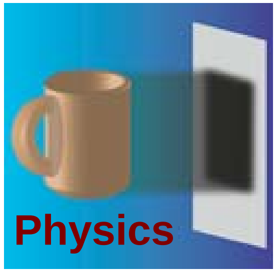
Quantum uncollapsing: theory and experiment

Alexander Korotkov

University of California, Riverside

<u>Andrew Jordan</u> (*Univ. of Rochester*, theory)

<u>Nadav Katz</u>, M. Neeley, M. Ansmann, R. Bialczak, M. Hofheinz, E. Lucero, A. O'Connell, H. Wang, A. Cleland, and <u>John Martinis</u> (*UC Santa Barbara*, experiment)



Support:

IARPA

PRL 101, 200401 (2008)
PRL 97, 166805 (2006)
Science 312, 1498 (2006)

Quantum eraser

Quantum eraser proposal by Scully and Drühl (PRA, 1982)

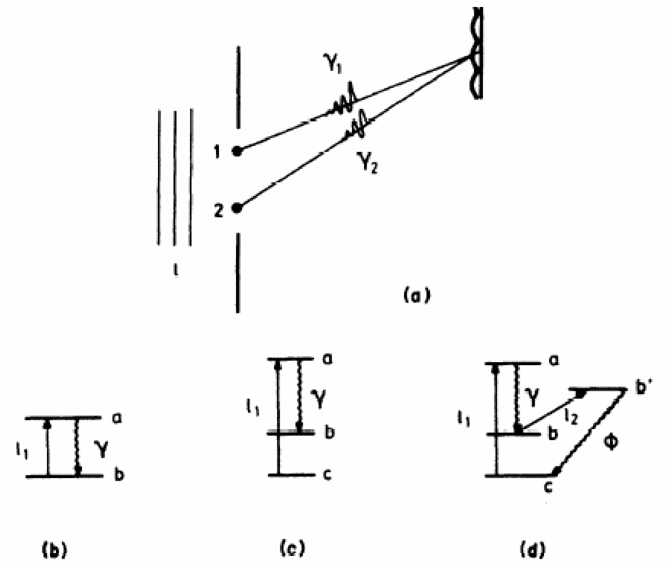


FIG. 1. (a) Figure depicting light impinging from left on atoms at sites 1 and 2. Scattered photons γ_1 and γ_2 produce interference pattern on screen. (b) Two-level atoms excited by laser pulse l_1, and emit γ photons in $a \rightarrow b$ transition. (c) Three-level atoms excited by pulse l_1 from $c \rightarrow a$ and emit photons in $a \rightarrow b$ transition. (d) Four-level system excited by pulse l_1 from $c \rightarrow a$ followed by emission of γ photons in $a \rightarrow b$ transition. Second pulse l_2 takes atoms from $b \rightarrow b'$. Decay from $b' \rightarrow c$ results in emission of ϕ photons.

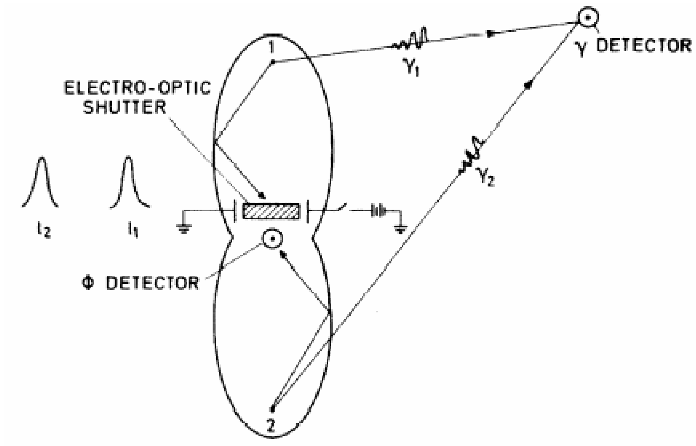


FIG. 2. Laser pulses l_1 and l_2 incident on atoms at sites 1 and 2. Scattered photons γ_1 and γ_2 result from $a \rightarrow b$ transition. Decay of atoms from $b' \rightarrow c$ results in ϕ photon emission. Elliptical cavities reflect ϕ photons onto common photodetector. Electro-optic shutter transmits ϕ photons only when switch is open. Choice of switch position determines whether we emphasize particle or wave nature of γ photons.

Interference fringes restored for two-detector correlations (since “which-path” information is erased)

Here only virtual information is erased. Can we really measure (extract information) and then uncollapse quantum state?

The problem

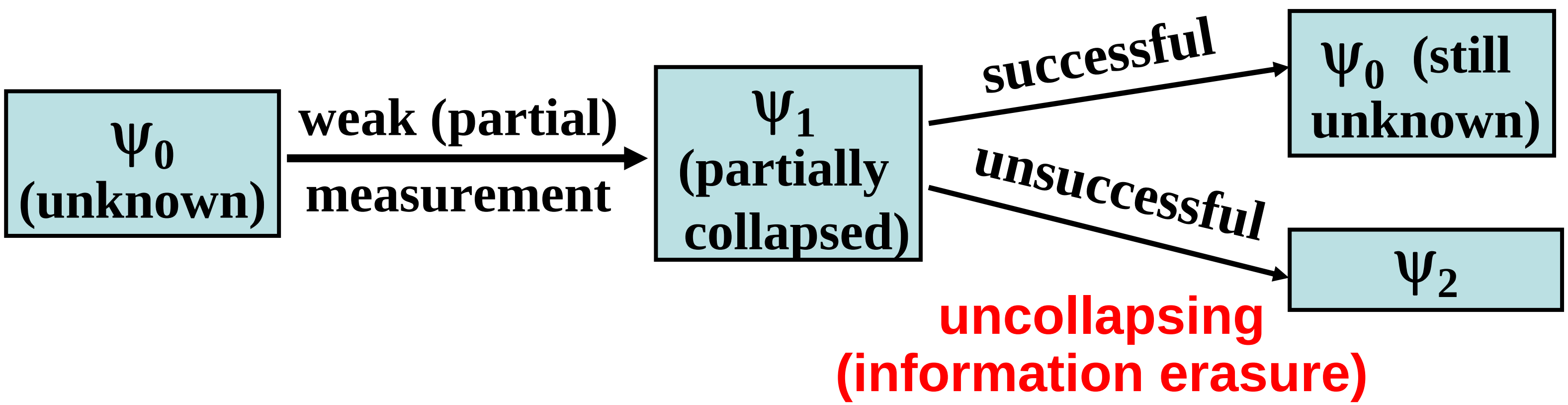
Korotkov & Jordan, 2006

It is impossible to undo "orthodox" quantum measurement (for an unknown initial state)

Is it possible to undo weak (partial) quantum measurement?

