Few topics in non-projective measurement of solid-state qubits

Alexander Korotkov

University of California, Riverside

Outline:

- Brief introduction
- Persistent Rabi oscillations and Leggett-Garg inequality
- Quantum feedback: several ideas
- Uncollapsing: theory and experiment
- Quantum efficiency of binary-outcome detectors

Support:

Quantum collapse due to measurement – fascinating puzzle of quantum mechanics since 1920s

"orthodox" projective collapse ⇒ Bell inequality (problem with causality)

Even more interesting: what is "inside" collapse
(nothing is instantaneous, matter of time scale ⇒ non-projective)

The same fascinating feature (as in Bell inequality):
"spooky" quantum back-action in the process of collapse

Proper question: "how", not "why"

Most importantly: "spookiness" of quantum measurement becomes an experimental subject in solid state systems
(3 experiments already: Santa Barbara & Sacley)

Possibly something useful (not only very interesting)?

In this talk: theory related to experiments (realized and future)
on non-projective collapse of solid-state qubits

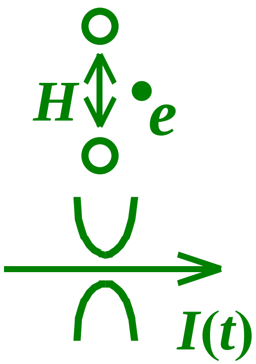
Quantum Bayesian formalism

A.K., 1998
(Caves-1986,
Gardiner-91)

Evolution due to measurement ("spooky" quantum back-action)

1) ρ_{ii} evolve as probabilities, i.e. according to the Bayes rule
(for $\psi=\alpha|1\rangle+\beta|2\rangle$, $|\alpha|^2$ and $|\beta|^2$ behave as probabilities)

2) $\rho_{ij}/(\rho_{ii}\,\rho_{jj})^{1/2}$ = const, i.e. pure state remains pure
(for $\psi=\alpha|1\rangle+\beta|2\rangle$, the phases of $\alpha(t)$ and $\beta(t)$ do not change)



Add physical (realistic) evolution

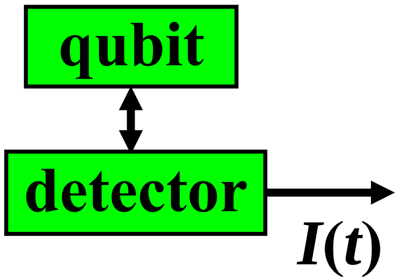
- Hamiltonian evolution, classical back-action, decoherence, etc.
(technically: add terms in the differential equation)

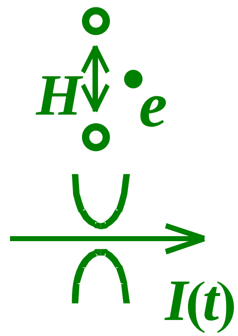
$$d\rho_{11}/dt = -d\rho_{22}/dt = -2(H/\hbar)\,\mathrm{Im}\,\rho_{12} + \rho_{11}\rho_{22}(2\Delta I/S_I)[I(t)-I_0]$$

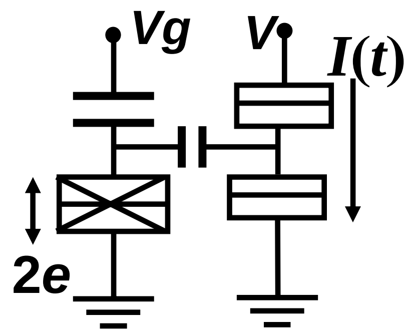
$$d\rho_{12}/dt = i(\varepsilon/\hbar)\rho_{12} + i(H/\hbar)(\rho_{11}-\rho_{22}) + \rho_{12}(\rho_{11}-\rho_{22})(\Delta I/S_I)[I(t)-I_0] - \gamma\rho_{12}$$

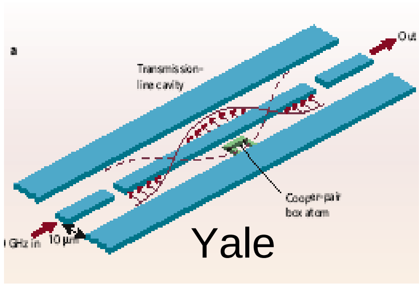
Same idea as in POVM, general quant. meas., quantum trajectories, etc.

Models of continuous measurement









$$H_{\rm qubit} = \frac{\varepsilon}{2}(|1\rangle\langle 1| - |2\rangle\langle 2|) + H(|1\rangle\langle 2| - |2\rangle\langle 1|)$$

Measurement: average signals I_1 and I_2, response $\Delta I = I_1 - I_2$, white noise S_I

Quantum efficiency: relation between “spooky” and non-spooky (ratio of “spooky” and total ensemble decoherence)

$$\eta = \frac{(\Delta I)^2/4S_I}{\Gamma}$$

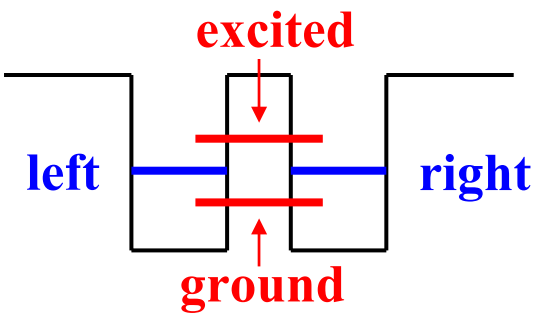
$\longleftarrow$ spooky (informational back-action)

$\longleftarrow$ total qubit dephasing

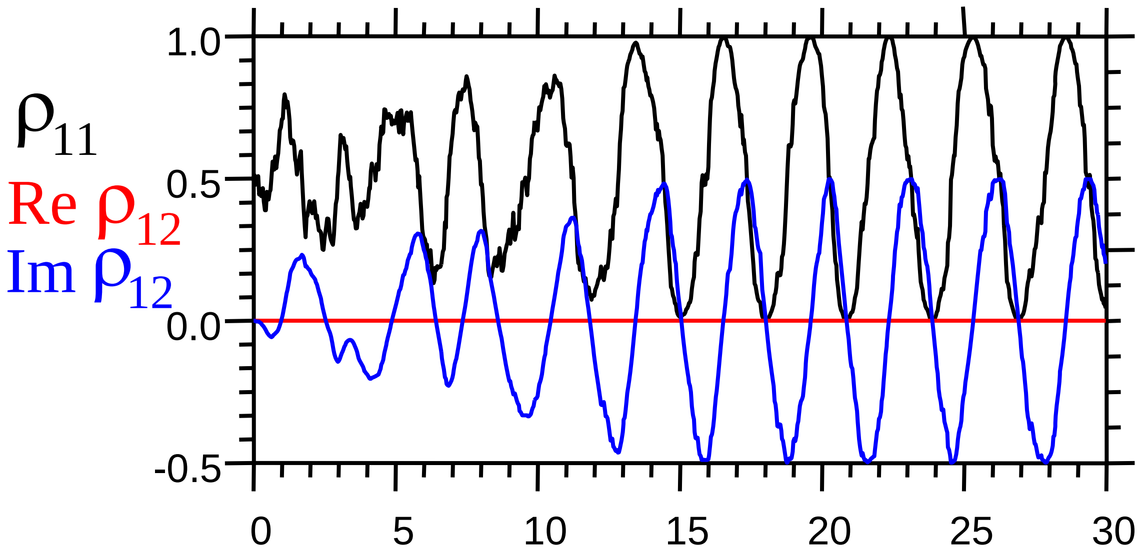
quantum limited: $\eta = 1$

(coincides with definition via energy sensitivity in units $\hbar/2$)

Persistent Rabi oscillations



- Relaxes to the ground state if left alone (low-T)
- Becomes fully mixed if coupled to a high-T (non-equilibrium) environment
- Oscillates persistently between left and right if (weakly) measured continuously



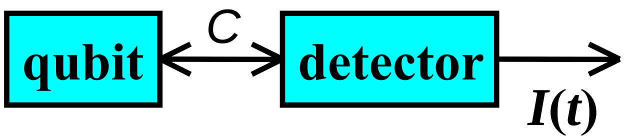
Phase of Rabi oscillations fluctuates (phase noise, dephasing)

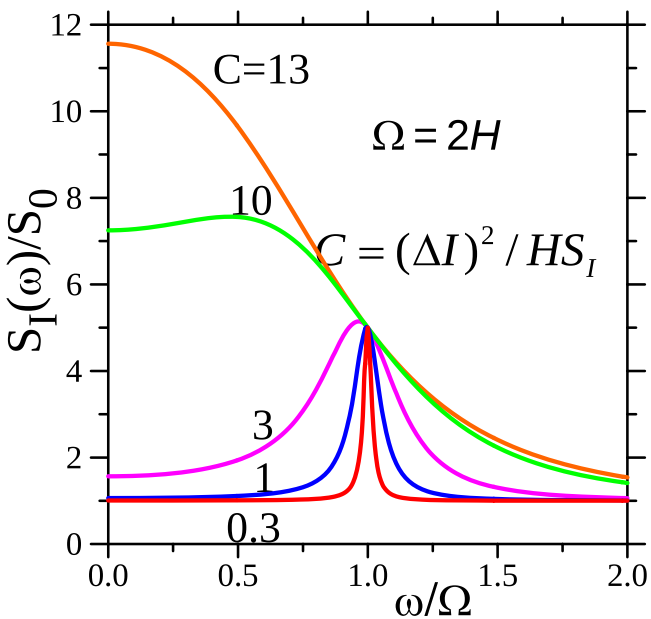
Direct experiment is difficult (quantum efficiency, bandwidth, control)

A.K., 1998

Indirect experiment: spectrum of persistent Rabi oscillations

A.K., LT'1999
A.K.-Averin, 2000





Ω - Rabi frequency

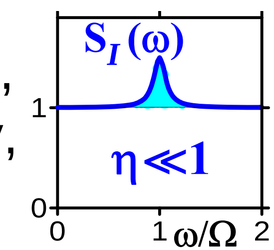
peak-to-pedestal ratio = $4\eta \leq 4$

$$S_I(\omega) = S_0 + \frac{\Omega^2 (\Delta I)^2 \Gamma}{(\omega^2 - \Omega^2)^2 + \Gamma^2 \omega^2}$$

$$I(t) = I_0 + \frac{\Delta I}{2} z(t) + \xi(t)$$

(const + signal + noise)

amplifier noise $\Rightarrow$ higher pedestal,
poor quantum efficiency,
but the peak is the same!!!



integral under the peak $\Leftrightarrow$ variance $\langle z^2 \rangle$

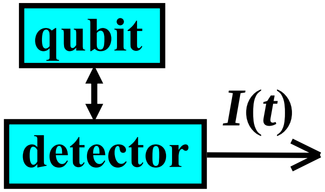
How to distinguish experimentally persistent from non-persistent? Easy!

perfect Rabi oscillations: $\langle z^2 \rangle = \langle \cos^2 \rangle = 1/2$
imperfect (non-persistent): $\langle z^2 \rangle \ll 1/2$
quantum (Bayesian) result: $\langle z^2 \rangle = 1$ (!!!)

(demonstrated in Saclay expt.)

How to understand $\langle z^2 \rangle = 1$?

$$I(t) = I_0 + \frac{\Delta I}{2} z(t) + \xi(t)$$



First way (mathematical)

We actually measure operator: $z \rightarrow \sigma_z$

$$z^2 \rightarrow \sigma_z{}^2 = 1$$

(What does it mean? Difficult to say…)

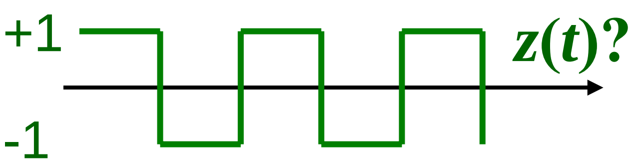
Second way (Bayesian)

$$S_I(\omega) = S_{\xi\xi} + \frac{\Delta I^2}{4} S_{zz}(\omega) + \frac{\Delta I}{2} S_{\xi z}(\omega)$$

quantum back-action changes z in accordance with the noise ξ

Equal contributions (for weak coupling and η=1)

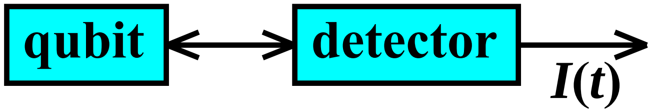
Can we explain it in a more reasonable way (without spooks/ghosts)?



No (under assumptions of macrorealism; Leggett-Garg, 1985)

or some other $z(t)$?

Leggett-Garg-type inequalities for continuous measurement of a qubit



Ruskov-A.K.-Mizel, PRL-2006
Jordan-A.K.-Büttiker, PRL-2006

Assumptions of macrorealism (similar to Leggett-Garg'85):

$$I(t) = I_0 + (\Delta I/2)\, z(t) + \xi(t)$$

$$|z(t)| \le 1, \quad \langle \xi(t)\, z(t+\tau)\rangle = 0$$

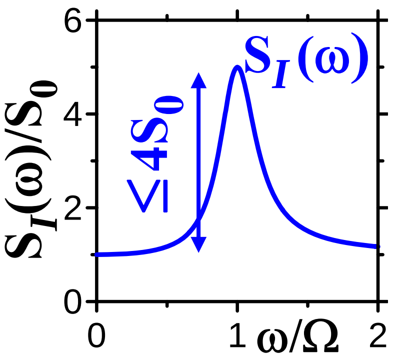
Leggett-Garg,1985

$K_{ij} = \langle Q_i Q_j \rangle$

if $Q = \pm 1$, then

$1 + K_{12} + K_{23} + K_{13} \ge 0$

$K_{12} + K_{23} + K_{34} - K_{14} \le 2$



Then for correlation function

$$K(\tau) = \langle I(t)\, I(t+\tau)\rangle$$

$$K(\tau_1) + K(\tau_2) - K(\tau_1 + \tau_2) \le (\Delta I/2)^2$$

quantum result: $\frac{3}{2}(\Delta I/2)^2$ — violation: $\times \frac{3}{2}$

and for area under narrow spectral peak

$$\int [S_I(f) - S_0]\, df \le (8/\pi^2)(\Delta I/2)^2$$

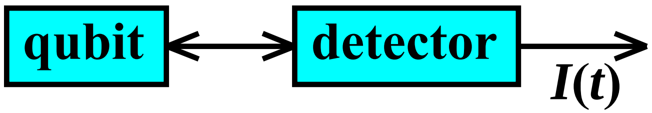
quantum result: $(\Delta I/2)^2$ — violation: $\times \frac{\pi^2}{8}$

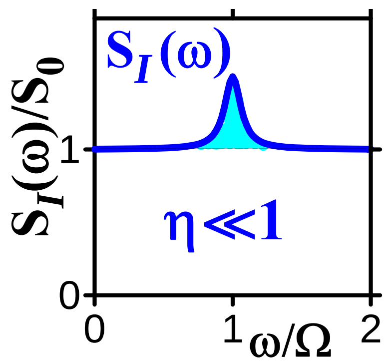
η is not important!

Experimentally measurable violation

(Saclay experiment)

May be a physical (realistic) back-action?





$$I(t) = I_0 + \frac{\Delta I}{2} z(t) + \xi(t)$$

OK, cannot explain without back-action

$$\langle \xi(t)\, z(t+\tau) \rangle \neq 0$$

But may be this is a simple classical back-action from the noise?

In principle, classical explanation cannot be ruled out (e.g. computer-generated $I(t)$; no non-locality as in optics)

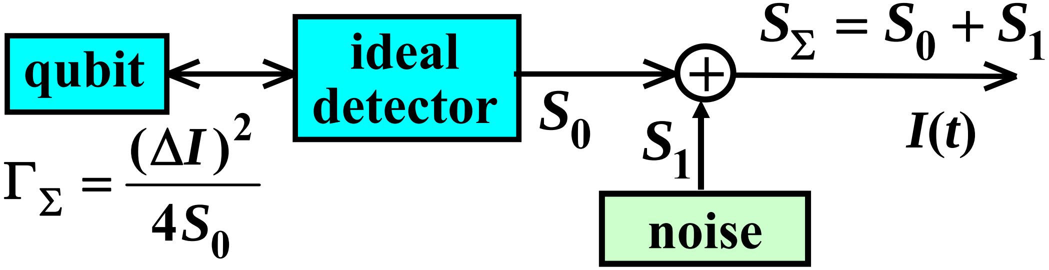
Try reasonable models: linear modulation of the qubit parameters (H and ε) by noise $\xi(t)$

No, does not work!

