

Non-projective measurement of solid-state qubits: theory and experiments

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Outline:

- Brief introduction
- Bayesian theory of quantum measurements, various derivations
- Confirming experiments: partial collapse, uncollapsing, persistent Rabi oscillations

Ackn.:

R. Ruskov, A. Jordan (theory)
N. Katz, J. Martinis, P. Bertet (expt.)

Support:



Quantum collapse due to measurement – most puzzling part of quantum mechanics since 1920s

First puzzle: nonlocality (EPR, Bell inequality)
($\Rightarrow$ collapse cannot in principle be described by Schr. Eq.)

Puzzle solved (we know correct result, though still philosophical questions). Now discussed in practically all modern textbooks.

Second puzzle: what is “inside” collapse (if stopped half-way)?
(nothing is instantaneous, matter of time scale)

Now we know the answer (at least for some simple systems)
(Hopefully will be in textbooks few decades later)

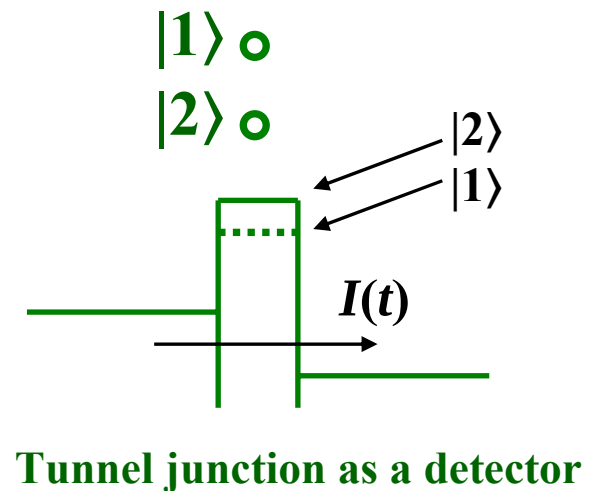
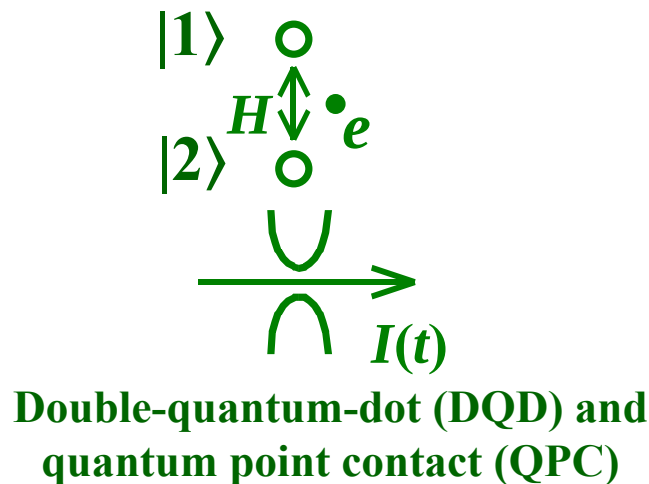
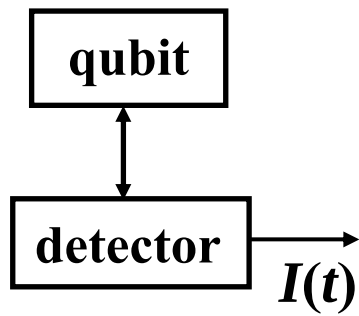
In this talk: Bayesian theory for measurement
of solid-state qubits (1998) (+ 3 experiments)

However, many similar theories much earlier (POVM, quant. trajet., etc.):

Alexander Holevo, Viatcheslav Belavkin, Michael Mensky,
Davies, Kraus, Caves, Gardiner, Walls, Gisin, Carmichael, Milburn, etc.



The system: qubit + detector



$$H = H_{\text{QB}} + H_{\text{DET}} + H_{\text{INT}}$$

$$H_{\text{QB}} = (\varepsilon/2)(c_1^\dagger c_1 - c_2^\dagger c_2) + H(c_1^\dagger c_2 + c_2^\dagger c_1) \quad \varepsilon - \text{asymmetry, } H - \text{tunneling}$$

$$H_{\text{DET}} = \sum_l E_l a_l^\dagger a_l + \sum_r E_r a_r^\dagger a_r + \sum_{l,r} T(a_r^\dagger a_l + a_l^\dagger a_r)$$

$$H_{\text{INT}} = \sum_{l,r} \Delta T (c_1^\dagger c_1 - c_2^\dagger c_2)(a_r^\dagger a_l + a_l^\dagger a_r) \quad S_I = 2eI \quad (\text{Gurvitz, 1997})$$

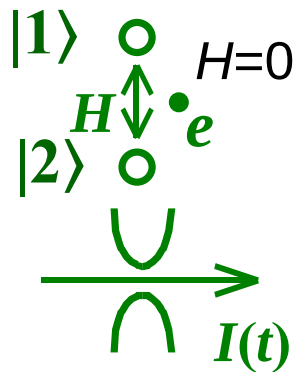
Two levels of average detector current: I_1 for qubit state $|1\rangle$, I_2 for $|2\rangle$

Response: $\Delta I = I_1 - I_2$ Detector noise: white, spectral density S_I

How the qubit state evolves in the process of measurement?

Bayesian formalism for DQD-QPC system

Qubit evolution due to measurement (quantum back-action):



$$\psi(t) = \alpha(t) |1\rangle + \beta(t) |2\rangle \quad \text{or} \quad \rho_{ij}(t)$$

- 1) $|\alpha(t)|^2$ and $|\beta(t)|^2$ evolve as probabilities,
i.e. according to the **Bayes rule** (same for ρ_{ij})
- 2) phases of $\alpha(t)$ and $\beta(t)$ do not change
(**no decoherence!**), $\rho_{12}/(\rho_{11} \rho_{22})^{1/2} = \text{const}$

(A.K., 1998)

Bayes rule (1763, Laplace-1812):

$$\underbrace{P(A_i | \text{res})}_{\text{posterior probability}} = \frac{\underbrace{P(A_i)}_{\text{prior probab.}} \underbrace{P(\text{res} | A_i)}_{\text{likelihood}}}{\sum_k P(A_k) P(\text{res} | A_k)}$$

So simple because:

- 1) QPC happens to be an ideal detector
- 2) no Hamiltonian evolution of the qubit

Large $t \Rightarrow$ textbook projection
Average over result $\Rightarrow$ decoherence

Add “physical” evolution (Hamiltonian, classical back-action, decoherence):

$$d\rho_{11}/dt = -d\rho_{22}/dt = -2H \text{Im} \rho_{12} + \rho_{11}\rho_{22} (2\Delta I / S_I) [\underline{I(t)} - I_0]$$

$$d\rho_{12}/dt = i\varepsilon\rho_{12} + iH(\rho_{11} - \rho_{22}) + \rho_{12}(\rho_{11} - \rho_{22}) (\Delta I / S_I) [\underline{I(t)} - I_0] + iK[\underline{I(t)} - I_0]\rho_{12} - \gamma\rho_{12}$$

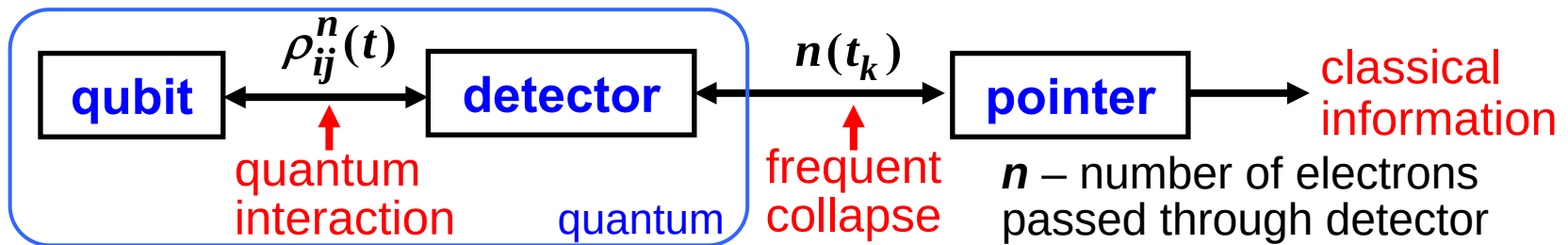


Assumptions needed for the Bayesian formalism:

- Detector voltage is much larger than the qubit energies involved
 $eV \gg \hbar\Omega, eV \gg \hbar\Gamma, \hbar/eV \ll (1/\Omega, 1/\Gamma)$
(no coherence in the detector, classical output, Markovian approximation)
- Simpler if weak response, $|\Delta| \ll I_0$, (coupling $C \sim \Gamma/\Omega$ is arbitrary)

Derivations:

- 1) “logical”: via correspondence principle and comparison with decoherence approach (A.K., 1998)
- 2) “microscopic”: Schr. eq. + collapse of the detector (A.K., 2000)



- 3) from “quantum trajectory” formalism developed for quantum optics (Goan-Milburn, 2001; also: Wiseman, Sun, Oxtoby, etc.)
- 4) from POVM formalism (Jordan-A.K., 2006)
- 5) (*related*) from Keldysh formalism (Wei-Nazarov, 2007)



“Informational” derivation of the Bayesian formalism

(A.K., 1998)

Step 1. Assume $H = \varepsilon = 0$ (“frozen” qubit).

Since ρ_{12} is not involved, evolution of ρ_{11} and ρ_{22} should be the same as in the classical case, i.e. Bayes formula (correspondence principle).

Step 2. Assume $H = \varepsilon = 0$ and pure initial state: $\rho_{12}(0) = [\rho_{11}(0) \rho_{22}(0)]^{1/2}$.

For any realization $|\rho_{12}(t)| \leq [\rho_{11}(t) \rho_{22}(t)]^{1/2}$. Then averaging over realizations gives $|\rho_{12}^{\text{av}}(t)| \leq \rho_{12}^{\text{av}}(0) \exp[-(\Delta I^2/4S_I) t]$.

Compare with conventional (ensemble) result (Gurvitz-1997, Aleiner et al.)

for QPC: $\rho_{12}^{\text{av}}(t) = \rho_{12}^{\text{av}}(0) \exp[-(\Delta I^2/4S_I) t]$. Exactly the upper bound!

Therefore, pure state remains pure: $\rho_{12}(t) = [\rho_{11}(t) \rho_{22}(t)]^{1/2}$.

Step 3. Account of a mixed initial state

Result: the degree of purity $\rho_{12}(t) / [\rho_{11}(t) \rho_{22}(t)]^{1/2}$ is conserved.

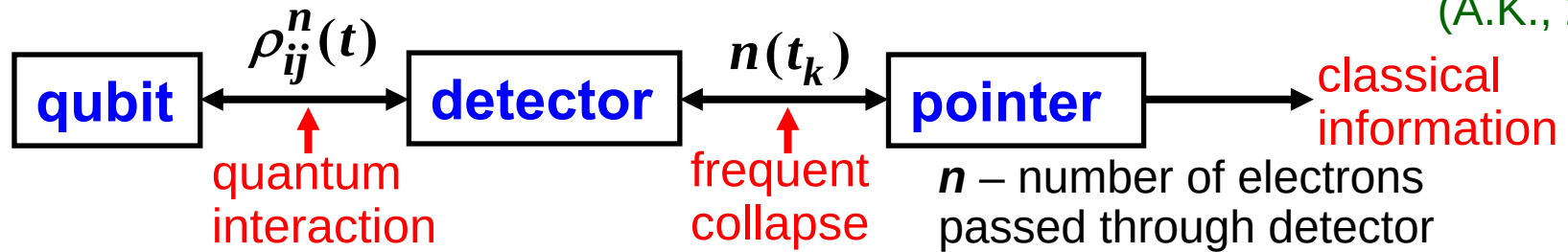
Step 4. Add qubit evolution due to H and ε .

Step 5. Add extra dephasing due to detector nonideality (i.e., for SET).



