Non-projective measurement of solid-state qubits: collapse and uncollapse
(what is “inside” collapse)

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Outline:

- Long introduction (collapse, solid-state qubits)
- Bayesian formalism for quantum measurement
- Some experimental predictions
- Recent experiments on partial collapse and uncollapse

Ackn.:

R. Ruskov, A. Jordan (theory)
N. Katz, J. Martinis, P. Bertet (expt.)

Funding:

Niels Bohr:

"If you are not confused by quantum physics then you haven't really understood it"

Richard Feynman:

"I think I can safely say that nobody understands quantum mechanics"

Quantum mechanics =

Schrödinger equation

+

collapse postulate

1) Probability of measurement result $p_r = |\langle \psi \,|\, \psi_r \rangle|^2$

2) Wavefunction after measurement = ψ_r

- State collapse follows from common sense
- Does not follow from Schr. Eq. (contradicts; Schr. cat, random vs. deterministic)

What is "inside" collapse?

(What if measurement is continuous, as typical for solid-state experiments?)

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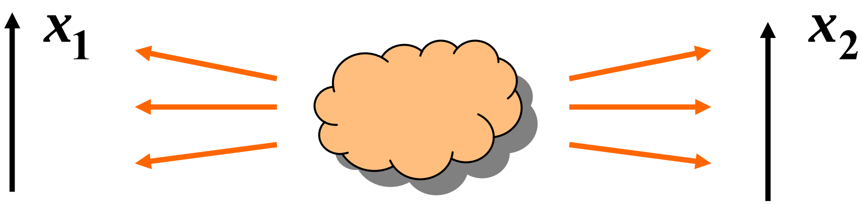
Einstein-Podolsky-Rosen (EPR) paradox

Phys. Rev., 1935

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system.

$$\psi(x_1, x_2) = \sum_n \psi_n(x_2)\, u_n(x_1)$$ *(nowadays we call it entangled state)*

$$\psi(x_1, x_2) = \int_{-\infty}^{\infty} \exp[(i/\hbar)(x_1 - x_2)p]\,dp \sim \delta(x_1 - x_2)$$



Measurement of particle 1 cannot affect particle 2, while QM says it affects (contradicts causality)

=> Quantum mechanics is incomplete

Bohr's reply **(Phys. Rev., 1935)** (seven pages, one formula: $\Delta p\, \Delta q \sim h$)

It is shown that a certain "criterion of physical reality" formulated … by A. Einstein, B. Podolsky and N. Rosen contains an essential ambiguity when it is applied to quantum phenomena.

Crudely: No need to understand QM, just use the result

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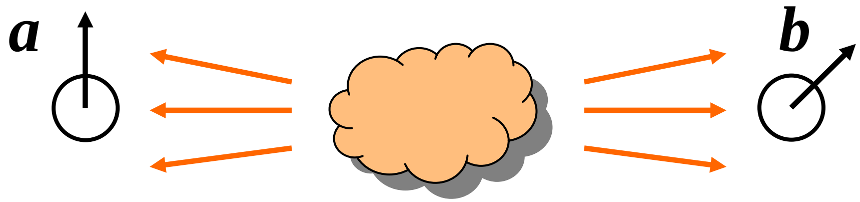
Bell's inequality (John Bell, 1964)



(setup by David Bohm)

$$\psi = \frac{1}{\sqrt{2}}(\uparrow_1 \downarrow_2 - \downarrow_1 \uparrow_2)$$

Perfect anticorrelation of results for same meas. directions, $\vec{a} = \vec{b}$

Is it possible to explain the QM result assuming local realism and hidden variables (without superluminal collapse)? No!!!

Assume: $A(\vec{a}, \lambda) = \pm 1, \quad B(\vec{b}, \lambda) = \pm 1$ (deterministic result with hidden variable λ)

Then: $|P(\vec{a}, \vec{b}) - P(\vec{a}, \vec{c})| \le 1 + P(\vec{b}, \vec{c})$

where $P \equiv P(++) + P(--) - P(+-) - P(-+)$

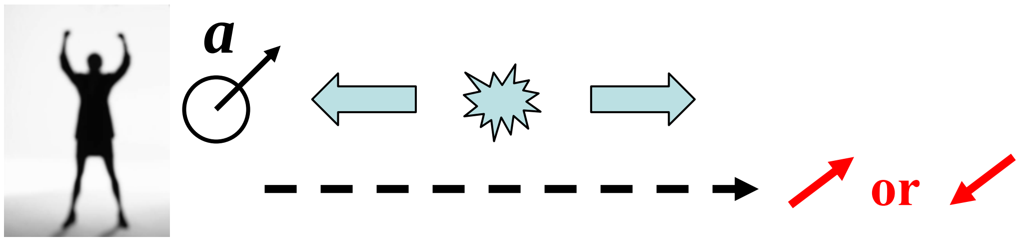
QM: $P(\vec{a}, \vec{b}) = -\vec{a} \bullet \vec{b}$ For 0°, 90°, and 45°: $0.71 \not\le 1 - 0.71$ **violation!**

Experiment (Aspect et al., 1982; photons instead of spins, CHSH):

yes, "spooky action-at-a-distance"

What about causality?

Actually, not too bad: you cannot transmit your own information choosing a particular measurement direction ***a***



Result of the other measurement does not depend on direction ***a***

Randomness saves causality

Collapse is still instantaneous: OK, just our recipe, not an “objective reality”, not a “physical” process

Consequence of causality: No-cloning theorem

Wootters-Zurek, 1982; Dieks, 1982; Yurke

You cannot copy an unknown quantum state

Proof: Otherwise get information on direction ***a*** (and causality violated)

Application: quantum cryptography

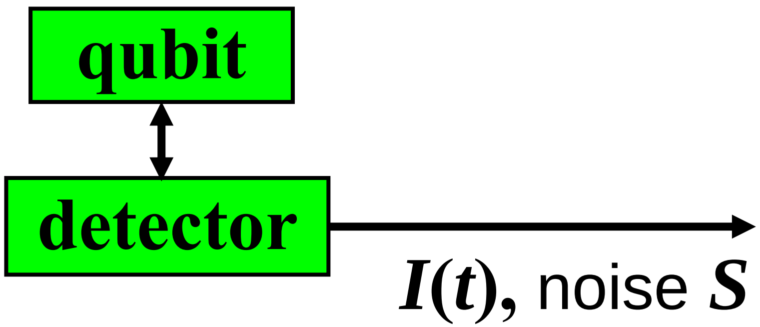
Information is an important concept in quantum mechanics

Quantum measurement in solid-state systems

No violation of locality – too small distances

However, interesting issue of continuous measurement (weak coupling, noise ⇒ gradual collapse)

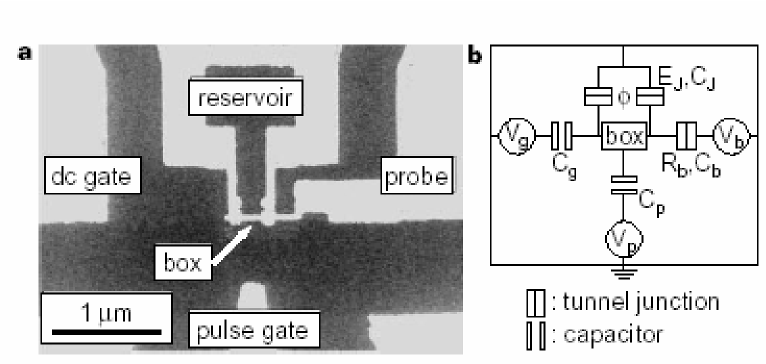
Starting point:



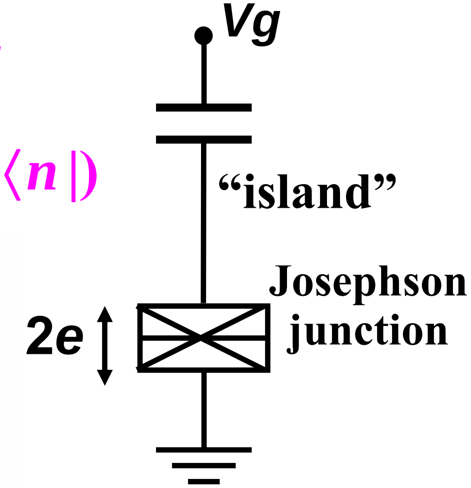
What happens to a solid-state qubit (two-level system) during its continuous measurement by a detector?

Superconducting "charge" qubit

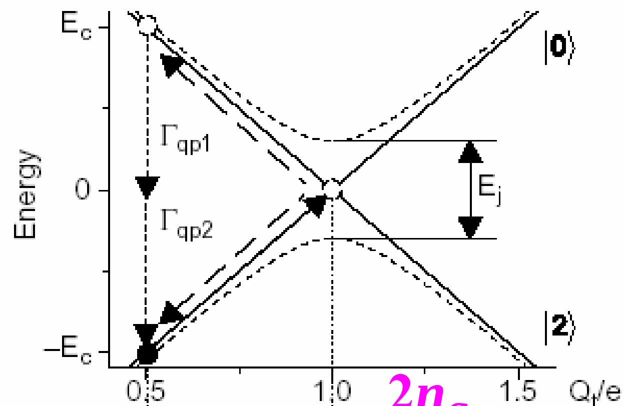
Y. Nakamura, Yu. Pashkin, and J.S. Tsai (Nature, 1998)



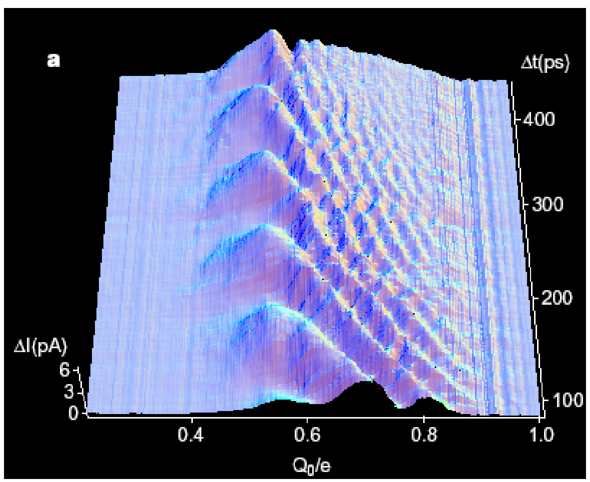
$$\hat{H} = \frac{(2e)^2}{2C}(\hat{n} - n_g)^2 - \frac{E_J}{2}(|n\rangle\langle n+1| + |n+1\rangle\langle n|)$$

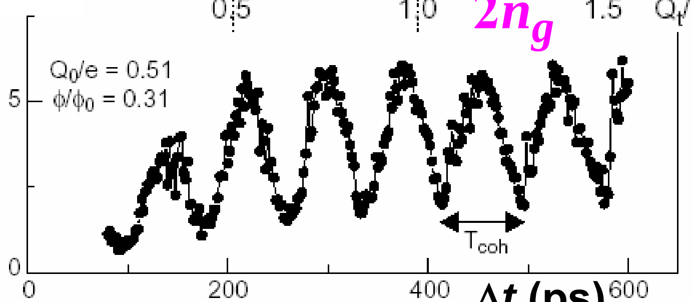


Single Cooper pair box



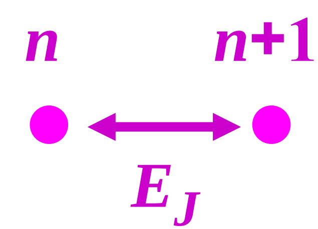
$2n_g$





Δ*t* (ps)

Quantum coherent (Rabi) oscillations

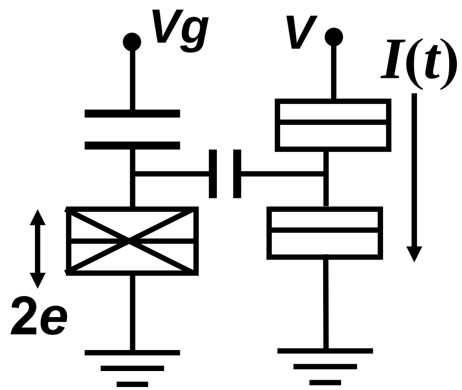


***n*: number of Cooper pairs on the island**

Vion et al. (Devoret's group); Science, 2002

Q-factor of coherent (Rabi) oscillations = 25,000

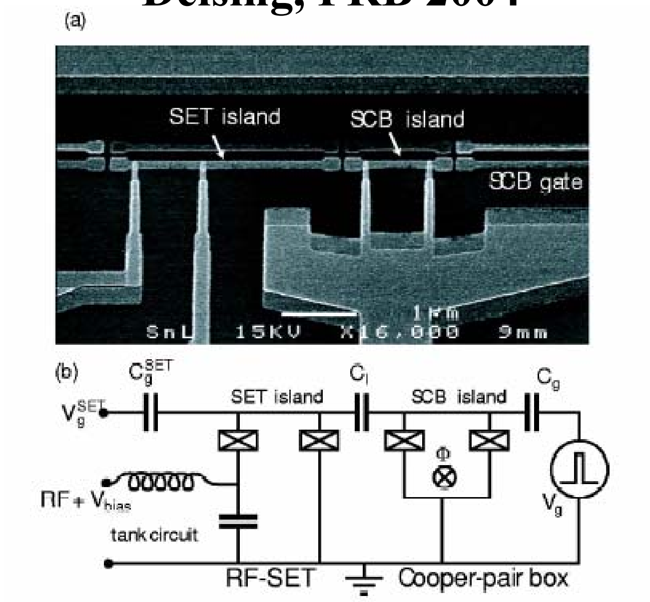
More of superconducting charge qubits

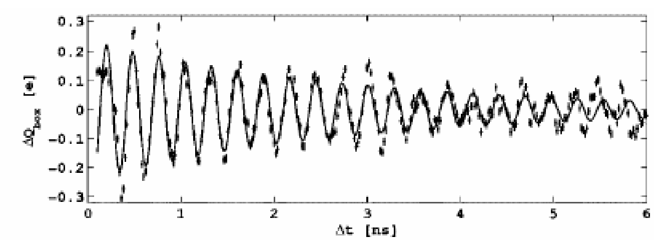


Cooper-pair box measured by single-electron transistor (rf-SET)

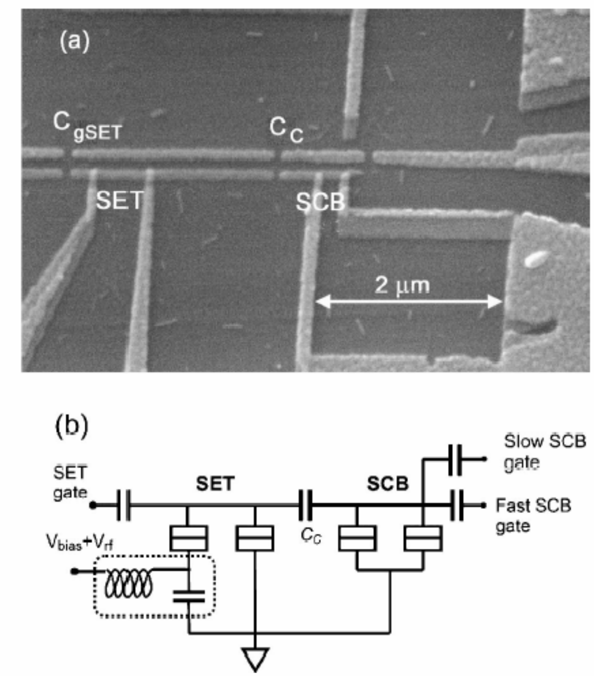
Setup can be used for continuous measurements

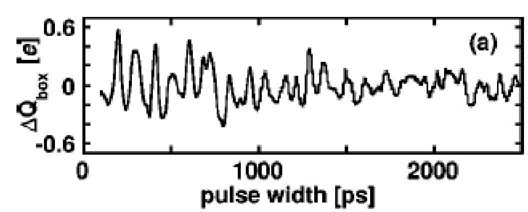
Duty, Gunnarsson, Bladh, Delsing, PRB 2004





Guillaume et al. (Echternach's group), PRB 2004



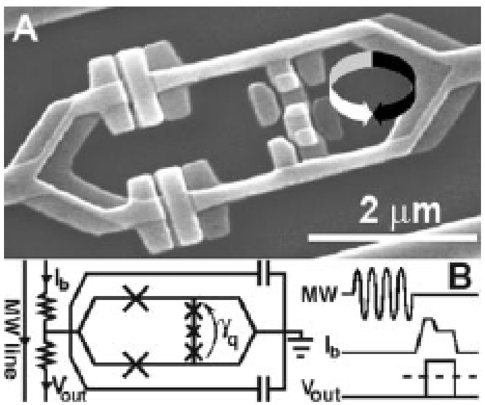


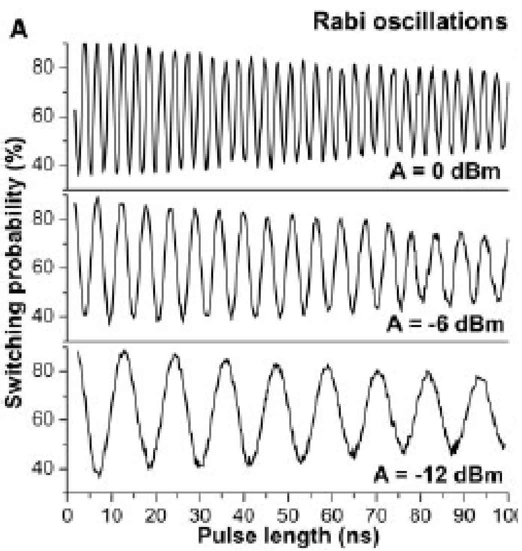
All results are averaged over many measurements (not "single-shot")

Some other superconducting qubits

Flux qubit

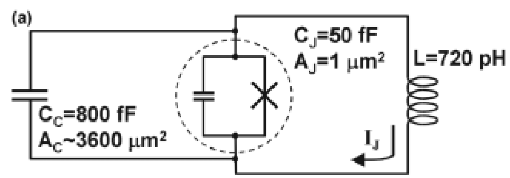
Mooij et al. (Delft)

