Quantum feedback control of solid-state qubits

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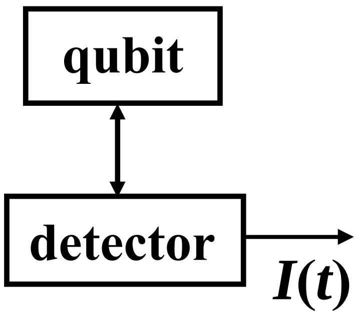
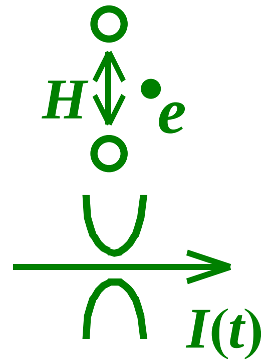
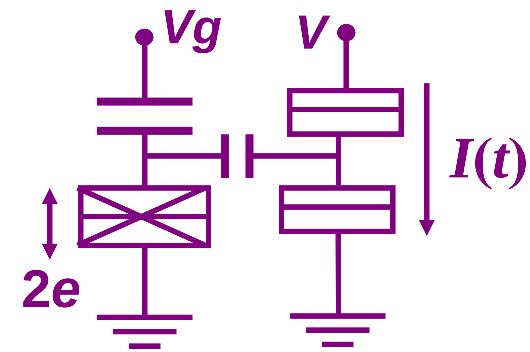
The system we consider: qubit + detector





Double-quantum-qot (DQD) and quantum point contact (QPC)



Cooper-pair box (CPB) and single-electron transistor (SET)

$H = H_{QB} + H_{DET} + H_{INT}$

$H_{QB} = (\varepsilon/2)(c_1^+c_1 - c_2^+c_2) + H(c_1^+c_2 + c_2^+c_1)$ ε – asymmetry, H – tunneling

$\Omega = (4H^2+\varepsilon^2)^{1/2}/\hbar$ – frequency of quantum coherent (Rabi) oscillations

Two levels of average detector current: I_1 for qubit state $|1\rangle$, I_2 for $|2\rangle$

Response: $\Delta I = I_1 - I_2$ Detector noise: white, spectral density S_I

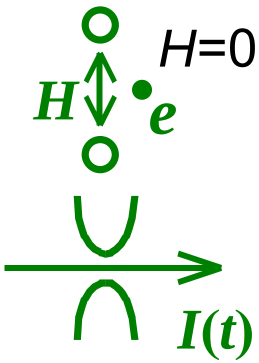
DQD and QPC
(setup due to Gurvitz, 1997)

$$H_{DET} = \sum_l E_l a_l^\dagger a_l + \sum_r E_r a_r^\dagger a_r + \sum_{l,r} T(a_r^\dagger a_l + a_l^\dagger a_r)$$

$$H_{INT} = \sum_{l,r} \Delta T\,(c_1^\dagger c_1 - c_2^\dagger c_2)(a_r^\dagger a_l + a_l^\dagger a_r)$$

$$S_I = 2eI$$

Bayesian formalism for DQD-QPC (qubit-detector) system



Qubit evolution due to continuous measurement:

1) Diagonal matrix elements of the qubit density matrix evolve as classical probabilities (i.e. according to the classical Bayes rule)

2) Non-diagonal matrix elements evolve so that the degree of purity $\rho_{ij}/[\rho_{ii}\,\rho_{jj}]^{1/2}$ is conserved

(A.K., 1998)

Bayes rule:

$$P(A_i\,|\,R) = \frac{P(A_i)\,P(R\,|\,A_i)}{\sum_k P(A_k)\,P(R\,|\,A_k)}$$

So simple because:

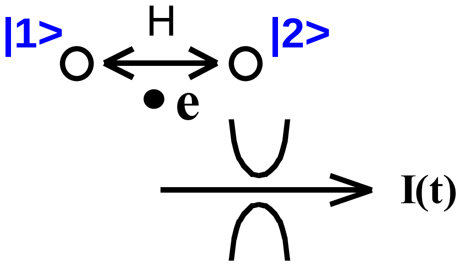
1) QPC happens to be an ideal detector

2) no Hamiltonian evolution of the qubit

Similar formalisms developed earlier. Key words: Imprecise, weak, selective, or conditional measurements, POVM, Quantum trajectories, Quantum jumps, Restricted path integral, etc.

Names: Davies, Kraus, Holevo, Belavkin, Mensky, Caves, Gardiner, Carmichael, Knight, Walls, Gisin, Percival, Milburn, Wiseman, Habib, etc. (very incomplete list)

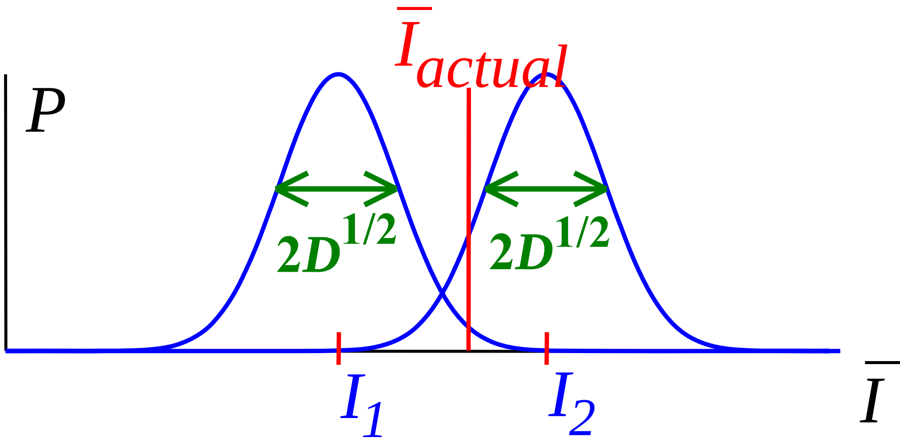
"Quantum Bayes theorem" (ideal detector assumed)



$H = \varepsilon = 0$
("frozen" qubit)

Initial state: $\begin{pmatrix} \rho_{11}(0) & \rho_{12}(0) \\ \rho_{21}(0) & \rho_{22}(0) \end{pmatrix}$

Measurement (during time τ):



$$\bar{I} \equiv \frac{1}{\tau}\int_0^\tau I(t)\,dt$$

$$P(\bar{I},\tau) = \rho_{11}(0)\, P_1(\bar{I},\tau) + \rho_{22}(0)\, P_2(\bar{I},\tau)$$

$$P_i(\bar{I},\tau) = \frac{1}{\sqrt{2\pi D}} \exp[-(\bar{I} - I_i)^2 / 2D],$$

$$D = S_I / 2\tau, \quad |I_1 - I_2| \ll I_i, \quad \tau \gg S_I / I_i^2$$

After the measurement during time τ, the probabilities should be updated using the standard Bayes formula:

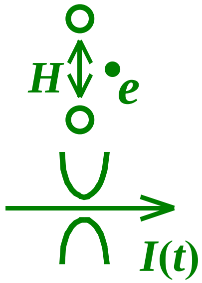
$$P(B_i \mid A) = \frac{P(B_i)P(A \mid B_i)}{\sum_k P(B_k)P(A \mid B_k)}$$

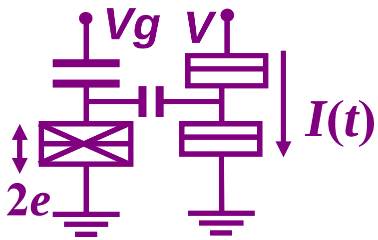
Quantum Bayes formulas:

$$\rho_{11}(\tau) = \frac{\rho_{11}(0) \exp[-(\bar{I} - I_1)^2 / 2D]}{\rho_{11}(0) \exp[-(\bar{I} - I_1)^2 / 2D] + \rho_{22}(0) \exp[-(\bar{I} - I_2)^2 / 2D]}$$

$$\frac{\rho_{12}(\tau)}{[\rho_{12}(\tau)\, \rho_{22}(\tau)]^{1/2}} = \frac{\rho_{12}(0)}{[\rho_{12}(0)\, \rho_{22}(0)]^{1/2}}, \qquad \rho_{22}(\tau) = 1 - \rho_{11}(\tau)$$

Bayesian formalism for a single qubit





$$\hat{H}_{QB} = \frac{\varepsilon}{2}(c_1^\dagger c_1 - c_2^\dagger c_2) + H(c_1^\dagger c_2 + c_2^\dagger c_1)$$