“Microscopic” derivation of the Bayesian formalism

(A.K., 2000)



Schrödinger evolution of “qubit + detector”
for a low- T QPC as a detector (Gurvitz, 1997)

$$\frac{d}{dt}\rho_{11}^n = -\frac{I_1}{e}\rho_{11}^n + \frac{I_1}{e}\rho_{11}^{n-1} - 2\frac{H}{\hbar}\text{Im}\rho_{12}^n$$

$$\frac{d}{dt}\rho_{22}^n = -\frac{I_2}{e}\rho_{22}^n + \frac{I_2}{e}\rho_{22}^{n-1} + 2\frac{H}{\hbar}\text{Im}\rho_{12}^n$$

$$\frac{d}{dt}\rho_{12}^n = i\frac{\varepsilon}{\hbar}\rho_{12}^n + i\frac{H}{\hbar}(\rho_{11}^n - \rho_{22}^n) - \frac{I_1 + I_2}{2e}\rho_{12}^n + \frac{\sqrt{I_1 I_2}}{e}\rho_{12}^{n-1}$$

If $H = \varepsilon = 0$,
this leads to

$$\rho_{11}(t) = \frac{\rho_{11}(0)P_1(n)}{\rho_{11}(0)P_1(n) + \rho_{22}(0)P_2(n)},$$

$$\rho_{12}(t) = \rho_{12}(0) \frac{[\rho_{11}(t)\rho_{22}(t)]^{1/2}}{[\rho_{11}(0)\rho_{22}(0)]^{1/2}},$$

$$\rho_{22}(t) = \frac{\rho_{22}(0)P_2(n)}{\rho_{11}(0)P_1(n) + \rho_{22}(0)P_2(n)}$$

$$P_i(n) = \frac{(I_i t / e)^n}{n!} \exp(-I_i t / e),$$

Detector collapse at $t = t_k$

Particular n_k is chosen at t_k

$$P(n) = \rho_{11}^n(t_k) + \rho_{22}^n(t_k)$$

$$\rho_{ij}^n(t_k + 0) = \delta_{n,nk} \rho_{ij}^n(t_k + 0)$$

$$\rho_{ij}(t_k + 0) = \frac{\rho_{ij}^{nk}(t_k)}{\rho_{11}^{nk}(t_k) + \rho_{22}^{nk}(t_k)}$$

which are exactly quantum Bayes formulas



Derivation via POVM

(Jordan, A.K., 2006)

General quantum measurement (POVM formalism) (Nielsen-Chuang, p. 85):

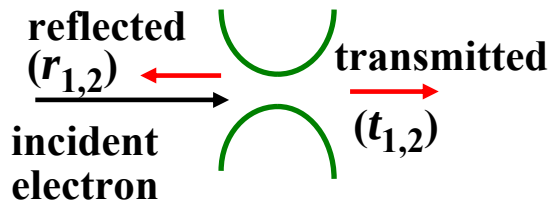
Measurement (Kraus) operator M_r (any linear operator in H.S.): $\psi \rightarrow \frac{M_r \psi}{\|M_r \psi\|}$ or $\rho \rightarrow \frac{M_r \rho M_r^\dagger}{\text{Tr}(M_r \rho M_r^\dagger)}$

Probability: $P_r = \|M_r \psi\|^2$ or $P_r = \text{Tr}(M_r \rho M_r^\dagger)$

Completeness: $\sum_r M_r^\dagger M_r = 1$ (People often prefer linear evolution and non-normalized states)

$|1\rangle \circ$ For each incident electron:
 $|2\rangle \circ$

$$|in\rangle(\alpha|1\rangle + \beta|2\rangle) \rightarrow \alpha(r_1|L\rangle + t_1|R\rangle)|1\rangle + \beta(r_2|L\rangle + t_2|R\rangle)|2\rangle$$



$$M_{\text{refl}} = \begin{pmatrix} r_1 & 0 \\ 0 & r_2 \end{pmatrix}, \quad M_{\text{trans}} = \begin{pmatrix} t_1 & 0 \\ 0 & t_2 \end{pmatrix}$$

For many incident electrons $\Rightarrow$ Bayesian formalism

Relation between POVM and quantum Bayesian formalism:

decomposition $M_r = \underbrace{U_r}_{\text{unitary}} \underbrace{\sqrt{M_r^\dagger M_r}}_{\text{Bayes}}$



Fundamental limit for ensemble decoherence

$$\Gamma = (\Delta I)^2 / 4S_I + \gamma$$

ensemble decoherence rate Γ (indicated by a red arrow)

single-qubit decoherence γ (indicated by a red arrow)

~ rate of information acquisition [bit/s] (indicated by a red arrow)

“measurement time” (S/N=1)
 $\tau_m = 2S_I / (\Delta I)^2$
 (Makhlin, Schon, Shnirman)

$$\Gamma \tau_m \geq \frac{1}{2}$$

$$\gamma \geq 0 \Rightarrow \boxed{\Gamma \geq (\Delta I)^2 / 4S_I}$$

A.K., 1998, 2000
 S. Pilgram et al., 2002
 A. Clerk et al., 2002
 D. Averin, 2003

$$\eta = 1 - \frac{\gamma}{\Gamma} = \frac{(\Delta I)^2 / 4S_I}{\Gamma}$$

detector ideality (quantum efficiency)
 $\eta \leq 100\%$

Translated into energy sensitivity: $(\epsilon_O \epsilon_{BA})^{1/2} \geq \hbar/2$

where ϵ_O is output-noise-limited sensitivity [J/Hz]

and ϵ_{BA} is back-action-limited sensitivity [J/Hz]

Sensitivity limitation is known since 1980s (Clarke, Tesche, Likharev, etc.);
 also Averin-2000, Clerk et al.-2002, Pilgram et al.-2002, etc.



Measurement vs. decoherence

Widely accepted point of view:

measurement = decoherence (environment)

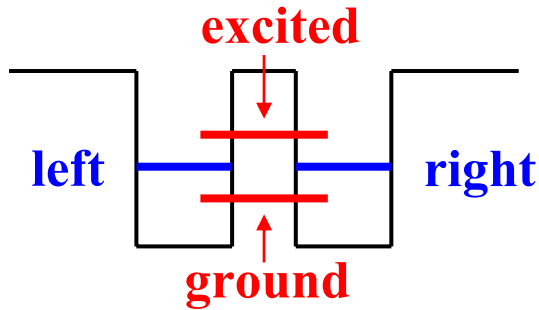
Is it true?

- **Yes**, if not interested in information from detector (ensemble-averaged evolution)
- **No**, if take into account measurement result (single quantum system)

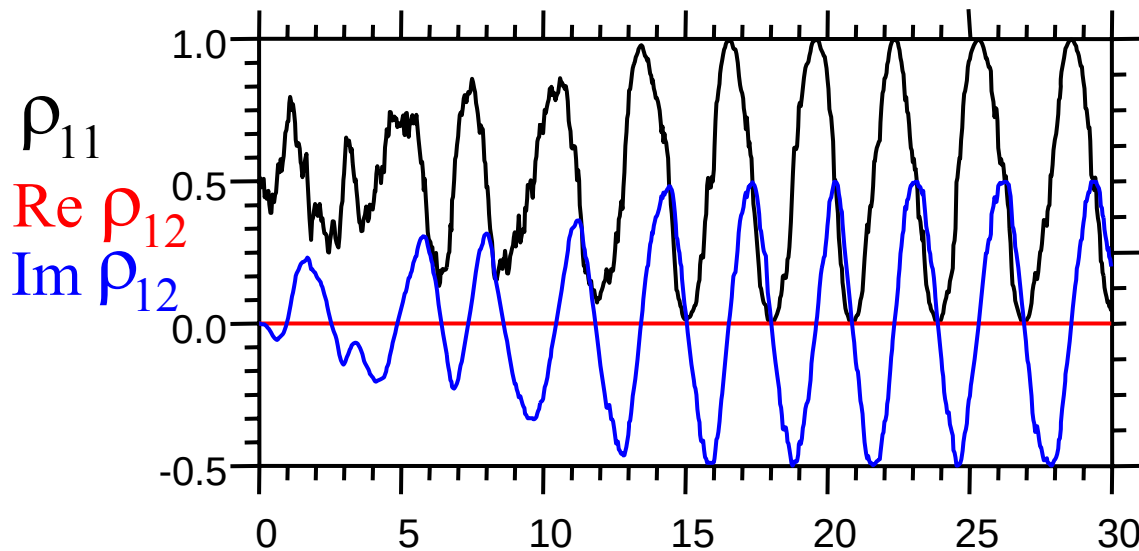
Measurement result obviously gives us more information about the measured system, so we know its quantum state better (ideally, a pure state instead of a mixed state)



Persistent Rabi oscillations



- Relaxes to the ground state if left alone (low-T)
- Becomes fully mixed if coupled to a high-T (non-equilibrium) environment
- Oscillates persistently between left and right if (weakly) measured continuously



Phase of Rabi oscillations fluctuates (phase noise, dephasing)

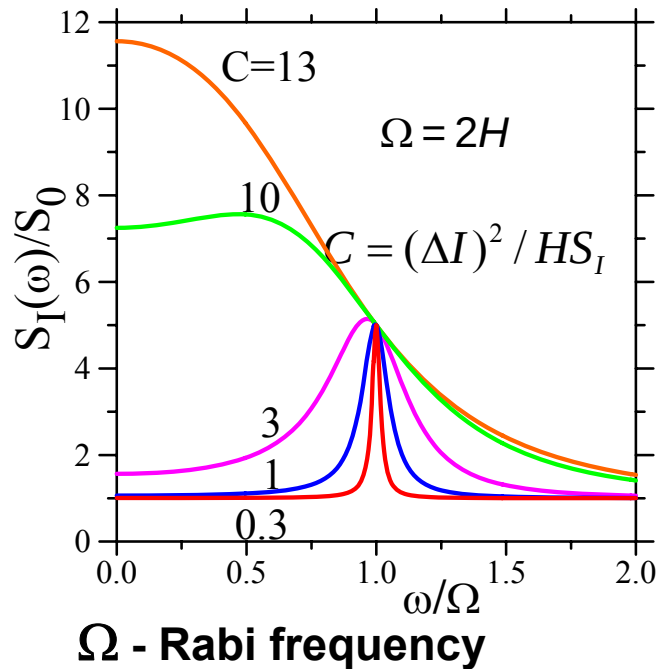
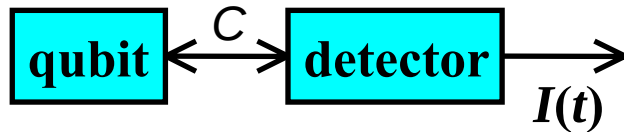
Direct experiment is difficult (quantum efficiency, bandwidth, control)

A.K., 1998



Indirect experiment: spectrum of persistent Rabi oscillations

A.K., LT'1999
A.K.-Averin, 2000



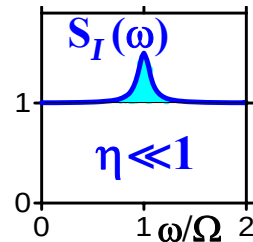
peak-to-pedestal ratio = $4\eta \leq 4$

$$S_I(\omega) = S_0 + \frac{\Omega^2 (\Delta I)^2 \Gamma}{(\omega^2 - \Omega^2)^2 + \Gamma^2 \omega^2}$$

$$I(t) = I_0 + \frac{\Delta I}{2} z(t) + \xi(t)$$

(const + signal + noise)

amplifier noise $\Rightarrow$ higher pedestal,
poor quantum efficiency,
but the peak is the same!!!



integral under the peak $\Leftrightarrow$ variance $\langle z^2 \rangle$

How to distinguish experimentally
persistent from non-persistent? **Easy!**

perfect Rabi oscillations: $\langle z^2 \rangle = \langle \cos^2 \rangle = 1/2$

imperfect (non-persistent): $\langle z^2 \rangle \ll 1/2$

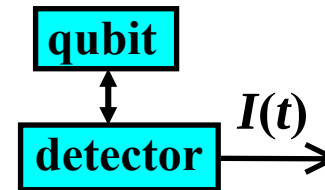
quantum (Bayesian) result: **$\langle z^2 \rangle = 1$ (!!!)**

(demonstrated in Saclay expt.)



