Quantum efficiency of binary-outcome solid-state detectors of qubits

Alexander Korotkov

University of California, Riverside

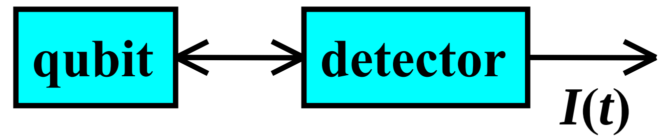
Outline:

- Intro: Quantum efficiency of linear detectors
- Definitions of quantum efficiency for binary-outcome detectors
- Quantum efficiency for several models
  - indirect projective measurements
  - linear detector in binary-output regime
  - detector for phase qubit
  - tunneling into continuum

Support:

Quantum efficiency of linear detectors

Idea of definition (Korotkov-1998):



$$\Gamma \geq 1/2\tau_m \qquad \text{(informational bound)}$$

ensemble decoherence rate — "measurement time" (to reach signal-to-noise =1)

Korotkov, 1998, 2000
Averin, 2000
Pilgram-Buttiker, 2002
Clerk-Girvin-Stone, 2002
. . .

$\Rightarrow$ **quantum efficiency (ideality)** $\eta = 1/2\Gamma\tau_m$

Ideal detector (η=1) does not decohere qubit (pure → pure)

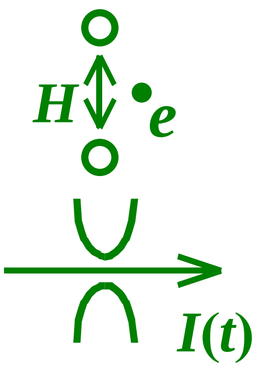
Slightly different definitions (A.K., 2000):

$$\Gamma \geq 1/2\tau_m + K^2 S/4$$

S – output noise
K – output-backaction noise correlation

$\Rightarrow$ **efficiency** $\tilde{\eta} = (1/2\tau_m + K^2 S/4)/\Gamma$

or $\tilde{\tilde{\eta}} = (1/2\tau_m)/(\Gamma - K^2 S/4)$



QPC is an ideal detector
η=1 (A.K., 1998)

Equivalent to the energy sensitivity limitations known since 1980s:

$(\epsilon_O \epsilon_B)^{1/2} \geq \hbar/2$, $(\epsilon_O \epsilon_B - \epsilon_{OB}^2)^{1/2} \geq \hbar/2$, **(Clarke, Tesche, Likharev, Caves, etc.)**

Now general binary-output detector (try to use the same idea)

We consider realistic detectors (not the "orthodox" projective measurement!)

Measurement fidelities F_0 and F_1

F_0 = probability to get result 0 for a qubit in state $|0\rangle$,
F_1 = probability to get result 1 for a qubit in state $|1\rangle$

Ideal detector (pure qubit state → pure)

Use POVM language: linear measurement operators M_0 and M_1 (result 0 or 1)

$$\rho \rightarrow \frac{M_i \rho M_i^\dagger}{\mathrm{Tr}(M_i \rho M_i^\dagger)} \quad \text{for result } i, \quad \text{probability } P_i = \mathrm{Tr}(M_i \rho M_i^\dagger)$$

Each operator M_i : 8 -1(phase) =7 real parameters

7+7=14, but completeness ($M_0^+ M_0 + M_1^+ M_1 = 1$), so 14 – 4=10

Ideal binary-output detector of a qubit is described by <u>10</u> real parameters (including fidelities F_0 and F_1)

Non-ideal binary-output detectors

Again use POVM language, now arbitrary one-qubit quantum operation (superoperator) for each measurement result

$\Rightarrow$ 16 +16 **–** 4 = 28 real parameters for a general (non-ideal) detector

28 (general) – 10 (ideal) = 18 (quantum efficiencies)

Therefore, quantum efficiency (ideality) of a general binary-outcome detector is described by <u>18</u> real parameters.

Too many!!! Impractical. What to do?

Consider only "QND" detectors

(qubit does not evolve itself during measurement, σ_z-coupling)

$|0\rangle \rightarrow |0\rangle$, $|1\rangle \rightarrow |1\rangle$

Try to use the informational bound (as for linear detectors)

Decoherence bound for a QND detector

General description of a QND detector: only 6 parameters

(fidelities F_0 and F_1, decoherences D_0 and D_1, and angles ϕ_0 and ϕ_1)

result 0: $$\begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix} \to \frac{1}{P_0} \begin{pmatrix} F_0\rho_{00} & \sqrt{F_0(1-F_1)}\, e^{-D_0} e^{i\phi_0} \rho_{01} \\ c.c. & (1-F_1)\rho_{11} \end{pmatrix}$$

result 1: $$\begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix} \to \frac{1}{P_1} \begin{pmatrix} (1-F_0)\rho_{00} & \sqrt{(1-F_0)F_1}\, e^{-D_1} e^{i\phi_1} \rho_{01} \\ c.c. & F_1\rho_{11} \end{pmatrix}$$

simple Bayes!

