Partial quantum collapse and uncollapsing

Alexander Korotkov

University of California, Riverside

Outline:

- Introduction (textbook collapse and Bell inequality)
- Beyond the textbook collapse: non-projective quantum measurement
- Uncollapsing (reversal of weak measurement)
- Recent experiments on partial collapse and "wavefunction uncollapsing"

Support:

Niels Bohr:

"If you are not confused by quantum physics then you haven't really understood it"

Richard Feynman:

"I think I can safely say that nobody understands quantum mechanics"

Quantum mechanics =

Schrödinger equation

+

collapse postulate

1) Probability of measurement result $p_r = |\langle \psi | \psi_r \rangle|^2$

2) Wavefunction after measurement $= \psi_r$

- State collapse follows from common sense
- Does not follow from Schrödinger equation
  (contradicts; random vs. deterministic)

Collapse postulate is controversial since 1920s
(needs an observer, contradicts causality)

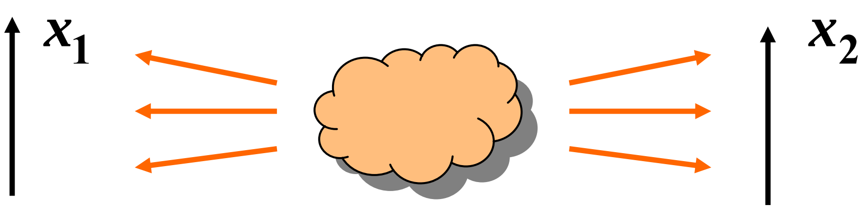
Einstein-Podolsky-Rosen (EPR) paradox

Phys. Rev., 1935

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system.

$$\psi(x_1, x_2) = \sum_n \psi_n(x_2)\, u_n(x_1)$$ *(nowadays we call it entangled state)*

$$\psi(x_1, x_2) = \int_{-\infty}^{\infty} \exp[(i/\hbar)(x_1 - x_2)p]\,dp \sim \delta(x_1 - x_2)$$



Measurement of particle 1 cannot affect particle 2, while QM says it affects (contradicts causality)

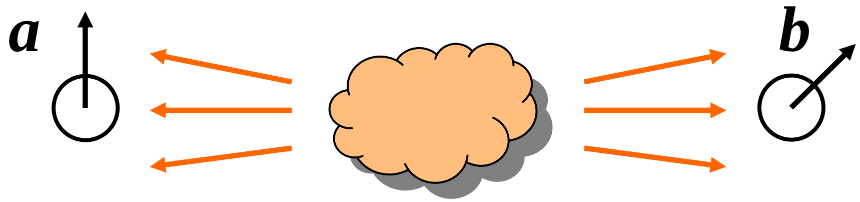
=> Quantum mechanics is incomplete

Bohr's reply **(Phys. Rev., 1935)** (seven pages, one formula: $\Delta p\, \Delta q \sim h$)

It is shown that a certain "criterion of physical reality" formulated … by A. Einstein, B. Podolsky and N. Rosen contains an essential ambiguity when it is applied to quantum phenomena.

Crudely: No need to understand QM, just use the result

Bell's inequality (John Bell, 1964)



(setup due to David Bohm)

$$\psi = \frac{1}{\sqrt{2}}(\uparrow_1\downarrow_2 - \downarrow_1\uparrow_2)$$

Perfect anticorrelation of measurement results for the same measurement directions, $\vec{a} = \vec{b}$

Is it possible to explain the QM result assuming local realism and hidden variables *or* collapse "propagates" instantaneously (faster than light, "spooky action-at-a-distance")?

Assume: $A(\vec{a},\lambda) = \pm 1, \quad B(\vec{b},\lambda) = \pm 1$ (deterministic result with hidden variable λ)

Then: $|P(\vec{a},\vec{b}) - P(\vec{a},\vec{c})| \le 1 + P(\vec{b},\vec{c})$

where $P \equiv P(++) + P(--) - P(+-) - P(-+)$

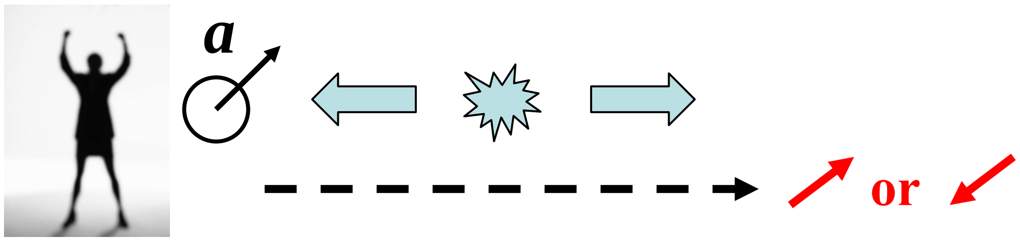
QM: $P(\vec{a},\vec{b}) = -\vec{a}\bullet\vec{b}$ For 0°, 90°, and 45°: $0.71 \not\le 1 - 0.71$ **violation!**

Experiment (Aspect et al., 1982; photons instead of spins, CHSH):

yes, "spooky action-at-a-distance"

What about causality?

Actually, not too bad: you cannot transmit your own information choosing a particular measurement direction ***a***



Result of the other measurement does not depend on direction ***a***

Randomness saves causality

Collapse is still instantaneous: OK, just our recipe, not an “objective reality”, not a “physical” process

Consequence of causality: No-cloning theorem

Wootters-Zurek, 1982; Dieks, 1982; Yurke

You cannot copy an unknown quantum state

Proof: Otherwise get information on direction ***a*** (and causality violated)

Application: quantum cryptography

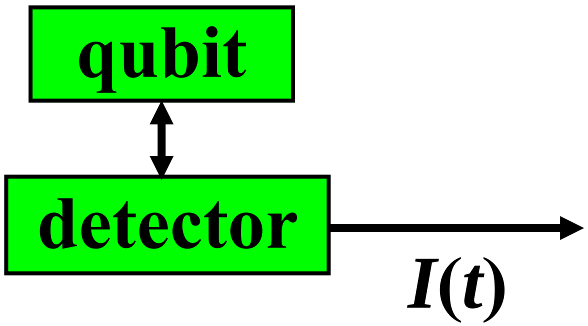
Information is an important concept in quantum mechanics

Quantum measurement in solid-state systems

No violation of locality – too small distances

However, interesting informational aspects of continuous quantum measurement (weak coupling, noise ⇒ gradual collapse)

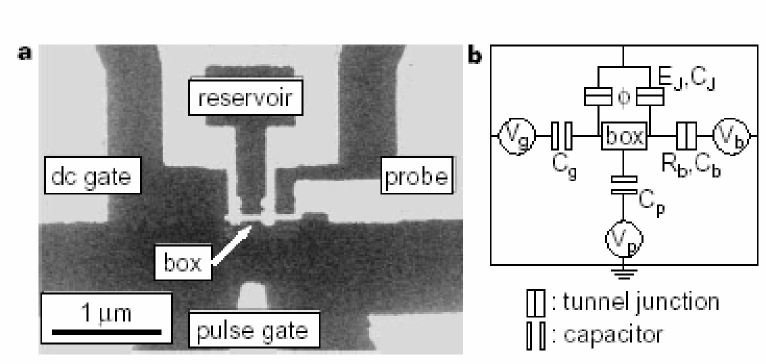
Starting point:



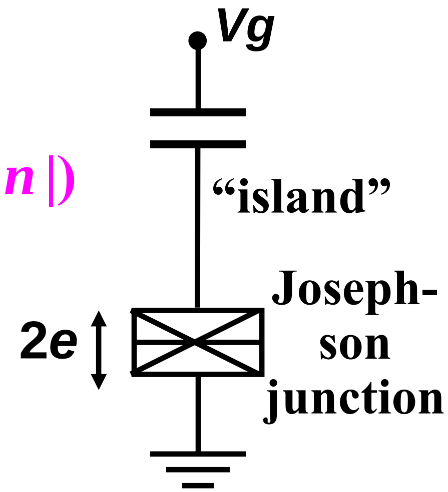
What happens to a solid-state qubit (two-level system) during its continuous (weak) measurement by a detector?

Superconducting "charge" qubit

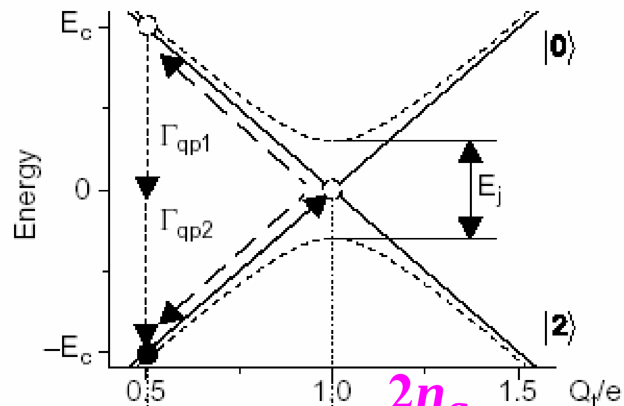
Y. Nakamura, Yu. Pashkin, and J.S. Tsai (Nature, 1998)



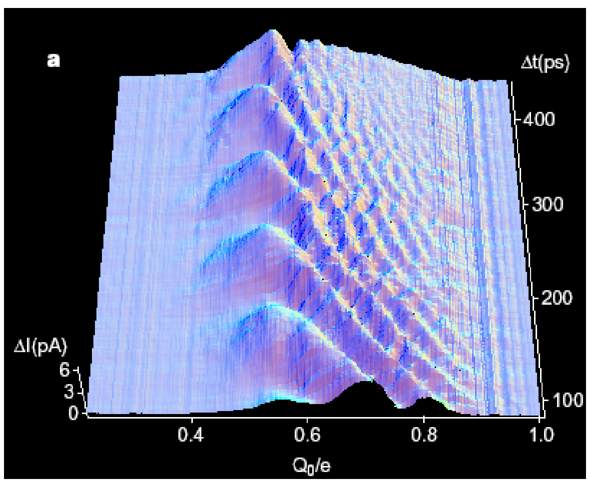
$$\hat{H} = \frac{(2e)^2}{2C}(\hat{n} - n_g)^2 - \frac{E_J}{2}(|n\rangle\langle n+1| + |n+1\rangle\langle n|)$$