How to understand $\langle z^2 \rangle = 1$?

$$I(t) = I_0 + \frac{\Delta I}{2} z(t) + \xi(t)$$



First way (mathematical)

We actually measure operator: $z \rightarrow \sigma_z$

$$z^2 \rightarrow \sigma_z^2 = 1$$

(What does it mean?
Difficult to say...)

Second way (Bayesian)

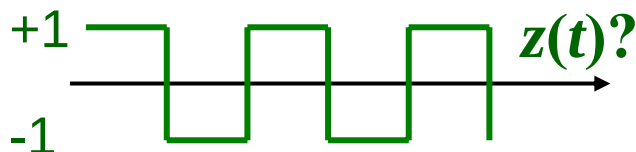
$$S_I(\omega) = S_{\xi\xi} + \frac{\Delta I^2}{4} S_{zz}(\omega) + \frac{\Delta I}{2} S_{\xi z}(\omega)$$



quantum back-action changes z
in accordance with the noise ξ

Equal contributions (for weak
coupling and $\eta=1$)

Can we explain it in a more reasonable way (without spooks/ghosts)?

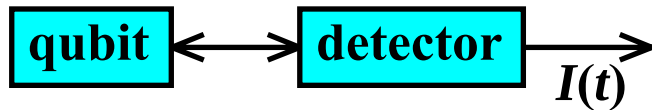


or some other $z(t)$?

No (under assumptions of macrorealism;
Leggett-Garg, 1985)



Leggett-Garg-type inequalities for continuous measurement of a qubit



Ruskov-A.K.-Mizel, PRL-2006
Jordan-A.K.-Büttiker, PRL-2006

Assumptions of macrorealism
(similar to Leggett-Garg'85):

$$I(t) = I_0 + (\Delta I / 2) z(t) + \xi(t)$$

$$|z(t)| \leq 1, \quad \langle \xi(t) z(t + \tau) \rangle = 0$$

Then for correlation function

$$K(\tau) = \langle I(t) I(t + \tau) \rangle$$

$$K(\tau_1) + K(\tau_2) - K(\tau_1 + \tau_2) \leq (\Delta I / 2)^2$$

and for area under narrow spectral peak

$$\int [S_I(f) - S_0] df \leq (8 / \pi^2) (\Delta I / 2)^2$$

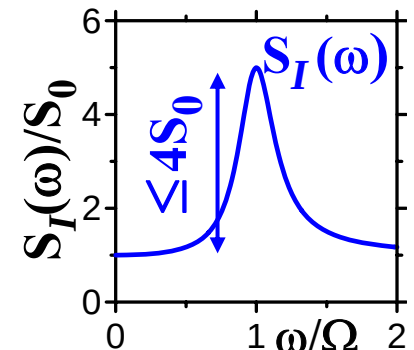
Leggett-Garg, 1985

$$K_{ij} = \langle Q_i Q_j \rangle$$

if $Q = \pm 1$, then

$$1 + K_{12} + K_{23} + K_{13} \geq 0$$

$$K_{12} + K_{23} + K_{34} - K_{14} \leq 2$$



quantum result

$$\frac{3}{2} (\Delta I / 2)^2$$

violation

$$\times \frac{3}{2}$$

$$(\Delta I / 2)^2$$

$$\times \frac{\pi^2}{8}$$

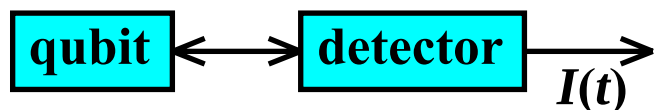
η is not important!

Experimentally measurable violation

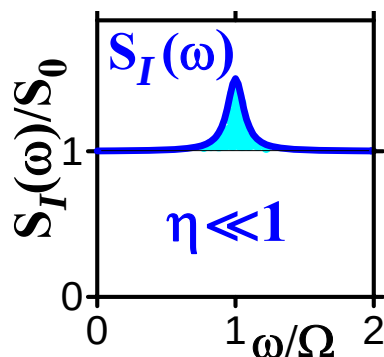
(Saclay experiment)



May be a physical (realistic) back-action?



$$I(t) = I_0 + \frac{\Delta I}{2} z(t) + \xi(t)$$



OK, cannot explain without back-action

$$\langle \xi(t) z(t + \tau) \rangle \neq 0$$

But may be this is a simple classical back-action from the noise?

In principle, classical explanation cannot be ruled out (e.g. computer-generated $I(t)$; no non-locality as in optics)

Try reasonable models: linear modulation of the qubit parameters (H and ε) by noise $\xi(t)$



No, does not work!

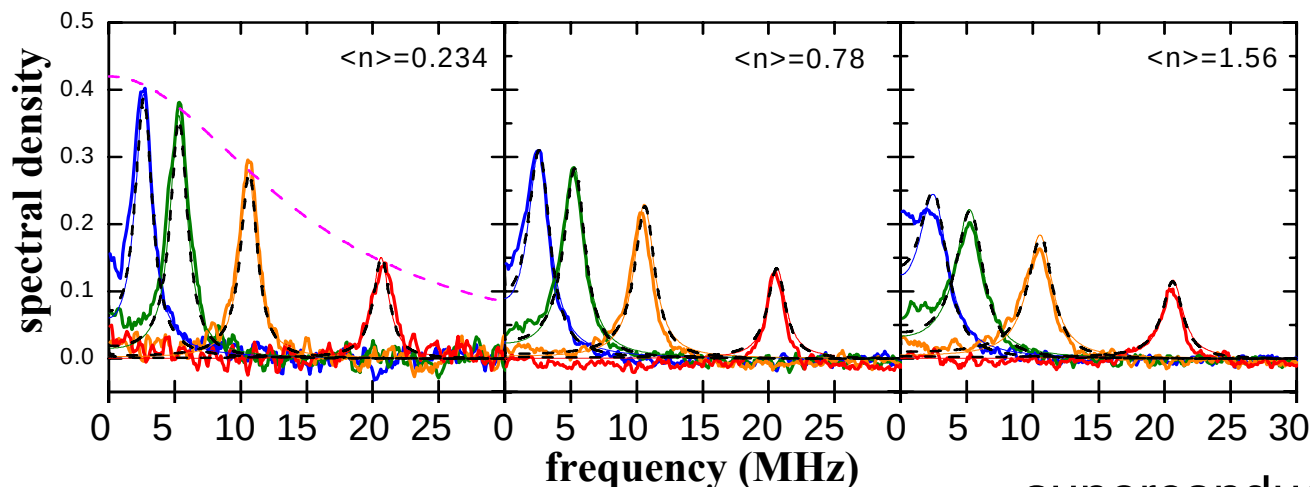
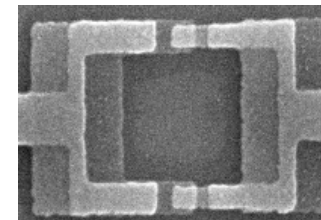
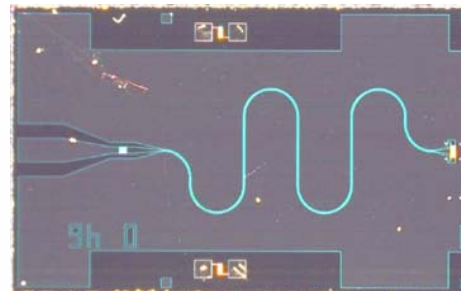
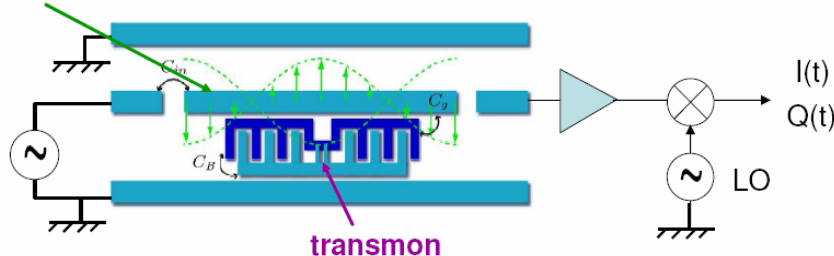
Our (spooky) back-action is quite peculiar: $\langle \xi(t) dz(t + 0) \rangle > 0$

“what you see is what you get”: observation becomes reality



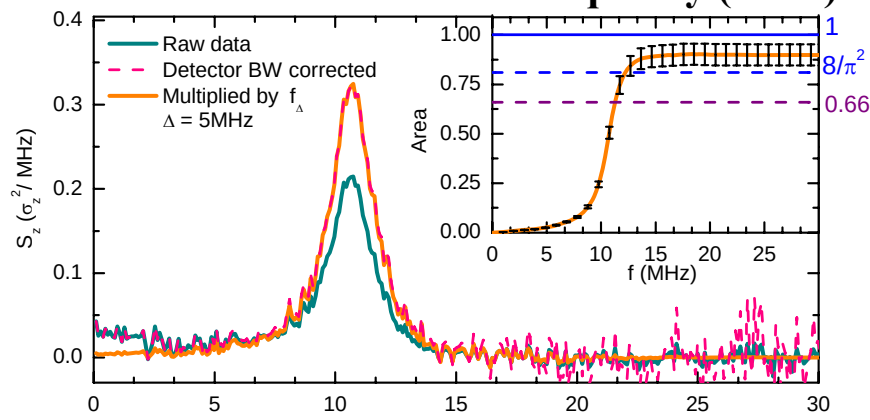
Recent experiment (Saclay group, unpub.)

Stripline resonator



A. Palacios-Laloy et al.
(unpublished)

courtesy of
Patrice Bertet



- superconducting charge qubit (transmon) in circuit QED setup (microwave reflection from cavity)
- driven Rabi oscillations

- perfect spectral peaks
- LGI violation



Previous experimental confirmation?

Durkan and Welland, 2001 (STM-ESR experiment similar to Manassen-1989)

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Electronic spin detection in molecules using scanning-tunneling-microscopy-assisted electron-spin resonance

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(Received 8 May 2001; accepted for publication 8 November 2001)

By combining the spatial resolution of a scanning-tunneling microscope (STM) with the electronic spin sensitivity of electron-spin resonance, we show that it is possible to detect the presence of localized spins on surfaces. The principle is that a STM is operated in a magnetic field, and the resulting component of the tunnel current at the Larmor (precession) frequency is measured. This component is nonzero whenever there is tunneling into or out of a paramagnetic entity. We have

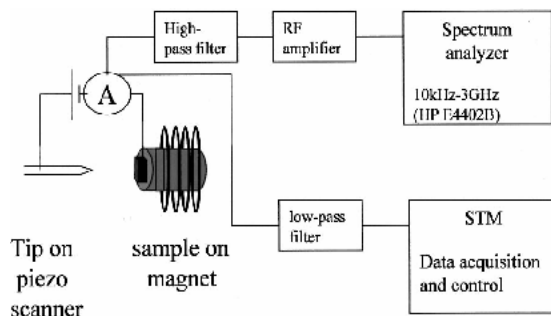


FIG. 1. Schematic of the electronics used in STM-ESR.

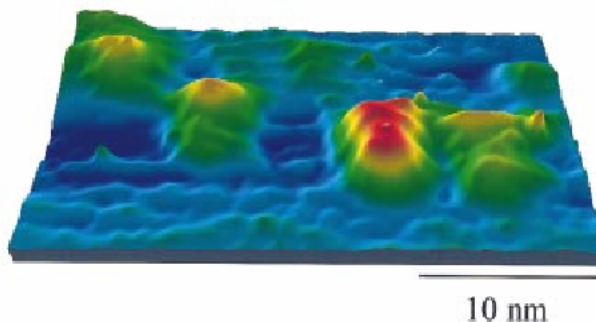


FIG. 2. (Color) STM image of a 250 Å x 150 Å area of HOPG with four adsorbed BDPA molecules.

