Wavefunction uncollapsing: theory and experiment

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In collaboration with:

<u>Andrew Jordan</u> (*Univ. of Rochester*, theory)

<u>Nadav Katz</u>, M. Neeley, M. Ansmann, R. Bialczak, M. Hofheinz, E. Lucero, A. O'Connell, H. Wang, A. Cleland, and <u>John Martinis</u> (*UC Santa Barbara*, experiment)

Support:

I A R P A

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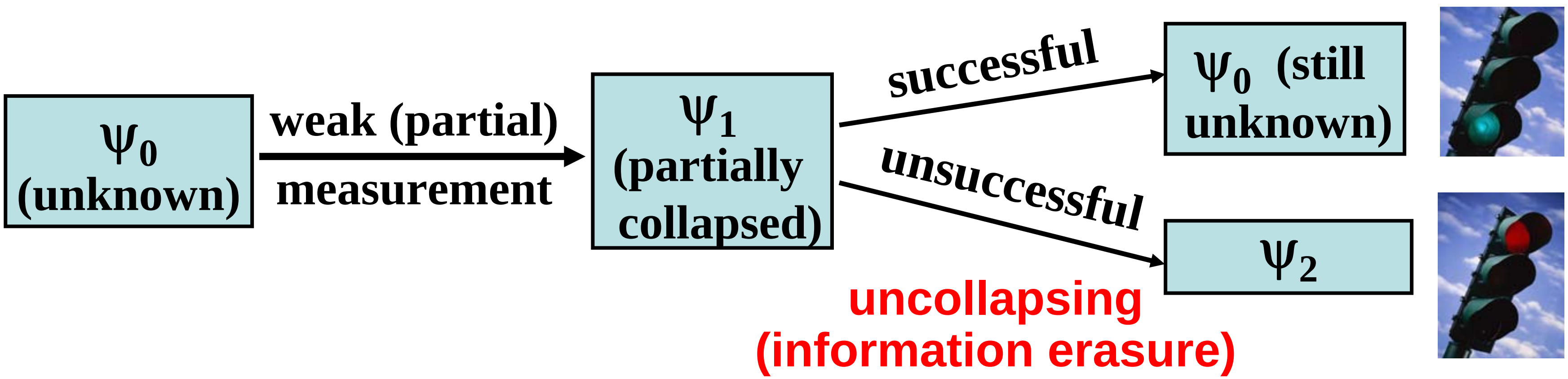
The problem

It is impossible to undo "orthodox" quantum measurement (for an unknown initial state)

Is it possible to undo weak (partial) quantum measurement?

Yes! (but with a finite probability)

If uncollapsing is successful, an unknown state is **fully** restored



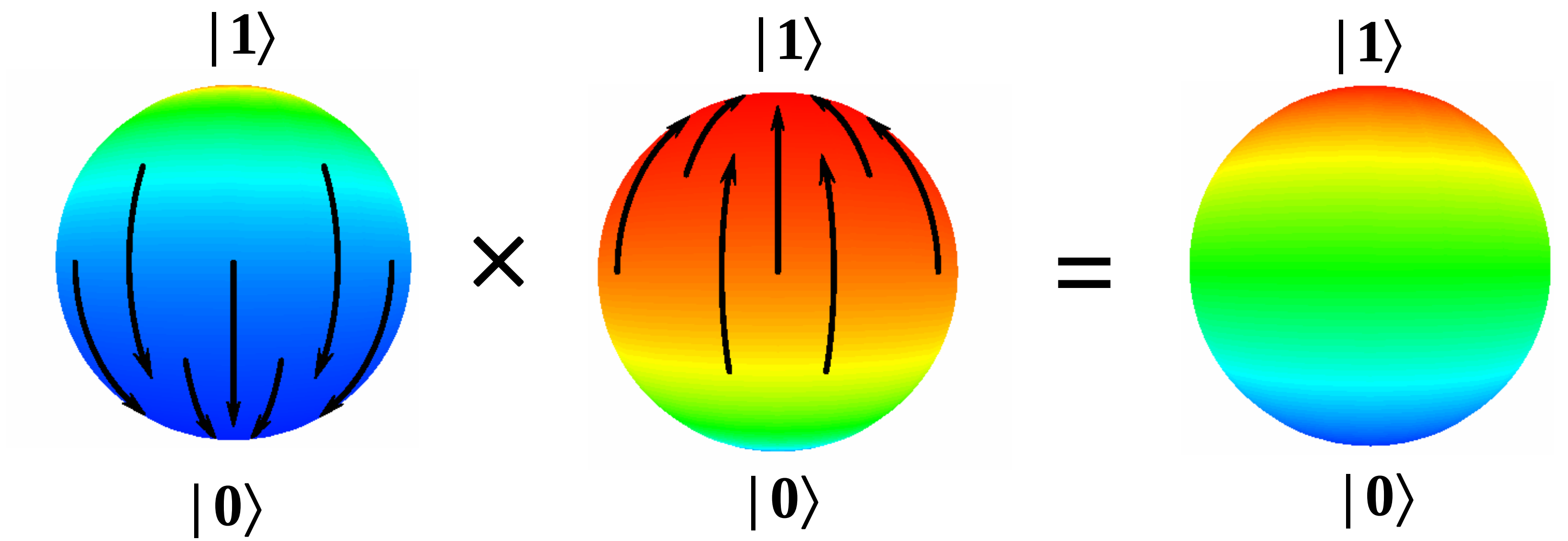
"Quantum Un-Demolition measurement"

(Not a "quantum eraser"!)

Uncollapsing of a qubit state

Evolution due to partial (weak, continuous, etc.) measurement is **non-unitary** (though coherent if detector is good!), therefore it is impossible to undo it by Hamiltonian dynamics.

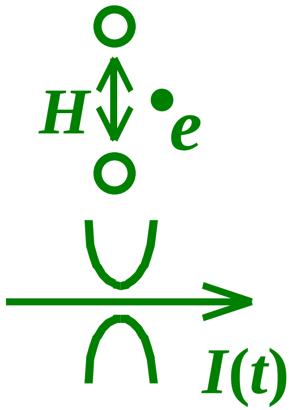
How to uncollapse? One more measurement!



(similar to Koashi-Ueda, PRL-1999)

(Figure partially adopted from A. Jordan, A. Korotkov, and M. Büttiker, PRL-2006

First example: double-dot qubit with no tunneling, measured by QPC



$$\hat{H}_{QB} = (\varepsilon/2)(c_1^\dagger c_1 - c_2^\dagger c_2) + H(c_1^\dagger c_2 + c_2^\dagger c_1)$$

Assume "frozen" qubit: $\varepsilon = H = 0$

Bayesian evolution due to measurement (Korotkov-1998)

1) Diagonal matrix elements of the density matrix evolve according to the classical Bayes rule

2) Non-diagonal matrix elements evolve so that the degree of purity $\rho_{ij}/[\rho_{ii}\,\rho_{jj}]^{1/2}$ is conserved

$$\rho_{11}(\tau) = \frac{\rho_{11}(0)\exp[-(\bar{I}-I_1)^2/2D]}{\rho_{11}(0)\exp[-(\bar{I}-I_1)^2/2D] + \rho_{22}(0)\exp[-(\bar{I}-I_2)^2/2D]}$$