Our (spooky) back-action is quite peculiar: $\langle \xi(t)\, dz(t+0) \rangle > 0$

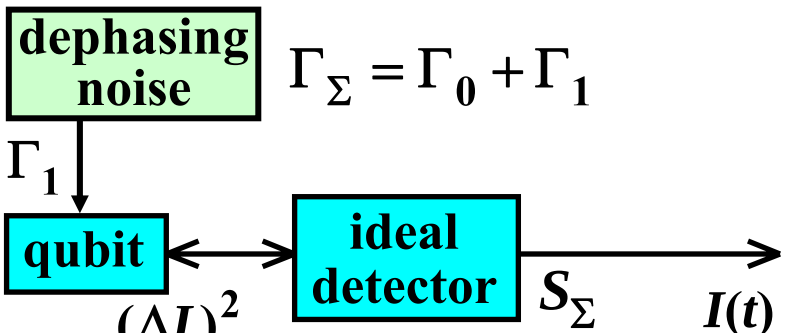
"what you see is what you get": observation becomes reality

Two ways to think about η<1



$$\Gamma_\Sigma = \frac{(\Delta I)^2}{4S_0}$$

$$\eta = \frac{(\Delta I)^2/4S_\Sigma}{\Gamma_\Sigma}$$



$$\Gamma_\Sigma = \Gamma_0 + \Gamma_1$$

$$\Gamma_0 = \frac{(\Delta I)^2}{4S_\Sigma}$$

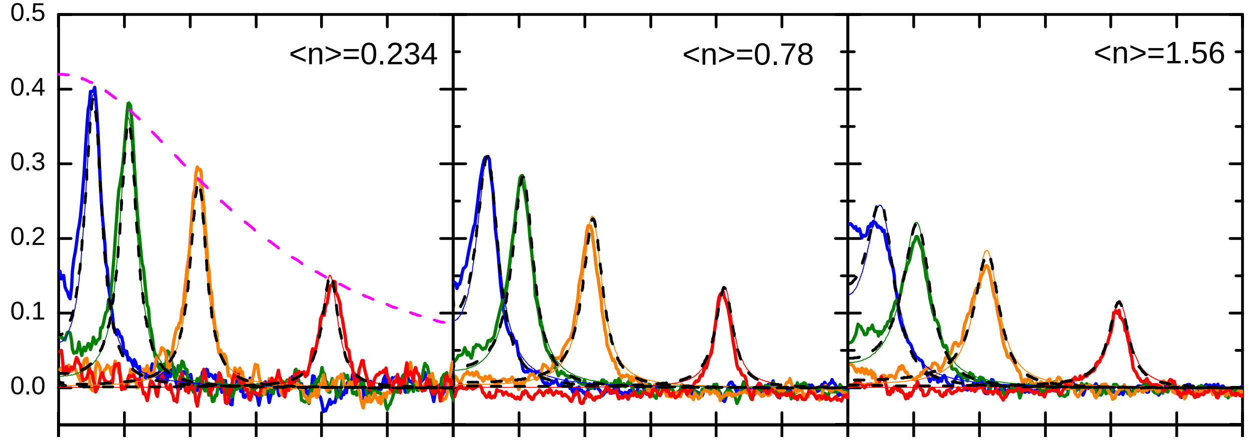
These ways are equivalent
(same results for any expt.)
⇒ **matter of convenience**

A.K., 2002

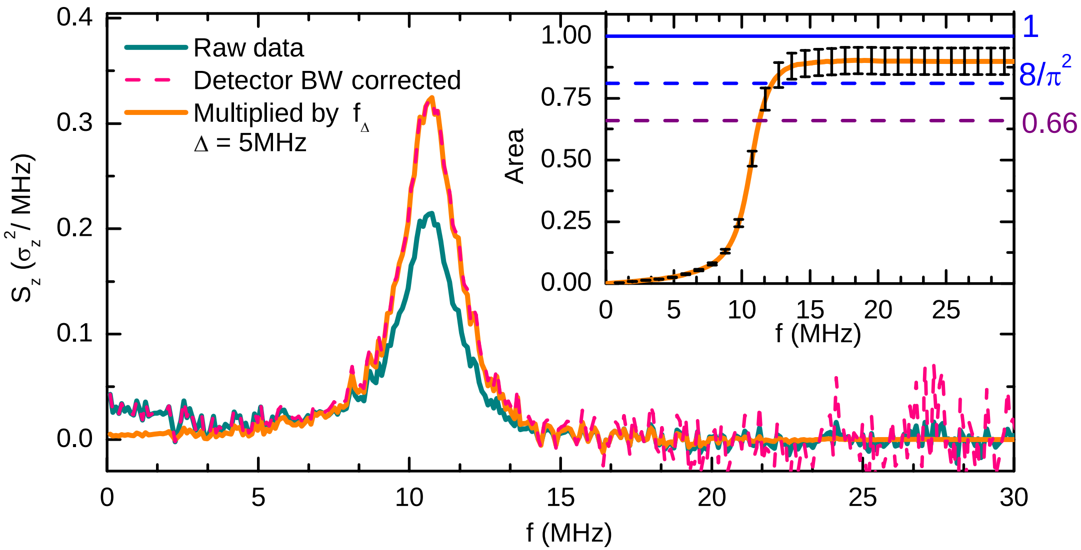
For spectrum: $$S_I(\omega) = S_\Sigma + \frac{\Delta I^2}{4} S_{zz}(\omega) + \frac{\Delta I}{2} S_{\xi z}(\omega)$$

different relative contributions
in the two approaches

Recent experiment on persistent Rabi oscillations (P. Bertet *et al.*, Saclay group)



courtesy of Patrice Bertet



- superconducting qubit monitored by microwave reflection from cavity
- driven Rabi oscillations
- perfect spectral peaks (η~0.02≪1)
- LGI violation

<u>First</u> demonstration of <u>persistent</u> Rabi oscillations (?)

Previous experimental confirmation?

Durkan and Welland, 2001 (STM-ESR experiment similar to Manassen-1989)

APPLIED PHYSICS LETTERS VOLUME 80, NUMBER 3 21 JANUARY 2002

Electronic spin detection in molecules using scanning-tunneling-microscopy-assisted electron-spin resonance

C. Durkan[a] and M. E. Welland
Nanoscale Science Laboratory, Department of Engineering, University of Cambridge, Trumpington Street, Cambridge CB2 1PZ, United Kingdom

(Received 8 May 2001; accepted for publication 8 November 2001)

By combining the spatial resolution of a scanning-tunneling microscope (STM) with the electronic spin sensitivity of electron-spin resonance, we show that it is possible to detect the presence of localized spins on surfaces. The principle is that a STM is operated in a magnetic field, and the resulting component of the tunnel current at the Larmor (precession) frequency is measured. This component is nonzero whenever there is tunneling into or out of a paramagnetic entity. We have

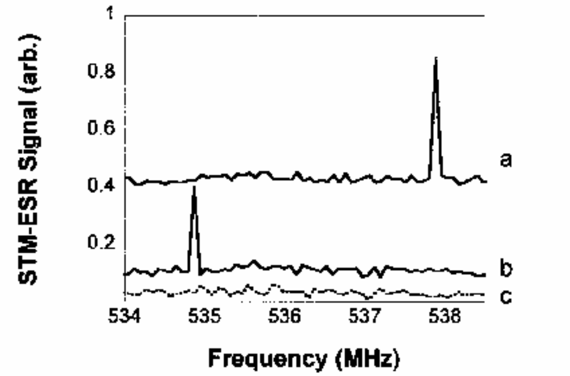


FIG. 3. STM-ESR spectra of (a), (b) two different areas (a few nm apart) of the molecule-covered sample and (c) bare HOPG. The graphs are shifted vertically for clarity.

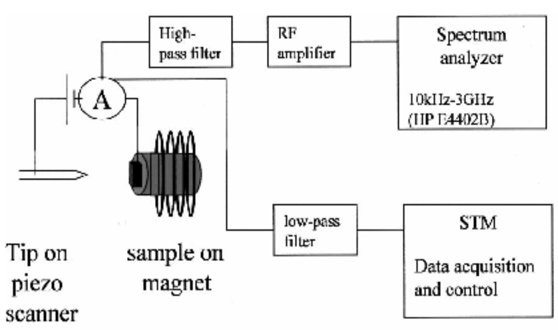


FIG. 1. Schematic of the electronics used in STM-ESR.

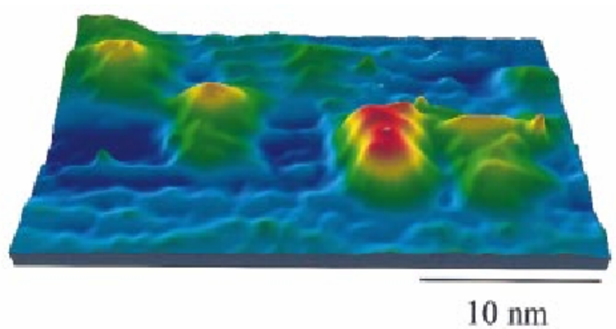


FIG. 2. (Color) STM image of a 250 Å×150 Å area of HOPG with four adsorbed BDPA molecules.

$$\frac{\text{peak}}{\text{noise}} \leq 3.5$$

(Colm Durkan, private comm.)

Recently reproduced:
Messina et al., JAP-2007

Questionable

Somewhat similar experiment

"Continuous monitoring of Rabi oscillations in a Josephson flux qubit"

$$H = -\frac{1}{2}(\Delta\sigma_x + \varepsilon\sigma_z) - W\,\sigma_z \cos\omega_{HF} t$$

$$(\omega_{HF} \approx \sqrt{\Delta^2 + \varepsilon^2}; \quad \varepsilon \neq 0)$$

E. Il'ichev *et al.*, PRL, 2003

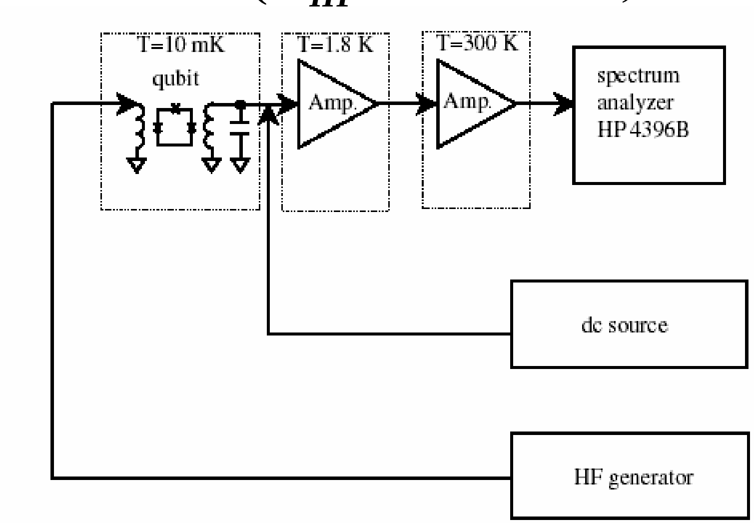


FIG. 1. Measurement setup. The flux qubit is inductively coupled to a tank circuit. The dc source applies a constant flux $\Phi_e \approx \frac{1}{2}\Phi_0$. The HF generator drives the qubit through a separate coil at a frequency close to the level separation $\Delta/h = 868$ MHz. The output voltage at the resonant frequency of the tank is measured as a function of HF power.

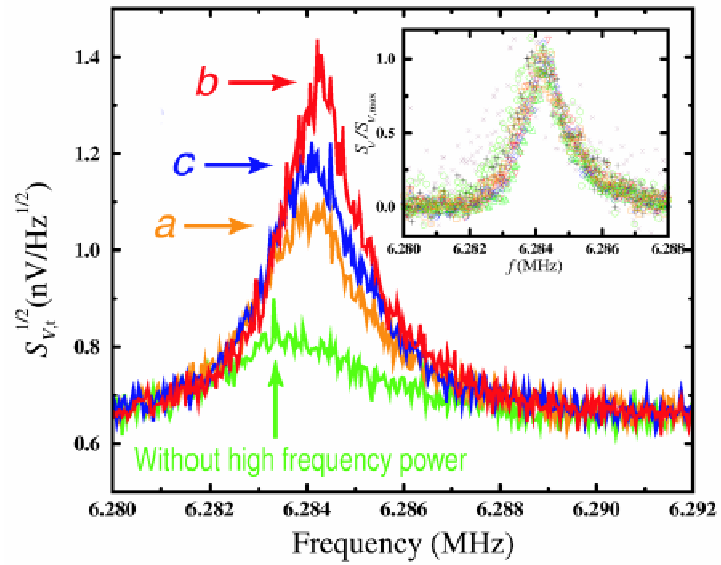


FIG. 3 (color online). The spectral amplitude of the tank voltage for HF powers $P_a < P_b < P_c$ at 868 MHz, detected using the setup of Fig. 1. The bottom curve corresponds to the background noise without an HF signal. The inset shows normalized voltage spectra for seven values of HF power, with background subtracted. The shape of the resonance, being determined by the tank circuit, is essentially the same in each

low-bandwidth tank ⇒ qubit monitoring is impossible

Next topic: Quantum feedback for persistent Rabi oscillations

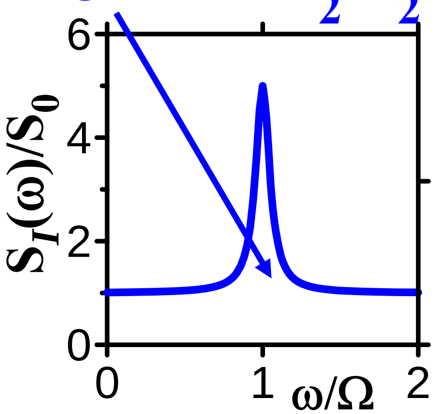
In simple monitoring the phase of persistent Rabi oscillations fluctuates randomly:

$$z(t) = \cos[\Omega t + \varphi(t)] \qquad \text{for } \eta=1$$

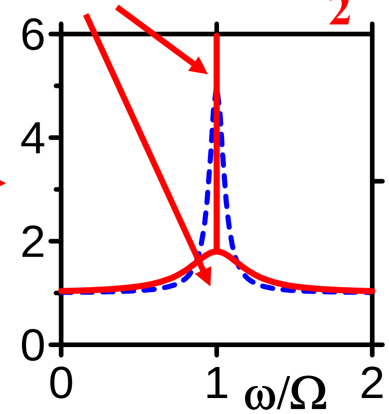
phase noise $\Rightarrow$ finite linewidth of the spectrum

Goal: produce persistent Rabi oscillations without phase noise by synchronizing with a classical signal $z_{\text{desired}}(t) = \cos(\Omega t)$

integral $\langle z^2 \rangle = \frac{1}{2} + \frac{1}{2} = 1$



integral $\langle z^2 \rangle = \frac{1}{2}$



$$I(t) = I_0 + \frac{\Delta I}{2} z(t) + \xi(t)$$

$$S_I = S_0 + \frac{\Delta I^2}{4} S_{zz} + \frac{\Delta I}{2} S_{\xi z}$$

synchronized

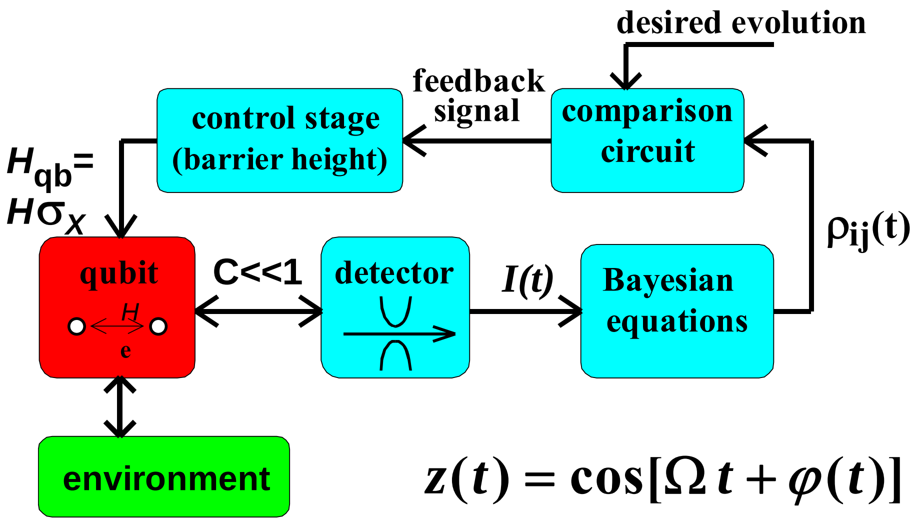
cannot synchronize

Several ways to organize quantum feedback

First idea: Bayesian feedback

(most straightforward but most difficult experimentally)

The wavefunction is monitored via Bayesian equations, and then usual (linear) feedback of the Rabi phase



$$z(t) = \cos[\Omega t + \varphi(t)]$$

$$\Delta H_{FB}/H = -F\times\varphi$$

$$\Delta\Omega/\Omega = -F\times\varphi$$

Ruskov & A.K., 2002

How to characterize feedback efficiency/fidelity?