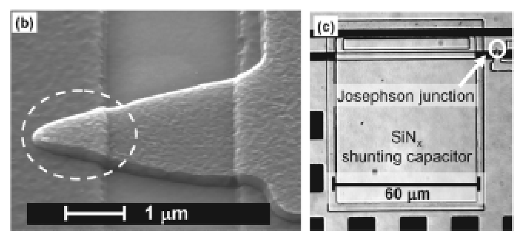


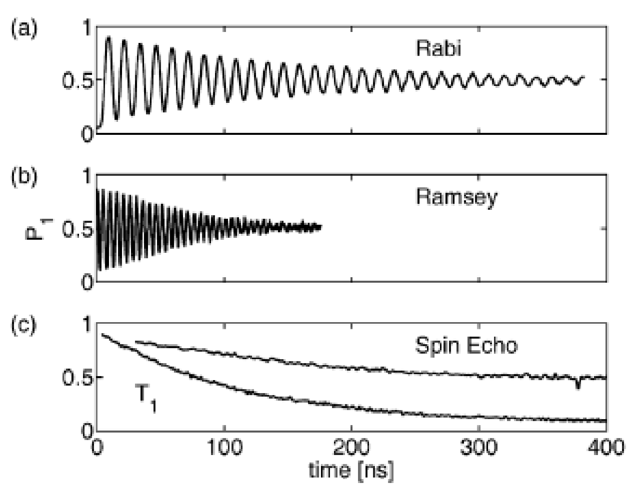


Phase qubit

J. Martinis et al.
(UCSB and NIST)

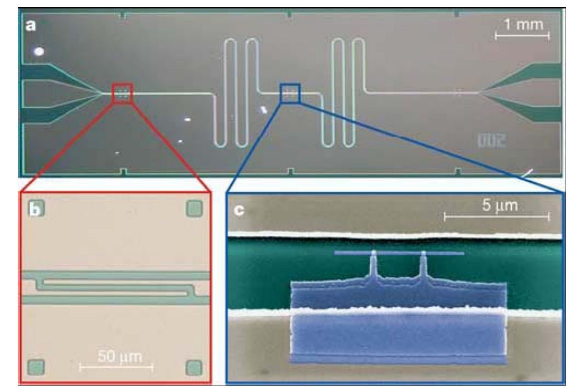


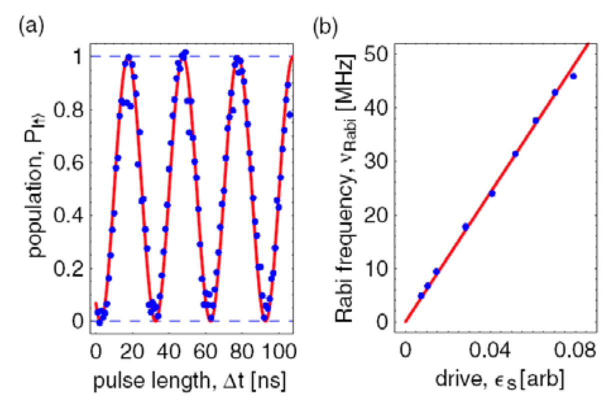




Charge qubit with circuit QED

R. Schoelkopf et al. (Yale)

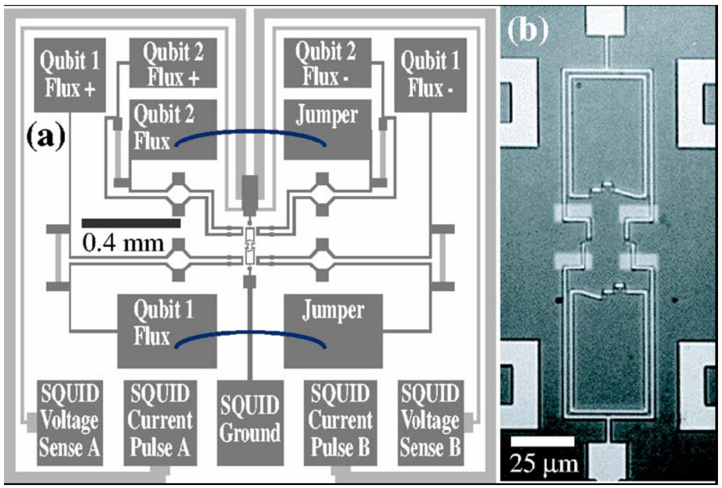


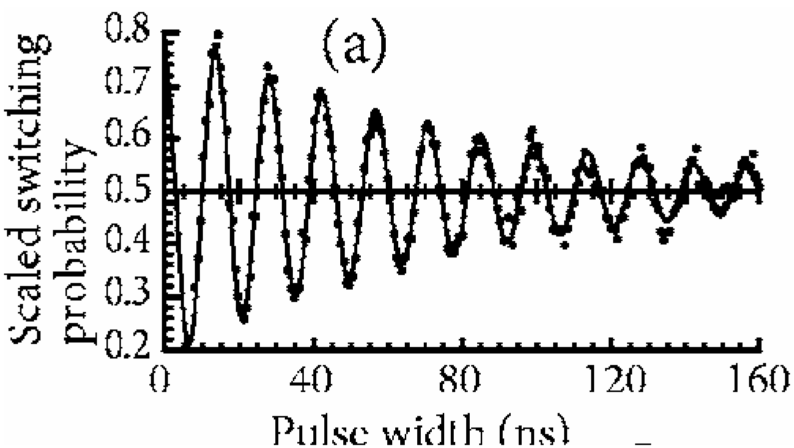


Some other superconducting qubits

Flux qubit

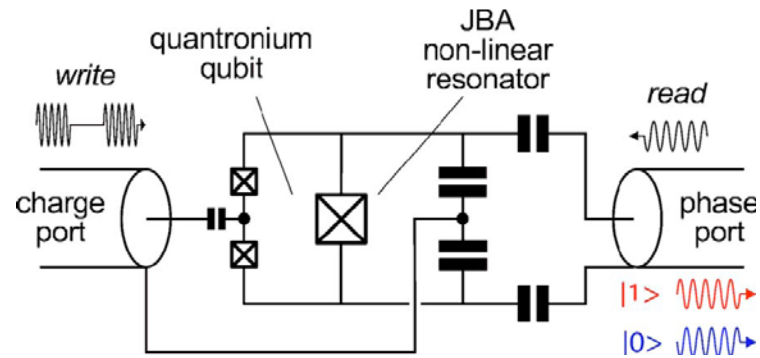
J. Clarke et al. (Berkeley)

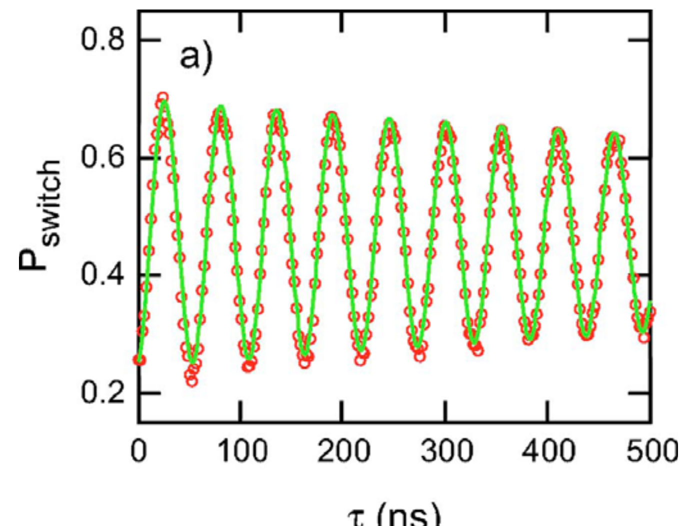




"Quantronium" qubit

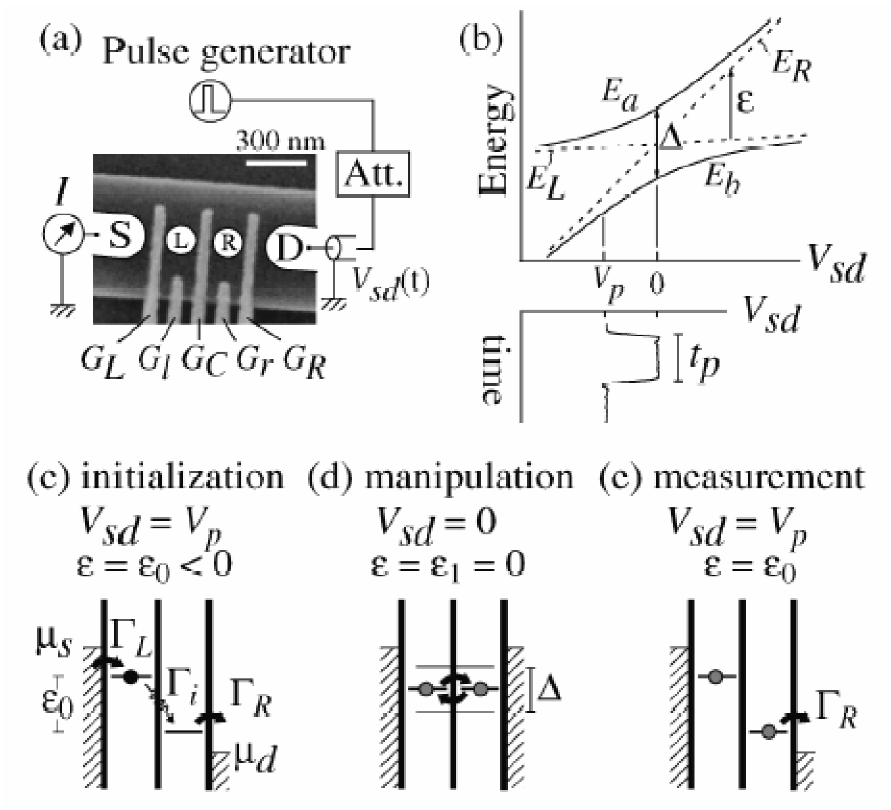
I. Siddiqi, R. Schoelkopf,
M. Devoret, et al. (Yale)

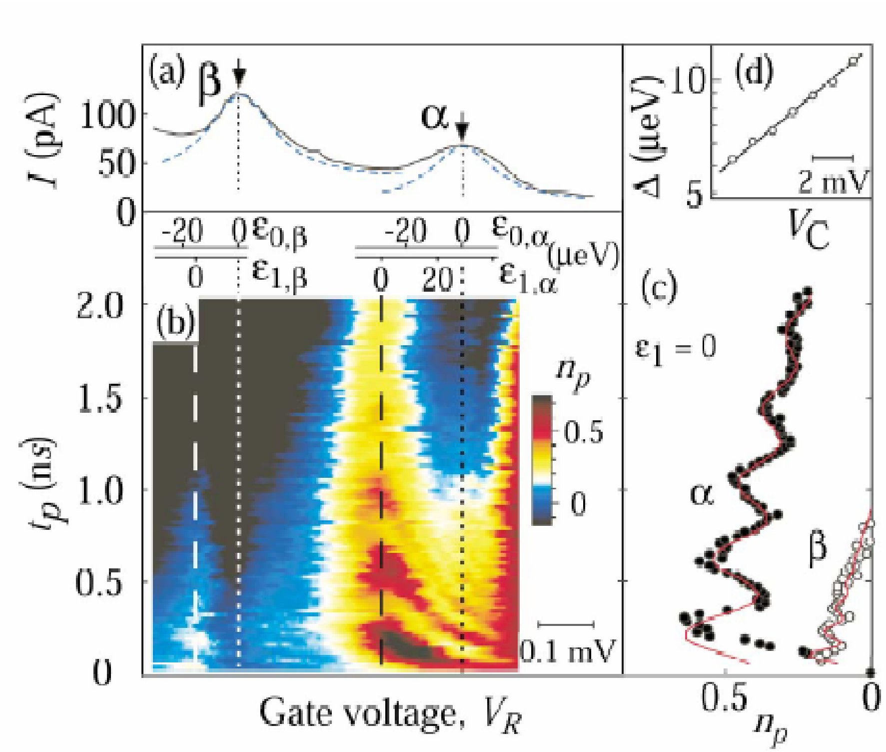




Semiconductor (double-dot) qubit

T. Hayashi et al., PRL 2003





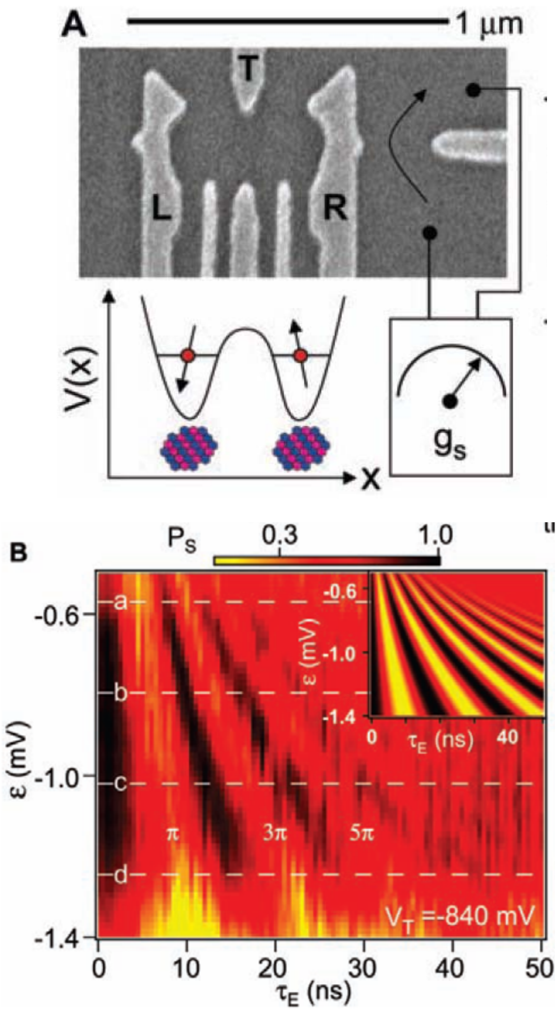
Rabi oscillations

Detector is not separated from qubit,
also possible to use a separate detector

Some other semiconductor qubits

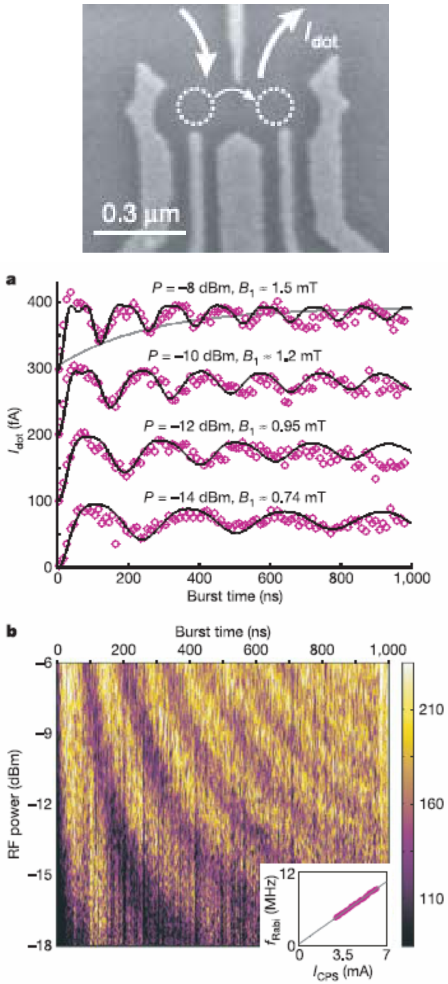
Spin qubit

C. Marcus et al. (Harvard)



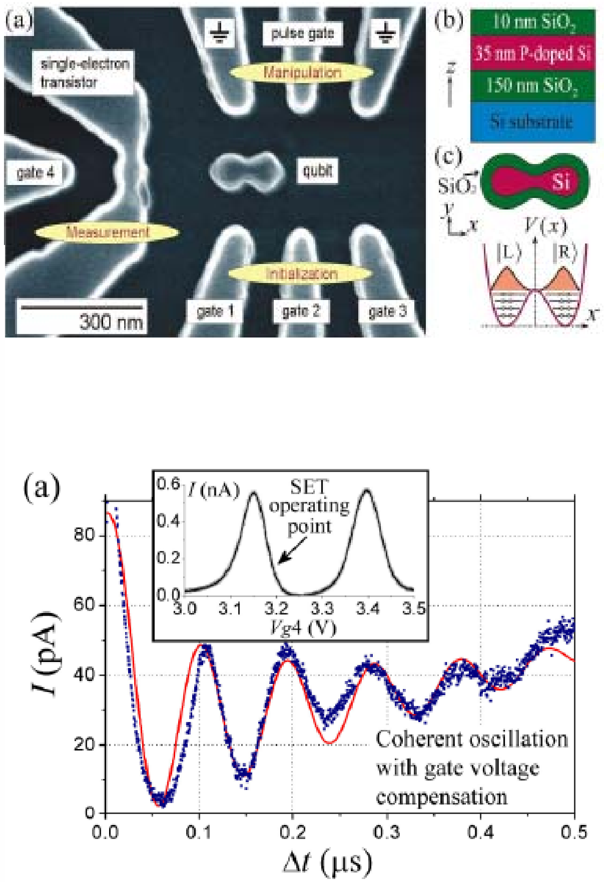
Spin qubit

L. Kouwenhoven et al. (Delft)



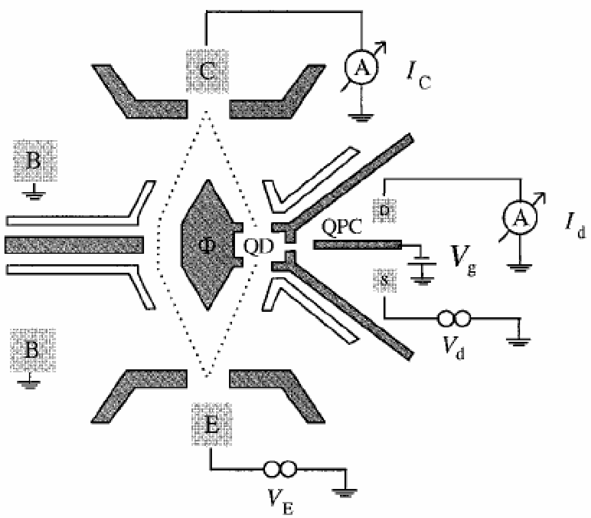
Double-dot qubit

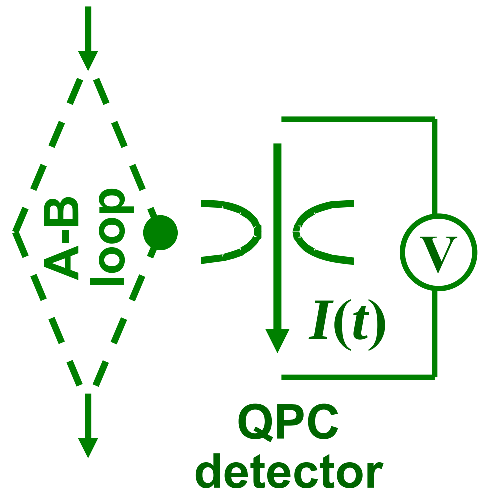
Gorman, Hasko, Williams (Cambridge)

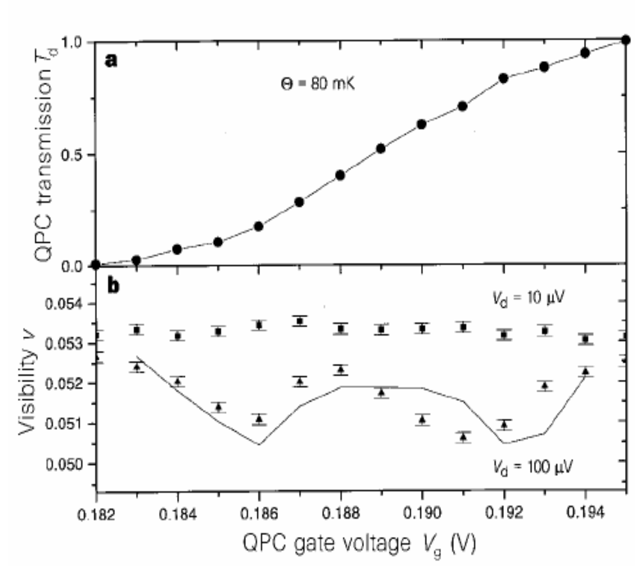


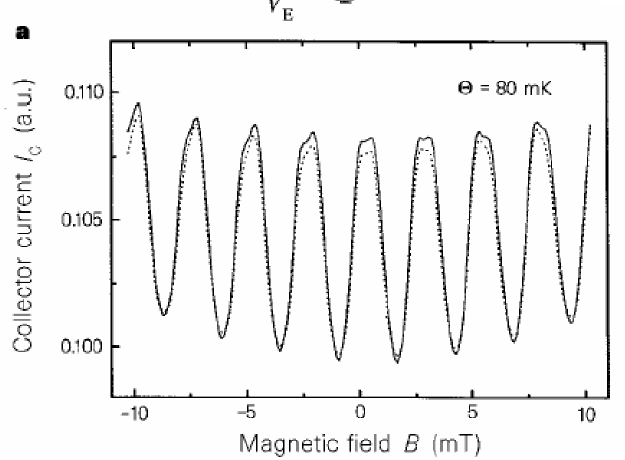
"Which-path detector" experiment

Buks, Schuster, Heiblum, Mahalu, and Umansky, Nature 1998









Dephasing rate: $$\Gamma = \frac{eV}{h}\frac{(\Delta T)^2}{T(1-T)} = \frac{(\Delta I)^2}{4S_I}$$

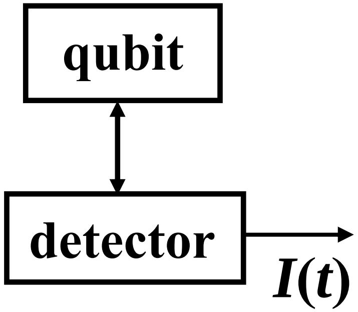
ΔI – detector response, S_I – shot noise

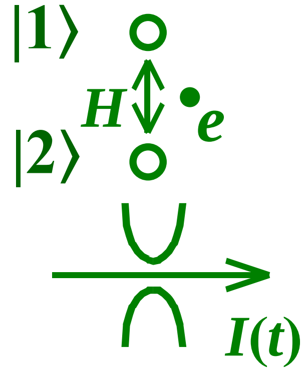
The larger noise, the smaller dephasing!!!