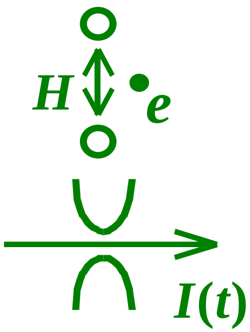
$$\frac{\rho_{12}(\tau)}{[\rho_{12}(\tau)\,\rho_{22}(\tau)]^{1/2}} = \frac{\rho_{12}(0)}{[\rho_{12}(0)\,\rho_{22}(0)]^{1/2}}, \qquad \rho_{22}(\tau) = 1 - \rho_{11}(\tau)$$

where $\bar{I} = \frac{1}{\tau}\int_0^\tau I(t)\,dt, \quad D = S_I/2\tau$

$\Delta I = I_1 - I_2$ - response

$S_I = 2eI(1-T)$ - shot noise

Graphical representation of the evolution

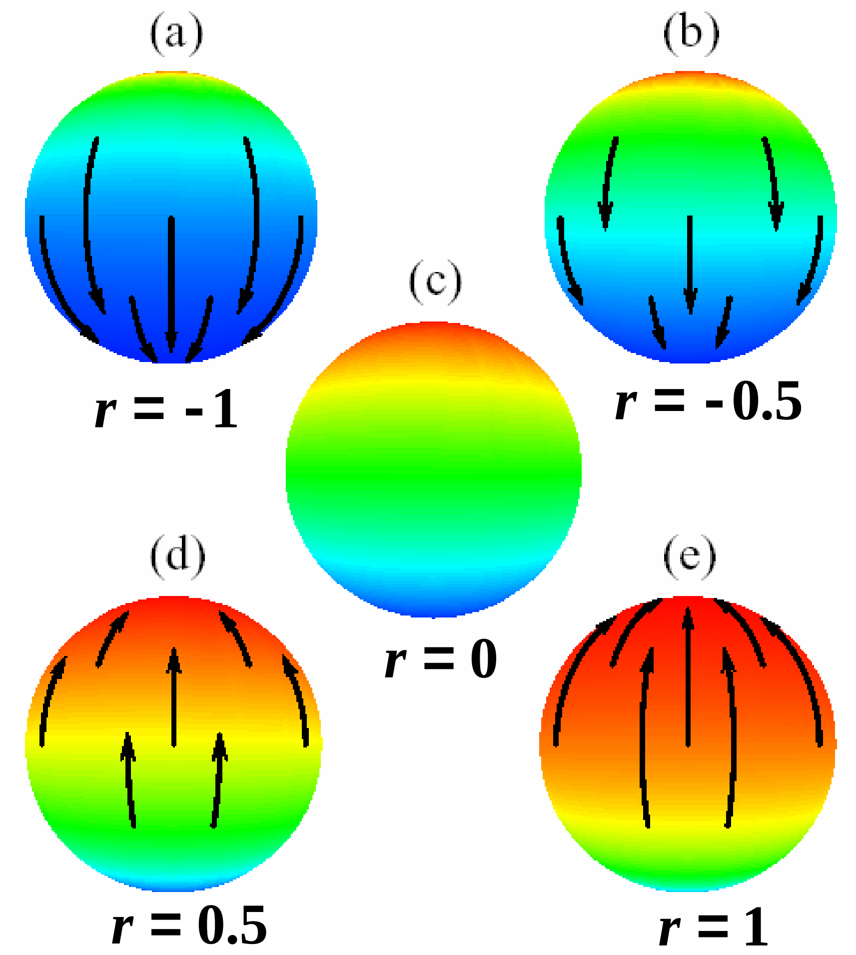


$$\frac{\rho_{11}(t)}{\rho_{22}(t)} = \frac{\rho_{11}(0)}{\rho_{22}(0)} \exp[2r(t)]$$

$$\frac{\rho_{12}(t)}{\sqrt{\rho_{11}(t)\rho_{22}(t)}} = \text{const}$$

where measurement result $r(t)$ is

$$r(t) = \frac{\Delta I}{S_I}\left[\int_0^t I(t')\,dt' - I_0 t\right]$$

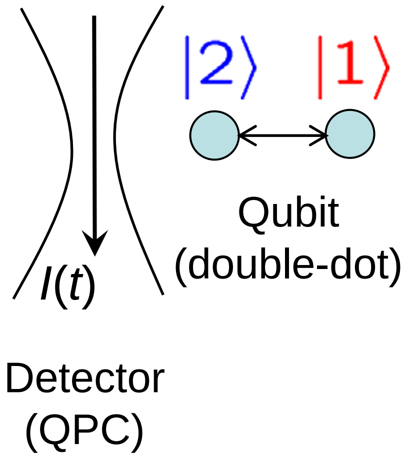


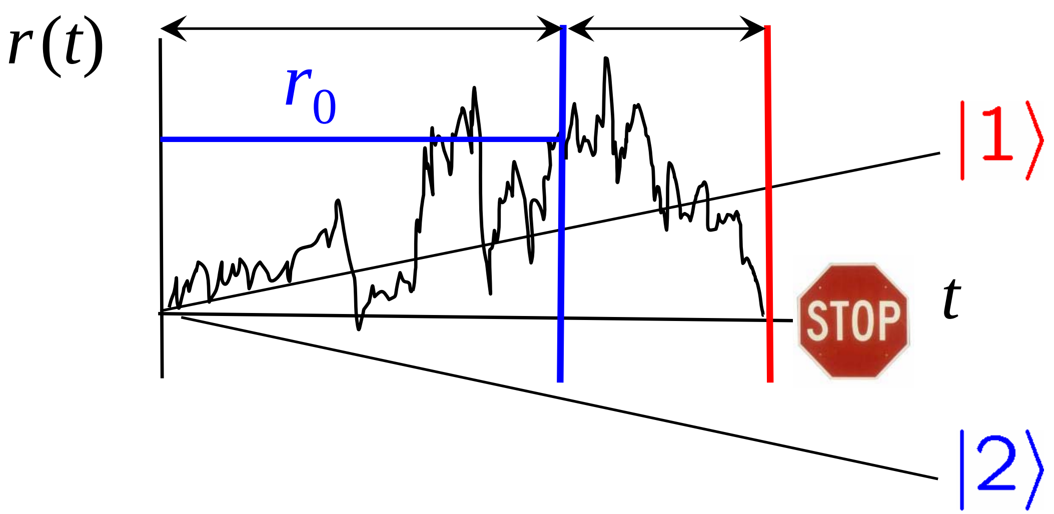
Jordan-Korotkov-Büttiker, PRL-06

If $r = 0$, then no information and no evolution!

Uncollapsing for qubit-QPC system

Jordan and Korotkov, PRL-2006





Simple strategy: continue measuring until *r*(*t*) becomes zero!

Then any unknown initial state is fully restored.

(same for an entangled qubit)

It may happen though that $r = 0$ never happens;
then undoing procedure is unsuccessful.