D = average scalar product of desired and actual vectors on Bloch sphere

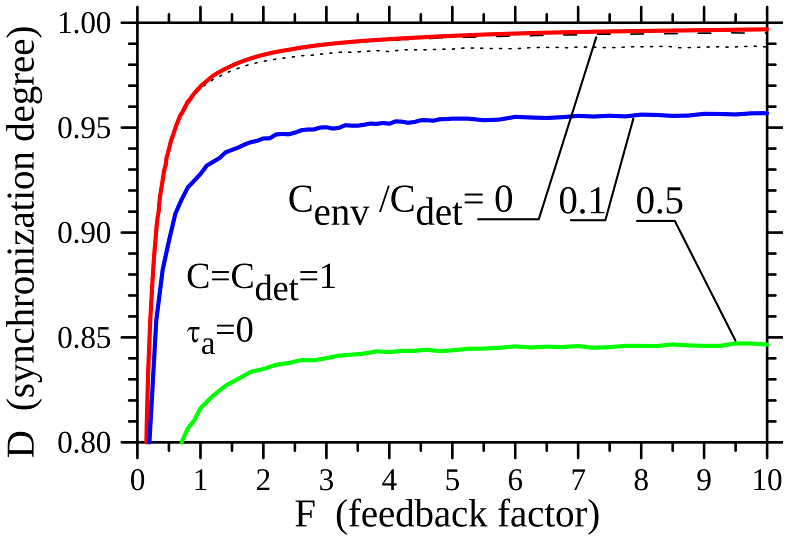
$$D = 2\langle \mathrm{Tr}\rho_{desired}\,\rho\rangle - 1$$

Experimental difficulties:

- necessity of **very fast** real-time solution of Bayesian equations
- **wide bandwidth** ($\gg\Omega$, GHz-range) of the line delivering noisy signal $I(t)$ to the "processor"

Performance of Bayesian feedback

Feedback fidelity vs. feedback strength



$C=\hbar(\Delta I)^2/S_I H$ – **coupling**

F – **feedback strength**

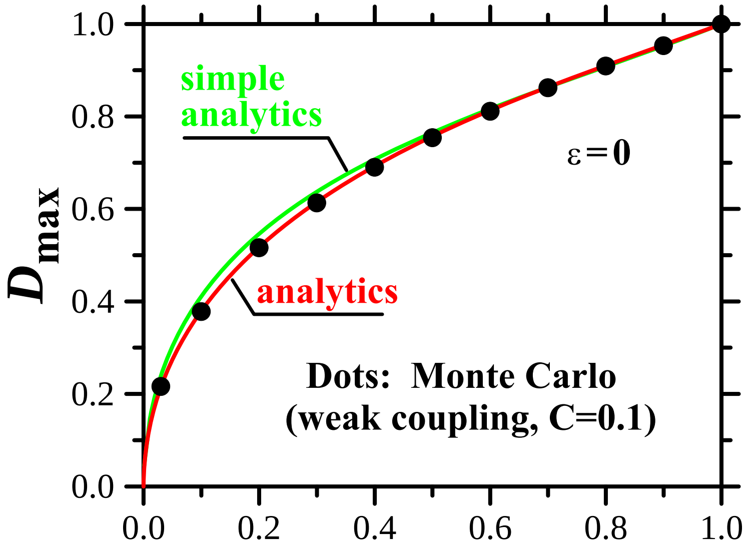
$D=2\langle \mathrm{Tr}\,\rho_{\text{desired}}\rho\rangle-1$

For ideal detector and wide bandwidth, feedback fidelity can be close to 100%

$D=\exp(-C/32F)$

Ruskov & A.K., 2002

Feedback fidelity vs. detector efficiency



$$\eta \ll 1 \;\Rightarrow\; D_{\max}\approx 1.25\sqrt{\eta}$$

$$\eta \approx 1 \;\Rightarrow\; D_{\max}\approx (1+\eta)/2$$

Zhang, Ruskov, A.K., 2005

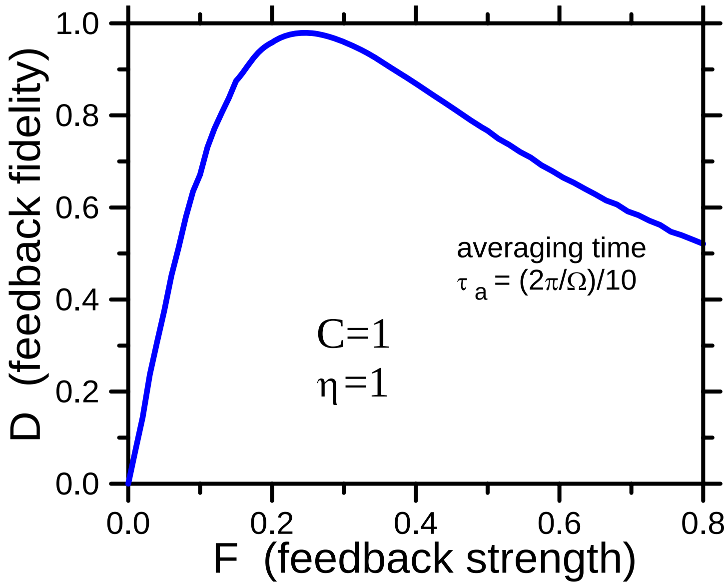
Bayesian quantum feedback gives the best possible performance, but very difficult experimentally

Second idea: direct feedback

(similar to Wiseman-Milburn, 1993)

Squeezing of an optical cavity field by feedback of the homodyne detection signal (Wiseman-Milburn, 1993) **feedback ~ $I(t)$-I_0**

$$\frac{\Delta H_{\mathrm{fb}}}{H} = \frac{\Delta\Omega}{\Omega} = F \times \left(\frac{I(t) - I_0}{\Delta I / 2} - \cos(\Omega t) \right) \times \sin(\Omega t)$$



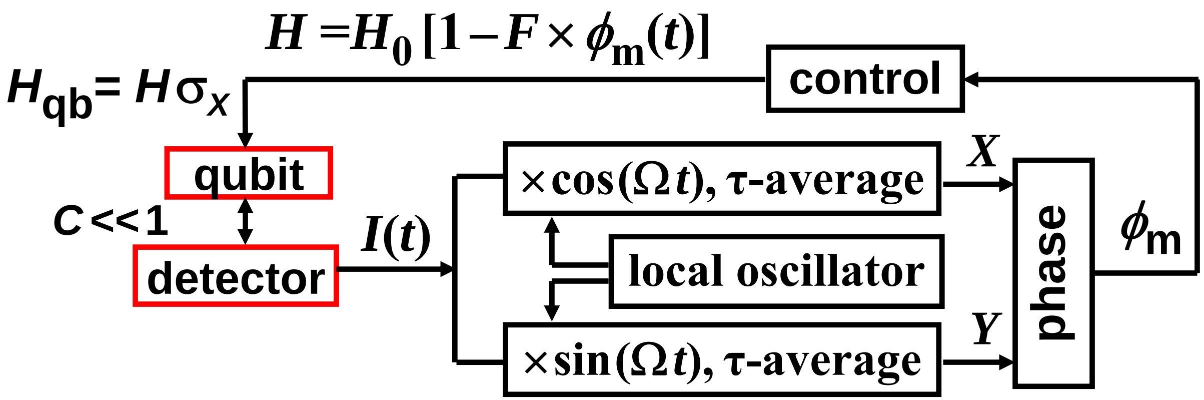
requires optimal feedback strength

Ruskov & A.K., 2002

We did not study much this type of feedback

Third idea: “Simple” quantum feedback

(A.K., 2005)



Idea: **use two quadrature components of the detector current $I(t)$ to monitor approximately the phase of qubit oscillations**
(a very natural way for usual classical feedback!)

$$X(t) = \int_{-\infty}^{t} [I(t') - I_0]\,\cos(\Omega t')\,\exp[-(t-t')/\tau]\,dt'$$

$$Y(t) = \int_{-\infty}^{t} [I(t') - I_0]\,\sin(\Omega t')\,\exp[-(t-t')/\tau]\,dt'$$

$$\phi_m = -\arctan(Y/X)$$

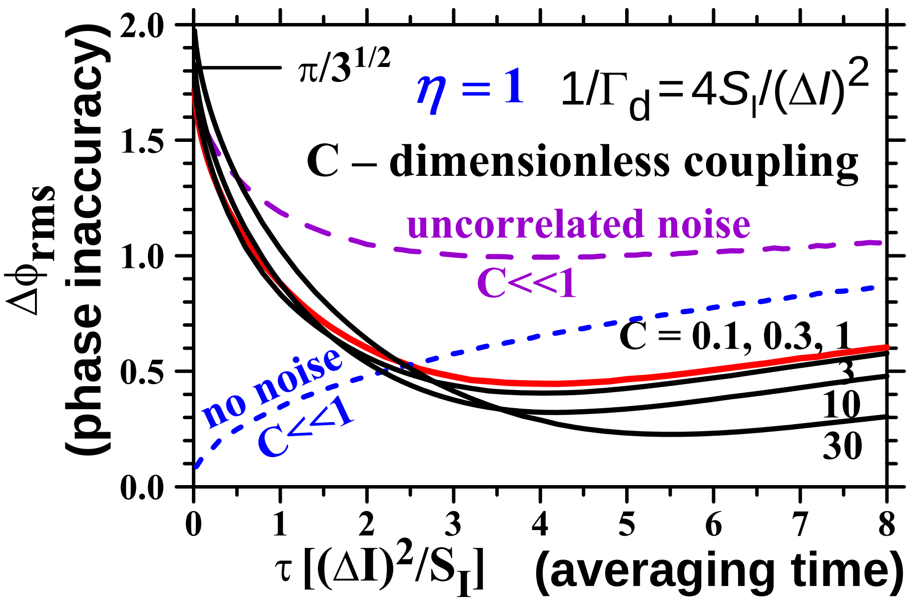
(similar formulas for a tank circuit instead of mixing with local oscillator)

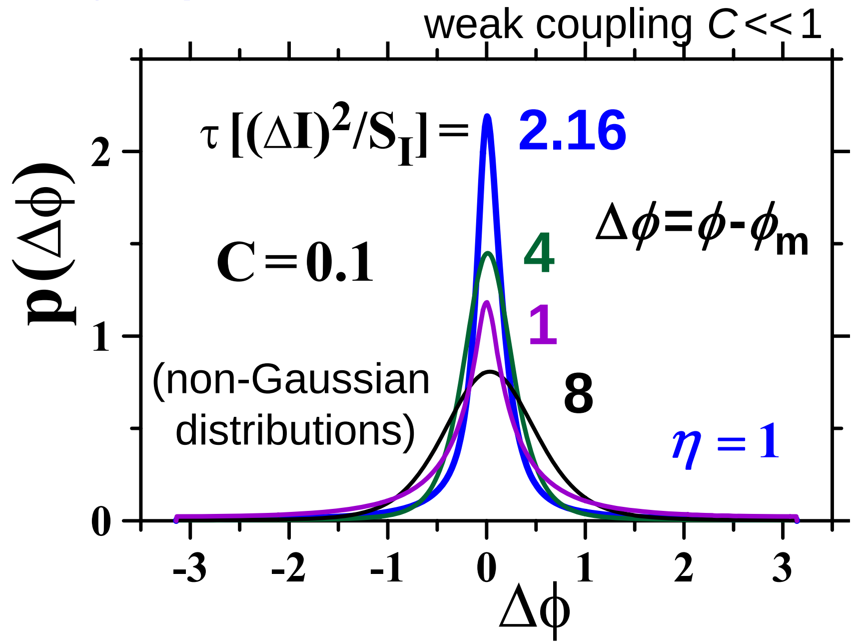
Advantage: **simplicity and relatively narrow bandwidth** $(1/\tau \sim \Gamma_d \ll \Omega)$

Essentially classical feedback. Does it really work?

Accuracy of phase monitoring via quadratures

(no feedback yet)





Noise improves the monitoring accuracy!

(purely quantum effect, "reality follows observations")

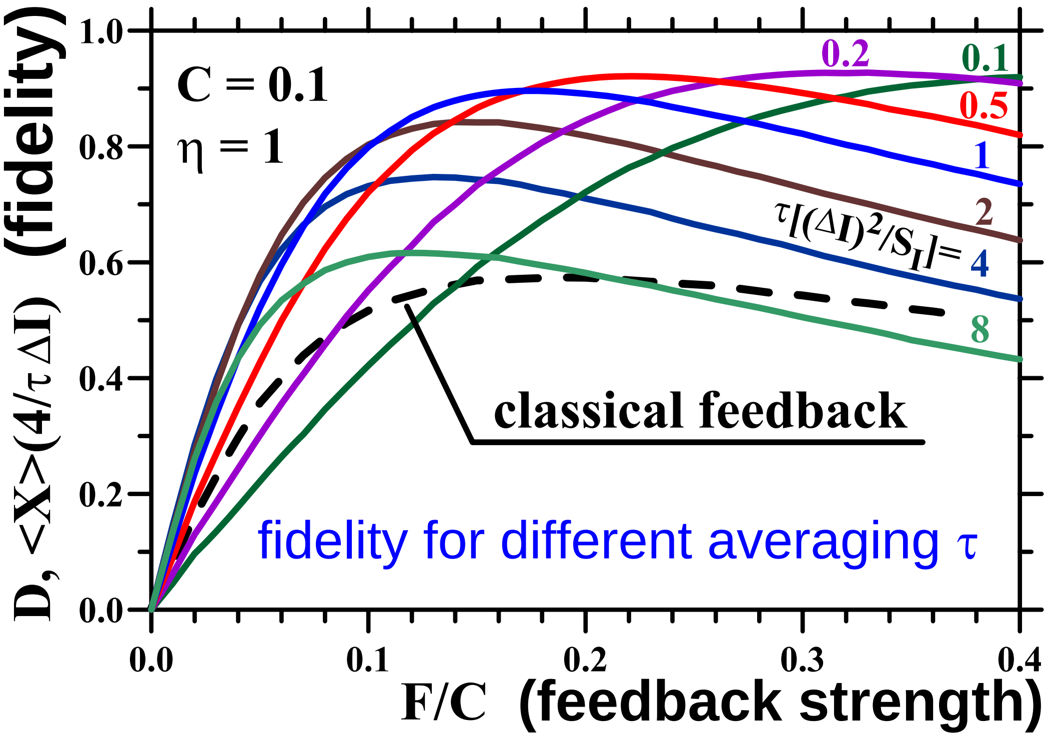
$$d\phi / dt = -[I(t) - I_0] \sin(\Omega t + \phi)(\Delta I / S_I)$$ (actual phase shift, ideal detector)

$$d\phi_m / dt = -[I(t) - I_0] \sin(\Omega t + \phi_m)/(X^2 + Y^2)^{1/2}$$ (observed phase shift)

Noise enters the actual and observed phase evolution in a similar way

Quite accurate monitoring! $\cos(0.44) \approx 0.9$

Simple quantum feedback



weak coupling C

D – feedback efficiency

$$D \equiv 2F_Q - 1$$

$$F_Q \equiv \langle \mathrm{Tr}\, \rho(t)\, \rho_{des}(t) \rangle$$

$D_{max} \approx 90\%$

How to verify feedback operation experimentally?

