Non-projective measurement of solid-state qubits
(what is "inside" collapse)

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Outline:
- Bayesian formalism for quantum measurement
- Persistent Rabi oscillations (+expt.)
- Wavefunction uncollapse (+expts.)
- New experimental proposals
  - decoherence suppression by uncollapsing
  - persistent Rabi oscillations revealed via noise

Ackn.:

Theory: R. Ruskov, A. Jordan, K. Keane
Expt.: UCSB (J. Martinis, N. Katz et al.),
Saclay (D. Esteve, P. Bertet et al.)

Funding:

Quantum mechanics =

Schrödinger equation + measurement postulate

1) Probability of measurement result r: $p_r = |\langle \psi | \psi_r \rangle|^2$

where $\hat{A}|\psi_r\rangle = r|\psi_r\rangle$

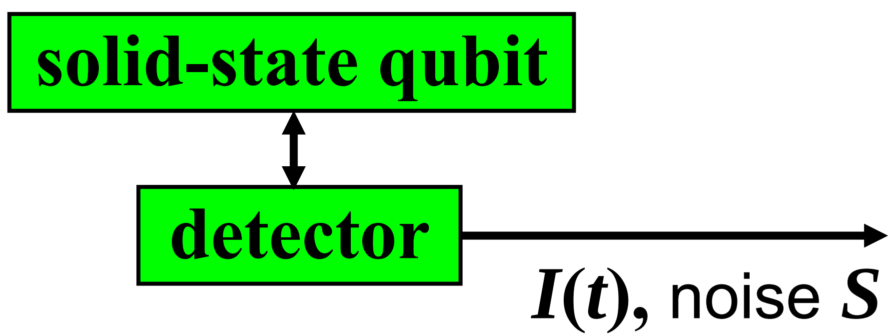
2) Wavefunction after measurement $= |\psi_r\rangle$ (collapse)

Instantaneous collapse is surely an approximation (though often OK in optics, also the main point in Bell's ineq.), especially obvious for solid-state systems

What is the evolution due to measurement?
(What is "inside" collapse?)

(controversial for last 80 years, many wrong answers, many correct answers)

Our limited scope:
(simplest system, experimental setups)



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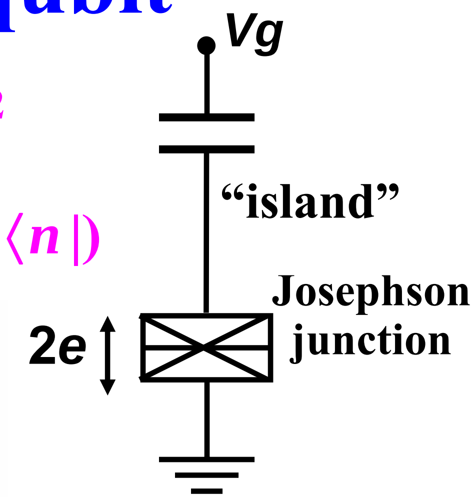
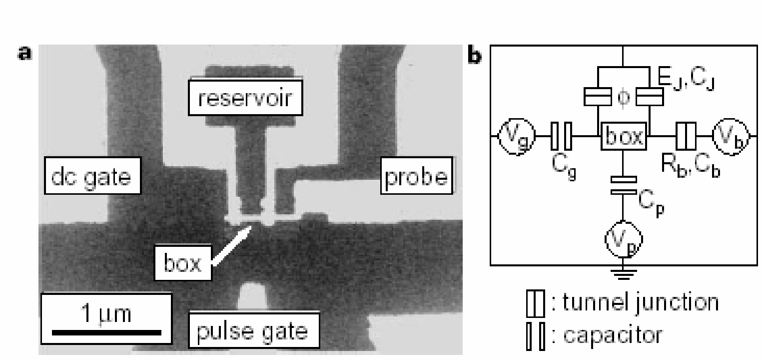
Superconducting "charge" qubit

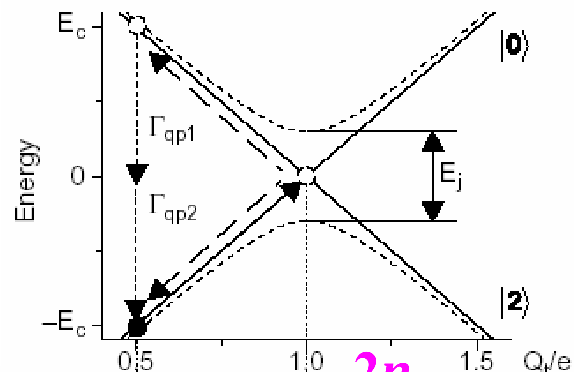
Y. Nakamura, Yu. Pashkin, and J.S. Tsai (Nature, 1998)

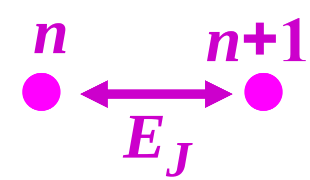
$$\hat{H} = \frac{(2e)^2}{2C}(\hat{n} - n_g)^2 - \frac{E_J}{2}(|n\rangle\langle n+1| + |n+1\rangle\langle n|)$$



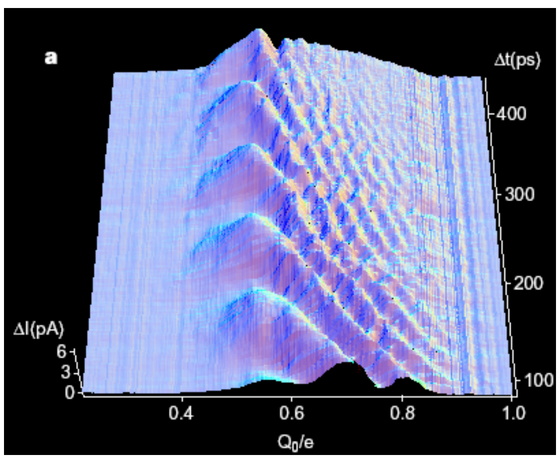
Single Cooper pair box

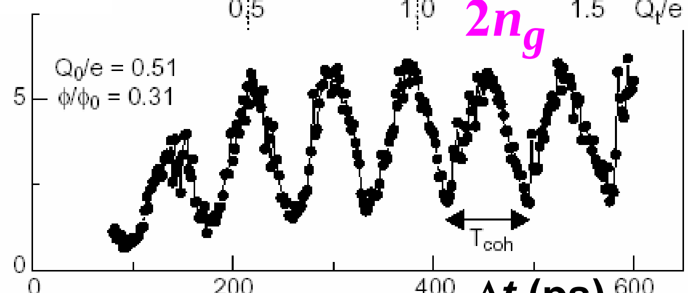




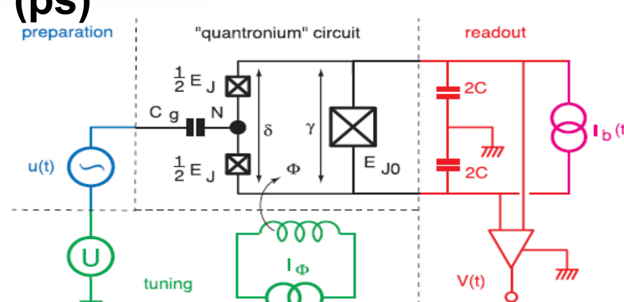


***n*: number of Cooper pairs on the island**





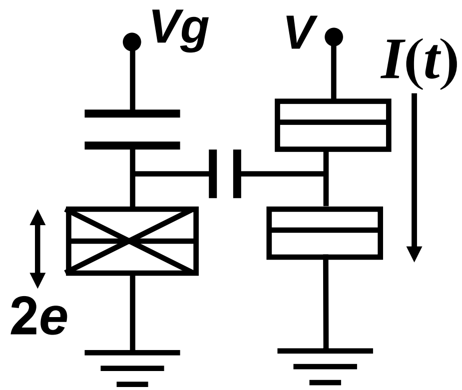
Quantum coherent (Rabi) oscillations



Vion et al. (Saclay group); Science, 2002

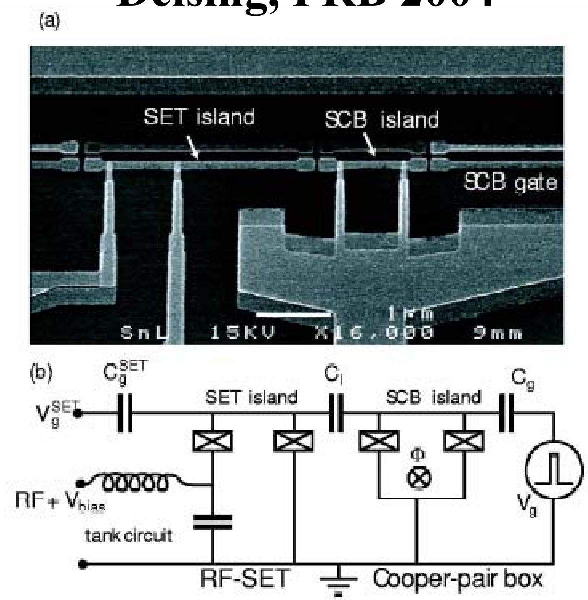
Q-factor of coherent (Rabi) oscillations = 25,000 ("quantronium")

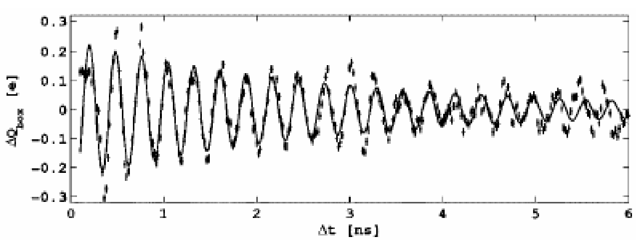
Charge qubits with SET readout



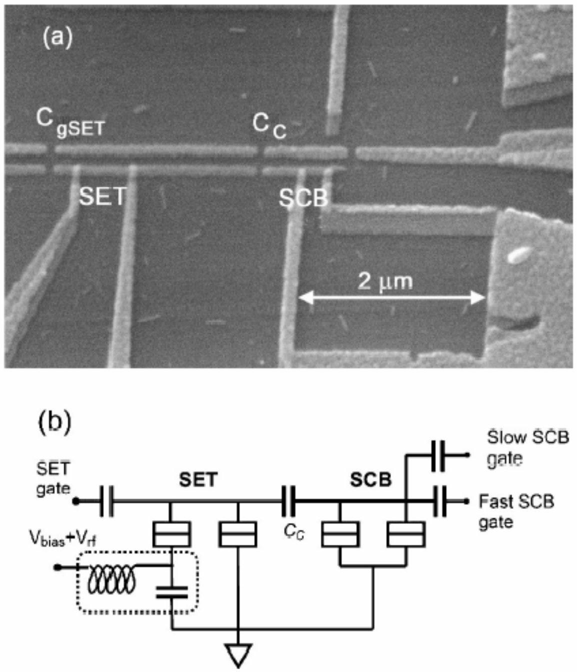
Cooper-pair box measured by single-electron transistor (rf-SET)

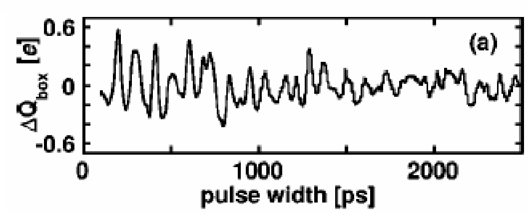
Duty, Gunnarsson, Bladh, Delsing, PRB 2004





Guillaume et al. (Echternach's group), PRB 2004





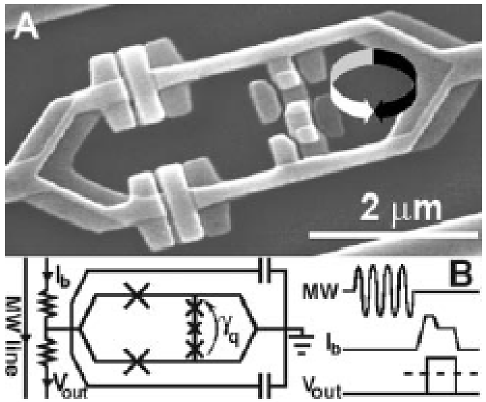
Setup can be used for continuous measurements

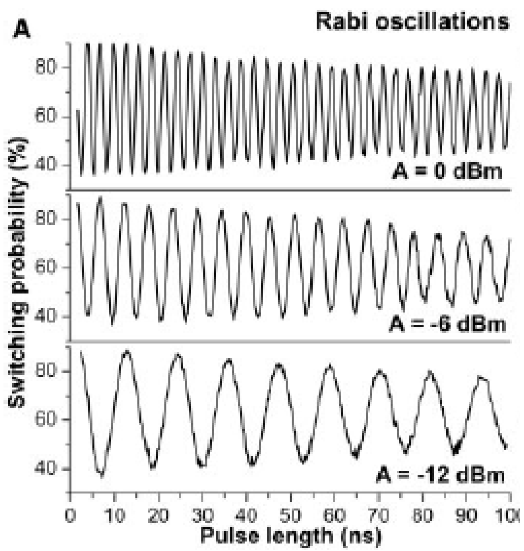
All results are averaged over many measurements (not "single-shot")

Some other superconducting qubits

Flux qubit

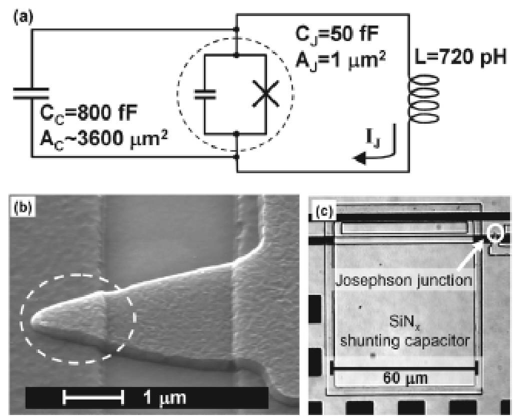
Mooij et al. (Delft)

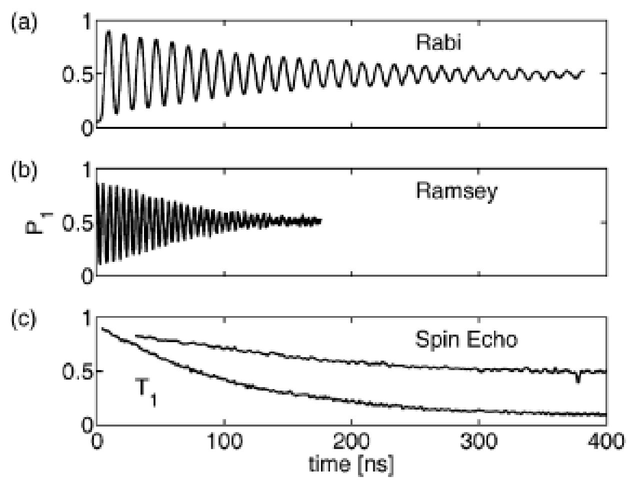




Phase qubit

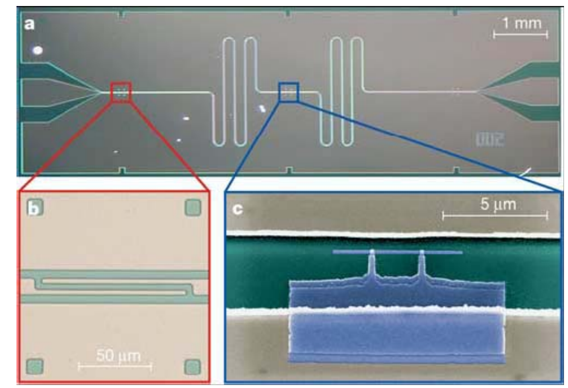
J. Martinis et al.
(UCSB and NIST)

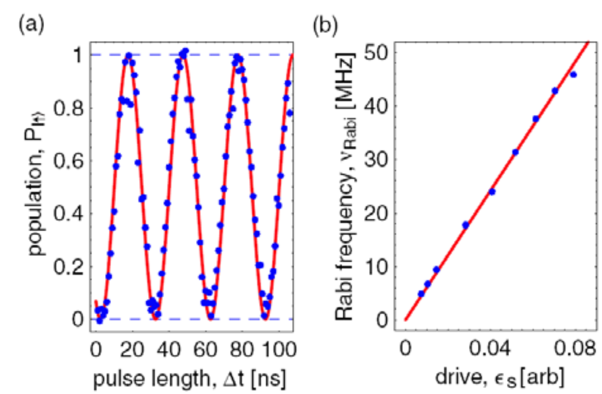




Charge qubit with circuit QED

R. Schoelkopf et al. (Yale)

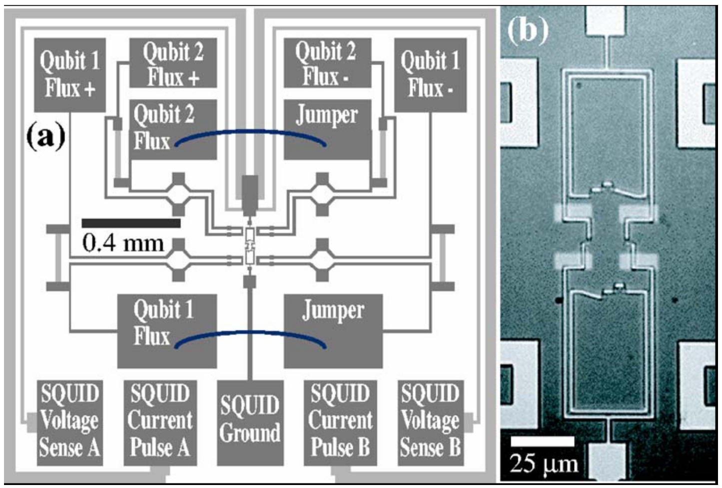


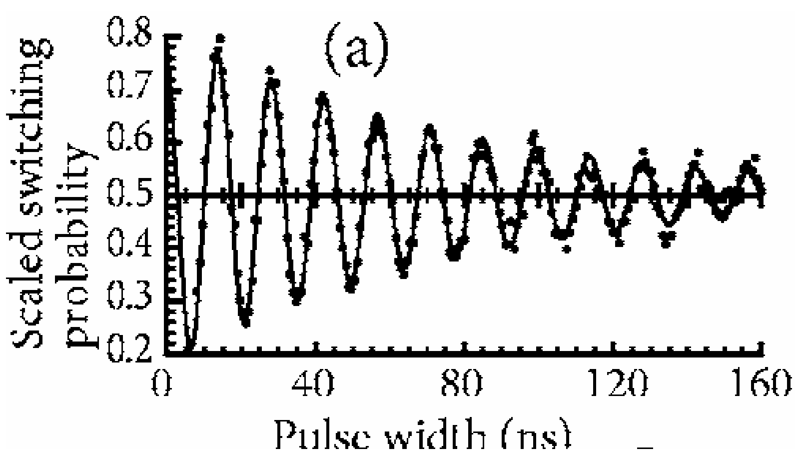


Some other superconducting qubits

Flux qubit

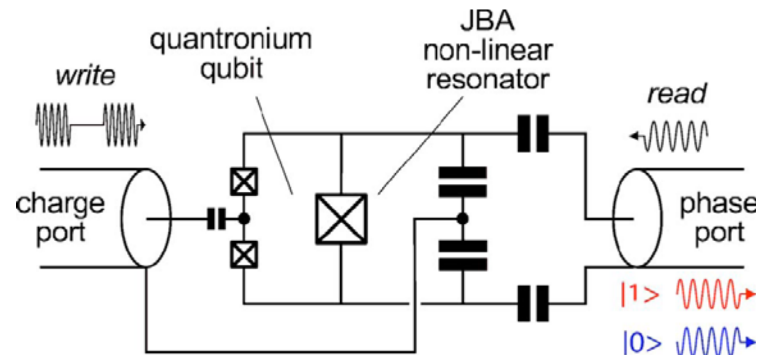
J. Clarke et al. (Berkeley)