Probability of success

Trick: since non-diagonal matrix elements are not directly involved, we can analyze classical probabilities (as if qubit is in some certain, but unknown state); then simple diffusion with drift

Results:

Probability of successful uncollapsing

$$P_S = \frac{e^{-|r_0|}}{e^{|r_0|}\rho_{11}(0) + e^{-|r_0|}\rho_{22}(0)}$$

where r_0 is the result of the measurement to be undone, and ρ(0) is our knowledge about an unknown initial state; in case of an entangled qubit ρ(0) is traced over other qubits

Averaged probability of success (over result r_0)

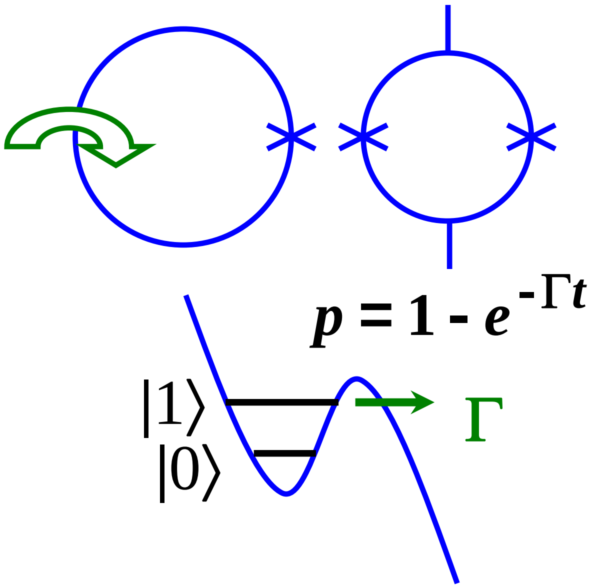
$$P_{av} = 1 - \mathrm{erf}[\sqrt{t/2T_m}\,]$$

(does not depend on initial state!)

where $T_m = 2S_I/(\Delta I)^2$ ("measurement time")

Second example: uncollapsing of a superconducting phase qubit

1) Start with an unknown state
2) Partial measurement of strength *p*
3) π-pulse (exchange |0> ↔ |1>)
4) One more measurement with the **same strength *p***
5) π-pulse



N. Katz et al., Science-2006

This is what was demonstrated experimentally

(in more detail later)

General theory of uncollapsing

Measurement operator M_r: $$\rho \to \frac{M_r \rho M_r^\dagger}{\mathrm{Tr}(M_r \rho M_r^\dagger)}$$ (POVM formalism)

Undoing measurement operator: $C \times M_r^{-1}$ (to satisfy completeness, eigenvalues cannot be >1)

$$\max(C) = \min_i \sqrt{p_i}\,, \quad p_i = \mathrm{Tr}(M_r^\dagger M_r |i\rangle\langle i|)$$

p_i – probability of the measurement result r for initial state $|i\rangle$

Probability of success: $$P_S \le \frac{\min_i p_i}{\sum_i p_i \rho_{ii}(0)} = \frac{\min P_r}{P_r(\rho(0))}$$

$P_r(\rho(0))$ – probability of result r for initial state $\rho(0)$,

min P_r – probability of result r minimized over all possible initial states

Averaged (over r) probability of success: $P_{av} \le \sum_r \min P_r$

(independent of initial state)

(similar to Koashi-Ueda, PRL, 1999)

Comparison of the general bound for uncollapsing success with two examples

General bound: $$P_S \le \frac{\min P_r}{P_r(\rho(0))}$$

First example (DQD+QPC)

$$P_S \le \frac{\min(p_1, p_2)}{p_1\rho_{11}(0) + p_2\rho_{22}(0)}$$

where $p_i = (\pi S_I / t)^{-1/2} \exp[-(\bar{I} - I_i)^2 t / S_I]\, d\bar{I}$

Coincides with the actual result, so the upper bound is reached, **therefore uncollapsing strategy is optimal**

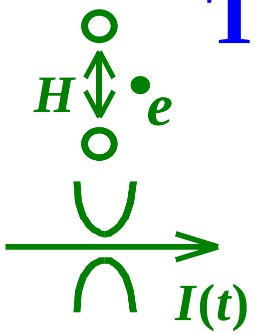
Second example (phase qubit)

Probabilities of no-tunneling are 1 and exp(-Γt)=1-p

$$P_S \le \frac{1-p}{\rho_{00}(0) + (1-p)\rho_{11}(0)}$$

uncollapsing for phase qubit is also optimal

Third example: evolving charge qubit



$$\hat{H}_{QB} = (\varepsilon/2)(c_1^\dagger c_1 - c_2^\dagger c_2) + H(c_1^\dagger c_2 + c_2^\dagger c_1)$$

(now non-zero *H* and ε, qubit evolves during measurement)

1) Bayesian equations to calculate measurement operator
2) unitary operation, measurement by QPC, unitary operation

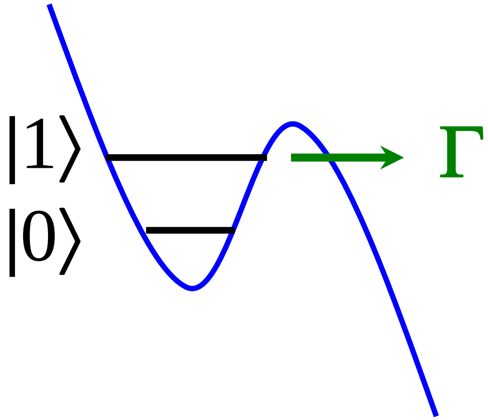
Fourth example: general uncollapsing for entangled charge qubits

1) unitary transformation of *N* qubits
2) null-result measurement of a certain strength by a strongly nonlinear QPC (tunneling only for state |11..1⟩)
3) repeat 2^N times, sequentially transforming the basis vectors of the diagonalized measurement operator into |11..1⟩

(also reaches the upper bound for success probability)

Partial collapse of a phase qubit

N. Katz, M. Ansmann, R. Bialczak, E. Lucero, R. McDermott, M. Neeley, M. Steffen, E. Weig, A. Cleland, J. Martinis, A. Korotkov, Science-06



How does a coherent state evolve in time before tunneling event?

(What happens when nothing happens?)

Qubit "ages" in contrast to a radioactive atom!

Main idea:

$$\psi = \alpha|0\rangle + \beta|1\rangle \;\rightarrow\; \psi(t) = \begin{cases} |out\rangle, \text{ if tunneled} \\ \dfrac{\alpha|0\rangle + \beta e^{-\Gamma t/2} e^{i\varphi}|1\rangle}{\sqrt{|\alpha|^2 + |\beta|^2 e^{-\Gamma t}}}, \text{ if not tunneled} \end{cases}$$

(better theory: Leonid Pryadko & A.K., 2007)

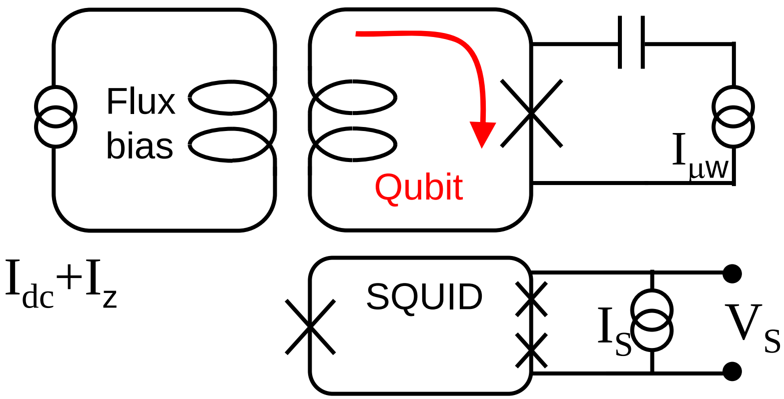
amplitude of state |0> grows without physical interaction