Simple: just check that in-phase quadrature $\langle X \rangle$ of the detector current is positive $D = \langle X \rangle (4/\tau \Delta I)$

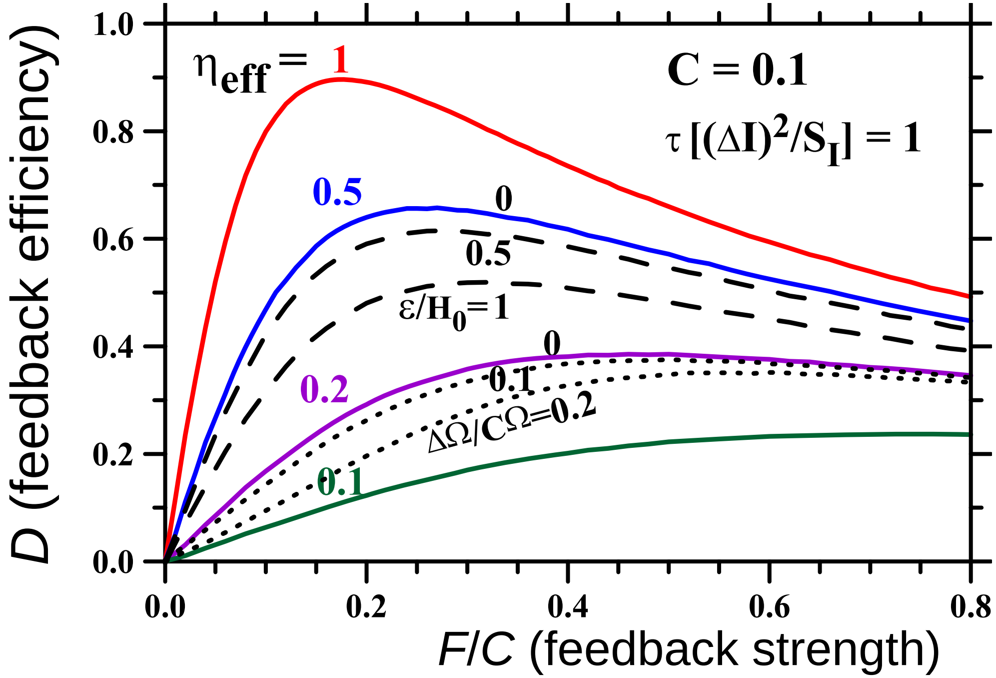
$\langle X \rangle$=0 for *any* non-feedback Hamiltonian control of the qubit

Effect of nonidealities

- nonideal detectors (finite quantum efficiency η)
- qubit energy asymmetry ε
- frequency mismatch $\Delta\Omega$

Quantum feedback still works quite well

(feedback loop must be faster than decoherence)



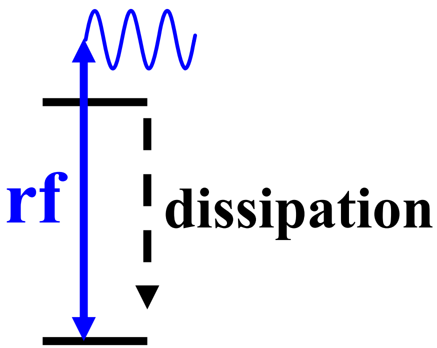
Main features:

- **Fidelity D up to ~90% achievable (for η=1)**
- **Natural, practically classical feedback setup**
- **Averaging $\tau \sim 1/\Gamma \gg 1/\Omega$ (narrow bandwidth!)**
- **Detector efficiency (ideality) $\eta \sim 0.1$ still OK**
- **Robust to asymmetry ε and frequency shift $\Delta\Omega$**
- **Simple verification: positive in-phase quadrature $\langle X \rangle$**

Simple enough experiment?!

One more idea

(not really a feedback, but synchronization caused by significant dissipation)



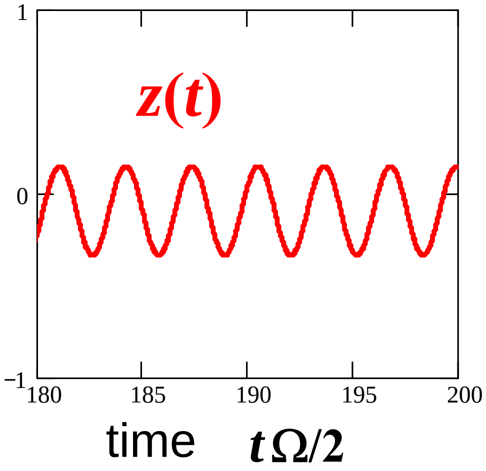
Greenberg and Il'ichev, 2004

(actually my modification of their mechanism)

driven Rabi oscillations

detuning oscillates in time

$$\omega_{\mathrm{rf}} = \omega_{\mathrm{qb}} + \delta + A\cos(\Omega t)$$



$\delta/\Omega=0.35$

$A/\Omega=0.15$

$1/T_1\Omega=0.075$

(not quite interesting for me personally)

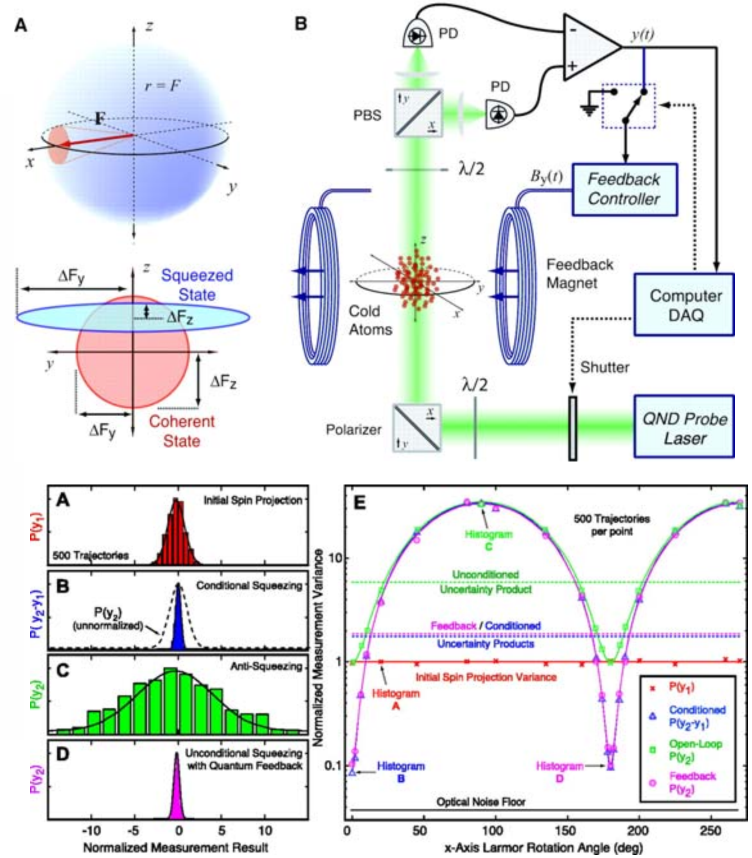
Quantum feedback in optics

First experiment: Science 304, 270 (2004)

Real-Time Quantum Feedback Control of Atomic Spin-Squeezing

JM Geremia,* John K. Stockton, Hideo Mabuchi

Real-time feedback performed during a quantum nondemolition measurement of atomic spin-angular momentum allowed us to influence the quantum statistics of the measurement outcome. We showed that it is possible to harness measurement backaction as a form of actuation in quantum control, and thus we describe a valuable tool for quantum information science. Our feedback-mediated procedure generates spin-squeezing, for which the reduction in quantum uncertainty and resulting atomic entanglement are not conditioned on the measurement outcome.



First detailed theory:

H.M. Wiseman and G. J. Milburn,
Phys. Rev. Lett. 70, 548 (**1993**)

Quantum feedback in optics

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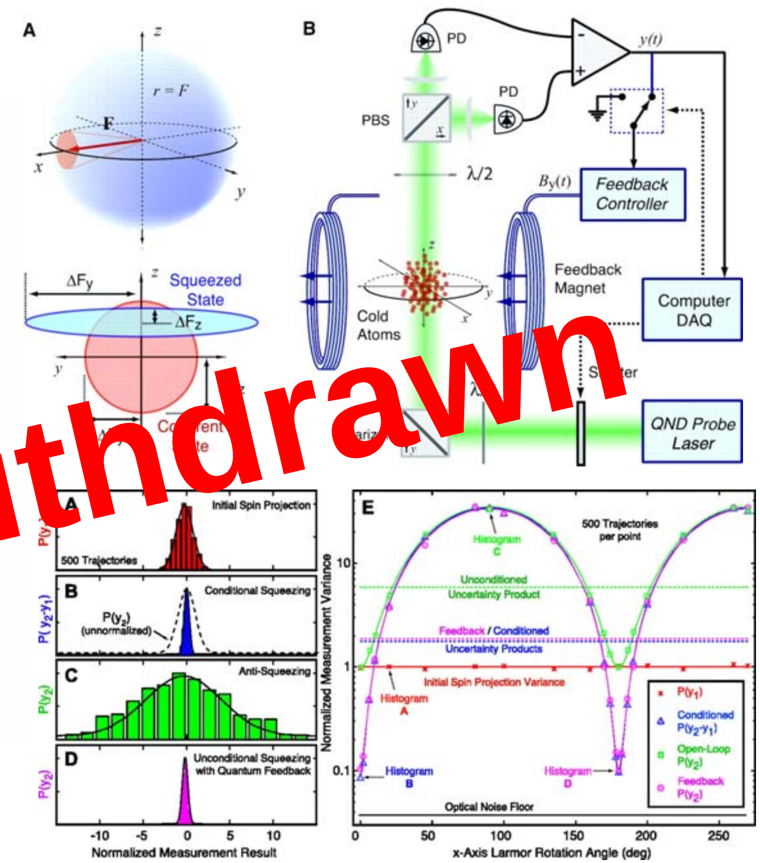
paper withdrawn



PRL 94, 203002 (2005) also withdrawn

First detailed theory:
H.M. Wiseman and G. J. Milburn,
Phys. Rev. Lett. 70, 548 (**1993**)

Recent experiment:
Cook, Martin, Geremia,
Nature 446, 774 (2007)
(coherent state discrimination)

Next topic: Undoing a weak measurement of a qubit (quantum uncollapsing)

A.K. & Jordan, PRL-2006

It is impossible to undo "orthodox" quantum measurement (for an unknown initial state)

Is it possible to undo partial quantum measurement?

Yes! (but with a finite probability)

If undoing is successful, an unknown state is **fully** restored

ψ_0 (unknown) $\xrightarrow{\text{weak (partial) measurement}}$ ψ_1 (partially collapsed)

ψ_1 $\xrightarrow{\text{successful}}$ ψ_0 (still unknown)

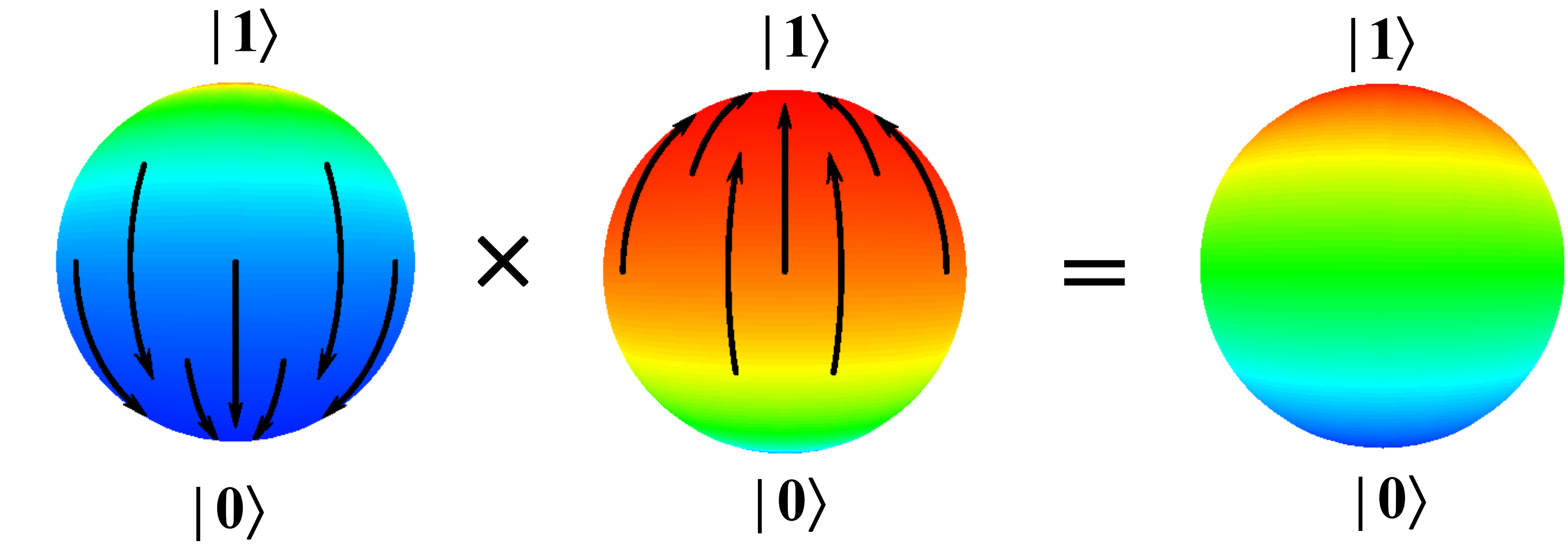
ψ_1 $\xrightarrow{\text{unsuccessful}}$ ψ_2

undoing (information erasure)

Uncollapsing of a qubit state

Evolution due to partial (weak, continuous, etc.) measurement is **non-unitary** (though coherent if detector is good!), therefore it is impossible to undo it by Hamiltonian dynamics.

How to undo? One more measurement!

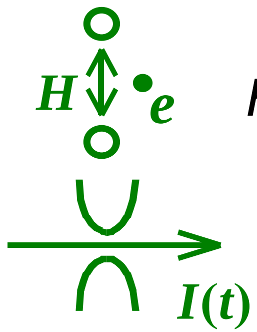


need ideal (quantum-limited) detector

(similar to Koashi-Ueda, PRL-1999)

(Figure partially adopted from Jordan-A.K.-Büttiker, PRL-06)

Evolution of a charge qubit with *H*=0



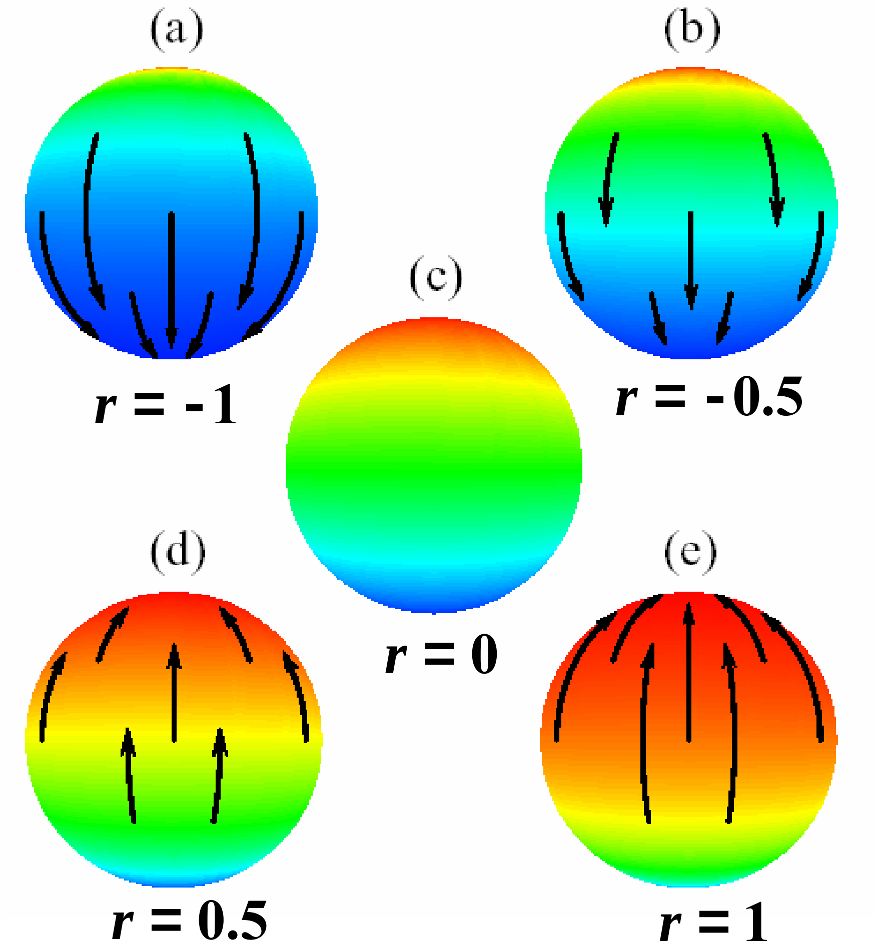
H=0

$$\frac{\rho_{11}(t)}{\rho_{22}(t)} = \frac{\rho_{11}(0)}{\rho_{22}(0)} \exp[2r(t)]$$