$(\Delta I)^2/4S_I$ ~ rate of "information flow"

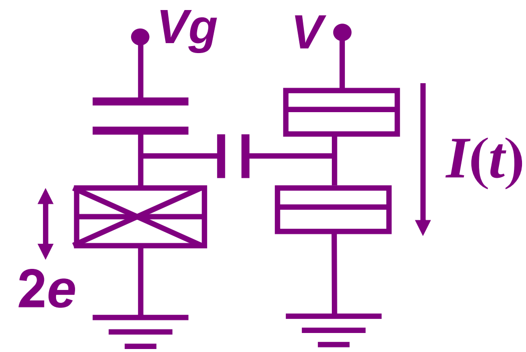
Theory: Aleiner, Wingreen, and Meir, PRL 1997

The system we consider: qubit + detector





Double-quantum-qot (DQD) and quantum point contact (QPC)



Cooper-pair box (CPB) and single-electron transistor (SET)

$H = H_{\mathrm{QB}} + H_{\mathrm{DET}} + H_{\mathrm{INT}}$

$H_{\mathrm{QB}} = (\varepsilon/2)(c_1^+c_1 - c_2^+c_2) + H(c_1^+c_2 + c_2^+c_1)$ ε – asymmetry, H – tunneling

$\Omega = (4H^2+\varepsilon^2)^{1/2}/\hbar$ – frequency of quantum coherent (Rabi) oscillations

Two levels of average detector current: I_1 for qubit state $|1\rangle$, I_2 for $|2\rangle$

Response: $\Delta I = I_1 - I_2$ Detector noise: white, spectral density S_I

DQD and QPC (setup due to Gurvitz, 1997)

$$H_{DET} = \sum_l E_l a_l^\dagger a_l + \sum_r E_r a_r^\dagger a_r + \sum_{l,r} T(a_r^\dagger a_l + a_l^\dagger a_r)$$

$$H_{INT} = \sum_{l,r} \Delta T\,(c_1^\dagger c_1 - c_2^\dagger c_2)(a_r^\dagger a_l + a_l^\dagger a_r)$$

$$S_I = 2eI$$

What happens to a qubit state during measurement?

Start with density matrix evolution due to measurement only ($H=\varepsilon=0$)

"Orthodox" answer

$$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \begin{matrix} \nearrow \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \\ \searrow \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \end{matrix}$$

|1> or |2>, depending on the result

"Conventional" (decoherence) answer

$$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \rightarrow \begin{pmatrix} \frac{1}{2} & \frac{\exp(-\Gamma t)}{2} \\ \frac{\exp(-\Gamma t)}{2} & \frac{1}{2} \end{pmatrix} \rightarrow \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

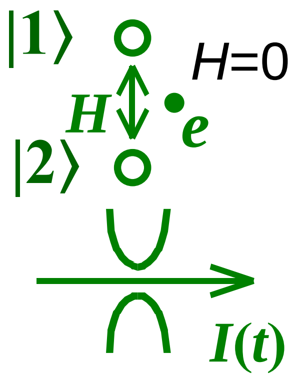
no measurement result! (ensemble averaged)

Orthodox and decoherence answers contradict each other!

applicable for:	single quant. system	continuous meas.
Orthodox	yes	no
Decoherence (ensemble)	no	yes
Bayesian, POVM, quant. traject., etc.	yes	yes

Bayesian (POVM, etc.) formalism describes gradual collapse of a single quantum system, **taking into account noisy detector output $I(t)$**

Bayesian formalism for DQD-QPC system



Qubit evolution due to measurement (quantum back-action):

$$\psi(t) = \alpha(t)\,|1\rangle + \beta(t)\,|2\rangle \quad \text{or} \quad \rho_{ij}(t)$$

1) $|\alpha(t)|^2$ and $|\beta(t)|^2$ evolve as probabilities,
i.e. according to the **Bayes rule** (same for ρ_{ii})

2) phases of $\alpha(t)$ and $\beta(t)$ do not change
(**no decoherence!**), $\rho_{ij}/(\rho_{ii}\,\rho_{jj})^{1/2} = \text{const}$

(A.K., 1998)

Bayes rule (1763, Laplace-1812):

$$\underbrace{P(A_i\,|\,\text{res})}_{\text{posterior probability}} = \frac{\overbrace{P(A_i)}^{\text{prior probab.}}\ \overbrace{P(\text{res}\,|\,A_i)}^{\text{likelihood}}}{\sum_k P(A_k)\,P(\text{res}\,|\,A_k)}$$

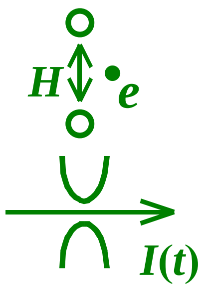
So simple because:

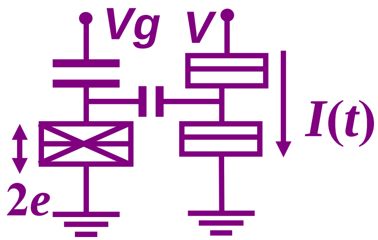
1) QPC happens to be an ideal detector
2) no Hamiltonian evolution of the qubit

Similar formalisms developed earlier. Key words: Imprecise, weak, selective, or conditional measurements, POVM, Quantum trajectories, Quantum jumps, Restricted path integral, etc.

Names: Davies, Kraus, Holevo, Mensky, Caves, Gardiner, Carmichael, Plenio, Knight, Walls, Gisin, Percival, Milburn, Wiseman, Habib, etc. (very incomplete list)

Bayesian formalism for a single qubit





$$\hat{H}_{QB} = \frac{\varepsilon}{2}(c_1^\dagger c_1 - c_2^\dagger c_2) + H(c_1^\dagger c_2 + c_2^\dagger c_1)$$

$|1\rangle \to I_1,\ |2\rangle \to I_2,\ \Delta I = I_1 - I_2,\ I_0 = (I_1+I_2)/2$

S_I – detector noise

$$\dot{\rho}_{11} = -\dot{\rho}_{22} = -2(H/\hbar)\,\mathrm{Im}\,\rho_{12} + \rho_{11}\rho_{22}(2\Delta I/S_I)[\underline{\underline{I(t)}} - I_0]$$

$$\dot{\rho}_{12} = i(\varepsilon/\hbar)\rho_{12} + i(H/\hbar)(\rho_{11}-\rho_{22}) + \rho_{12}(\rho_{11}-\rho_{22})(\Delta I/S_I)[\underline{\underline{I(t)}} - I_0] - \gamma\rho_{12}$$

(A.K., 1998)

$\gamma = \Gamma - (\Delta I)^2/4S_I,\quad \Gamma$ – ensemble decoherence

$\eta = 1 - \gamma/\Gamma = (\Delta I)^2/4S_I\Gamma$ – detector ideality (efficiency), $\eta \le 100\%$

Ideal detector (η=1, as QPC) does not decohere a qubit, then random evolution of qubit *wavefunction* can be monitored

Averaging over result $I(t)$ leads to conventional master equation:

$$d\rho_{11}/dt = -d\rho_{22}/dt = -2(H/\hbar)\,\mathrm{Im}\,\rho_{12}$$

$$d\rho_{12}/dt = i(\varepsilon/\hbar)\rho_{12} + i(H/\hbar)(\rho_{11}-\rho_{22}) - \Gamma\rho_{12}$$

Ensemble averaging includes averaging over measurement result!

Assumptions needed for the Bayesian formalism:

- Detector voltage is much larger than the qubit energies involved

$$eV \gg \hbar\Omega,\ eV \gg \hbar\Gamma,\ \ \hbar/eV \ll (1/\Omega,\ 1/\Gamma)$$

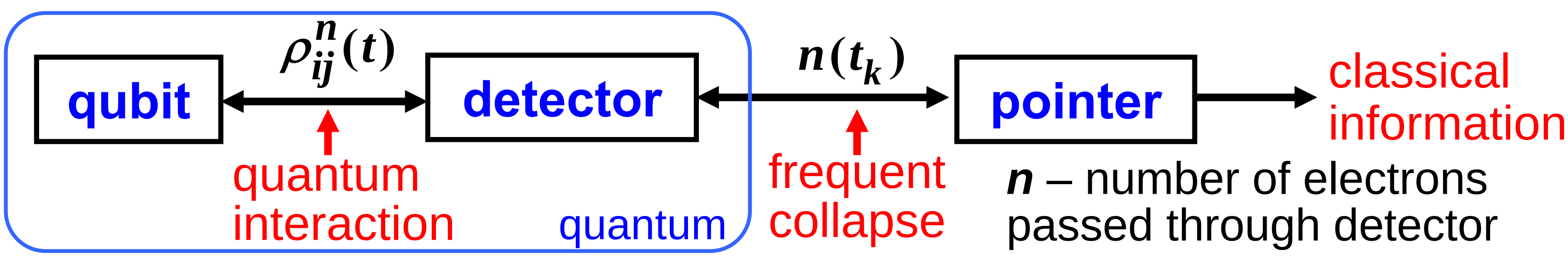
(no coherence in the detector, classical output, Markovian approximation)

- Simpler if weak response, $|\Delta I| \ll I_0$, (coupling $C \sim \Gamma/\Omega$ is arbitrary)

Derivations:

1) "logical": via correspondence principle and comparison with decoherence approach (A.K., 1998)

2) "microscopic": Schr. eq. + collapse of the detector (A.K., 2000)



3) from "quantum trajectory" formalism developed for quantum optics (Goan-Milburn, 2001; also: Wiseman, Sun, Oxtoby, etc.)

4) from POVM formalism (Jordan-A.K., 2006)

5) from Keldysh formalism (Wei-Nazarov, 2007)

Fundamental limit for ensemble decoherence

$$\Gamma = (\Delta I)^2/4S_I + \gamma$$

ensemble decoherence rate → Γ

~ rate of information acquisition [bit/s] → $(\Delta I)^2/4S_I$

single-qubit decoherence → γ

$$\gamma \geq 0 \Rightarrow \boxed{\Gamma \geq (\Delta I)^2/4S_I} \Rightarrow \Gamma\tau_m \geq \frac{1}{2}$$

A.K., 1998, 2000
S. Pilgram et al., 2002
A. Clerk et al., 2002
D. Averin, 2003

$$\eta = 1 - \frac{\gamma}{\Gamma} = \frac{(\Delta I)^2/4S_I}{\Gamma}$$

detector ideality (quantum efficiency) $\eta \leq 100\%$

Translated into energy sensitivity: $(\epsilon_O\, \epsilon_{BA})^{1/2} \geq \hbar/2$

where ϵ_O is output-noise-limited sensitivity [J/Hz]
and ϵ_{BA} is back-action-limited sensitivity [J/Hz]

Sensitivity limitation is known since 1980s (Clarke, Tesche, Likharev, etc.); also Averin-2000, Clerk et al.-2002, Pilgram et al.-2002, etc.

Measurement vs. decoherence

Widely accepted point of view:

measurement = decoherence (environment)

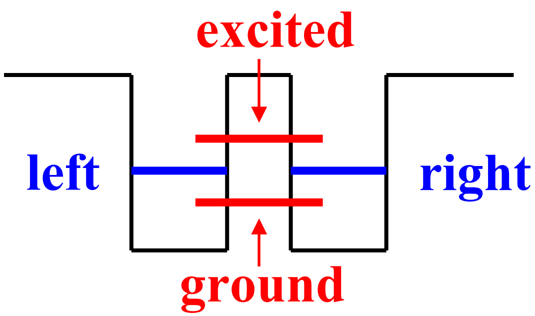
Is it true?

- **Yes**, if not interested in information from detector (ensemble-averaged evolution)
- **No**, if take into account measurement result (single quantum system)

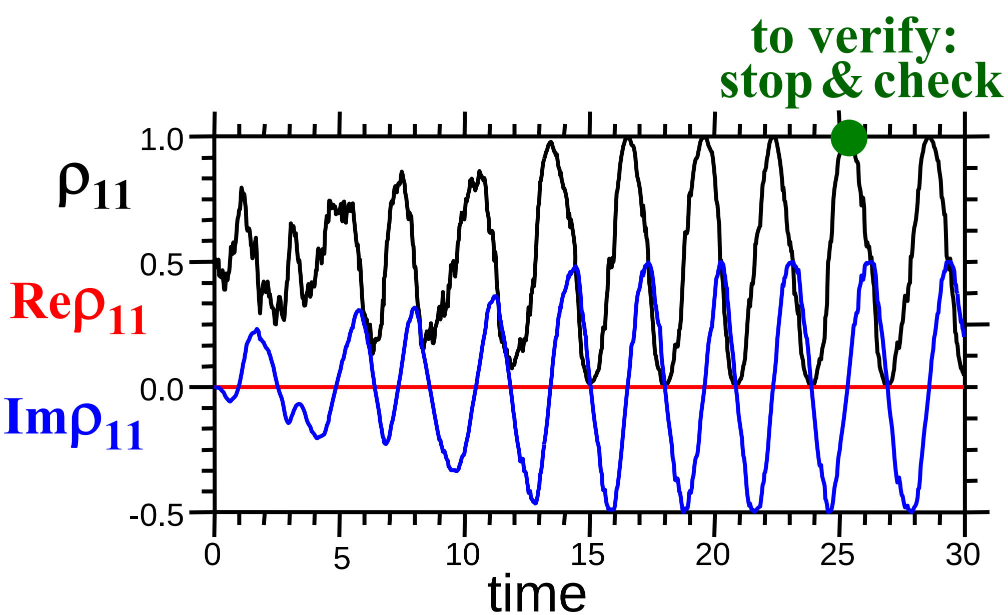
Experimental predictions and proposals from Bayesian formalism

- Direct experimental verification (1998)
- Measured spectral density of Rabi oscillations (1999, 2000, 2002)
- Bell-type correlation experiment (2000)
- Quantum feedback control of a qubit (2001)
- Entanglement by measurement (2002)
- Measurement by a quadratic detector (2003)
- Simple quantum feedback of a qubit (2004)
- Squeezing of a nanomechanical resonator (2004)
- Violation of Leggett-Garg inequality (2005)
- Partial collapse of a phase qubit (2005)
- Undoing of a weak measurement (2006)

Persistent Rabi oscillations



- Relaxes to the ground state if left alone (low-T)
- Becomes fully mixed if coupled to a high-T (non-equilibrium) environment
- Oscillates persistently between left and right if (weakly) measured continuously

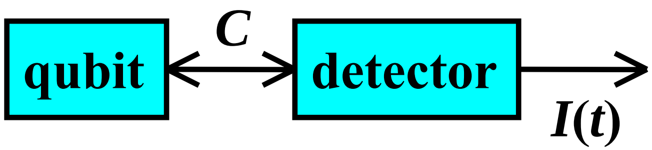


A.K., 1998

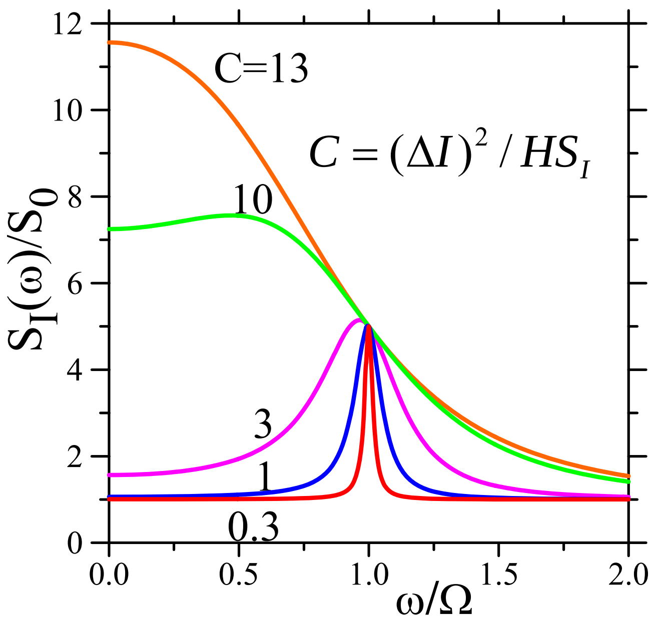
Phase of Rabi oscillations fluctuates (dephasing)

Direct experiment is difficult (good quantum efficiency, bandwidth, control)

Measured spectrum of Rabi oscillations



What is the spectral density $S_I(\omega)$ of detector current?



$$I(t) = I_0 + \frac{\Delta I}{2} z(t) + \xi(t)$$

(const + signal + noise)

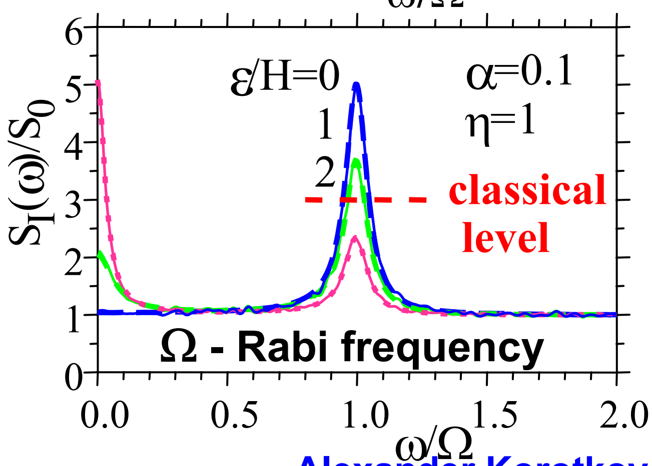
Assume classical output, eV » $\hbar\Omega$

$$\varepsilon = 0, \quad \Gamma = \eta^{-1} (\Delta I)^2 / 4S_0$$

$$S_I(\omega) = S_0 + \frac{\Omega^2 (\Delta I)^2 \Gamma}{(\omega^2 - \Omega^2)^2 + \Gamma^2 \omega^2}$$