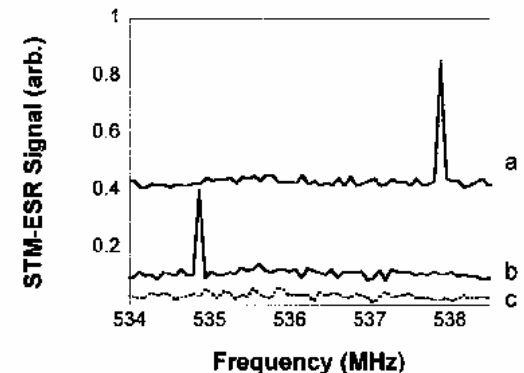


FIG. 3. STM-ESR spectra of (a), (b) two different areas (a few nm apart) of the molecule-covered sample and (c) bare HOPG. The graphs are shifted vertically for clarity.

$$\frac{\text{peak}}{\text{noise}} \leq 3.5$$

(Colm Durkan,
private comm.)

Questionable

Recently reproduced:
Messina et al., JAP-2007



Somewhat similar experiment

“Continuous monitoring of Rabi oscillations in a Josephson flux qubit”

$$H = -\frac{1}{2}(\Delta \sigma_x + \varepsilon \sigma_z) - W \sigma_z \cos \omega_{HF} t$$

$$(\omega_{HF} \approx \sqrt{\Delta^2 + \varepsilon^2}; \varepsilon \neq 0)$$

E. Il'ichev et al., PRL, 2003

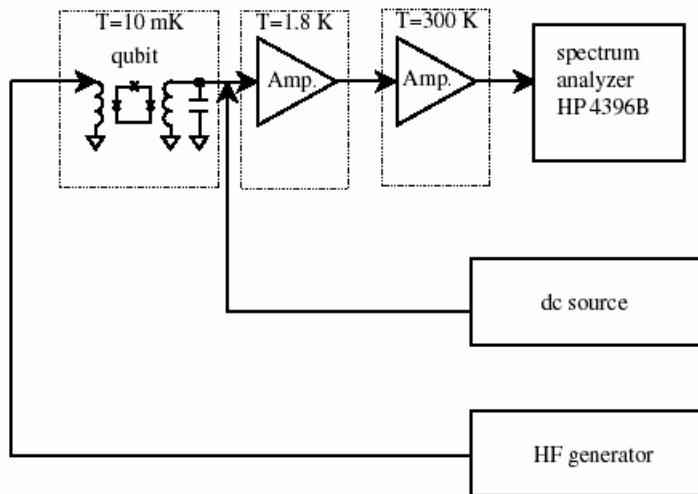


FIG. 1. Measurement setup. The flux qubit is inductively coupled to a tank circuit. The dc source applies a constant flux $\Phi_e \approx \frac{1}{2}\Phi_0$. The HF generator drives the qubit through a separate coil at a frequency close to the level separation $\Delta/h = 868$ MHz. The output voltage at the resonant frequency of the tank is measured as a function of HF power.

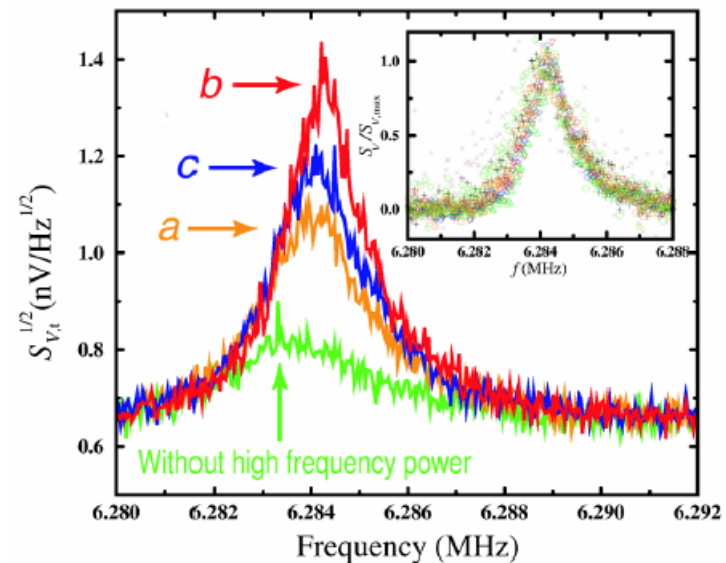


FIG. 3 (color online). The spectral amplitude of the tank voltage for HF powers $P_a < P_b < P_c$ at 868 MHz, detected using the setup of Fig. 1. The bottom curve corresponds to the background noise without an HF signal. The inset shows normalized voltage spectra for seven values of HF power, with background subtracted. The shape of the resonance, being determined by the tank circuit, is essentially the same in each

low-bandwidth tank $\Rightarrow$ qubit monitoring is impossible

Quantum uncollapsing (undoing a weak measurement of a qubit)

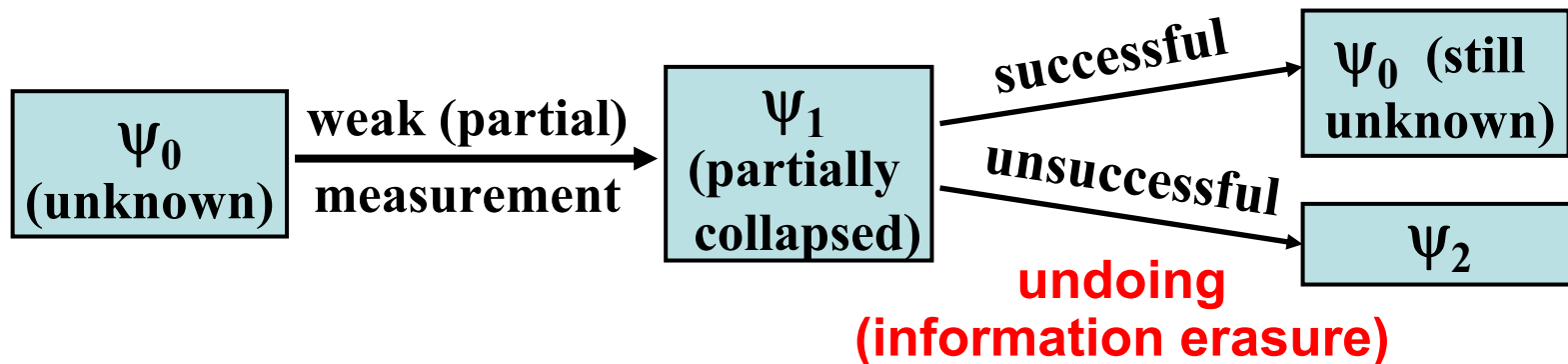
A.K. & Jordan, PRL-2006



It is impossible to undo “orthodox” quantum measurement (for an unknown initial state)

Is it possible to undo partial quantum measurement?
Yes! (but with a finite probability)

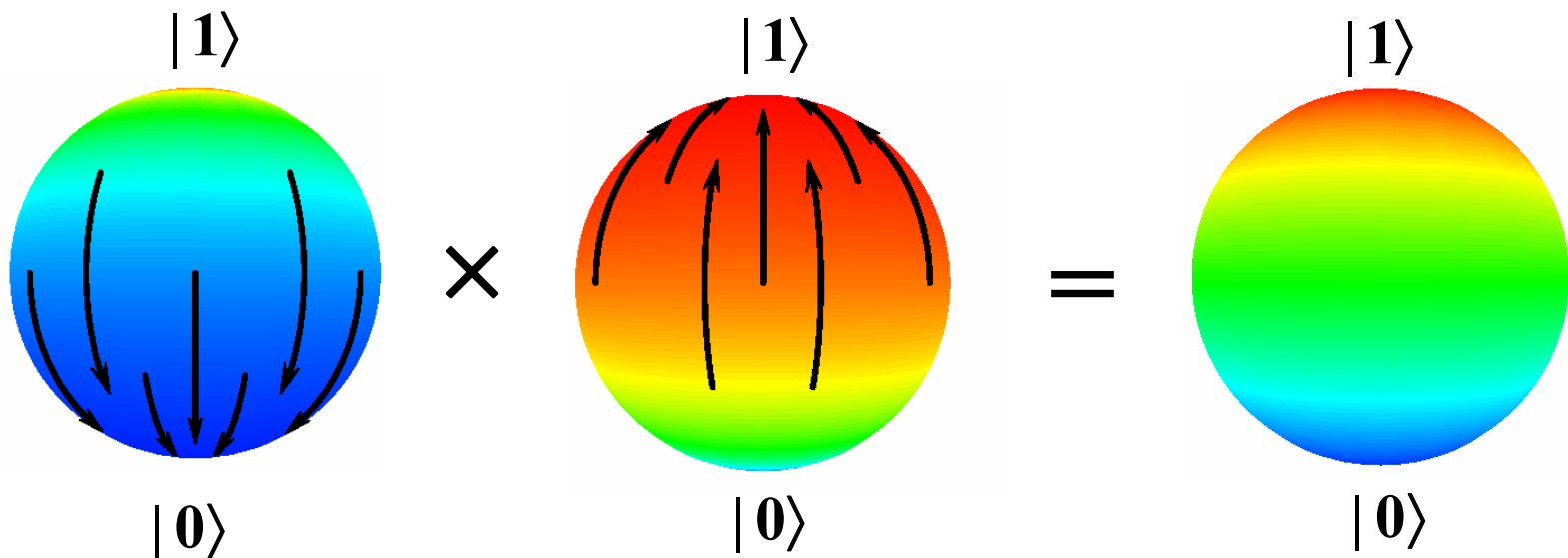
If undoing is successful, an unknown state is **fully** restored



Uncollapsing of a qubit state

Evolution due to partial (weak, continuous, etc.) measurement is **non-unitary** (though coherent if detector is good!), therefore it is impossible to undo it by Hamiltonian dynamics.

How to undo? One more measurement!



need ideal (quantum-limited) detector

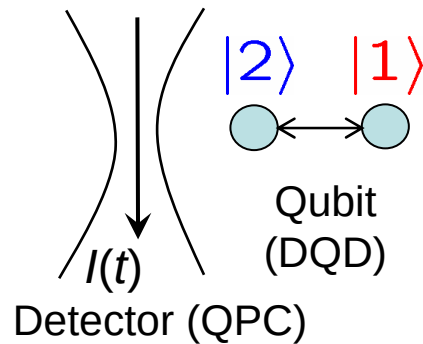
(similar to Koashi-Ueda, PRL-1999)

(Figure partially adopted from
Jordan-A.K.-Büttiker, PRL-06)



Uncollapsing for DQD-QPC system

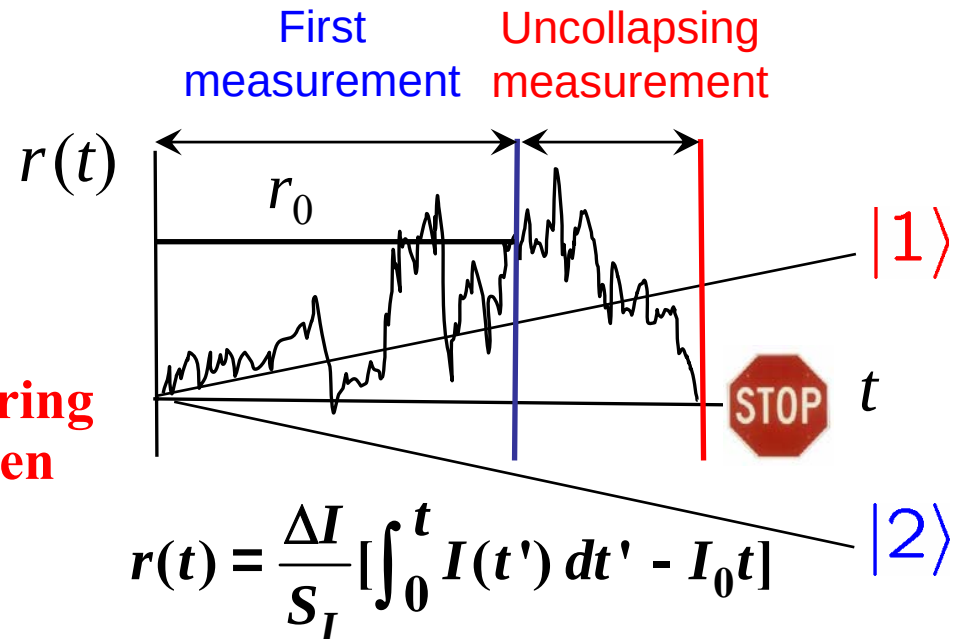
A.K. & Jordan



Simple strategy: continue measuring until result $r(t)$ becomes zero. Then any initial state is fully restored.