probabilities: $P_0 = F_0\rho_{00} + (1-F_1)\rho_{11}, \; P_1 = (1-F_0)\rho_{00} + F_1\rho_{11}$

Average over results: $$\begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix} \to \begin{pmatrix} \rho_{00} & e^{-D_{av}} e^{i\phi_{av}} \rho_{01} \\ c.c. & \rho_{11} \end{pmatrix}$$

$$e^{-D_{av}} e^{i\phi_{av}} = \sqrt{F_0(1-F_1)}\, e^{-D_0} e^{i\phi_0} + \sqrt{(1-F_0)F_1}\, e^{-D_1} e^{i\phi_1}$$

⇒ ensemble decoherence bound $$D_{av} \geq D_{\min} = -\ln[\sqrt{F_0(1-F_1)} + \sqrt{(1-F_0)F_1}\,]$$

Definitions of quantum efficiency (actual decoherence vs. informational bound)

Similar to the first definition for linear detectors

$$\eta = D_{\min} / D_{av}$$

Taking into account phase correlation:

$$\tilde{\eta} = \frac{-\ln | \sqrt{F_0(1-F_1)} + \sqrt{(1-F_0)F_1}\, e^{i(\phi_1 - \phi_0)} |}{D_{av}}$$

or

$$\tilde{\tilde{\eta}} = \frac{-\ln[\sqrt{F_0(1-F_1)} + \sqrt{(1-F_0)F_1}\,]}{-\ln[\sqrt{F_0(1-F_1)}\, e^{-D_0} + \sqrt{(1-F_0)F_1}\, e^{-D_1}\,]}$$

Also meaningful to define quantum efficiency for each result of the measurement:

$$1-\eta_0 = \frac{D_0}{D_0 - \ln\sqrt{F_0(1-F_1)}}, \quad 1-\eta_1 = \frac{D_1}{D_1 - \ln\sqrt{(1-F_0)F_1}}$$

(useful for “asymmetric” and “half-destructive” detectors, as for phase qubits)

Quantum efficiency for several detector models

Model 1: Indirect projective measurement

$$\boxed{\text{qubit}} \longleftrightarrow \boxed{\text{ancilla qubit}} \xleftrightarrow[\text{measurement}]{\text{projective}} \boxed{\text{detector}}$$

Evolution:

$$(\alpha|0\rangle+\beta|1\rangle)|0_a\rangle \to \alpha|0\rangle(c_{00}|0_a\rangle+c_{10}|1_a\rangle)+\beta|1\rangle(c_{01}|0_a\rangle+c_{11}|1_a\rangle) \to$$

$$\to \begin{cases} (\alpha c_{00}|0\rangle+\beta c_{01}|1\rangle)/\text{Norm}, & \text{if result 0} \\ (\alpha c_{10}|0\rangle+\beta c_{11}|1\rangle)/\text{Norm}, & \text{if result 1} \end{cases}$$

Then

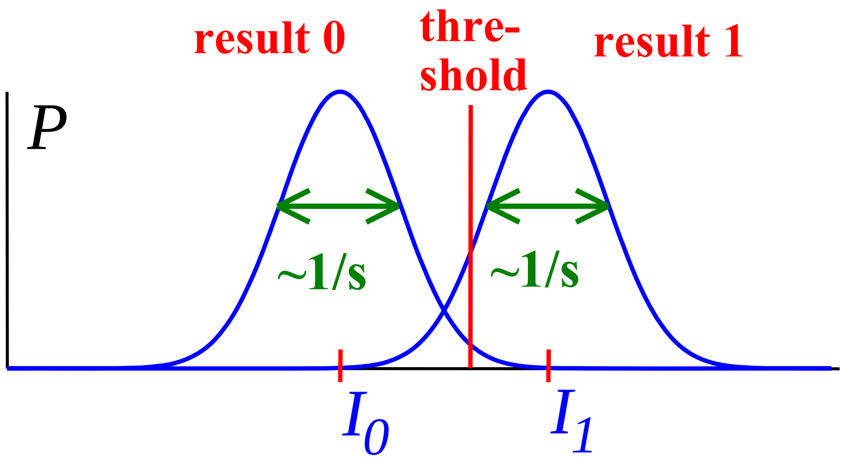
$$F_0=|c_{00}|^2,\ \ F_1=|c_{11}|^2,\ \ \phi_0=\arg(c_{00}c_{01}^*),\ \ \phi_1=\arg(c_{10}c_{11}^*),\ \ D_0=0,\ \ D_1=0$$