$$\frac{\rho_{12}(t)}{\sqrt{\rho_{11}(t)\rho_{22}(t)}} = \text{const}$$

where measurement result $r(t)$ is

$$r(t) = \frac{\Delta I}{S_I}\left[\int_0^t I(t')\,dt' - I_0 t\right]$$

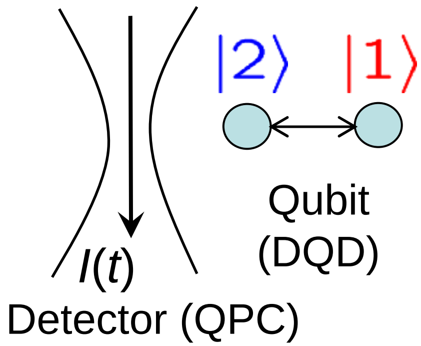


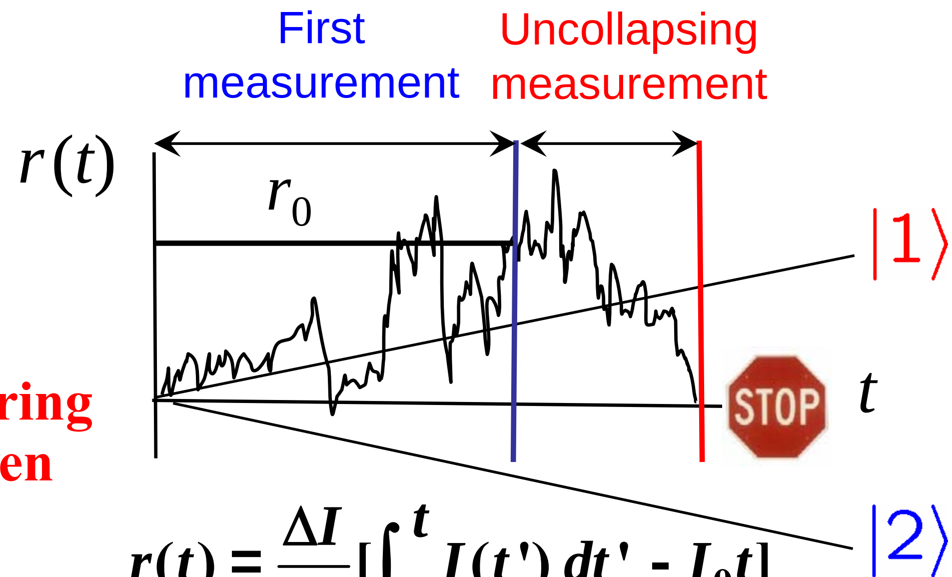
Jordan-Korotkov-Büttiker, PRL-06

If $r = 0$, then no information and no evolution!

Uncollapsing for DQD-QPC system

A.K. & Jordan





Simple strategy: continue measuring until result *r*(*t*) becomes zero. Then any initial state is fully restored.

(same for an entangled qubit)

$$r(t) = \frac{\Delta I}{S_I}\left[\int_0^t I(t')\,dt' - I_0 t\right]$$

It may happen though that $r = 0$ never crossed; then undoing procedure is unsuccessful.

Probability of success:

$$P_S = \frac{e^{-|r_0|}}{e^{|r_0|}\rho_{11}(0) + e^{-|r_0|}\rho_{22}(0)}$$

Averaged probability of success (over result r_0):

$$P_{av} = 1 - \mathrm{erf}[\sqrt{t/2T_m}\,], \qquad T_m = 2S_I/(\Delta I)^2$$

(does not depend on initial state)

General theory of uncollapsing

POVM formalism
(Nielsen-Chuang, p.85)

Measurement operator M_r: $\rho \to \dfrac{M_r \rho M_r^\dagger}{\mathrm{Tr}(M_r \rho M_r^\dagger)}$

Probability: $P_r = \mathrm{Tr}(M_r \rho M_r^\dagger)$ Completeness: $\sum_r M_r^\dagger M_r = 1$

Uncollapsing operator: $C \times M_r^{-1}$ (to satisfy completeness, eigenvalues cannot be >1)

$$\max(C) = \min_i \sqrt{p_i}\,,\ \ p_i \text{ – eigenvalues of } M_r^\dagger M_r$$

Probability of success: $P_S \le \dfrac{\min P_r}{P_r(\rho_{\rm in})}$

$P_r(\rho_{in})$ – probability of result r for initial state ρ_{in},

min P_r – probability of result r minimized over all possible initial states

Averaged (over r) probability of success: $P_{av} \le \sum_r \min P_r$

(cannot depend on initial state, otherwise get information)

(similar to Koashi-Ueda, 1999)

General bound for DQD-QPC system

General bound: $$P_S \le \frac{\min P_r}{P_r(\rho_{\rm in})}$$

DQD+QPC system

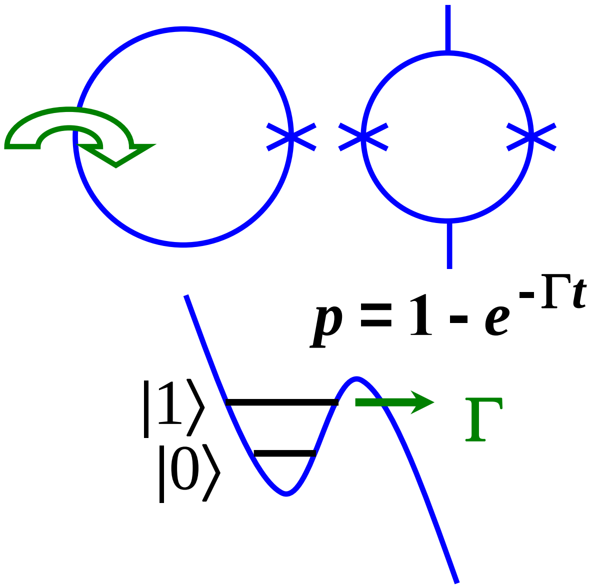
$$P_S \le \frac{\min(p_1, p_2)}{p_1\rho_{11}(0) + p_2\rho_{22}(0)}$$

where $$p_i = (\pi S_I / t)^{-1/2} \exp[-(\bar{I} - I_i)^2 t / S_I]\, d\bar{I}$$

Coincides with the actual result, so the upper bound is reached, **therefore uncollapsing strategy is optimal**

Second example: uncollapsing of a superconducting phase qubit

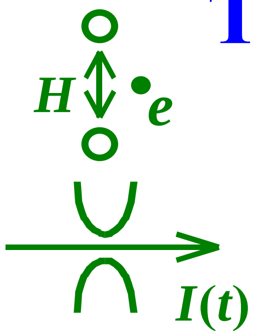
1) Start with an unknown state
2) Partial measurement of strength p
3) π-pulse (exchange $|0\rangle \leftrightarrow |1\rangle$)
4) One more measurement with the **same strength** p
5) π-pulse



N. Katz et al., Science-2006, PRL-2008

This is what was demonstrated experimentally

(in more detail later)



Third example: evolving charge qubit

$$\hat{H}_{QB} = (\varepsilon/2)(c_1^\dagger c_1 - c_2^\dagger c_2) + H(c_1^\dagger c_2 + c_2^\dagger c_1)$$

(now non-zero *H* and ε, qubit evolves during measurement)

1) Bayesian equations to calculate measurement operator
2) unitary operation, measurement by QPC, unitary operation

Fourth example: general uncollapsing for entangled charge qubits

1) unitary transformation of *N* qubits
2) null-result measurement of a certain strength by a strongly nonlinear QPC (tunneling only for state $|11..1\rangle$)
3) repeat 2^N times, sequentially transforming the basis vectors of the diagonalized measurement operator into $|11..1\rangle$

(also reach the upper bound for success probability)

Quantum erasers in optics

Quantum eraser proposal by Scully and Drühl, PRA (1982)

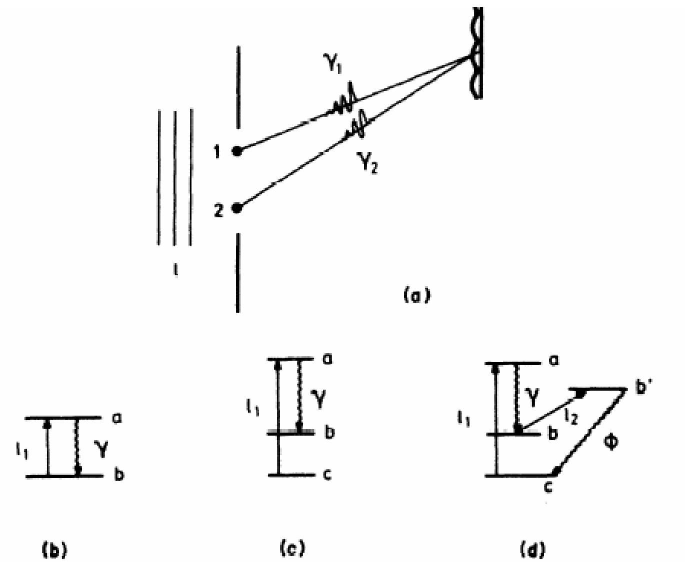


FIG. 1. (a) Figure depicting light impinging from left on atoms at sites 1 and 2. Scattered photons γ_1 and γ_2 produce interference pattern on screen. (b) Two-level atoms excited by laser pulse l_1, and emit γ photons in $a \rightarrow b$ transition. (c) Three-level atoms excited by pulse l_1 from $c \rightarrow a$ and emit photons in $a \rightarrow b$ transition. (d) Four-level system excited by pulse l_1 from $c \rightarrow a$ followed by emission of γ photons in $a \rightarrow b$ transition. Second pulse l_2 takes atoms from $b \rightarrow b'$. Decay from $b' \rightarrow c$ results in emission of ϕ photons.

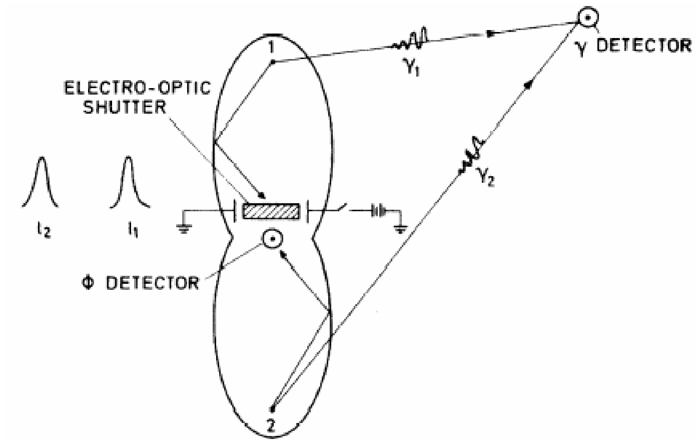
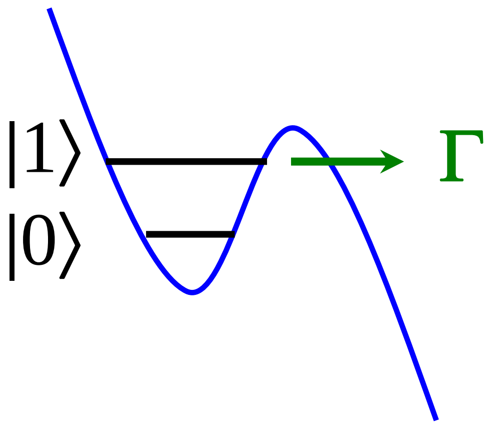


FIG. 2. Laser pulses l_1 and l_2 incident on atoms at sites 1 and 2. Scattered photons γ_1 and γ_2 result from $a \rightarrow b$ transition. Decay of atoms from $b' \rightarrow c$ results in ϕ photon emission. Elliptical cavities reflect ϕ photons onto common photodetector. Electro-optic shutter transmits ϕ photons only when switch is open. Choice of switch position determines whether we emphasize particle or wave nature of γ photons.

Interference fringes restored for two-detector correlations (since "which-path" information is erased)

Our idea of uncollapsing is quite different:
we really extract information and then erase it

Partial collapse of a phase qubit



N. Katz, M. Ansmann, R. Bialczak, E. Lucero, R. McDermott, M. Neeley, M. Steffen, E. Weig, A. Cleland, J. Martinis, A. Korotkov, Science-06

How does a coherent state evolve in time before tunneling event?

(What happens when nothing happens?)

Qubit "ages" in contrast to a radioactive atom!

Main idea:

$$\psi = \alpha|0\rangle + \beta|1\rangle \;\rightarrow\; \psi(t) = \begin{cases} |out\rangle, \text{ if tunneled} \\ \dfrac{\alpha|0\rangle + \beta e^{-\Gamma t/2} e^{i\varphi}|1\rangle}{\sqrt{|\alpha|^2 + |\beta|^2 e^{-\Gamma t}}}, \text{ if not tunneled} \end{cases}$$

(better theory: Leonid Pryadko & A.K., 2007)

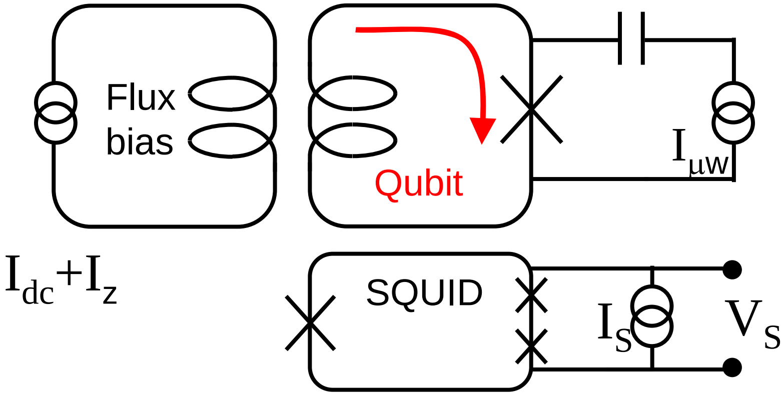
amplitude of state |0⟩ grows without physical interaction

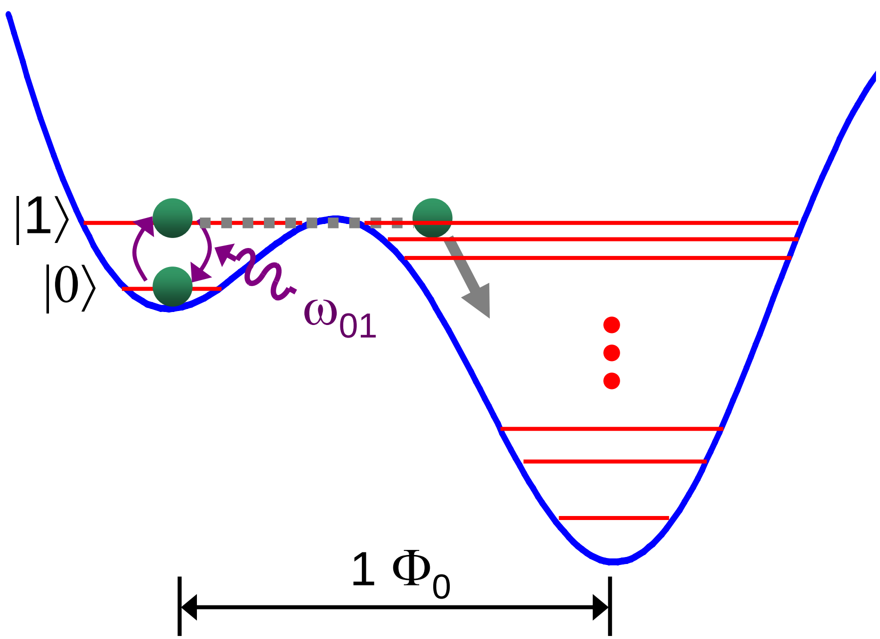
continuous null-result collapse

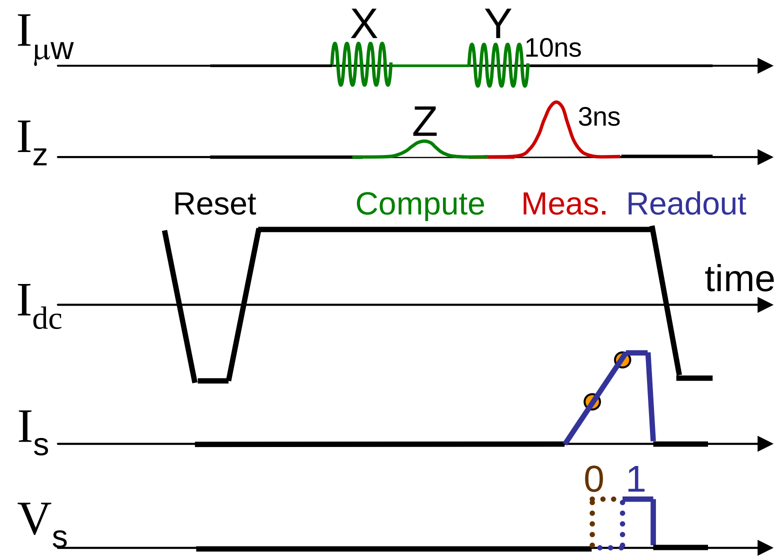
(similar to optics, Dalibard-Castin-Molmer, PRL-1992)