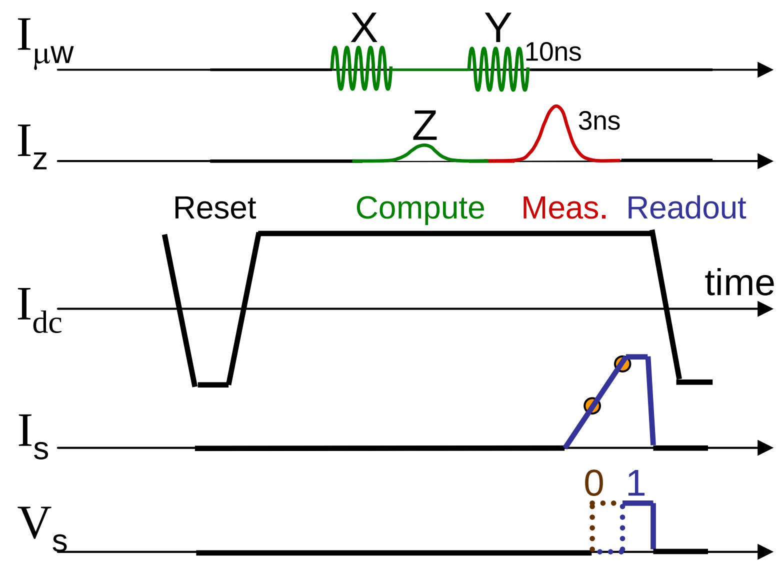
continuous null-result collapse

(similar to optics, Dalibard-Castin-Molmer, PRL-1992)

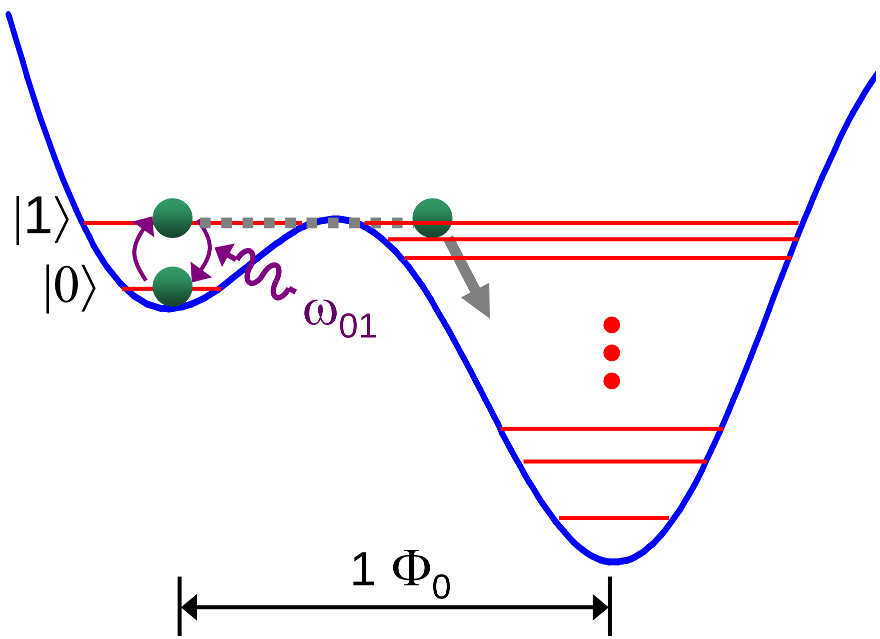
Superconducting phase qubit at UCSB

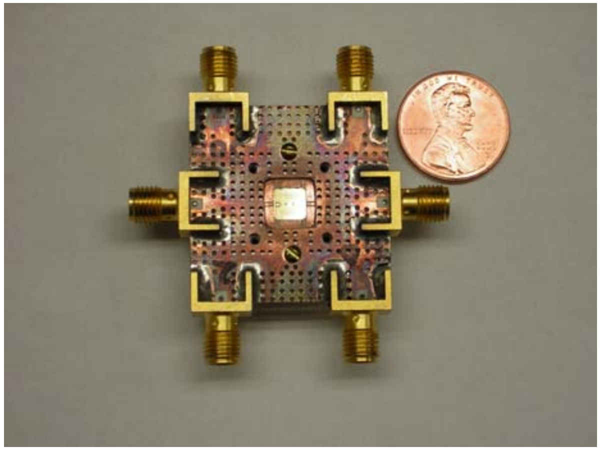
Courtesy of Nadav Katz (UCSB)





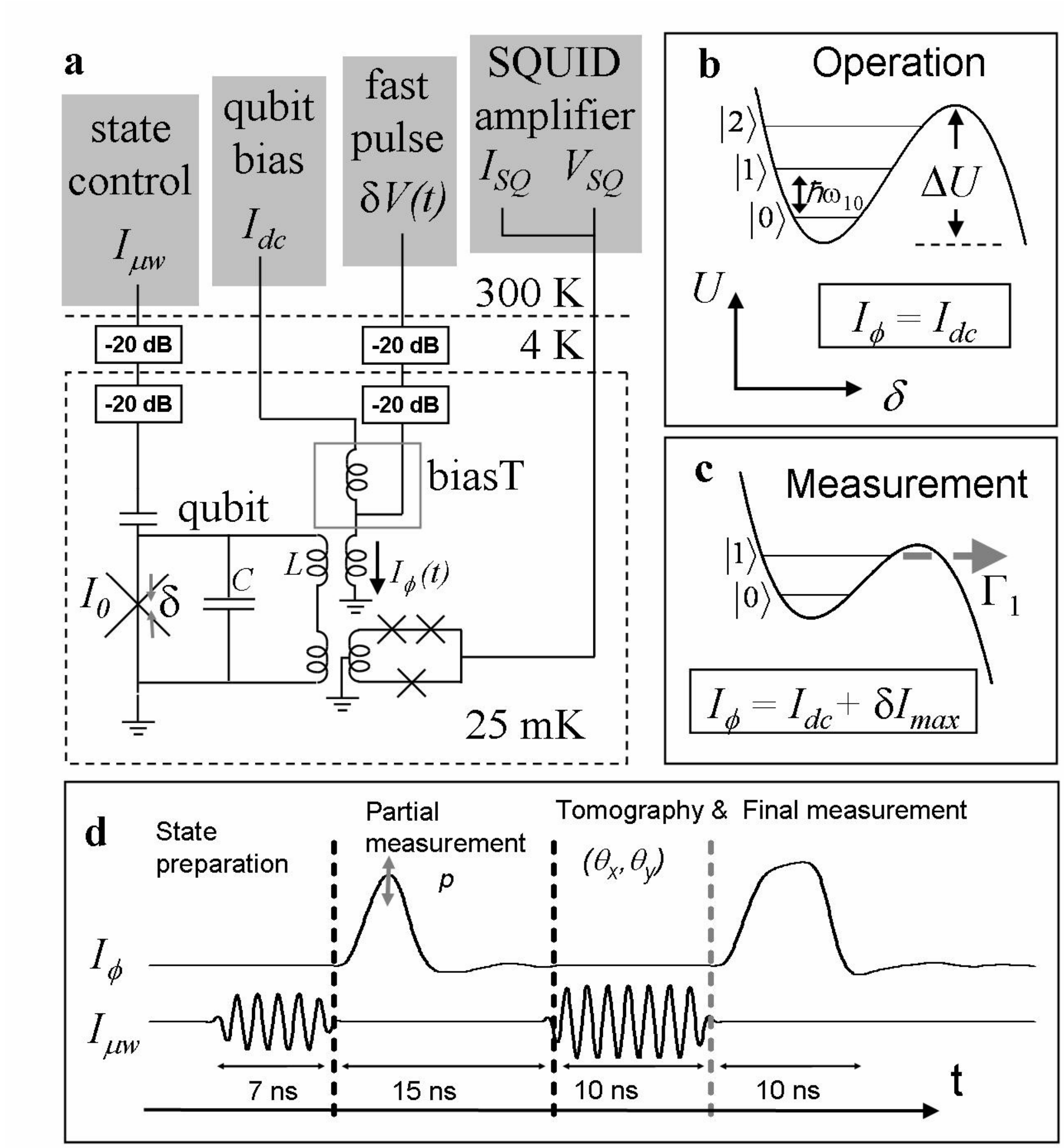
Repeat 1000x
prob. 0,1





Experimental technique for partial collapse

Nadav Katz *et al.*
(John Martinis' group)