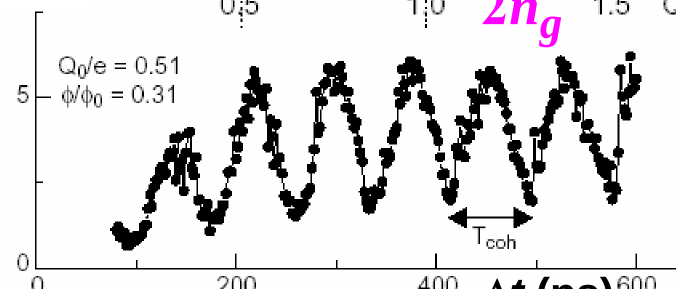


Single Cooper pair box



$2n_g$





Quantum coherent (Rabi) oscillations

n $\longleftrightarrow$ $n+1$

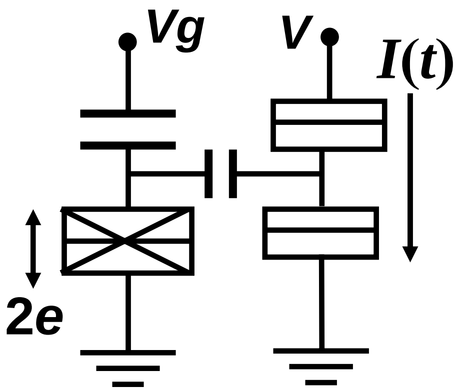
E_J

n: number of Cooper pairs on the island

Vion et al. (Devoret's group); Science, 2002

Q-factor of coherent (Rabi) oscillations = 25,000

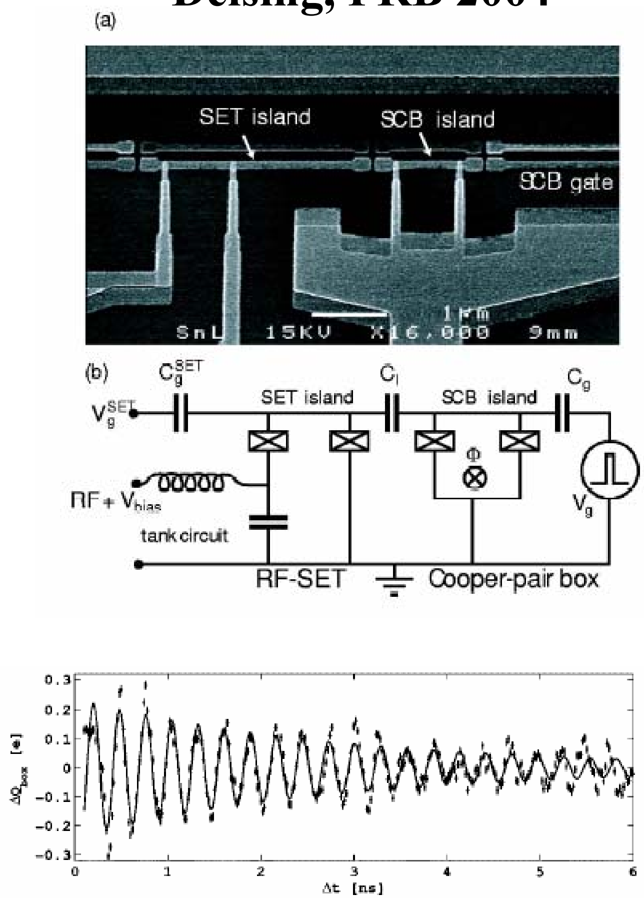
More of superconducting charge qubits



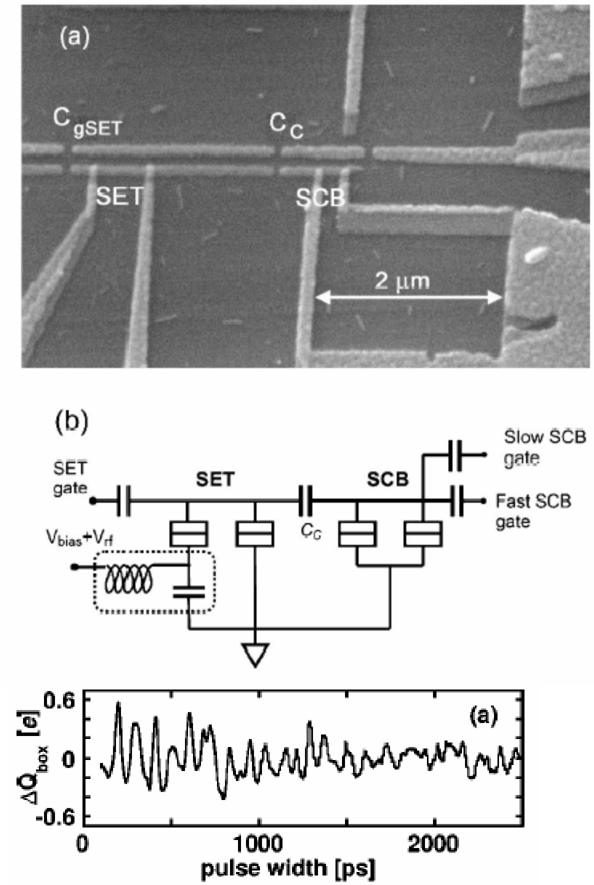
Cooper-pair box measured by single-electron transistor (SET)
(actually, RF-SET)

Setup can be used for continuous measurements

Duty, Gunnarsson, Bladh, Delsing, PRB 2004



Guillaume et al. (Echternach's group), PRB 2004

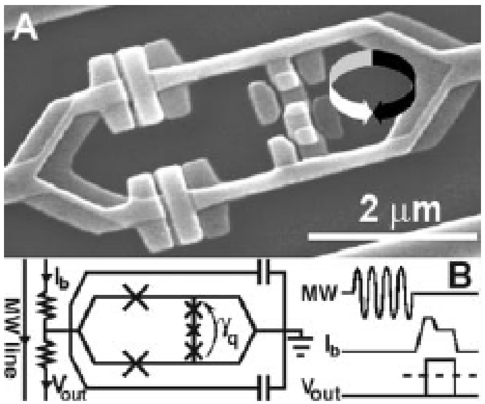


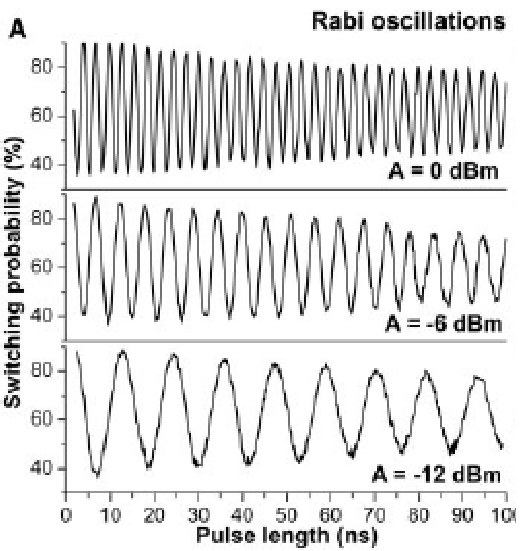
All results are averaged over many measurements (not "single-shot")

Some other superconducting qubits

Flux qubit

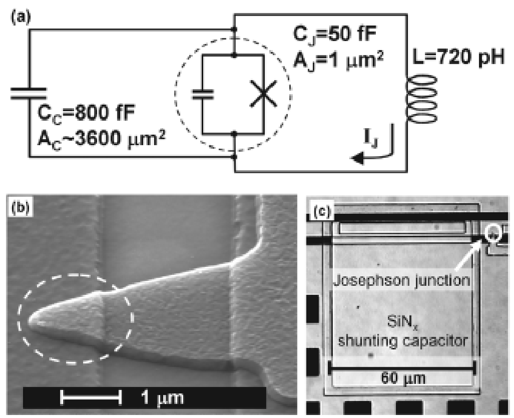
Mooij et al. (Delft)

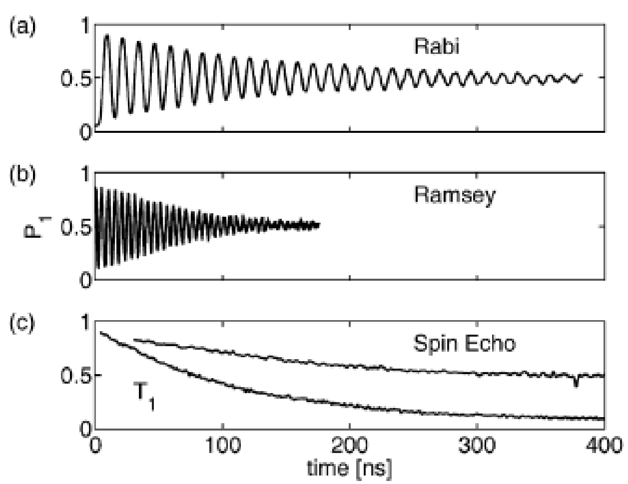




Phase qubit

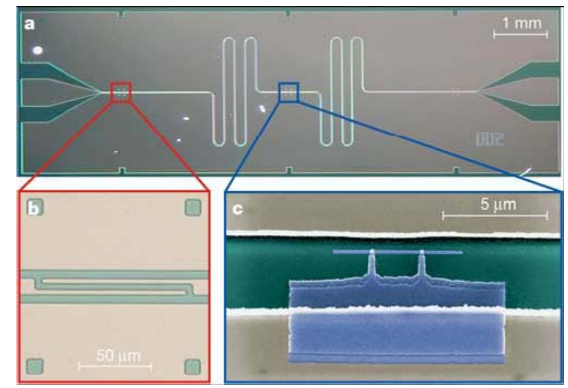
J. Martinis et al.
(UCSB and NIST)

