Undoing a weak quantum measurement of a solid-state qubit
(*Quantum Un-Demolition measurement*)

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We propose an experiment which demonstrates the undoing of a weak continuous measurement of a solid-state qubit, so that any unknown initial state is fully restored. The undoing procedure has only a finite probability of success because of the non-unitary nature of quantum measurement, though it is accompanied by a clear experimental indication of whether or not the undoing has been successful. The probability of success decreases with increasing strength of the measurement, reaching zero for a traditional projective measurement. Measurement undoing (``quantum un-demolition'') may be interpreted as a kind of a quantum eraser, in which the information obtained from the first measurement is erased by the second measurement, which is an essential part of the undoing procedure. The experiment can be realized using quantum dot (charge) or superconducting (phase) qubits.

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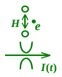
Graphical representation of the evolution

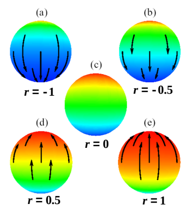


$$\frac{\rho_{11}(t)}{\rho_{22}(t)} = \frac{\rho_{11}(0)}{\rho_{22}(0)}\exp[2r(t)]$$

$$\frac{\rho_{12}(t)}{\sqrt{\rho_{11}(t)\rho_{22}(t)}} = \text{const}$$

where measurement result $r(t)$ is

$$r(t) = \frac{\Delta I}{S_I}\left[\int_0^t I(t')\,dt' - I_0 t\right]$$



Jordan-Korotkov-Büttiker, PRL-06

If $r = 0$, then no information and no evolution!

Probability of successful measurement undoing for phase qubit

Success probability if no tunneling during first measurement:

$$P_S = \frac{e^{-\Gamma t}}{\rho_{00}(0) + e^{-\Gamma t}\rho_{11}(0)} = \frac{1-p}{\rho_{00}(0) + (1-p)\rho_{11}(0)}$$

where ρ(0) is the density matrix of the initial state (either averaged unknown state or an entangled state traced over all other qubits)

Total (averaged) success probability: $P_{av} = 1 - p$

For measurement strength p increasing to 1, success probability decreases to zero (orthodox collapse), but still exact undoing

Such an experiment is only slightly more difficult than recent experiment on partial collapse (N. Katz et al., 2006).
Can be realized experimentally pretty soon!!!

Quantum erasers in optics

Quantum eraser proposal by Scully and Drühl, PRA (1982)

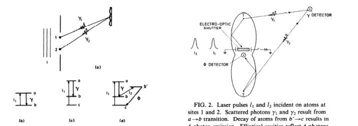


FIG. 1. (a) Figure depicting light impinging from left on atoms at sites 1 and 2. Scattered photons γ_1 and γ_2 produce interference pattern on screen. (b) Two-level atoms excited by laser pulse l_1, and emit γ photons in $a \to b$ transition. (c) Three-level atoms excited by pulse l_1 from $c \to a$ and emit photons in $a \to b$ transition. (d) Four-level system excited by pulse l_1 from $c \to a$ followed by emission of γ photons in $a \to b$ transition. Second pulse l_2 takes atoms from $b \to b'$. Decay from $b' \to c$ results in emission of ϕ photons.

FIG. 2. Laser pulses l_1 and l_2 incident on atoms at sites 1 and 2. Scattered photons γ_1 and γ_2 result from $a \to b$ transition. Decay of atoms from $b' \to c$ results in ϕ photon emission. Elliptical cavities reflect ϕ photons onto common photodetector. Electro-optic shutter transmits ϕ photons only when switch is open. Choice of switch position determines whether we emphasize particle or wave nature of γ photons.