Protocol:

1) State preparation by applying microwave pulse (via Rabi oscillations)

2) Partial measurement by lowering barrier for time *t*

3) State tomography (micro-wave + full measurement)

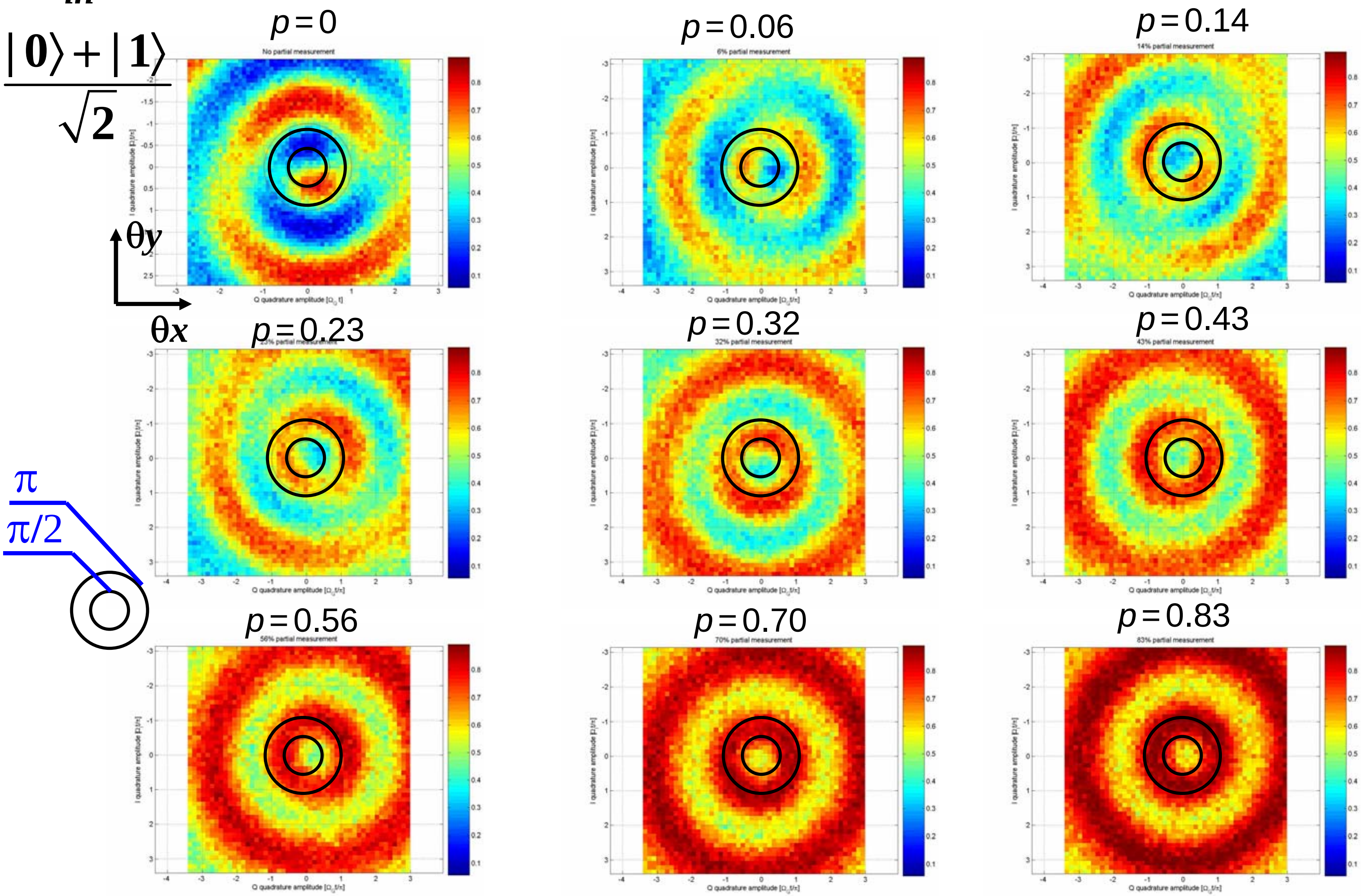
Measurement strength
$p = 1 - \exp(-\Gamma t)$
is actually controlled by Γ, not by *t*

***p* = 0: no measurement**
***p* = 1: orthodox collapse**

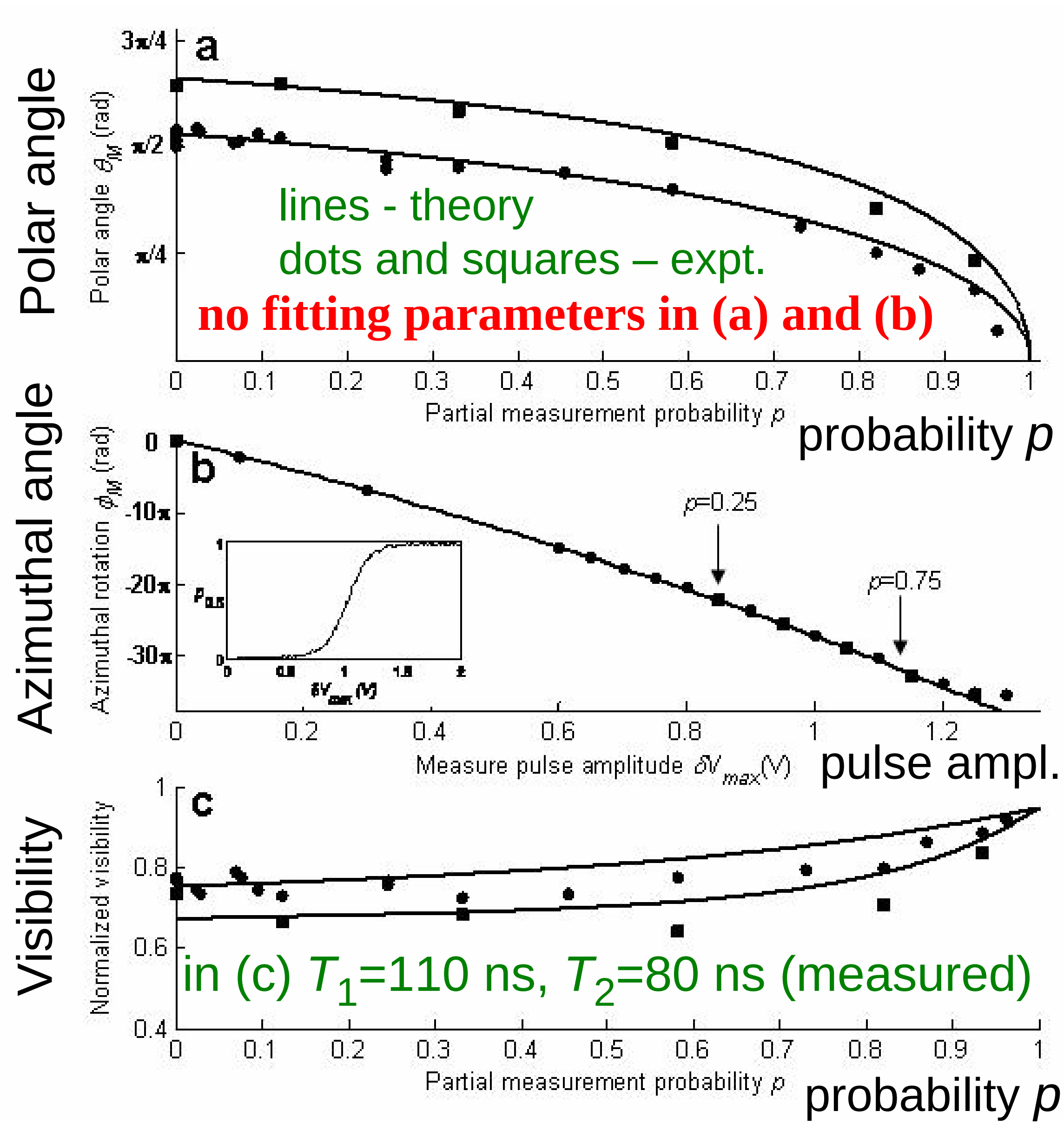
Experimental tomography data

Nadav Katz *et al*. (UCSB)