$|1\rangle \to I_1,\ |2\rangle \to I_2,\ \Delta I = I_1 - I_2,\ I_0 = (I_1 + I_2)/2$

S_I – detector noise

$$\dot{\rho}_{11} = -\dot{\rho}_{22} = -2(H/\hbar)\,\mathrm{Im}\,\rho_{12} + \rho_{11}\rho_{22}(2\Delta I/S_I)[\underline{\underline{I(t)}} - I_0]$$

$$\dot{\rho}_{12} = i(\varepsilon/\hbar)\rho_{12} + i(H/\hbar)(\rho_{11} - \rho_{22}) + \rho_{12}(\rho_{11} - \rho_{22})(\Delta I/S_I)[\underline{\underline{I(t)}} - I_0] - \gamma\rho_{12}$$

$$I(t) - I_0 = (\rho_{22} - \rho_{11})\Delta I/2 + \xi(t), \quad S_\xi = S_I$$

(A.K., 1998)

$\gamma = \Gamma - (\Delta I)^2/4S_I$, Γ – ensemble decoherence

$\eta = 1 - \gamma/\Gamma = (\Delta I)^2/4S_I\Gamma$ – detector ideality (efficiency), $\eta \le 100\%$

Ideal detector (η=1, as QPC) does not decohere a qubit, then random evolution of qubit *wavefunction* can be monitored

Averaging over result $I(t)$ leads to conventional master equation:

$$d\rho_{11}/dt = -d\rho_{22}/dt = -2(H/\hbar)\,\mathrm{Im}\,\rho_{12}$$

$$d\rho_{12}/dt = i(\varepsilon/\hbar)\rho_{12} + i(H/\hbar)(\rho_{11} - \rho_{22}) - \Gamma\rho_{12}$$

Assumptions needed for the Bayesian formalism

- Detector voltage is much larger than the qubit energies involved
  $eV >> \hbar\Omega$, $eV >> \hbar\Gamma$ (no coherence in the detector, $\hbar/eV << (1/\Omega, 1/\Gamma)$; Markovian approximation)

- Small detector response, $|\Delta I| << I_0$, $\Delta I = I_1 - I_2$, $I_0 = (I_1 + I_2)/2$
  Many electrons pass through detector before qubit evolves noticeably.
  (Not a really important condition, but simplifies formalism.)

Coupling $C \sim \Gamma/\Omega$ is arbitrary [we define $C = \hbar(\Delta I)^2/S_I H$]

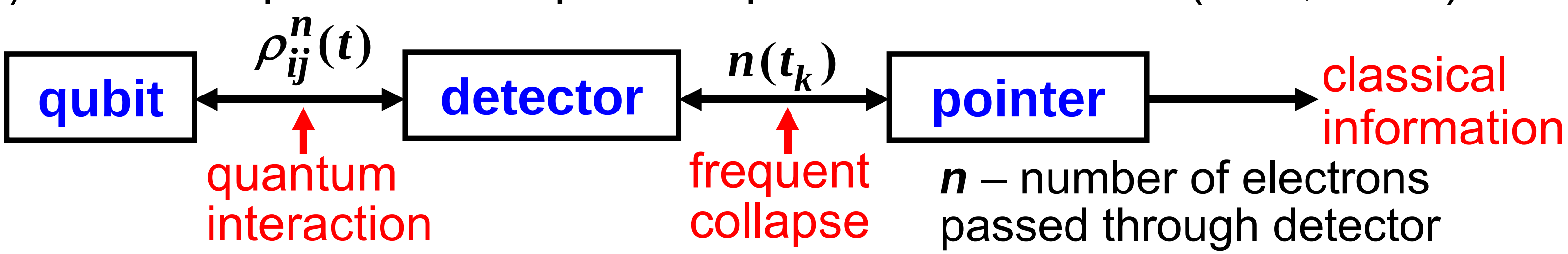
$$\frac{d}{dt}\rho_{11} = -\frac{d}{dt}\rho_{22} = -2\frac{H}{\hbar}\,\mathrm{Im}\,\rho_{12} + \rho_{11}\rho_{22}\frac{2\Delta I}{S_I}[I(t) - I_0]$$

$$\frac{d}{dt}\rho_{12} = i\frac{\varepsilon}{\hbar}\rho_{12} + i\frac{H}{\hbar}(\rho_{11} - \rho_{22}) + \rho_{12}(\rho_{11} - \rho_{22})\frac{\Delta I}{S_I}[I(t) - I_0] - \gamma\rho_{12}$$

Derivations

1) "logical": via correspondence principle and comparison with decoherence approach (A.K., 1998)

2) "microscopic": Schr. eq. + collapse of the detector (A.K., 2000)



3) from "quantum trajectory" formalism developed for quantum optics (Goan-Milburn, 2001; also: Wiseman, Sun, Oxtoby, etc.)

4) from POVM formalism (Jordan-A.K., 2006)

"Informational" derivation of the Bayesian formalism

Step 1. Assume $H = \varepsilon = 0$, "frozen" qubit
Since ρ_{12} is not involved, evolution of ρ_{11} and ρ_{22} should be the same as in the classical case, i.e. Bayes formula (correspondence principle).

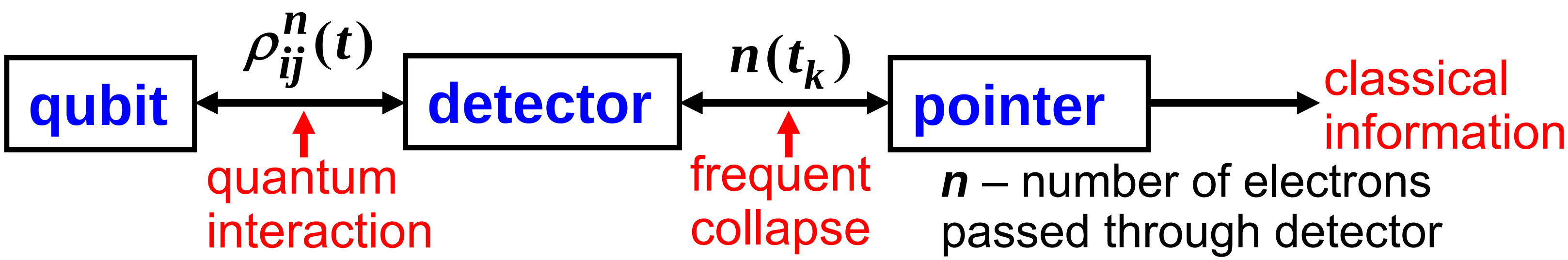
Step 2. Assume $H = \varepsilon = 0$ and pure initial state, $\rho_{12}(0) = [\rho_{11}(0)\,\rho_{22}(0)]^{1/2}$
For any realization $|\rho_{12}(t)| \leq [\rho_{11}(t)\,\rho_{22}(t)]^{1/2}$. Hence, averaging over ensemble of realizations gives $|\rho_{12}^{av}(t)| \leq \rho_{12}^{av}(0)\exp[-(\Delta I^2/4S_I)\,t]$
However, conventional (ensemble) result (Gurvitz-1997, Aleiner *et al*.-1997) for QPC is exactly the upper bound: $\rho_{12}^{av}(t) = \rho_{12}^{av}(0)\exp[-(\Delta I^2/4S_I)\,t]$.
Therefore, pure state remains pure: $\rho_{12}(t) = [\rho_{11}(t)\,\rho_{22}(t)]^{1/2}$.

Step 3. Account of a mixed initial state
Result: the degree of purity $\rho_{12}(t)/[\rho_{11}(t)\,\rho_{22}(t)]^{1/2}$ is conserved.

Step 4. Add qubit evolution due to H and ε.

Step 5. Add extra dephasing due to detector nonideality (i.e., for SET).