(same for an entangled qubit)

It may happen though that $r = 0$ never crossed; then undoing procedure is unsuccessful.



Probability of success:

$$P_s = \frac{e^{-|r_0|}}{e^{|r_0|} \rho_{11}(0) + e^{-|r_0|} \rho_{22}(0)}$$

Averaged probability of success (over result r_0):

$$P_{av} = 1 - \text{erf}[\sqrt{t/2T_m}], \quad T_m = 2S_I / (\Delta I)^2$$

(does not depend on initial state)



General theory of uncollapsing

POVM formalism
(Nielsen-Chuang, p.85)

Measurement operator M_r : $\rho \rightarrow \frac{M_r \rho M_r^\dagger}{\text{Tr}(M_r \rho M_r^\dagger)}$

Probability: $P_r = \text{Tr}(M_r \rho M_r^\dagger)$ Completeness: $\sum_r M_r^\dagger M_r = 1$

Uncollapsing operator: $C \times M_r^{-1}$ (to satisfy completeness, eigenvalues cannot be >1)

$\max(C) = \min_i \sqrt{p_i}$, p_i – eigenvalues of $M_r^\dagger M_r$

Probability of success: $P_s \leq \frac{\min P_r}{P_r(\rho_{\text{in}})}$

$P_r(\rho_{\text{in}})$ – probability of result r for initial state ρ_{in} ,

$\min P_r$ – probability of result r minimized over all possible initial states

Averaged (over r) probability of success: $P_{av} \leq \sum_r \min P_r$

(cannot depend on initial state, otherwise get information)

(similar to Koashi-Ueda, 1999)



Quantum erasers in optics

Quantum eraser proposal by Scully and Drühl, PRA (1982)

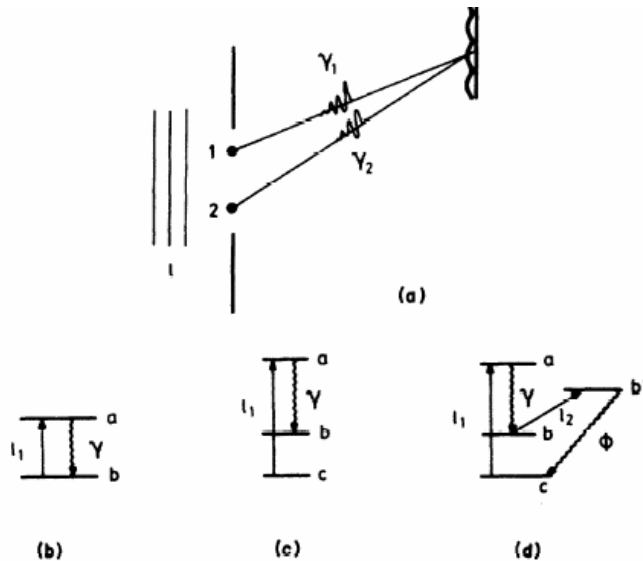


FIG. 1. (a) Figure depicting light impinging from left on atoms at sites 1 and 2. Scattered photons γ_1 and γ_2 produce interference pattern on screen. (b) Two-level atoms excited by laser pulse l_1 , and emit γ photons in $a \rightarrow b$ transition. (c) Three-level atoms excited by pulse l_1 from $c \rightarrow a$ and emit photons in $a \rightarrow b$ transition. (d) Four-level system excited by pulse l_1 from $c \rightarrow a$ followed by emission of γ photons in $a \rightarrow b$ transition. Second pulse l_2 takes atoms from $b \rightarrow b'$. Decay from $b' \rightarrow c$ results in emission of ϕ photons.

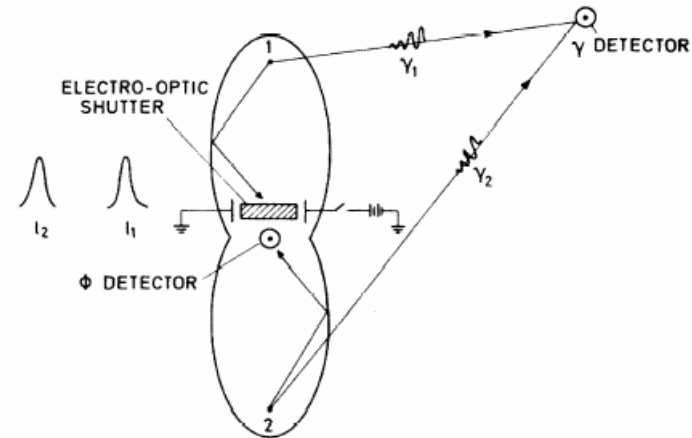


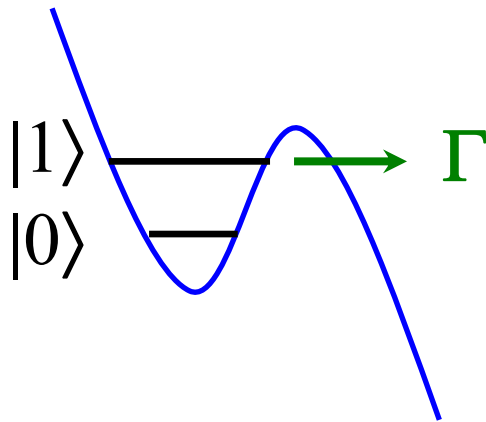
FIG. 2. Laser pulses l_1 and l_2 incident on atoms at sites 1 and 2. Scattered photons γ_1 and γ_2 result from $a \rightarrow b$ transition. Decay of atoms from $b' \rightarrow c$ results in ϕ photon emission. Elliptical cavities reflect ϕ photons onto common photodetector. Electro-optic shutter transmits ϕ photons only when switch is open. Choice of switch position determines whether we emphasize particle or wave nature of γ photons.

Interference fringes restored for two-detector correlations (since “which-path” information is erased)

Our idea of uncollapsing is quite different:
we really extract information and then erase it

Partial collapse of a phase qubit

N. Katz, M. Ansmann, R. Bialczak, E. Lucero,
R. McDermott, M. Neeley, M. Steffen, E. Weig,
A. Cleland, J. Martinis, A. Korotkov, Science-06



**How does a coherent state evolve
in time before tunneling event?**

(What happens when nothing happens?)

Qubit “ages” in contrast to a radioactive atom!

Main idea:

$$\psi = \alpha |0\rangle + \beta |1\rangle \rightarrow \psi(t) = \begin{cases} |out\rangle, & \text{if tunneled} \\ \frac{\alpha |0\rangle + \beta e^{-\Gamma t/2} e^{i\varphi} |1\rangle}{\sqrt{|\alpha|^2 + |\beta|^2 e^{-\Gamma t}}}, & \text{if not tunneled} \end{cases}$$

(better theory: Leonid Pryadko & A.K., 2007)

amplitude of state $|0\rangle$ grows without physical interaction

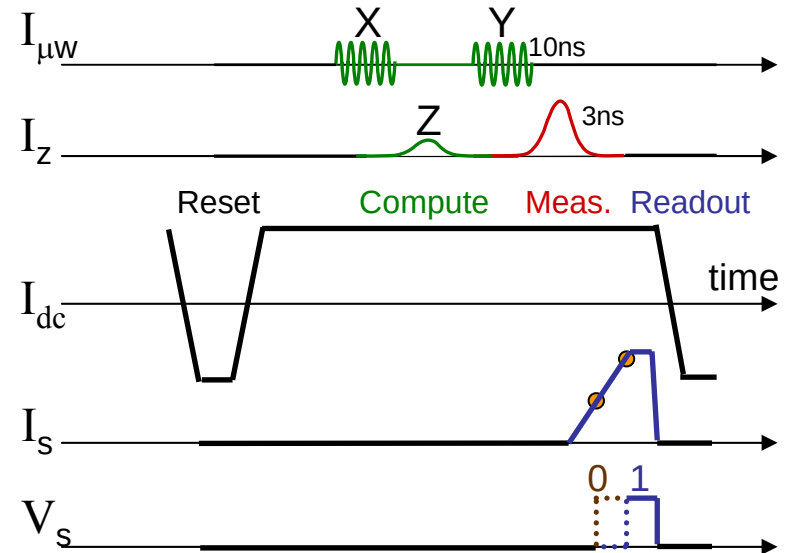
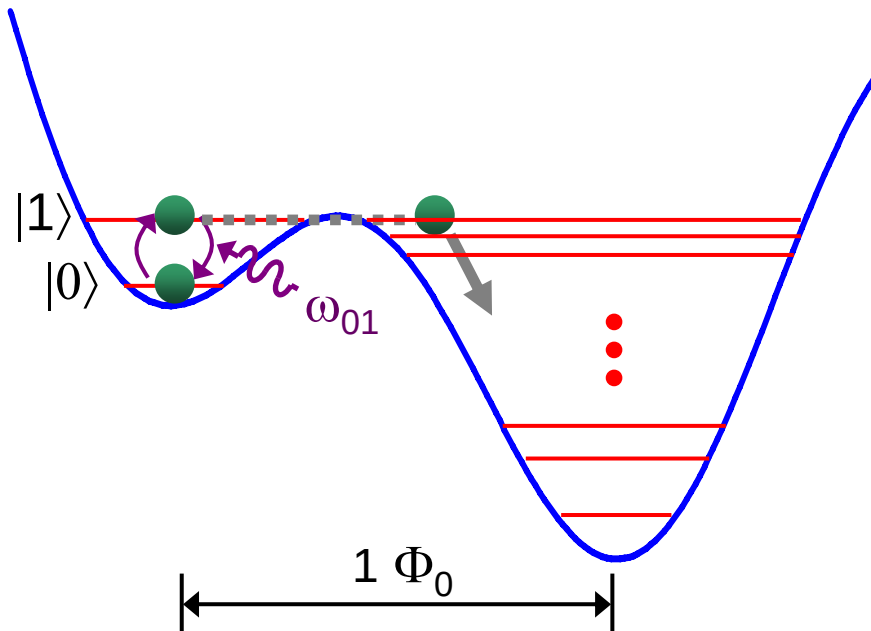
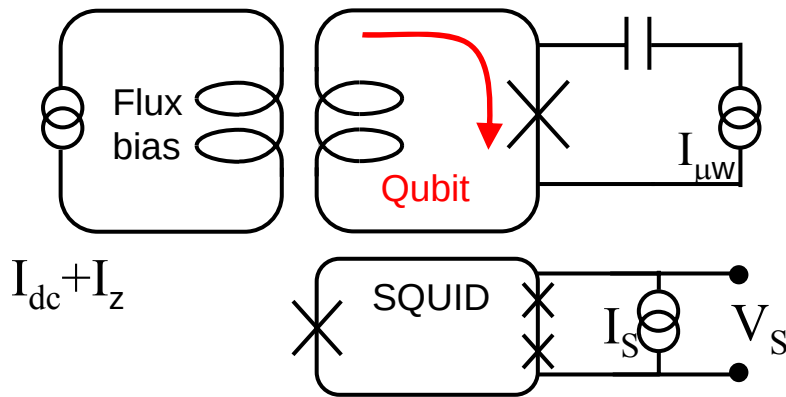
continuous null-result collapse

(similar to optics, Dalibard-Castin-Molmer, PRL-1992)

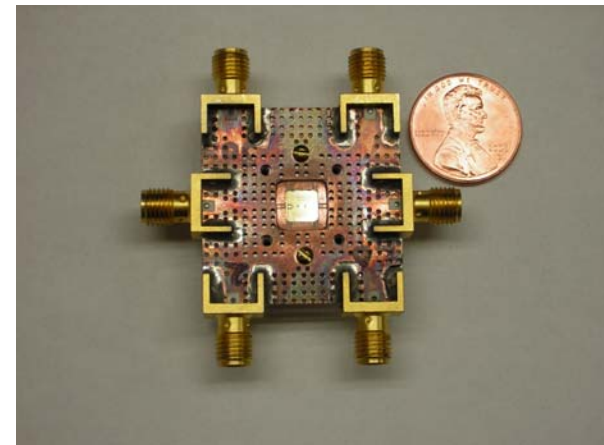


Superconducting phase qubit at UCSB

Courtesy of Nadav Katz (UCSB)