$\psi_{in} = \dfrac{|0\rangle + |1\rangle}{\sqrt{2}}$



Partial collapse: experimental results



N. Katz *et al.*, Science-06

- **In case of no tunneling (null-result measurement) phase qubit evolves**
- **This evolution is well described by a simple Bayesian theory, without fitting parameters**
- **Phase qubit remains fully coherent in the process of continuous collapse (experimentally ~80% raw data, ~96% after account for T1 and T2)**

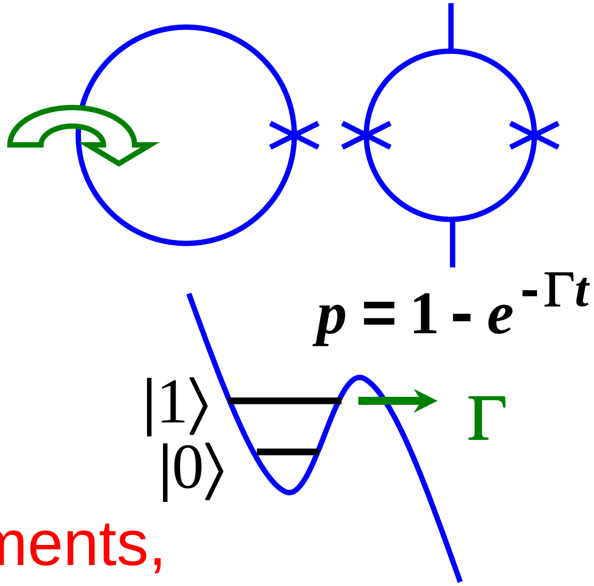
quantum efficiency

$\eta_0 > 0.8$

Uncollapsing of a phase qubit state

Jordan and Korotkov, PRL-2006

1) Start with an unknown state
2) Partial measurement of strength p
3) π-pulse (exchange |0> ↔ |1>)
4) One more measurement with the **same strength** p
5) π-pulse



If no tunneling for both measurements, then initial state is fully restored!

$$\alpha|0\rangle + \beta|1\rangle \rightarrow \frac{\alpha|0\rangle + e^{i\phi}\beta e^{-\Gamma t/2}|1\rangle}{\text{Norm}} \rightarrow$$

$$\frac{e^{i\phi}\alpha e^{-\Gamma t/2}|0\rangle + e^{i\phi}\beta e^{-\Gamma t/2}|1\rangle}{\text{Norm}} = e^{i\phi}(\alpha|0\rangle + \beta|1\rangle)$$

phase is also restored (spin echo)

Probability of success

Success probability if no tunneling during first measurement:

$$P_S = \frac{e^{-\Gamma t}}{\rho_{00}(0) + e^{-\Gamma t}\rho_{11}(0)} = \frac{1-p}{\rho_{00}(0) + (1-p)\rho_{11}(0)}$$

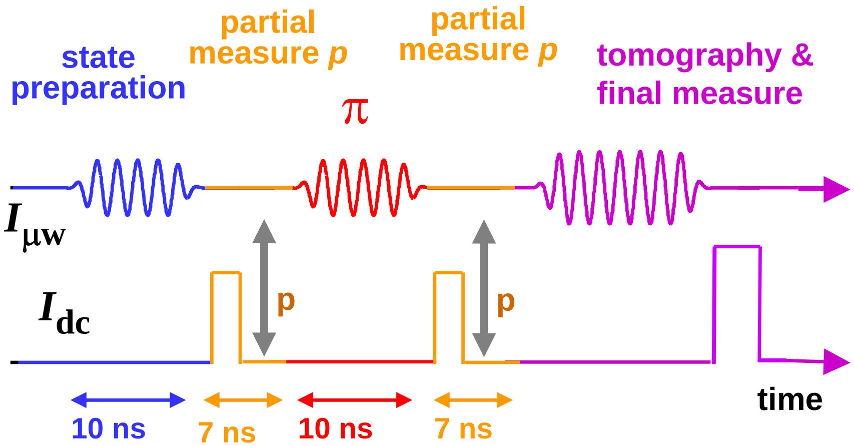
where ρ(0) is the density matrix of the initial state (either averaged unknown state or an entangled state traced over all other qubits)

Total (averaged) success probability: $P_{\rm av} = 1-p$

For measurement strength *p* increasing to 1, success probability decreases to zero (orthodox collapse), but still exact undoing

Optimal uncollapsing (reaches fundamental upper bound)

Experiment on wavefunction uncollapsing



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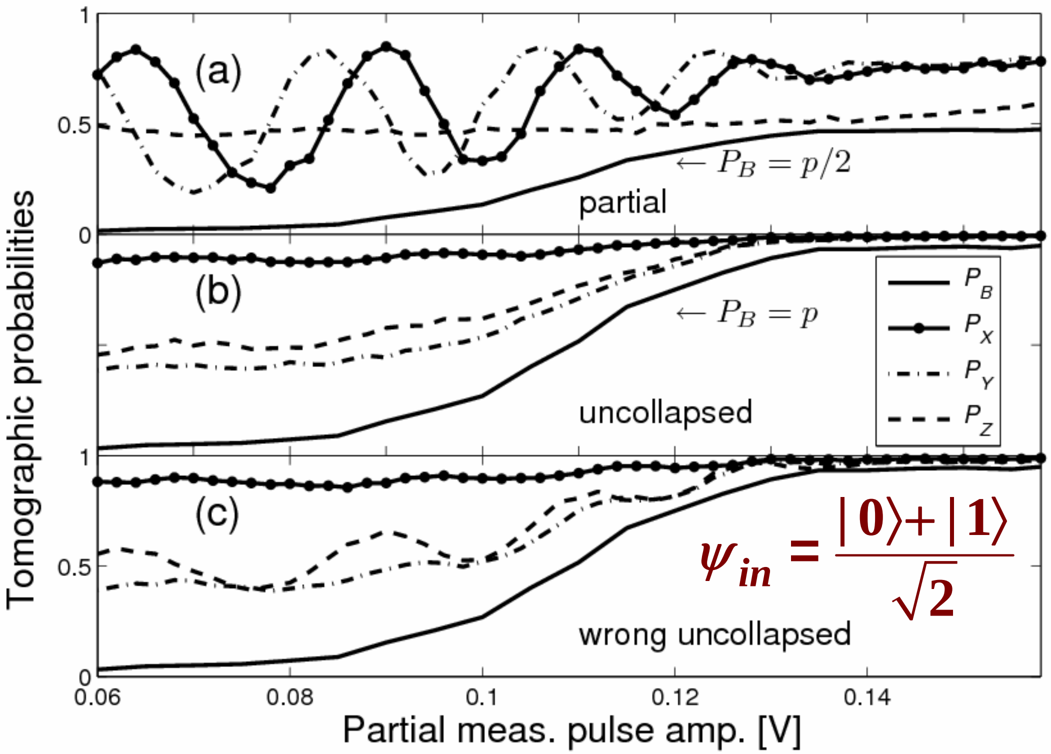


