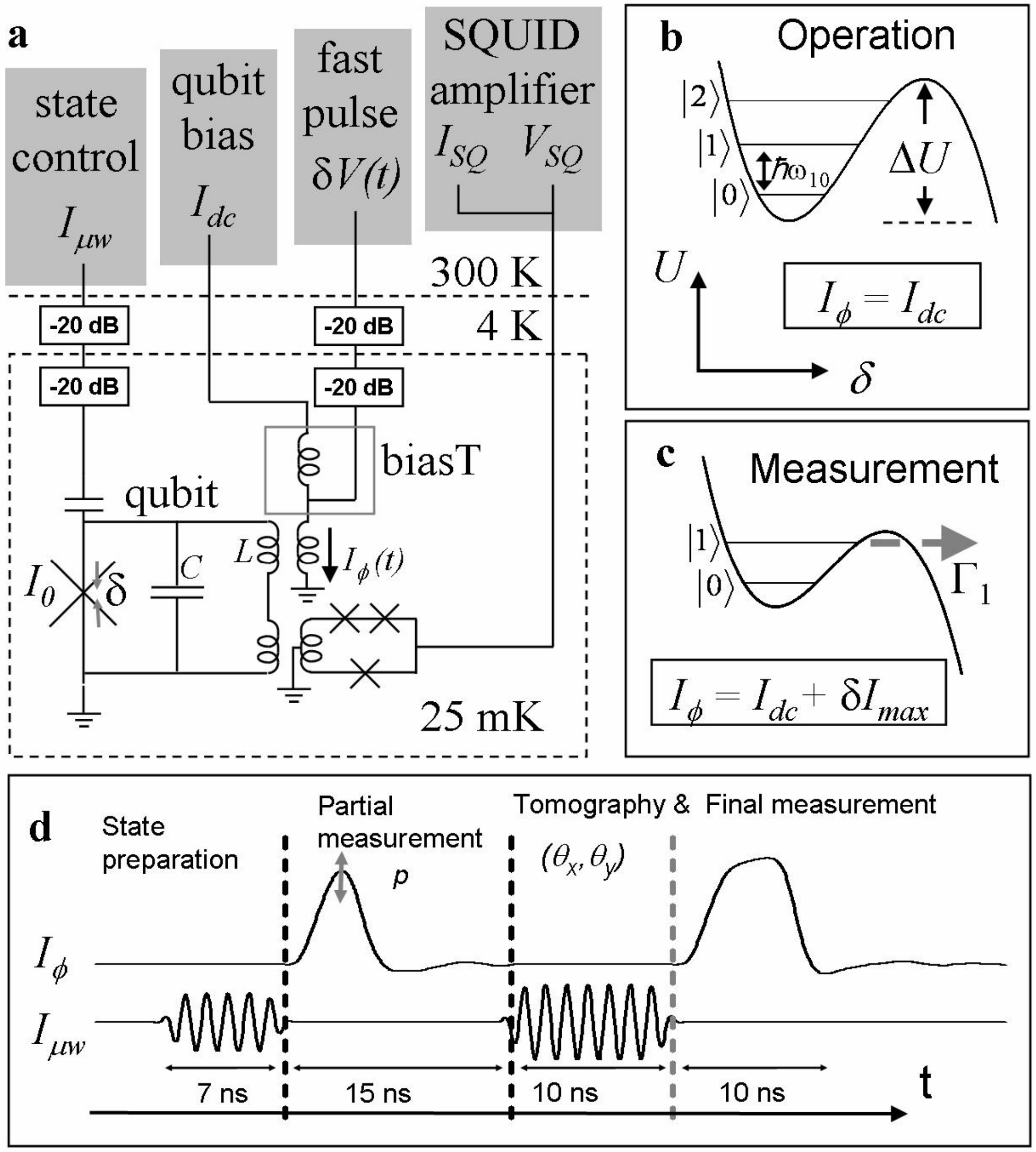
(better theory: Pryadko & A.K., 2007)

amplitude of state |0> grows without any physical interaction (!)

continuous null-result collapse

(similar to optics (never realized) Dalibard-Castin-Molmer,1992)

Experimental technique for partial collapse



Nadav Katz *et al.*
(John Martinis' group)

Protocol:

1) State preparation (microwave pulse)
2) Partial measurement by lowering barrier for time *t*
3) State tomography (micro-wave + full measurement)