Superconducting phase qubit at UCSB

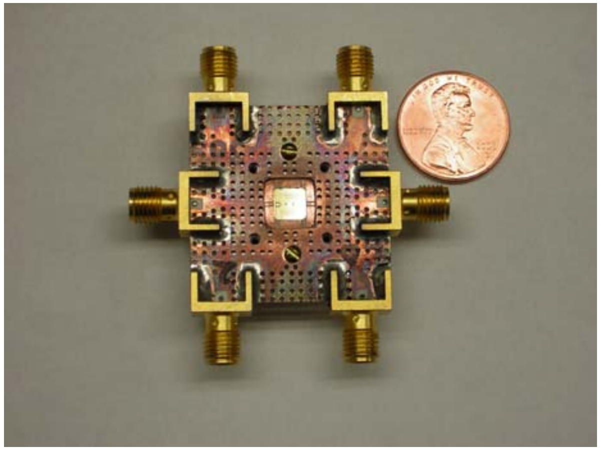
Courtesy of Nadav Katz (UCSB)





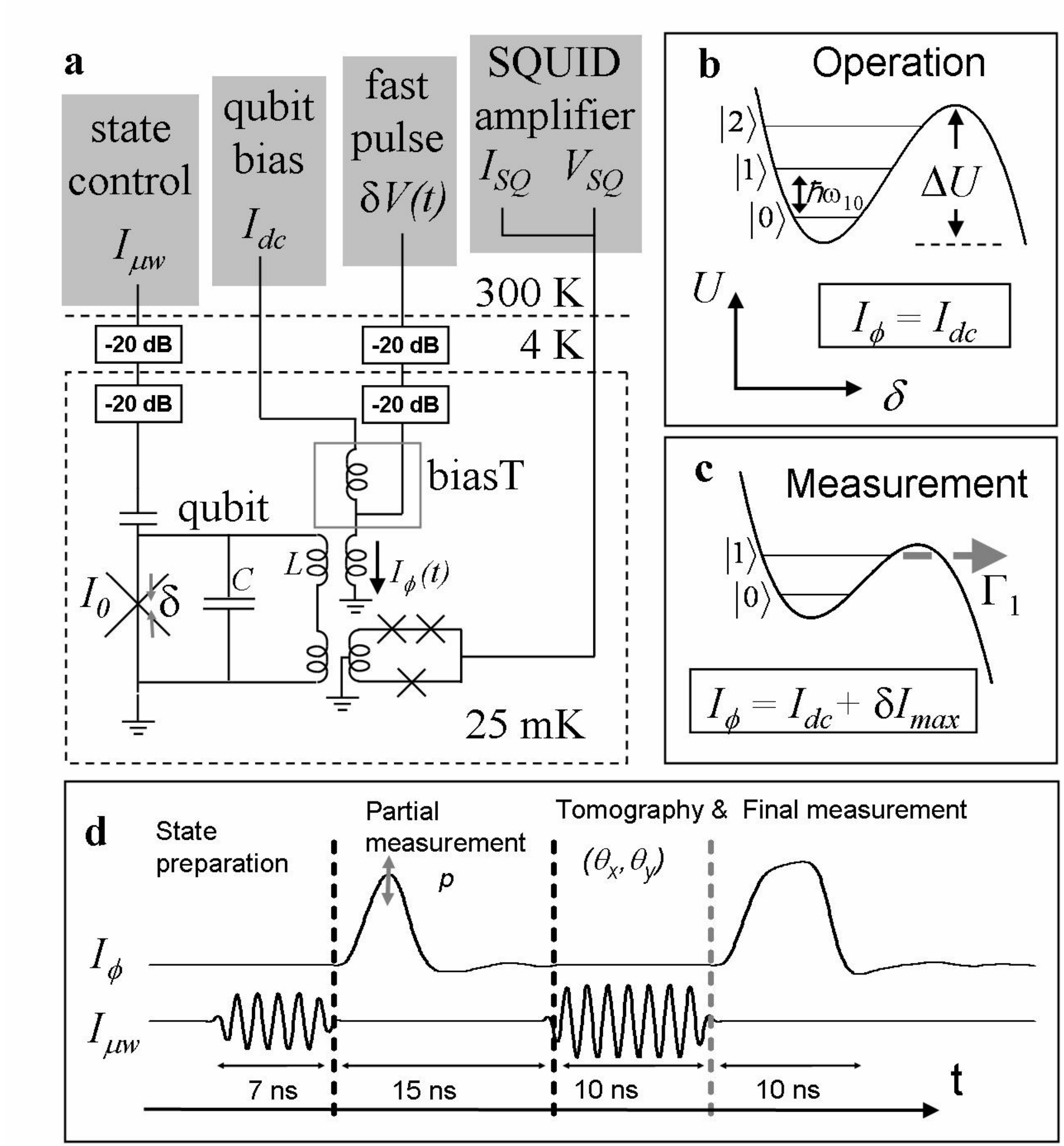


Repeat 1000x
prob. 0,1



Experimental technique for partial collapse

Nadav Katz *et al.*
(John Martinis' group)



Protocol:

1) State preparation by applying microwave pulse (via Rabi oscillations)

2) Partial measurement by lowering barrier for time *t*

3) State tomography (micro-wave + full measurement)

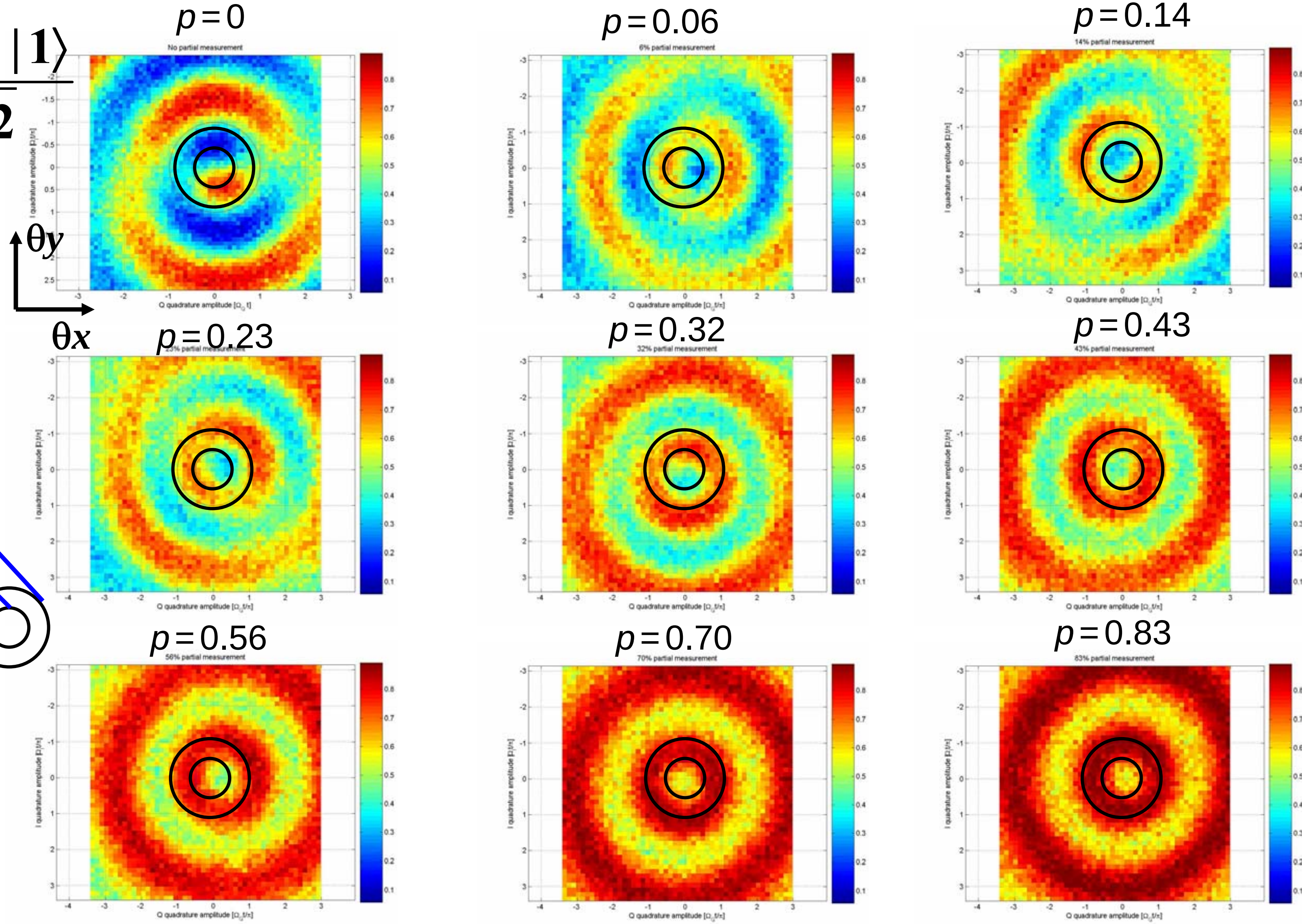
Measurement strength
$p = 1 - \exp(-\Gamma t)$
is actually controlled by Γ, not by t

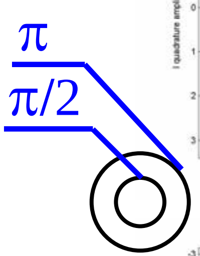
***p*=0: no measurement**
***p*=1: orthodox collapse**

Experimental tomography data

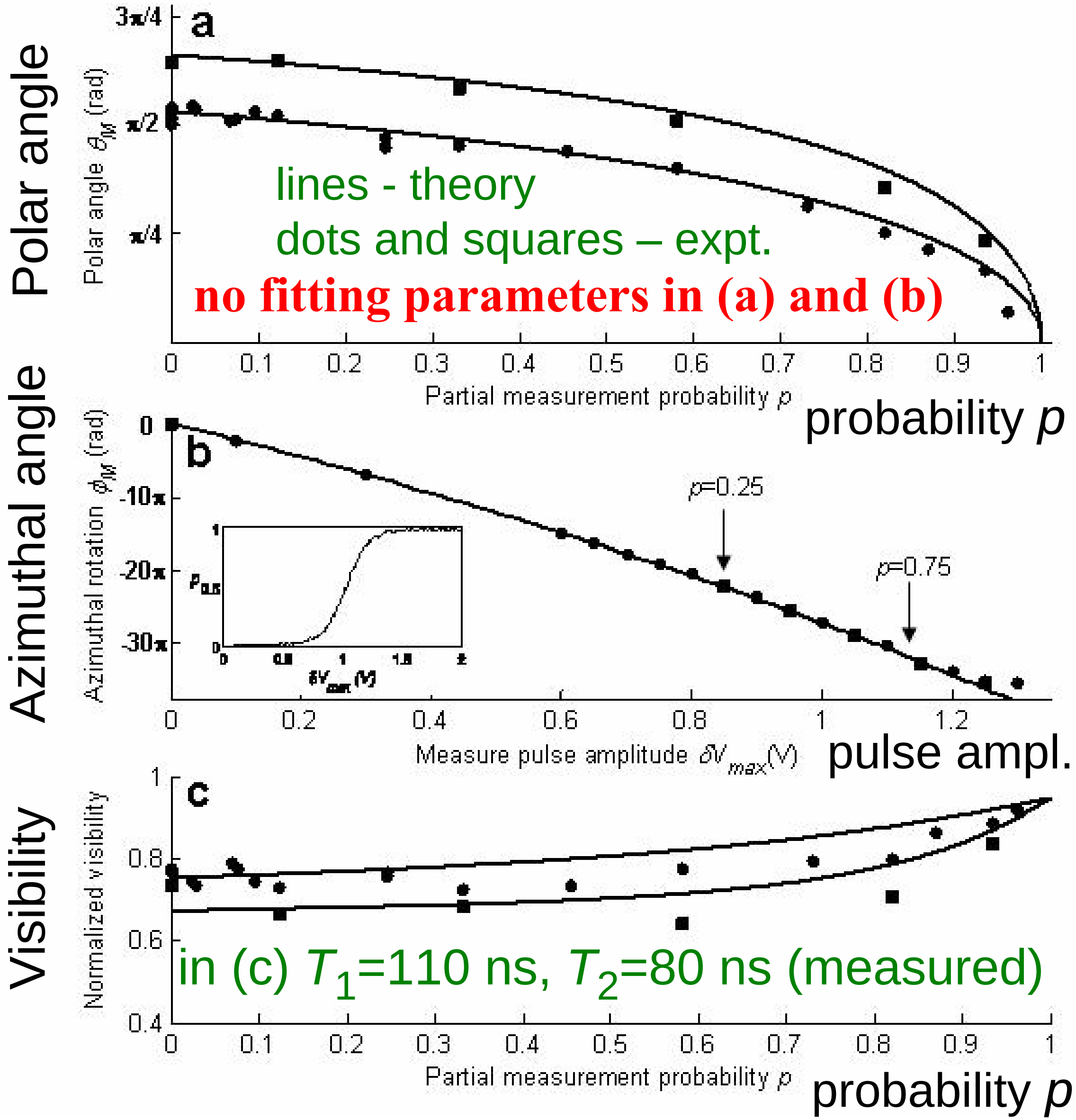
Nadav Katz *et al.* (UCSB)

$$\psi_{in} = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$





Partial collapse: experimental results



N. Katz *et al*., Science-06

- In case of no tunneling (null-result measurement) phase qubit evolves
- This evolution is well described by a simple Bayesian theory, without fitting parameters
- Phase qubit remains fully coherent in the process of continuous collapse (experimentally ~80% raw data, ~96% after account for T1 and T2)

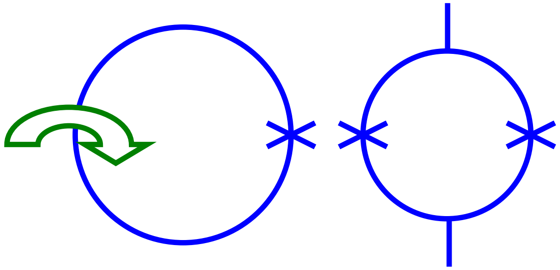
quantum efficiency

$\eta_0 > 0.8$

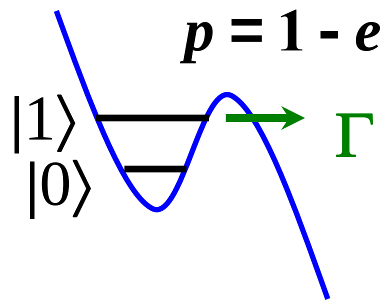
Uncollapsing of a phase qubit state

A.K. & Jordan, 2006

1) Start with an unknown state
2) Partial measurement of strength p
3) π-pulse (exchange $|0\rangle \leftrightarrow |1\rangle$)
4) One more measurement with the **same strength** p
5) π-pulse



$$p = 1 - e^{-\Gamma t}$$



If no tunneling for both measurements, then initial state is fully restored!

$$\alpha\,|0\rangle + \beta\,|1\rangle \rightarrow \frac{\alpha\,|0\rangle + e^{i\phi}\beta\, e^{-\Gamma t/2}\,|1\rangle}{\text{Norm}} \rightarrow$$

$$\frac{e^{i\phi}\alpha\, e^{-\Gamma t/2}\,|0\rangle + e^{i\phi}\beta\, e^{-\Gamma t/2}\,|1\rangle}{\text{Norm}} = e^{i\phi}(\alpha\,|0\rangle + \beta\,|1\rangle)$$

phase is also restored (spin echo)