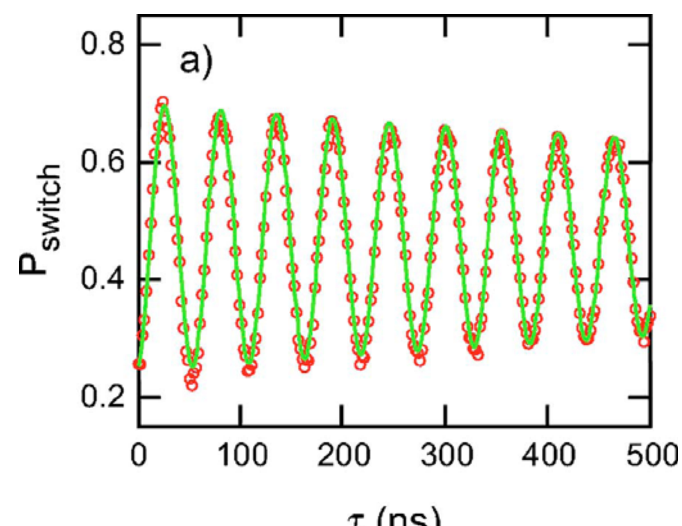




"Quantronium" qubit

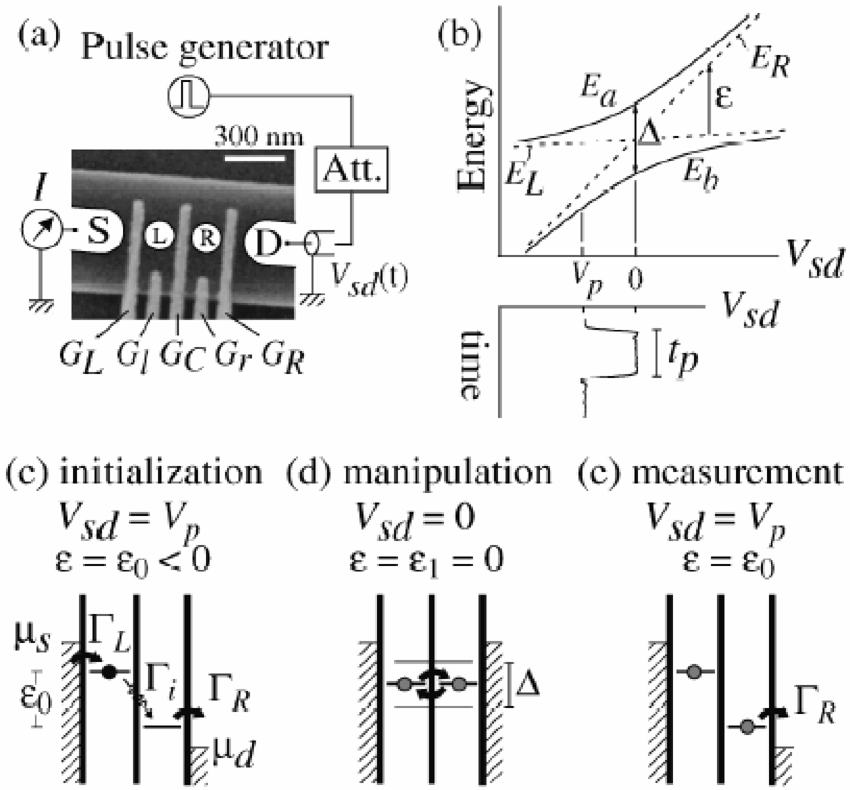
I. Siddiqi, R. Schoelkopf, M. Devoret, et al. (Yale)

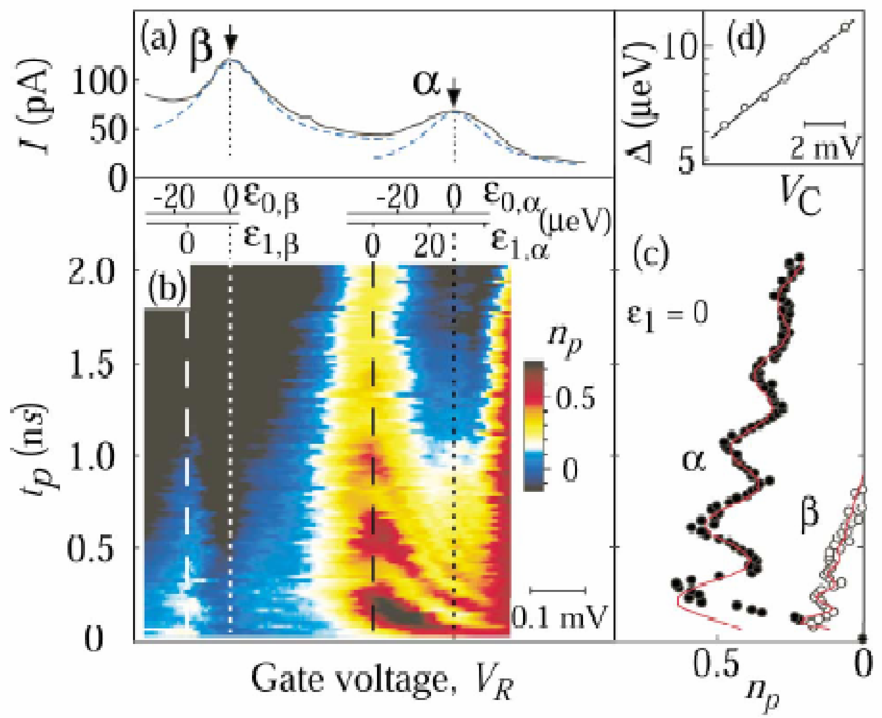




Semiconductor (double-dot) qubit

T. Hayashi et al., PRL 2003





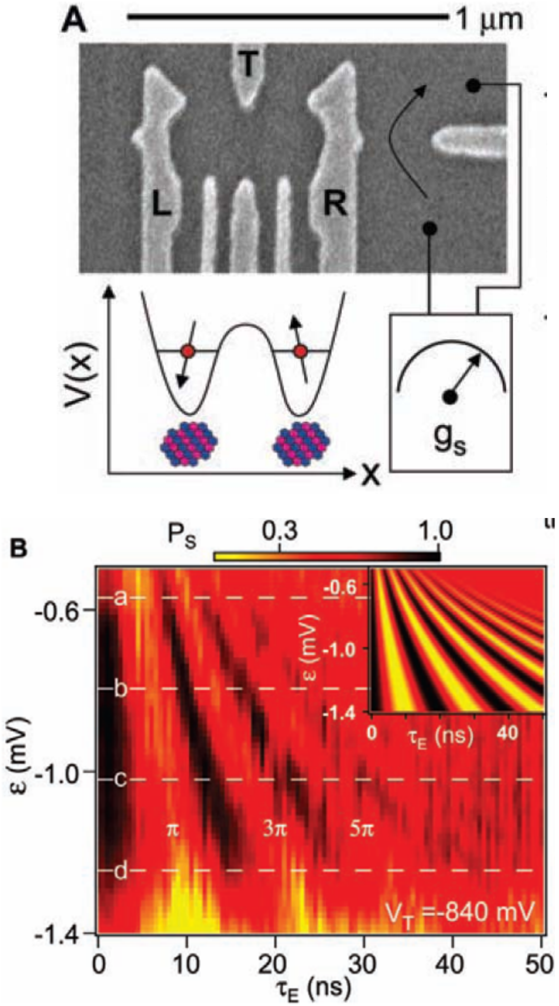
Rabi oscillations

Detector is not separated from qubit,
also possible to use a separate detector

Some other semiconductor qubits

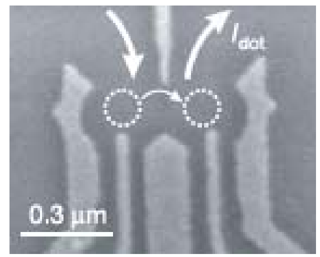
Spin qubit (QPC meas.)

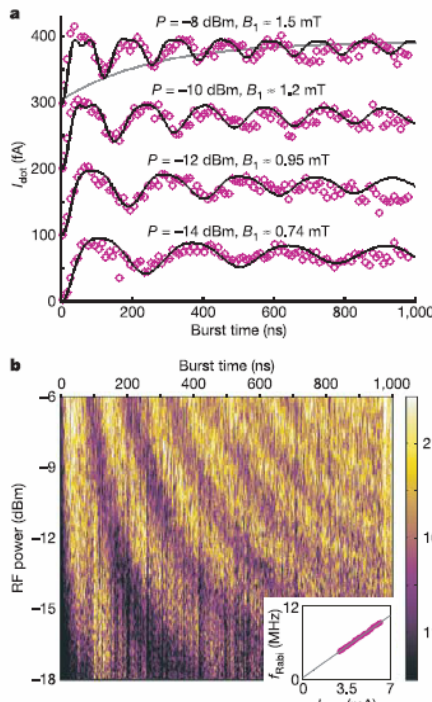
C. Marcus et al. (Harvard)



Spin qubit

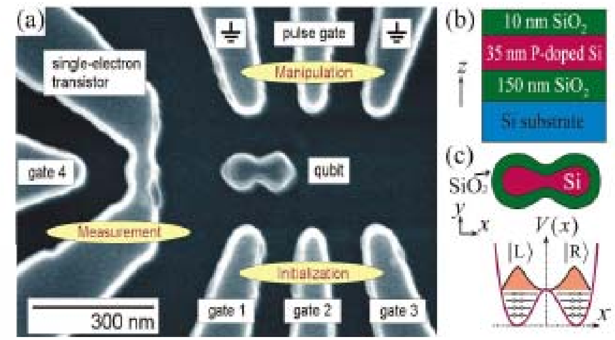
L. Kouwenhoven et al. (Delft)

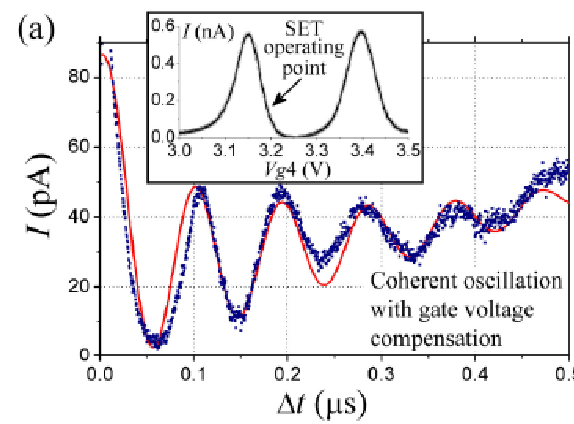




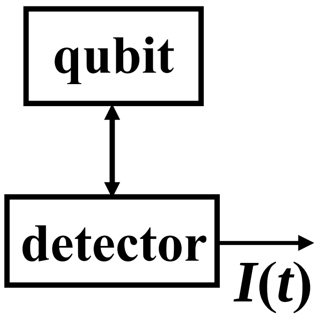
Double-dot qubit

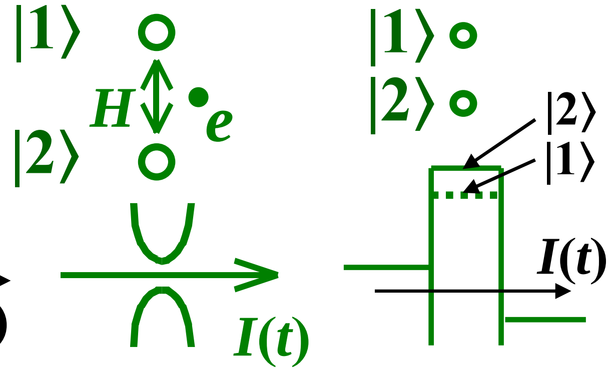
Gorman, Hasko, Williams (Cambridge)



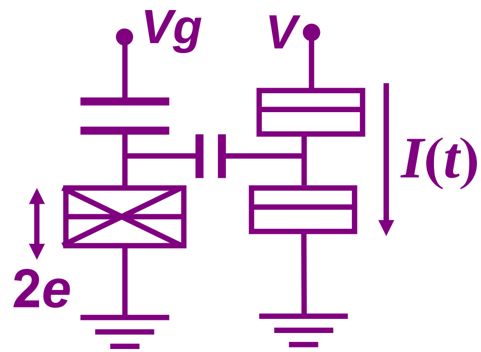


The system we consider: qubit + detector

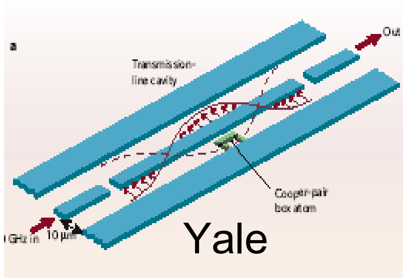




Double-quantum-qot and quantum point contact



Cooper-pair box and single-electron transistor



Charge qubit with circuit QED readout

$H = H_{QB} + H_{DET} + H_{INT}$

$H_{QB} = (\varepsilon/2)(c_1^+c_1 - c_2^+c_2) + H(c_1^+c_2 + c_2^+c_1)$ **ε – asymmetry, H – tunneling**

$\Omega = (4H^2 + \varepsilon^2)^{1/2}/\hbar$ – frequency of quantum coherent (Rabi) oscillations

Two levels of average detector current: I_1 for qubit state $|1\rangle$, I_2 for $|2\rangle$

Response: $\Delta I = I_1 - I_2$ Detector noise: white, spectral density S_I

DQD and QPC
(setup due to Gurvitz, 1997)

$$H_{DET} = \sum_l E_l a_l^\dagger a_l + \sum_r E_r a_r^\dagger a_r + \sum_{l,r} T(a_r^\dagger a_l + a_l^\dagger a_r)$$

$$H_{INT} = \sum_{l,r} \Delta T\,(c_1^\dagger c_1 - c_2^\dagger c_2)(a_r^\dagger a_l + a_l^\dagger a_r)$$

$$S_I = 2eI$$

What happens to a qubit state during measurement?

Start with density matrix **evolution due to measurement only** ($H=\varepsilon=0$)

"Orthodox" answer

$$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \begin{matrix} \nearrow \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \\ \searrow \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \end{matrix}$$

|1> or |2>, depending on the result

"Decoherence" answer

$$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \to \begin{pmatrix} \frac{1}{2} & \frac{\exp(-\Gamma t)}{2} \\ \frac{\exp(-\Gamma t)}{2} & \frac{1}{2} \end{pmatrix} \to \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

no measurement result! (ensemble averaged)

Decoherence has nothing to do with collapse!

applicable for:	single quant. system	continuous meas.
Orthodox	yes	no
Decoherence (ensemble)	no	yes
Bayesian, POVM, quant. traject., etc.	yes	yes

Bayesian (POVM, quant. traj., etc.) formalism describes gradual collapse of a single quantum system, **taking into account measurement result**

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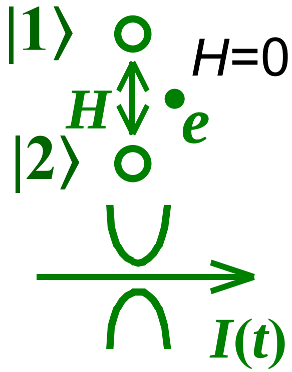
Bayesian formalism for DQD-QPC system



Qubit evolution due to measurement (quantum back-action):

$$\psi(t) = \alpha(t)\,|1\rangle + \beta(t)\,|2\rangle \quad \text{or} \quad \rho_{ij}(t)$$

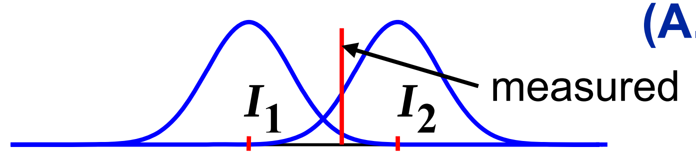
1) $|\alpha(t)|^2$ and $|\beta(t)|^2$ evolve as probabilities,
   i.e. according to the **Bayes rule** (same for ρ_{ii})

2) phases of $\alpha(t)$ and $\beta(t)$ do not change
   (**no dephasing!**), $\rho_{ij}/(\rho_{ii}\,\rho_{jj})^{1/2} = \text{const}$

Bayes rule (1763, Laplace-1812):

$$\underbrace{P(A_i\,|\,\text{res})}_{\text{posterior probability}} = \frac{\underbrace{P(A_i)}_{\text{prior probab.}}\;\underbrace{P(\text{res}\,|\,A_i)}_{\text{likelihood}}}{\sum_k P(A_k)\,P(\text{res}\,|\,A_k)}$$

(A.K., 1998)



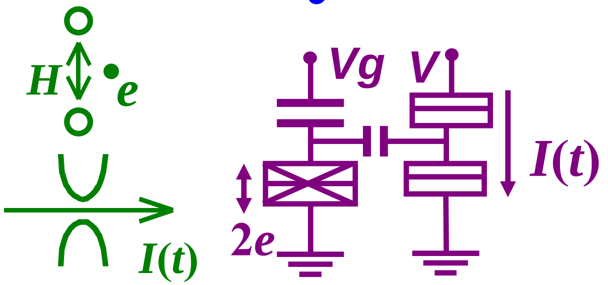
So simple because:

1) QPC happens to be an ideal detector
2) no Hamiltonian evolution of the qubit

Similar formalisms developed earlier. Key words: Imprecise, weak, selective, or conditional measurements, POVM, Quantum trajectories, Quantum jumps, Restricted path integral, etc.