Uncollapse protocol:

- partial collapse
- π-pulse
- partial collapse (same strength)

State tomography with *X*, *Y*, and *Z* pulses

Background P_B should be subtracted to find qubit density matrix

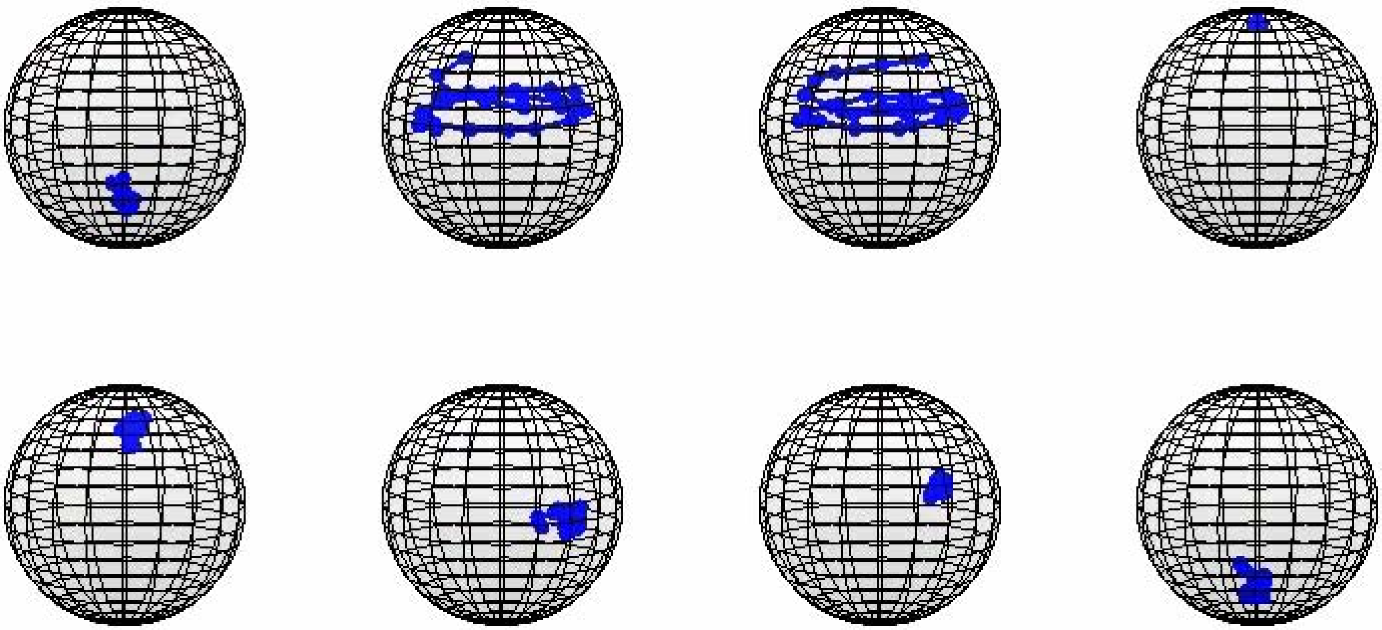
Experimental results on Bloch sphere

N. Katz et al.

Initial state	$\|1\rangle$	$\frac{\|0\rangle + \|1\rangle}{\sqrt{2}}$	$\frac{\|0\rangle + i\|1\rangle}{\sqrt{2}}$	$\|0\rangle$

Partial collapse

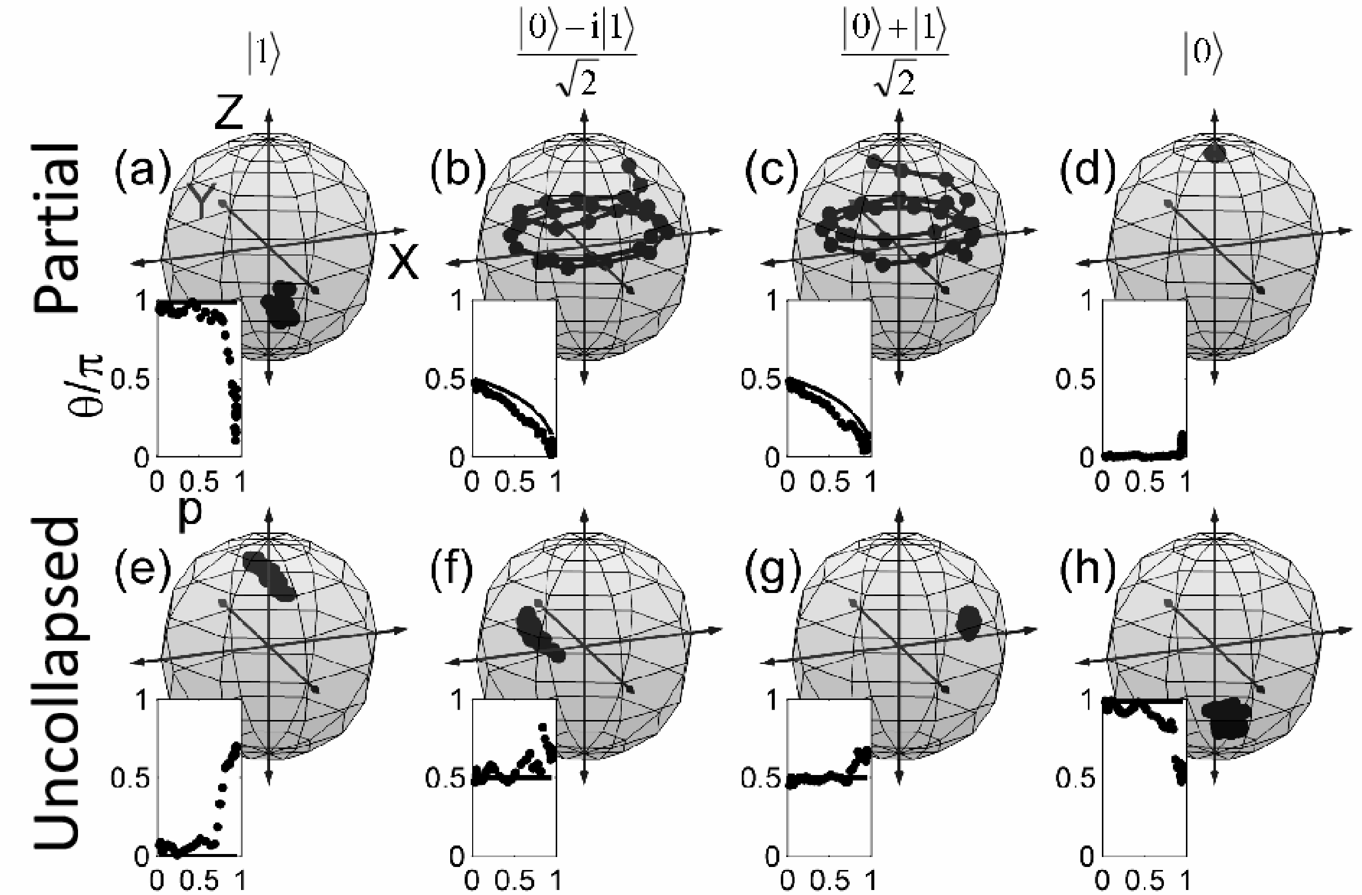
Uncollapsed



Collapse strength: $0.05 < p < 0.7$

uncollapsing works well!