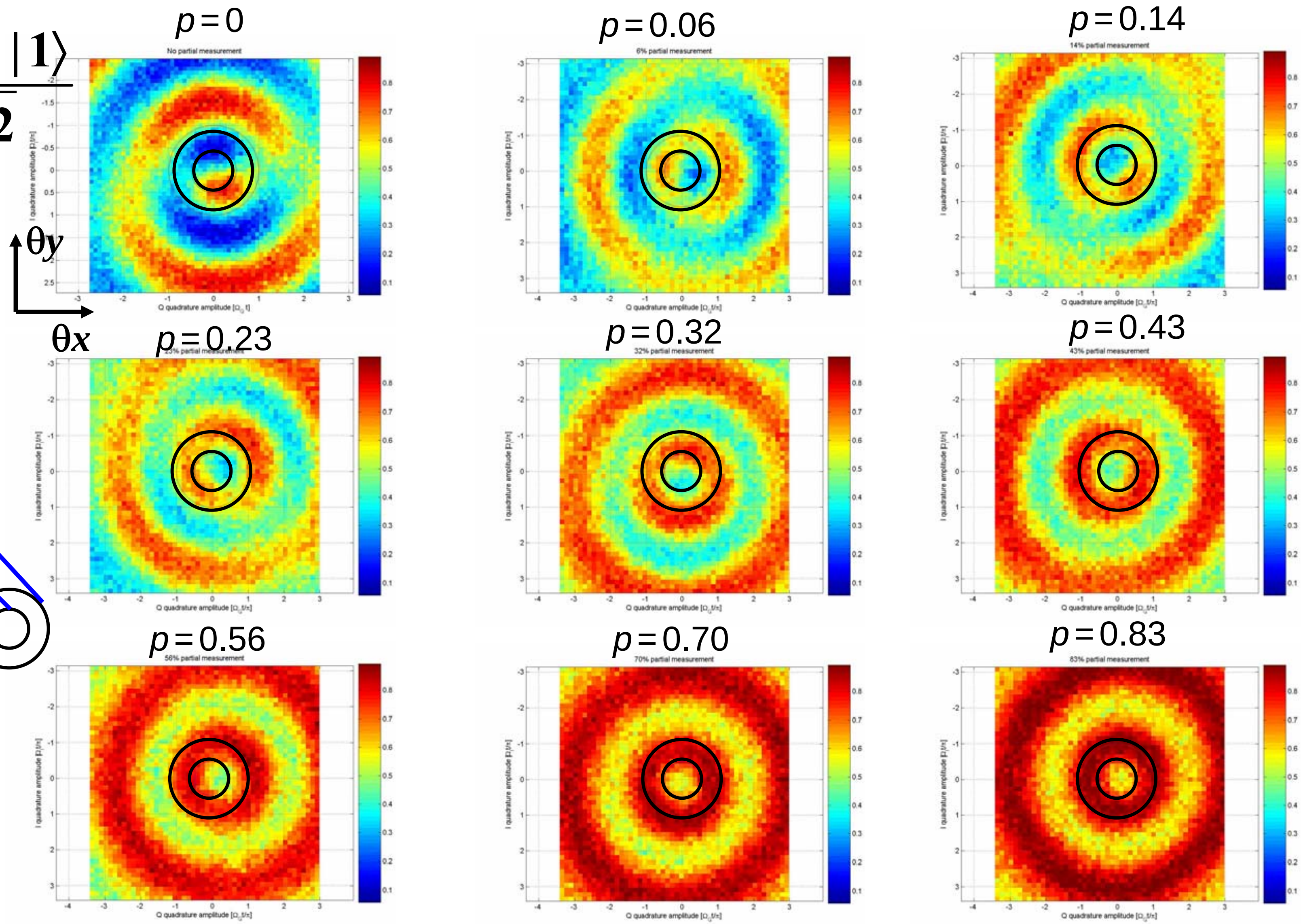
Measurement strength
$p = 1 - \exp(-\Gamma t)$
(actually controlled by Γ)

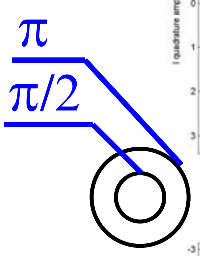
***p*=0: no measurement**
***p*=1: orthodox collapse**
anything in between!