Names: Davies, Kraus, Holevo, Mensky, **Caves**, Gardiner, Carmichael, Plenio, Knight, Walls, Gisin, Percival, Milburn, Wiseman, Habib, etc. (very incomplete list)

Bayesian formalism for a single qubit



- Time derivative of the quantum Bayes rule
- Add unitary evolution of the qubit
- Add decoherence (if any)

$$\dot{\rho}_{11} = -\dot{\rho}_{22} = -2(H/\hbar)\,\mathrm{Im}\,\rho_{12} + \rho_{11}\rho_{22}(2\Delta I/S_I)[\underline{\underline{I(t)}} - I_0]$$

$$\dot{\rho}_{12} = i(\varepsilon/\hbar)\rho_{12} + i(H/\hbar)(\rho_{11}-\rho_{22}) + \rho_{12}(\rho_{11}-\rho_{22})(\Delta I/S_I)[\underline{\underline{I(t)}} - I_0] - \gamma\rho_{12}$$

$$\hat{H}_{QB} = (\varepsilon/2)(c_1^\dagger c_1 - c_2^\dagger c_2) + H(c_1^\dagger c_2 + c_2^\dagger c_1)$$

(A.K., 1998)

$|1\rangle \to I_1$, $|2\rangle \to I_2$, $\Delta I = I_1 - I_2$, $I_0 = (I_1+I_2)/2$, S_I – detector noise

$$\gamma = \Gamma - (\Delta I)^2/4S_I, \quad \Gamma \text{ – ensemble decoherence}$$

Evolution of qubit *wavefunction* can be monitored if $\gamma=0$ (quantum-limited)

Averaging over result $I(t)$ leads to conventional master equation:

$$d\rho_{11}/dt = -d\rho_{22}/dt = -2(H/\hbar)\,\mathrm{Im}\,\rho_{12}$$

$$d\rho_{12}/dt = i(\varepsilon/\hbar)\rho_{12} + i(H/\hbar)(\rho_{11}-\rho_{22}) - \Gamma\rho_{12}$$

Ensemble averaging includes averaging over measurement result!

Assumptions needed for the Bayesian formalism:

- Detector voltage is much larger than the qubit energies involved

$$eV \gg \hbar\Omega,\ eV \gg \hbar\Gamma,\ \ \hbar/eV \ll (1/\Omega,\ 1/\Gamma)$$

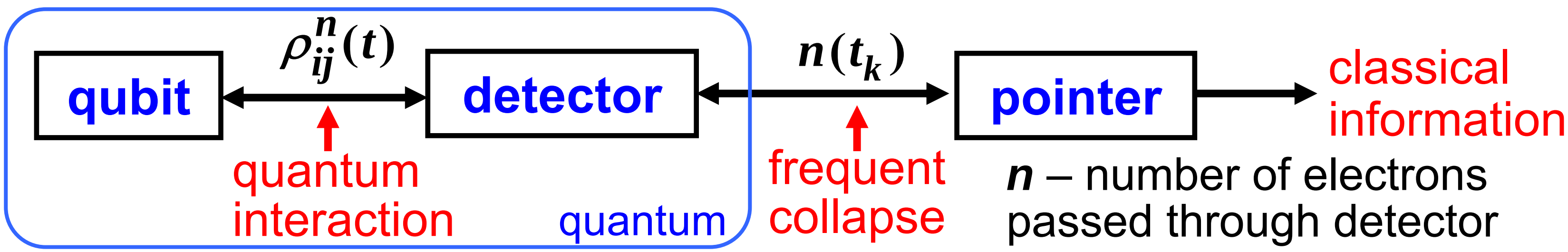
(no coherence in the detector, **classical output**, Markovian approximation)

- Simpler if weak response, $|\Delta I| \ll I_0$, (coupling $C \sim \Gamma/\Omega$ is arbitrary)

Derivations:

1) "logical": via correspondence principle and comparison with decoherence approach (A.K., 1998)

2) "microscopic": Schr. eq. + collapse of the detector (A.K., 2000)

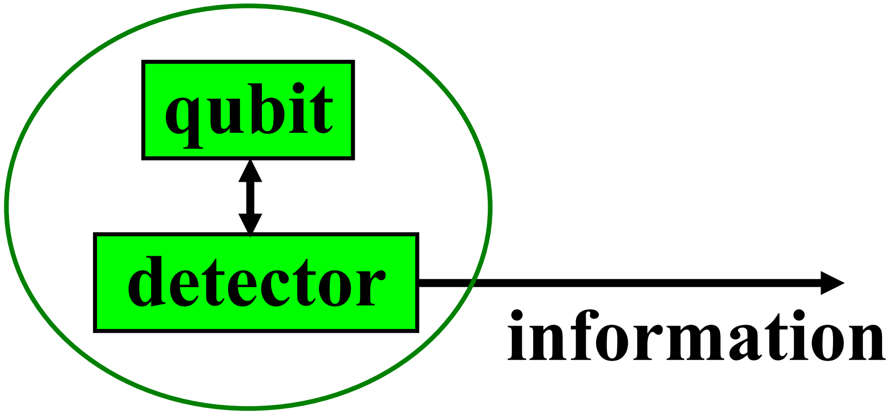


3) from "quantum trajectory" formalism developed for quantum optics (Goan-Milburn, 2001; also: Wiseman, Sun, Oxtoby, etc.)

4) from POVM formalism (Jordan-A.K., 2006)

5) from Keldysh formalism (Wei-Nazarov, 2007)

Why not just use Schrödinger equation for the whole system?



Impossible in principle!

Technical reason: Outgoing information (measurement result) makes it an open system

Philosophical reason: Random measurement result, but deterministic Schrödinger equation

Einstein: God does not play dice

Heisenberg: unavoidable quantum-classical boundary

Fundamental limit for ensemble decoherence

$$\Gamma = (\Delta I)^2/4S_I + \gamma$$

ensemble decoherence rate

~ information flow [bit/s]

single-qubit decoherence

"measurement time" (S/N=1)

$$\tau_m = 2S_I/(\Delta I)^2$$

(Shnirman & Schön, 1998)

$$\Gamma\tau_m \geq \frac{1}{2}$$

$$\gamma \geq 0 \;\Rightarrow\; \boxed{\Gamma \geq (\Delta I)^2/4S_I}$$

A.K., 1998, 2000
S. Pilgram et al., 2002
A. Clerk et al., 2002
D. Averin, 2000,2003

$$\eta = 1 - \frac{\gamma}{\Gamma} = \frac{(\Delta I)^2/4S_I}{\Gamma}$$

detector ideality (quantum efficiency)
$\eta \leq 100\%$

Translated into energy sensitivity: $(\varepsilon_O\, \varepsilon_{BA})^{1/2} \geq \hbar/2$

where ε_O is output-noise-limited sensitivity [J/Hz]
and ε_{BA} is back-action-limited sensitivity [J/Hz]

Danilov, Likharev, Zorin, 1983

$$\eta = \frac{\hbar^2/4}{\varepsilon_O \varepsilon_{BA}} = \eta_{\rm opt}$$

Sensitivity limitation is known since 1980s (Caves, Clarke, Likharev, Zorin, Vorontsov, Khalili, etc.); also Zorin-1996, Averin-2000, Clerk et al.-2002, etc.

$$(\varepsilon_O\, \varepsilon_{BA} - \varepsilon_{O,BA}{}^2)^{1/2} \geq \hbar/2 \;\Leftrightarrow\; \Gamma \geq (\Delta I)^2/4S_I + K^2 S_I/4$$

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POVM vs. Bayesian formalism

General quantum measurement (POVM formalism) (Nielsen-Chuang, p. 85,100):

Measurement (Kraus) operator M_r (any linear operator in H.S.): $\psi \rightarrow \dfrac{M_r \psi}{\| M_r \psi \|}$ or $\rho \rightarrow \dfrac{M_r \rho M_r^\dagger}{\mathrm{Tr}(M_r \rho M_r^\dagger)}$

Probability: $P_r = \| M_r \psi \|^2$ or $P_r = \mathrm{Tr}(M_r \rho M_r^\dagger)$

Completeness: $\sum_r M_r^\dagger M_r = 1$

(People often prefer linear evolution and non-normalized states)

- POVM is essentially a projective measurement in an extended Hilbert space
- Easy to derive: interaction with ancilla + projective measurement of ancilla
- For extra decoherence: incoherent sum over subsets of results

Relation between POVM and quantum Bayesian formalism:

decomposition $M_r = \underset{\text{unitary}}{U_r} \underbrace{\sqrt{M_r^\dagger M_r}}_{\text{Bayes}}$

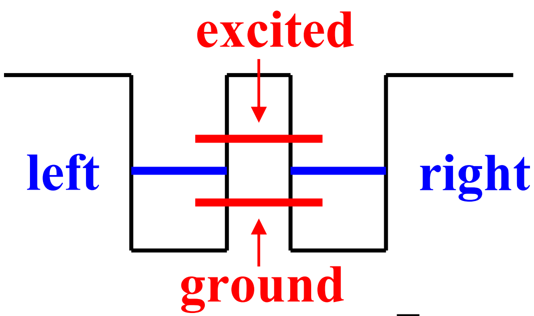
So, mathematically, POVM and quantum Bayes are very close (Caves was possibly first to notice)

We emphasize not mathematical structures, but particular setups (goal: find a proper description) and experimental consequences

Experimental predictions and proposals from Bayesian formalism

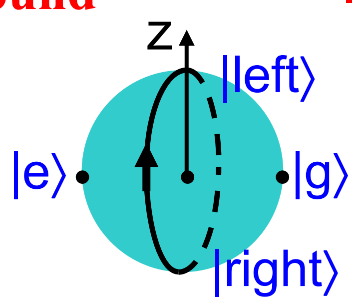
- Direct experimental verification (1998)
- Measured spectrum of Rabi oscillations (1999, 2000, 2002)
- Bell-type correlation experiment (2000)
- Quantum feedback control of a qubit (2001, 2004, 2009)
- Entanglement by measurement (2002)
- Measurement by a quadratic detector (2003)
- Squeezing of a nanomechanical resonator (2004)
- Violation of Leggett-Garg inequality (2005)
- Partial collapse of a phase qubit (2005)
- Undoing of a weak measurement (2006, 2008)
- Decoherence suppression by uncollapsing (2009)

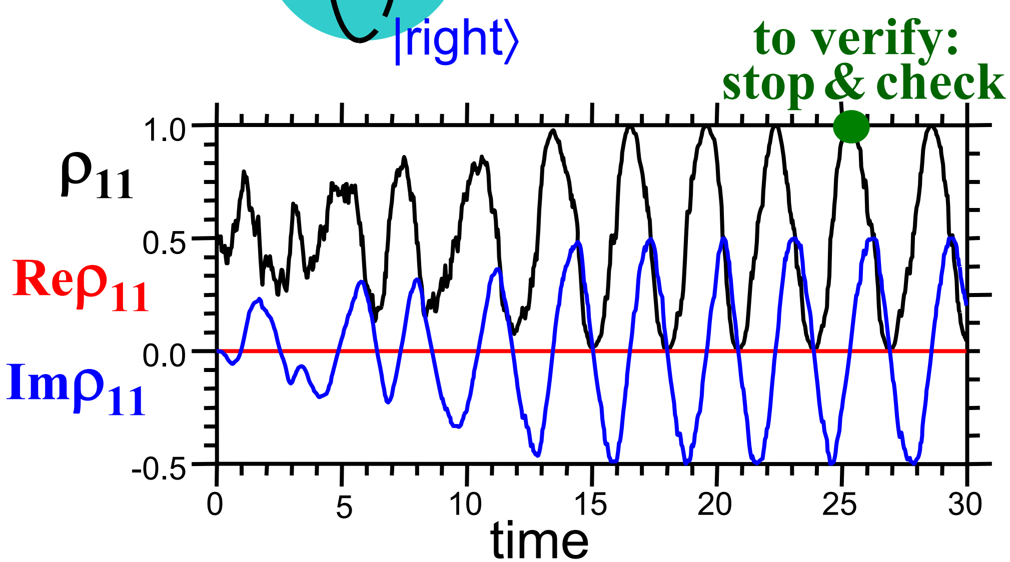
Persistent Rabi oscillations



- Relaxes to the ground state if left alone (low-*T*)
- Becomes fully mixed if coupled to a high-*T* (non-equilibrium) environment
- Oscillates persistently between left and right if (weakly) measured continuously

(“reason”: attraction to two points on the Bloch sphere great circle)





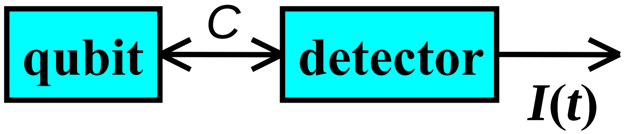
A.K., 1999

Phase of Rabi oscillations fluctuates (dephasing)

Direct experiment is difficult (good quantum efficiency, bandwidth, control)

Indirect experiment: spectrum of persistent Rabi oscillations

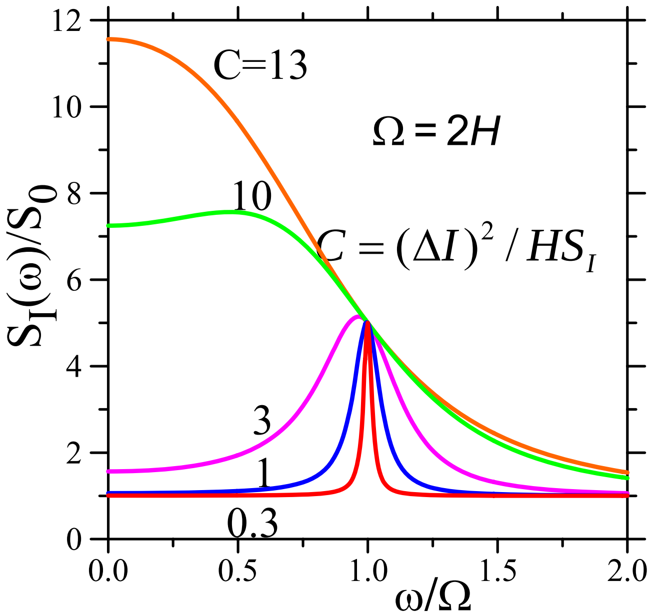
A.K., LT'1999
A.K.-Averin, 2000



$$I(t) = I_0 + \frac{\Delta I}{2} z(t) + \xi(t)$$

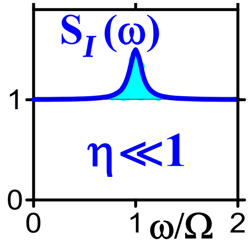
(const + signal + noise)

z is Bloch coordinate



Ω - Rabi frequency

amplifier noise ⇒ higher pedestal, poor quantum efficiency, but the peak is the same!!!



integral under the peak ⇔ variance $\langle z^2 \rangle$

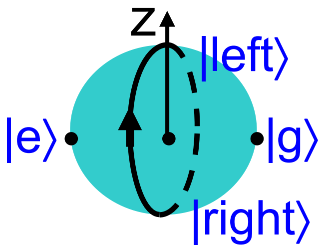
peak-to-pedestal ratio = $4\eta \leq 4$

$$S_I(\omega) = S_0 + \frac{\Omega^2 (\Delta I)^2 \Gamma}{(\omega^2 - \Omega^2)^2 + \Gamma^2 \omega^2}$$

How to distinguish experimentally persistent from non-persistent? Easy!

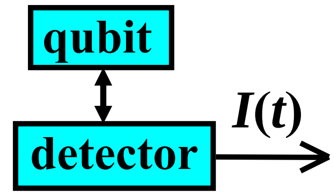
perfect Rabi oscillations: $\langle z^2 \rangle = \langle \cos^2 \rangle = 1/2$
imperfect (non-persistent): $\langle z^2 \rangle \ll 1/2$
quantum (Bayesian) result: $\langle z^2 \rangle = 1$ (!!!)

(demonstrated in Saclay expt.)



How to understand $\langle z^2 \rangle = 1$?

$$I(t) = I_0 + \frac{\Delta I}{2} z(t) + \xi(t)$$



First way (mathematical)

We actually measure operator: $z \to \sigma_z$

$$z^2 \to \sigma_z^{\,2} = 1$$

(What does it mean? Difficult to say…)

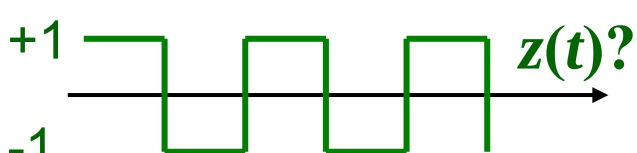
Second way (Bayesian)

$$S_I(\omega) = S_{\xi\xi} + \frac{\Delta I^2}{4} S_{zz}(\omega) + \frac{\Delta I}{2} S_{\xi z}(\omega)$$

quantum back-action changes z in accordance with the noise ξ (what you see becomes reality)

Equal contributions (for weak coupling and η=1)

Can we explain it in a more reasonable way (without spooks/ghosts)?

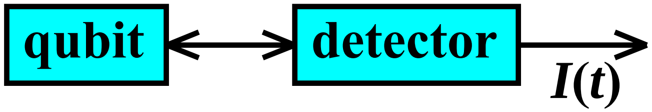


or some other $z(t)$?

No (under assumptions of macrorealism; Leggett-Garg, 1985)

Alexander Korotkov
University of California, Riverside

Leggett-Garg-type inequalities for continuous measurement of a qubit



Ruskov-A.K.-Mizel, PRL-2006
Jordan-A.K.-Büttiker, PRL-2006

Assumptions of macrorealism (similar to Leggett-Garg'85):

$$I(t) = I_0 + (\Delta I/2)\, z(t) + \xi(t)$$

$$|z(t)| \le 1, \quad \langle \xi(t)\; z(t+\tau)\rangle = 0$$

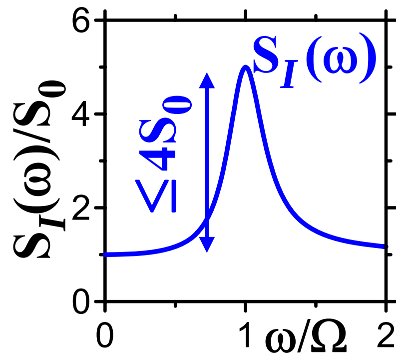
Leggett-Garg, 1985

$K_{ij} = \langle Q_i Q_j \rangle$

if $Q = \pm 1$, then

$1 + K_{12} + K_{23} + K_{13} \ge 0$

$K_{12} + K_{23} + K_{34} - K_{14} \le 2$



Then for correlation function

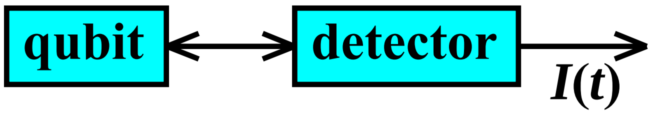
$$K(\tau) = \langle I(t)\, I(t+\tau)\rangle$$

	quantum result	violation
$K(\tau_1) + K(\tau_2) - K(\tau_1+\tau_2) \le (\Delta I/2)^2$	$\frac{3}{2}(\Delta I/2)^2$	$\times\frac{3}{2}$
and for area under narrow spectral peak $\int [S_I(f) - S_0]\, df \le (8/\pi^2)(\Delta I/2)^2$	$(\Delta I/2)^2$	$\times\frac{\pi^2}{8}$

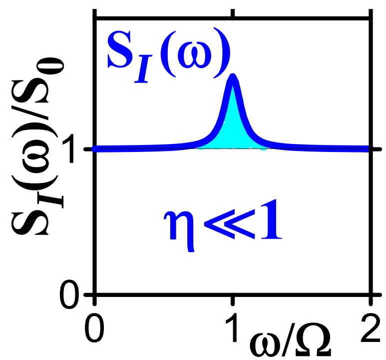
η is not important! **Experimentally measurable violation**

(Saclay experiment)

May be a physical (realistic) back-action?



$$I(t) = I_0 + \frac{\Delta I}{2} z(t) + \xi(t)$$



OK, cannot explain without back-action

$$\langle \xi(t)\, z(t+\tau) \rangle \neq 0$$

But may be there is a simple classical back-action from the noise?