"Microscopic" derivation of the Bayesian formalism



Schrödinger evolution of "qubit + detector"

for a low-*T* QPC as a detector (Gurvitz, 1997)

$$\frac{d}{dt}\rho_{11}^{n} = -\frac{I_1}{e}\rho_{11}^{n} + \frac{I_1}{e}\rho_{11}^{n-1} - 2\frac{H}{\hbar}\mathrm{Im}\,\rho_{12}^{n}$$

$$\frac{d}{dt}\rho_{22}^{n} = -\frac{I_2}{e}\rho_{22}^{n} + \frac{I_2}{e}\rho_{22}^{n-1} + 2\frac{H}{\hbar}\mathrm{Im}\,\rho_{12}^{n}$$

$$\frac{d}{dt}\rho_{12}^{n} = i\frac{\varepsilon}{\hbar}\rho_{12}^{n} + i\frac{H}{\hbar}(\rho_{11}^{n} - \rho_{22}^{n}) - \frac{I_1 + I_2}{2e}\rho_{12}^{n} + \frac{\sqrt{I_1 I_2}}{e}\rho_{12}^{n-1}$$

Detector collapse at $t = t_k$

Particular n_k is chosen at t_k

$$P(n) = \rho_{11}^{n}(t_k) + \rho_{22}^{n}(t_k)$$

$$\rho_{ij}^{n}(t_k + 0) = \delta_{n,nk}\,\rho_{ij}(t_k + 0)$$

$$\rho_{ij}(t_k + 0) = \frac{\rho_{ij}^{nk}(t_k)}{\rho_{11}^{nk}(t_k) + \rho_{22}^{nk}(t_k)}$$

If $H = \varepsilon = 0$, this leads to

$$\rho_{11}(t) = \frac{\rho_{11}(0)P_1(n)}{\rho_{11}(0)P_1(n) + \rho_{22}(0)P_2(n)}, \quad \rho_{22}(t) = \frac{\rho_{22}(0)P_2(n)}{\rho_{11}(0)P_1(n) + \rho_{22}(0)P_2(n)}$$

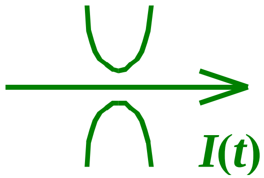
$$\rho_{12}(t) = \rho_{12}(0)\frac{[\rho_{11}(t)\rho_{22}(t)]^{1/2}}{[\rho_{11}(0)\rho_{22}(0)]^{1/2}}, \quad P_i(n) = \frac{(I_i t/e)^n}{n!}\exp(-I_i t/e),$$

which are exactly quantum Bayes formulas

Quantum efficiency of solid-state detectors

(ideal detector does not cause single qubit decoherence)

1. Quantum point contact



Theoretically, ideal quantum detector, $\eta = 1$

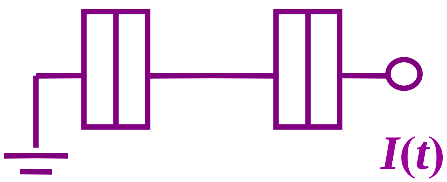
A.K., 1998 (Gurvitz, 1997; Aleiner *et al.*, 1997)

Averin, 2000; Pilgram et al., 2002, Clerk et al., 2002

Experimentally, $\eta > 80\%$

(using Buks *et al.*, 1998)

2. SET-transistor



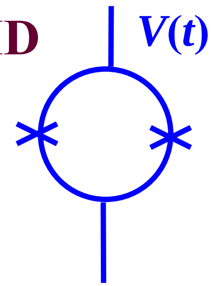
Very non-ideal in usual operation regime, $\eta \ll 1$

Shnirman-Schön, 1998; A.K., 2000, Devoret-Schoelkopf, 2000

However, reaches ideality, $\eta = 1$ if:

- in deep cotunneling regime (Averin, van den Brink, 2000)
- S-SET, using supercurrent (Zorin, 1996)
- S-SET, double-JQP peak ($\eta \sim 1$) (Clerk *et al.*, 2002)
- resonant-tunneling SET, low bias (Averin, 2000)

3. SQUID



Can reach ideality, $\eta = 1$

(Danilov-Likharev-Zorin, 1983; Averin, 2000)

4. FET ?? HEMT ??
ballistic FET/HEMT ??

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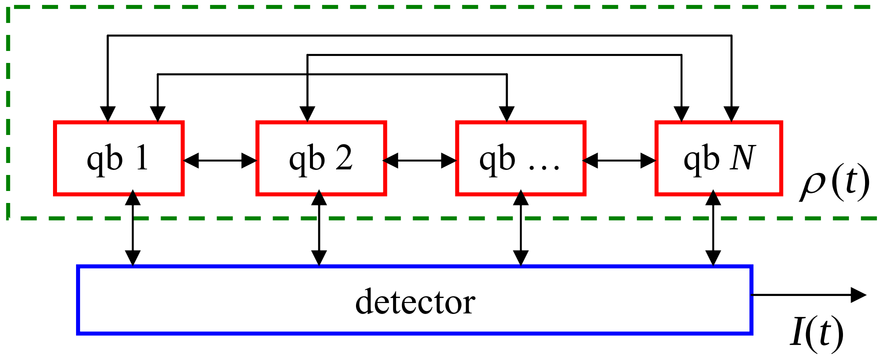
Bayesian formalism for N entangled qubits measured by one detector



Up to 2^N levels of current

$$\frac{d}{dt}\rho_{ij} = \frac{-i}{\hbar}[\hat{H}_{qb},\rho]_{ij} + \rho_{ij}\frac{1}{S}\sum_k \rho_{kk}\Big[\big(I(t)-\frac{I_k+I_i}{2}\big)(I_i-I_k) + \big(I(t)-\frac{I_k+I_j}{2}\big)(I_j-I_k)\Big] - \gamma_{ij}\rho_{ij}$$

(Stratonovich form)

$$\gamma_{ij} = (\eta^{-1}-1)(I_i-I_j)^2/4S_I \qquad I(t) = \sum_i \rho_{ii}(t) I_i + \xi(t)$$

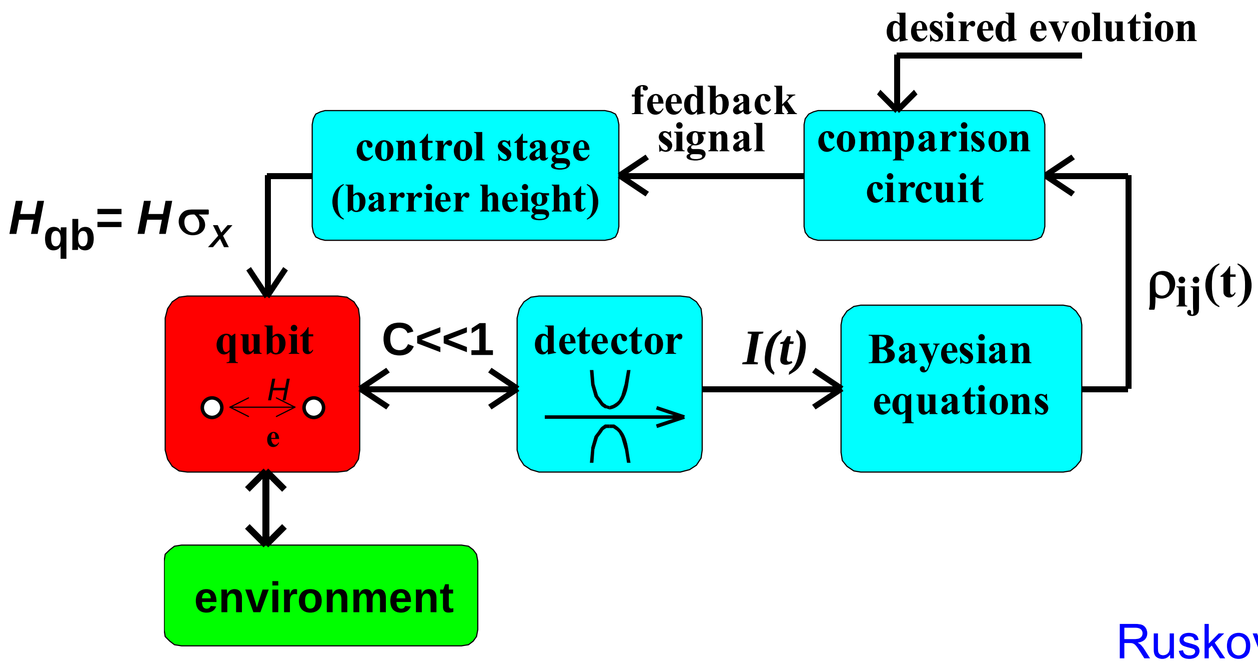
Averaging over ξ(t) ⟹ master equation

No measurement-induced dephasing between states |i⟩ and |j⟩ if $I_i = I_j$!

A.K., PRA 65 (2002), PRB 67 (2003)

Bayesian quantum feedback control of a qubit

Since qubit state can be monitored, the feedback is possible!