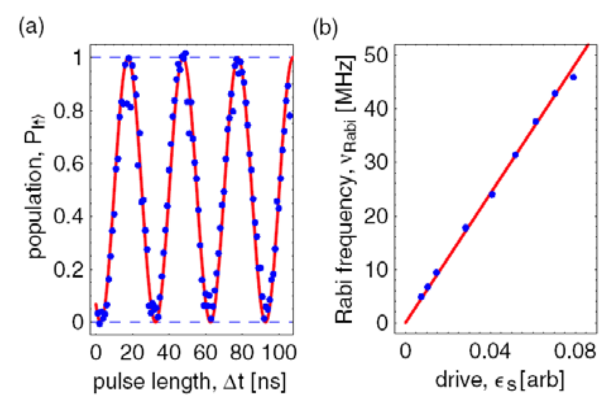




Charge qubit with circuit QED

R. Schoelkopf et al. (Yale)

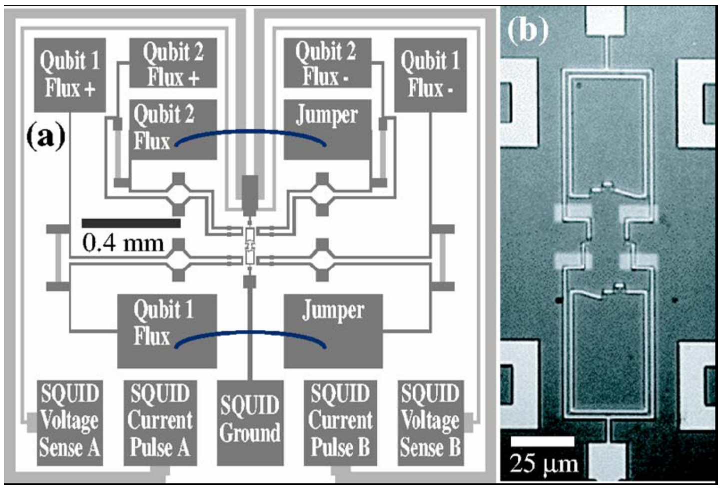


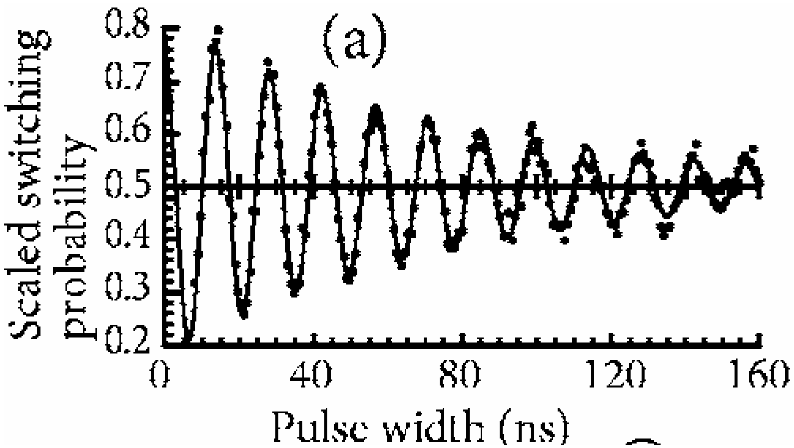


Some other superconducting qubits

Flux qubit

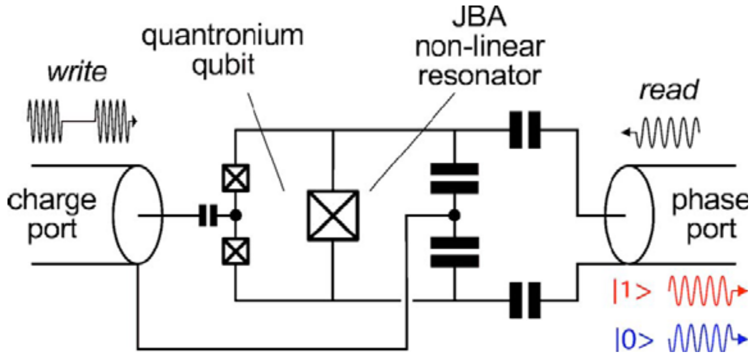
J. Clarke et al. (Berkeley)

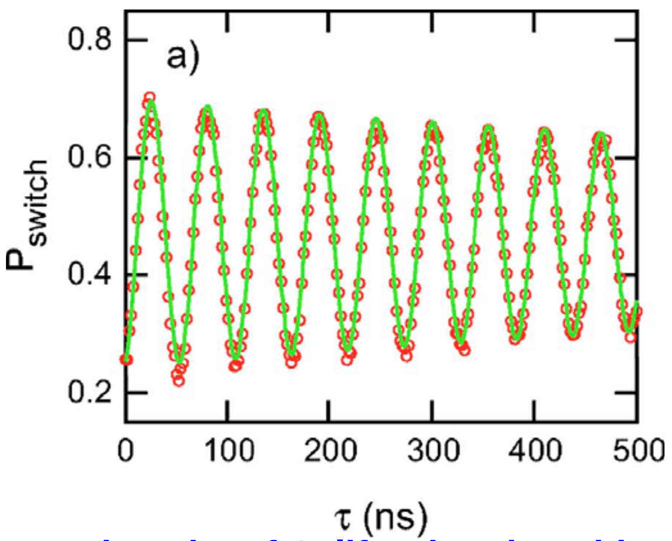




"Quantronium" qubit

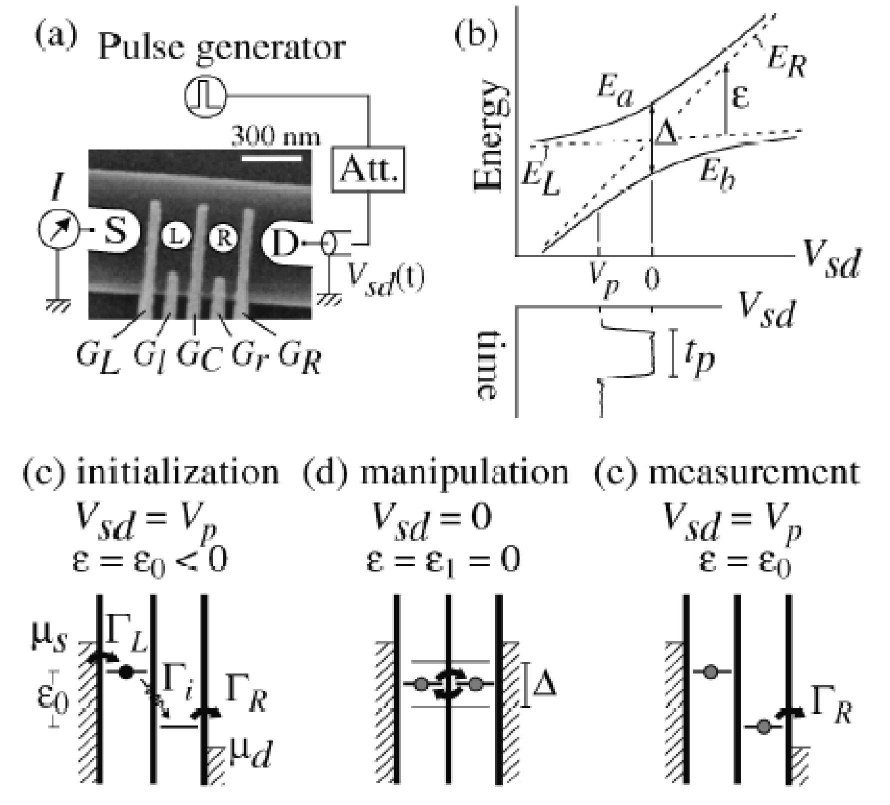
I. Siddiqi, R. Schoelkopf,
M. Devoret, et al. (Yale)

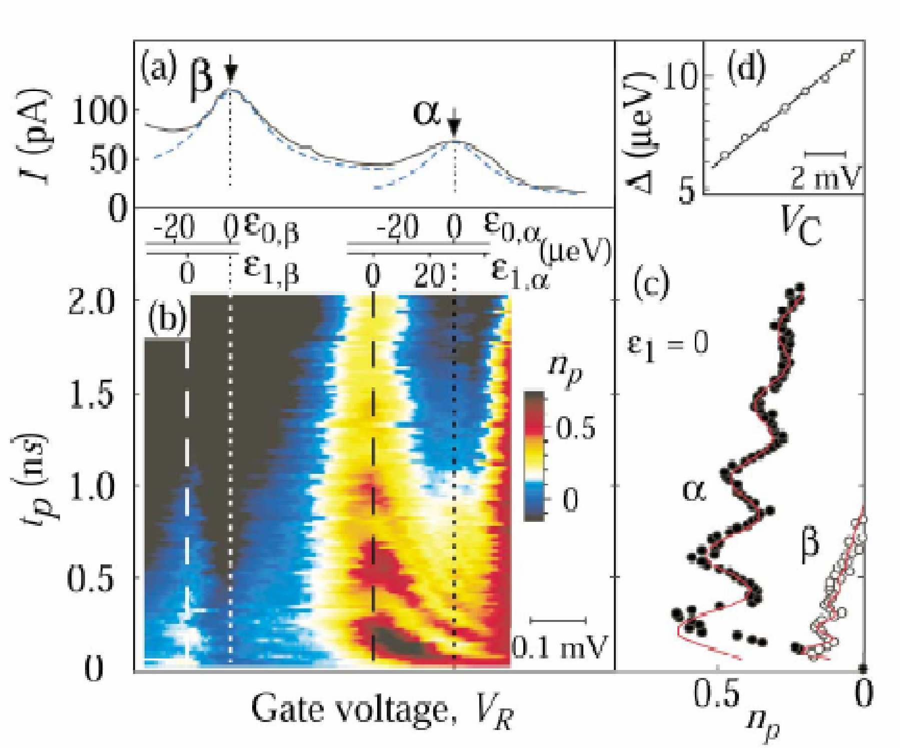




Semiconductor (double-dot) qubit

T. Hayashi et al., PRL 2003





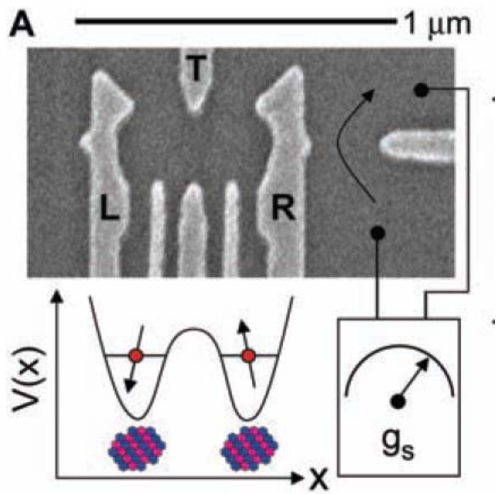
Rabi oscillations

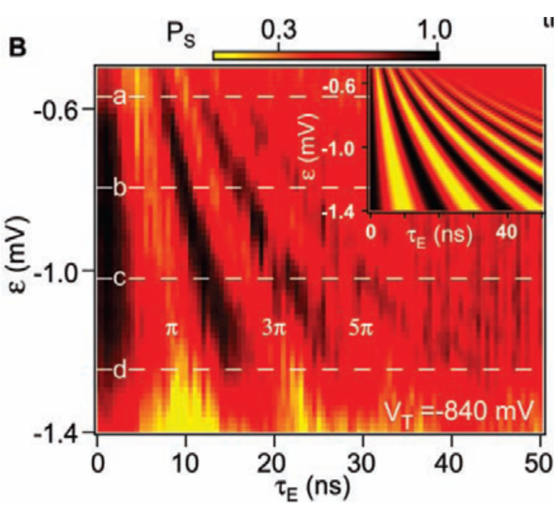
Detector is not separated from qubit,
also possible to use a separate detector