In principle, classical explanation cannot be ruled out (e.g. computer-generated $I(t)$; no non-locality as in optics)

Try reasonable models: linear modulation of the qubit parameters (H and ε) by noise $\xi(t)$

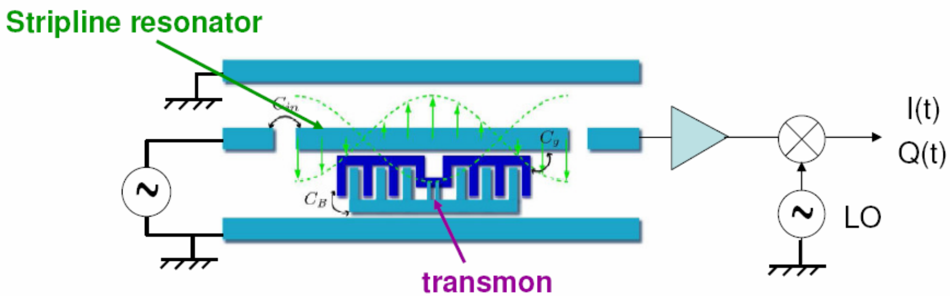
No, does not work!

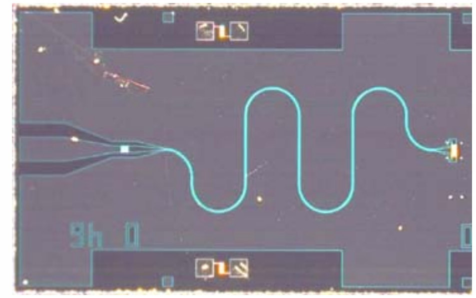
Our (spooky) back-action is quite peculiar: $\langle \xi(t)\, dz(t+0) \rangle > 0$

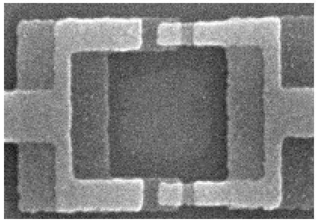
“what you see is what you get”: observation becomes reality

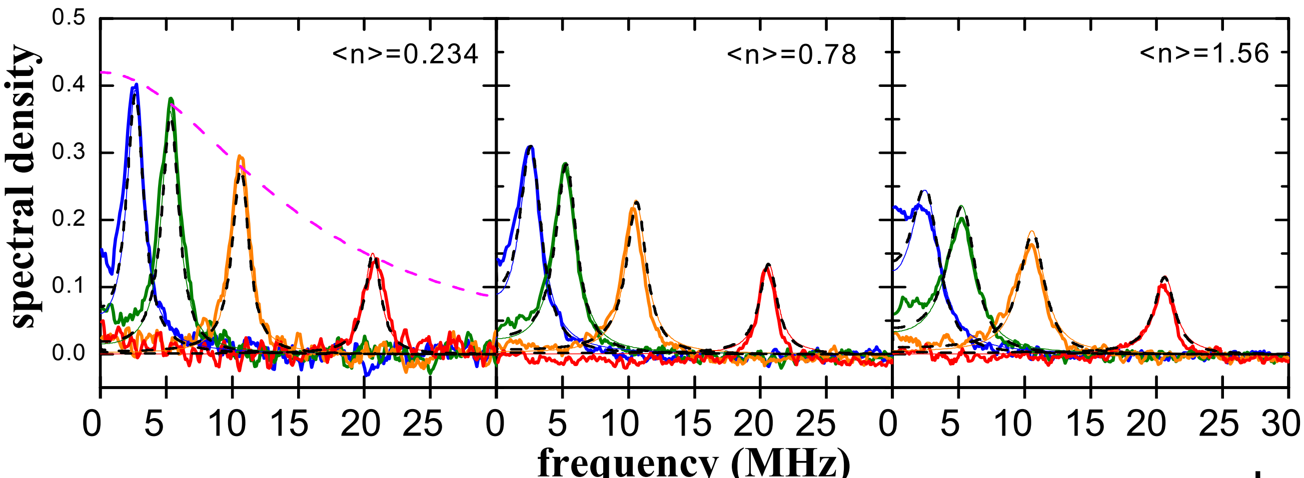
Alexander Korotkov University of California, Riverside

Recent experiment (Saclay group, unpub.)



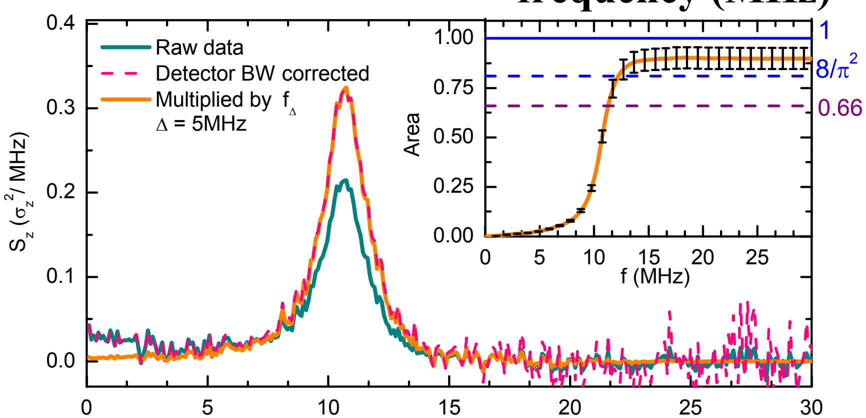






A. Palacios-Laloy et al. (unpublished)

courtesy of Patrice Bertet



- superconducting charge qubit (transmon) in circuit QED setup (microwave reflection from cavity)
- driven Rabi oscillations
- perfect spectral peaks
- LGI violation (both *K* and *S*)

Next step: quantum feedback?

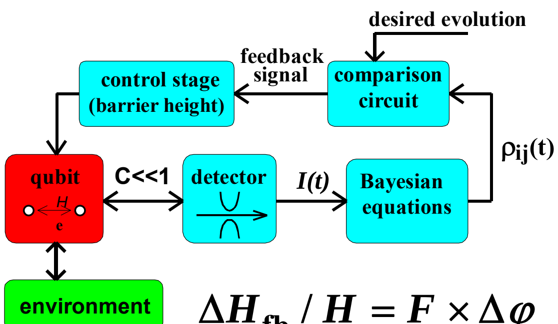
Goal: persistent Rabi oscillations with zero linewidth (synchronized)

Types of quantum feedback:

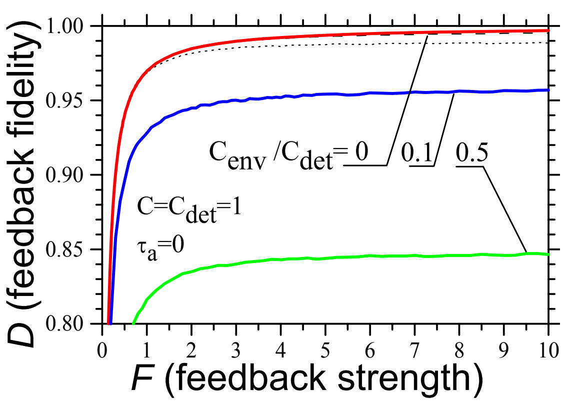
Bayesian

Best but very difficult

(monitor quantum state and control deviation)



$$\Delta H_{\mathrm{fb}} / H = F \times \Delta\varphi$$



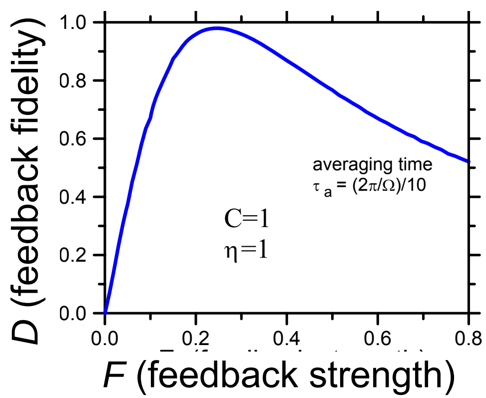
Ruskov & A.K., 2002

Direct

a la Wiseman-Milburn (1993)

(apply measurement signal to control with minimal processing)

$$\Delta H_{\mathrm{fb}} / H = F \sin(\Omega t) \times \left(\frac{I(t) - I_0}{\Delta I / 2} - \cos\Omega t \right)$$



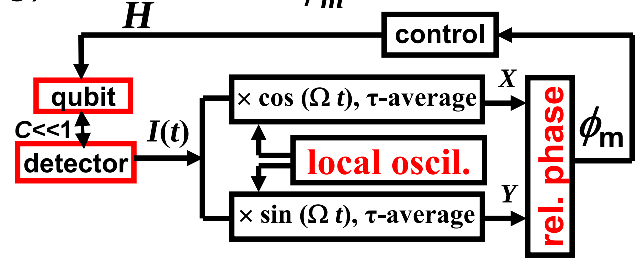
Ruskov & A.K., 2002

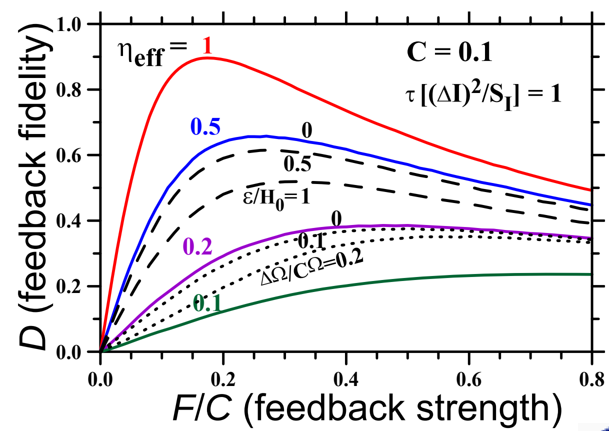
"Simple"

Imperfect but simple

(do as in usual classical feedback)

$$\frac{\Delta H_{\mathrm{fb}}}{H} = F \times \phi_m$$





A.K., 2005

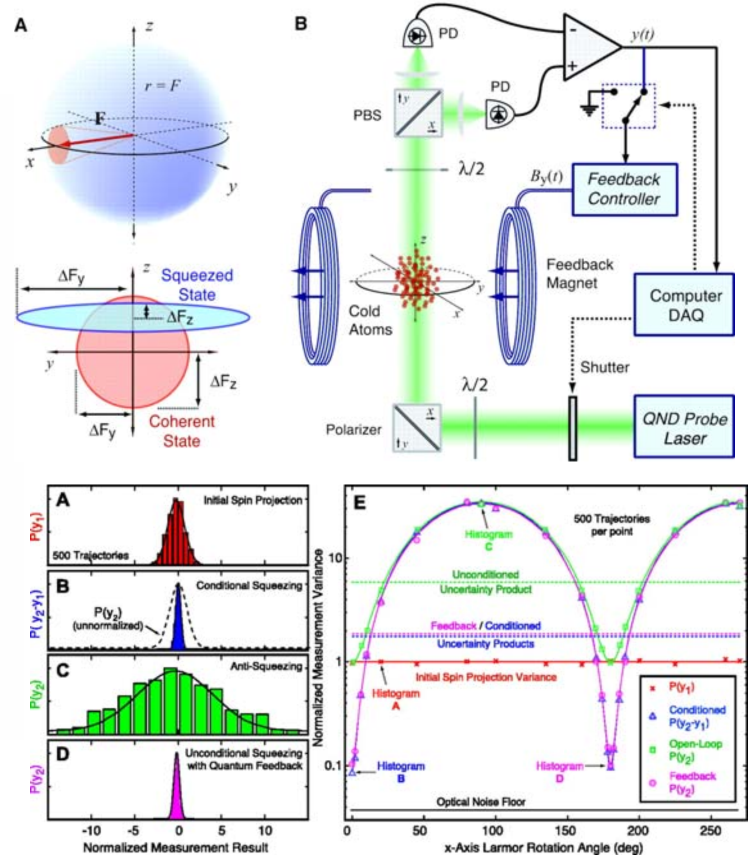
Quantum feedback in optics

First experiment: Science 304, 270 (2004)

Real-Time Quantum Feedback Control of Atomic Spin-Squeezing

JM Geremia,* John K. Stockton, Hideo Mabuchi

Real-time feedback performed during a quantum nondemolition measurement of atomic spin-angular momentum allowed us to influence the quantum statistics of the measurement outcome. We showed that it is possible to harness measurement backaction as a form of actuation in quantum control, and thus we describe a valuable tool for quantum information science. Our feedback-mediated procedure generates spin-squeezing, for which the reduction in quantum uncertainty and resulting atomic entanglement are not conditioned on the measurement outcome.



First detailed theory:

H.M. Wiseman and G. J. Milburn, Phys. Rev. Lett. 70, 548 (**1993**)

Quantum feedback in optics

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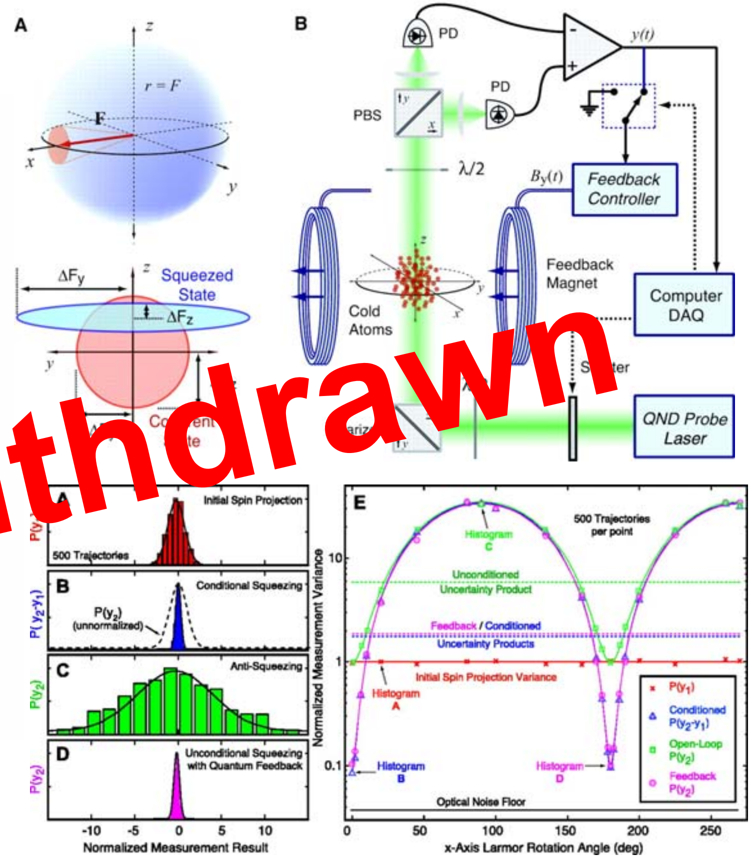
paper withdrawn



PRL 94, 203002 (2005) also withdrawn

First detailed theory:

H.M. Wiseman and G. J. Milburn, Phys. Rev. Lett. 70, 548 (**1993**)

Recent experiment:

Cook, Martin, Geremia, Nature 446, 774 (2007)
(coherent state discrimination)

Undoing a weak measurement of a qubit (“uncollapse”)

A.K. & Jordan, PRL-2006

It is impossible to undo “orthodox” quantum measurement (for an unknown initial state)

Is it possible to undo partial quantum measurement?
(To restore a “precious” qubit accidentally measured)

Yes! (but with a finite probability)

If undoing is successful, an unknown state is **fully** restored

ψ_0 (unknown) $\xrightarrow{\text{weak (partial) measurement}}$ ψ_1 (partially collapsed)

ψ_1 $\xrightarrow{\text{successful}}$ ψ_0 (still unknown)

ψ_1 $\xrightarrow{\text{unsuccessful}}$ ψ_2

undoing (information erasure)

Quantum erasers in optics

Quantum eraser proposal by Scully and Drühl, PRA (1982)

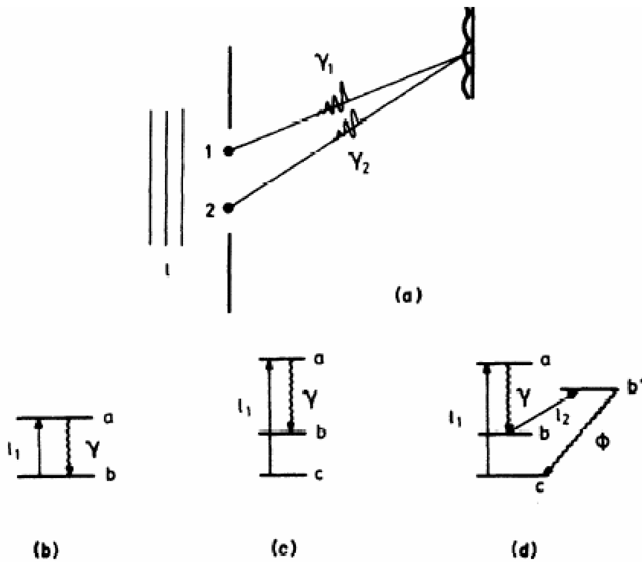


FIG. 1. (a) Figure depicting light impinging from left on atoms at sites 1 and 2. Scattered photons γ_1 and γ_2 produce interference pattern on screen. (b) Two-level atoms excited by laser pulse l_1, and emit γ photons in $a \rightarrow b$ transition. (c) Three-level atoms excited by pulse l_1 from $c \rightarrow a$ and emit photons in $a \rightarrow b$ transition. (d) Four-level system excited by pulse l_1 from $c \rightarrow a$ followed by emission of γ photons in $a \rightarrow b$ transition. Second pulse l_2 takes atoms from $b \rightarrow b'$. Decay from $b' \rightarrow c$ results in emission of ϕ photons.

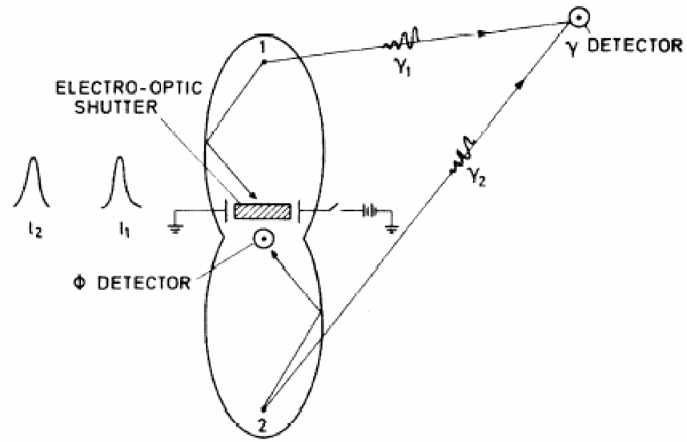


FIG. 2. Laser pulses l_1 and l_2 incident on atoms at sites 1 and 2. Scattered photons γ_1 and γ_2 result from $a \rightarrow b$ transition. Decay of atoms from $b' \rightarrow c$ results in ϕ photon emission. Elliptical cavities reflect ϕ photons onto common photodetector. Electro-optic shutter transmits ϕ photons only when switch is open. Choice of switch position determines whether we emphasize particle or wave nature of γ photons.