Yes! (but with a finite probability)

If uncollapsing is successful, an unknown state is **fully** restored

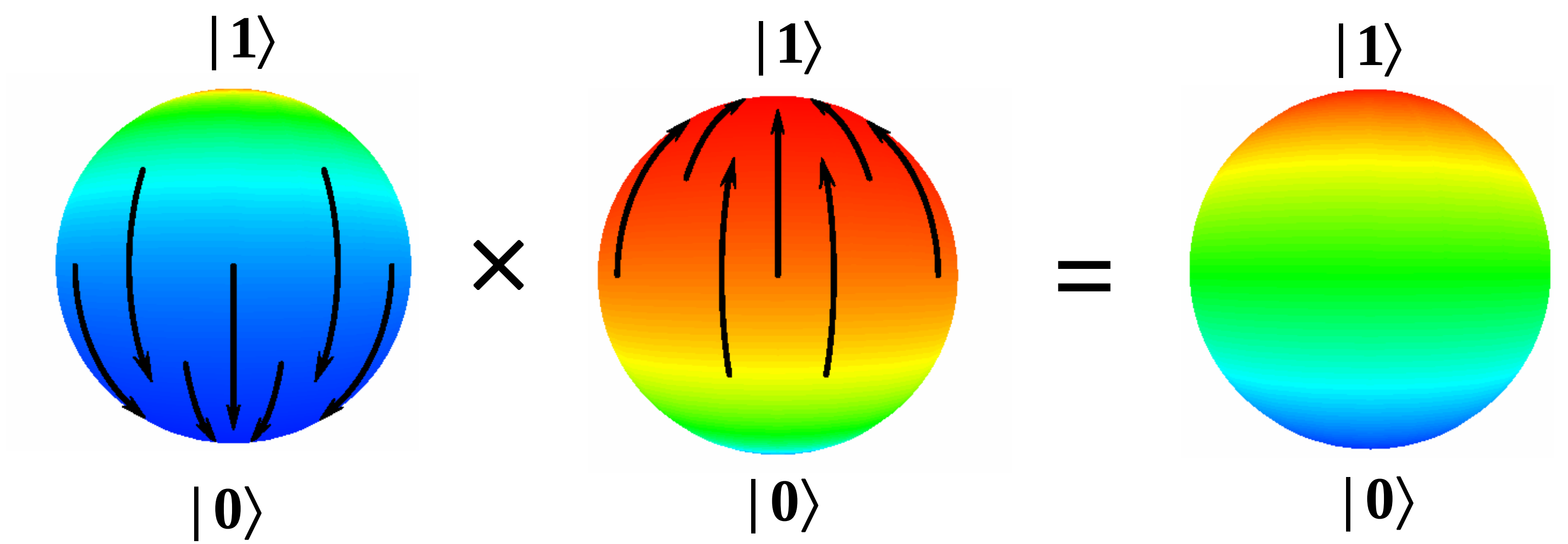


Information is erased by another measurement with "exactly contradicting" result

Uncollapsing of a qubit state

Evolution due to partial (weak, continuous, etc.) measurement is **non-unitary** (though coherent if detector is good!), therefore it is impossible to undo it by Hamiltonian dynamics.

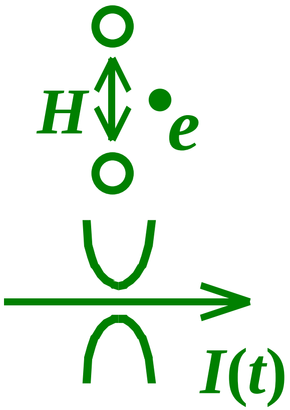
How to uncollapse? One more measurement!



(similar to Koashi-Ueda, PRL-1999)

(Figure partially adopted from A. Jordan, A. Korotkov, and M. Büttiker, PRL-2006

First example: double-dot qubit with no tunneling, measured by QPC



$$\hat{H}_{QB} = (\varepsilon/2)(c_1^\dagger c_1 - c_2^\dagger c_2) + H(c_1^\dagger c_2 + c_2^\dagger c_1)$$

Assume “frozen” qubit: $\varepsilon = H = 0$

Bayesian evolution due to measurement (Korotkov-1998)

1) Diagonal matrix elements of the density matrix evolve according to the classical Bayes rule

2) Non-diagonal matrix elements evolve so that the degree of purity $\rho_{ij}/[\rho_{ii}\,\rho_{jj}]^{1/2}$ is conserved

$$\rho_{11}(\tau) = \frac{\rho_{11}(0)\exp[-(\bar{I}-I_1)^2/2D]}{\rho_{11}(0)\exp[-(\bar{I}-I_1)^2/2D] + \rho_{22}(0)\exp[-(\bar{I}-I_2)^2/2D]}$$

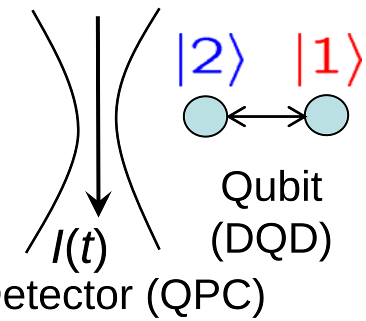
$$\frac{\rho_{12}(\tau)}{[\rho_{12}(\tau)\,\rho_{22}(\tau)]^{1/2}} = \frac{\rho_{12}(0)}{[\rho_{12}(0)\,\rho_{22}(0)]^{1/2}}\,, \qquad \rho_{22}(\tau) = 1 - \rho_{11}(\tau)$$

where $\bar{I} = \frac{1}{\tau}\int_0^\tau I(t)\,dt, \quad D = S_I/2\tau$ $\quad \Delta I = I_1 - I_2$ - response

$S_I = 2eI(1-T)$ - shot noise

Uncollapsing for DQD-QPC system

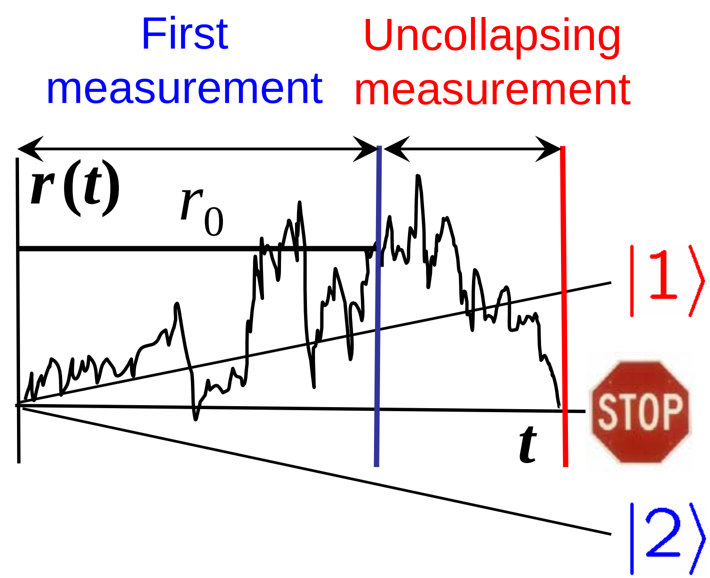
Korotkov-Jordan, PRL-2006



Measurement result:

$$r(t) = \frac{\Delta I}{S_I}\left[\int_0^t I(t')\,dt' - I_0 t\right]$$

If $r = 0$, then no information and no evolution!