Some other semiconductor qubits

Spin qubit

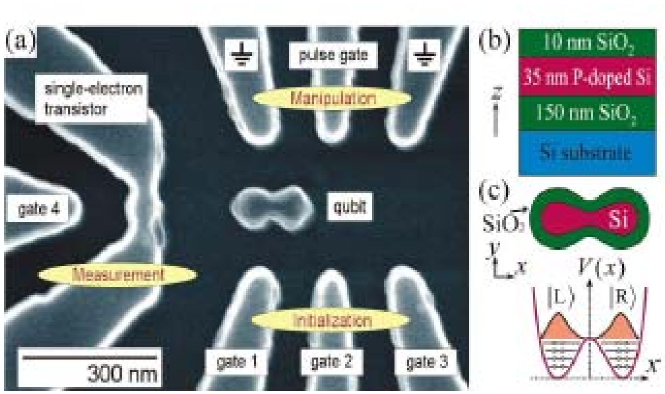
C. Marcus et al. (Harvard)

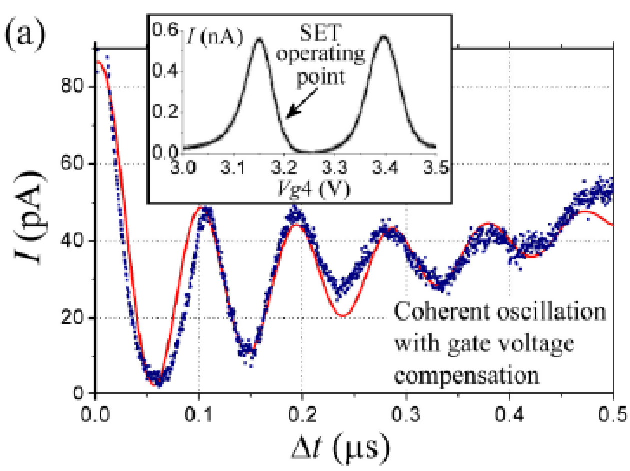




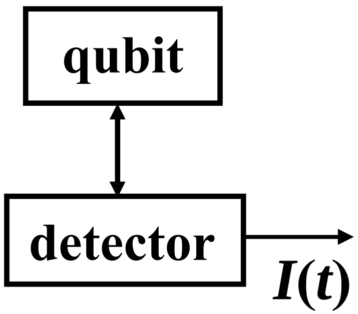
Double-dot qubit

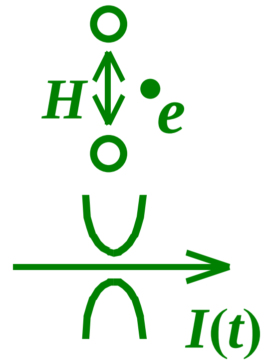
J. Gorman et al. (Cambridge)



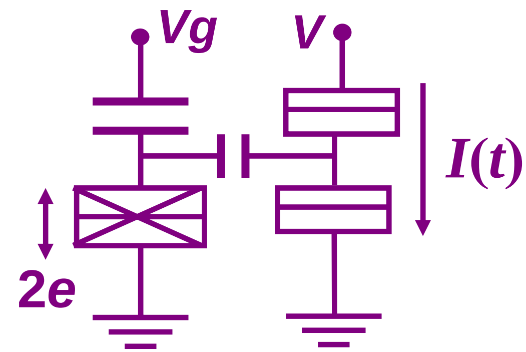


The system we consider: qubit + detector





Double-quantum-qot (DQD) and quantum point contact (QPC)



Cooper-pair box (CPB) and single-electron transistor (SET)

$H = H_{\mathrm{QB}} + H_{\mathrm{DET}} + H_{\mathrm{INT}}$

$H_{\mathrm{QB}} = (\varepsilon/2)(c_1^+c_1 - c_2^+c_2) + H(c_1^+c_2 + c_2^+c_1)$ ε – asymmetry, H – tunneling

$\Omega = (4H^2 + \varepsilon^2)^{1/2}/\hbar$ – frequency of quantum coherent (Rabi) oscillations

Two levels of average detector current: I_1 for qubit state $|1\rangle$, I_2 for $|2\rangle$

Response: $\Delta I = I_1 - I_2$ Detector noise: white, spectral density S_I

DQD and QPC (setup due to Gurvitz, 1997)

$$H_{DET} = \sum_l E_l a_l^\dagger a_l + \sum_r E_r a_r^\dagger a_r + \sum_{l,r} T(a_r^\dagger a_l + a_l^\dagger a_r)$$

$$H_{INT} = \sum_{l,r} \Delta T\,(c_1^\dagger c_1 - c_2^\dagger c_2)(a_r^\dagger a_l + a_l^\dagger a_r)$$

$$S_I = 2eI$$

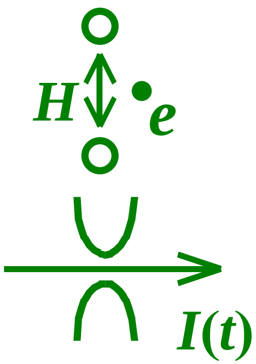
Quantum Bayesian formalism

A.K., 1998

Evolution due to measurement ("spooky" quantum back-action)

1) ρ_{ii} evolve as probabilities, i.e. according to the Bayes rule
(for $\psi=\alpha|1\rangle+\beta|2\rangle$, $|\alpha(t)|^2$ and $|\beta(t)|^2$ behave as probabilities)

2) $\rho_{ij}/(\rho_{ii}\,\rho_{jj})^{1/2}=$ const, i.e. pure state remains pure
(for $\psi=\alpha|1\rangle+\beta|2\rangle$, the phases of $\alpha(t)$ and $\beta(t)$ do not change)



Bayes rule (1763, 1812):

$$P(A_i\,|\,R)=\frac{P(A_i)\,P(R\,|\,A_i)}{\sum_k P(A_k)\,P(R\,|\,A_k)}$$

Add physical (realistic) evolution

- Hamiltonian evolution, classical back-action, decoherence, etc.
  (technically: add terms in the differential equation)

Same idea as in POVM, general quant. meas., quantum trajectories, etc.

Even more general formalism

POVM, general quantum measurement, etc. (known since 1960s)

Nielsen and Chuang, “Quantum information and quantum computation”, p. 85

Measurement with a result *r* is characterized by a linear operator M_r:

$$|\psi\rangle \rightarrow \frac{M_r\,|\psi\rangle}{\sqrt{\langle\psi|\,M_r^\dagger M_r\,|\psi\rangle}}$$

Probability: $P_r = \langle\psi|\,M_r^\dagger M_r\,|\psi\rangle$

Completeness: $\sum_r M_r^\dagger M_r = 1$

Textbook collapse: when M_r is a projection operator

POVM collapse is equivalent to a projective collapse in a larger Hilbert space (including detector)

Measurement vs. decoherence

Widely accepted point of view:

measurement = decoherence (environment)

Is it true?

- **Yes**, if not interested in information from detector (ensemble-averaged evolution)

- **No**, if take into account measurement result (single quantum system)

Measurement result obviously gives us more information about the measured system, so we know its quantum state better (ideally, a pure state instead of a mixed state)

Undoing a weak measurement of a qubit (quantum uncollapsing)

A.K. & Jordan, PRL-2006

It is impossible to undo "orthodox" quantum measurement (for an <u>unknown</u> initial state)

Is it possible to undo partial quantum measurement?

Yes! (but with a finite probability)

If undoing is successful, an unknown state is **fully** restored

ψ_0 (unknown) → **weak (partial) measurement** → **ψ_1 (partially collapsed)**

successful → **ψ_0 (still unknown)**

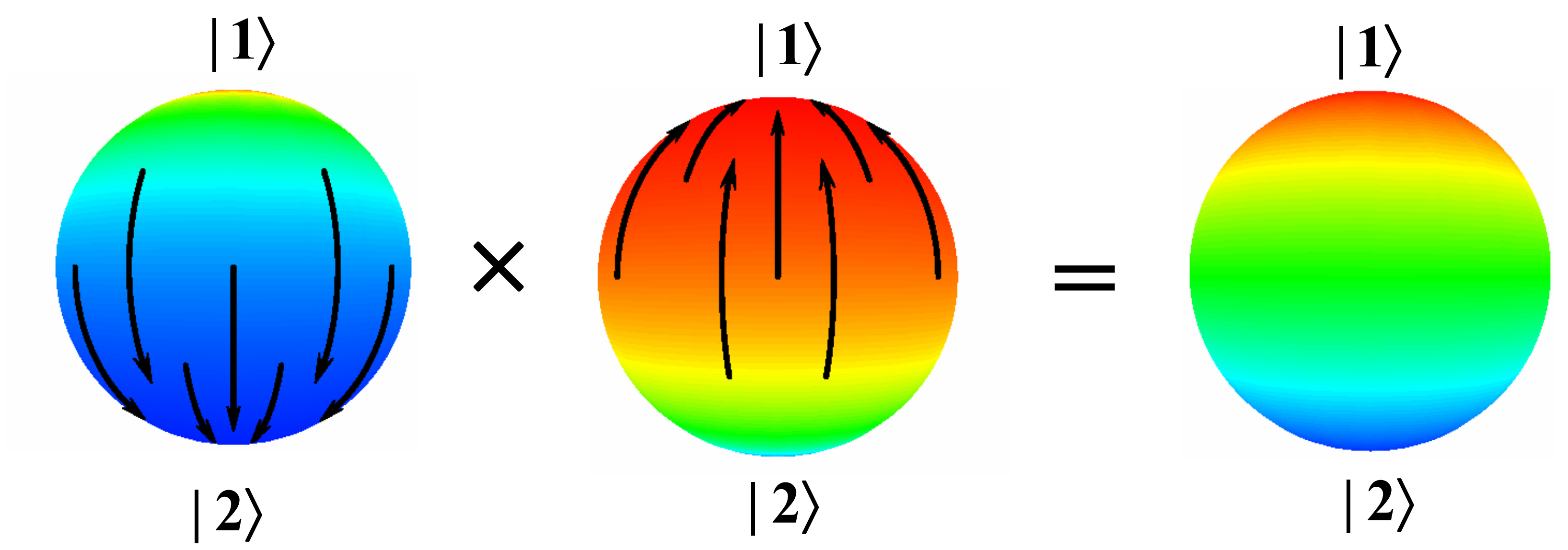
unsuccessful → **ψ_2**

undoing (information erasure)

Uncollapsing of a qubit state

Evolution due to partial (weak, continuous, etc.) measurement is **non-unitary** (though coherent if detector is good!), therefore it is impossible to undo it by Hamiltonian dynamics.