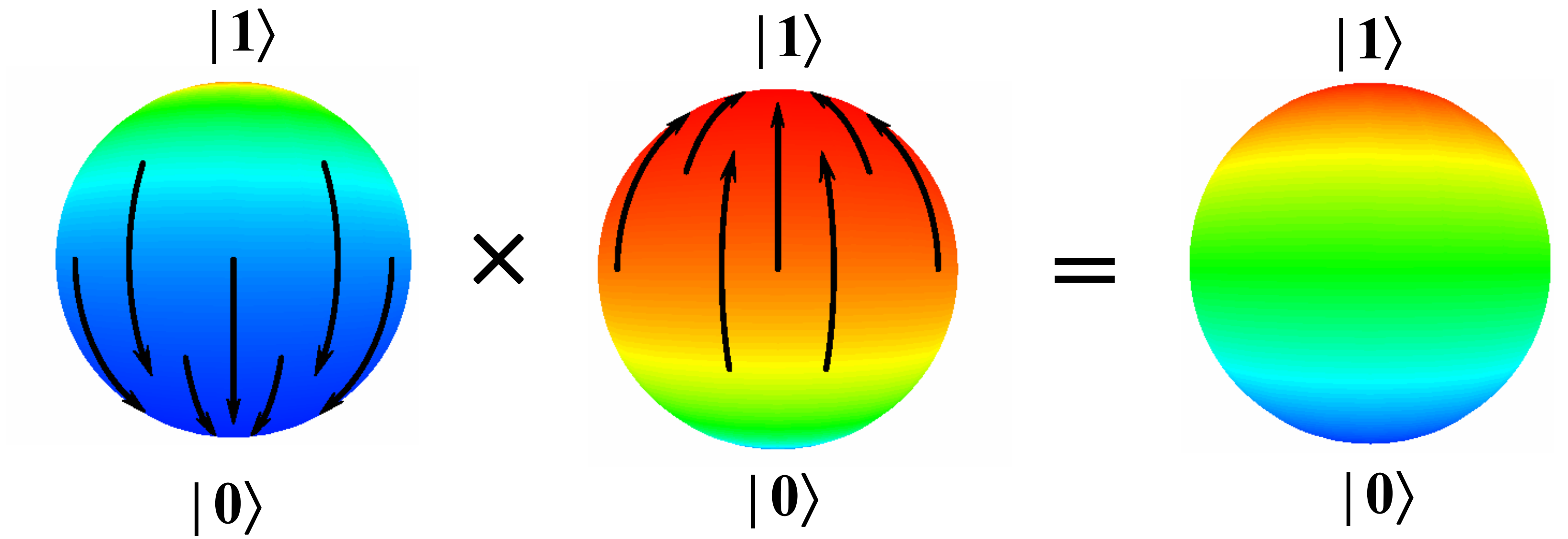
Interference fringes restored for two-detector correlations (since “which-path” information is erased)

Our idea of uncollapsing is quite different:
we really extract quantum information and then erase it

Uncollapse of a qubit state

Evolution due to partial (weak, continuous, etc.) measurement is **non-unitary** (though coherent if detector is good!), therefore it is impossible to undo it by Hamiltonian dynamics.

How to undo? One more measurement!



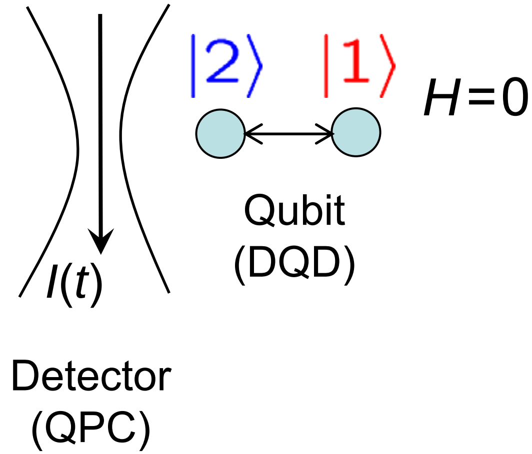
need ideal (quantum-limited) detector

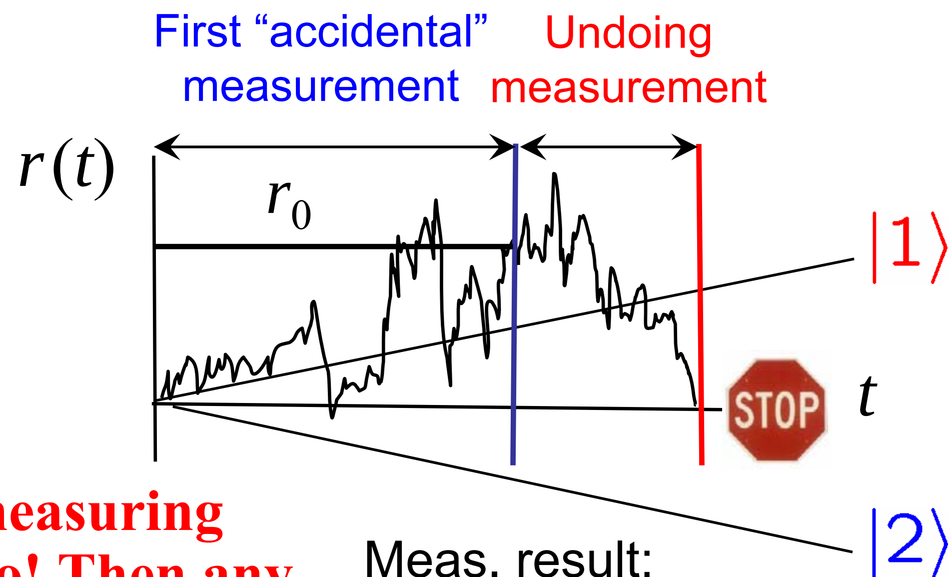
(similar to Koashi-Ueda, PRL-1999, also Nielsen-Caves-1997, Royer-1994, etc.)

(Figure partially adopted from Jordan-A.K.-Büttiker, PRL-06)

Uncollapsing for DQD-QPC system

A.K. & Jordan, PRL-2006





Simple strategy: continue measuring until result $r(t)$ becomes zero! Then any unknown initial state is fully restored.

(same for an entangled qubit)

However, if $r = 0$ never happens, then undoing procedure is unsuccessful.

Meas. result:

$$r(t) = \frac{\Delta I}{S_I}\left[\int_0^t I(t')\,dt' - I_0 t\right]$$

If $r = 0$, then no information and no evolution!

Probability of success:

$$P_S = \frac{e^{-|r_0|}}{e^{|r_0|}\rho_{11}(0) + e^{-|r_0|}\rho_{22}(0)}$$

General theory of uncollapsing

POVM formalism
(Nielsen-Chuang, p.100)

Measurement operator M_r: $\rho \to \dfrac{M_r \rho M_r^\dagger}{\mathrm{Tr}(M_r \rho M_r^\dagger)}$

Probability: $P_r = \mathrm{Tr}(M_r \rho M_r^\dagger)$ Completeness: $\sum_r M_r^\dagger M_r = 1$

Uncollapsing operator: $C \times M_r^{-1}$ (to satisfy completeness, eigenvalues cannot be >1)

$\max(C) = \min_i \sqrt{p_i}$, p_i – eigenvalues of $M_r^\dagger M_r$

Probability of success: $\boxed{P_S \leq \dfrac{\min P_r}{P_r(\rho_{\mathrm{in}})}}$ A.K. & Jordan, 2006

$P_r(\rho_{in})$ – probability of result r for initial state ρ_{in},

min P_r – probability of result r minimized over all possible initial states

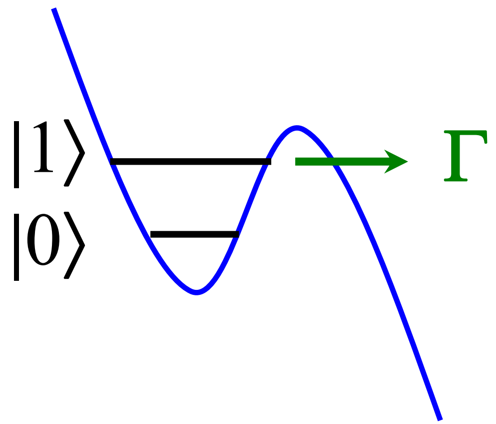
Averaged (over r) probability of success: $P_{av} \leq \sum_r \min P_r$

(cannot depend on initial state, otherwise get information)

(similar to Koashi-Ueda, 1999)

Partial collapse of a Josephson phase qubit

N. Katz, M. Ansmann, R. Bialczak, E. Lucero, R. McDermott, M. Neeley, M. Steffen, E. Weig, A. Cleland, J. Martinis, A. Korotkov, Science-06



How does a qubit state evolve in time before tunneling event?

(What happens when nothing happens?)

Qubit "ages" in contrast to a radioactive atom!

Main idea:

$$\psi = \alpha\,|0\rangle + \beta\,|1\rangle \;\rightarrow\; \psi(t) = \begin{cases} |out\rangle, \text{ if tunneled} \\ \dfrac{\alpha\,|0\rangle + \beta\, e^{-\Gamma t/2} e^{i\varphi}\,|1\rangle}{\sqrt{|\alpha|^2 + |\beta|^2\, e^{-\Gamma t}}}, \text{ if not tunneled} \end{cases}$$

(better theory: Pryadko & A.K., 2007)

amplitude of state |0> grows without physical interaction

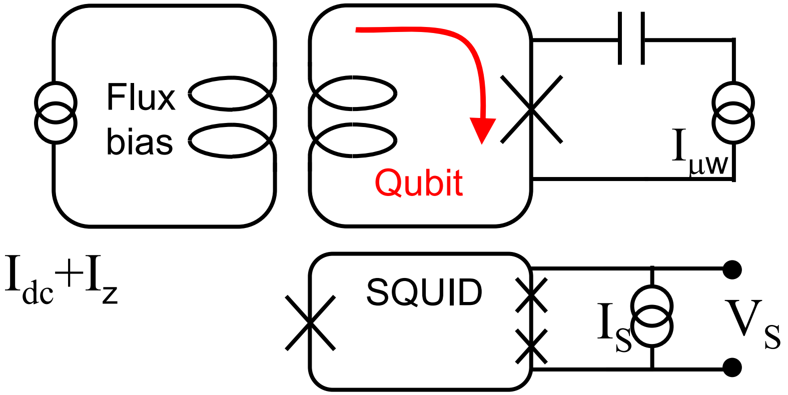
finite linewidth only after tunneling

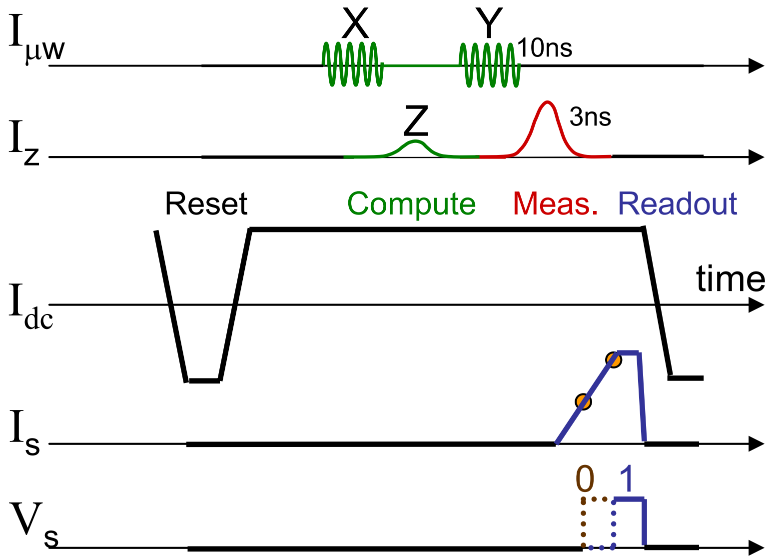
continuous null-result collapse

(similar to optics, Dalibard-Castin-Molmer, PRL-1992)

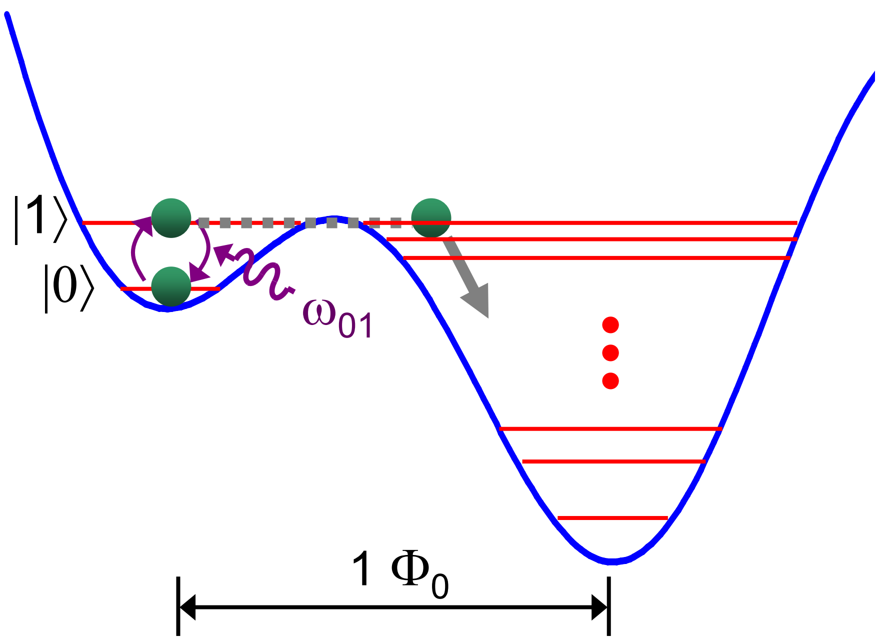
Superconducting phase qubit at UCSB

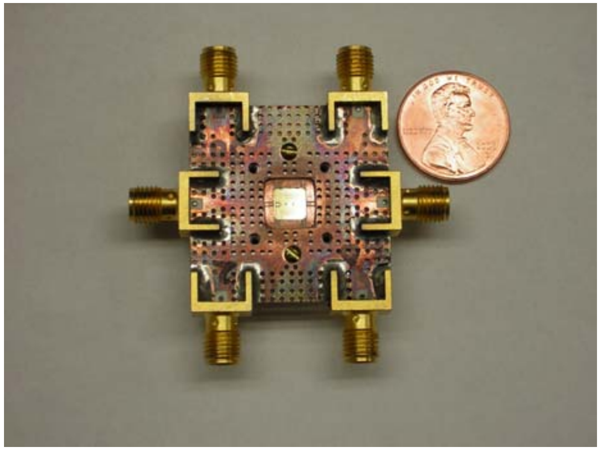
Courtesy of Nadav Katz (UCSB)



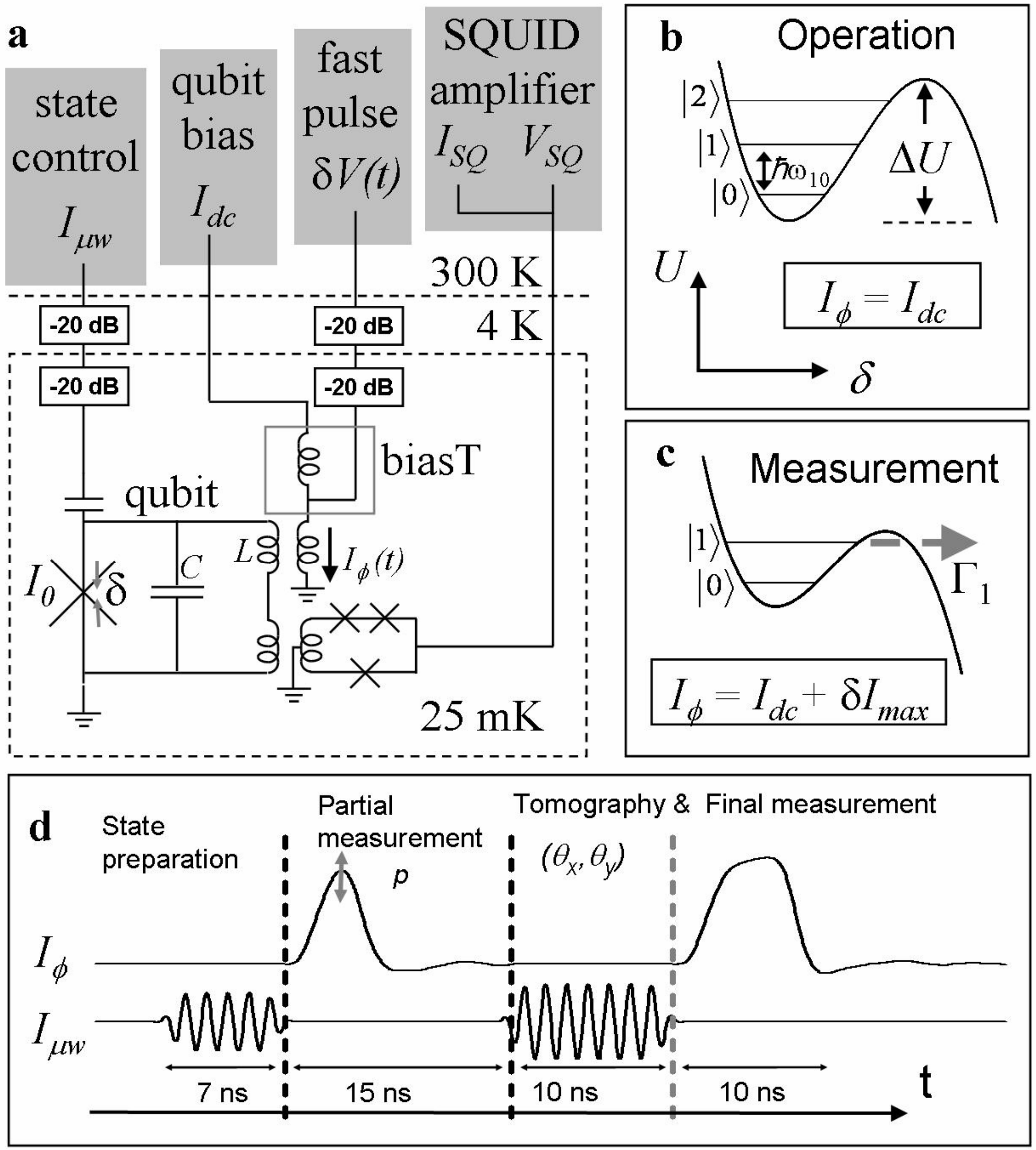


Repeat 1000x
prob. 0,1





Experimental technique for partial collapse



Nadav Katz *et al.*
(John Martinis group)