Simple strategy: continue measuring until result $r(t)$ becomes zero. Then any initial state is fully restored.

(same for an entangled qubit)

It may happen though that $r = 0$ never crossed; then undoing procedure is unsuccessful.

Probability of success:

$$P_S = \frac{e^{-|r_0|}}{e^{|r_0|}\rho_{11}(0) + e^{-|r_0|}\rho_{22}(0)}$$

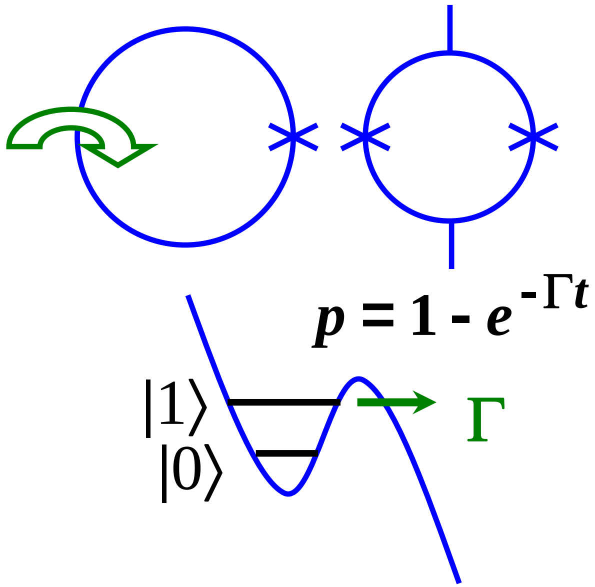
Averaged probability of success (over result r_0):

$$P_{av} = 1 - \mathrm{erf}\left[\sqrt{t/2T_m}\right], \qquad T_m = 2S_I/(\Delta I)^2$$

(does not depend on initial state)

Second example: uncollapsing of a superconducting phase qubit

1) Start with an unknown state
2) Partial measurement of strength p
3) π-pulse (exchange |0> ↔ |1>)
4) One more measurement with the **same strength** p
5) π-pulse



This is what was demonstrated experimentally

(in more detail later)

General theory of uncollapsing

Measurement operator M_r: $\rho \to \frac{M_r \rho M_r^\dagger}{\mathrm{Tr}(M_r \rho M_r^\dagger)}$ (POVM formalism)

Undoing measurement operator: $C \times M_r^{-1}$ (to satisfy completeness, eigenvalues cannot be >1)

$\max(C) = \min_i \sqrt{p_i}$, p_i are eigenvalues of $M_r^\dagger M_r$

Probability of success: $P_S \leq \frac{\min P_r}{P_r(\rho_{in})}$

$P_r(\rho_{in})$ – probability of result r for initial state ρ_{in},

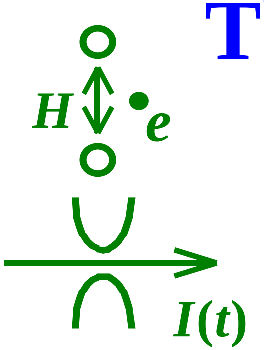
$\min P_r$ – probability of result r minimized over all possible initial states

Averaged (over r) probability of success: $P_{av} \leq \sum_r \min P_r$

(independent of initial state, otherwise get information)

(similar to Koashi-Ueda, PRL, 1999)

Third example: evolving charge qubit



$$\hat{H}_{QB} = (\varepsilon/2)(c_1^\dagger c_1 - c_2^\dagger c_2) + H(c_1^\dagger c_2 + c_2^\dagger c_1)$$

(now non-zero *H* and ε, qubit evolves during measurement)

1) Bayesian equations to calculate measurement operator
2) unitary operation, measurement by QPC, unitary operation

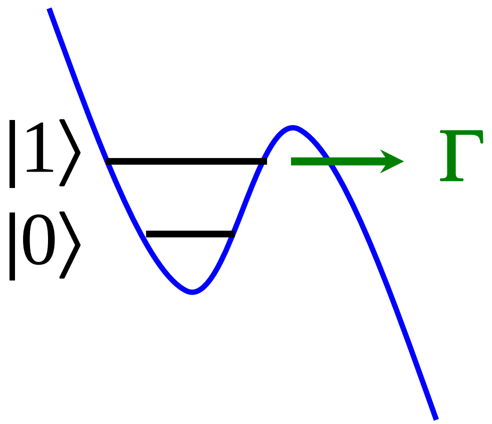
Fourth example: general uncollapsing for entangled charge qubits

1) unitary transformation of *N* qubits
2) null-result measurement of a certain strength by a strongly nonlinear QPC (tunneling only for state |11..1〉)
3) repeat 2^N times, sequentially transforming the basis vectors of the diagonalized measurement operator into |11..1〉

In all four examples the success probability P_S reaches the upper bound (optimal uncollapsing)

Partial collapse of superconducting phase qubit

N. Katz, M. Ansmann, R. Bialczak, E. Lucero, R. McDermott, M. Neeley, M. Steffen, E. Weig, A. Cleland, J. Martinis, A. Korotkov, Science-06



How does a coherent state evolve in time before tunneling event?

(What happens when nothing happens?)

Main idea:

$$\psi = \alpha|0\rangle + \beta|1\rangle \;\rightarrow\; \psi(t) = \begin{cases} |out\rangle, \text{ if tunneled} \\ \dfrac{\alpha|0\rangle + \beta e^{-\Gamma t/2} e^{i\varphi}|1\rangle}{\sqrt{|\alpha|^2 + |\beta|^2 e^{-\Gamma t}}}, \text{ if not tunneled} \end{cases}$$

(better theory: Pryadko & A.K., 2007)

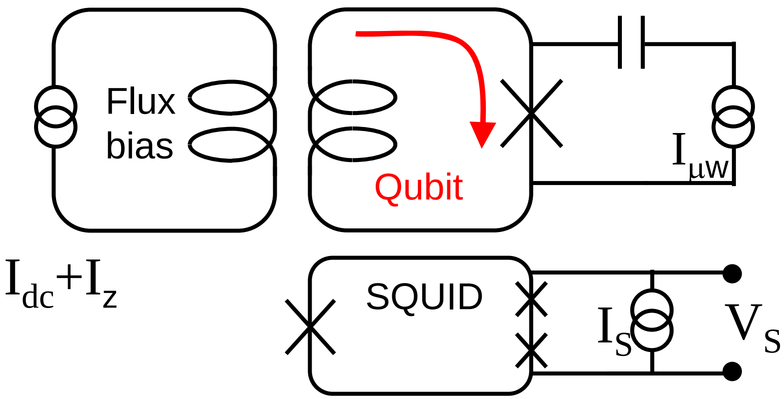
amplitude of state |0> grows without physical interaction