Ruskov & A.K., 2001

Goal: maintain perfect Rabi oscillations forever

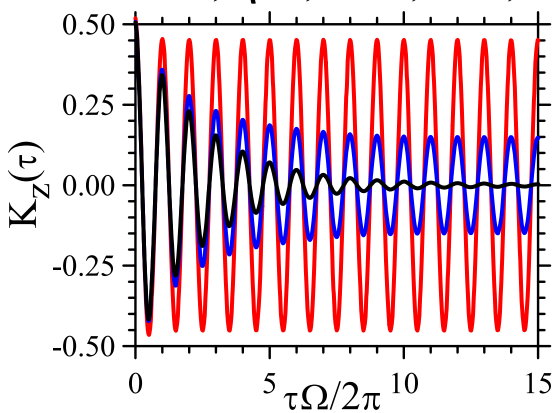
Idea: monitor the Rabi phase ϕ by continuous measurement and apply feedback control of the qubit barrier height, $\Delta H_{FB}/H = -F\times\Delta\phi$

To monitor phase ϕ we plug detector output $I(t)$ into Bayesian equations

Performance of quantum feedback

Qubit correlation function

C=1, η=1, F=0, 0.05, 0.5



$$K_z(\tau) = \frac{\cos\Omega t}{2} \exp\left[\frac{C}{16F}(e^{-2FH\tau/\hbar} - 1)\right]$$

$C = \hbar(\Delta I)^2 / S_I H$ – coupling

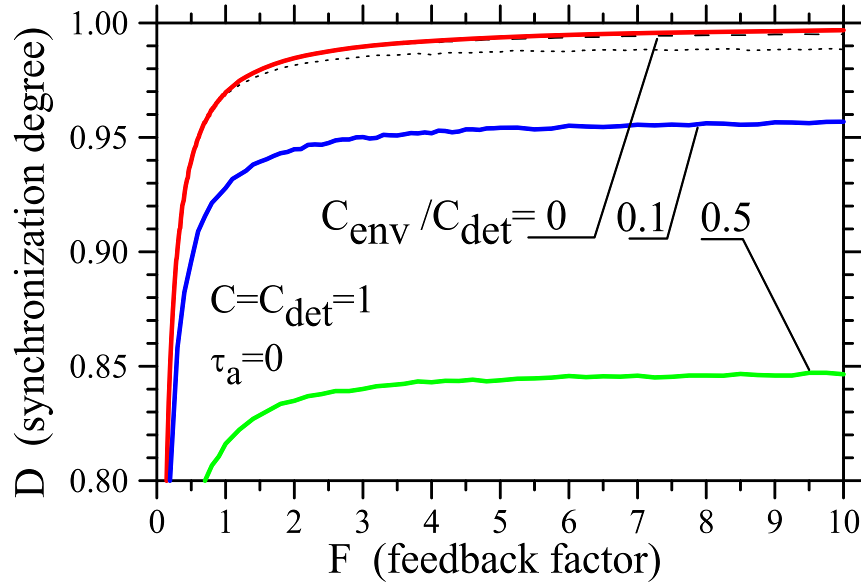
F – feedback strength

$D = 2\langle \mathrm{Tr}\rho\rho_{desir}\rangle - 1$

For ideal detector and wide bandwidth, fidelity can be arbitrarily close to 100%

$D = \exp(-C/32F)$

Fidelity (synchronization degree)



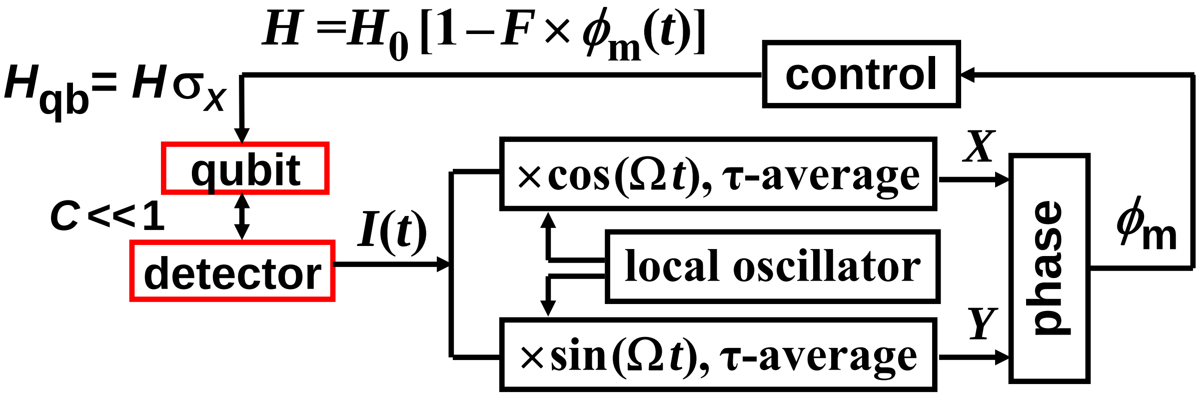
Experimental difficulties:

- necessity of **very fast** real-time solution of Bayesian equations
- **wide bandwidth** (>>Ω, GHz-range) of the line delivering noisy signal $I(t)$ to the "processor"

Ruskov & A.K., PRB-2002

Simple quantum feedback of a solid-state qubit

(A.K., 2005)



Goal: maintain coherent (Rabi) oscillations for arbitrarily long time

Idea: **use two quadrature components of the detector current *I*(*t*) to monitor approximately the phase of qubit oscillations**
(a very natural way for usual classical feedback!)

$$X(t) = \int_{-\infty}^{t} [I(t') - I_0] \cos(\Omega t') \exp[-(t-t')/\tau]\, dt$$
$$Y(t) = \int_{-\infty}^{t} [I(t') - I_0] \sin(\Omega t') \exp[-(t-t')/\tau]\, dt$$

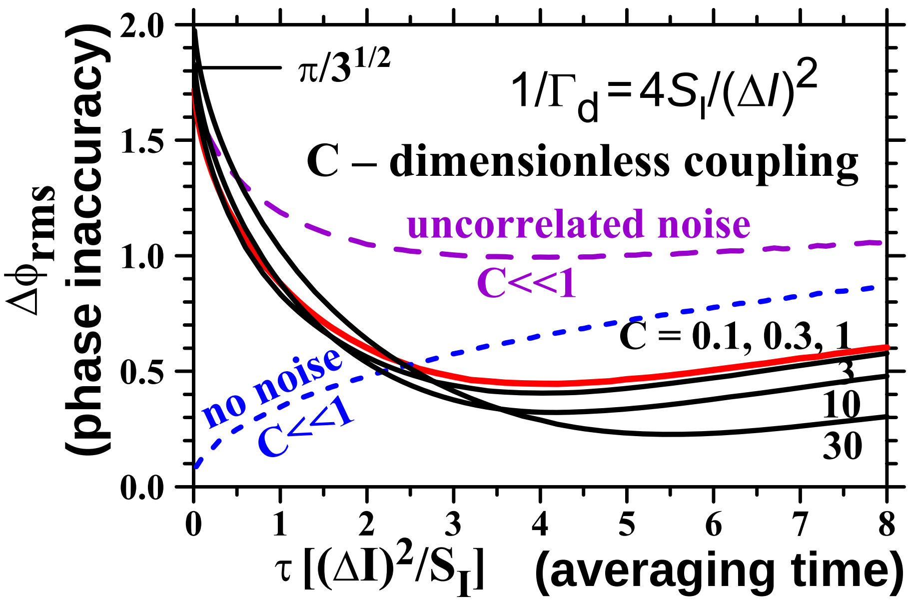
$$\phi_m = -\arctan(Y/X)$$

(similar formulas for a tank circuit instead of mixing with local oscillator)

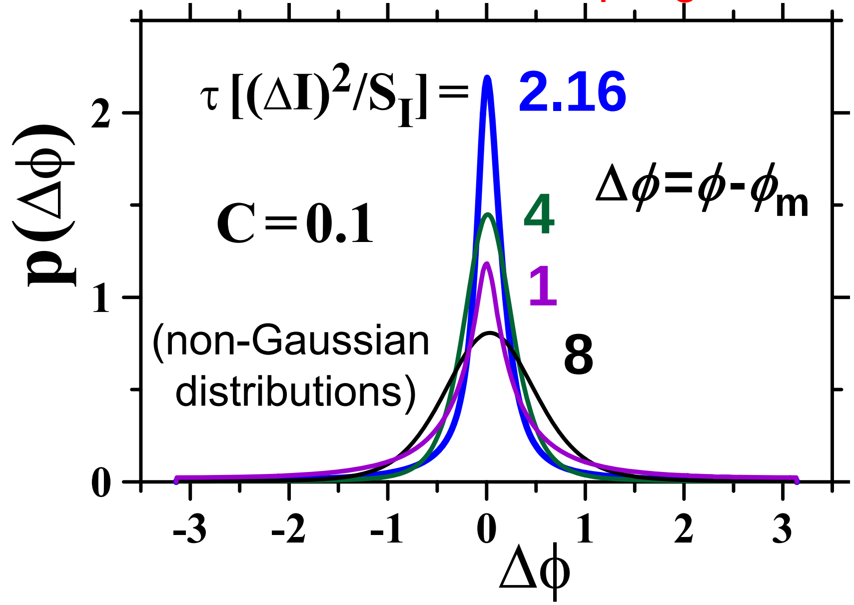
Advantage: **simplicity and relatively narrow bandwidth** $(1/\tau \sim \Gamma_d << \Omega)$

Essentially classical feedback. Does it really work?