Spectral peak can be seen, but peak-to-pedestal ratio ≤ 4η ≤ 4

(result can be obtained using various methods, not only Bayesian method)

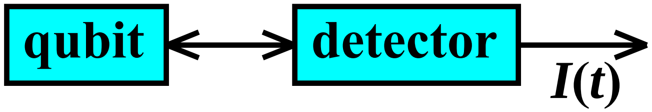


Expt. confirmed (Saclay)

A.K., LT'99
A.K.-Averin, 2000
A.K., 2000
Averin, 2000
Goan-Milburn, 2001
Makhlin et al., 2001
Balatsky-Martin, 2001
Ruskov-A.K., 2002
Mozyrsky et al., 2002
Balatsky et al., 2002
Bulaevskii et al., 2002
Shnirman et al., 2002
Bulaevskii-Ortiz, 2003
Shnirman et al., 2003

Contrary:
Stace-Barrett, PRL-2004

Bell-type (Leggett-Garg-type) inequalities for continuous measurement of a qubit



Ruskov-A.K.-Mizel, PRL-2006
Jordan-A.K.-Büttiker, PRL-2006

Assumptions of macrorealism
(similar to Leggett-Garg'85):

$$I(t) = I_0 + (\Delta I/2)\,Q(t) + \xi(t)$$

$$|Q(t)| \le 1, \quad \langle \xi(t)\, Q(t+\tau)\rangle = 0$$

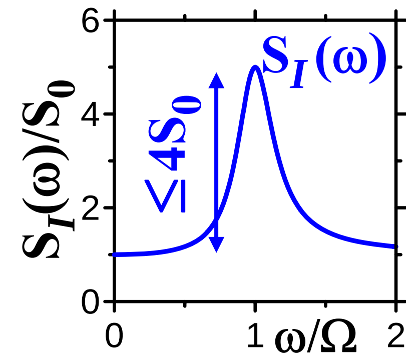
Leggett-Garg,1985

$K_{ij} = \langle Q_i Q_j \rangle$

if $Q = \pm 1$, then

$1 + K_{12} + K_{23} + K_{13} \ge 0$

$K_{12} + K_{23} + K_{34} - K_{14} \le 2$



Then for correlation function

$$K(\tau) = \langle I(t)\, I(t+\tau)\rangle$$

	quantum result	violation
$K(\tau_1) + K(\tau_2) - K(\tau_1+\tau_2) \le (\Delta I/2)^2$	$\frac{3}{2}(\Delta I/2)^2$	$\times\frac{3}{2}$

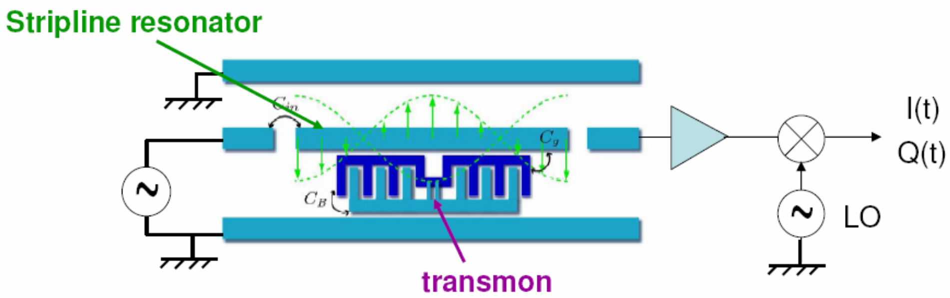
and for area under spectral peak

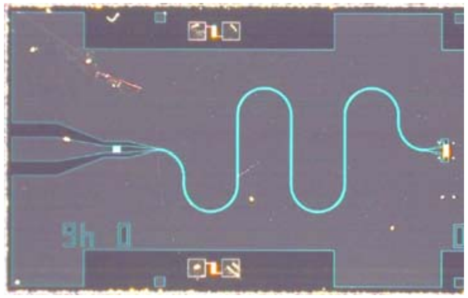
$\int [S_I(f) - S_0]\, df \le (8/\pi^2)(\Delta I/2)^2$	$(\Delta I/2)^2$	$\times\frac{\pi^2}{8}$

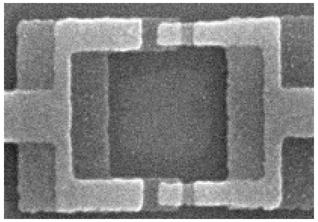
Experimentally measurable violation of classical bound

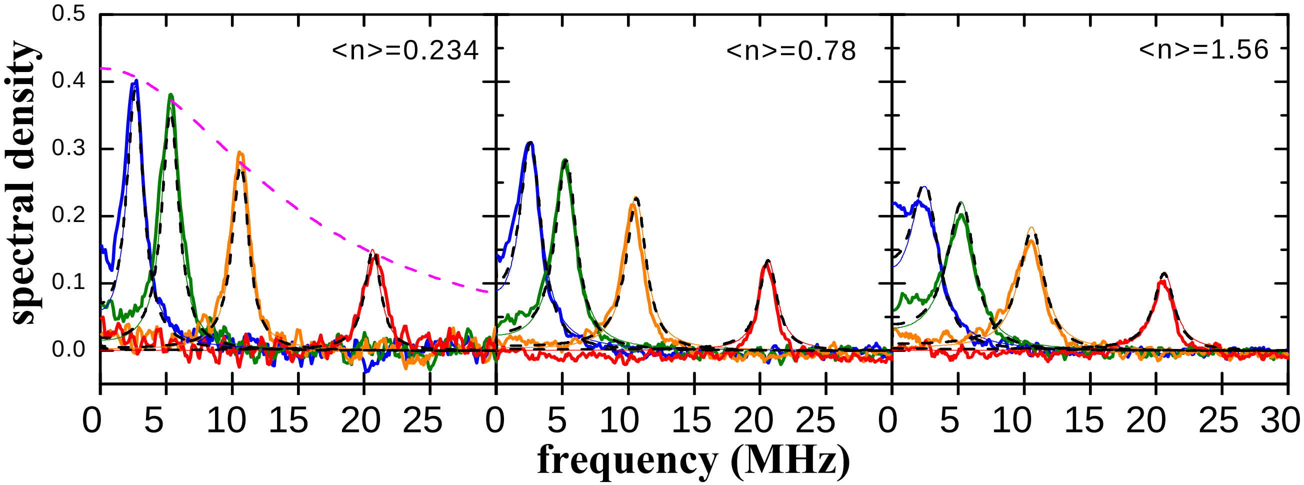
Alexander Korotkov University of California, Riverside

Recent experiment (Saclay group, unpub.)



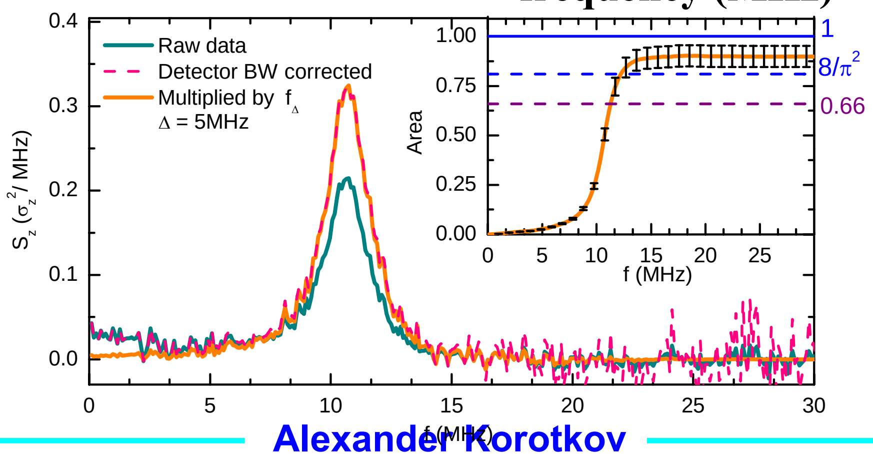






A. Palacios-Laloy et al. (unpublished)

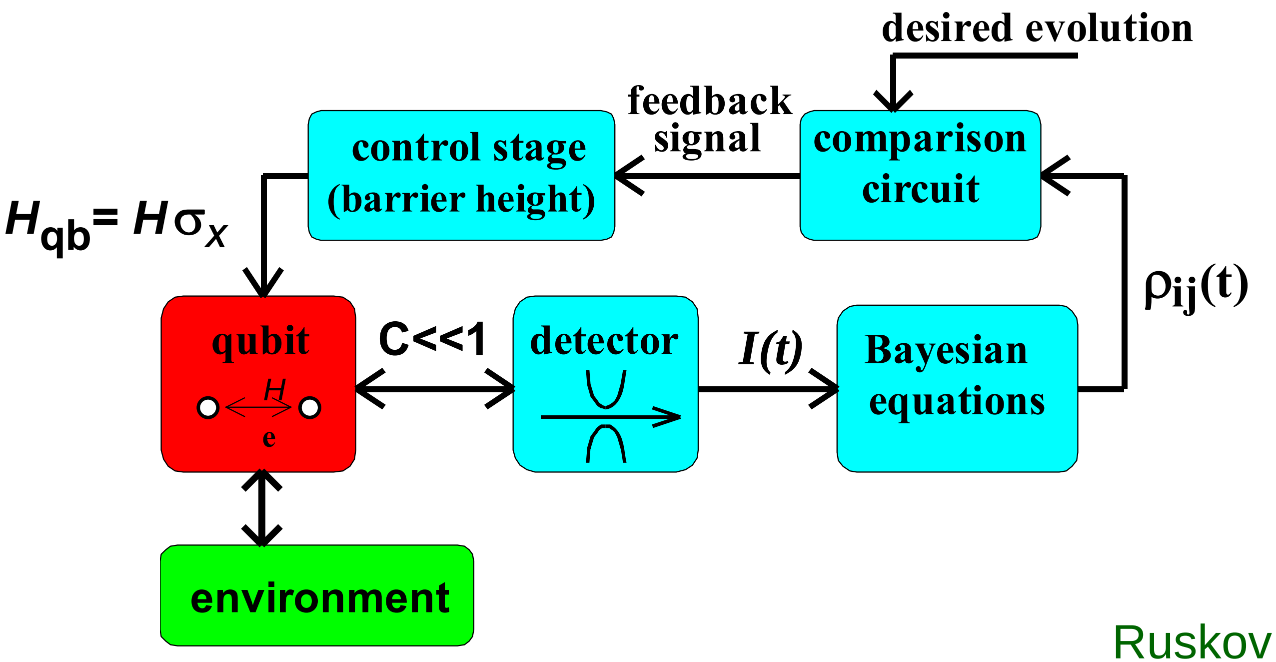
courtesy of Patrice Bertet



- superconducting charge qubit (transmon) in circuit QED setup
- driven Rabi oscillations
  - perfect spectral peaks
  - LGI violation

Quantum feedback control of a qubit

Since qubit state can be monitored, the feedback is possible!



Ruskov & A.K., 2001

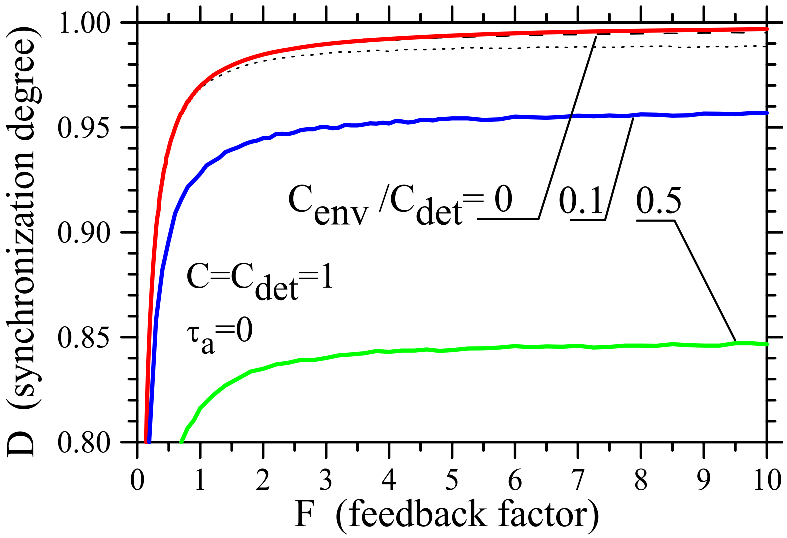
Goal: persistent Rabi oscillations with perfect phase

Idea: monitor the Rabi phase ϕ by continuous measurement and apply feedback control of the qubit barrier height, $\Delta H_{FB}/H = -F\times\Delta\phi$

To monitor phase ϕ we plug detector output $I(t)$ into Bayesian equations

Performance of Bayesian feedback

Feedback fidelity vs. feedback strength



$C = \hbar(\Delta I)^2 / S_I H$ – coupling

F – feedback strength

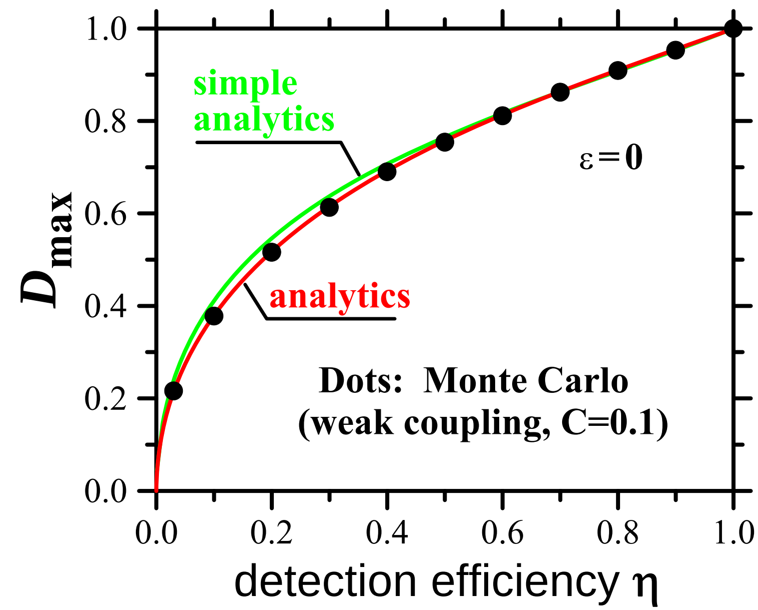
$D = 2\langle \mathrm{Tr}\,\rho_{desired}\,\rho\rangle - 1$

For ideal detector and wide bandwidth, feedback fidelity can be close to 100%

$D = \exp(-C/32F)$

Ruskov & A.K., 2002

Feedback fidelity vs. detector efficiency



$$\eta \ll 1 \Rightarrow D_{max} \approx 1.25\sqrt{\eta}$$

$$\eta \approx 1 \Rightarrow D_{max} \approx (1+\eta)/2$$

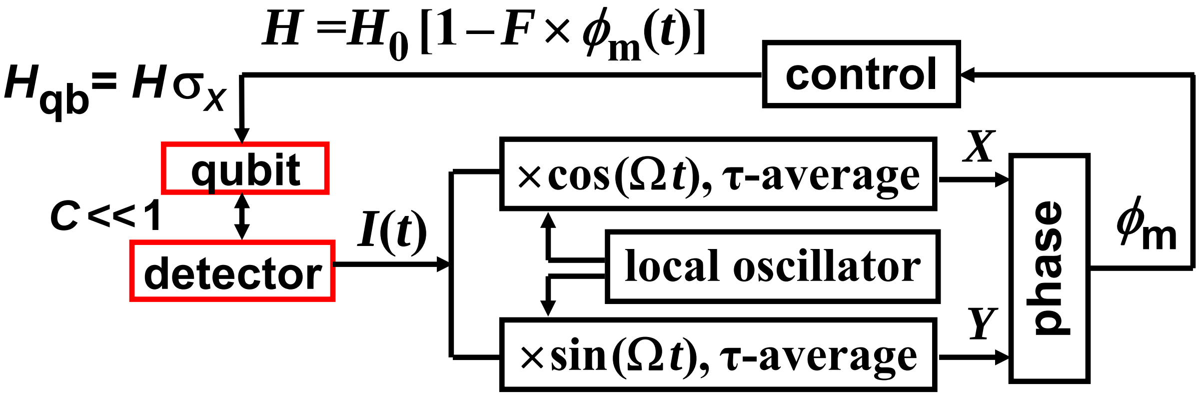
Zhang, Ruskov, A.K., 2005

Experimental difficulties:

- need real-time solution of Bayesian eqs.
- wide bandwidth ($\gg\Omega$) of the output $I(t)$

Simple quantum feedback of a solid-state qubit

(A.K., 2005)



Goal: maintain coherent (Rabi) oscillations for arbitrarily long time

Idea: **use two quadrature components of the detector current *I*(*t*) to monitor approximately the phase of qubit oscillations**
(a very natural way for usual classical feedback!)

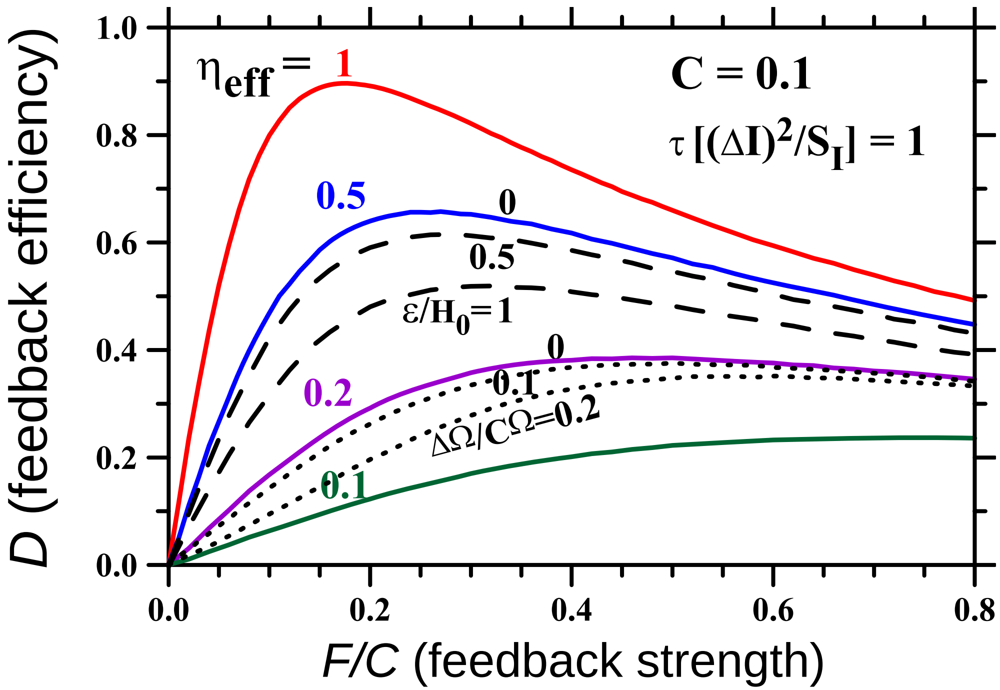
$$X(t) = \int_{-\infty}^{t} [I(t') - I_0] \cos(\Omega t') \exp[-(t-t')/\tau]\, dt'$$
$$Y(t) = \int_{-\infty}^{t} [I(t') - I_0] \sin(\Omega t') \exp[-(t-t')/\tau]\, dt'$$
$$\phi_m = -\arctan(Y/X)$$

(similar formulas for a tank circuit instead of mixing with local oscillator)

Advantage: **simplicity and relatively narrow bandwidth** $(1/\tau \sim \Gamma_d << \Omega)$

Essentially classical feedback. Does it really work?