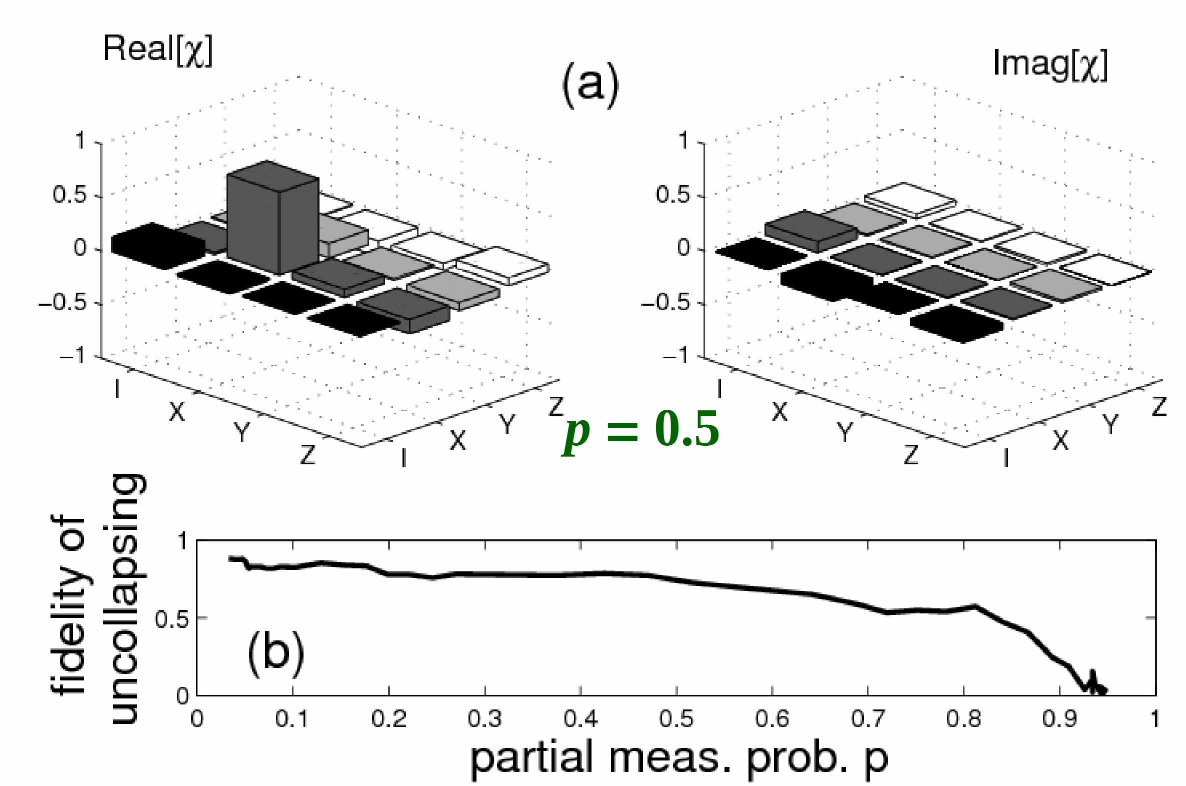
Same with polar angle dependence
(another experimental run)



Both spin echo (azimuth) and uncollapsing (polar angle)

Difference: spin echo – undoing of an unknown unitary evolution, uncollapsing – undoing of a known, but non-unitary evolution

Quantum process tomography



Why getting worse at $p>0.6$?

Energy relaxation $\boldsymbol{p_r = t/T_1 = 45\text{ns}/450\text{ns} = 0.1}$

Selection affected when $\boldsymbol{1-p \sim p_r}$

Overall: uncollapsing is well-confirmed experimentally

Quantum erasers in optics

Quantum eraser proposal by Scully and Drühl, PRA (1982)

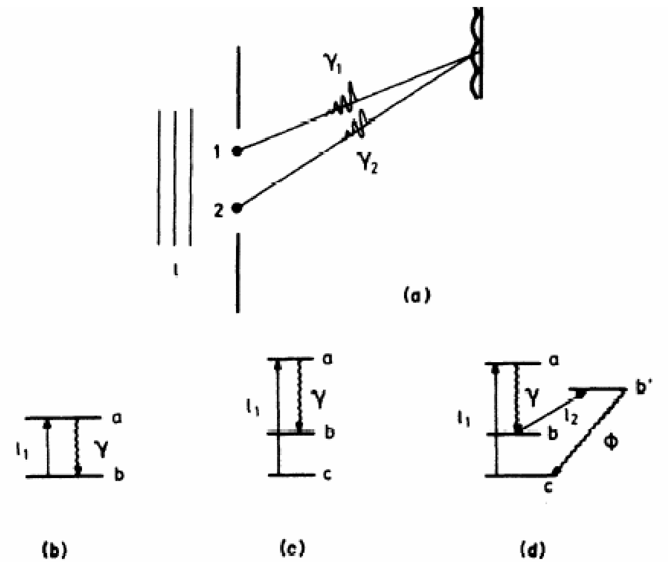


FIG. 1. (a) Figure depicting light impinging from left on atoms at sites 1 and 2. Scattered photons γ_1 and γ_2 produce interference pattern on screen. (b) Two-level atoms excited by laser pulse l_1, and emit γ photons in $a \rightarrow b$ transition. (c) Three-level atoms excited by pulse l_1 from $c \rightarrow a$ and emit photons in $a \rightarrow b$ transition. (d) Four-level system excited by pulse l_1 from $c \rightarrow a$ followed by emission of γ photons in $a \rightarrow b$ transition. Second pulse l_2 takes atoms from $b \rightarrow b'$. Decay from $b' \rightarrow c$ results in emission of ϕ photons.

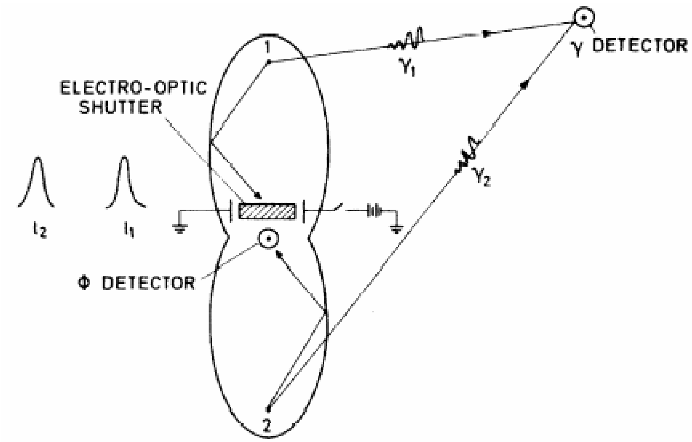


FIG. 2. Laser pulses l_1 and l_2 incident on atoms at sites 1 and 2. Scattered photons γ_1 and γ_2 result from $a \rightarrow b$ transition. Decay of atoms from $b' \rightarrow c$ results in ϕ photon emission. Elliptical cavities reflect ϕ photons onto common photodetector. Electro-optic shutter transmits ϕ photons only when switch is open. Choice of switch position determines whether we emphasize particle or wave nature of γ photons.

Interference fringes restored for two-detector correlations (since "which-path" information is erased)

Our idea of uncollapsing is quite different:
we really extract information and then erase it

Conclusions

- Partial (weak, etc.) quantum measurement can be undone, though with a finite probability P_S, which decreases with increasing strength of measurement ($P_S = 0$ for orthodox case)

- Arbitrary initial state is uncollapsed exactly in the case of success (need a detector with perfect quantum efficiency)

- Uncollapsing is different from the quantum eraser

- Uncollapsing for a superconducting phase qubit has been demonstrated, extending the previous experiment on partial collapse (would be very interesting to demonstrate for a charge qubit as well; SET cannot be used)