Accuracy of phase monitoring via quadratures
(no feedback yet)



weak coupling $C<<1$



Noise improves the monitoring accuracy!
(purely quantum effect, "reality follows observations")

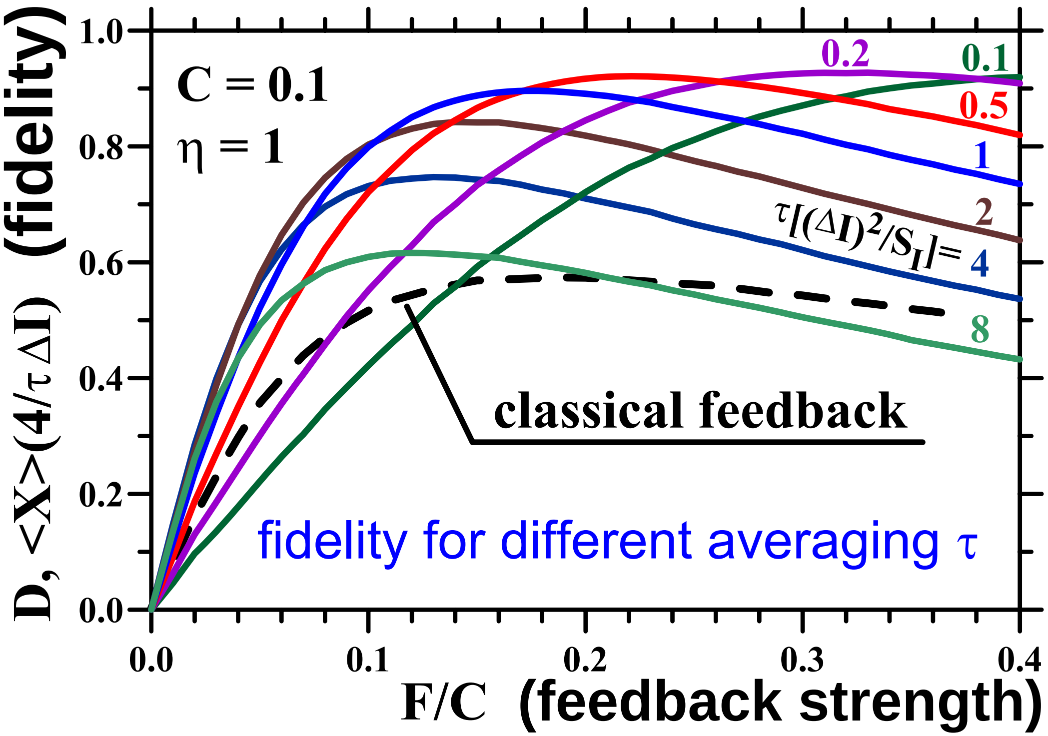
$$d\phi/dt = -[I(t)-I_0]\sin(\Omega t+\phi)(\Delta I/S_I)$$ (actual phase shift, ideal detector)

$$d\phi_m/dt = -[I(t)-I_0]\sin(\Omega t+\phi_m)/(X^2+Y^2)^{1/2}$$ (observed phase shift)

Noise enters the actual and observed phase evolution in a similar way

Quite accurate monitoring! $\cos(0.44)\approx 0.9$

Simple quantum feedback



weak coupling C

D – feedback efficiency

$D \equiv 2F_Q - 1$

$F_Q \equiv \langle \mathrm{Tr}\, \rho(t)\, \rho_{des}(t) \rangle$

$D_{max} \approx 90\%$

($F_Q \approx 95\%$)

How to verify feedback operation experimentally?

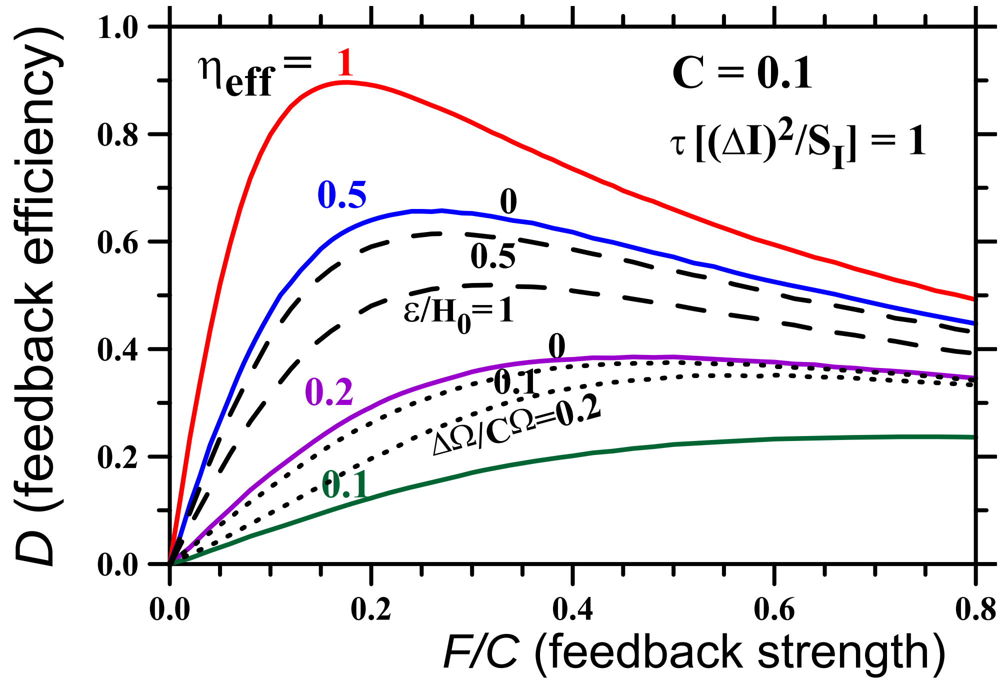
Simple: just check that in-phase quadrature $\langle X \rangle$ of the detector current is positive $D = \langle X \rangle (4/\tau \Delta I)$

$\langle X \rangle$=0 for *any* non-feedback Hamiltonian control of the qubit

Effect of nonidealities

- nonideal detectors (finite quantum efficiency η) and environment
- qubit energy asymmetry ε
- frequency mismatch ΔΩ

Quantum feedback still works quite well



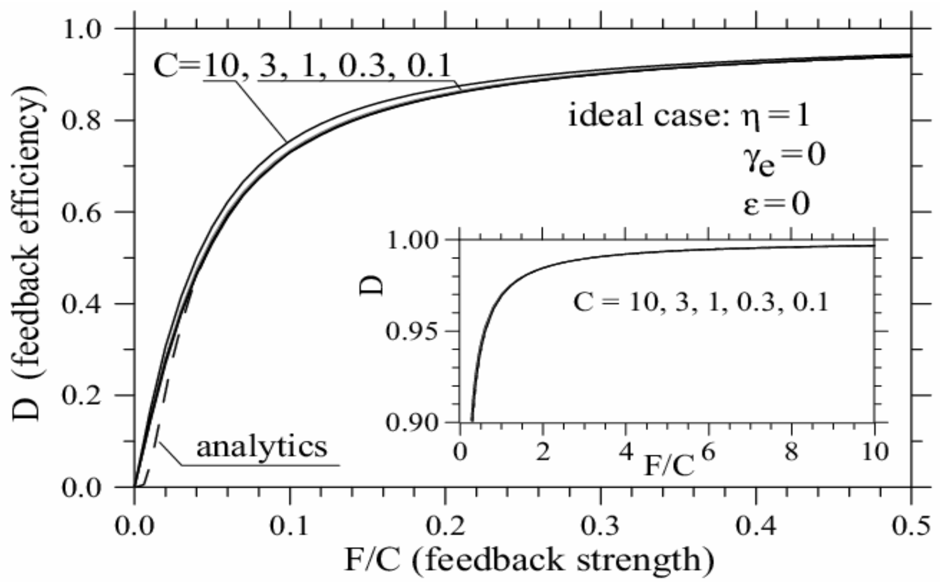
Main features:

- **Fidelity F_Q up to ~95% achievable (D~90%)**
- **Natural, practically classical feedback setup**
- **Averaging $\tau \sim 1/\Gamma >> 1/\Omega$ (narrow bandwidth!)**
- **Detector efficiency (ideality) η~0.1 still OK**
- **Robust to asymmetry ε and frequency shift ΔΩ**
- **Simple verification: positive in-phase quadrature $\langle X \rangle$**

Simple enough experiment?!

Bayesian quantum feedback in more detail

Ideal case (ideal detector, no extra dephasing, infinite bandwidth and no delay)



$C = \hbar(\Delta I)^2 / S_I H$, **Coupling**

D **- Feedback efficiency**

Analytics: *D*=exp(-*C*/32*F*)

***D*(*F*/C>>1) ≈1 – works very well in ideal case**

We have also analyzed the following effects:

Non-ideal detector & extra dephasing

Finite bandwidth

Time delay in the feedback loop

Qubit parameter deviations (*ε* & *H*)

Feedback of a qubit with ε≠0 (needs special controller)

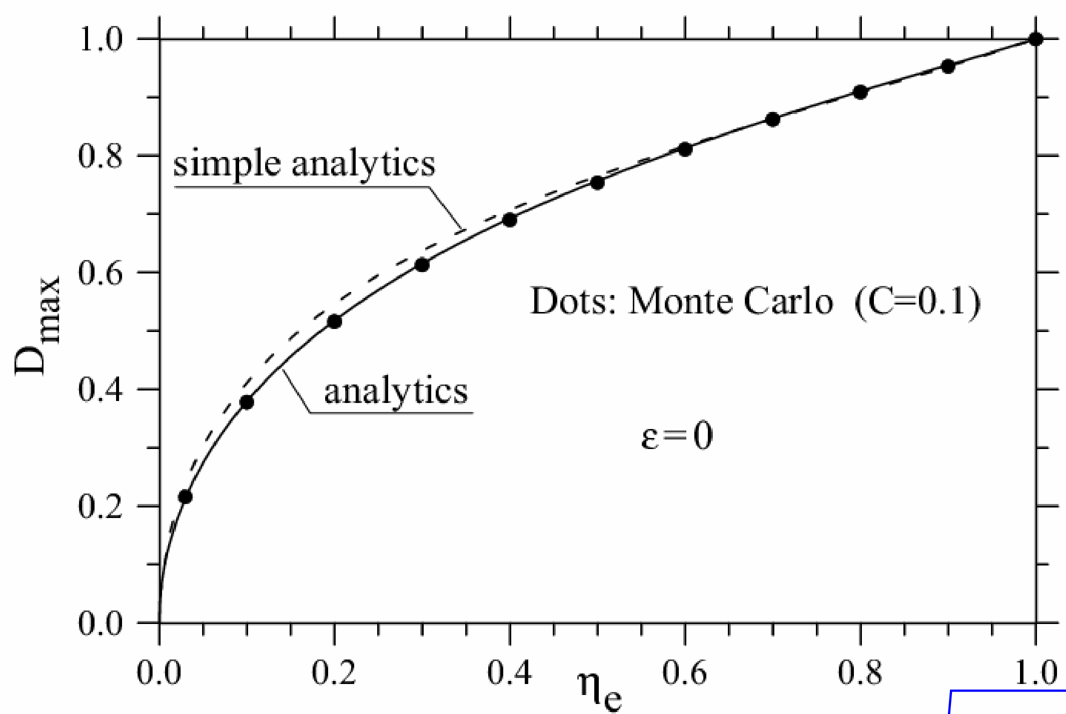
Detector nonideality and dephasing

→**Effect of imperfect detector and extra dephasing due to coupling to environment**

- **Imperfection & extra dephasing ⇒ $D_{max}(\eta_e)<1$**

  D_{max}: maximum feedback efficiency

  $\eta_e=[1+4\gamma S_I/(\Delta I)^2]^{-1}$ -- quantum efficiency



Analytical $D_{max} = \langle P \rangle$

P - purity

$$D_{max} = \frac{\int_0^1 P^2 G(P^2)dp}{\int_0^1 P G(P^2)dp}$$

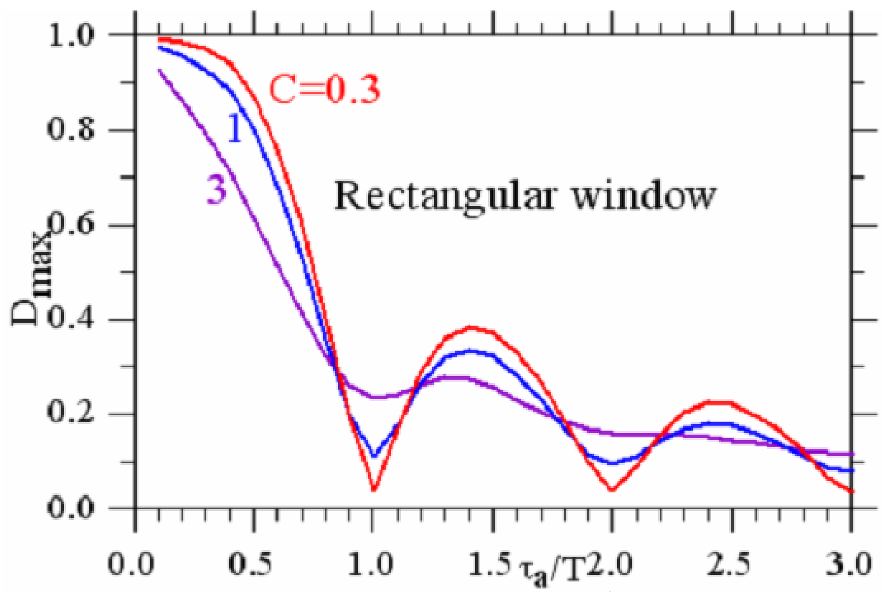
$$G(P^2) = (1-P^2)^{-5/2} A$$

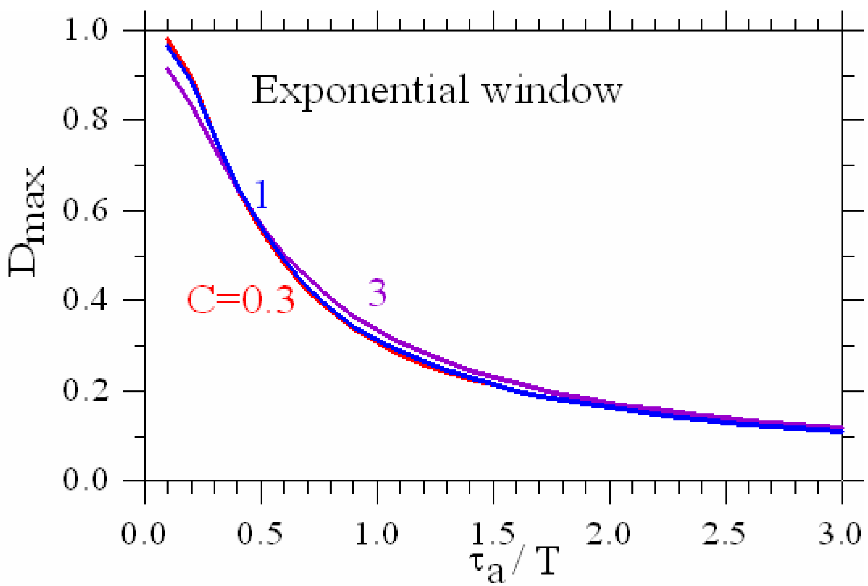
$$A = \exp[-(\eta_e^{-1}-1)/2(1-P^2)]$$

Simple $D_{max} = \sqrt{1+\frac{1}{2\eta_e}-\sqrt{[(1+\frac{1}{2\eta_e})^2-2]}}$

Effect of finite detector bandwidth

- Available detector output signal $I_a(t)$ differs form the "true" one ⇒ A modified signal to compensate this implicit time delay, $\Delta\phi = \phi_a - \Omega(t - \kappa\tau_a)$, $D_{\max} = D(k = k_{opt})$
- $I_a(t) = \tau_a^{-1} \int_{-\infty}^{t} I(t') \exp^{-(t-t')/\tau_a} dt'$ – exponential window
- $I_a(t) = \tau_a^{-1} \int_{t-\tau_a}^{t} I(t') dt'$ – rectangular window