Interference fringes restored for two-detector correlations (since "which-path" information is erased)

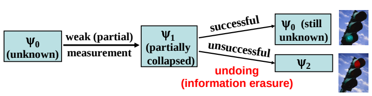
Our idea of measurement undoing is quite different: we really extract quantum information and then erase it

The problem

It is impossible to undo "orthodox" quantum measurement (for an unknown initial state)

Is it possible to undo partial quantum measurement?
(To restore a "precious" qubit accidentally measured)
Yes! (but with a finite probability)

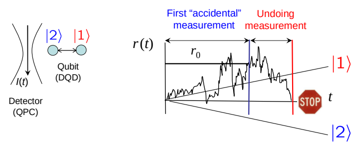
If undoing is successful, an unknown state is **fully** restored



"Quantum Un-Demolition (QUD) measurement"

Measurement undoing for DQD-QPC system

Jordan and Korotkov, 2006



Simple strategy: continue measuring until $r(t)$ becomes zero!
Then any unknown initial state is fully restored.

(same for an entangled qubit)

It may happen though that $r = 0$ never happens; then undoing procedure is unsuccessful.

General theory of quantum measurement undoing

Measurement operator M_r: $\rho \to \dfrac{M_r \rho M_r^\dagger}{\mathrm{Tr}(M_r \rho M_r^\dagger)}$ (POVM formalism)

Undoing measurement operator: $C \times M_r^{-1}$ (to satisfy completeness, eigenvalues cannot be >1)

$$\max(C) = \min_i \sqrt{p_i}, \quad p_i = \mathrm{Tr}(M_r^\dagger M_r |i\rangle\langle i|)$$

p_i – probability of the measurement result r for initial state $|i\rangle$

Probability of success: $P_S \le \dfrac{\min_i p_i}{\Sigma_i p_i \rho_{ii}(0)} = \dfrac{\min P_r}{P_r(\rho(0))}$

$P_r(\rho(0))$ – probability of result r for initial state $\rho(0)$,
$\min(P_r)$ – probability of result r minimized over all possible initial states

Averaged (over r) probability of success: $P_{av} \le \sum_r \min P_r$

(similar to Koashi-Ueda, PRL, 1999)

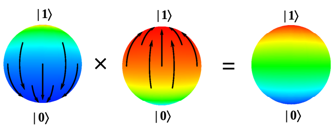
Conclusions

- **Partial (incomplete, weak, etc.) quantum measurement can be undone, though with a finite probability P_s, decreasing with increasing strength of the measurement ($P_s = 0$ for orthodox case)**
- **Though somewhat similar to the quantum eraser, undoing idea is actually quite different:**
  "Quantum Un-Demolition" (QUD) measurement
- **Measurement undoing for single phase qubit is realizable now, experiment with a charge qubit will hopefully be possible soon (difficulty to use SET: need an ideal quantum detector)**

Measurement undoing

Evolution due to partial (weak, continuous, etc.) measurement is **non-unitary** (though coherent if detector is good!), therefore it is impossible to undo it by Hamiltonian dynamics.

How to undo? One more measurement!



(similar to Koashi-Ueda, PRL-1999)

(Figure partially adopted from A. Jordan, A. Korotkov, and M. Büttiker, PRL-2006)

Probability of success

Trick: since non-diagonal matrix elements are not directly involved, we can analyze classical probabilities (as if qubit is in some certain, but unknown state); then simple diffusion with drift

Results:

Probability of successful undoing $$P_S = \frac{e^{-|r_0|}}{e^{|r_0|}\rho_{11}(0) + e^{-|r_0|}\rho_{22}(0)}$$

where r_0 is the result of the measurement to be undone, and $\rho(0)$ is our knowledge about an unknown initial state; in case of an entangled qubit $\rho(0)$ is traced over other qubits

Average time to wait $T_{undo} = T_m |r_0|$ where $T_m = 2S_I/(\Delta I)^2$ ("measurement time")

Averaged probability of success (over result r_0) $P_{av} = 1 - \mathrm{erf}[\sqrt{t/2T_m}]$

(does not depend on initial state!)