Protocol:

1) State preparation by applying microwave pulse (via Rabi oscillations)

2) Partial measurement by lowering barrier for time *t*

3) State tomography (micro-wave + full measurement)

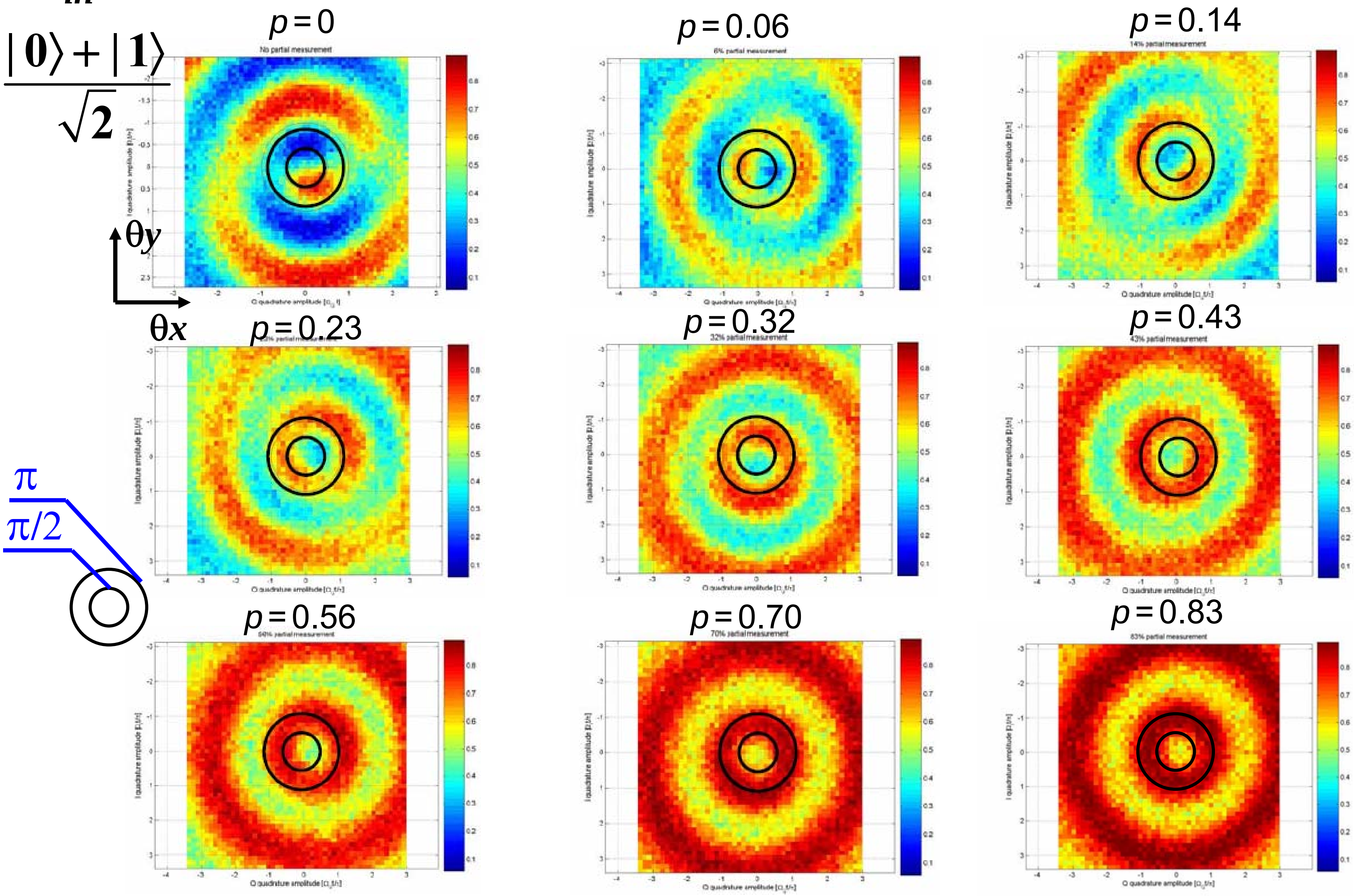
Measurement strength
$p = 1 - \exp(-\Gamma t)$
is actually controlled by Γ, not by t

$p=0$: no measurement
$p=1$: orthodox collapse

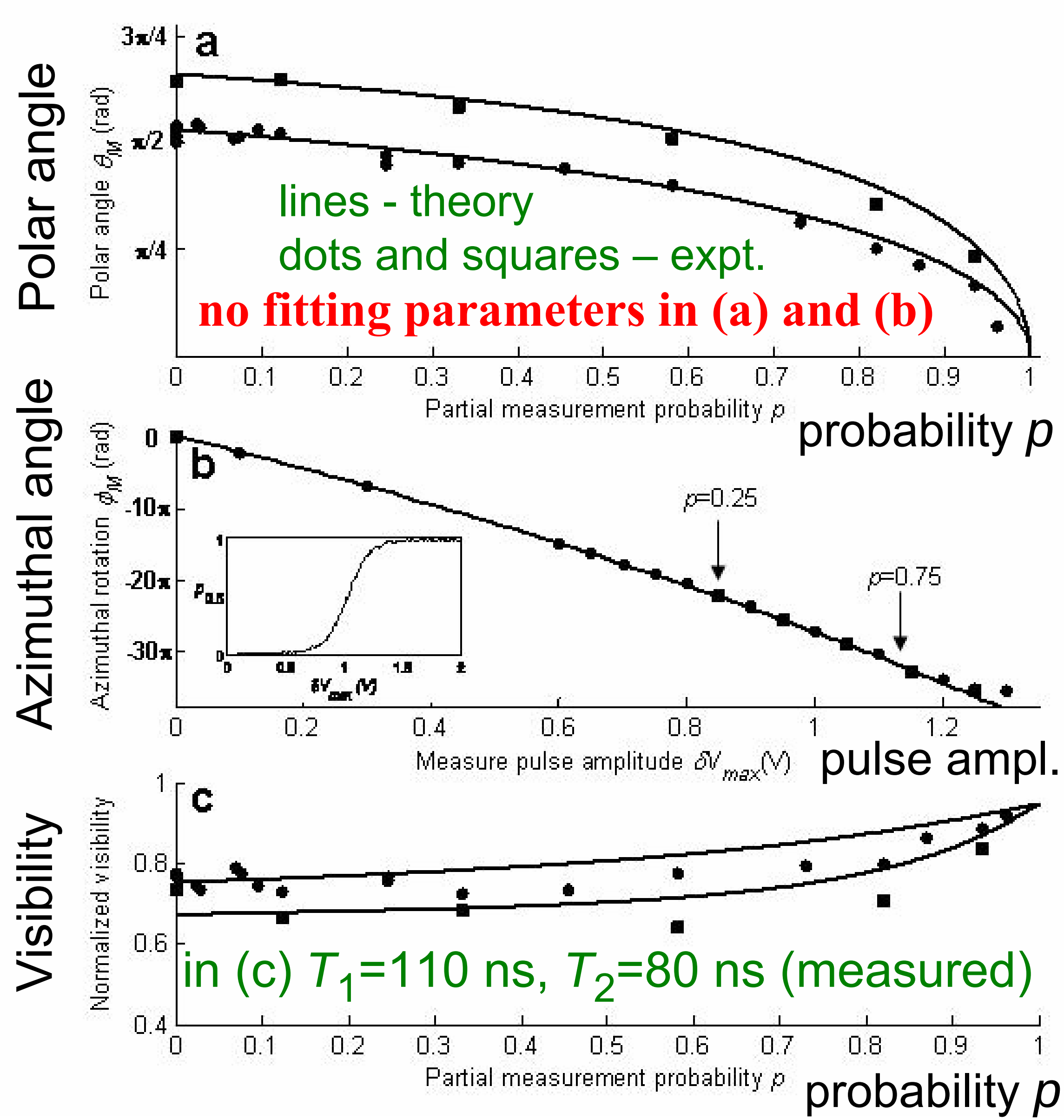
Experimental tomography data

Nadav Katz *et al*. (UCSB, 2005)

$$\psi_{in} = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$



Partial collapse: experimental results



N. Katz *et al.*, Science-06

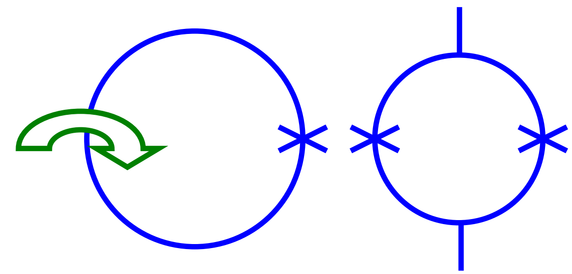
- In case of no tunneling (null-result measurement) phase qubit evolves
- This evolution is well described by a simple Bayesian theory, without fitting parameters
- Phase qubit remains fully coherent in the process of continuous collapse (experimentally ~80% raw data, ~96% after account for *T*1 and *T*2)

quantum efficiency
$\eta_0 > 0.8$

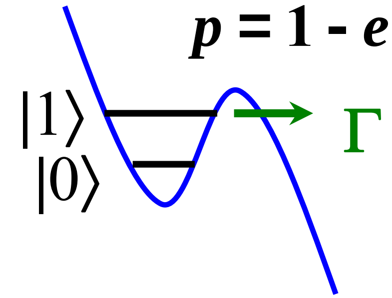
Uncollapse of a phase qubit state

A.K. & Jordan, 2006

1) Start with an unknown state
2) Partial measurement of strength p
3) π-pulse (exchange $|0\rangle \leftrightarrow |1\rangle$)
4) One more measurement with the **same strength** p
5) π-pulse



$$p = 1 - e^{-\Gamma t}$$



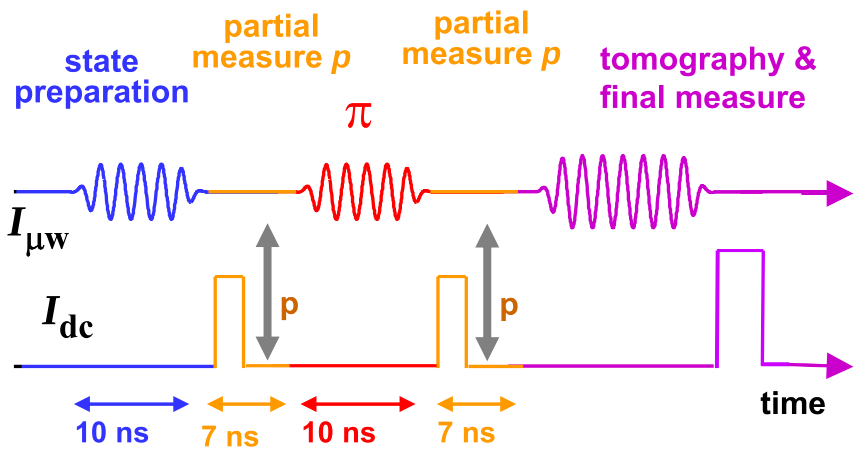
If no tunneling for both measurements, then initial state is fully restored!

$$\alpha|0\rangle + \beta|1\rangle \rightarrow \frac{\alpha|0\rangle + e^{i\phi}\beta e^{-\Gamma t/2}|1\rangle}{\text{Norm}} \rightarrow$$

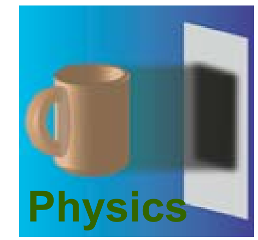
$$\frac{e^{i\phi}\alpha e^{-\Gamma t/2}|0\rangle + e^{i\phi}\beta e^{-\Gamma t/2}|1\rangle}{\text{Norm}} = e^{i\phi}(\alpha|0\rangle + \beta|1\rangle)$$

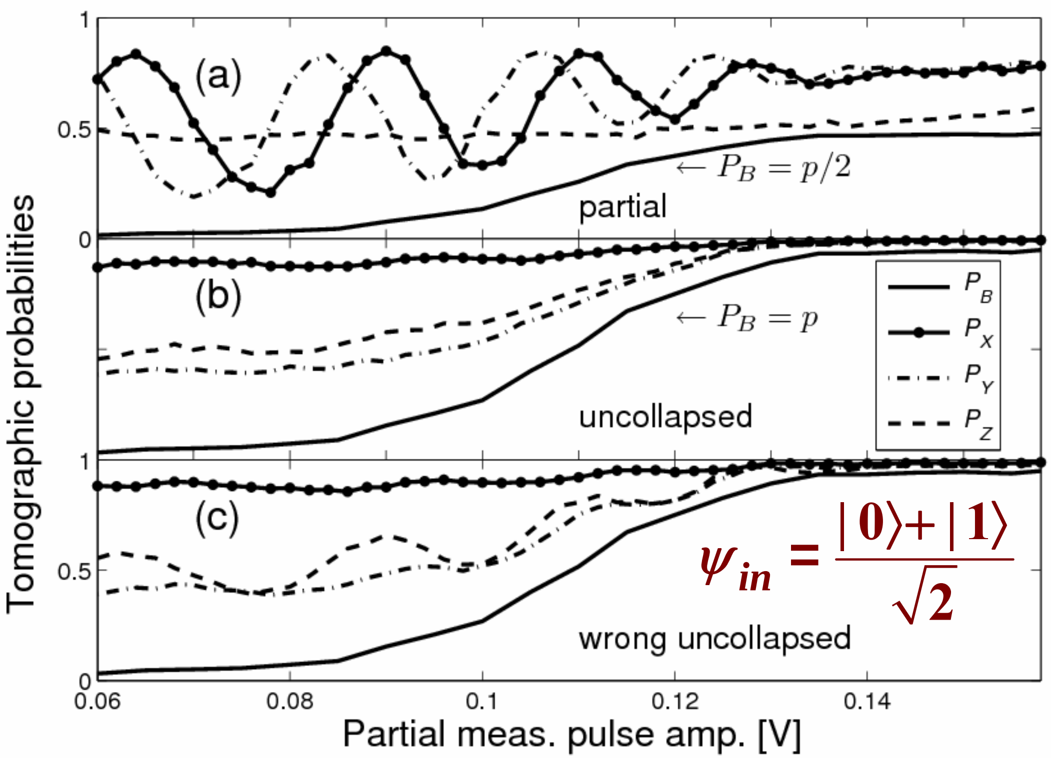
phase is also restored (spin echo)

Experiment on wavefunction uncollapse



N. Katz, M. Neeley, M. Ansmann, R. Bialzak, E. Lucero, A. O'Connell, H. Wang, A. Cleland, J. Martinis, and A. Korotkov, PRL-2008





Uncollapse protocol:

- partial collapse
- π-pulse
- partial collapse (same strength)

State tomography with *X*, *Y*, and no pulses

Background P_B should be subtracted to find qubit density matrix

Experimental results on the Bloch sphere

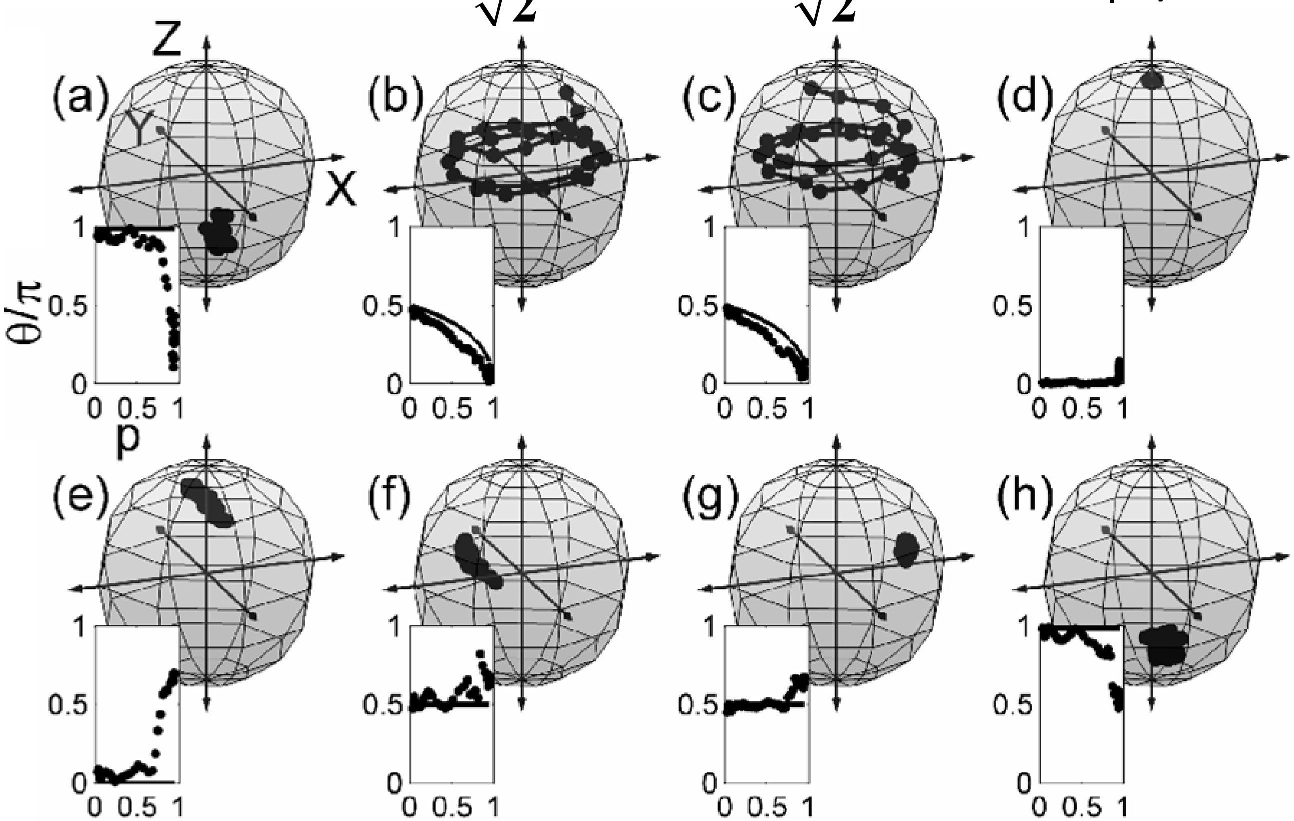
N. Katz et al.

Initial state: $|1\rangle$, $\frac{|0\rangle - i|1\rangle}{\sqrt{2}}$, $\frac{|0\rangle + |1\rangle}{\sqrt{2}}$, $|0\rangle$

Partially collapsed

Uncollapsed

uncollapsing works well!