How to undo? One more measurement!



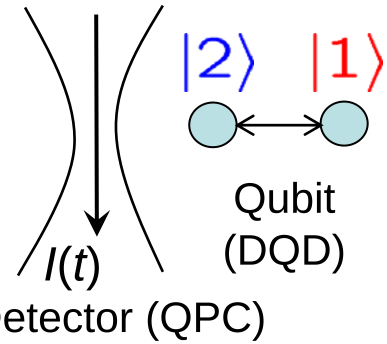
(similar to Koashi-Ueda, PRL-1999)

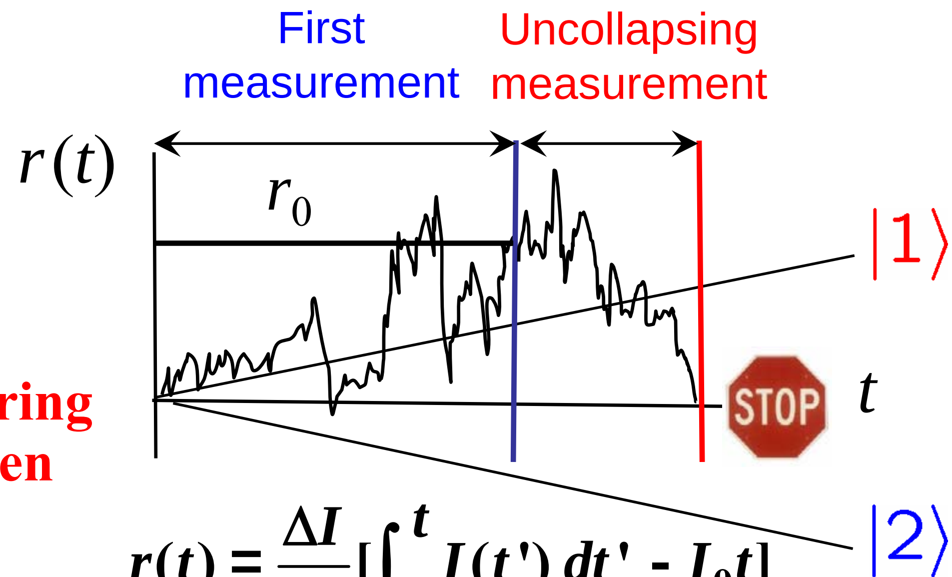
(Figure partially adopted from Jordan-A.K.-Büttiker, PRL-06)

Uncollapsing for DQD-QPC system

A.K. & Jordan

$$\psi = \alpha|1\rangle + \beta|2\rangle$$





Simple strategy: continue measuring until result $r(t)$ becomes zero. Then any initial state is fully restored.

(same for an entangled qubit)

$$r(t) = \frac{\Delta I}{S_I}\left[\int_0^t I(t')\,dt' - I_0 t\right]$$

It may happen though that $r = 0$ never crossed; then undoing procedure is unsuccessful.

Probability of success:

$$P_S = \frac{e^{-|r_0|}}{e^{|r_0|}|\alpha_{\rm in}|^2 + e^{-|r_0|}|\beta_{\rm in}|^2}$$

Averaged probability of success (over result r_0):

$$P_{\rm av} = 1 - \mathrm{erf}[\sqrt{t/2T_m}\,], \qquad T_m = 2S_I/(\Delta I)^2$$

(does not depend on initial state)

General theory of uncollapsing

Uncollapsing operator: $C \times M_r^{-1}$ (to satisfy completeness, eigenvalues cannot be >1)

$$\max(C) = \min_i \sqrt{p_i}\,, \quad p_i \text{ – eigenvalues of } M_r^\dagger M_r$$

Probability of success: $P_S \le \dfrac{\min P_r}{P_r(\psi_{\text{in}})}$

$P_r(\psi_{\text{in}})$ – probability of result r for initial state ψ_{in},

min P_r – probability of result r minimized over all possible initial states

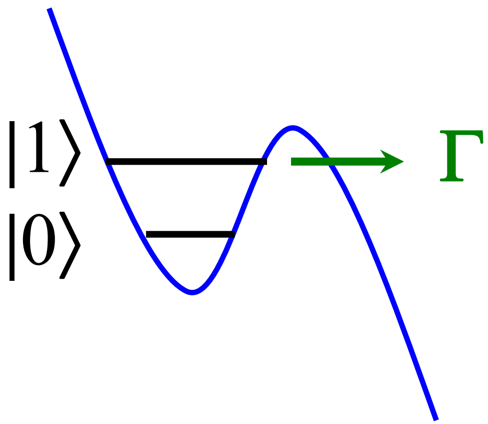
Averaged (over r) probability of success: $P_{av} \le \sum_r \min P_r$

(cannot depend on initial state, otherwise get information)

(similar to Koashi-Ueda, 1999)

Partial collapse of a phase qubit

N. Katz, M. Ansmann, R. Bialczak, E. Lucero, R. McDermott, M. Neeley, M. Steffen, E. Weig, A. Cleland, J. Martinis, A. Korotkov, Science-06



How does a coherent state evolve in time before tunneling event?

(What happens when nothing happens?)

Main idea:

$$\psi = \alpha|0\rangle + \beta|1\rangle \;\rightarrow\; \psi(t) = \begin{cases} |out\rangle, \text{ if tunneled} \\ \boxed{\dfrac{\alpha|0\rangle + \beta e^{-\Gamma t/2} e^{i\varphi}|1\rangle}{\sqrt{|\alpha|^2 + |\beta|^2 e^{-\Gamma t}}}}, \text{ if not tunneled} \end{cases}$$

(better theory: Leonid Pryadko & A.K., 2007)

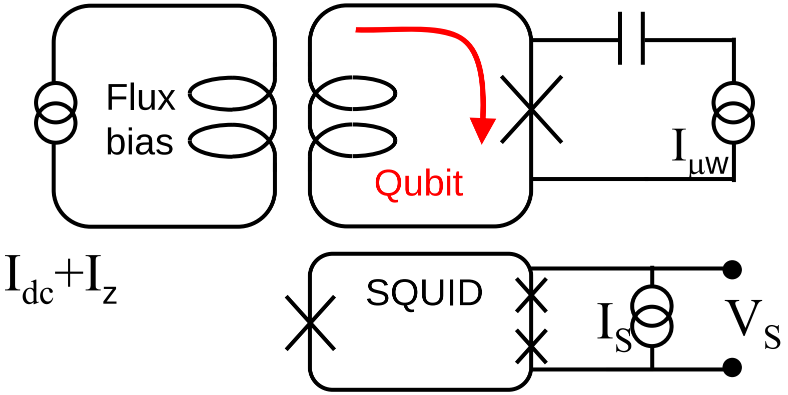
amplitude of state |0⟩ grows without physical interaction

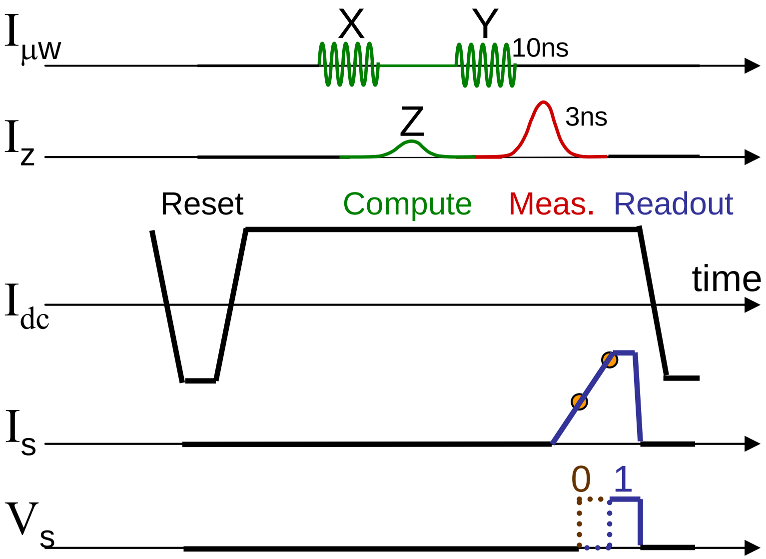
continuous null-result collapse

(similar to optics, Dalibard-Castin-Molmer, PRL-1992)

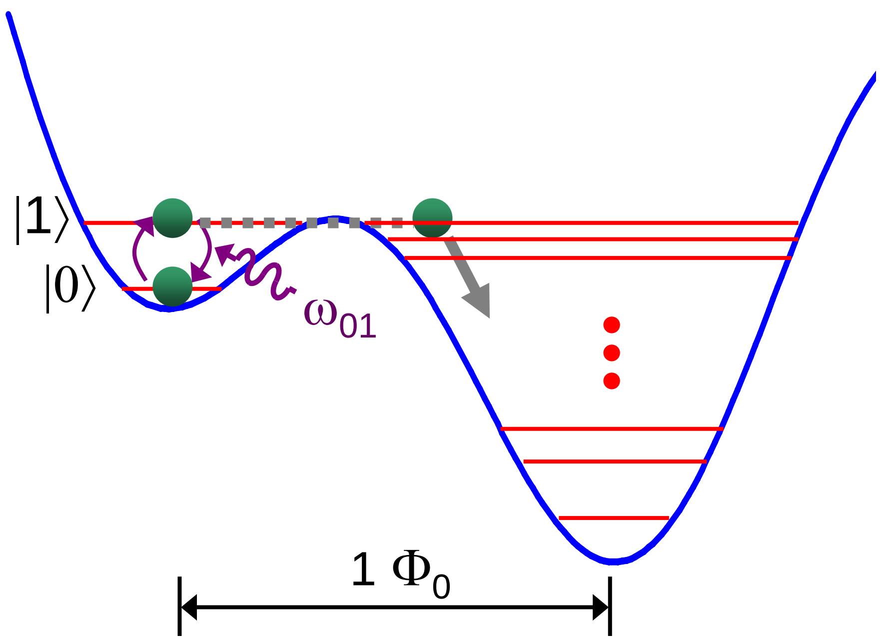
Superconducting phase qubit at UCSB

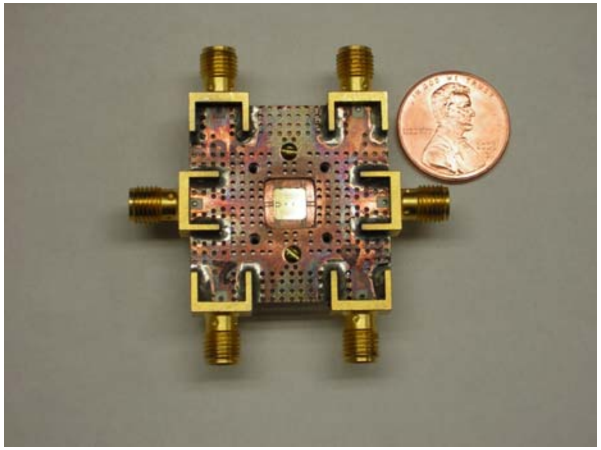
Courtesy of Nadav Katz (UCSB)