Fidelity of simple quantum feedback



$D_{max} \approx 90\%$

$D \equiv 2F_Q - 1$

$F_Q \equiv \langle \mathrm{Tr}\, \rho(t)\, \rho_{des}(t) \rangle$

Robust to imperfections
(inefficient detector, frequency mismatch, qubit asymmetry)

How to verify feedback operation experimentally?

Simple: just check that in-phase quadrature $\langle X \rangle$ of the detector current is positive $D = \langle X \rangle (4/\tau\Delta I)$

$\langle X \rangle$=0 for *any* non-feedback Hamiltonian control of the qubit

Simple enough for real experiment!

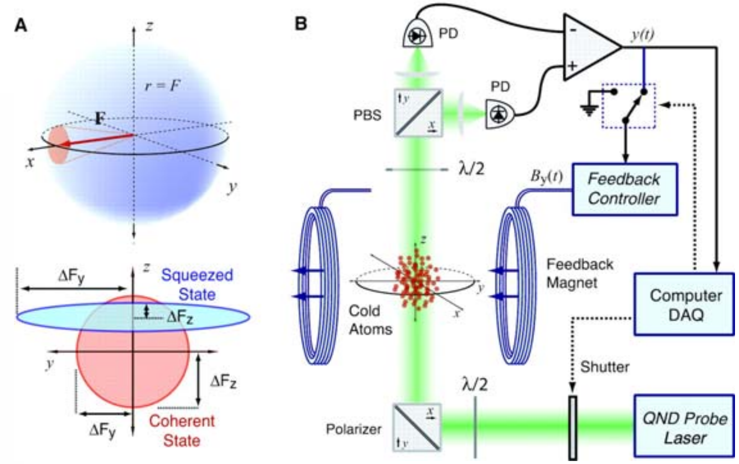
Quantum feedback in optics

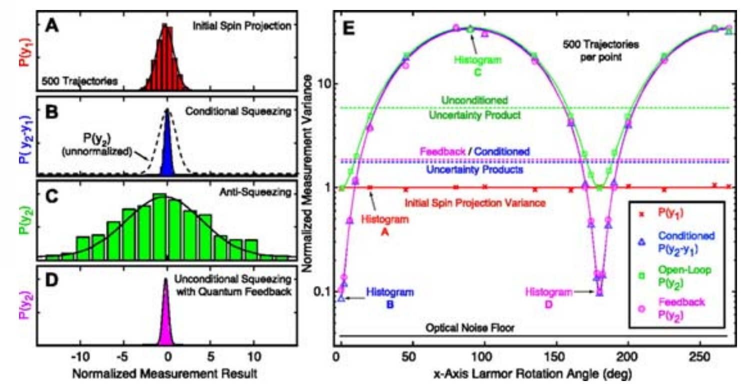
First experiment: Science 304, 270 (2004)

Real-Time Quantum Feedback Control of Atomic Spin-Squeezing

JM Geremia,* John K. Stockton, Hideo Mabuchi

Real-time feedback performed during a quantum nondemolition measurement of atomic spin-angular momentum allowed us to influence the quantum statistics of the measurement outcome. We showed that it is possible to harness measurement backaction as a form of actuation in quantum control, and thus we describe a valuable tool for quantum information science. Our feedback-mediated procedure generates spin-squeezing, for which the reduction in quantum uncertainty and resulting atomic entanglement are not conditioned on the measurement outcome.





First detailed theory:

H.M. Wiseman and G. J. Milburn,
Phys. Rev. Lett. 70, 548 (**1993**)

Quantum feedback in optics

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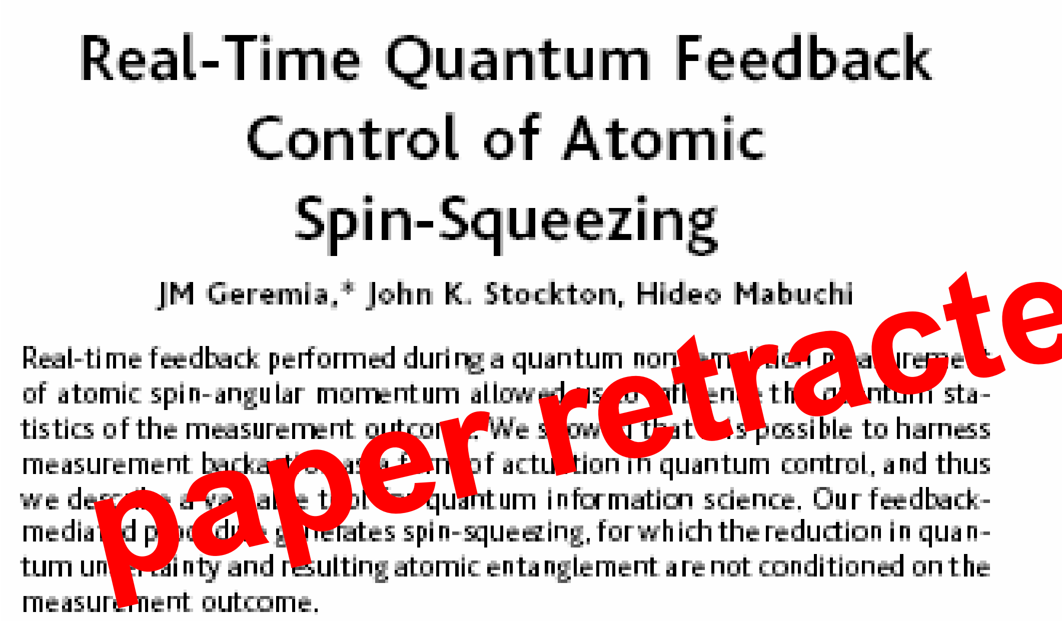

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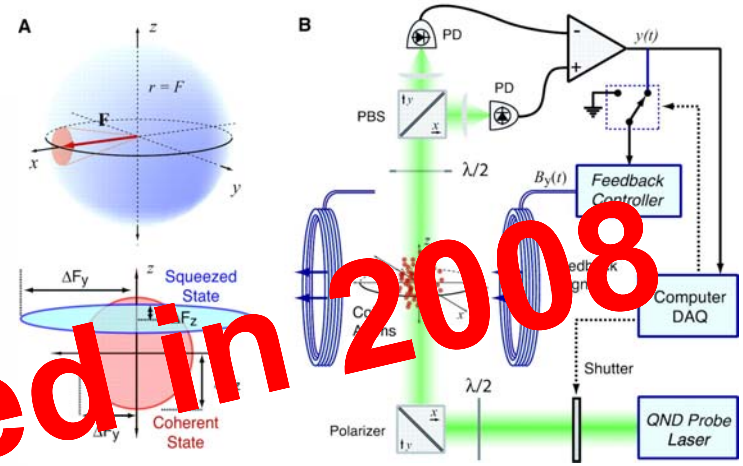
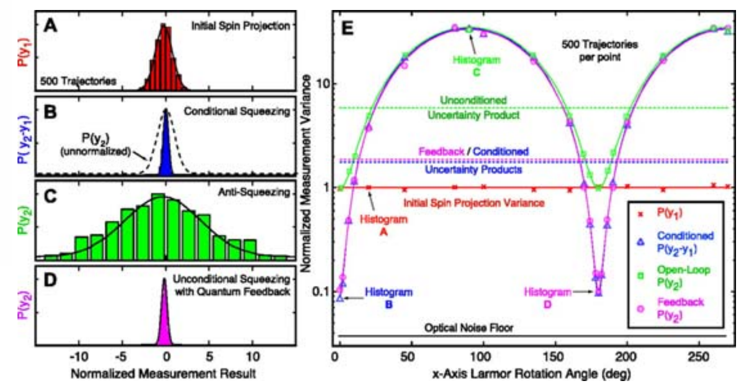
paper retracted in 2008





PRL 94, 203002 (2005) also withdrawn

First detailed theory:

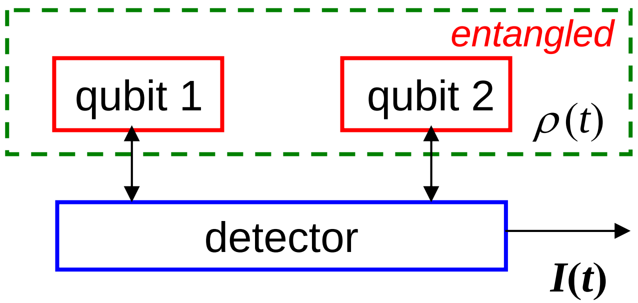
H.M. Wiseman and G. J. Milburn,
Phys. Rev. Lett. 70, 548 (**1993**)

Recent experiment:

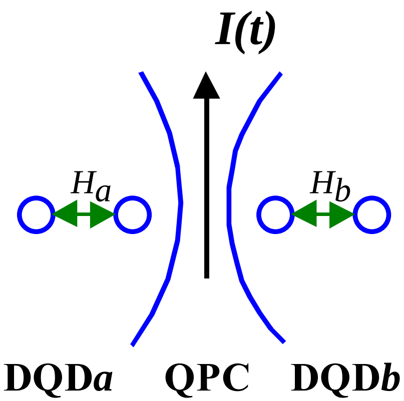
Cook, Martin, Geremia,
Nature 446, 774 (2007)
(coherent state discrimination)

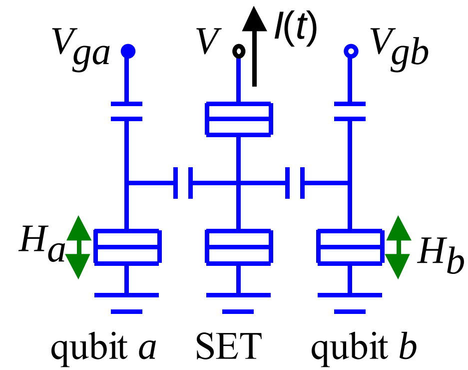
Two-qubit entanglement by measurement

Ruskov & A.K., 2002

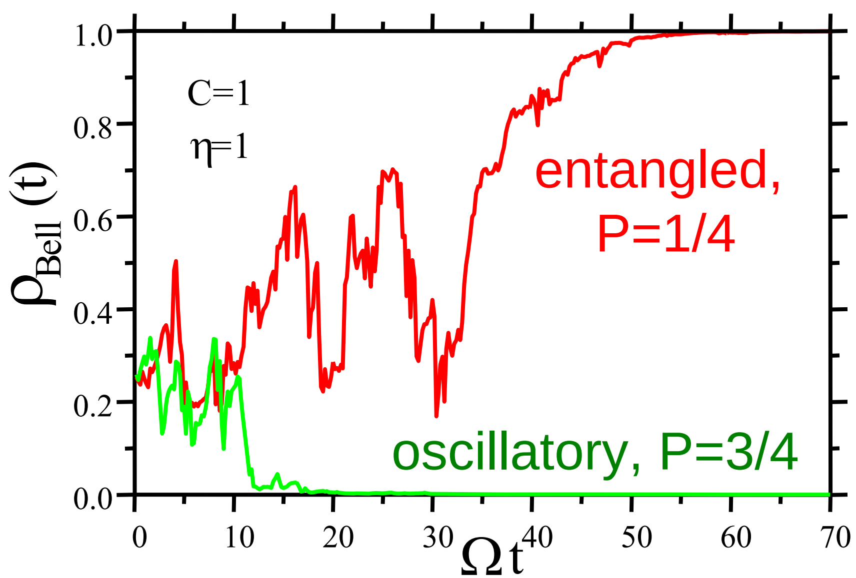


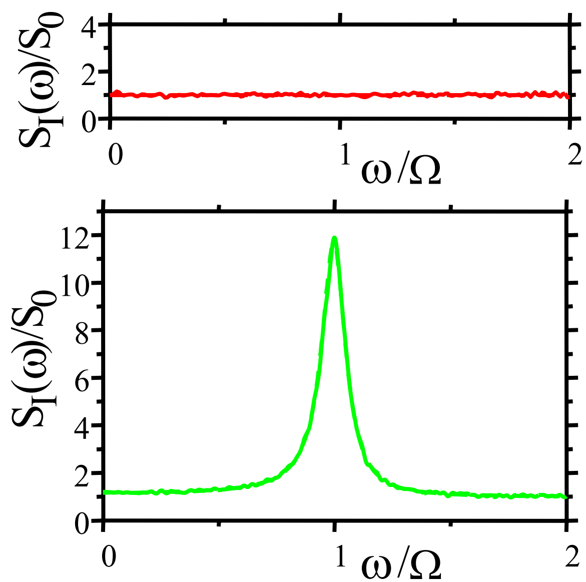
Symmetric setup, no qubit interaction





Two evolution scenarios:



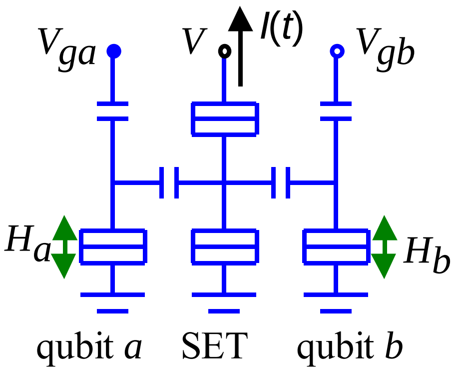


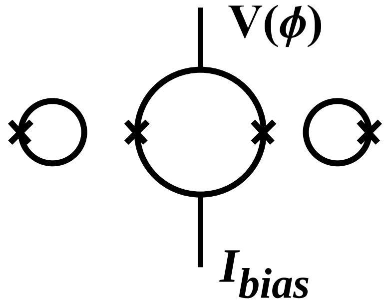
Peak/noise = (32/3)η

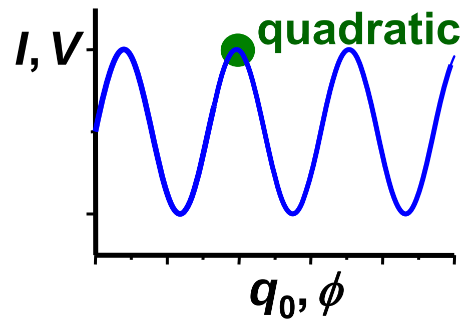
Collapse into |Bell⟩ state (spontaneous entanglement) with probability 1/4 starting from fully mixed state

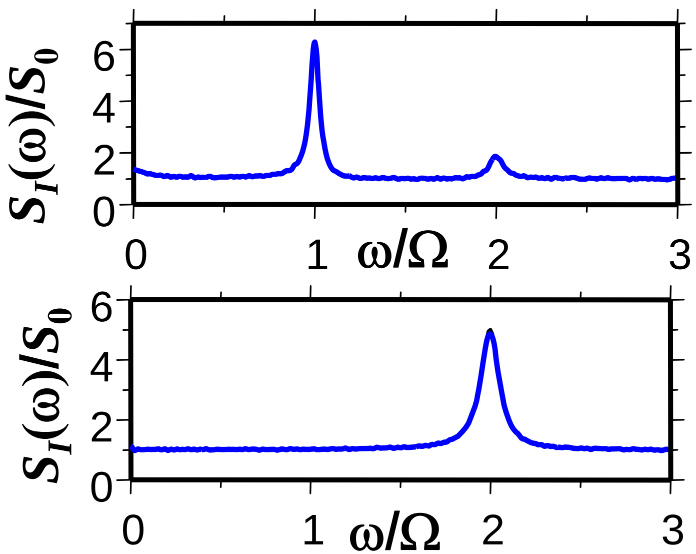
Quadratic quantum detection

Mao, Averin, Ruskov, & A.K., PRL-2004









Nonlinear detector:

spectral peaks at Ω, 2Ω and 0

Quadratic detector:

Peak only at 2Ω, peak/noise = 4η

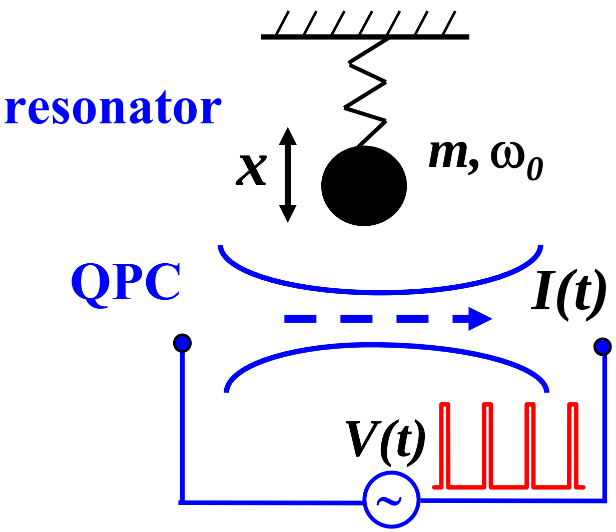
$$S_I(\omega) = S_0 + \frac{4\Omega^2(\Delta I)^2\Gamma}{(\omega^2 - 4\Omega^2)^2 + \Gamma^2\omega^2}$$

Three evolution scenarios: 1) collapse into $|\uparrow\downarrow - \downarrow\uparrow\rangle$, current $I_{\uparrow\downarrow}$, flat spectrum 2) collapse into $|\uparrow\uparrow - \downarrow\downarrow\rangle$, current $I_{\uparrow\uparrow}$, flat spectrum; 3) collapse into remaining subspace, current $(I_{\uparrow\downarrow} + I_{\uparrow\uparrow})/2$, spectral peak at 2Ω

Entangled states distinguished by average detector current

QND squeezing of a nanomechanical resonator

Ruskov, Schwab, & A.K., PRB-2005



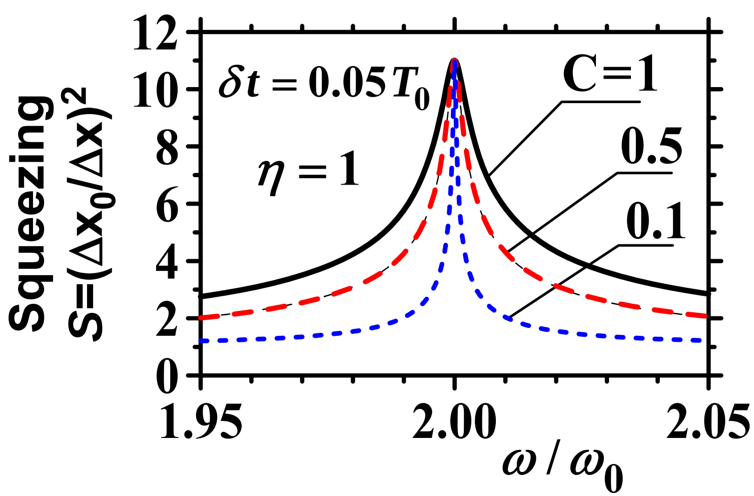
Experimental status:

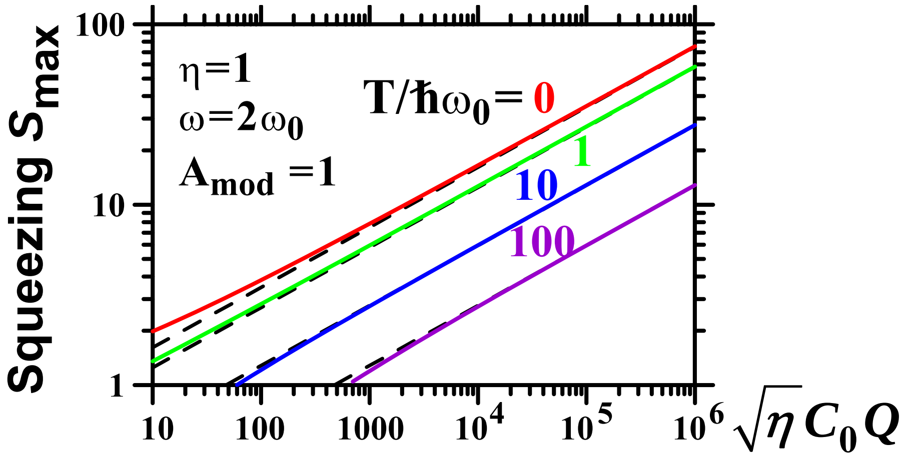
$\omega_0/2\pi \sim 1$ GHz ($\hbar\omega_0 \sim 80$ mK), Roukes' group, 2003

$\Delta x/\Delta x_0 \sim 5$ [SQL $\Delta x_0 = (\hbar/2m\omega_0)^{1/2}$], Schwab's group, 2004

$$S_{max} = \frac{3}{4}\left[\frac{\sqrt{\eta}\,C_0 Q}{\coth(\hbar\omega_0/2T)}\right]^{1/3}$$

C_0 – coupling with detector, η – detector efficiency,
T – temperature, Q – resonator Q-factor





(So far in experiment $\eta^{1/2}C_0Q \sim 0.1$)

Potential application: ultrasensitive force measurements

Undoing a weak measurement of a qubit (“uncollapse”)

A.K. & Jordan, PRL-2006

It is impossible to undo “orthodox” quantum measurement (for an unknown initial state)

Is it possible to undo partial quantum measurement?
(To restore a “precious” qubit accidentally measured)
Yes! (but with a finite probability)

If undoing is successful, an unknown state is **fully** restored

ψ_0 (unknown) → weak (partial) measurement → ψ_1 (partially collapsed)

ψ_1 → successful → ψ_0 (still unknown)

ψ_1 → unsuccessful → ψ_2

undoing (information erasure)

Quantum erasers in optics

Quantum eraser proposal by Scully and Drühl, PRA (1982)

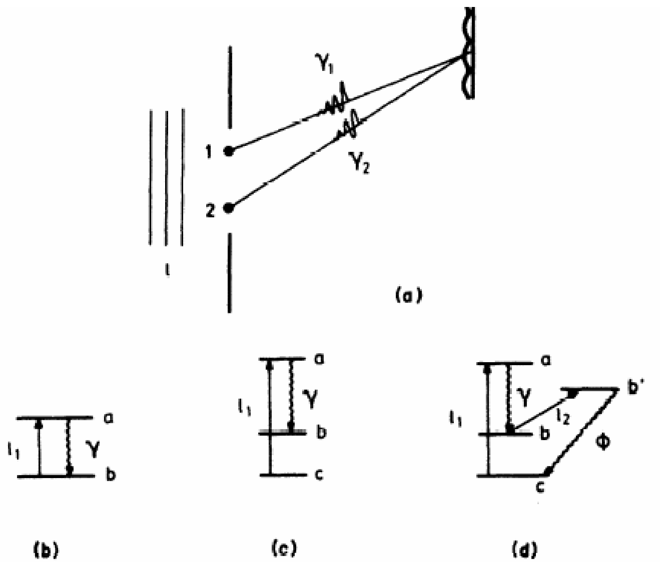


FIG. 1. (a) Figure depicting light impinging from left on atoms at sites 1 and 2. Scattered photons γ_1 and γ_2 produce interference pattern on screen. (b) Two-level atoms excited by laser pulse l_1, and emit γ photons in $a \rightarrow b$ transition. (c) Three-level atoms excited by pulse l_1 from $c \rightarrow a$ and emit photons in $a \rightarrow b$ transition. (d) Four-level system excited by pulse l_1 from $c \rightarrow a$ followed by emission of γ photons in $a \rightarrow b$ transition. Second pulse l_2 takes atoms from $b \rightarrow b'$. Decay from $b' \rightarrow c$ results in emission of ϕ photons.

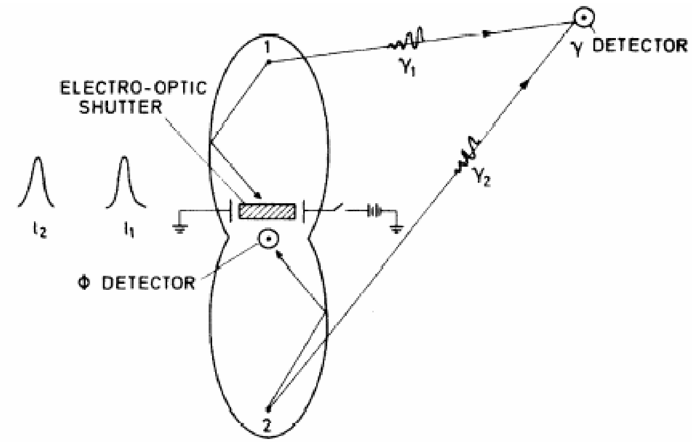


FIG. 2. Laser pulses l_1 and l_2 incident on atoms at sites 1 and 2. Scattered photons γ_1 and γ_2 result from $a \rightarrow b$ transition. Decay of atoms from $b' \rightarrow c$ results in ϕ photon emission. Elliptical cavities reflect ϕ photons onto common photodetector. Electro-optic shutter transmits ϕ photons only when switch is open. Choice of switch position determines whether we emphasize particle or wave nature of γ photons.

Interference fringes restored for two-detector correlations (since "which-path" information is erased)

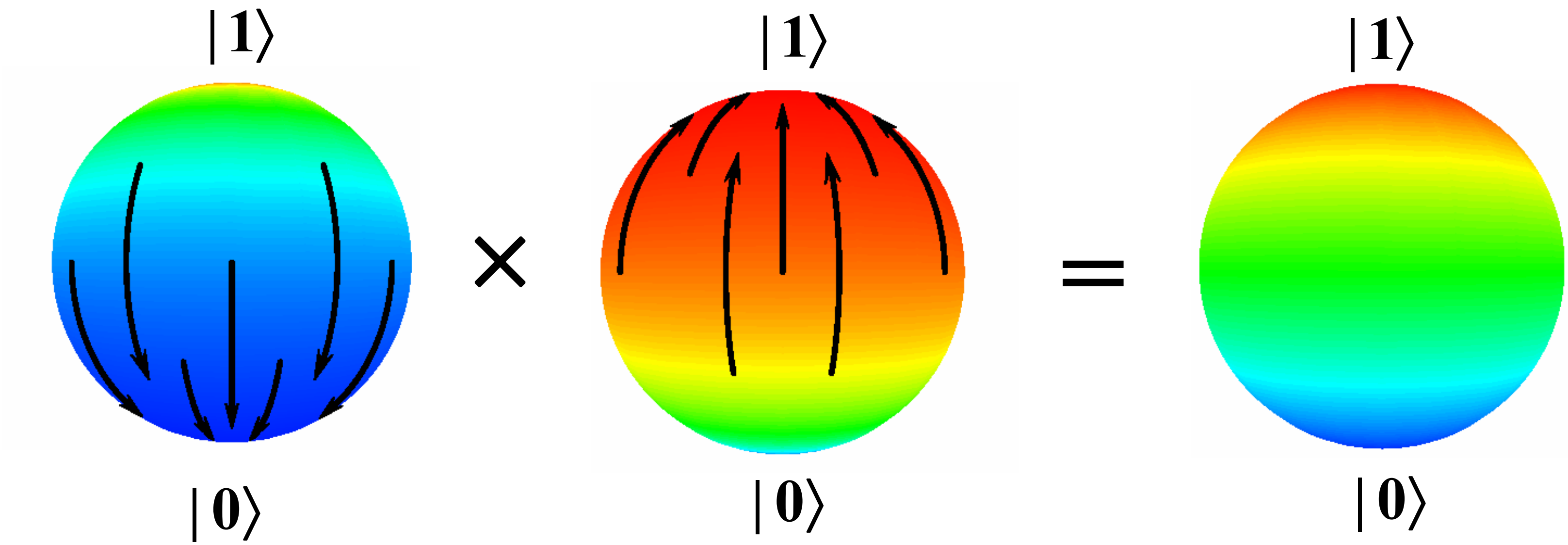
Our idea of uncollapsing is quite different:
we really extract quantum information and then erase it

Alexander Korotkov University of California, Riverside

Uncollapsing of a qubit state

Evolution due to partial (weak, continuous, etc.) measurement is **non-unitary** (though coherent if detector is good!), therefore it is impossible to undo it by Hamiltonian dynamics.

How to undo? One more measurement!

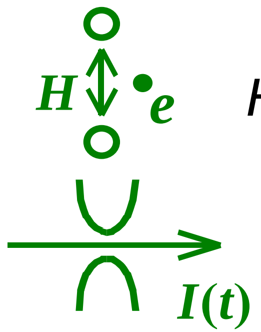


need ideal (quantum-limited) detector

(similar to Koashi-Ueda, PRL-1999)

(Figure partially adopted from Jordan-A.K.-Büttiker, PRL-06)

Evolution of a charge qubit



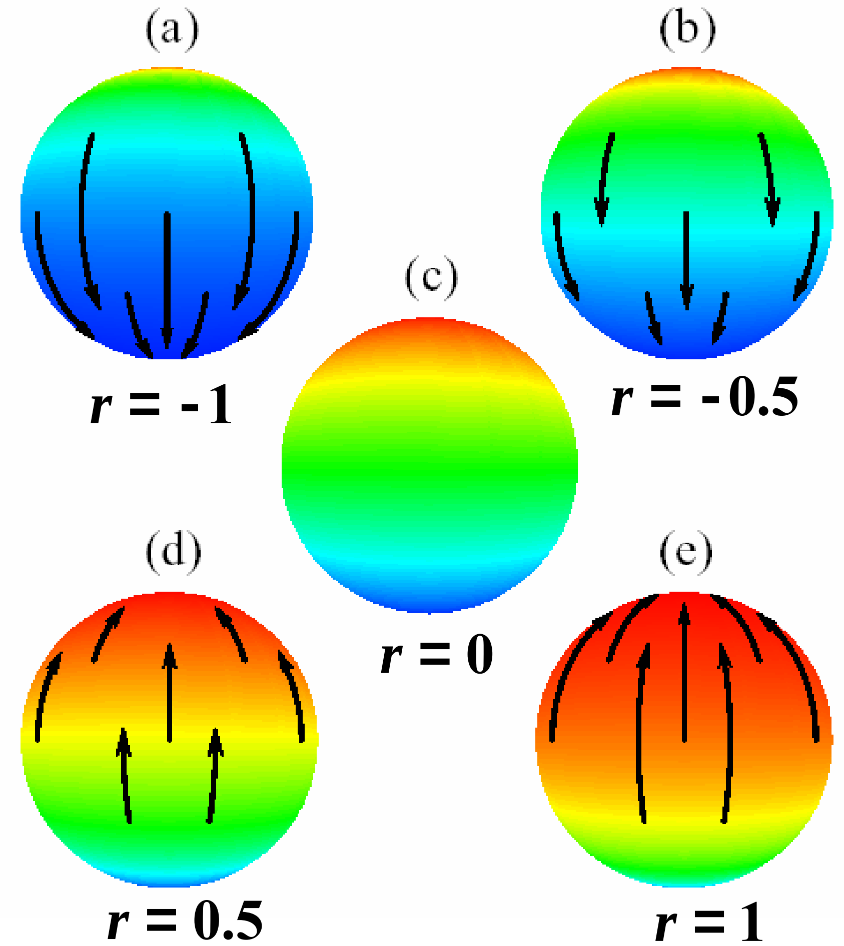
H=0

$$\frac{\rho_{11}(t)}{\rho_{22}(t)} = \frac{\rho_{11}(0)}{\rho_{22}(0)} \exp[2r(t)]$$

$$\frac{\rho_{12}(t)}{\sqrt{\rho_{11}(t)\rho_{22}(t)}} = \text{const}$$

where measurement result $r(t)$ is

$$r(t) = \frac{\Delta I}{S_I}\left[\int_0^t I(t')\,dt' - I_0 t\right]$$

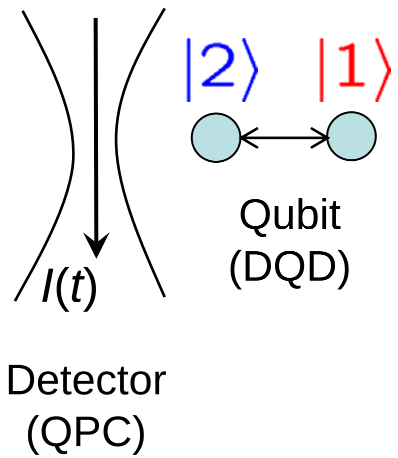


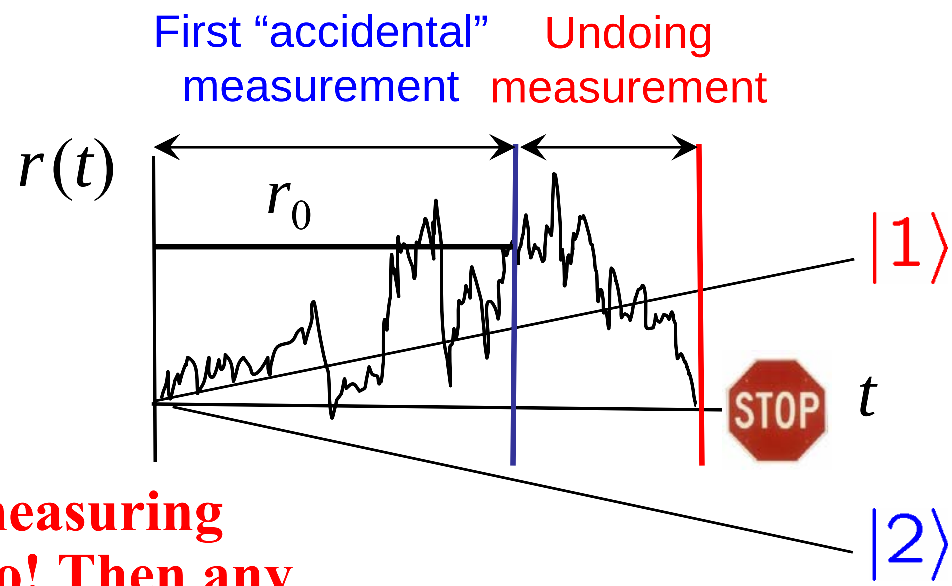
Jordan-Korotkov-Büttiker, PRL-06

If $r = 0$, then no information and no evolution!

Uncollapsing for DQD-QPC system

A.K. & Jordan, PRL-2006





Simple strategy: continue measuring until result $r(t)$ becomes zero! Then any unknown initial state is fully restored.

(same for an entangled qubit)

$$r(t) = \frac{\Delta I}{S_I}\left[\int_0^t I(t')\,dt' - I_0 t\right]$$

It may happen though that $r = 0$ never happens; then undoing procedure is unsuccessful.

Probability of success:
$$P_S = \frac{e^{-|r_0|}}{e^{|r_0|}\rho_{11}(0) + e^{-|r_0|}\rho_{22}(0)}$$

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General theory of uncollapsing

POVM formalism
(Nielsen-Chuang, p.85)

Measurement operator M_r: $\rho \to \dfrac{M_r \rho M_r^\dagger}{\mathrm{Tr}(M_r \rho M_r^\dagger)}$

Probability: $P_r = \mathrm{Tr}(M_r \rho M_r^\dagger)$ Completeness: $\sum_r M_r^\dagger M_r = 1$

Uncollapsing operator: $C \times M_r^{-1}$ (to satisfy completeness, eigenvalues cannot be >1)

$$\max(C) = \min_i \sqrt{p_i}\,,\quad p_i \text{ – eigenvalues of } M_r^\dagger M_r$$

Probability of success: $P_S \le \dfrac{\min P_r}{P_r(\rho_{\text{in}})}$ A.K. & Jordan, 2006

$P_r(\rho_{\text{in}})$ – probability of result r for initial state ρ_{in},

min P_r – probability of result r minimized over all possible initial states

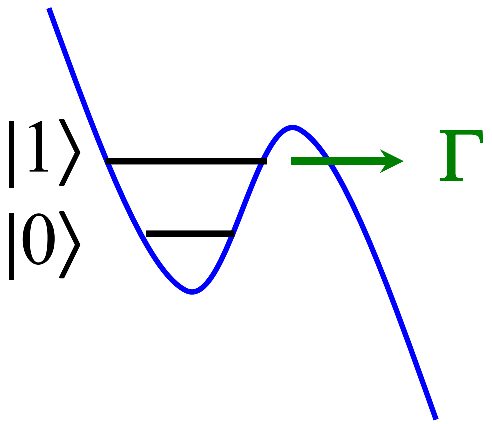
Averaged (over r) probability of success: $P_{av} \le \sum_r \min P_r$

(cannot depend on initial state, otherwise get information)

(similar to Koashi-Ueda, 1999)

Partial collapse of a "phase" qubit

N. Katz, M. Ansmann, R. Bialczak, E. Lucero, R. McDermott, M. Neeley, M. Steffen, E. Weig, A. Cleland, J. Martinis, A. Korotkov, Science-06



How does a coherent state evolve in time before tunneling event?

(What happens when nothing happens?)

Qubit "ages" in contrast to a radioactive atom!