Repeat 1000x
prob. 0,1



Experimental technique for partial collapse

Nadav Katz et al.
(John Martinis' group)

Protocol:

- 1) State preparation by applying microwave pulse (via Rabi oscillations)**
- 2) Partial measurement by lowering barrier for time t**
- 3) State tomography (microwave + full measurement)**

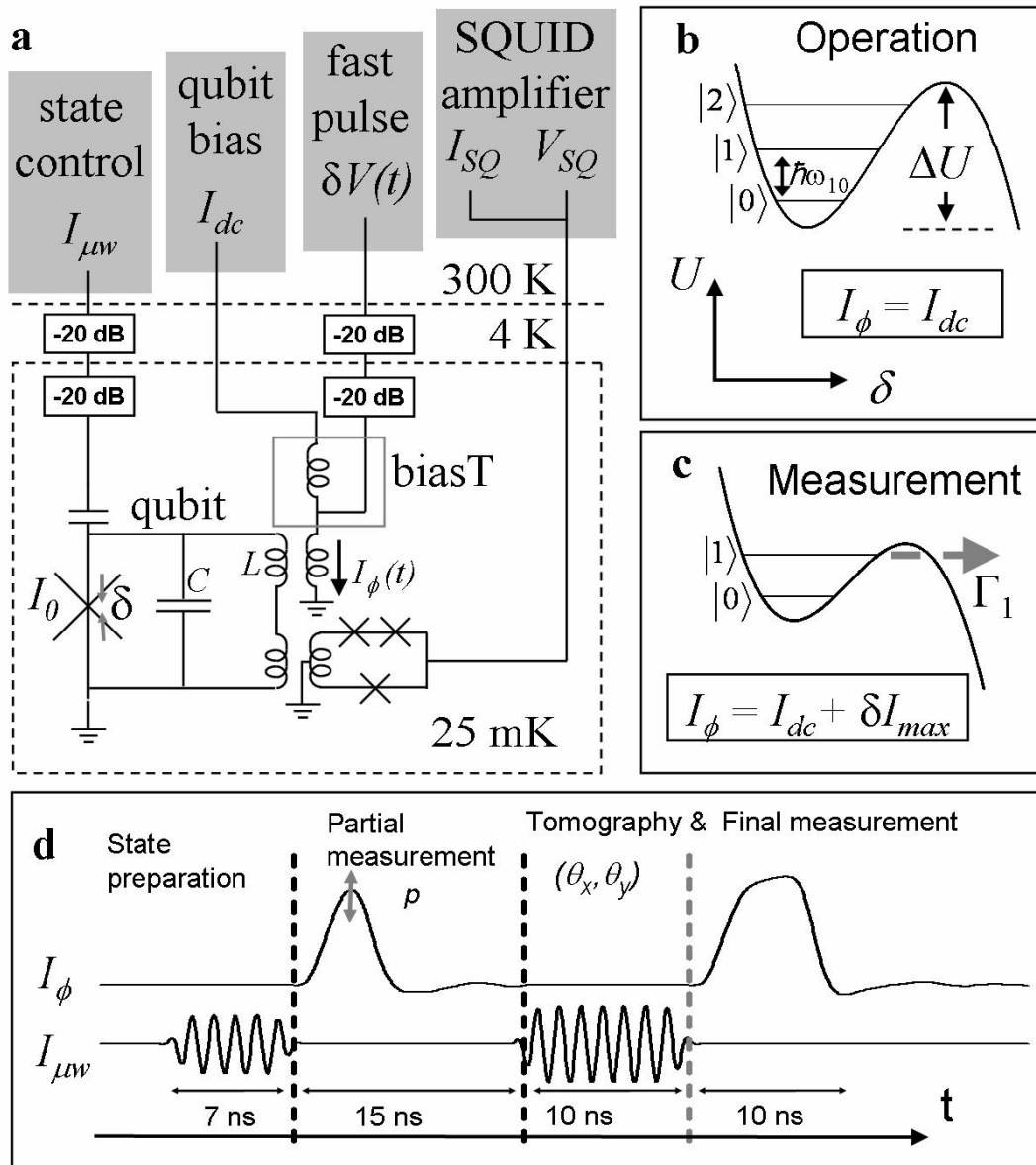
Measurement strength

$$p = 1 - \exp(-\Gamma t)$$

is actually controlled
by Γ , not by t

$p=0$: no measurement

$p=1$: orthodox collapse

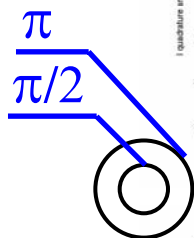
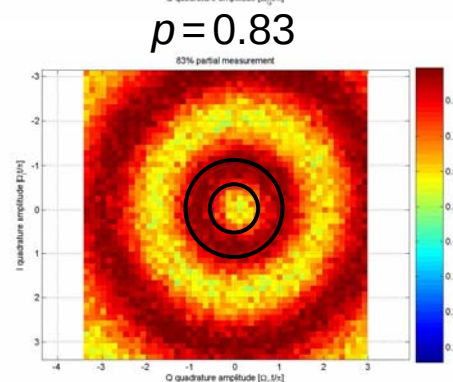
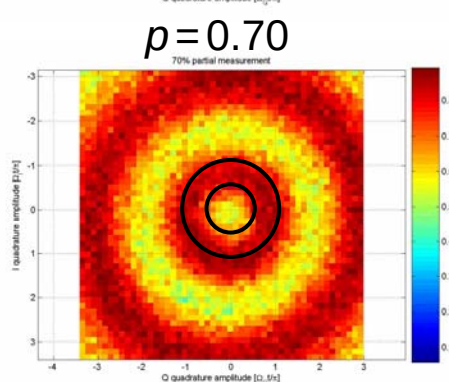
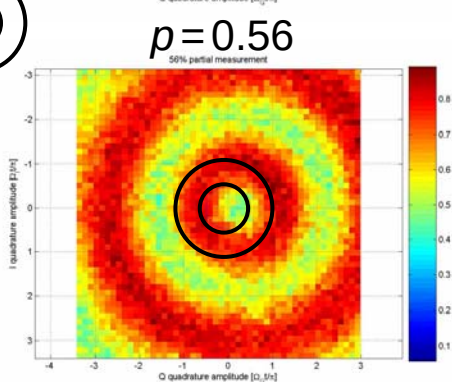
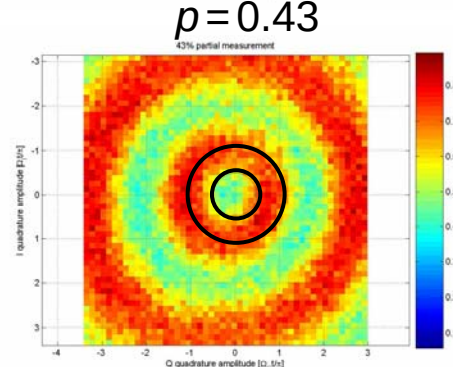
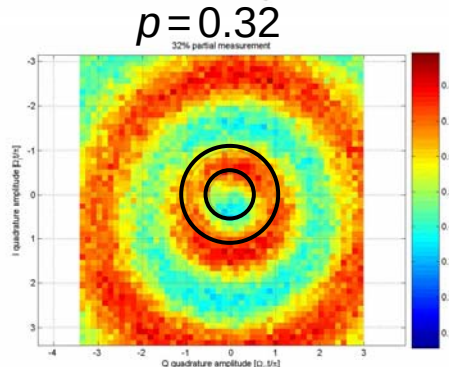
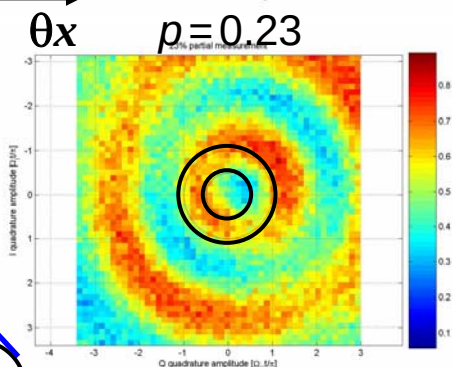
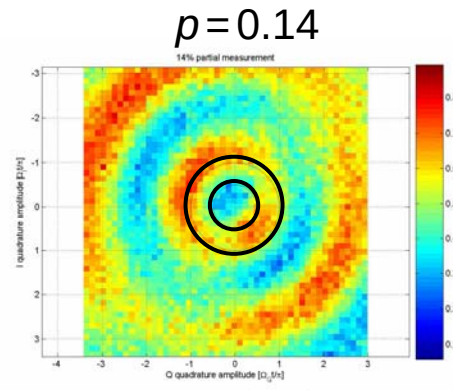
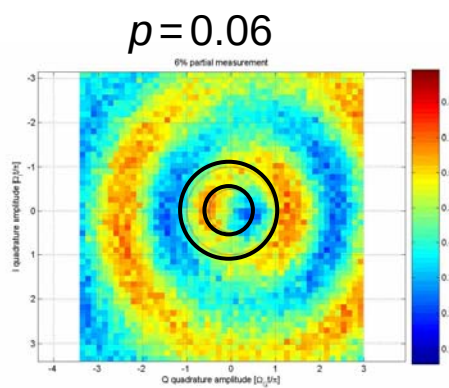
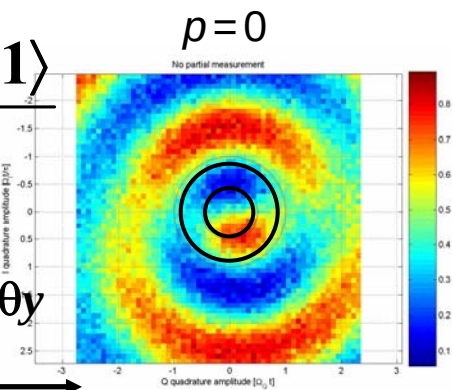


Experimental tomography data

Nadav Katz *et al.* (UCSB)

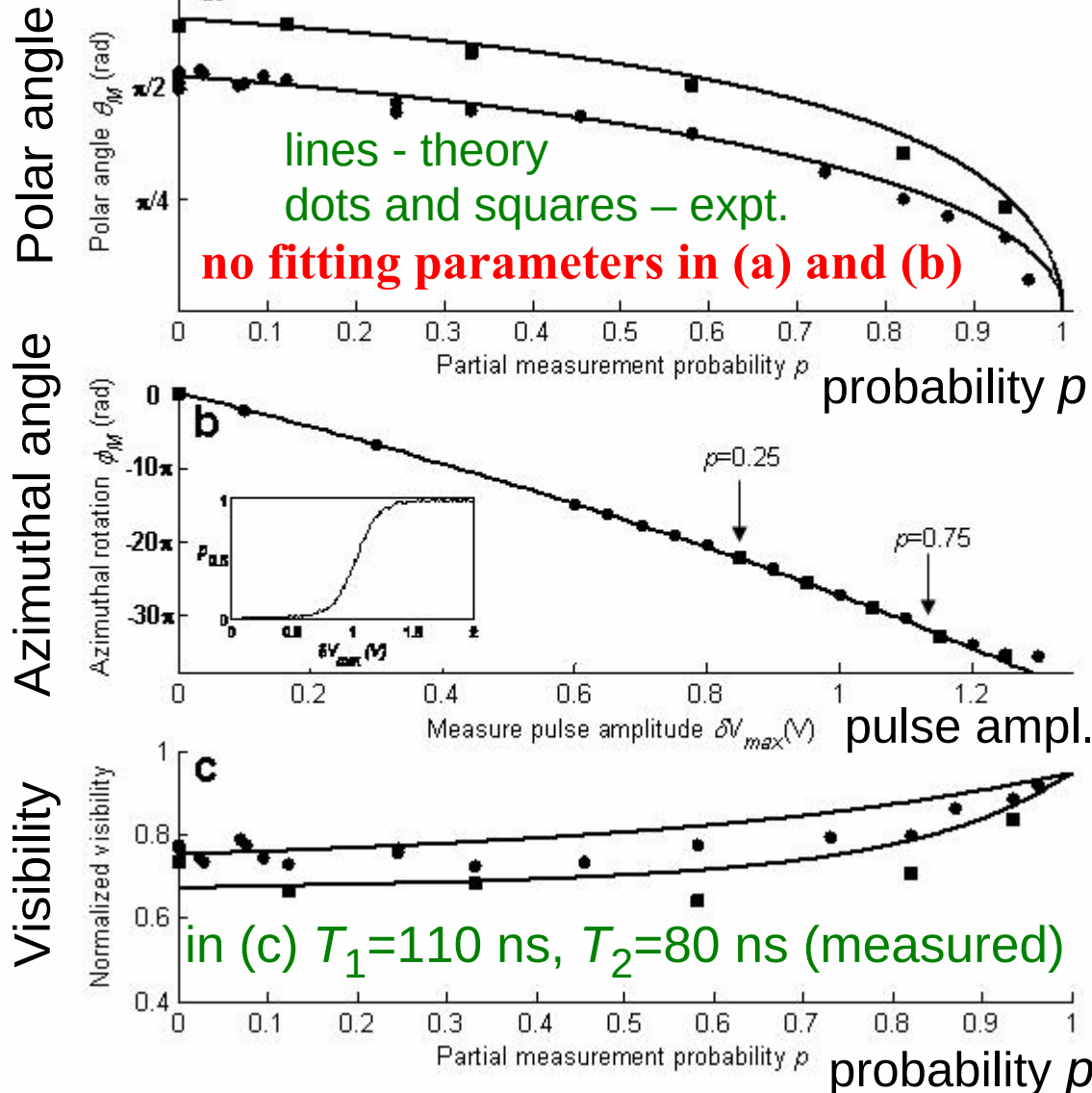
$$\psi_{in} = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