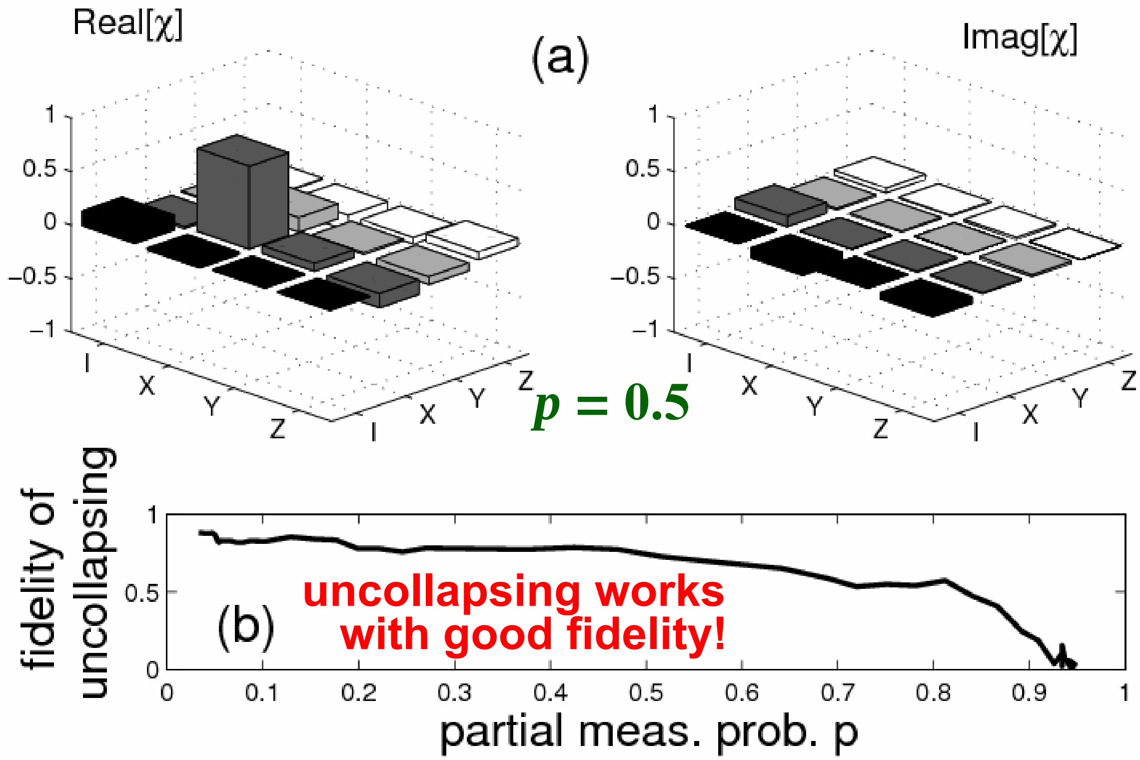


Both spin echo (azimuth) and uncollapsing (polar angle)

Difference: spin echo – undoing of an <u>unknown unitary</u> evolution, uncollapsing – undoing of a <u>known, but non-unitary</u> evolution

Quantum process tomography

N. Katz et al.
(Martinis group)



Why getting worse at p>0.6?

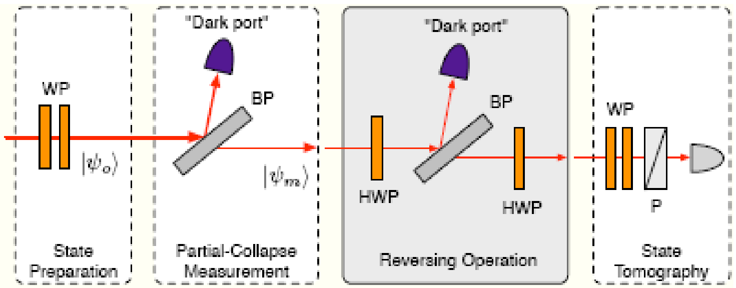
Energy relaxation $p_r = t/T_1$= 45ns/450ns = 0.1

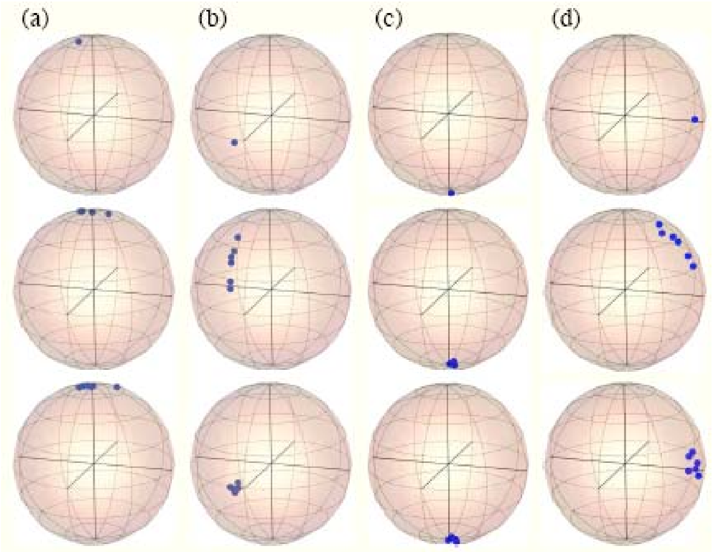
Selection affected when $1-p \sim p_r$

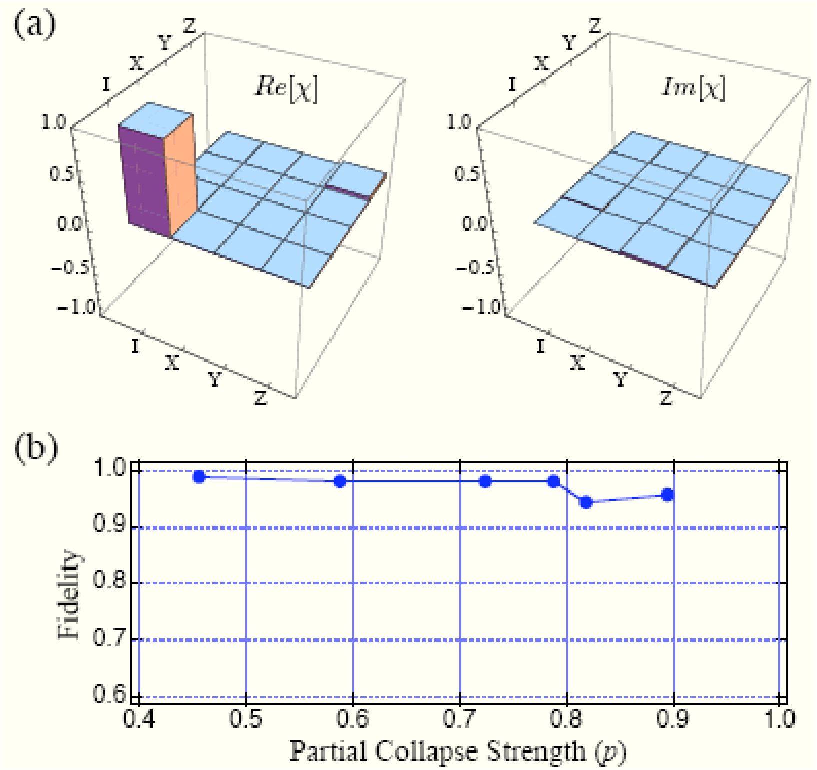
Overall: uncollapsing is well-confirmed experimentally

Recent experiment on uncollapsing using single photons

Kim et al., Opt. Expr.-2009



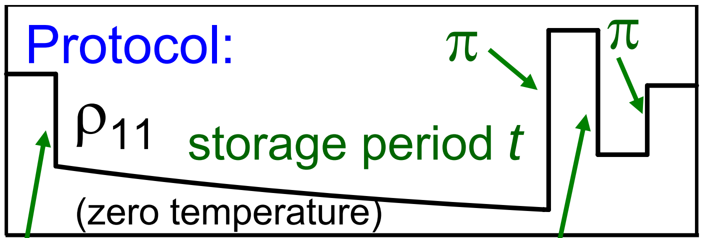




- very good fidelity of uncollapsing (>94%)
- measurement fidelity is probably not good (normalization by coincidence counts)

Suppression of T_1-decoherence by uncollapsing

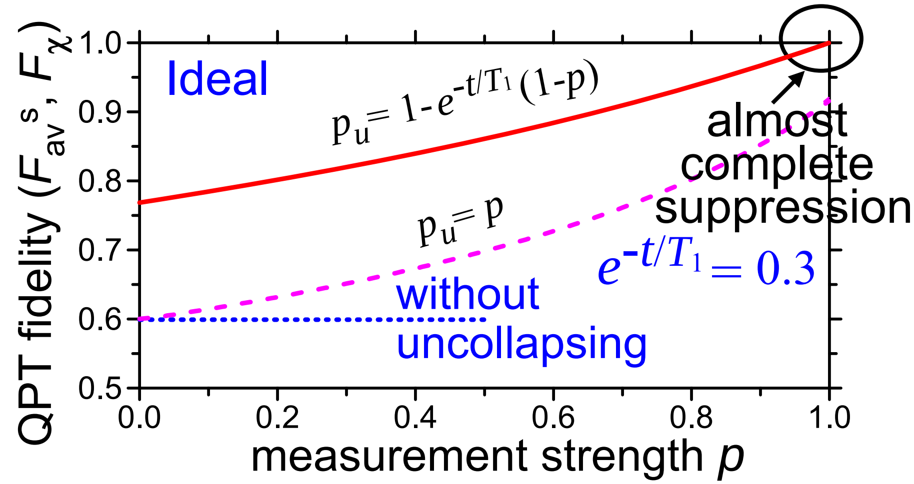
Korotkov & Keane, arXiv:0908.1134



partial collapse towards ground state (strength p)

uncollapse (measurem. strength p_u)

(almost same as existing experiment!)



Ideal case (T_1 during storage only, T=0)

for initial state $|\psi_{in}\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$

$|\psi_f\rangle = |\psi_{in}\rangle$ with probability $(1-p)\,e^{-t/T_1}$

$|\psi_f\rangle = |0\rangle$ with $(1-p)^2 |\beta|^2 e^{-t/T_1}(1-e^{-t/T_1})$

procedure preferentially selects events without energy decay

Trade-off: fidelity vs. selection probability

Unraveling of energy relaxation

$$\begin{pmatrix} |\beta|^2 e^{-t/T_1} & \alpha\beta^* e^{-t/2T_1} \\ \alpha^*\beta e^{-t/2T_1} & 1-|\beta|^2 e^{-t/T_1} \end{pmatrix} =$$

$$= p_\downarrow\,|0\rangle\langle 0| + (1-p_\downarrow)\,|\tilde\psi\rangle\langle\tilde\psi|$$

where $p_\downarrow = |\beta|^2 (1 - e^{-t/T_1})$

$$|\tilde\psi\rangle = (\alpha\,|0\rangle + \beta e^{-t/2T_1}\,|1\rangle)\,/\,Norm$$

$\Rightarrow$ optimum: $1 - p_u = e^{-t/T_1}(1-p)$

An issue with quantum process tomography (QPT)

QPT fidelity is usually $F_\chi = \mathrm{Tr}(\chi_{desired}\chi)$ where χ is the QPT matrix.

However, QPT is developed for a linear quantum process, while uncollapsing (after renormalization) is non-linear.

A better way: average state fidelity

$$F_{av} = \mathrm{Tr}(\rho_f U_0 |\psi_{in}\rangle\langle\psi_{in}|)\, d\,|\psi_{in}\rangle$$

Without selection

$$F_\chi = F_{av}^s = \frac{(d+1)F_{av} - 1}{d},\ \ d = 2$$

Another way: "naïve" QPT fidelity
(via 4 standard initial states)

The two ways practically coincide
(within line thickness)

Analytics for the ideal case

Average state fidelity

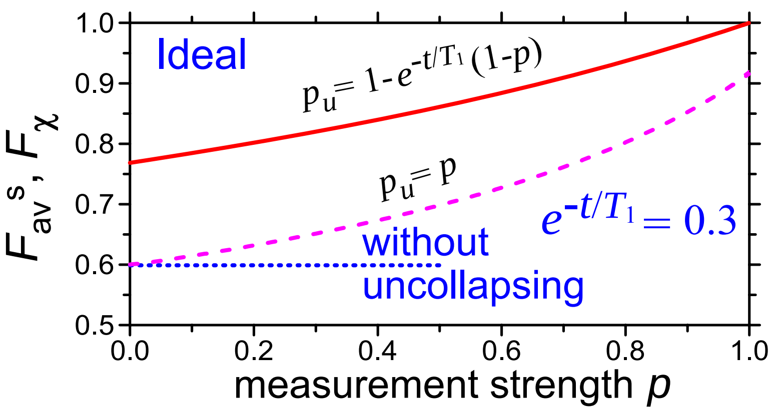
$$F_{av} = \frac{1}{2} + \frac{1}{C} + \frac{\ln(1+C)}{C^2}$$

"Naïve" QPT fidelity

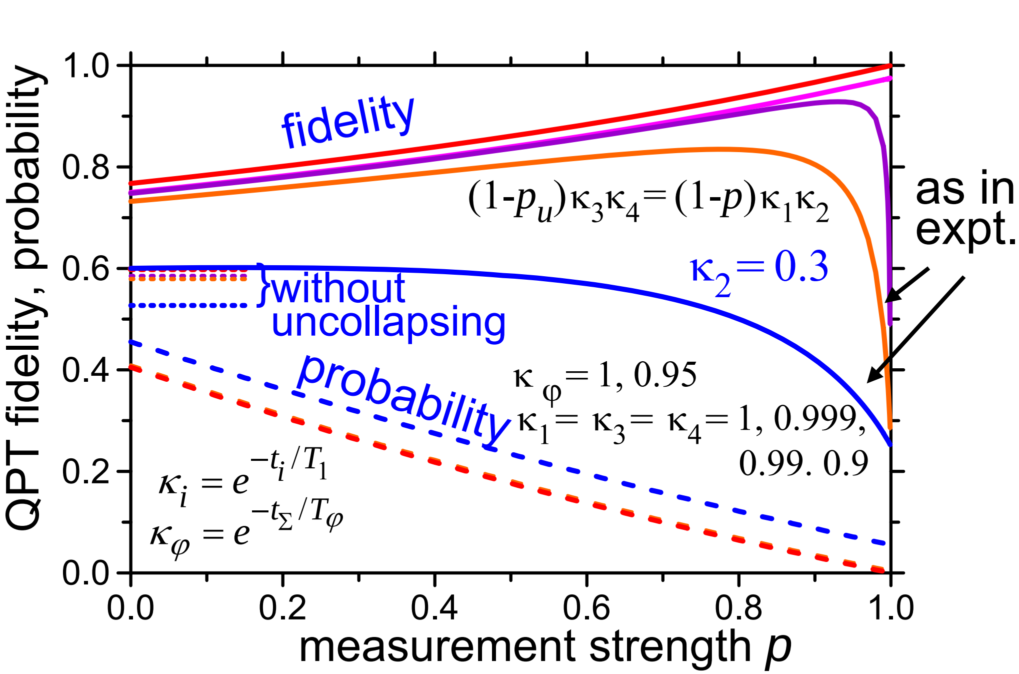
$$F_\chi = -\frac{1}{4} + \frac{1}{4(1+C)} + \frac{4+C}{2(2+C)}$$

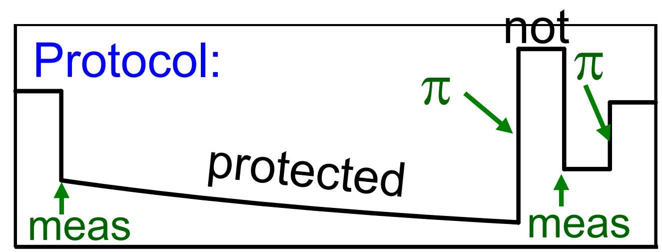
where $C = (1-p)(1-e^{-\Gamma t})$

$$p_u = 1 - e^{-\Gamma t}(1-p)$$



Realistic case (T_1 and T_φ at all stages)





- Easy to realize experimentally (similar to existing experiment)
- Increase of fidelity with p can be observed experimentally
- Improved fidelity can be observed with just one partial measurement

- decoherence due to pure dephasing is not affected
- T_1-decoherence between first π-pulse and second measurement causes decrease of fidelity at p close to 1

Trade-off: fidelity vs. selection probability

Uncollapse seems to be **the only way** to protect against T_1-decoherence without encoding in a larger Hilbert space (QEC, DFS)

A.K. & Keane, arXiv:0908.1134

One more experimental proposal:

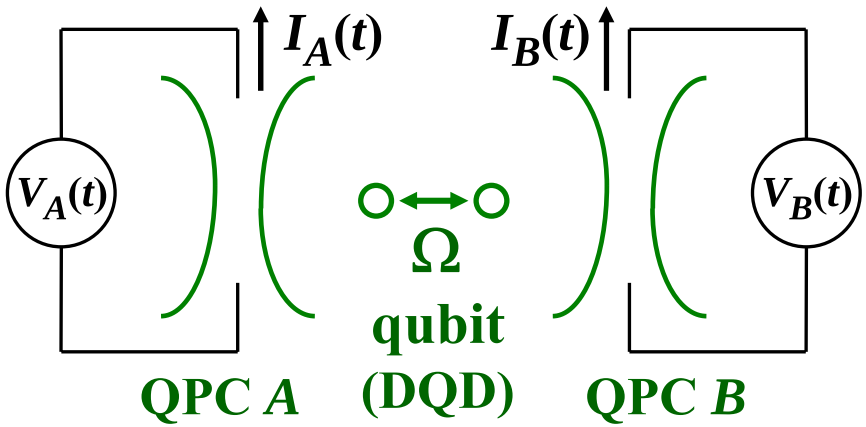
Persistent Rabi oscillations revealed in low-frequency noise

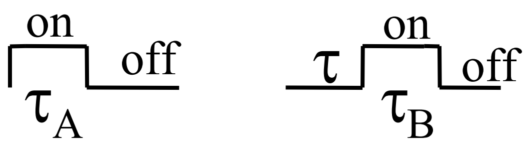
Hopefully, simple enough for semiconductor qubits

Goal: something easy for experiment, but still
with a non-trivial measurement effect

Alexander Korotkov University of California, Riverside

Setup: one qubit & two detectors





For single-shot measurements partial collapse can be revealed via **correlations** of $\int I_A$ and $\int I_B$.