Main idea:

$$\psi = \alpha\,|0\rangle + \beta\,|1\rangle \;\rightarrow\; \psi(t) = \begin{cases} |out\rangle, \text{ if tunneled} \\ \dfrac{\alpha\,|0\rangle + \beta\, e^{-\Gamma t/2} e^{i\varphi}\,|1\rangle}{\sqrt{|\alpha|^2 + |\beta|^2\, e^{-\Gamma t}}}, \text{ if not tunneled} \end{cases}$$

(better theory: Pryadko & A.K., 2007)

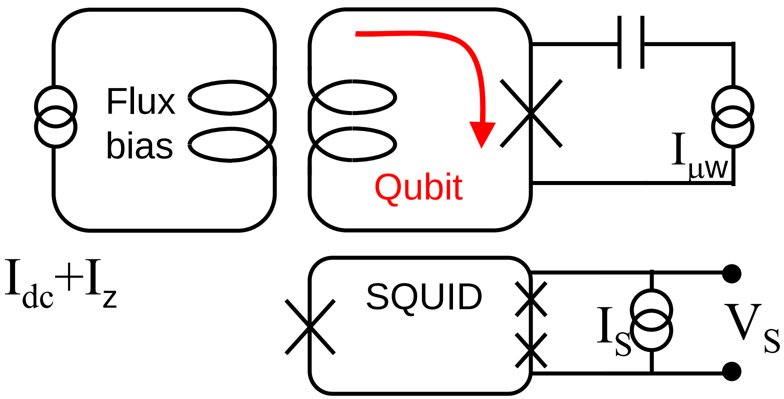
amplitude of state |0> grows without physical interaction

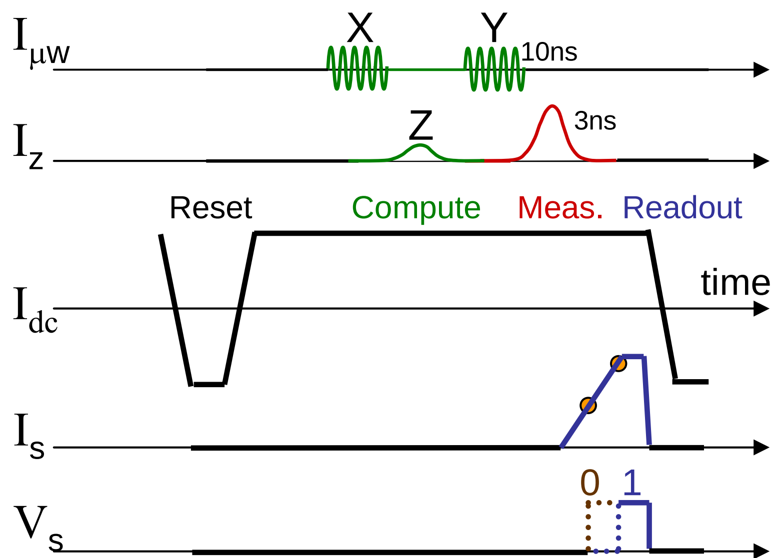
continuous null-result collapse

(similar to optics, Dalibard-Castin-Molmer, PRL-1992)

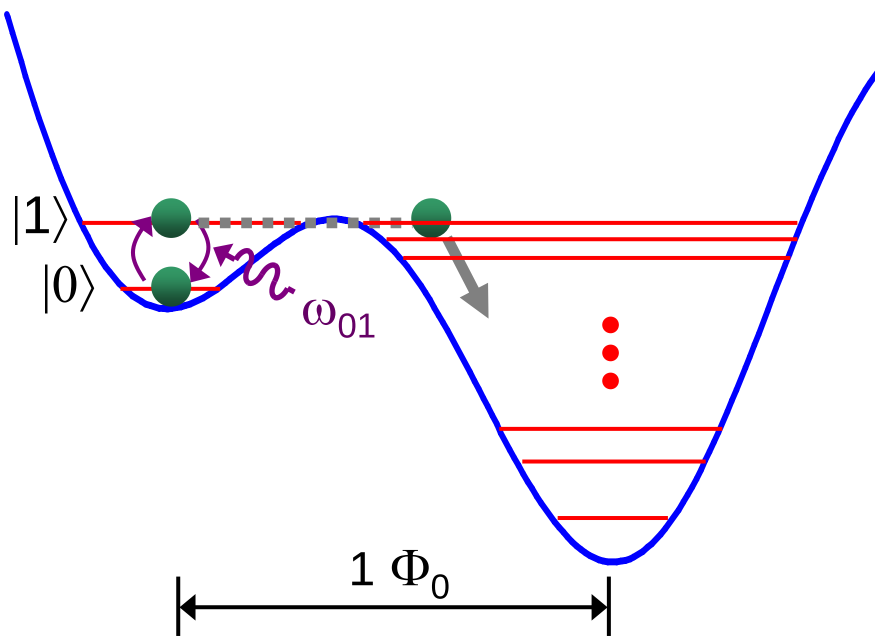
Superconducting phase qubit at UCSB

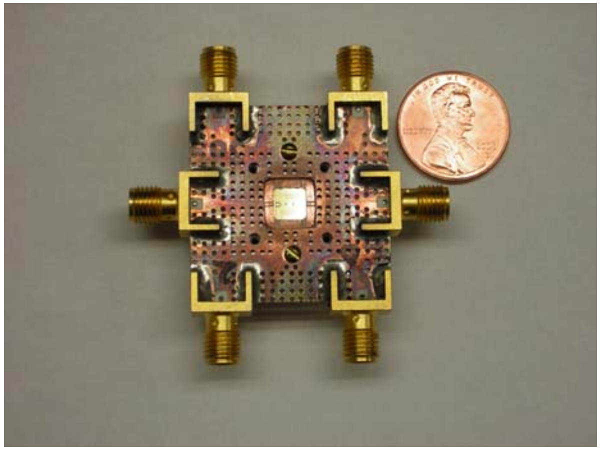
Courtesy of Nadav Katz (UCSB)



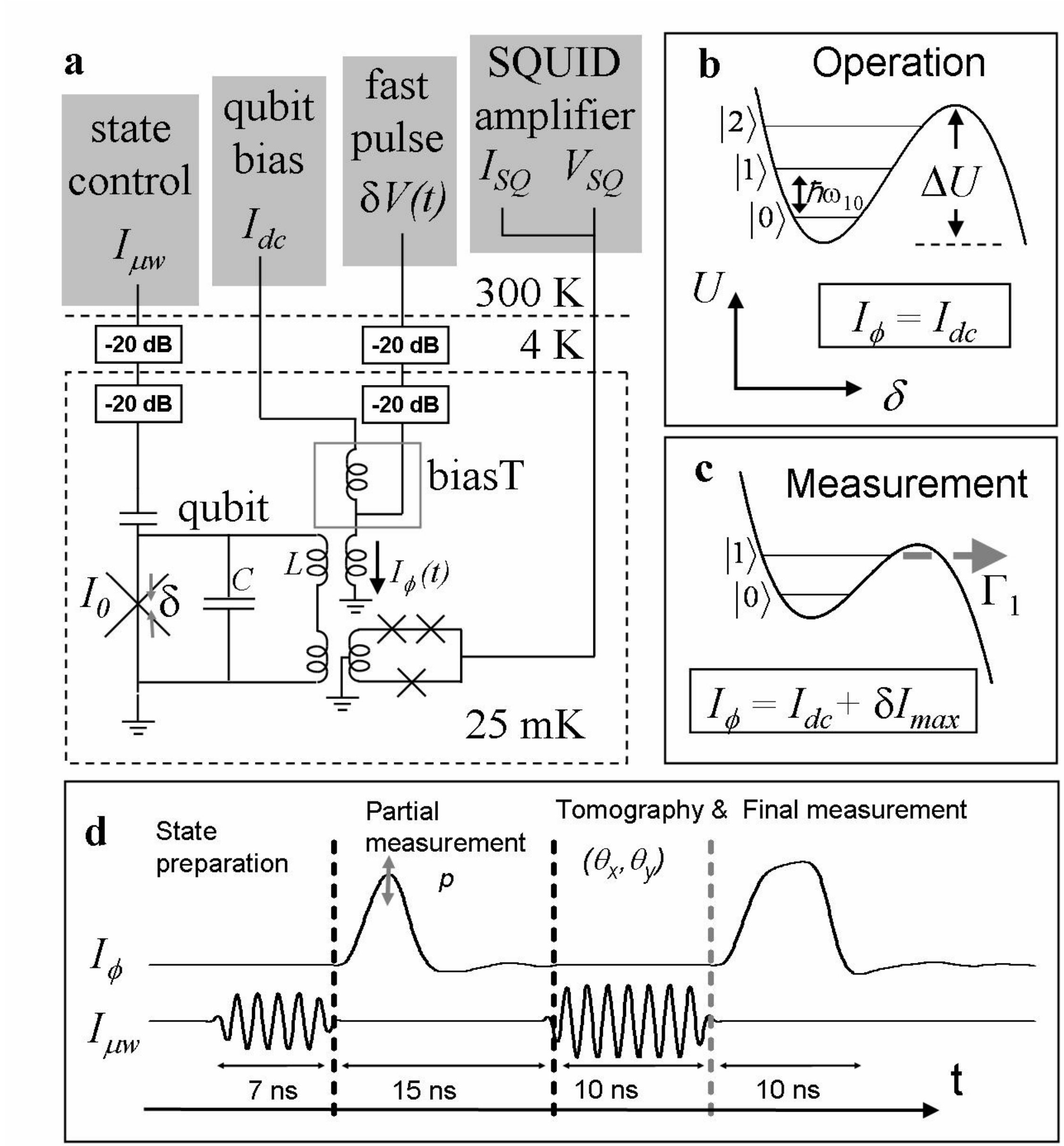


Repeat 1000x
prob. 0,1





Experimental technique for partial collapse



Nadav Katz *et al.*
(John Martinis' group)

Protocol:

1) **State preparation by applying microwave pulse (via Rabi oscillations)**

2) **Partial measurement by lowering barrier for time *t***

3) **State tomography (micro-wave + full measurement)**

Measurement strength

$p = 1 - \exp(-\Gamma t)$

is actually controlled by Γ, not by t

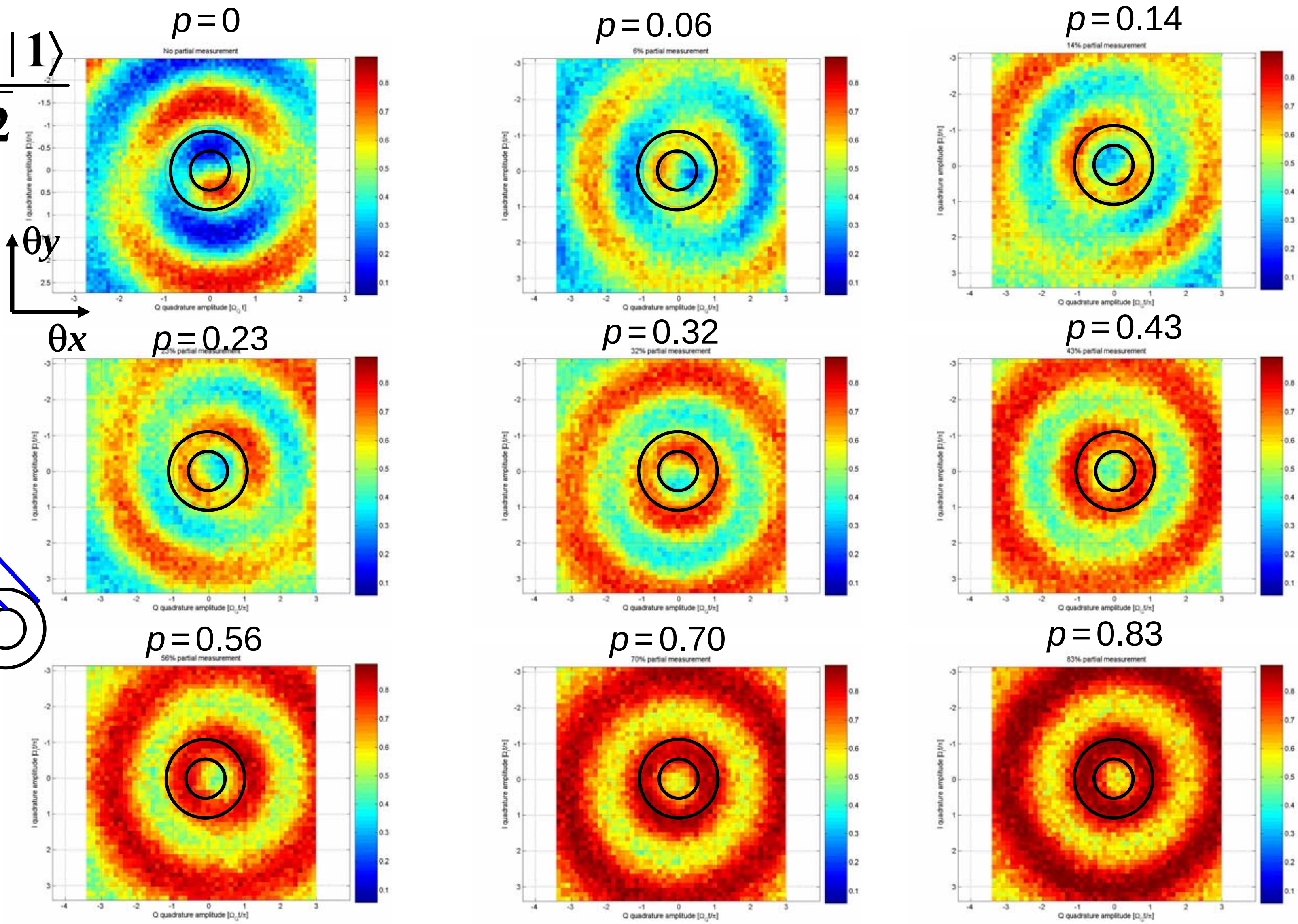
***p*=0: no measurement**

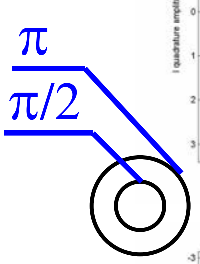
***p*=1: orthodox collapse**

Experimental tomography data

Nadav Katz *et al.* (UCSB)

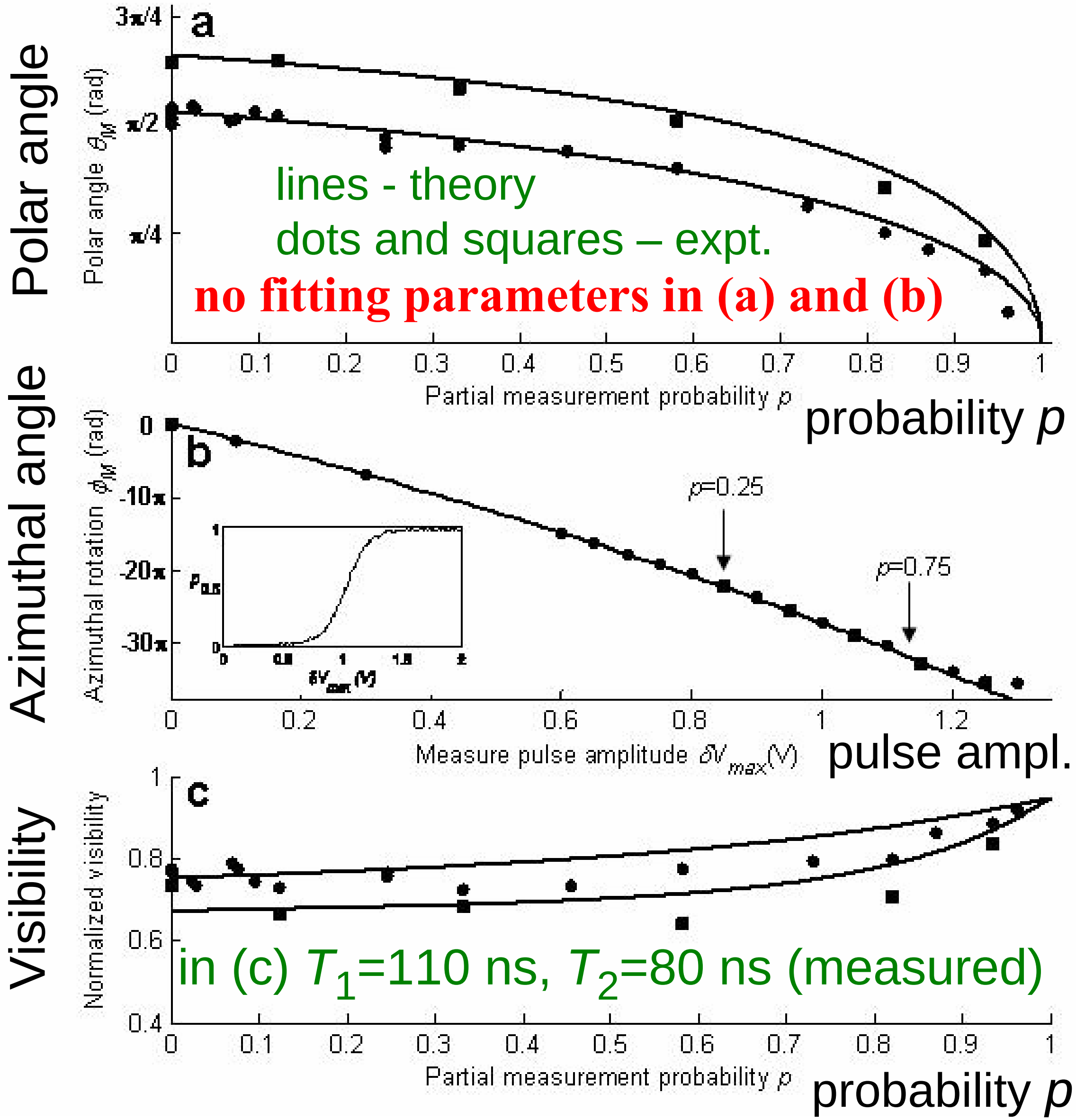
$$\psi_{in} = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$





Partial collapse: experimental results

N. Katz *et al.*, Science-06

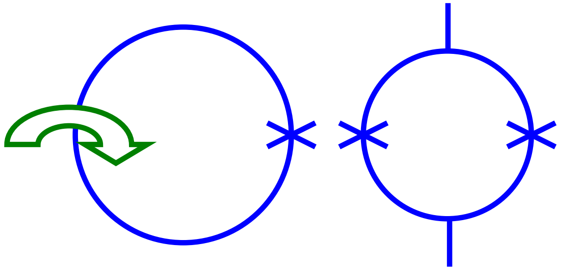


- **In case of no tunneling (null-result measurement) phase qubit evolves**
- **This evolution is well described by a simple Bayesian theory, without fitting parameters**
- **Phase qubit remains fully coherent in the process of continuous collapse (experimentally ~80% raw data, ~96% after account for T1 and T2)**

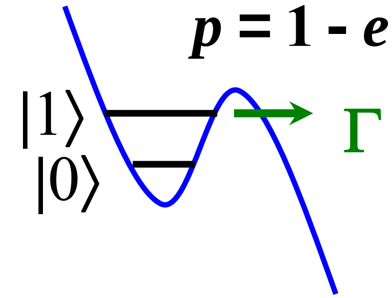
Uncollapsing of a phase qubit state

A.K. & Jordan, 2006

1) Start with an unknown state
2) Partial measurement of strength *p*
3) π-pulse (exchange $|0\rangle \leftrightarrow |1\rangle$)
4) One more measurement with the **same strength** *p*
5) π-pulse



$p = 1 - e^{-\Gamma t}$



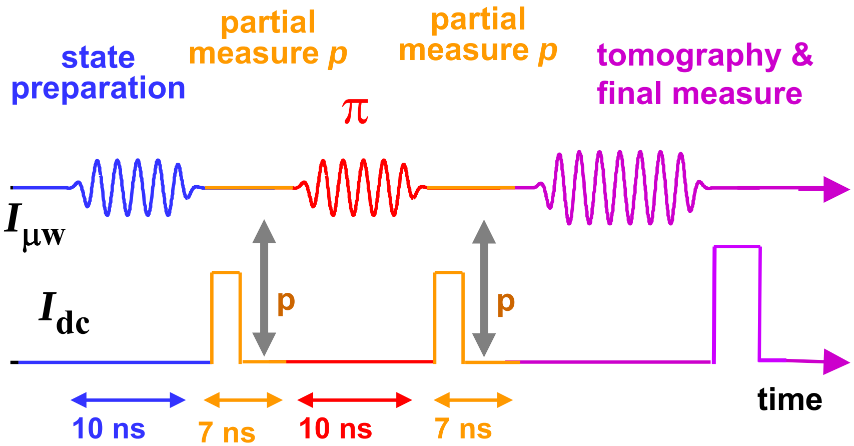
If no tunneling for both measurements, then initial state is fully restored!