Experimental tomography data

Nadav Katz *et al.* (UCSB)

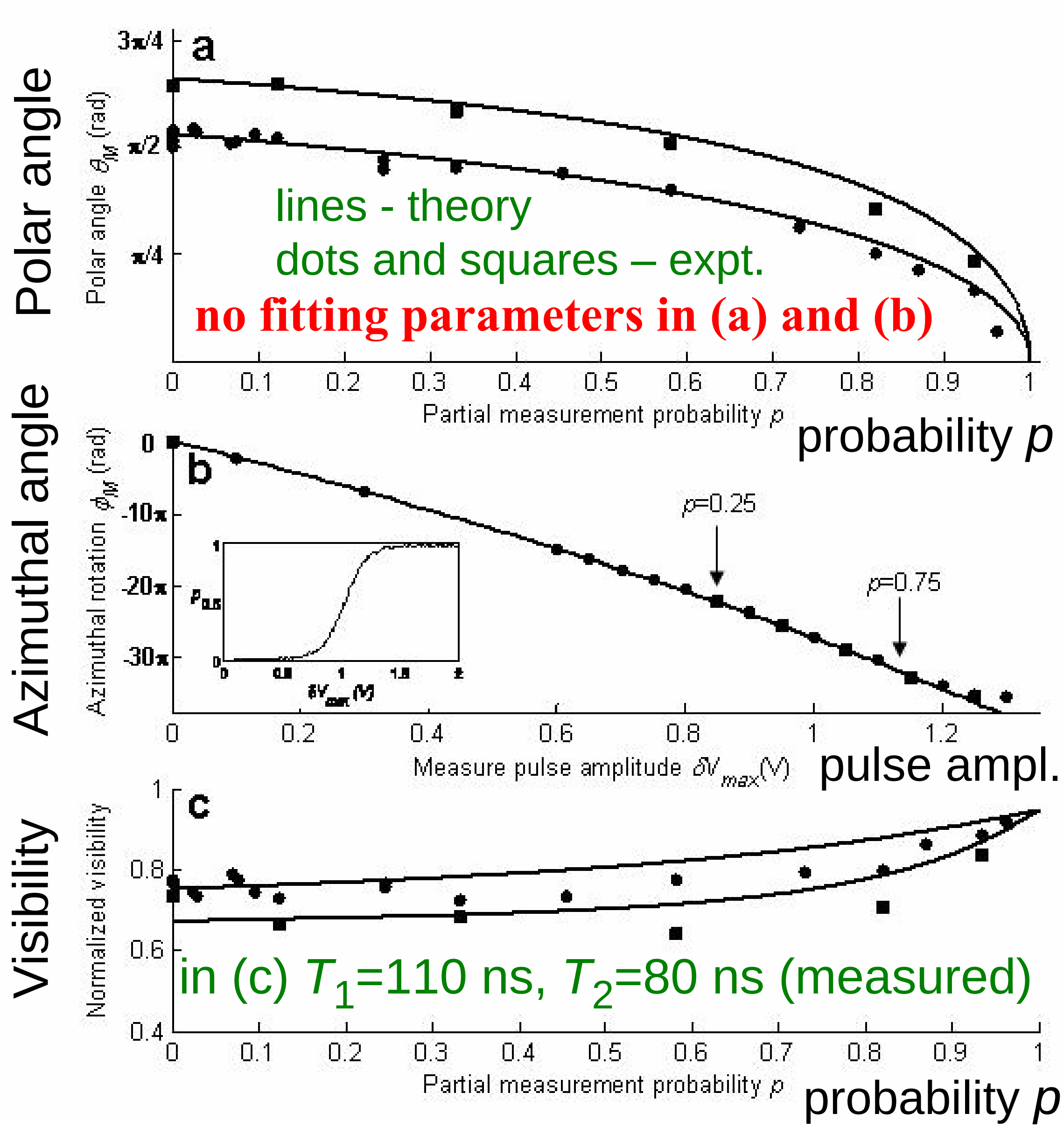
$$\psi_{in} = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$