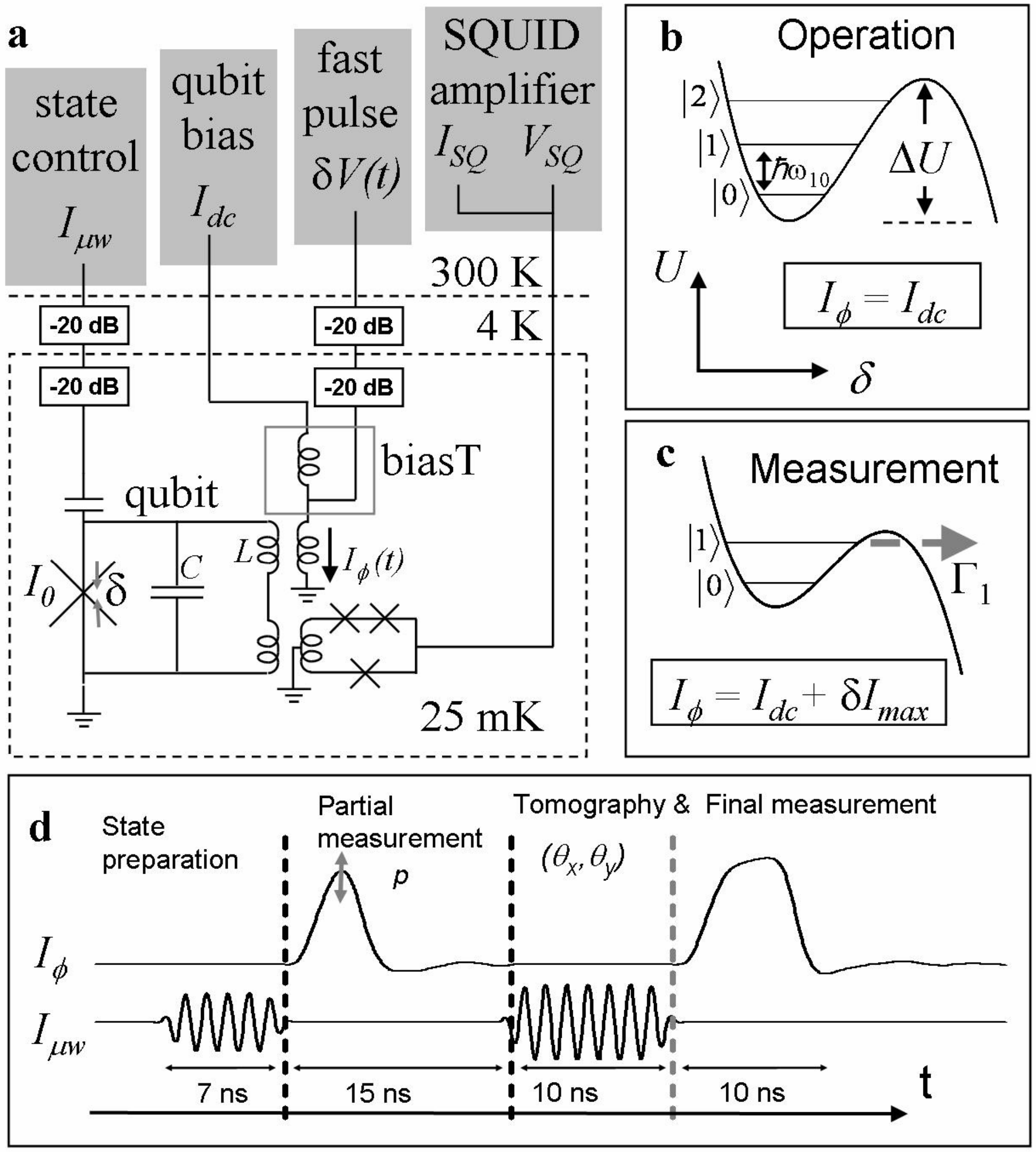


Repeat 1000x
prob. 0,1





Experimental technique for partial collapse



Nadav Katz *et al.*
(John Martinis' group)

Protocol:

1) State preparation by applying microwave pulse (via Rabi oscillations)

2) Partial measurement by lowering barrier for time t

3) State tomography (micro-wave + full measurement)

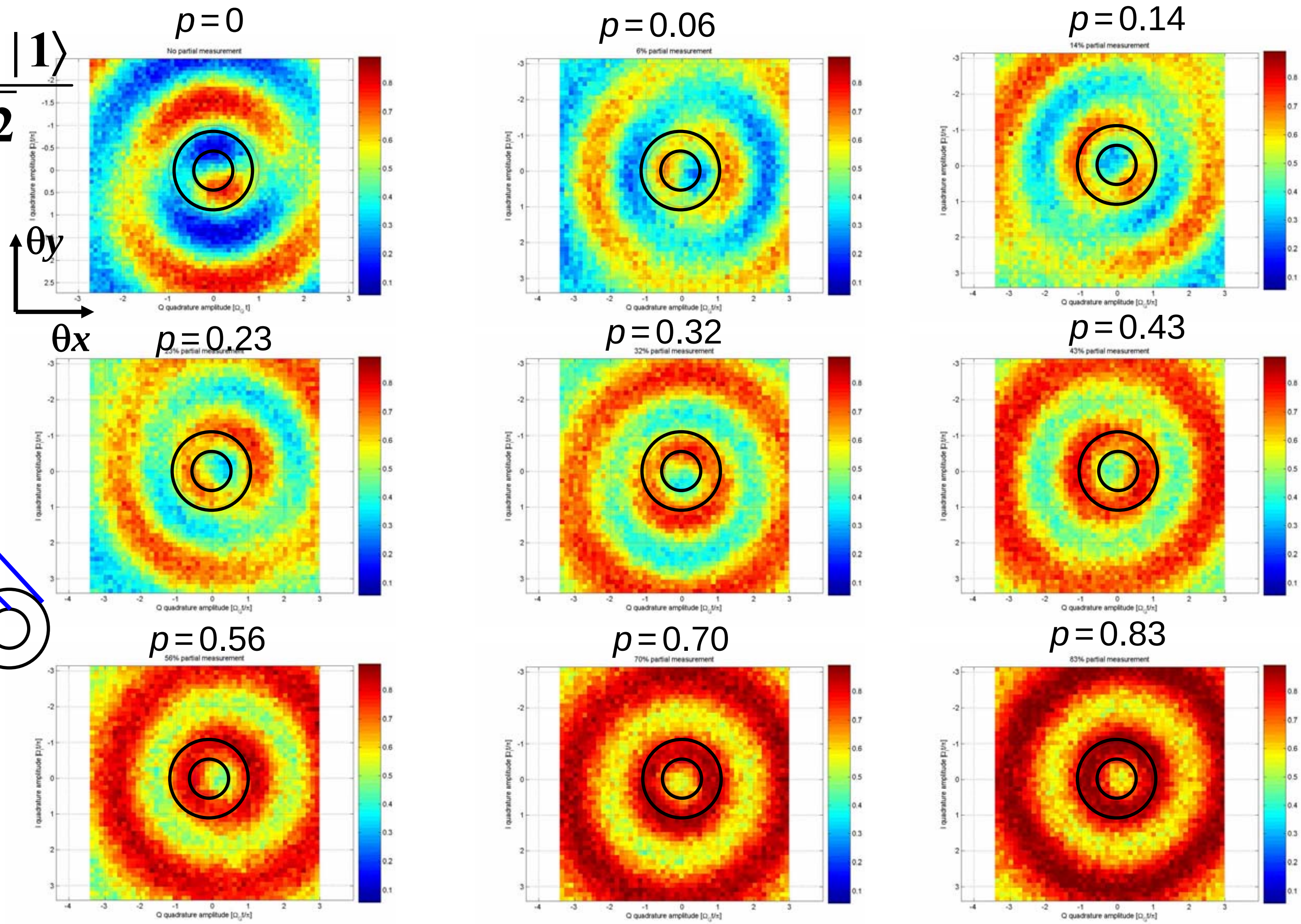
Measurement strength
$p = 1 - \exp(-\Gamma t)$
is actually controlled by Γ, not by t

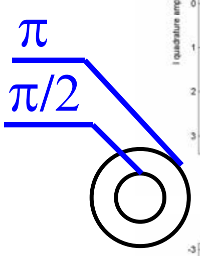
p=0: no measurement
p=1: orthodox collapse