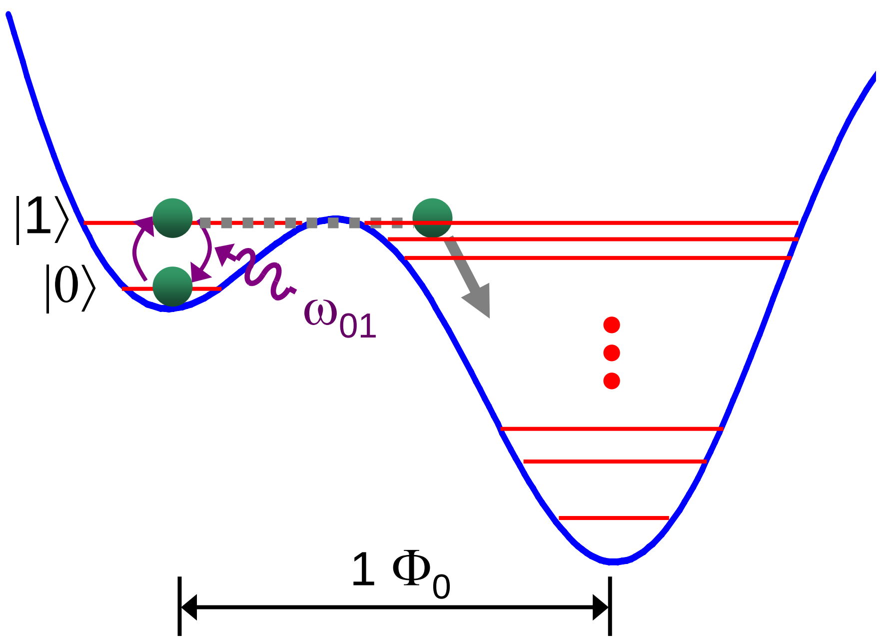
continuous null-result collapse

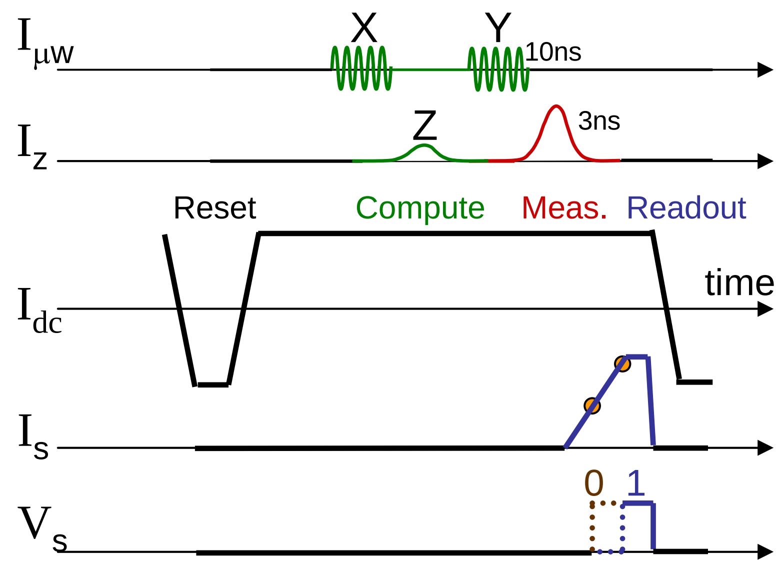
(similar to optics, Dalibard-Castin-Molmer, PRL-1992)

Superconducting phase qubit at UCSB

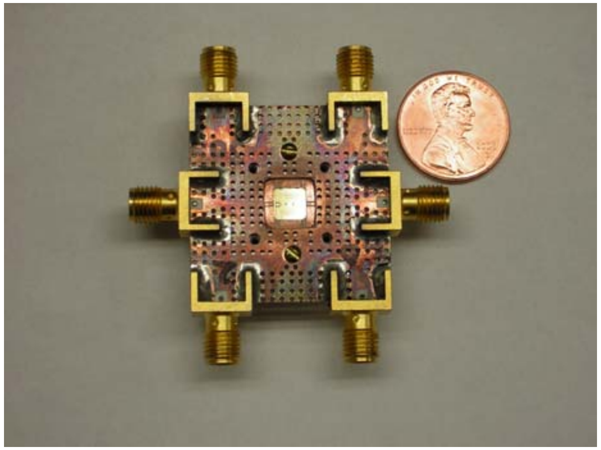
Courtesy of Nadav Katz (UCSB)