Partial collapse: experimental results

N. Katz *et al.*, Science-06



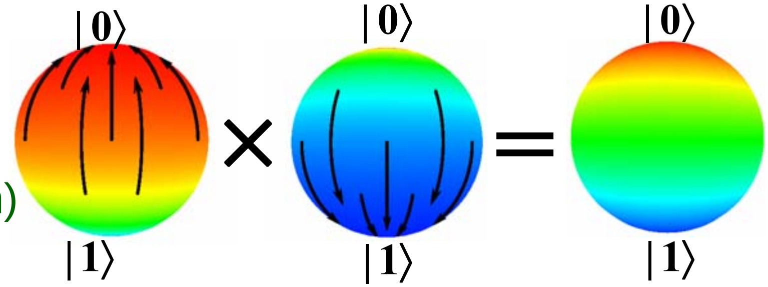
- **In case of no tunneling (null-result measurement) phase qubit evolves**
- **This evolution is well described by a simple Bayesian theory, without fitting parameters**
- **Phase qubit remains fully coherent in the process of continuous collapse (experimentally ~80% raw data, ~96% after account for T1 and T2)**

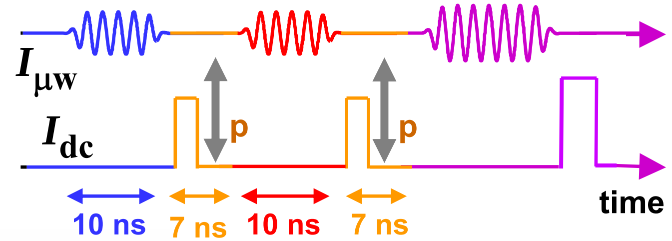
Experiment on wavefunction uncollapsing (QUD)

N. Katz et al. (J. Martinis' group)

Idea:

(A.K.& Jordan)



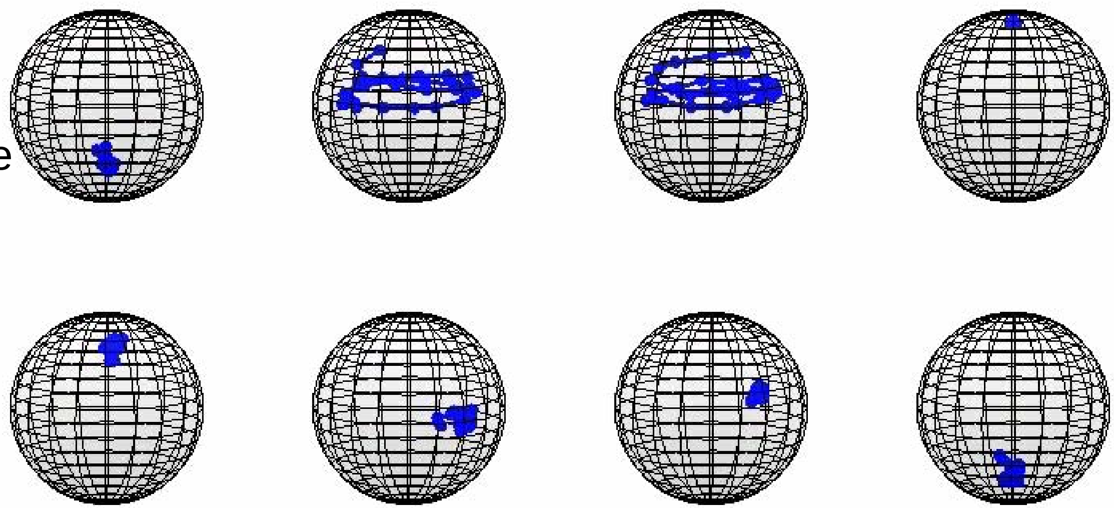


Expt. results:

Initial state	$|1\rangle$	$(|0\rangle+|1\rangle)/\sqrt{2}$	$(|0\rangle+i|1\rangle)/\sqrt{2}$	$|0\rangle$

Partial collapse

Erasure (QUD)



QUD protocol:

- partial collapse
- π-pulse
- partial collapse (same strength)

Collapse strength: $0.05 < p < 0.7$

Courtesy of Nadav Katz and John Martinis

uncollapsing works well!

Conclusions

- Continuous (weak) quantum measurement is a new interesting subject in solid-state mesoscopics; interest grows
- A number of experimental predictions have been made, two direct experiments have been realized

Future directions

- Understanding of weak (continuous) quantum measurement for particular systems
- Analysis of possible applications (quantum feedback, realistic non-QND qubit measurements, non-unitary quantum gates, continuous error correction, etc.)
- Experimental proposals (realistic and for future)
- Actual experiments (direct and indirect; to resolve controversies and towards applications)