$$\alpha|0\rangle + \beta|1\rangle \rightarrow \frac{\alpha|0\rangle + e^{i\phi}\beta e^{-\Gamma t/2}|1\rangle}{\text{Norm}} \rightarrow$$

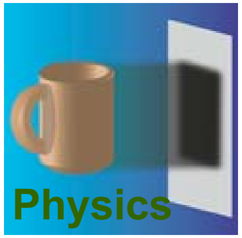
$$\frac{e^{i\phi}\alpha e^{-\Gamma t/2}|0\rangle + e^{i\phi}\beta e^{-\Gamma t/2}|1\rangle}{\text{Norm}} = e^{i\phi}(\alpha|0\rangle + \beta|1\rangle)$$

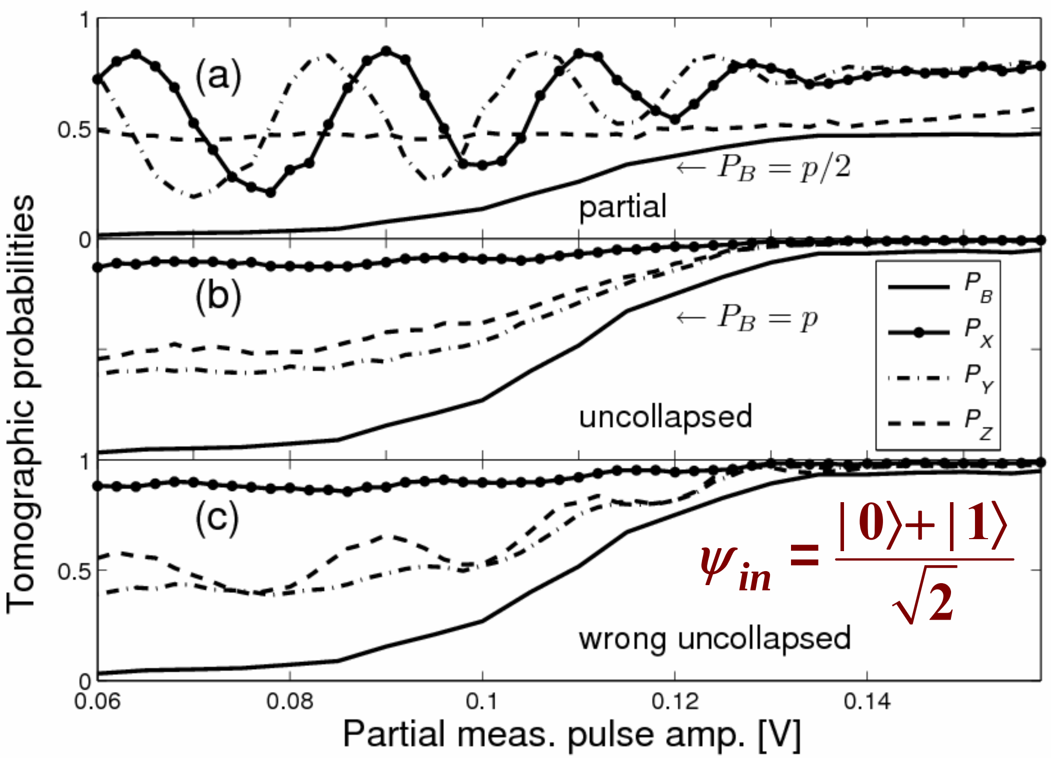
phase is also restored (spin echo)

Experiment on wavefunction uncollapsing



N. Katz, M. Neeley, M. Ansmann, R. Bialzak, E. Lucero, A. O'Connell, H. Wang, A. Cleland, J. Martinis, and A. Korotkov, PRL-2008





Uncollapse protocol:

- partial collapse
- π-pulse
- partial collapse (same strength)

State tomography with *X*, *Y*, and no pulses

Background P_B should be subtracted to find qubit density matrix

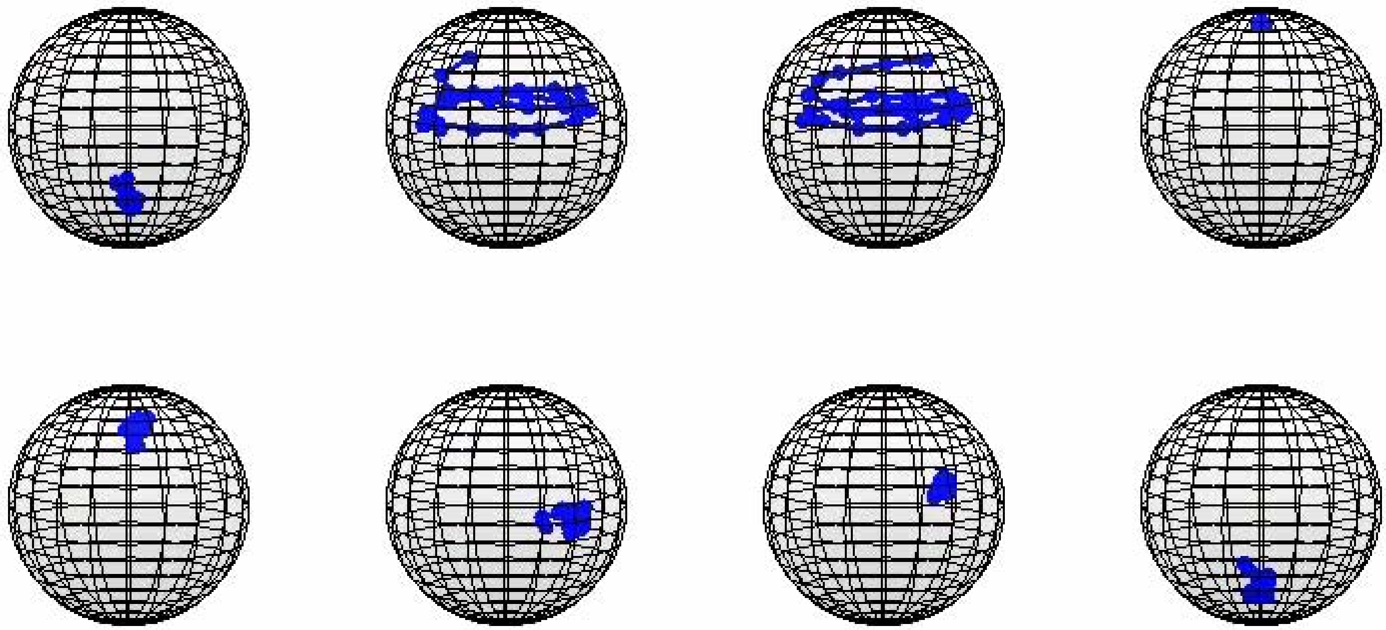
Experimental results on Bloch sphere

N. Katz et al.

Initial state: $|1\rangle$ $\quad \frac{|0\rangle+|1\rangle}{\sqrt{2}}$ $\quad \frac{|0\rangle+i|1\rangle}{\sqrt{2}}$ $\quad |0\rangle$

Partial collapse

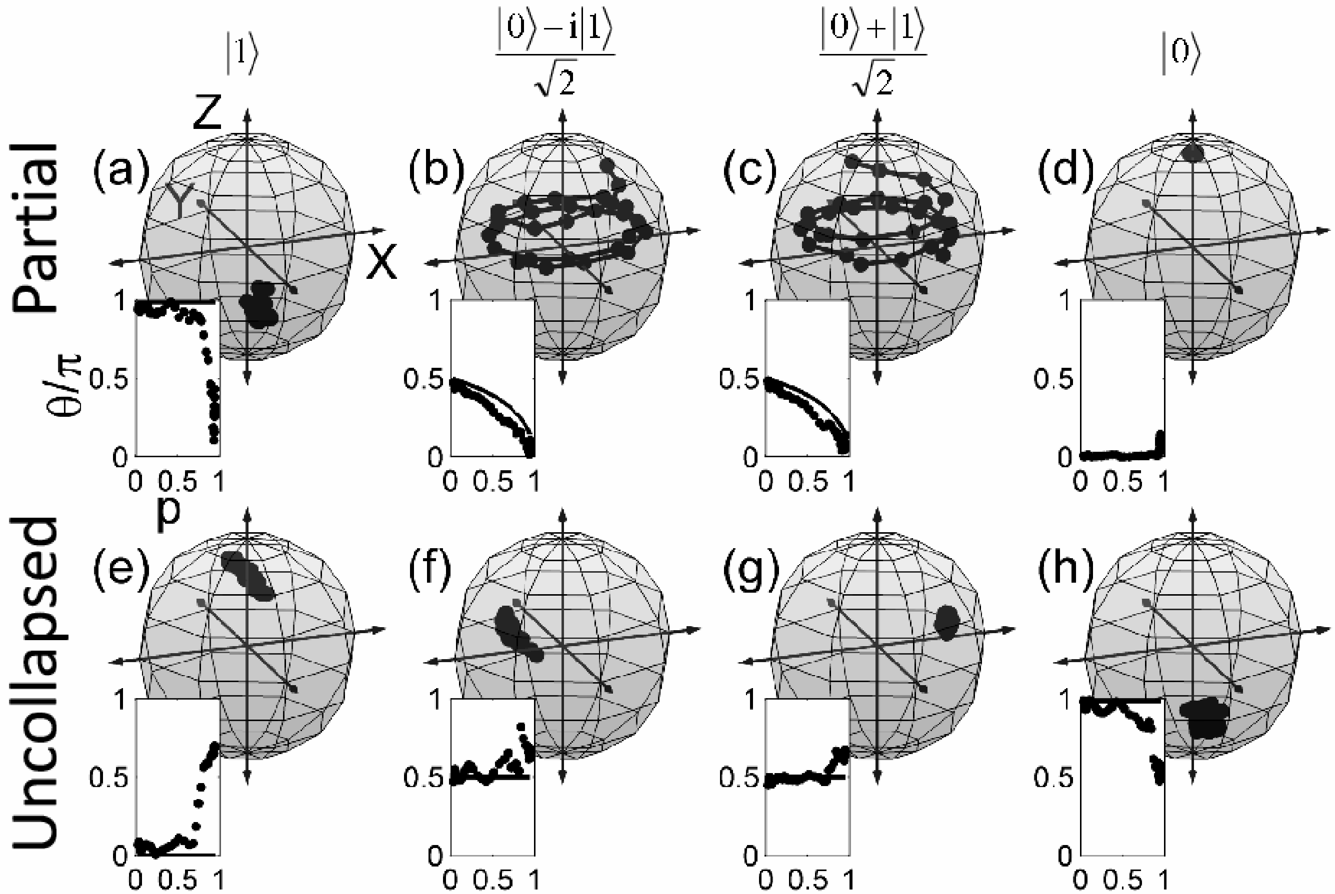
Uncollapsed



Collapse strength: $0.05 < p < 0.7$

uncollapsing works well!

Same with polar angle dependence
(another experimental run)



Both spin echo (azimuth) and uncollapsing (polar angle)

Difference: spin echo – undoing of an unknown unitary evolution, uncollapsing – undoing of a known, but non-unitary evolution

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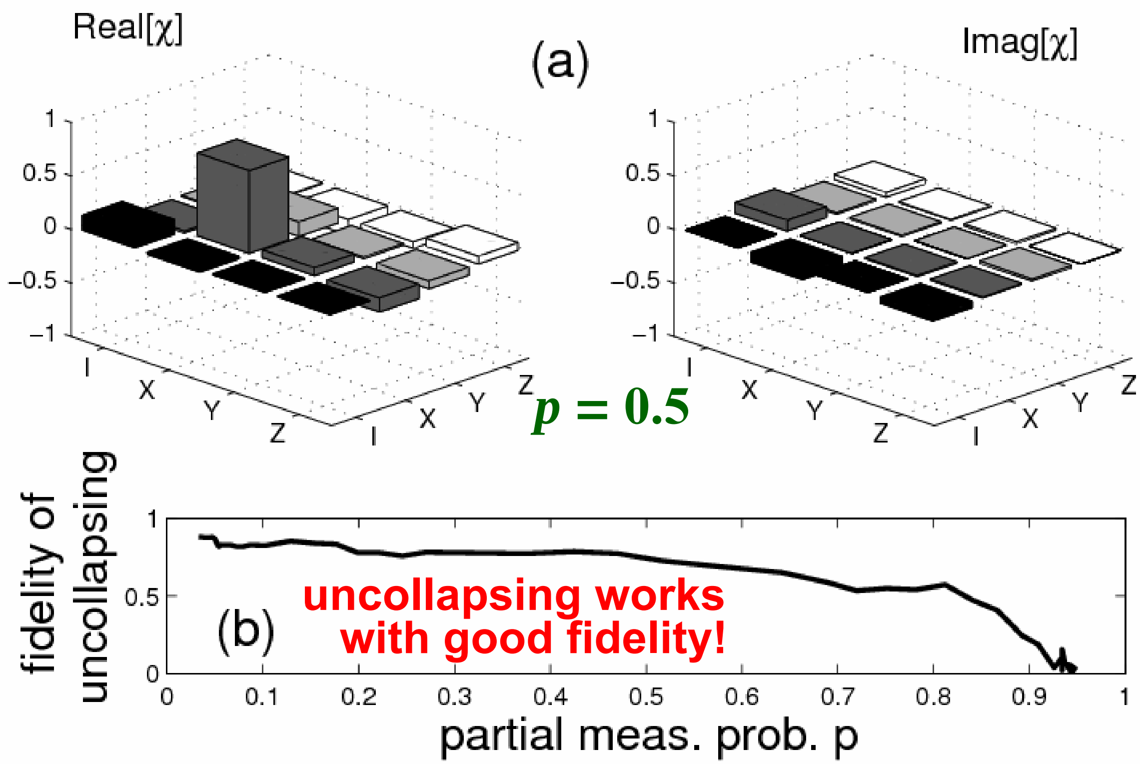
Quantum process tomography

N. Katz et al.
(Martinis group)



Why getting worse at p>0.6?

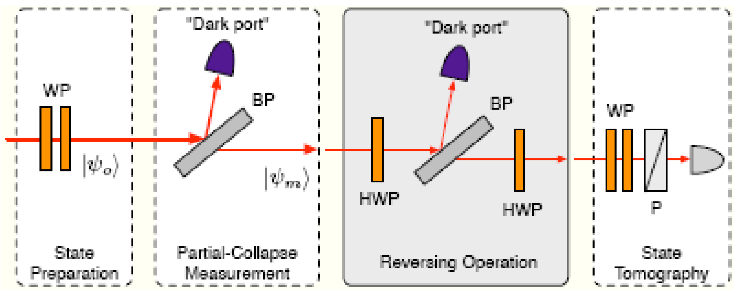
Energy relaxation $p_r = t/T_1 = 45\text{ns}/450\text{ns} = 0.1$

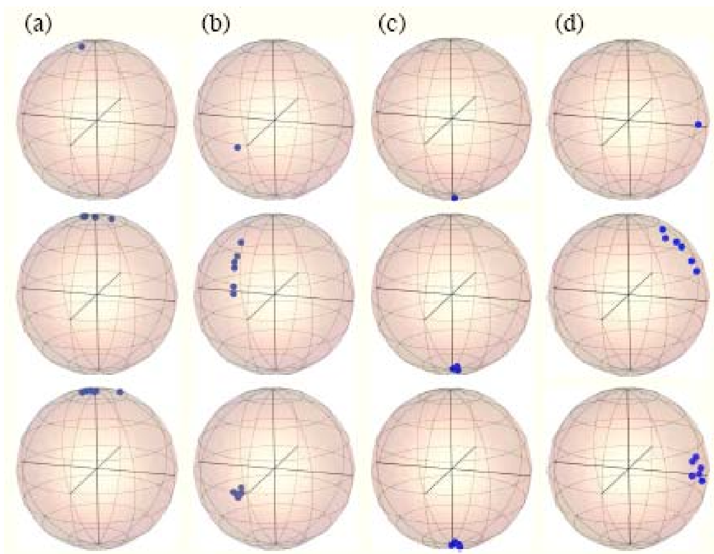
Selection affected when $1-p \sim p_r$

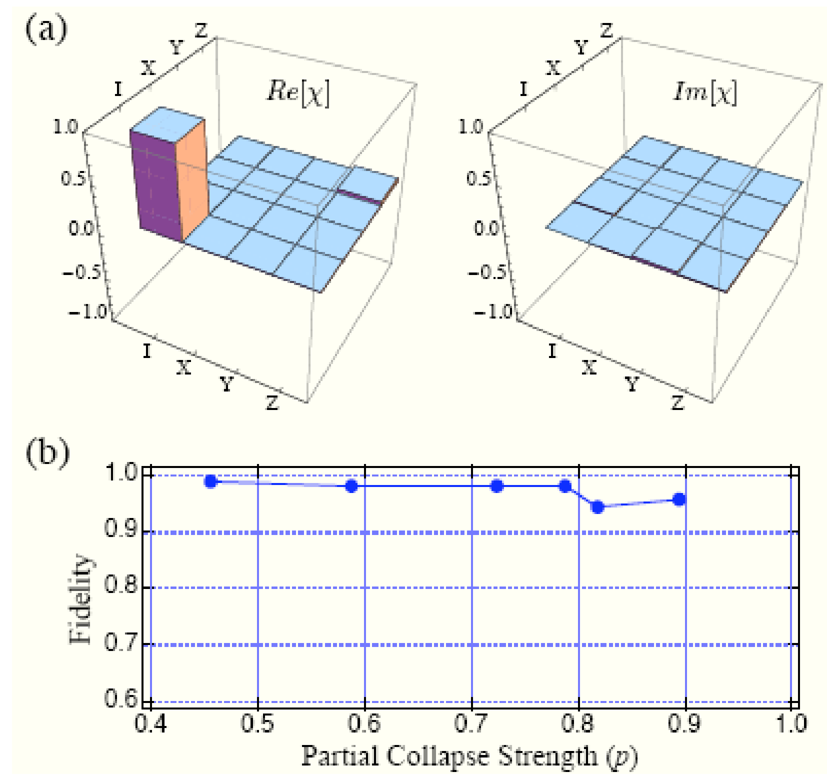
Overall: uncollapsing is well-confirmed experimentally

Recent experiment on uncollapsing using single photons

Kim, Cho, Ra, Kim, arXiv:0903.3077







- very good fidelity of uncollapsing (>94%)
- measurement fidelity is probably not good (normalization by coincidence counts)

Conclusions

- **Continuous quantum measurement is *not* equivalent to decoherence (environment) if detector output (information) is taken into account**

- **It is easy to see what is "inside" collapse: simple Bayesian formalism works for many solid-state setups**

- **Collapse can sometimes be undone (uncollapsing)**

- **A number of experimental predictions have been made**

- **Three direct solid-state experiments have been realized; hopefully, more experiments are coming soon**