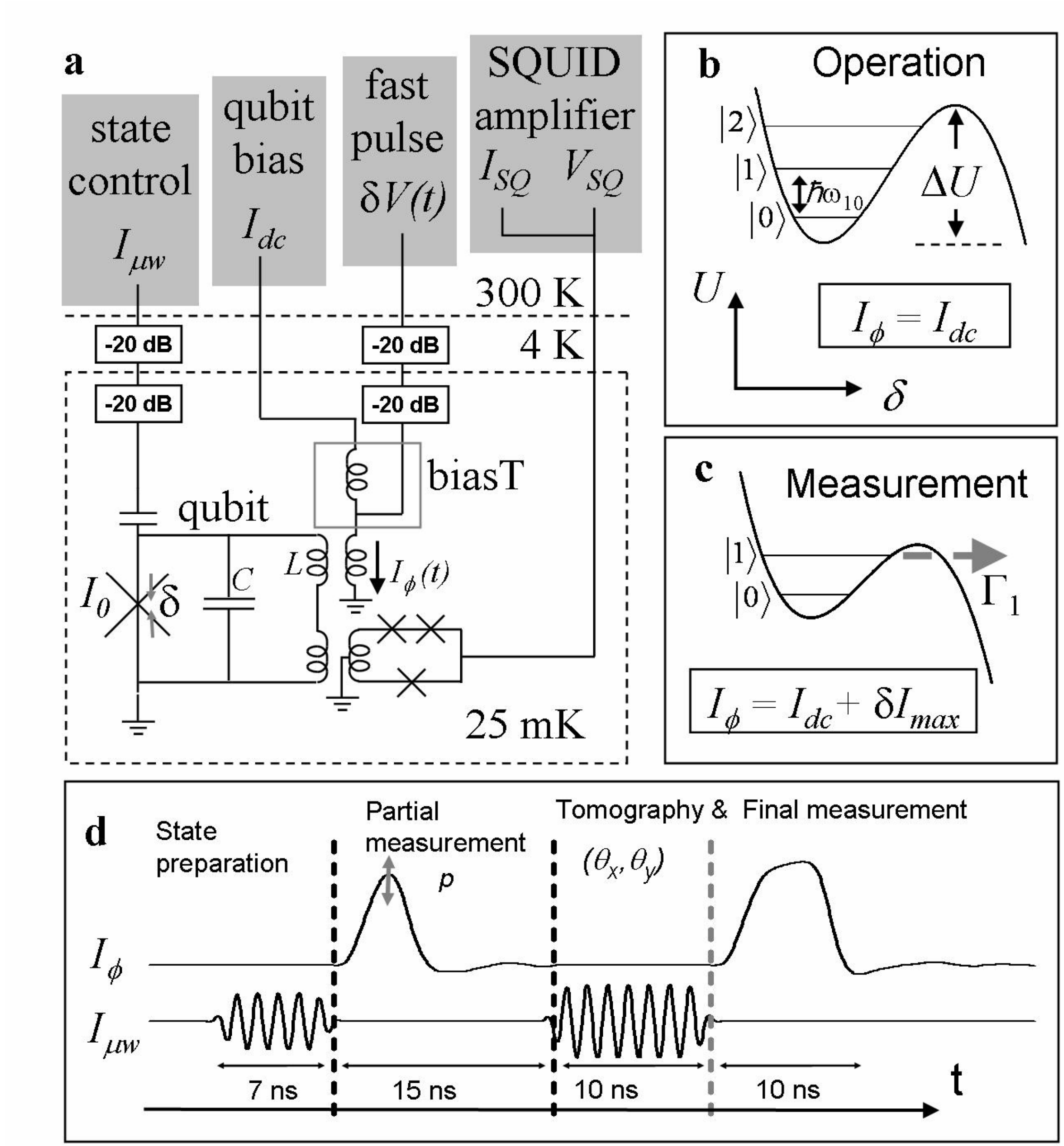




Repeat 1000x
prob. 0,1



Experimental technique for partial collapse



Nadav Katz *et al.*
(John Martinis' group)

Protocol:

1) State preparation by applying microwave pulse (via Rabi oscillations)

2) Partial measurement by lowering barrier for time *t*

3) State tomography (micro-wave + full measurement)

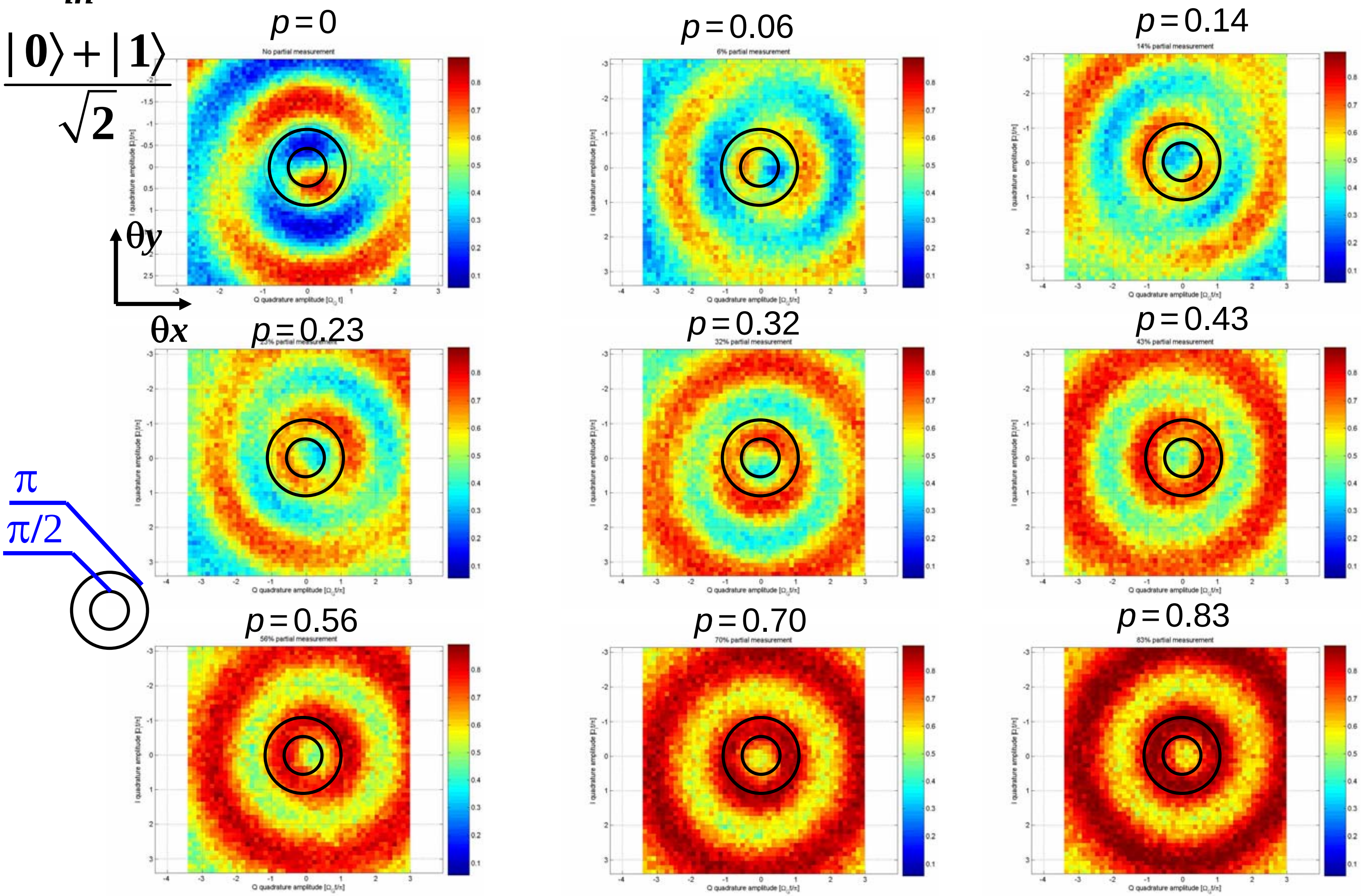
Measurement strength
$p = 1 - \exp(-\Gamma t)$
is actually controlled by Γ, not by t