Probability of success

Success probability if no tunneling during first measurement:

$$P_S = \frac{e^{-\Gamma t}}{\rho_{00}(0) + e^{-\Gamma t}\rho_{11}(0)} = \frac{1-p}{\rho_{00}(0) + (1-p)\rho_{11}(0)}$$

where $\rho(0)$ is the density matrix of the initial state (either averaged unknown state or an entangled state traced over all other qubits)

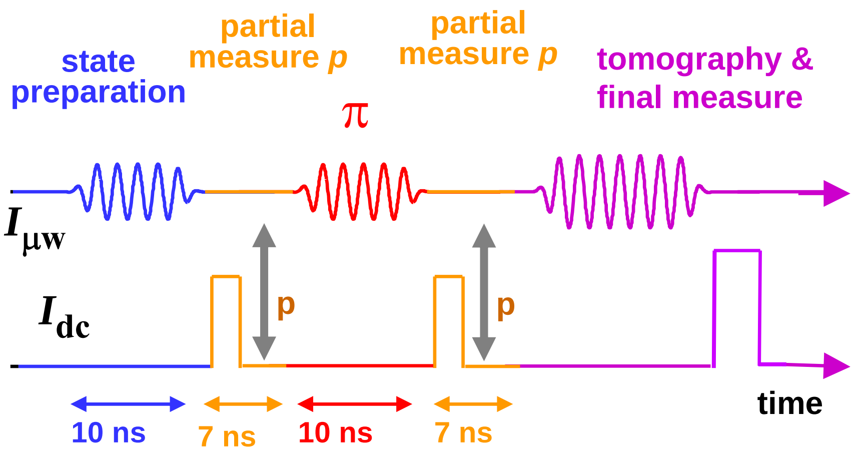
Total (averaged) success probability: $P_{\mathrm{av}} = 1 - p$

For measurement strength *p* increasing to 1, success probability decreases to zero (orthodox collapse), but still exact uncollapsing

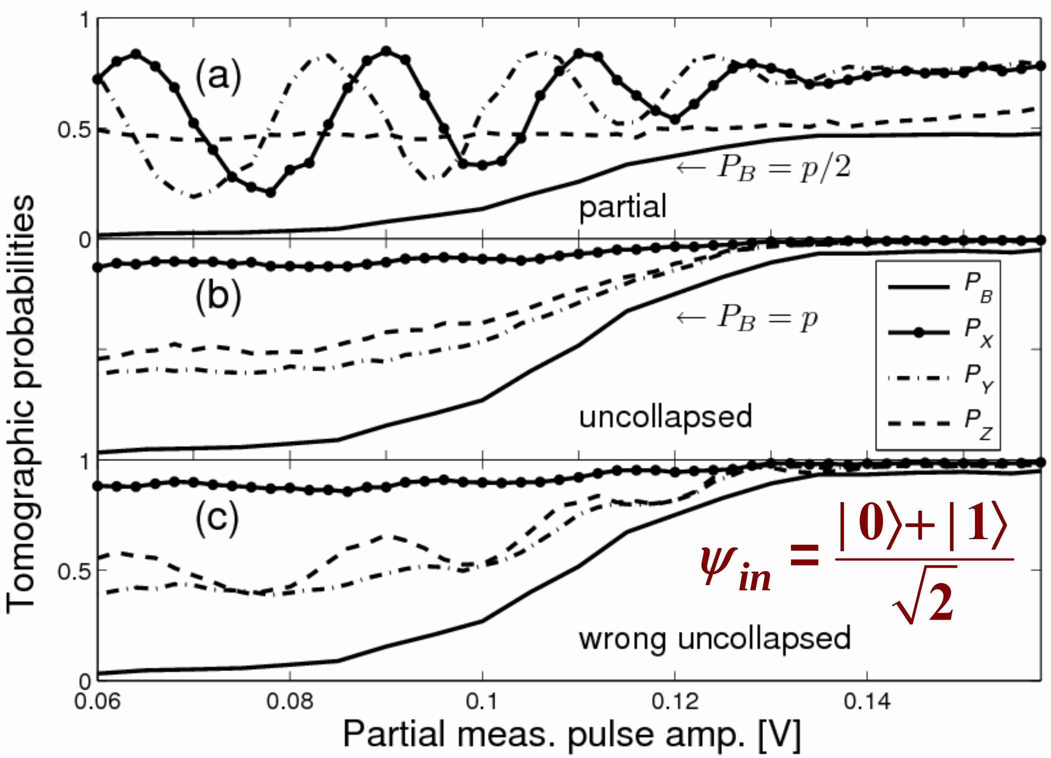
Compare with the general upper bound $P_S \le \dfrac{\min P_r}{P_r(\rho_0)}$

coincides ⇒ **optimal uncollapsing**

Experiment on wavefunction uncollapsing



N. Katz, M. Neeley, M. Ansmann, R. Bialzak, E. Lucero, A. O'Connell, H. Wang, A. Cleland, J. Martinis, and A. Korotkov, PRL-2008 (soon)



Uncollapse protocol:

- partial collapse
- π-pulse
- partial collapse (same strength)

State tomography with *X*, *Y*, and no pulses

Background P_B should be subtracted to find qubit density matrix

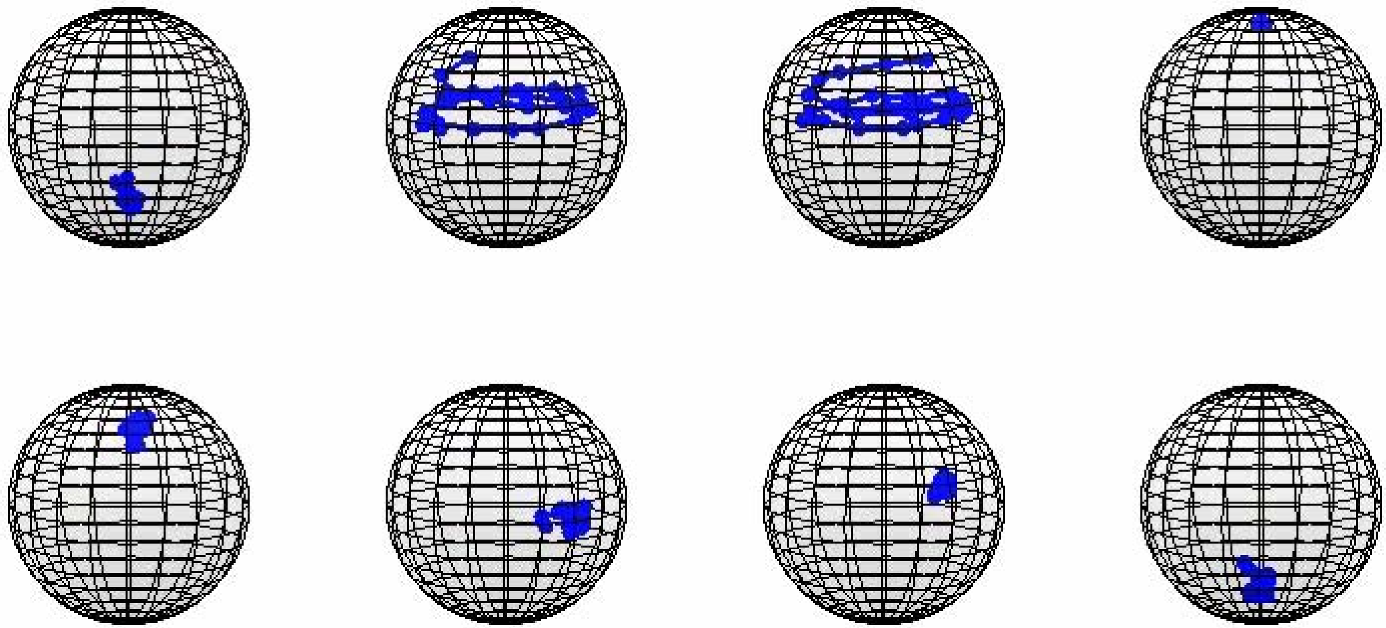
Experimental results on Bloch sphere

N. Katz et al.

Initial state: $|1\rangle$, $\dfrac{|0\rangle+|1\rangle}{\sqrt{2}}$, $\dfrac{|0\rangle+i|1\rangle}{\sqrt{2}}$, $|0\rangle$

Partial collapse

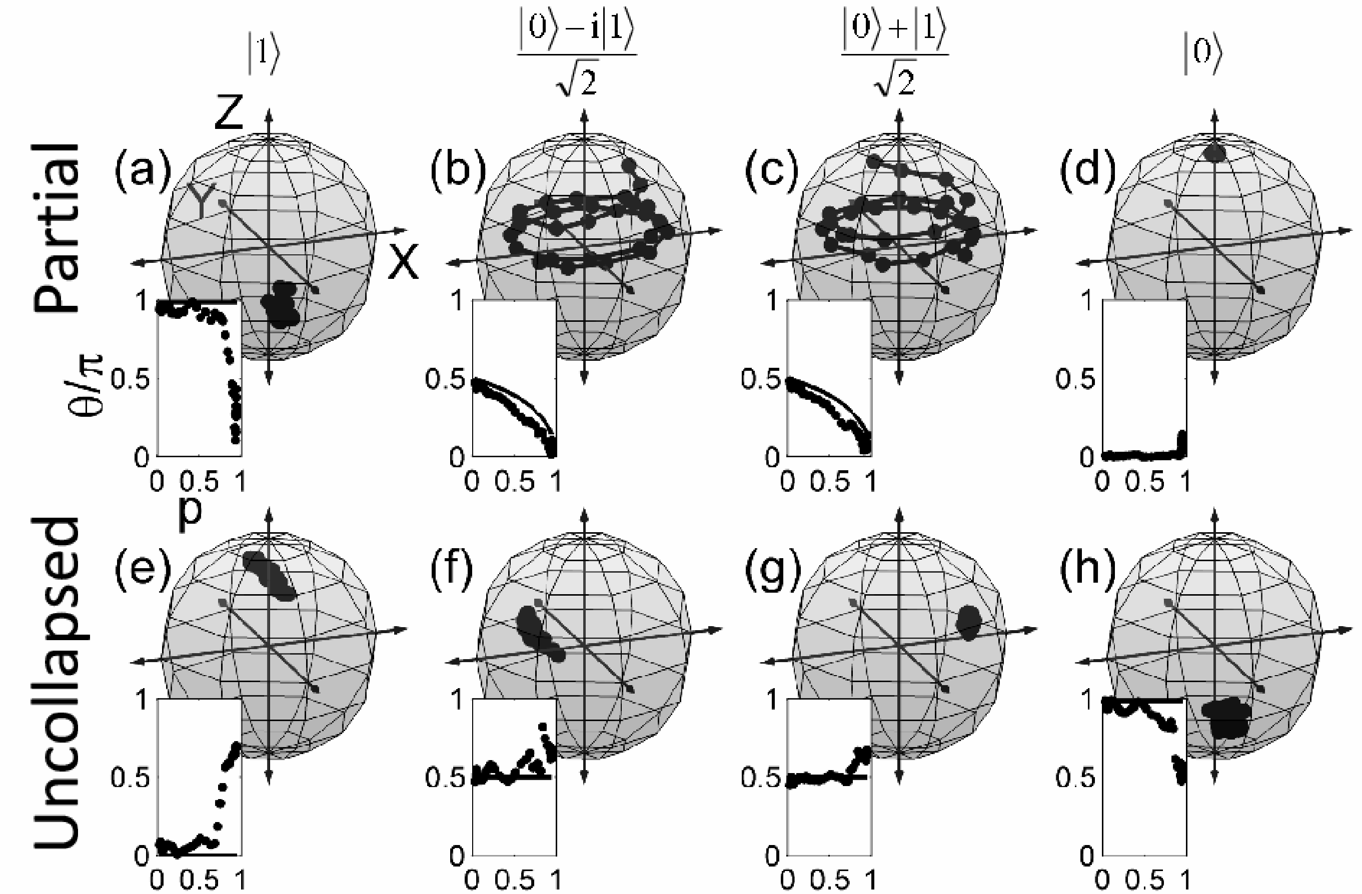
Uncollapsed



Collapse strength: $\boxed{0.05 < p < 0.7}$

uncollapsing works well!

Same with polar angle dependence
(another experimental run)

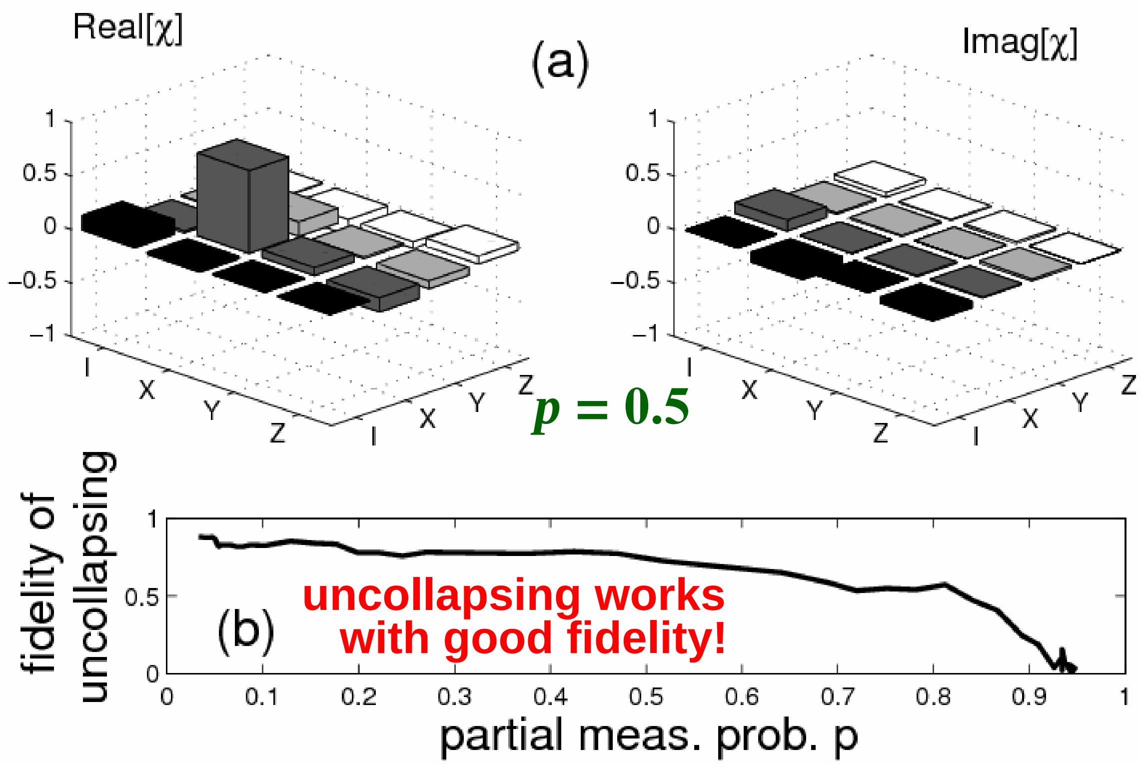


Both spin echo (azimuth) and uncollapsing (polar angle)

Difference: spin echo – undoing of an unknown unitary evolution, uncollapsing – undoing of a known, but non-unitary evolution

Quantum process tomography

N. Katz et al.
(Martinis group)



Why getting worse at p>0.6?

Energy relaxation $p_r = t/T_1 = 45\text{ns}/450\text{ns} = 0.1$

Selection affected when $1-p \sim p_r$

Overall: uncollapsing is well-confirmed experimentally

Last topic: quantum efficiency of binary-outcome qubit detectors

A.K., PRB-2008 (soon)

Measurement fidelities F_0 and F_1

F_0 = probability to get result 0 for a qubit in state $|0\rangle$,
F_1 = probability to get result 1 for a qubit in state $|1\rangle$

How to define quantum efficiency?

Let us use the same idea as for linear detectors:
by comparing actual with quantum-limited

$$\eta \leftrightarrow \frac{\text{quantum-limited ensemble decoherence}}{\text{actual (total) ensemble decoherence}}$$

Why need? For quantum feedback, non-unitary gates, etc.