θy
 θx



Partial collapse: experimental results

N. Katz *et al.*, Science-06



- In case of no tunneling (null-result measurement) phase qubit evolves
- This evolution is well described by a simple Bayesian theory, without fitting parameters
- Phase qubit remains fully coherent in the process of continuous collapse (experimentally ~80% raw data, ~96% after account for T1 and T2)

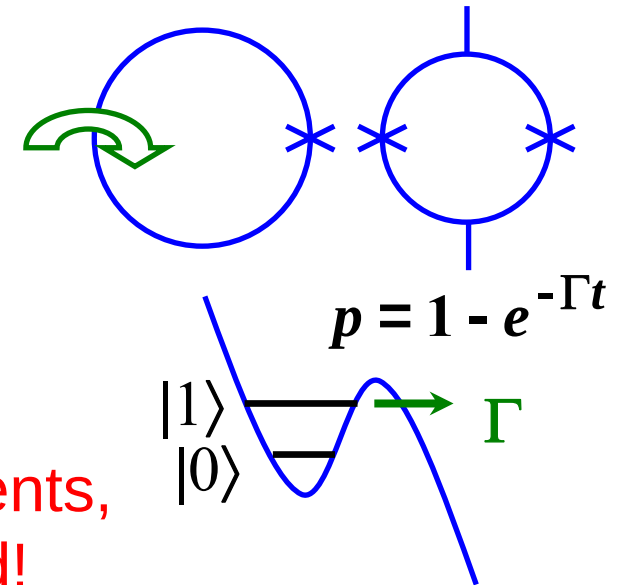
quantum efficiency
 $\eta_0 > 0.8$



Uncollapsing of a phase qubit state

A.K. & Jordan, 2006

- 1) Start with an unknown state
- 2) Partial measurement of strength p
- 3) π -pulse (exchange $|0\rangle \leftrightarrow |1\rangle$)
- 4) One more measurement with the same strength p
- 5) π -pulse



If no tunneling for both measurements,
then initial state is fully restored!

$$\alpha |0\rangle + \beta |1\rangle \rightarrow \frac{\alpha |0\rangle + e^{i\phi} \beta e^{-\Gamma t/2} |1\rangle}{\text{Norm}} \rightarrow$$

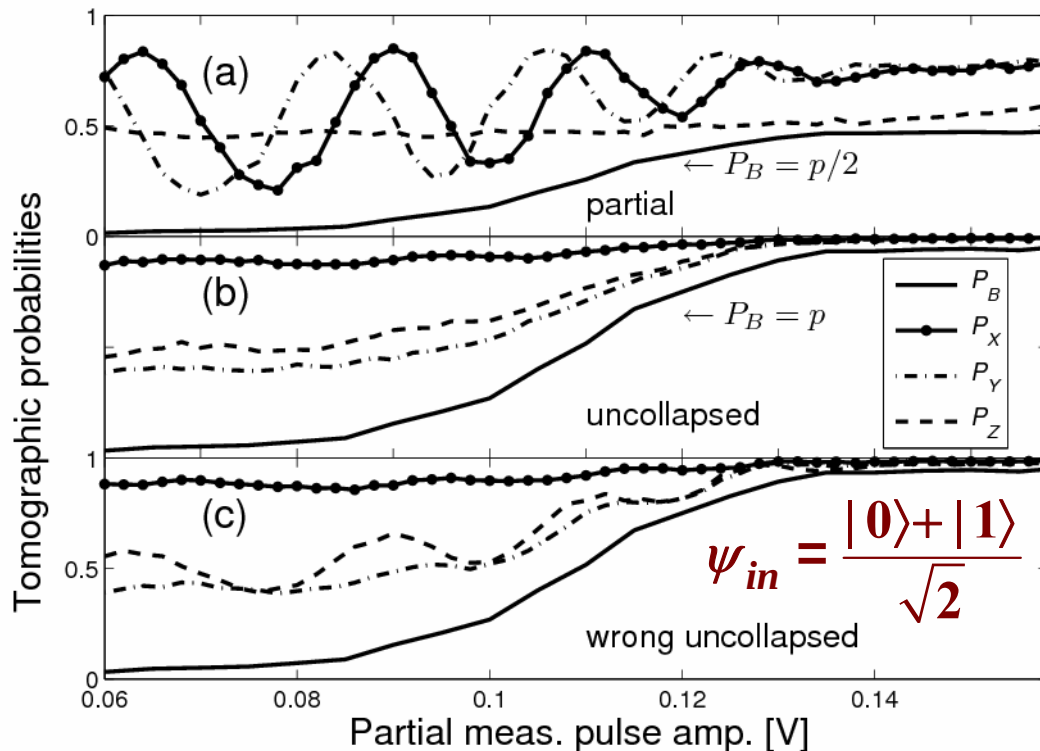
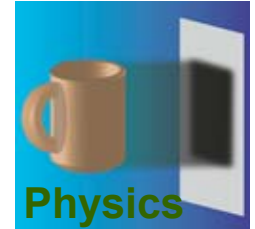
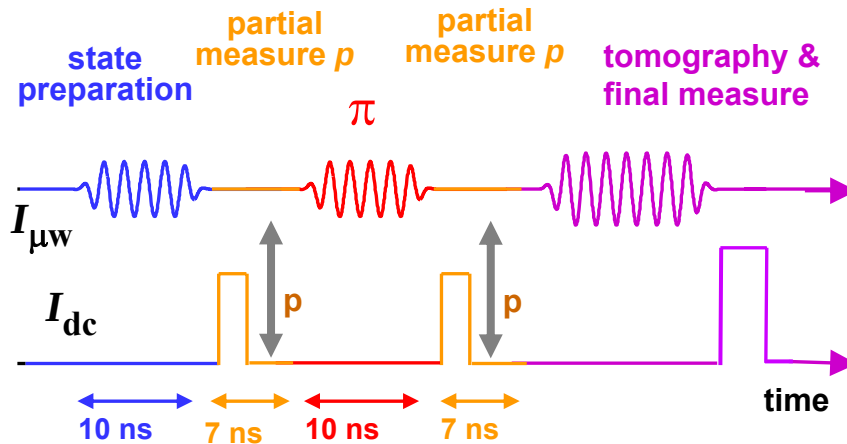
$$\frac{e^{i\phi} \alpha e^{-\Gamma t/2} |0\rangle + e^{i\phi} \beta e^{-\Gamma t/2} |1\rangle}{\text{Norm}} = e^{i\phi} (\alpha |0\rangle + \beta |1\rangle)$$

phase is also restored (spin echo)



Experiment on wavefunction uncollapsing

N. Katz, M. Neeley, M. Ansmann, R. Bialzak, E. Lucero, A. O'Connell, H. Wang, A. Cleland, J. Martinis, and A. Korotkov, PRL-2008



Uncollapse protocol:

- partial collapse
- π -pulse
- partial collapse (same strength)

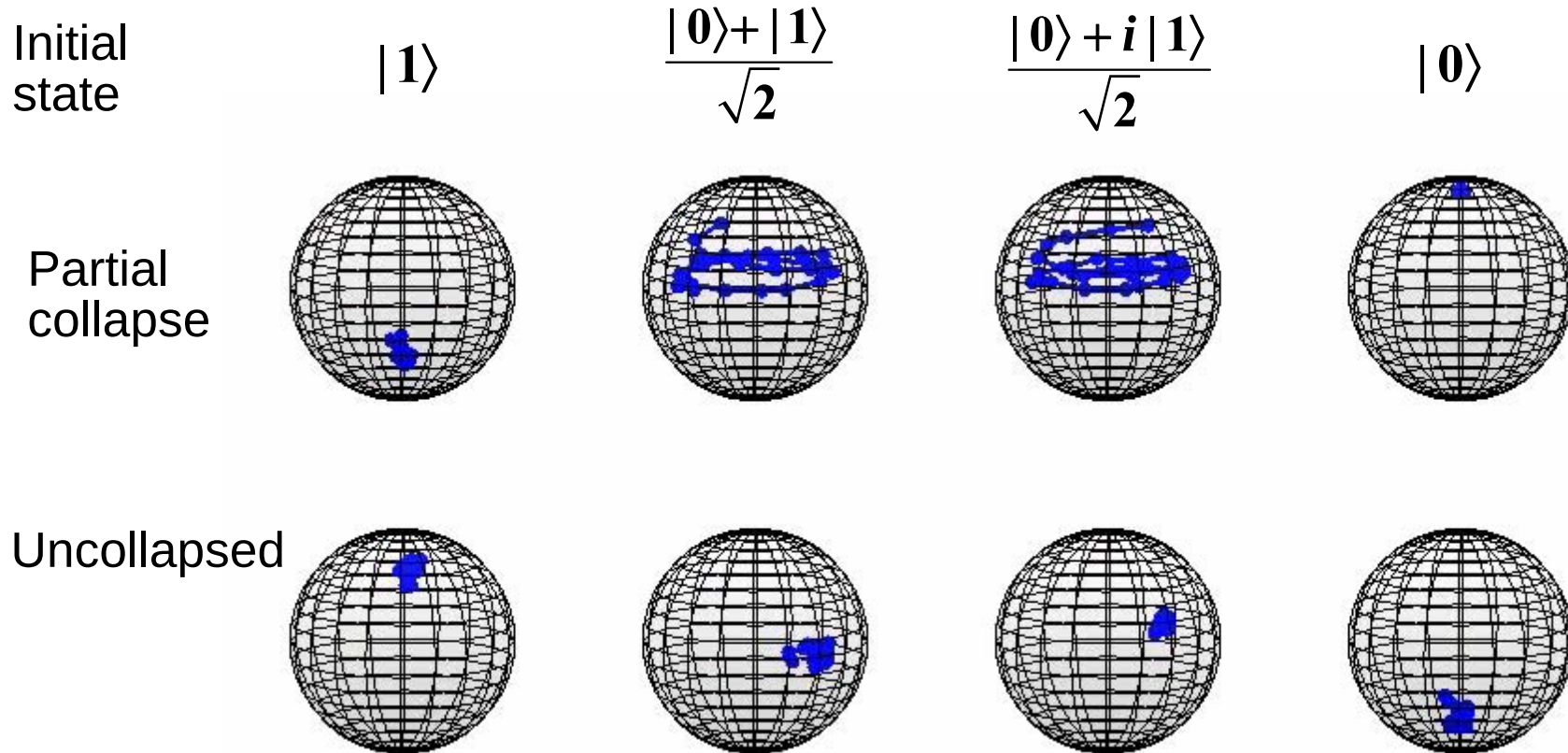
State tomography with X, Y, and no pulses