***p*=0: no measurement**
***p*=1: orthodox collapse**

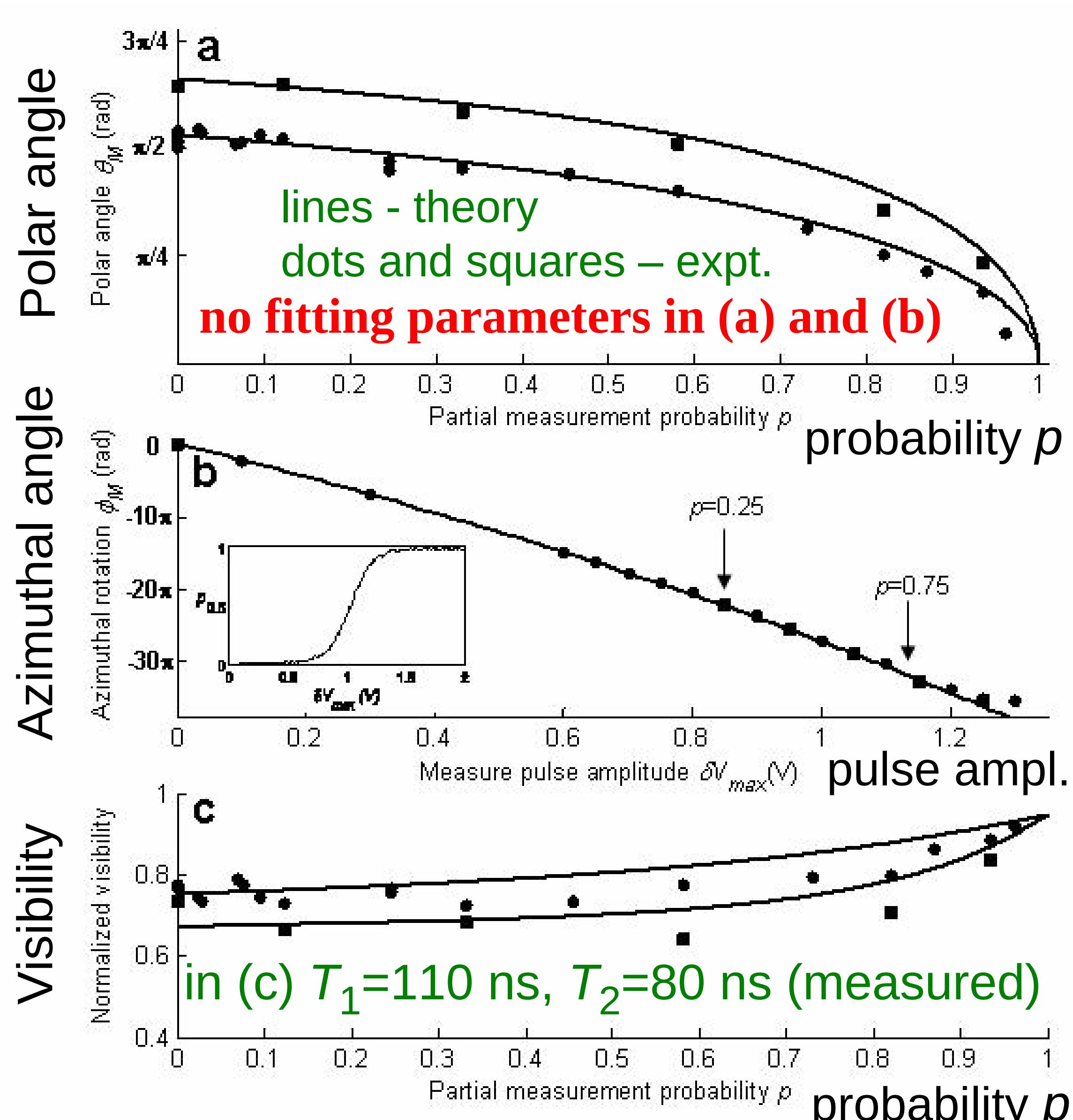
Experimental tomography data

Nadav Katz *et al.* (UCSB)

$\psi_{in} = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$



Partial collapse: experimental results



N. Katz *et al.*, Science-06

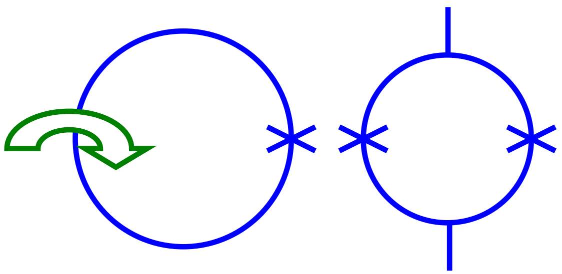
- **In case of no tunneling (null-result measurement) phase qubit evolves**
- **This evolution is well described by a simple Bayesian theory, without fitting parameters**
- **Phase qubit remains fully coherent in the process of continuous collapse (experimentally ~80% raw data, ~96% after account for T1 and T2)**

quantum efficiency
$\eta_0 > 0.8$

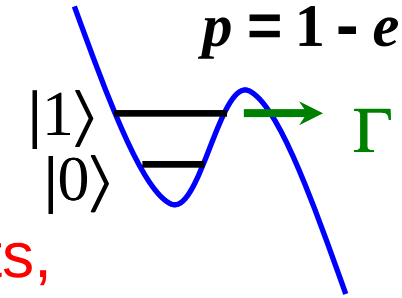
Uncollapsing of a phase qubit state

Korotkov & Jordan, 2006

1) Start with an unknown state
2) Partial measurement of strength *p*
3) π-pulse (exchange |0> ↔ |1>)
4) One more measurement with the **same strength** ***p***
5) π-pulse



$$p = 1 - e^{-\Gamma t}$$



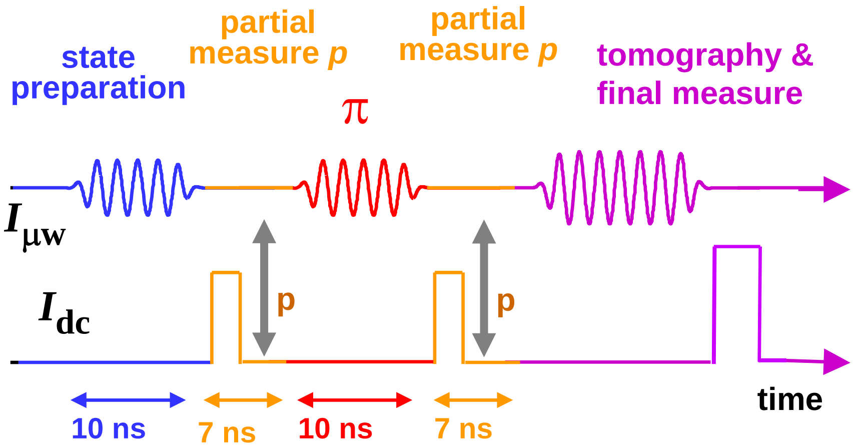
If no tunneling for both measurements, then initial state is fully restored!

$$\alpha|0\rangle + \beta|1\rangle \rightarrow \frac{\alpha|0\rangle + e^{i\phi}\beta e^{-\Gamma t/2}|1\rangle}{\text{Norm}} \rightarrow$$

$$\frac{e^{i\phi}\alpha e^{-\Gamma t/2}|0\rangle + e^{i\phi}\beta e^{-\Gamma t/2}|1\rangle}{\text{Norm}} = e^{i\phi}(\alpha|0\rangle + \beta|1\rangle)$$

phase is also restored (spin echo)