General binary-output qubit detector

Quantum-limited (ideal) detector: pure qubit state remains pure

Use POVM language: linear measurement operators M_0 and M_1 (result 0 or 1)

$\rho \to M_i \rho M_i^\dagger / \mathrm{Tr}(M_i \rho M_i^\dagger)$ for result i, probability $P_i = \mathrm{Tr}(M_i \rho M_i^\dagger)$

Each operator M_i : 8 -1(phase) =7 real parameters

7+7=14, but completeness ($M_0^+ M_0 + M_1^+ M_1 = 1$), so 14 – 4=10

Ideal binary-output qubit detector is described by
10 real parameters (including fidelities F_0 and F_1)

Non-ideal detector

Again use POVM, now arbitrary one-qubit superoperators for i=0,1
$\Rightarrow$ 16 +16 – 4 = 28 real parameters for a general (non-ideal) detector

28 (general) – 10 (ideal) = 18 (quantum efficiencies)

Quantum efficiency (ideality) of a general binary-outcome qubit detector is described by 18 real parameters.

Too many!!! Impractical. What to do? **Consider only "QND" detectors**

$|0\rangle \to |0\rangle$, $|1\rangle \to |1\rangle$ (qubit does not evolve itself, σ_z-coupling)

Decoherence bound for a QND detector

General description of a QND detector: only 6 parameters

(fidelities F_0 and F_1, decoherences D_0 and D_1, and angles ϕ_0 and ϕ_1)

result 0: $$\begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix} \to \frac{1}{P_0} \begin{pmatrix} F_0\rho_{00} & \sqrt{F_0(1-F_1)}\, e^{-D_0} e^{i\phi_0} \rho_{01} \\ c.c. & (1-F_1)\rho_{11} \end{pmatrix}$$

result 1: $$\begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix} \to \frac{1}{P_1} \begin{pmatrix} (1-F_0)\rho_{00} & \sqrt{(1-F_0)F_1}\, e^{-D_1} e^{i\phi_1} \rho_{01} \\ c.c. & F_1\rho_{11} \end{pmatrix}$$

simple Bayes!

probabilities: $P_0 = F_0\rho_{00} + (1-F_1)\rho_{11}, \; P_1 = (1-F_0)\rho_{00} + F_1\rho_{11}$

average over two results: $$\begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix} \to \begin{pmatrix} \rho_{00} & e^{-D_{\text{av}}} e^{i\phi_{\text{av}}} \rho_{01} \\ c.c. & \rho_{11} \end{pmatrix}$$

$$e^{-D_{\text{av}}} e^{i\phi_{\text{av}}} = \sqrt{F_0(1-F_1)}\, e^{-D_0} e^{i\phi_0} + \sqrt{(1-F_0)F_1}\, e^{-D_1} e^{i\phi_1}$$

⇒ ensemble decoherence bound

$$D_{\text{av}} \ge D_{\text{min}} = -\ln\left[\sqrt{F_0(1-F_1)} + \sqrt{(1-F_0)F_1}\,\right]$$

Definitions of quantum efficiency
(actual decoherence vs. informational bound)

Similar to the first definition for linear detectors

$$\eta = \frac{D_{\min}}{D_{\text{av}}}$$

Taking into account phase correlation:

$$\tilde{\eta} = \frac{-\ln\left|\sqrt{F_0(1-F_1)} + \sqrt{(1-F_0)F_1}\, e^{i(\phi_1-\phi_0)}\right|}{D_{\text{av}}}$$

or

$$\tilde{\tilde{\eta}} = \frac{-\ln[\sqrt{F_0(1-F_1)} + \sqrt{(1-F_0)F_1}\,]}{-\ln[\sqrt{F_0(1-F_1)}\, e^{-D_0} + \sqrt{(1-F_0)F_1}\, e^{-D_1}\,]}$$

Also meaningful to define quantum efficiency for each result of the measurement:

$$\eta_0 = \frac{D_{\min}}{D_0 + D_{\min}}, \quad \eta_1 = \frac{D_{\min}}{D_1 + D_{\min}}$$

(another definition possible)

(useful for “asymmetric” and “half-destructive” detectors, as for phase qubits)

Quantum efficiency for several detector models

Model 1: Indirect projective measurement

$$\boxed{\text{qubit}} \longleftrightarrow \boxed{\text{ancilla qubit}} \xleftrightarrow[\text{measurement}]{\text{projective}} \boxed{\text{detector}}$$

Evolution:

$$(\alpha|0\rangle+\beta|1\rangle)|0_a\rangle \rightarrow \alpha|0\rangle(c_{00}|0_a\rangle+c_{10}|1_a\rangle)+\beta|1\rangle(c_{01}|0_a\rangle+c_{11}|1_a\rangle) \rightarrow$$

$$\rightarrow \begin{cases} (\alpha c_{00}|0\rangle+\beta c_{01}|1\rangle)/\text{Norm}, & \text{if result 0} \\ (\alpha c_{10}|0\rangle+\beta c_{11}|1\rangle)/\text{Norm}, & \text{if result 1} \end{cases}$$

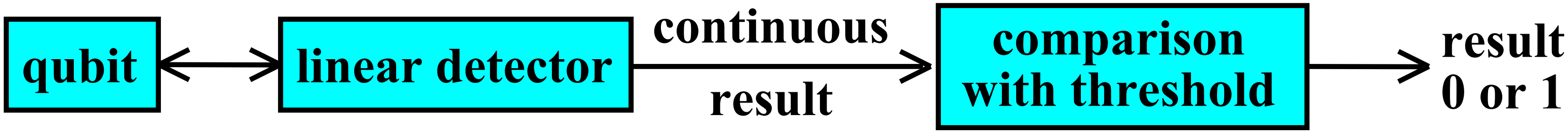
Then

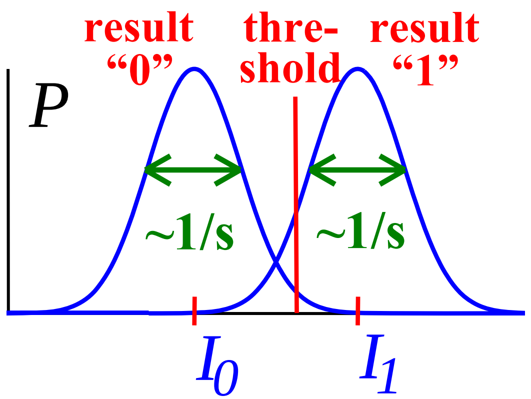
$$F_0=|c_{00}|^2,\;\; F_1=|c_{11}|^2,\;\; \phi_0=\arg(c_{00}c_{01}^*),\;\; \phi_1=\arg(c_{10}c_{11}^*),\;\; D_0=0,\;\; D_1=0$$

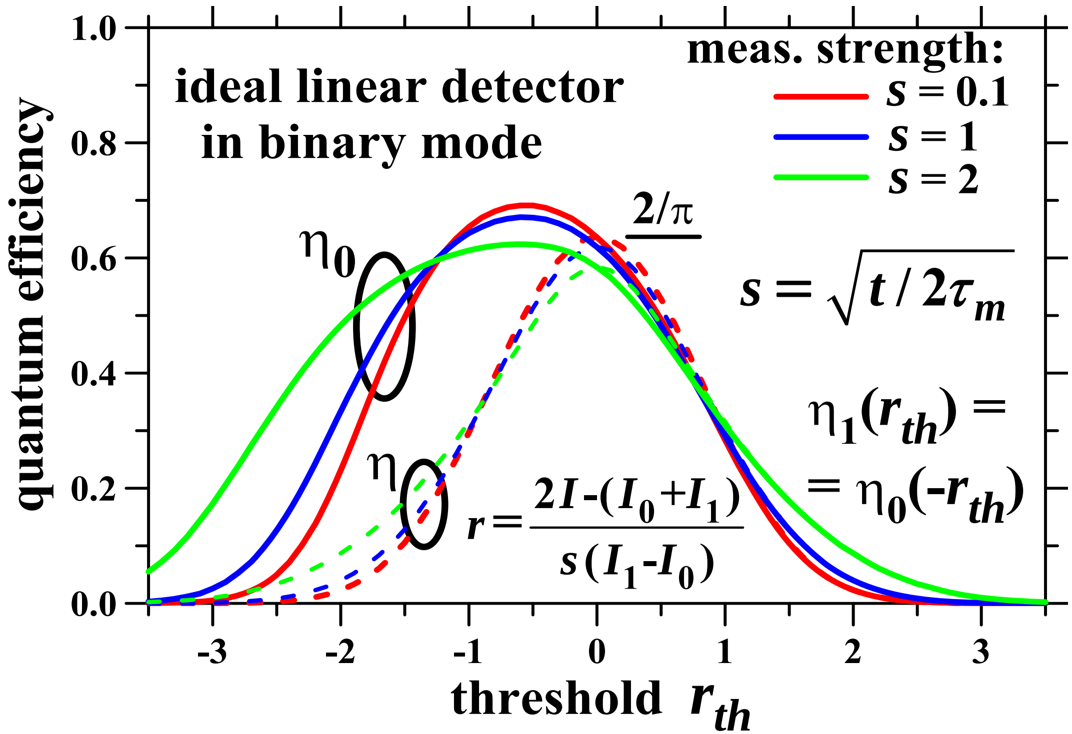
And so $\eta_0=\eta_1=1,\;\; \tilde{\eta}=\tilde{\tilde{\eta}}=1$ **(ideal, but not practical)**

but $\eta\neq 1,$ **if** $\phi_0\neq\phi_1$

Model 2: Linear detector in binary-output mode







$$F_{0,1} = [1 + \mathrm{erf}(\pm r_{th} + s)]/2$$

meas. strength $s = \sqrt{t/2\tau_m}$

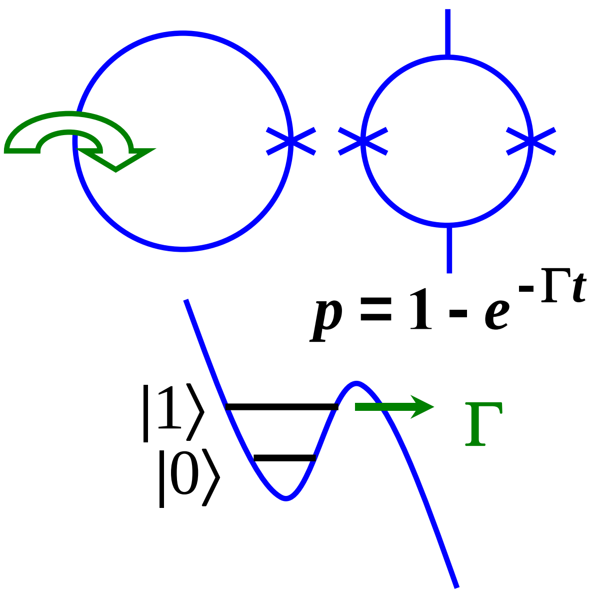
$$\eta = \frac{-\ln[\sqrt{F_0(1-F_1)} + \sqrt{(1-F_0)F_1}\,]}{s^2 + \gamma t}$$

$$\eta \le 2/\pi, \;\; \eta_{0,1} < 0.7$$

Even for an ideal linear detector, threshold mode is significantly non-ideal

Why? Because we loose information!

Model 3: Phase qubit detection



Experimentally

$\eta_0 \geq 0.8$

(using Katz et al., Science-06)

Result 0 (“null result”), then

$$\alpha\,|0\rangle + \beta\,|1\rangle \rightarrow \frac{\alpha\,|0\rangle + e^{i\phi}\beta\, e^{-\Gamma t/2}\,|1\rangle}{\text{Norm}}$$

Result 1, then qubit destroyed

Fidelity: $F_1 = p,\;\; F_0 = 1\;\; (\text{or } F_0 = 1 - p_0)$

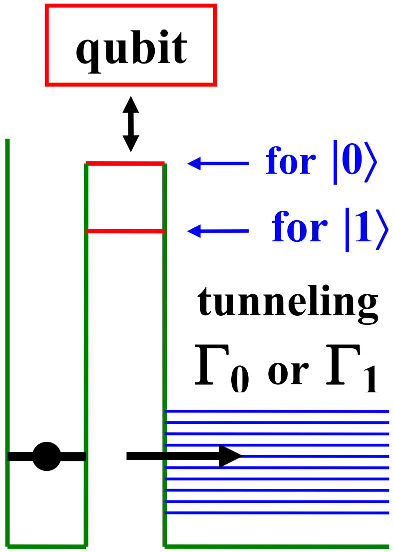
Quantum efficiency: $\eta_0 = 1$

while η_1 and η cannot be defined

(“half-destructive” measurement)

If imperfections are taken into account (Pryadko-A.K., 2007), then η_0<1

Model 4: Tunneling-into-continuum detector



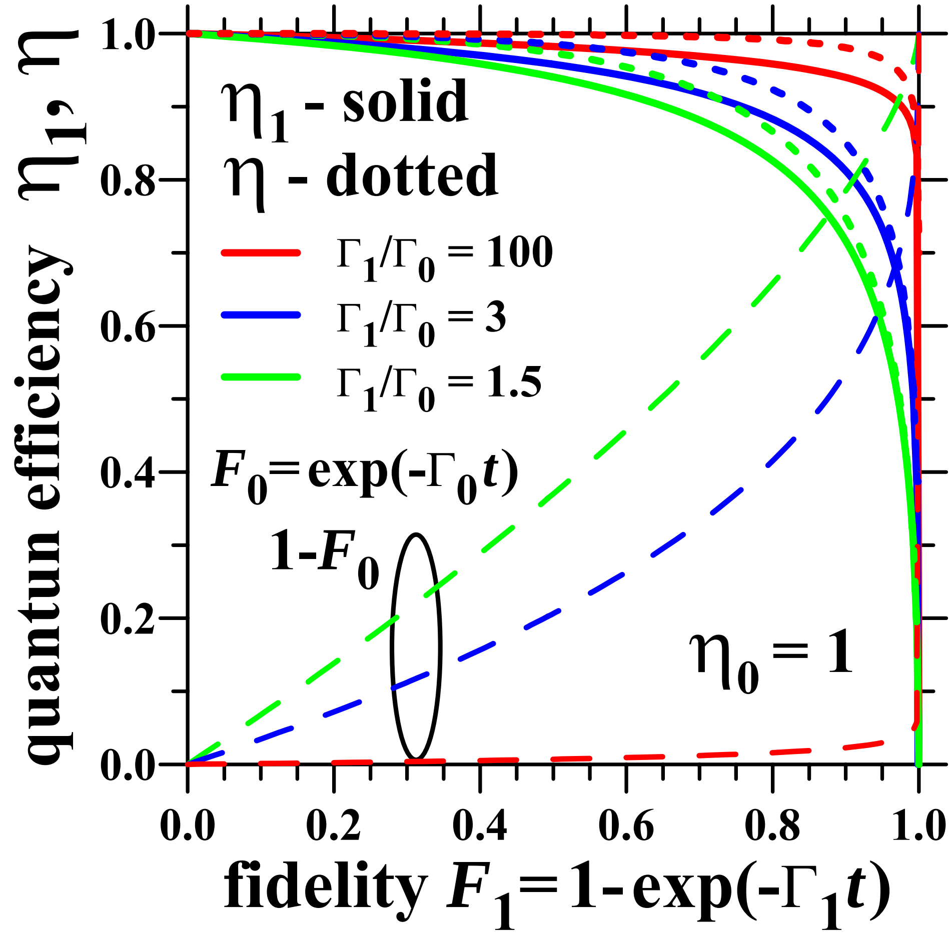
Non-destructive for both results

Ideal for null result: $\eta_0 = 1$

However, $\eta_1 < 1, \eta < 1$

Approaches ideality $(\eta_1 \simeq 1, \eta \simeq 1)$ only when $\Gamma_0 t < \Gamma_1 t \ll 1$ (otherwise lost information in excitation profile)

$\eta > 0.9$ is theoretically easy



Can such regime be realized by a real SQUID or by a bifurcation detector?

Summary

- Rabi oscillations are persistent if monitored (observed in Saclay, though with $\eta \ll 1$; LGI violated)

- Quantum feedback of Rabi oscillations: much harder, but doable (needs somewhat better η and feedback loop faster than decoherence)

- Quantum uncollapsing demonstrated in UCSB for phase qubit, as extension of partial-collapse expt.

- Perfect quantum efficiency seems to be difficult for binary-outcome detectors (except null-result), but η>0.9 is possible for tunneling-based detectors