Background P_B should be subtracted to find qubit density matrix



Experimental results on Bloch sphere

N. Katz et al.

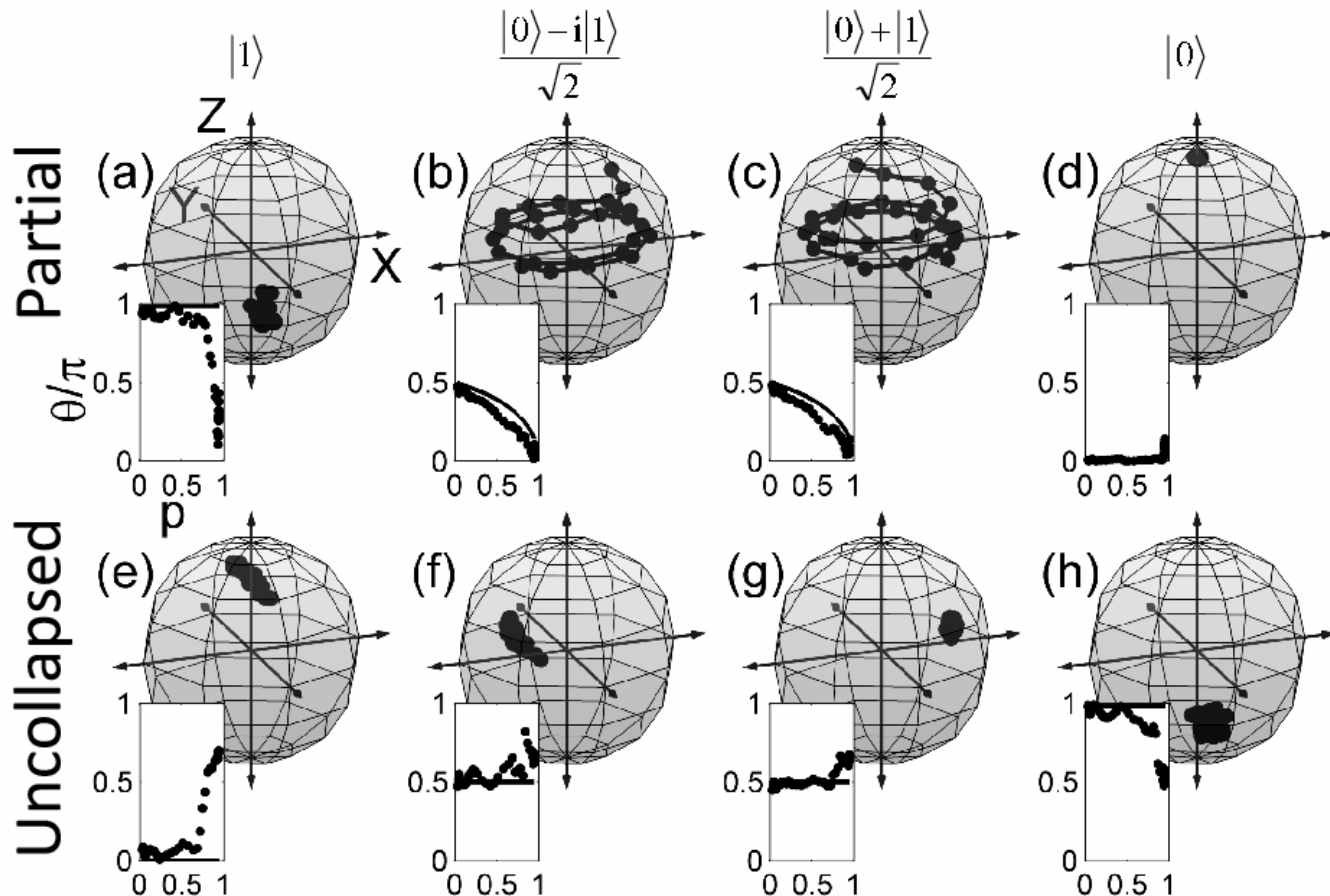


Collapse strength: $0.05 < p < 0.7$

uncollapsing works well!



Same with polar angle dependence (another experimental run)



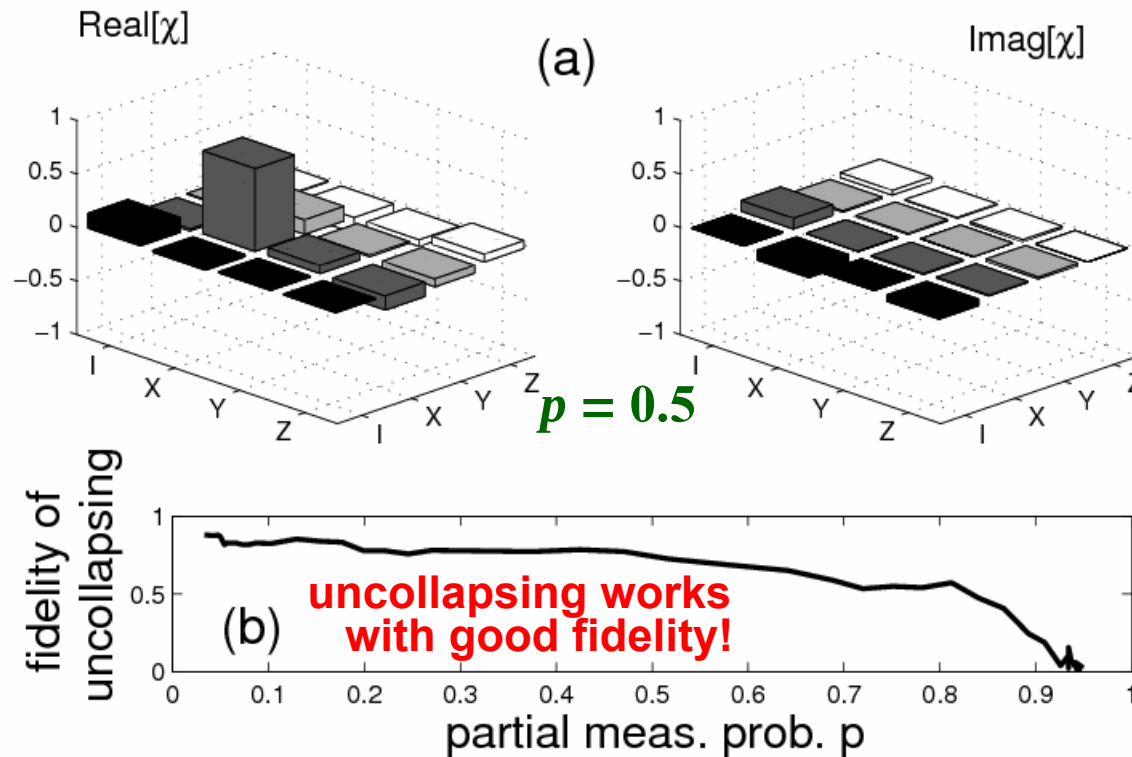
Both spin echo (azimuth) and uncollapsing (polar angle)

**Difference: spin echo – undoing of an unknown unitary evolution,
uncollapsing – undoing of a known, but non-unitary evolution**



Quantum process tomography

N. Katz et al.
(Martinis group)



Why getting worse at $p > 0.6$?

Energy relaxation $p_r = t/T_1 = 45\text{ns}/450\text{ns} = 0.1$

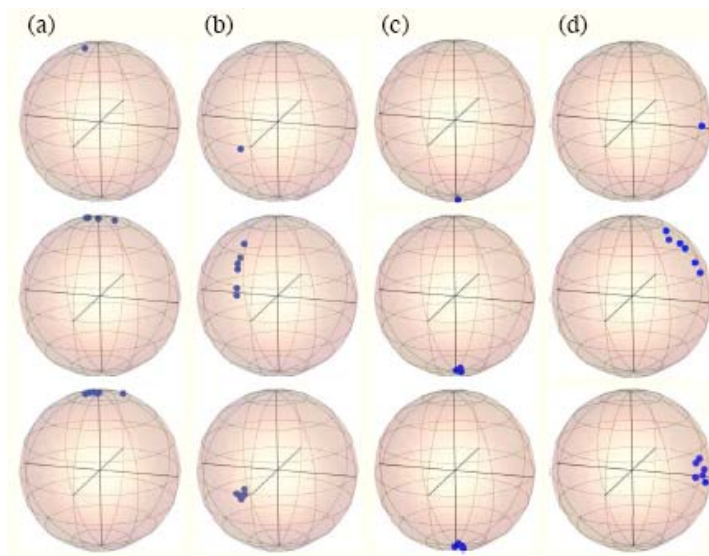
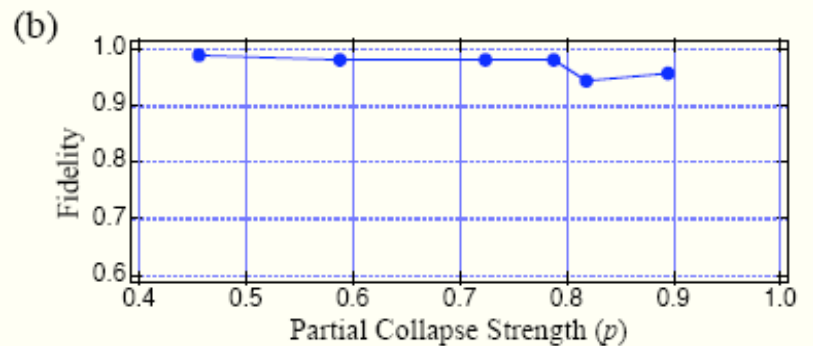
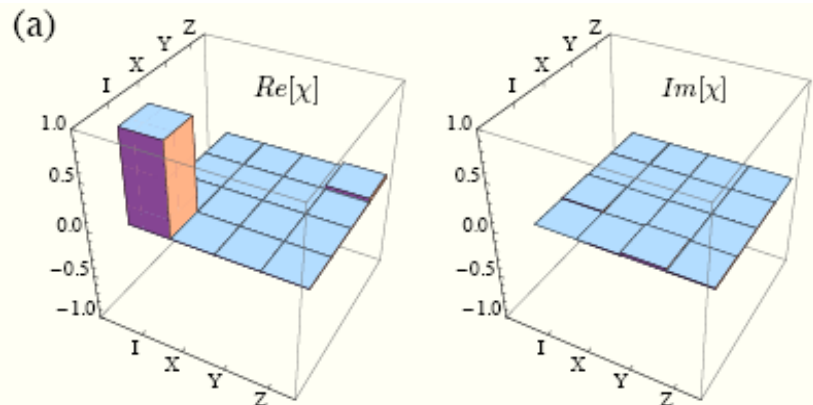
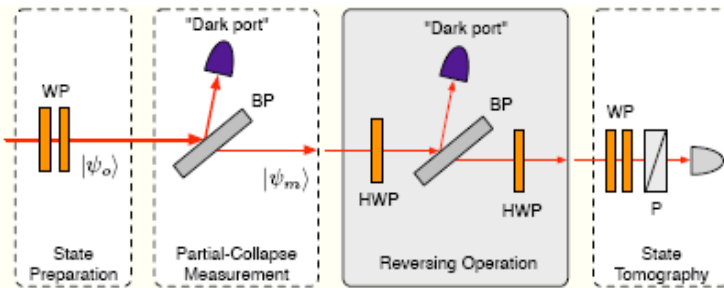
Selection affected when $1-p \sim p_r$

Overall: uncollapsing is well-confirmed experimentally



Recent experiment on uncollapsing using single photons

Kim, Cho, Ra, Kim, arXiv:0903.3077



- very good fidelity of uncollapsing ($>94\%$)
- measurement fidelity is probably not good (normalization by coincidence counts)



Conclusions

- Continuous quantum measurement is *not* equivalent to decoherence (environment) if detector output (information) is taken into account
- It is easy to see what is “inside” collapse: simple Bayesian formalism works for many solid-state setups
- Rabi oscillations are persistent if monitored
- Collapse can sometimes be undone (uncollapsing)
- Three direct solid-state experiments have been realized; hopefully, more experiments are coming soon