(Korotkov, PRB-2001)

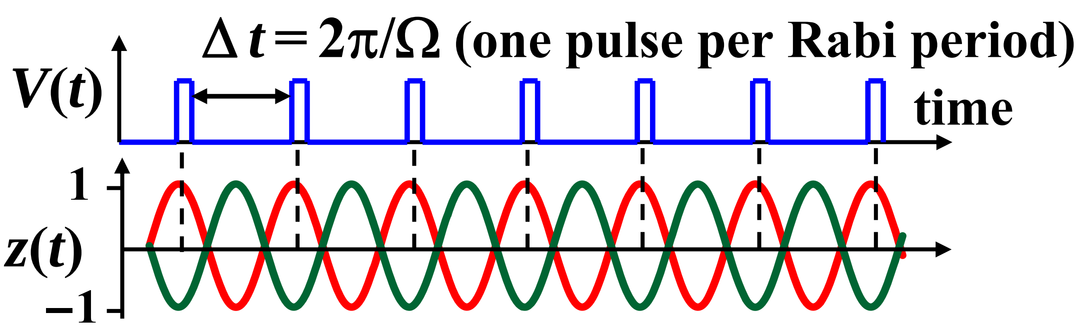
Same idea with another averaging → weak values

(Romito et al., PRL-2008)

Single-shot measurements are not yet available

⇒ use train (comb) of meas. pulses in QND regime

One-detector stroboscopic QND measurement



Stroboscopic QND:

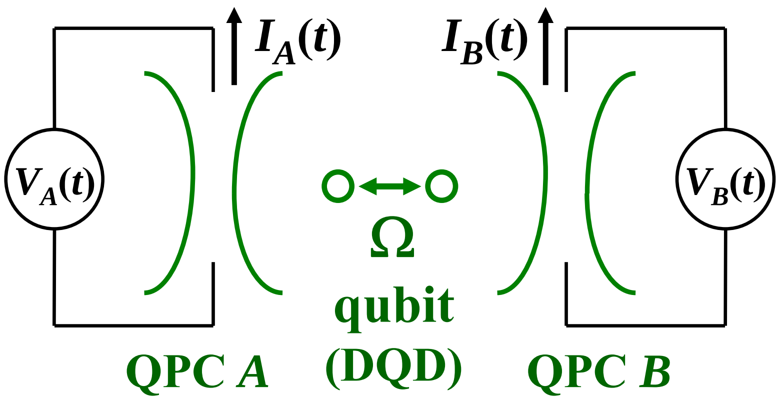
Braginsky, Vorontsov, Khalili, 1978

Jordan, Buttiker, 2005

Jordan, Korotkov, 2006

Stroboscopic QND measurement synchronizes (!) phase of persistent Rabi oscillations (attracts to either 0 or π)

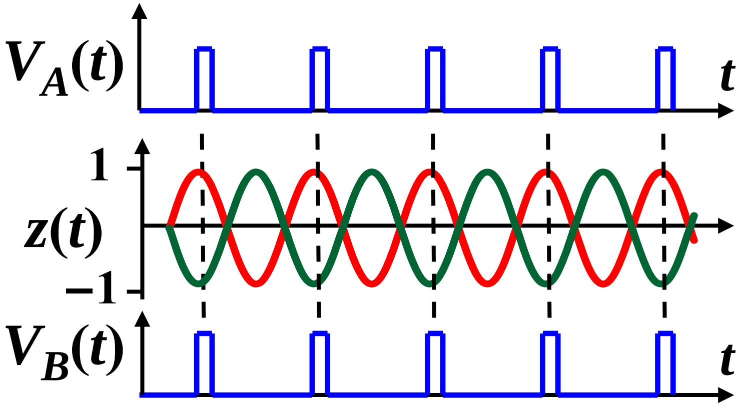
Idea of experiment



Perfect QND $\Rightarrow$ correlation/anticorr. between currents in two detectors

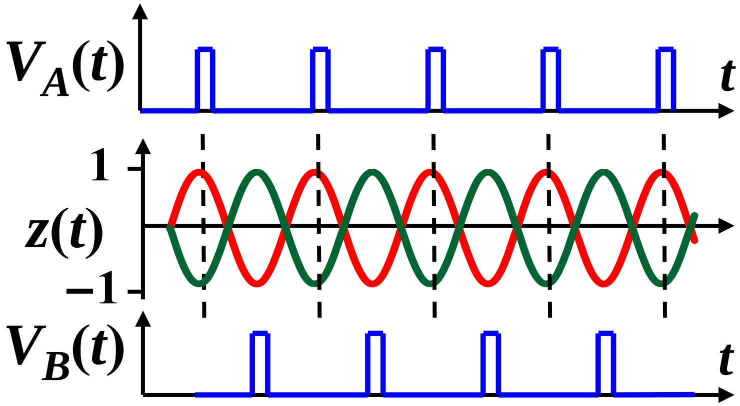
Imperfect QND $\Rightarrow$ random switching between two Rabi phases (0 and π) $\Rightarrow$ low-frequency telegraph noise

same combs on V_A and V_B



anticorrelation between I_A and I_B

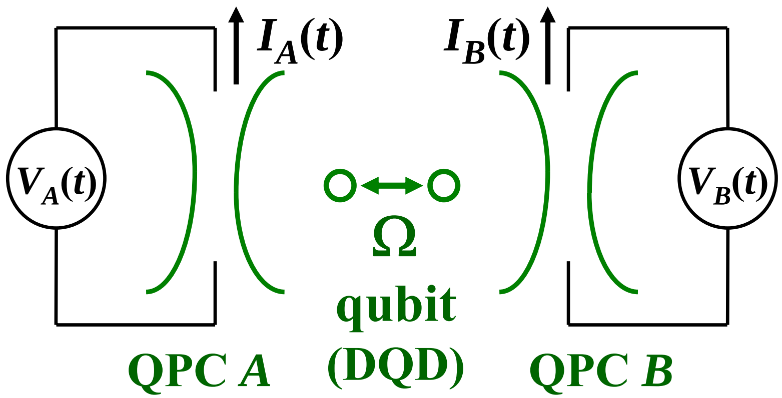
π-shifted combs on V_A and V_B

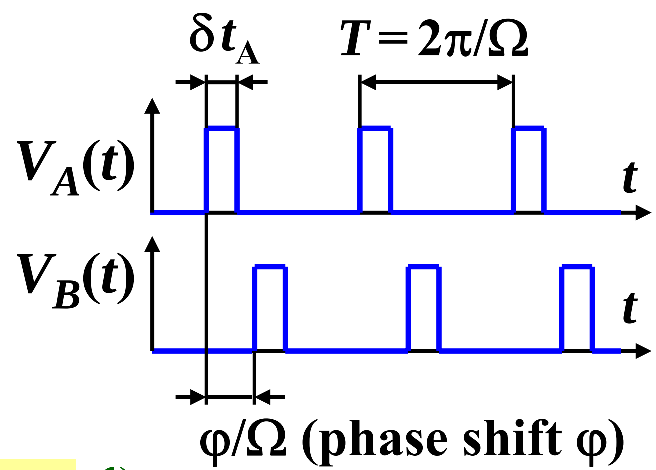


correlation (still QND!)

correlation/anticorrelation between low-frequency (telegraph) noises indicates presence of persistent Rabi oscillations

Analytical results for current noise





$$S_{IA}(\omega) \simeq \underbrace{S_A \frac{\delta t_A}{T}}_{\text{shot noise}} + \underbrace{\left(\frac{\delta t_A}{T}\right)^2 (\Delta I_A)^2 \frac{1/2\Gamma_S}{1+(\omega/2\Gamma_S)^2}}_{\text{telegraph noise}}$$

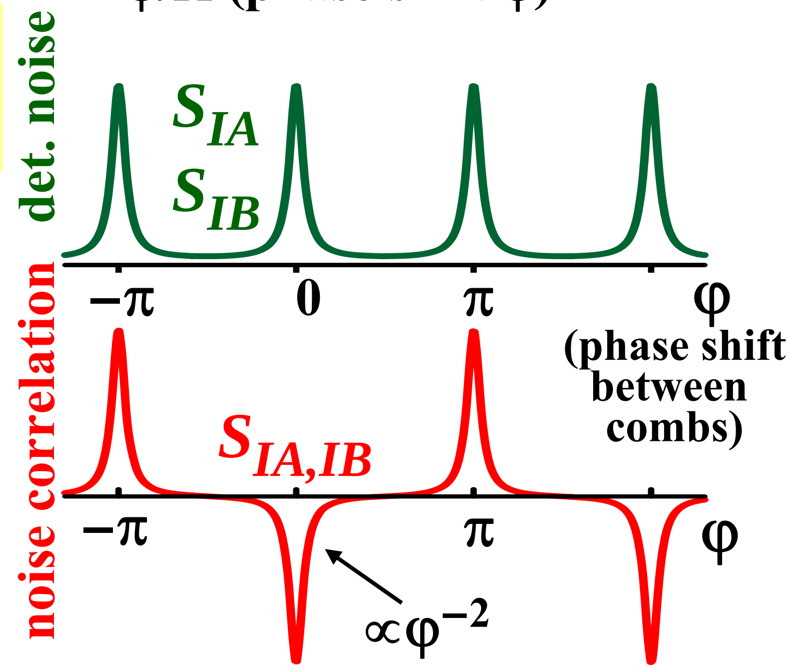
$$S_{IA,IB}(\omega) \simeq \pm \frac{\delta t_A}{T} \frac{\delta t_B}{T} \Delta I_A \, \Delta I_B \frac{1/2\Gamma_S}{1+(\omega/2\Gamma_S)^2}$$

(fully correlated/anticorrelated in first approx.)

$$\Gamma_S \simeq \frac{1}{4T_2} + \frac{\Omega}{4\pi}\left[\varphi^2 \frac{M_A M_B}{M_A+M_B} + \frac{\delta t_A^2 M_A + \delta t_B^2 M_B}{12\, T^2}\right]$$

$M_{A,B} = \delta t_{A,B} (\Delta I_{A,B})^2 / 4S_{A,B}, \;\; S_{A,B} = 2eI_{A,B}(1-T_{A,B})$

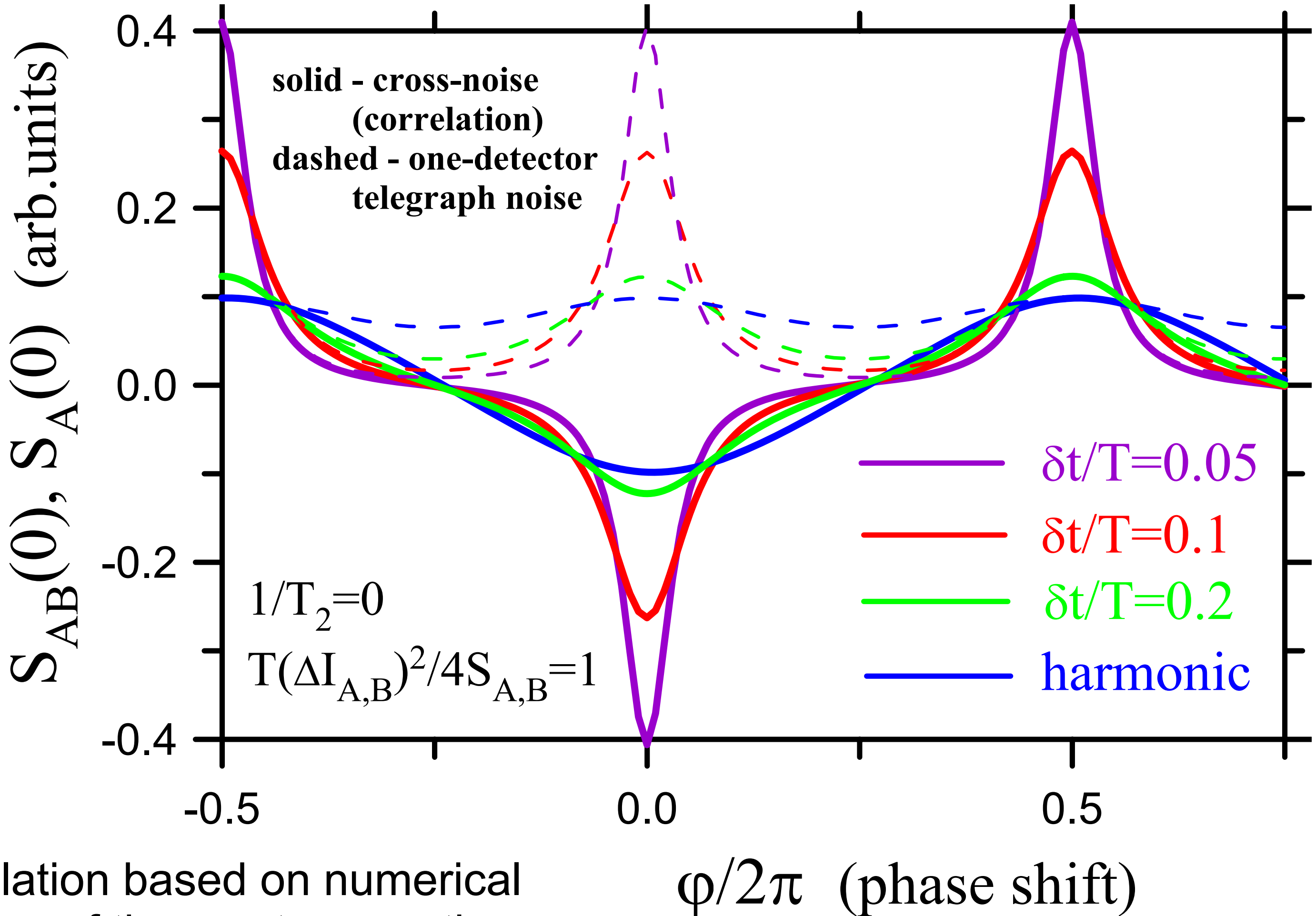
Assumed: $\varphi \ll 1, \;\; \delta t \ll T, \;\; \delta t \ll 4S/(\Delta I)^2, \;\; T_2 \gg T$



Correlat. factor $K \approx \pm 1$

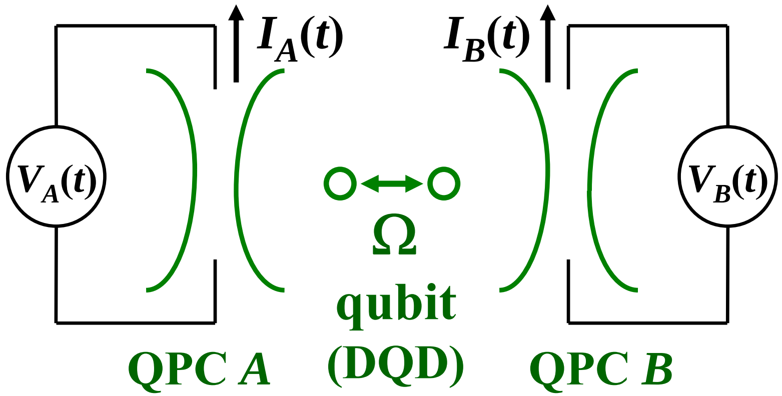
Numerical results

Low-frequency telegraph noise (dashed) and cross-noise (solid)



Calculation based on numerical solution of the master equation

Estimates



Assume:

QPC current $I = 100$ nA

response $\Delta I/I = 0.1$

duty cycle δt/T=0.2 (symmetric)

Rabi frequency ~ 2 GHz

Then:

"attraction" (collapse) time 1.5 ns (few Rabi periods)

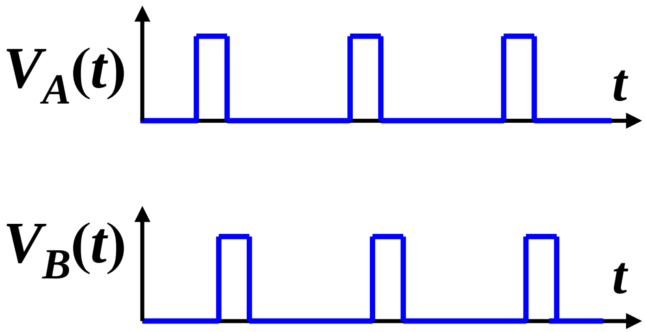
switching rate $\Gamma_s \simeq \frac{1}{4T_2} + \frac{1}{1\,\mu\mathrm{s}} + \frac{\varphi^2}{13\,\mathrm{ns}}$ (many Rabi periods)

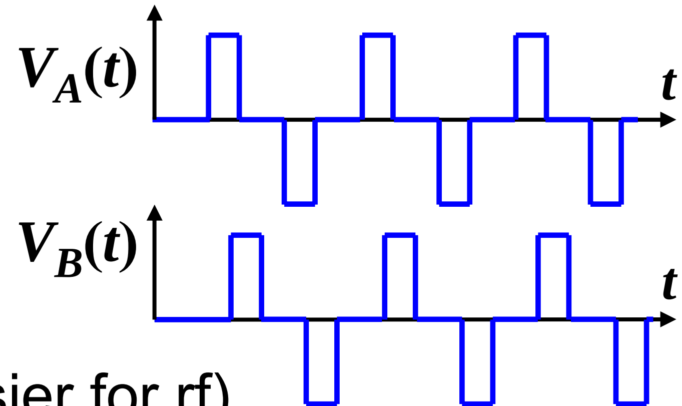
need $T_2 > 10\,\mathrm{ns}$

$$\frac{S_{\text{telegraph}}}{S_{\text{shot}}} \simeq 600 \times \min\left(\frac{T_2}{250\,\mathrm{ns}}, 1\right)$$ (relatively large noise signal)

seems to be reasonable and doable

Useful modification

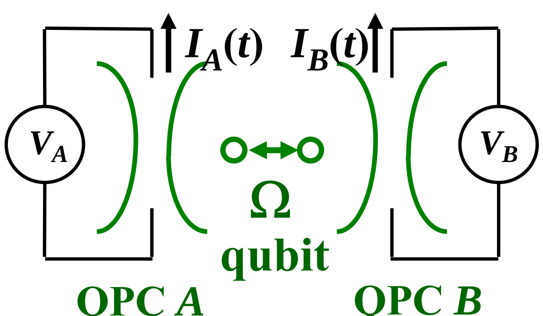


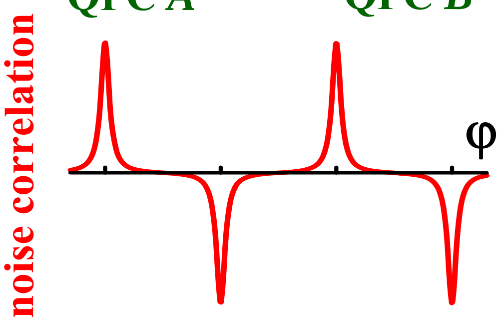


(zero average, easier for rf)

(harmonic rf is also OK)

Any alternative explanation?





1) no oscillations – then no corr./anticorr.

2) unsynchronized Rabi oscillations – then different dependence on φ (cos φ instead of φ^{-2}); also $\int S_{telegr}(f)\,df$ at least twice smaller

3) resonant frequency - driven Rabi?
Then oscillations between |g⟩ and |e⟩ (both do not give a signal) with different frequency. Driven Rabi decreases corr./anticorr. (not an alternative explanation, but should be avoided)

Good news: both phases insensitive to driven Rabi

Conclusions

- It is easy to see what is "inside" collapse: simple Bayesian formalism works for many solid-state setups
- Rabi oscillations are persistent if weakly measured
- Collapse can sometimes be undone (uncollapsing)
- Three direct solid-state experiments have been realized
- Many interesting experimental proposals are still waiting

  Two last proposals:

  - suppression of T_1-decoherence by uncollapsing
  - persistent Rabi oscillations revealed via noise correlation in two detectors