Experimental tomography data

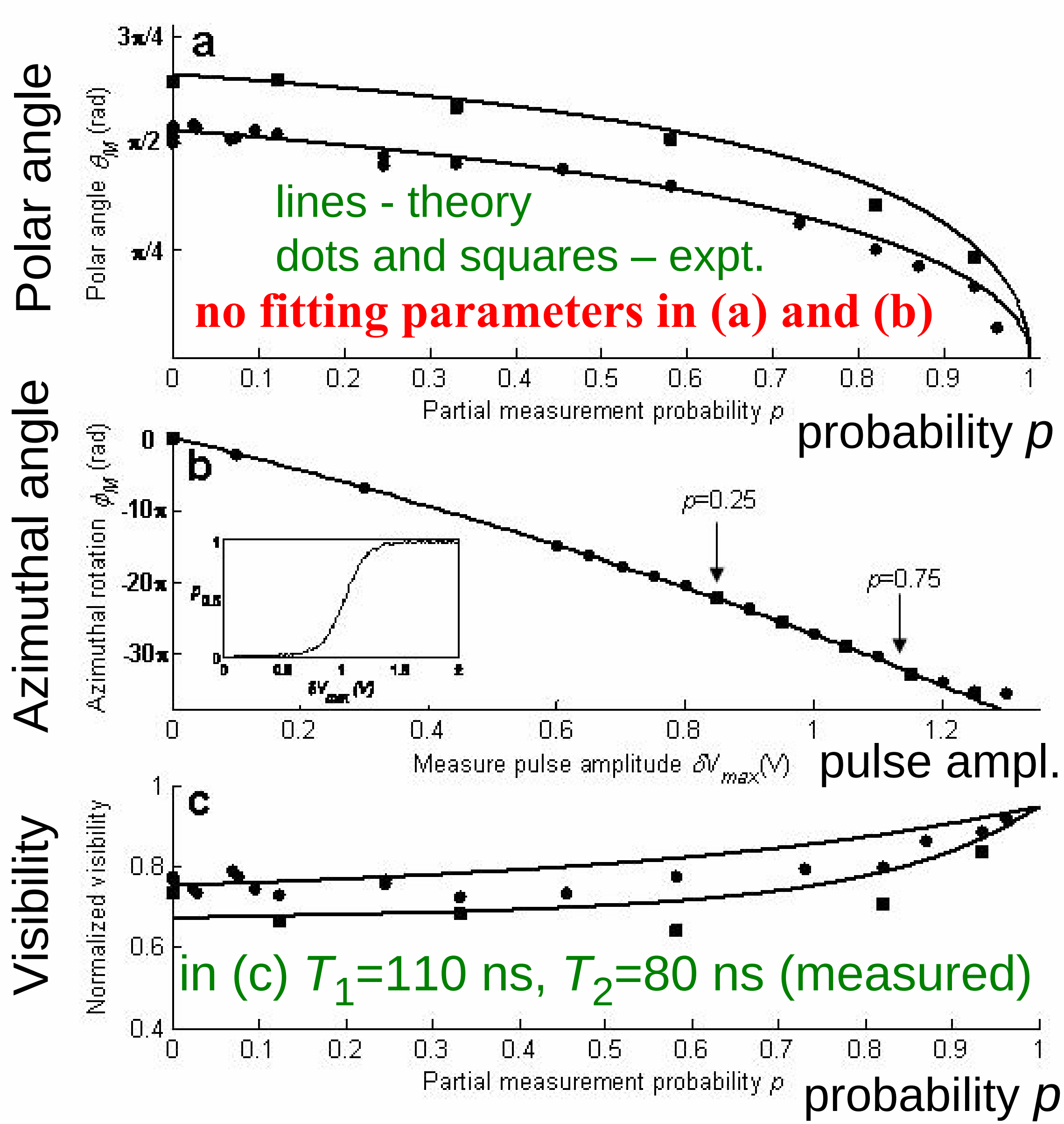
Nadav Katz *et al.* (UCSB)

$$\psi_{in} = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$





Partial collapse: experimental results



N. Katz *et al.*, Science-06

- **In case of no tunneling (null-result measurement) phase qubit evolves**
- **This evolution is well described by a simple Bayesian theory, without fitting parameters**
- **Phase qubit remains fully coherent in the process of continuous collapse (experimentally ~80% raw data, ~96% after account for T1 and T2)**

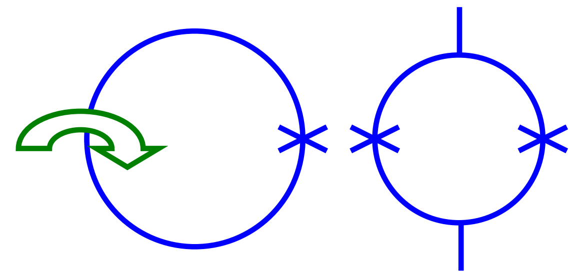
quantum efficiency

$\eta_0 > 0.8$

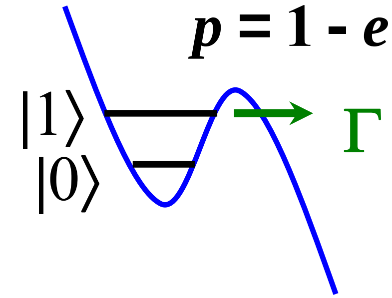
Uncollapsing of a phase qubit state

A.K. & Jordan, 2006

1) Start with an unknown state
2) Partial measurement of strength *p*
3) π-pulse (exchange $|0\rangle \leftrightarrow |1\rangle$)
4) One more measurement with the **same strength** *p*
5) π-pulse



$$p = 1 - e^{-\Gamma t}$$



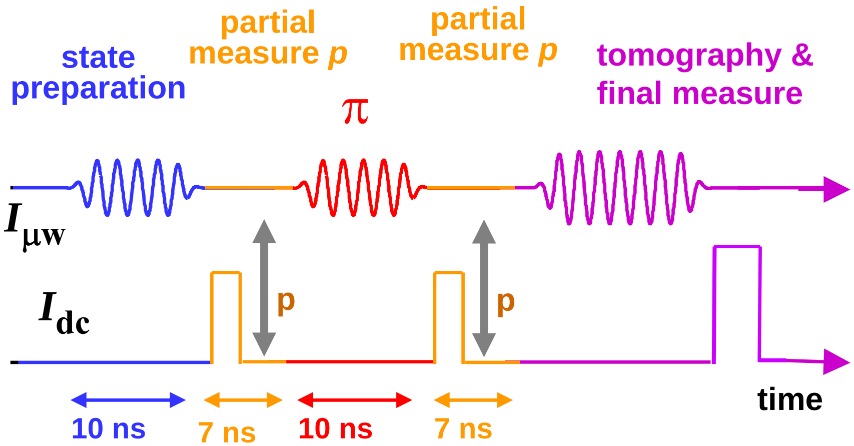
If no tunneling for both measurements, then initial state is fully restored!

$$\alpha|0\rangle + \beta|1\rangle \rightarrow \frac{\alpha|0\rangle + e^{i\phi}\beta e^{-\Gamma t/2}|1\rangle}{\text{Norm}} \rightarrow$$

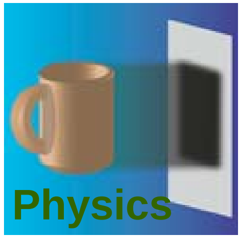
$$\frac{e^{i\phi}\alpha e^{-\Gamma t/2}|0\rangle + e^{i\phi}\beta e^{-\Gamma t/2}|1\rangle}{\text{Norm}} = e^{i\phi}(\alpha|0\rangle + \beta|1\rangle)$$

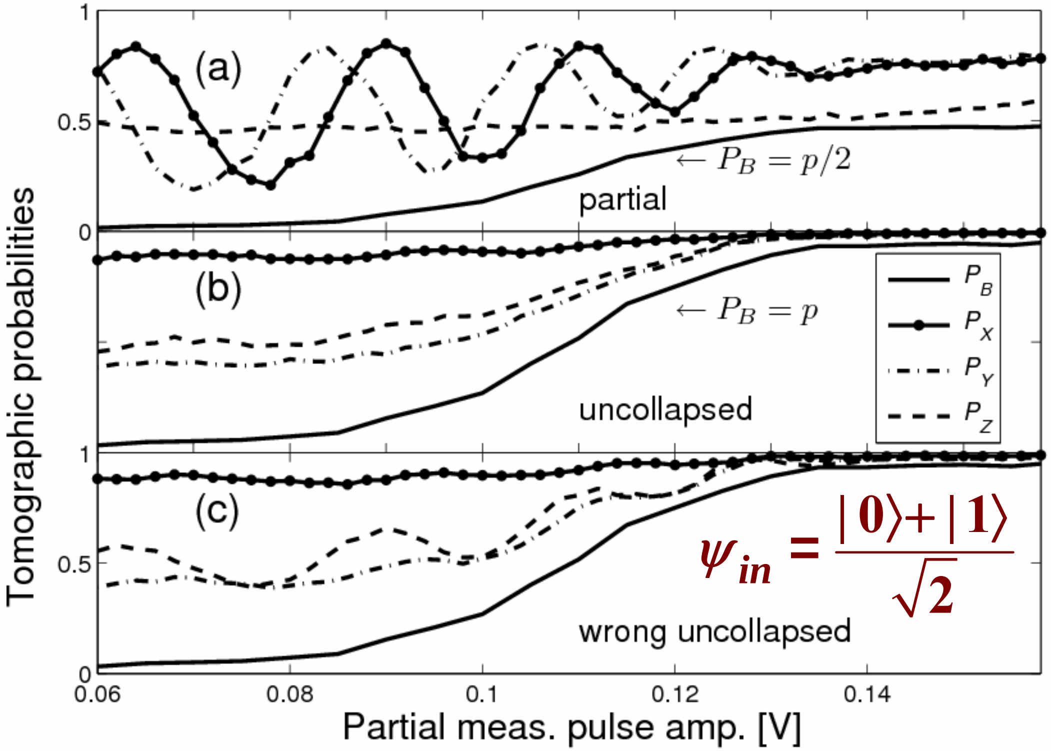
phase is also restored (spin echo)

Experiment on wavefunction uncollapsing



N. Katz, M. Neeley, M. Ansmann, R. Bialzak, E. Lucero, A. O'Connell, H. Wang, A. Cleland, J. Martinis, and A. Korotkov, PRL-2008





Uncollapse protocol:

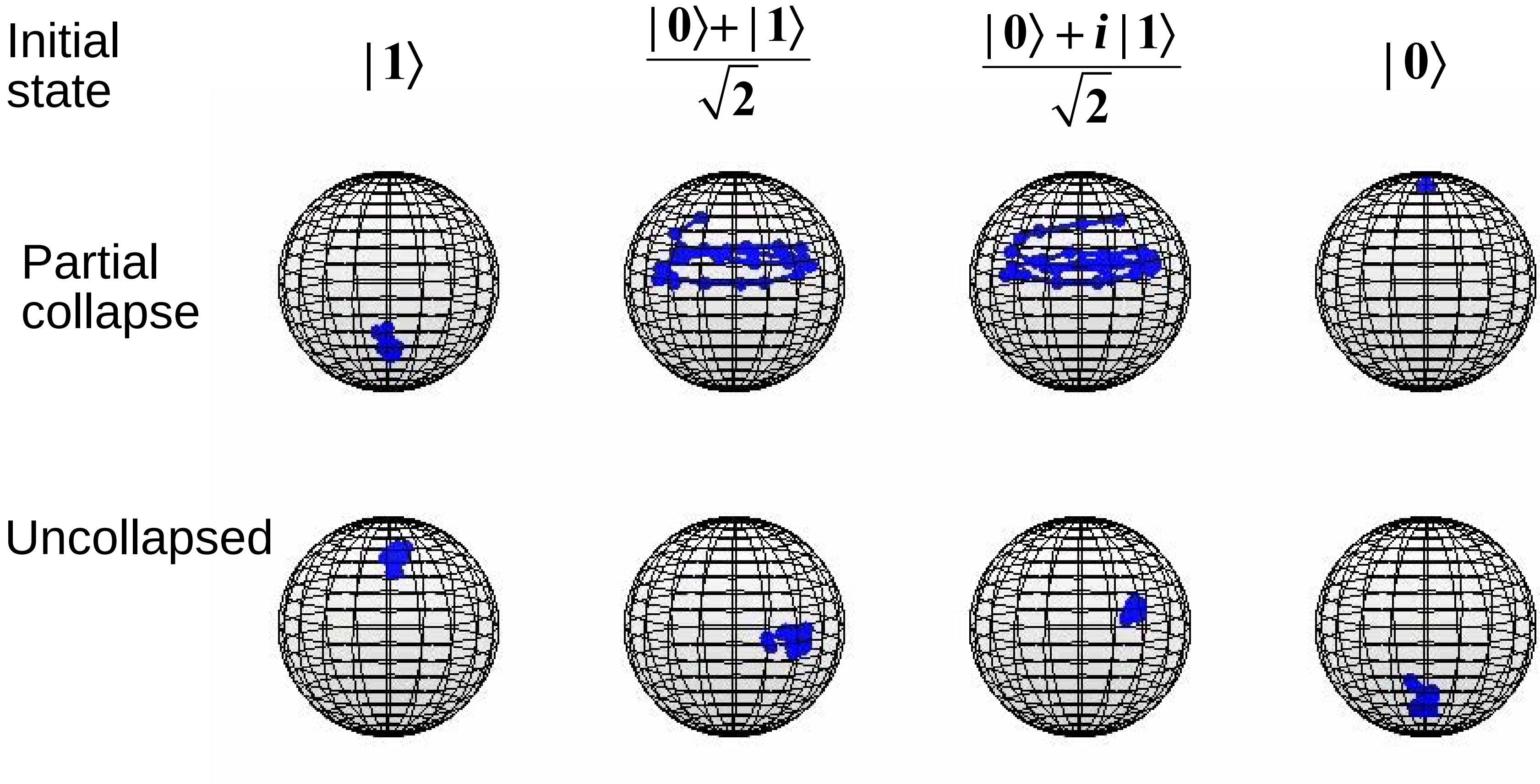
- partial collapse
- π-pulse
- partial collapse (same strength)

State tomography with *X*, *Y*, and no pulses

Background P_B should be subtracted to find qubit density matrix

Experimental results on Bloch sphere

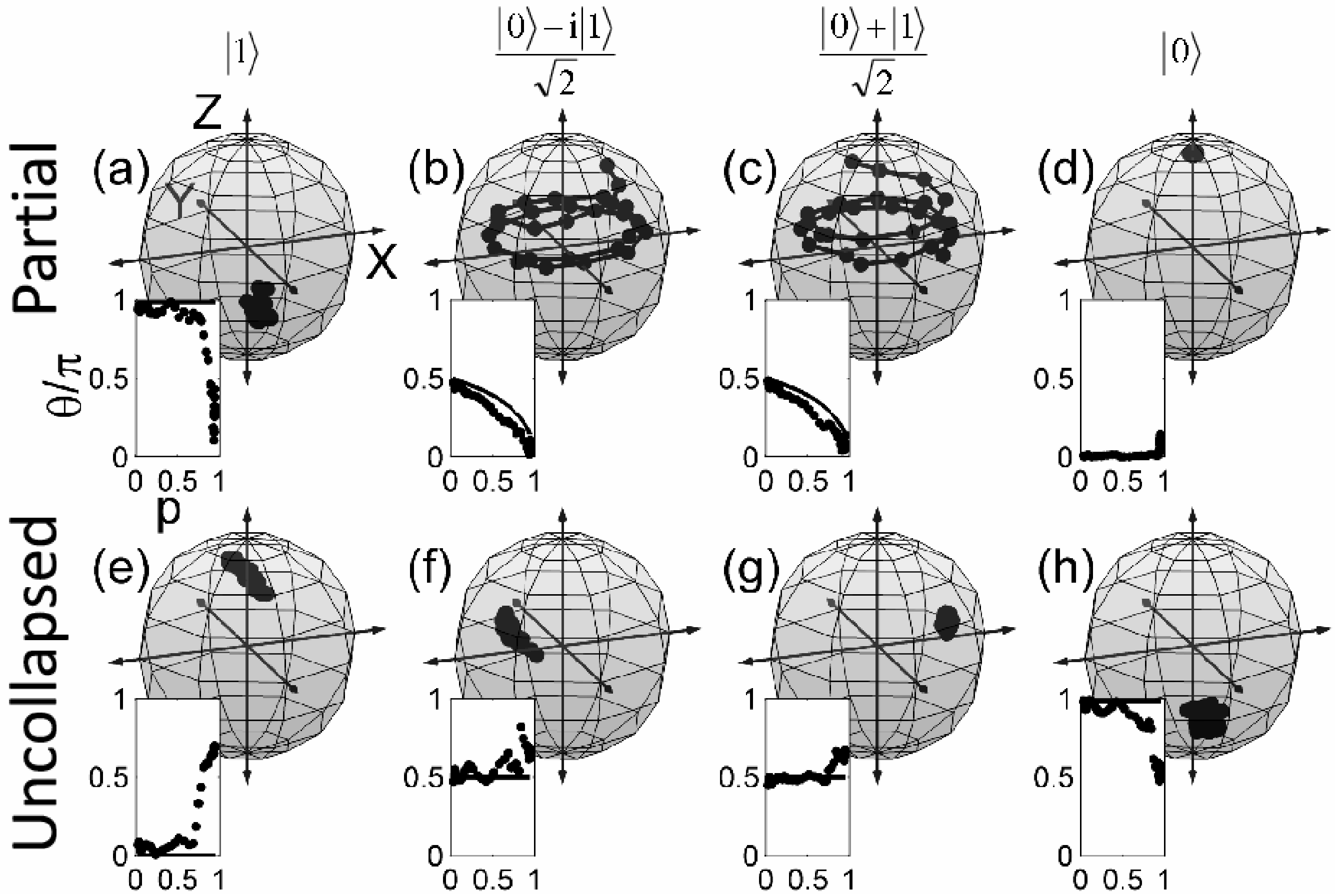
N. Katz et al.



Collapse strength: $\boxed{0.05 < p < 0.7}$

uncollapsing works well!

Same with polar angle dependence
(another experimental run)

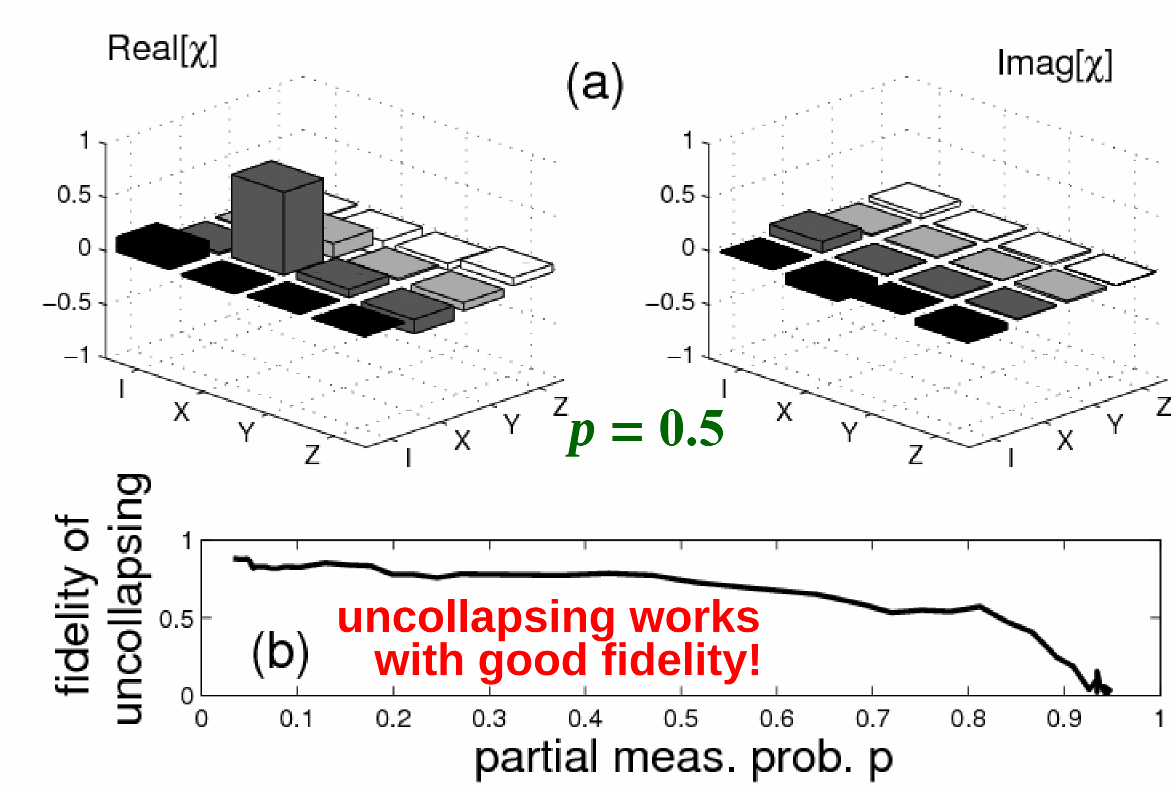


Both spin echo (azimuth) and uncollapsing (polar angle)

Difference: spin echo – undoing of an unknown unitary evolution, uncollapsing – undoing of a known, but non-unitary evolution

Quantum process tomography

N. Katz et al.
(Martinis group)



Why getting worse at p>0.6?

Energy relaxation $\boldsymbol{p_r = t/T_1 = 45\text{ns}/450\text{ns} = 0.1}$

Selection affected when $\boldsymbol{1-p \sim p_r}$

Overall: uncollapsing is well-confirmed experimentally

Conclusions

- **Quantum measurement is not as simple as in a textbook**
- **In many cases quantum collapse happens gradually (possible to describe *how* but impossible to understand *why*)**
- **A partial collapse can be reversed (uncollapsing), though with a probability less than 100%**
- **Partial collapse and uncollapsing have been recently demonstrated experimentally**