Comparison of the general bound for undoing success with examples

General bound: $P_S \le \dfrac{\min P_r}{P_r(\rho(0))}$

First example (DQD+QPC) $P_S \le \dfrac{\min(p_1, p_2)}{p_1\rho_{11}(0) + p_2\rho_{22}(0)}$

where $p_i = (\pi S_I/t)^{-1/2}\exp[-(\bar{I} - I_i)^2 t/S_I]\,d\bar{I}$

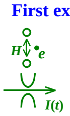
Coincides with the pervious result, so the upper bound is reached, **therefore undoing strategy is optimal**

Second example (phase qubit) Probabilities of no-tunneling are 1 and exp(-Γ*t*)=1-*p*

$$P_S \le \frac{1-p}{\rho_{00}(0) + (1-p)\rho_{11}(0)}$$

Again same as before, so measurement **undoing for phase qubit is also optimal**

First example: DQD qubit with no tunneling, measured by QPC



$$\hat{H}_{QB} = (\varepsilon/2)(c_1^\dagger c_1 - c_2^\dagger c_2) + H(c_1^\dagger c_2 + c_2^\dagger c_1)$$

Assume "frozen" qubit: $\varepsilon = H = 0$

Bayesian evolution due to measurement (Korotkov-1998)

1) Diagonal matrix elements of the density matrix evolve according to the classical Bayes rule

2) Non-diagonal matrix elements evolve so that the degree of purity $\rho_{ij}/[\rho_{ii}\rho_{jj}]^{1/2}$ is conserved

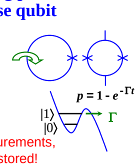
$$\rho_{11}(\tau) = \frac{\rho_{11}(0)\exp[-(\bar{I} - I_1)^2/2D]}{\rho_{11}(0)\exp[-(\bar{I} - I_1)^2/2D] + \rho_{22}(0)\exp[-(\bar{I} - I_2)^2/2D]}$$

$$\frac{\rho_{12}(\tau)}{[\rho_{12}(\tau)\,\rho_{22}(\tau)]^{1/2}} = \frac{\rho_{12}(0)}{[\rho_{12}(0)\,\rho_{22}(0)]^{1/2}}, \qquad \rho_{22}(\tau) = 1 - \rho_{11}(\tau)$$

where $\bar{I} = \frac{1}{\tau}\int_0^\tau I(t)\,dt$

Second example: Undoing partial measurement of a phase qubit

1) Start with an unknown state
2) Partial measurement of strength p
3) π-pulse (exchange |0> ↔ |1>)
4) One more measurement with the **same strength** p
5) π-pulse



If no tunneling for both measurements, then initial state is fully restored!

$$\alpha|0\rangle + \beta|1\rangle \to \frac{\alpha|0\rangle + e^{i\phi}\beta\sqrt{1-p}\,|1\rangle}{\text{Norm}} \to$$

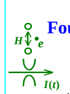
$$\frac{e^{i\phi}\alpha\sqrt{1-p}\,|0\rangle + e^{i\phi}\beta\sqrt{1-p}\,|1\rangle}{\text{Norm}} = e^{i\phi}(\alpha|0\rangle + \beta|1\rangle)$$

Third example: General undoing procedure for entangled charge qubits

1) unitary transformation of N qubits
2) null-result measurement of a certain strength by a strongly nonlinear QPC (tunneling only for state |11..1⟩)
3) repeat 2^N times, sequentially transforming the basis vectors of the measurement operator into |11..1⟩

(also reaches the upper bound for success probability)

Fourth example: Evolving charge qubit



$$\hat{H}_{QB} = (\varepsilon/2)(c_1^\dagger c_1 - c_2^\dagger c_2) + H(c_1^\dagger c_2 + c_2^\dagger c_1)$$

(now non-zero H and ε, qubit evolves during measurement)

1) Bayesian equations to calculate measurement operator
2) unitary operation, measurement by QPC, unitary operation