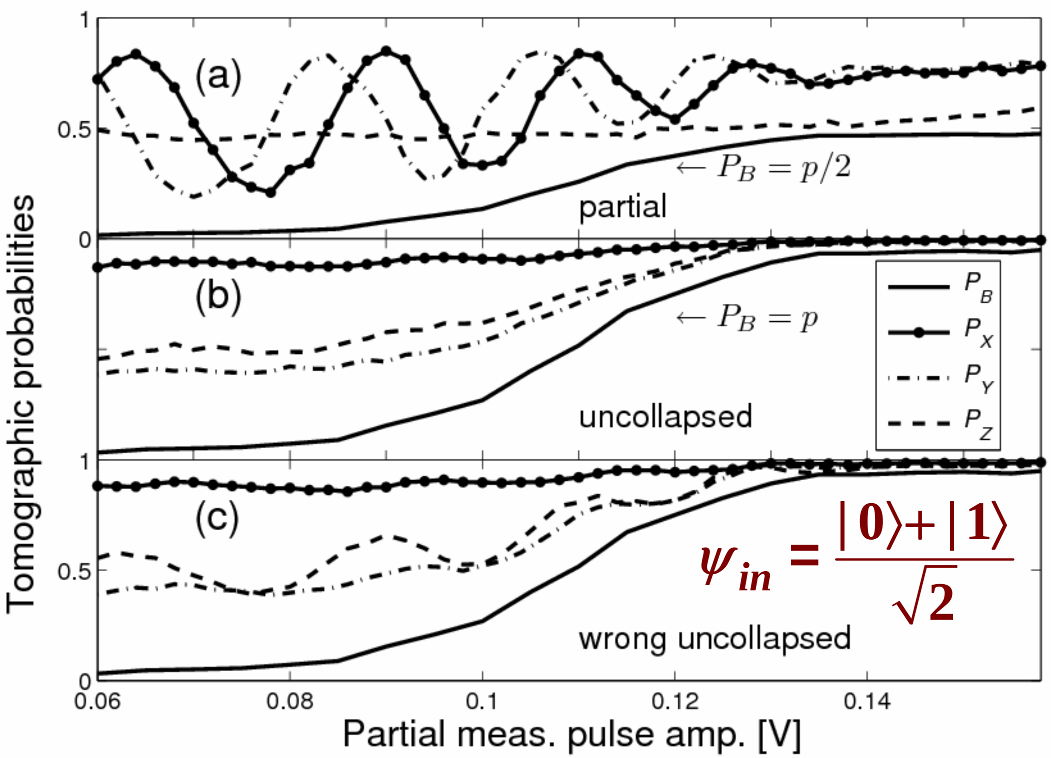
Experiment on wavefunction uncollapsing



N. Katz, M. Neeley, M. Ansmann, R. Bialzak, E. Lucero, A. O'Connell, H. Wang, A. Cleland, J. Martinis, and A. Korotkov, PRL-2008

Uncollapse protocol:

- partial collapse
- π-pulse
- partial collapse (same strength)



$$\psi_{in} = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

State tomography with *X*, *Y*, and *Z* pulses

Background P_B should be subtracted to find qubit density matrix

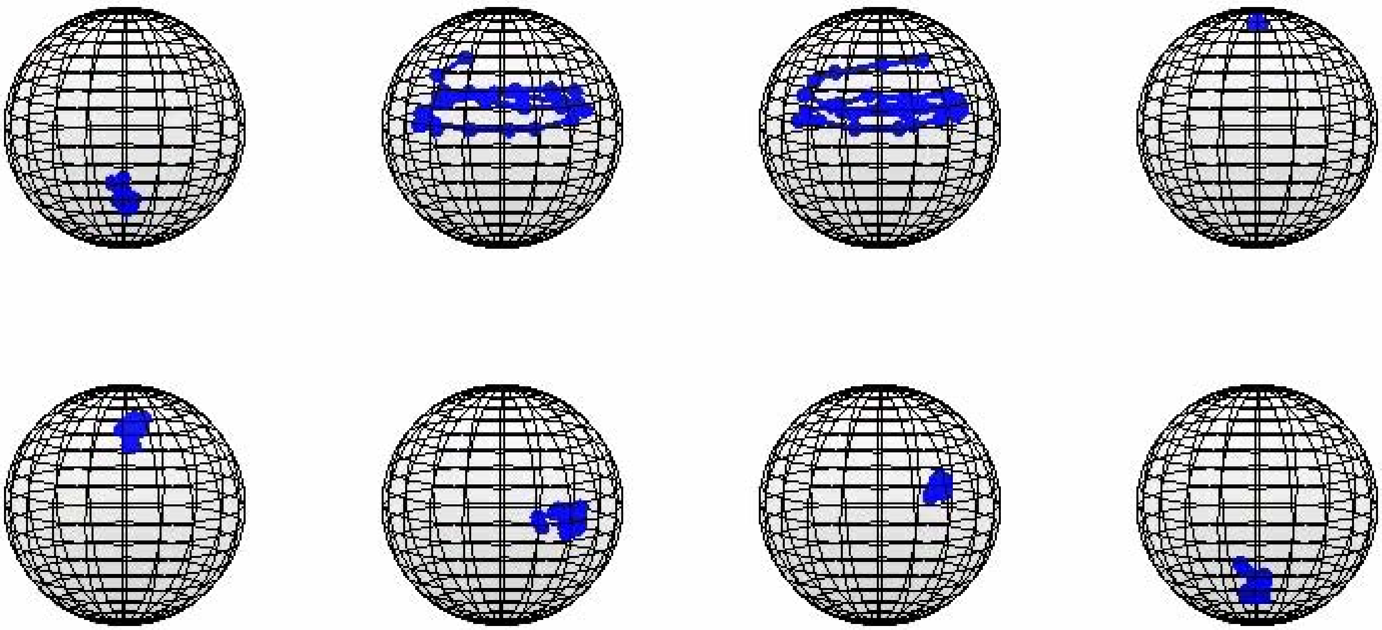
Experimental results on Bloch sphere

N. Katz et al.

Initial state	$\lvert 1\rangle$	$\frac{\lvert 0\rangle+\lvert 1\rangle}{\sqrt{2}}$	$\frac{\lvert 0\rangle+i\lvert 1\rangle}{\sqrt{2}}$	$\lvert 0\rangle$