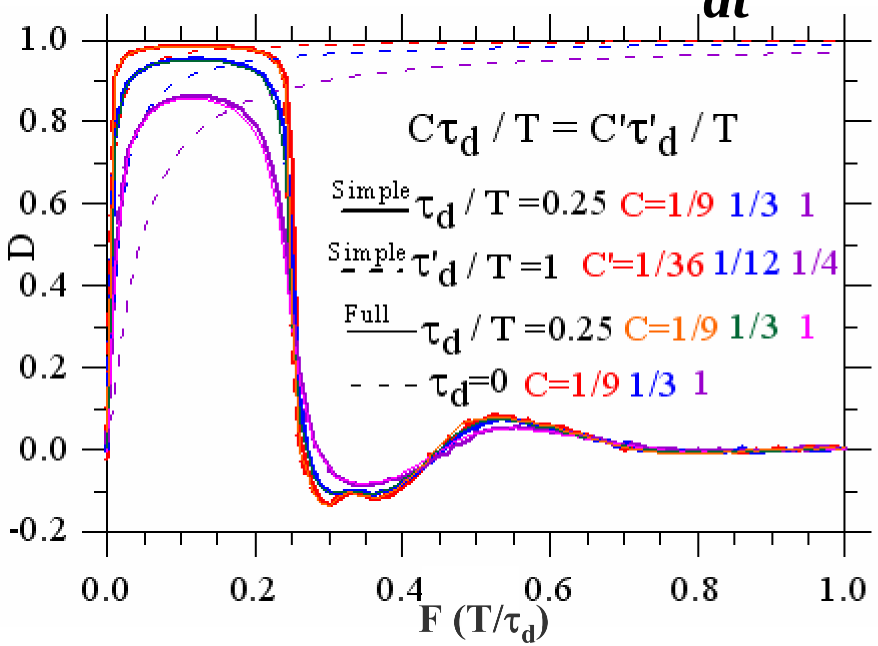


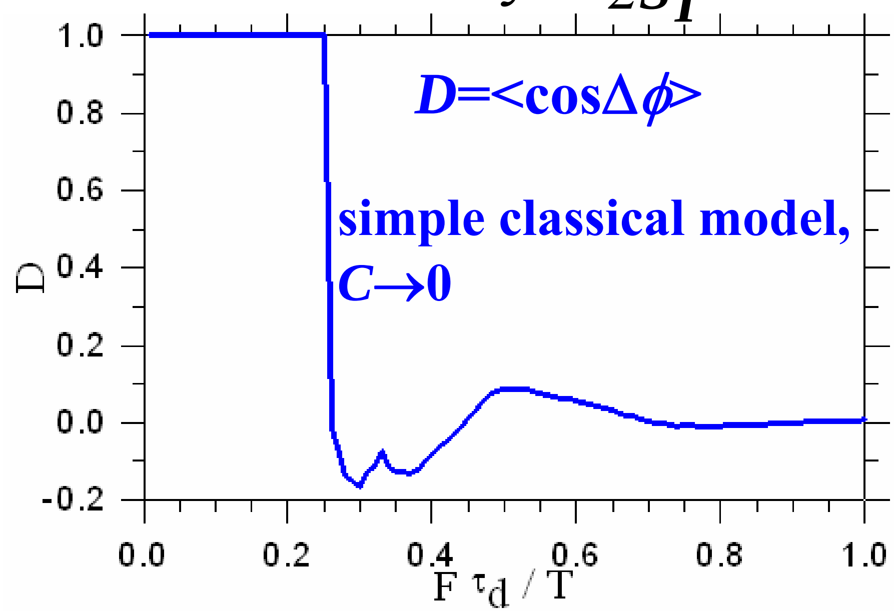


Narrower bandwidth (τ_a↑)⇒reduced efficiency (D_{max}↓), loss of information

Effect of time delay in the feedback loop

- **Signal from past time $t-\tau_d \Rightarrow \Delta H_{fb}/H = -F\times\Delta\phi(t-\tau_d) \Rightarrow D< D(\tau_d=0)$**
- **Full model:** $\dfrac{d\Delta\phi}{dt} = -\sin\phi \dfrac{\Delta I}{S_I}\left(\dfrac{\Delta I}{2}\cos\phi + \xi(t)\right) - 2FH\Delta\phi(t-\tau_d)$
- **Simple model ($C \ll 1$):** $\dfrac{d\Delta\phi}{dt} = \bar{\xi}(t) - 2FH\Delta\phi(t-\tau_d), S_{\bar{\xi}} = \dfrac{(\Delta I)^2}{2S_I}$





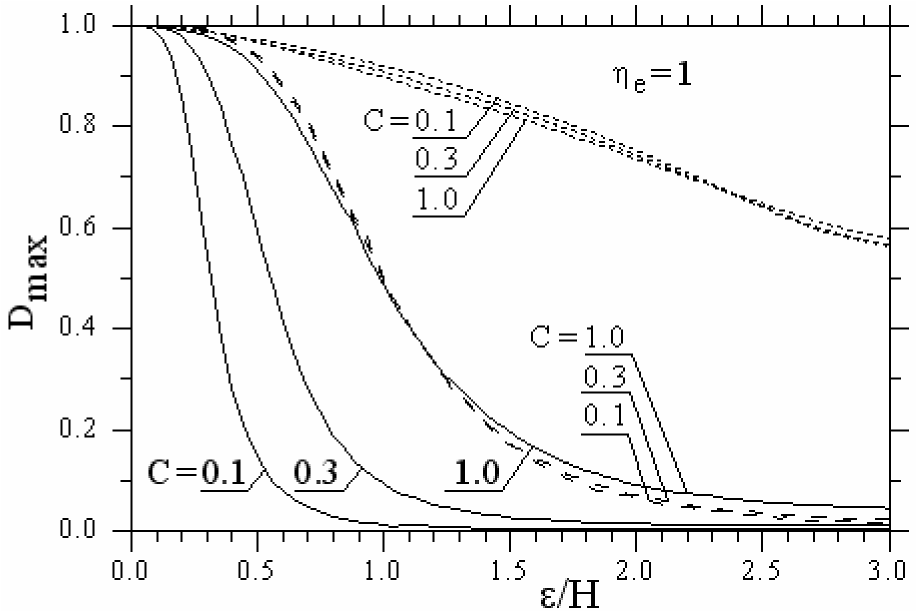
Sharp drop at $F\tau_d/T>1/4$ ---different from ideal case

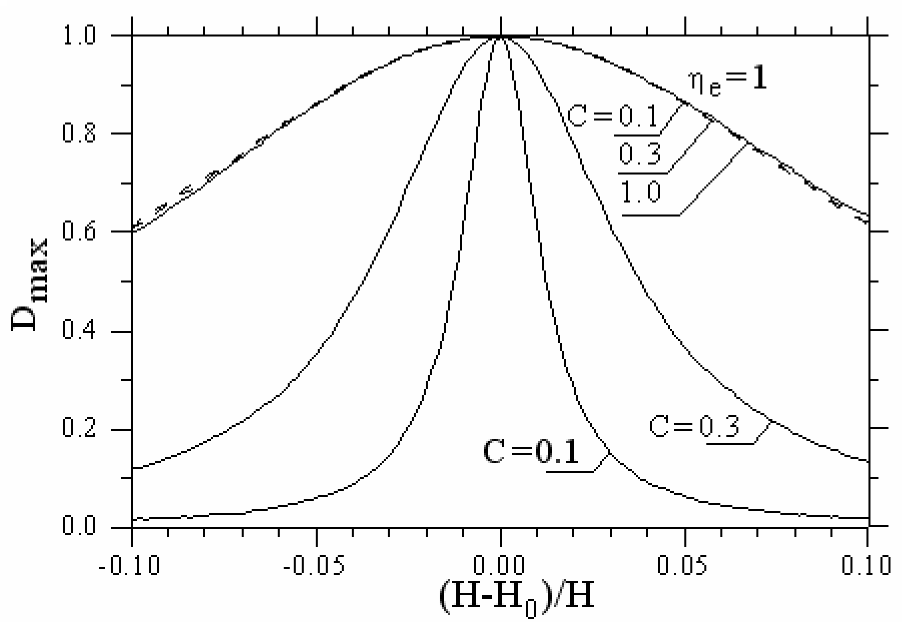
$F\tau_d/T=1/4$ separates stable region & unstable (oversteering) region