And so $\eta_0=\eta_1=1,\ \ \tilde{\eta}=\tilde{\tilde{\eta}}=1$ **(ideal)**

but $\eta\neq 1,$ **if** $\phi_0\neq\phi_1$

Model 2: Linear detector in binary-output regime

qubit ⟷ **linear detector** —continuous result→ **comparison with threshold** → **result 0 or 1**



$$F_0 = [1 + \mathrm{erf}(r + s)]/2$$

$$F_1 = [1 + \mathrm{erf}(-r + s)]/2$$

where r is threshold and

$s = \sqrt{t/2\tau_m}$ is measurement strength

Results:

$$\eta_0 = \frac{-\ln\sqrt{F_0(1-F_1)}}{-\ln[(1+\mathrm{erf}(r))/2] + s^2 + \gamma t}$$

$$\eta_1(r) = \eta_0(-r)$$

$$\eta = \frac{-\ln[\sqrt{F_0(1-F_1)} + \sqrt{(1-F_0)F_1}\,]}{s^2 + \gamma t}$$

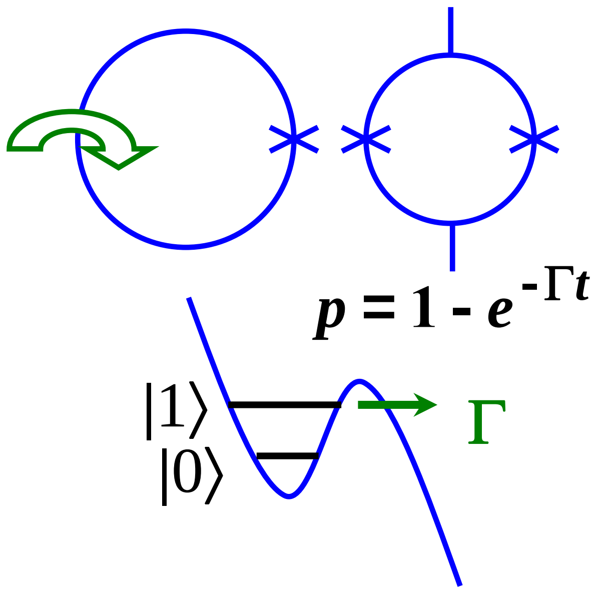
$$\eta \le 2/\pi$$

Even for an ideal linear detector the threshold detector is significantly non-ideal

Why? Because we loose information!

Model 3: Partial-collapse measurement of a phase qubit

N. Katz et al.,
Science-2006
(Martinis' group)



Result 0 ("null result"), then

$$\alpha|0\rangle + \beta|1\rangle \rightarrow \frac{\alpha|0\rangle + e^{i\phi}\beta e^{-\Gamma t/2}|1\rangle}{\text{Norm}}$$

Result 1, then qubit destroyed

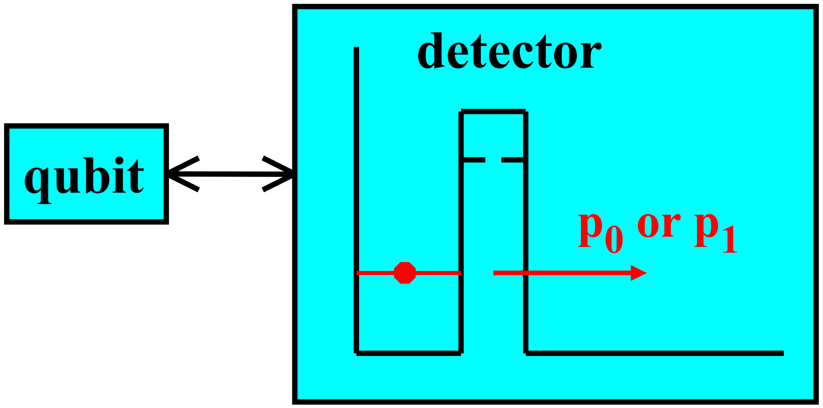
For this model $\eta_0 = 1$,

while η_1 **and** η **cannot be defined**

("half-destructive" measurement)

If imperfections are taken into account (Pryadko-Korotkov, 2007), then finite quantum efficiency for null-result case: $\eta_0 < 1$.

Model 4: Detector based on tunneling into continuum



probability to tunnel out (p_0 or p_1) depends on the qubit state

$$F_0 = 1 - p_0\,, \quad F_1 = p_1$$

non-destructive detector for both measurement results

In simple case (when $p_0 < p_1 << 1$):

Result 0 (no tunneling), then

$$\alpha\,|0\rangle + \beta\,|1\rangle \rightarrow \frac{\alpha\sqrt{1-p_0}\,|0\rangle + \beta\sqrt{1-p_1}\,e^{i\phi_0}\,|1\rangle}{\text{Norm}}$$

Result 1 (tunneling), then

$$\alpha\,|0\rangle + \beta\,|1\rangle \rightarrow \frac{\alpha\sqrt{p_0}\,|0\rangle + \beta\sqrt{p_1}\,e^{i\phi_1}\,|1\rangle}{\text{Norm}}$$

Then ideal detector:

$$\eta_0 = \eta_1 = 1,$$

$$\tilde{\eta} = \tilde{\tilde{\eta}} = 1$$

$$\eta = 1 \ \text{ if } \ \phi_0 = \phi_1$$

Can such regime be realized by a real SQUID or by a bifurcation detector?

Conclusions

- **Derived a simple informational bound on the qubit ensemble decoherence due to measurement by a binary-outcome detector**
- **Introduced corresponding definitions for the detector quantum efficiency**
- **Calculated the quantum efficiency in several models**