Partial collapse

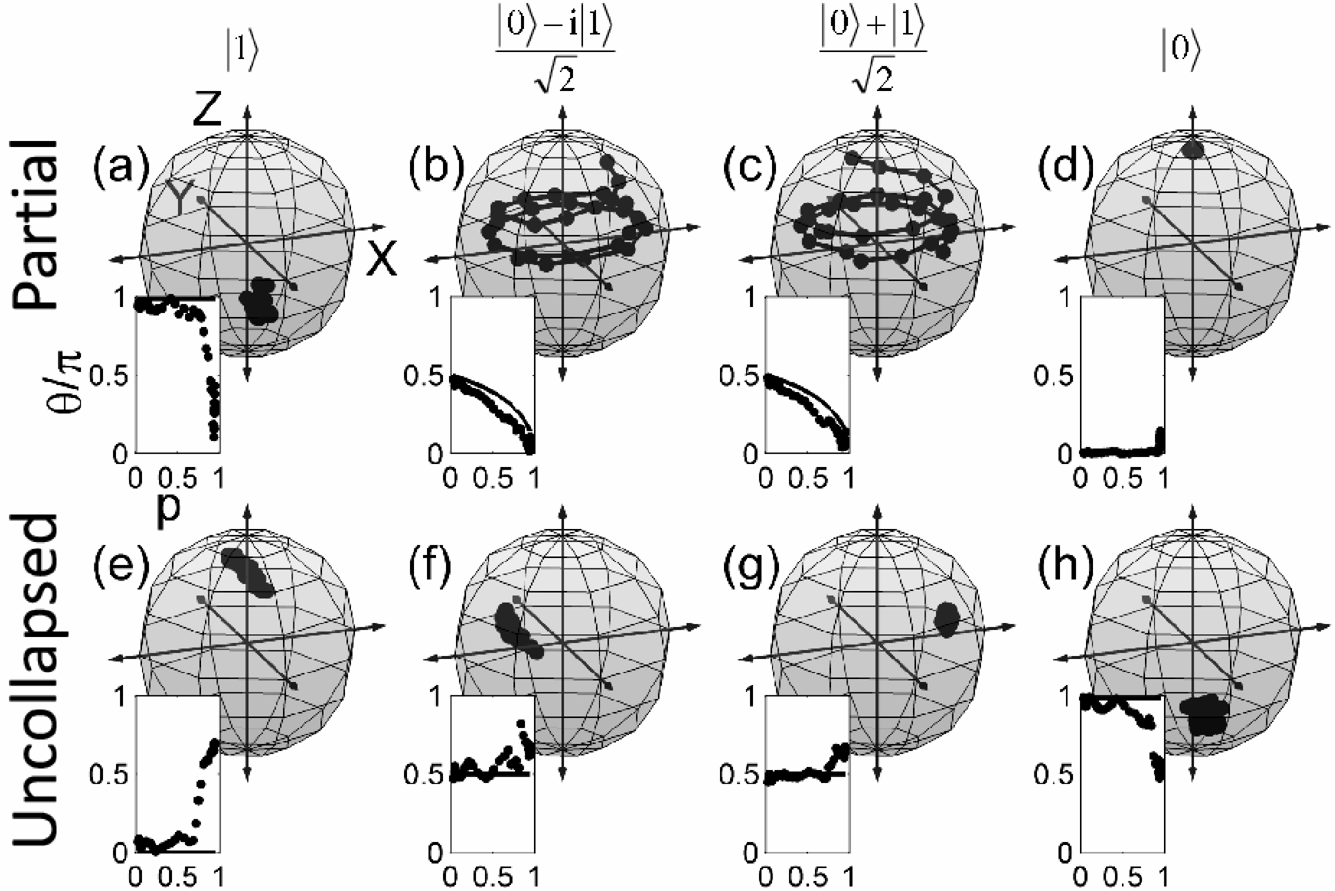
Uncollapsed



Collapse strength: $0.05 < p < 0.7$

uncollapsing works well!

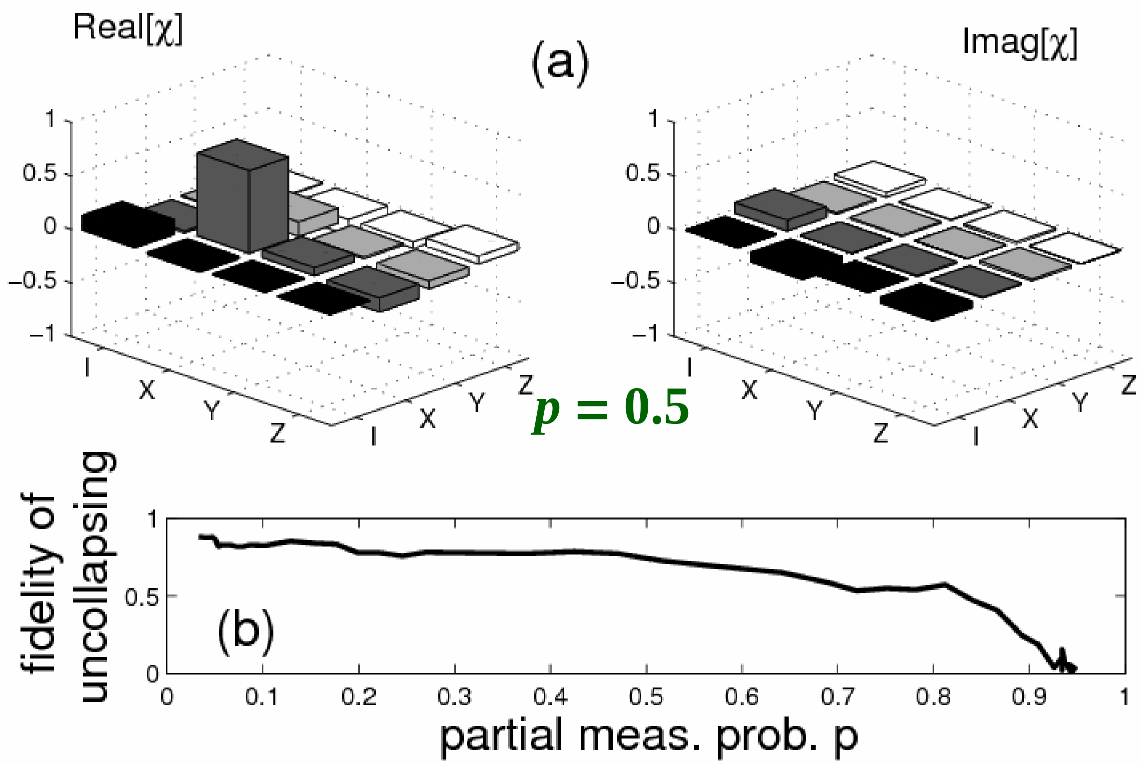
Same with polar angle dependence
(another experimental run)



Both spin echo (azimuth) and uncollapsing (polar angle)

Difference: spin echo – undoing of an unknown unitary evolution, uncollapsing – undoing of a known, but non-unitary evolution

Quantum process tomography



Why getting worse at p>0.6?

Energy relaxation $\boldsymbol{p_r = t/T_1 = 45\text{ns}/450\text{ns} = 0.1}$

Selection affected when $\boldsymbol{1-p \sim p_r}$

Overall: uncollapsing is well-confirmed experimentally

Conclusions

- Partial (weak, etc.) quantum measurement can be undone, though with some probability P_S, which decreases with increasing strength of measurement ($P_S = 0$ for orthodox case)

- Arbitrary initial state is uncollapsed exactly in the case of success (need a detector with perfect quantum efficiency)

- Uncollapsing for a superconducting phase qubit has been demonstrated, extending the previous experiment on partial collapse

- Solid-state experiments on non-projective quantum measurement are now competitive with (sometimes ahead of) optical experiments (*also, recent expt. on persistent Rabi oscillations*)