Simple model results ≈ full model results; Scaling behavior for $C\tau_d/T= C'\tau_d'/T$

Effect of ε & H deviation (from ε =0, H =H_0)

- mistake in qubit monitoring ⇒ obviously worsens D





Solid: small *ε* decreases *D* very little

Dashed: rescaling x-axis by $C^{1/2}$

Dotted: exact monitoring significantly improves *D*

Effect of H deviation quite similar to *ε*

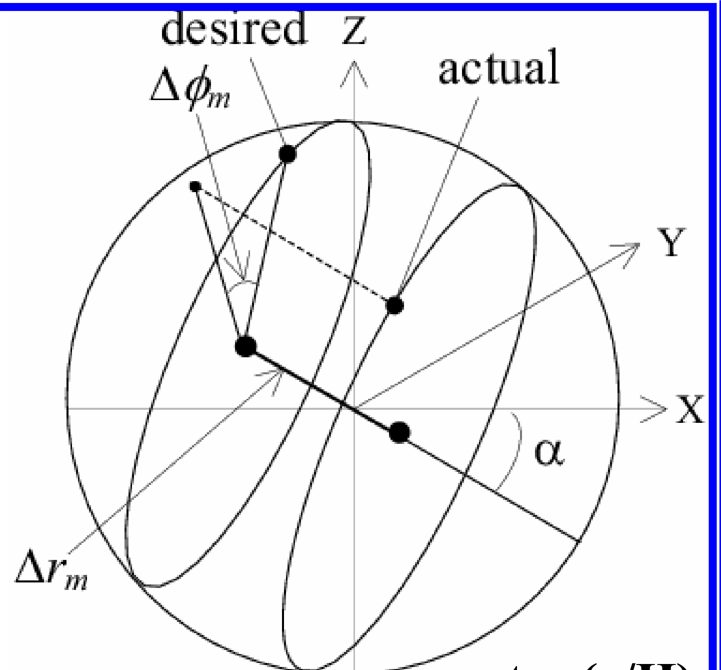
Dashed : rescaling x-axis by *C*

Feedback operation is robust against small unknown deviations of qubit parameters.

Feedback of an energy-asymmetric qubit ($\varepsilon \neq 0$)

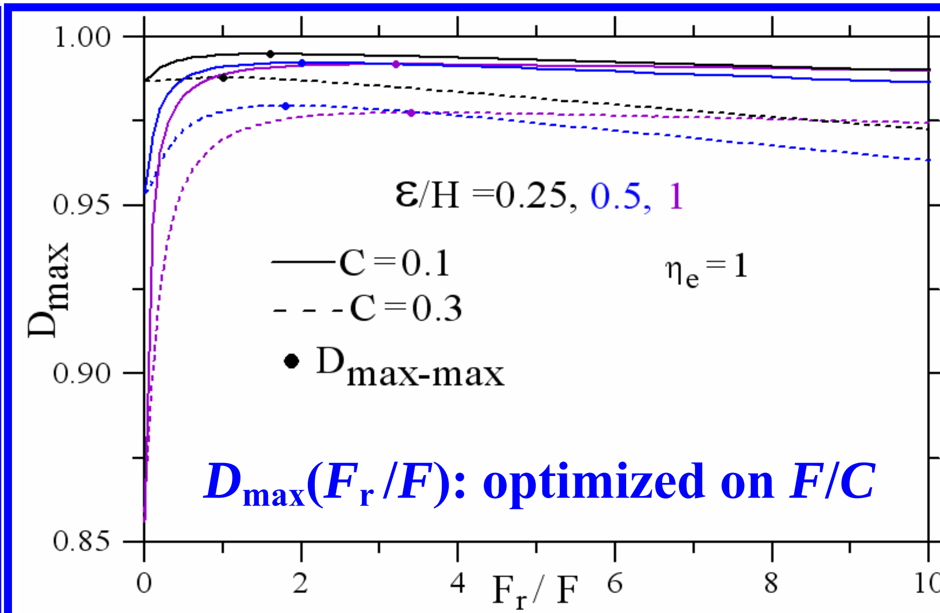
Despite the phase space becomes essentially two-dimensional, we still want to control only one parameter (H)

- **Special controller: $\Delta H_{fb}/H = -F\Delta\phi_m - F_r \sin\phi_m \, \Delta r_m$**
- **Change only one parameter H to reduce both $\Delta\phi_m$ and Δr_m**



α=atan(ε/H)

ρ^m & ρ^d are in two parallel planes



$D_{max\text{-}max}(\varepsilon/H, C{=}0.1) >> D_{max\text{-}max}(\varepsilon/H, C{=}0.3)$

$\Rightarrow D_{max\text{-}max}(\varepsilon/H, C{<<}1)$ can approach 1

This controller works very well!

Conclusions

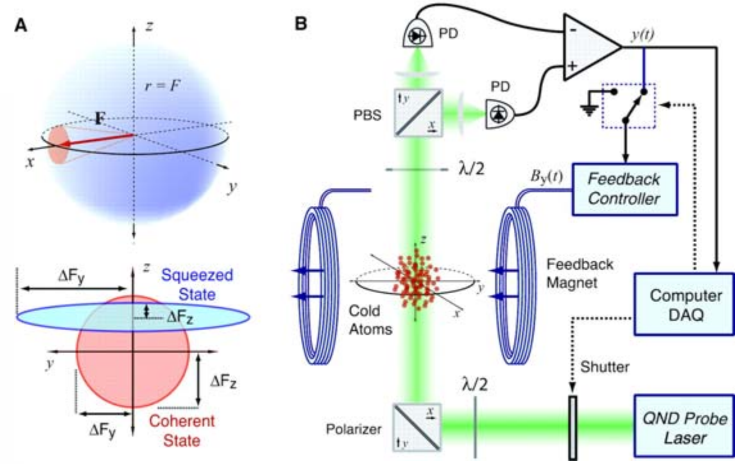
Quantum feedback in optics

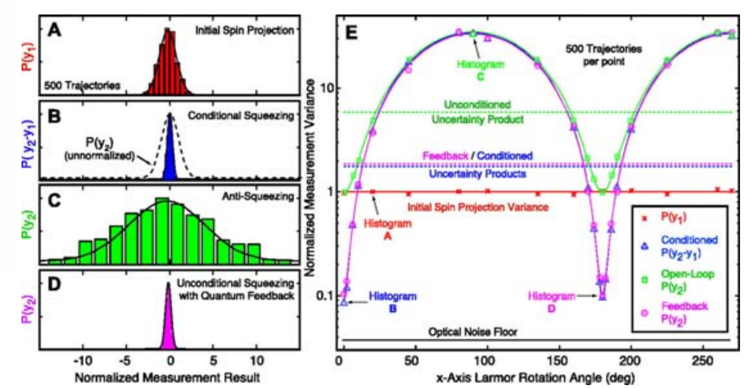
First experiment: Science 304, 270 (2004)

Real-Time Quantum Feedback Control of Atomic Spin-Squeezing

JM Geremia,* John K. Stockton, Hideo Mabuchi

Real-time feedback performed during a quantum nondemolition measurement of atomic spin-angular momentum allowed us to influence the quantum statistics of the measurement outcome. We showed that it is possible to harness measurement backaction as a form of actuation in quantum control, and thus we describe a valuable tool for quantum information science. Our feedback-mediated procedure generates spin-squeezing, for which the reduction in quantum uncertainty and resulting atomic entanglement are not conditioned on the measurement outcome.





First detailed theory:

H.M. Wiseman and G. J. Milburn,
Phys. Rev. Lett. 70, 548 (**1993**)