Wavefunction uncollapse: theory and experiments

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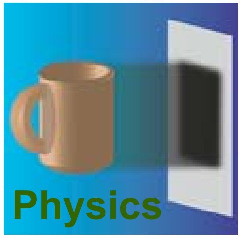


In collaboration with:

Theory: **Andrew Jordan** (***U. Rochester***), **Kyle Keane** (***UCR***)

Experiment: **Nadav Katz, M. Neeley, M. Ansmann, R. Bialczak, M. Hofheinz, E. Lucero, A. O'Connell, H. Wang, A. Cleland, and John Martinis** (***UC Santa Barbara***)

PRL 97, 166805 (2006); PRL 101, 200401 (2008); arXiv:0906.3468; arXiv:0908.1134

Outline:

- Theory of uncollapsing
- Experiments (phase qubit and optical qubit)
- Decoherence suppression by uncollapsing

Special thanks: K. K. Likharev, D. V. Averin

Funding:

The problem

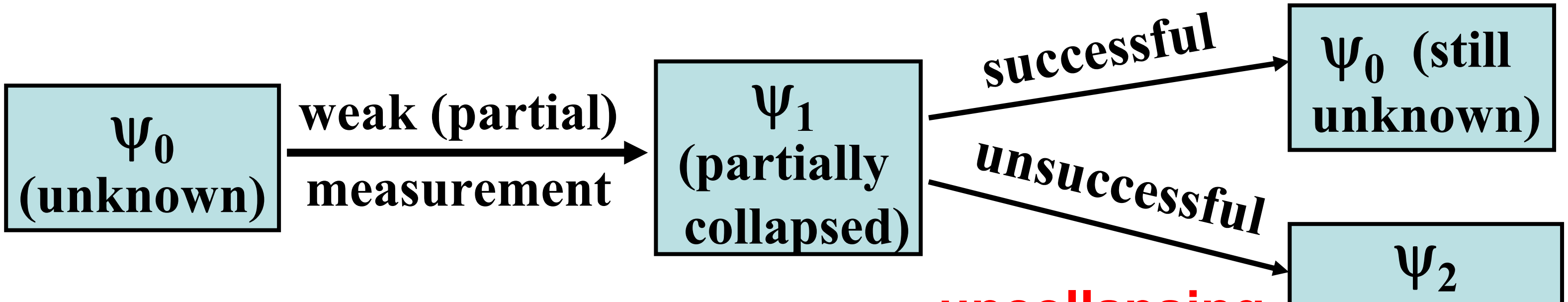
A.K. & Jordan, PRL-2006

It is impossible to undo “orthodox” quantum measurement (for an unknown initial state)

Is it possible to undo weak (partial) quantum measurement?

Yes! (but with a finite probability)

If uncollapsing is successful, an unknown state is **fully** restored



uncollapsing (information erasure)

“Quantum Un-Demolition measurement”

(Not a “quantum eraser”!)

Quantum erasers in optics

Quantum eraser proposal by Scully and Drühl, PRA-1982

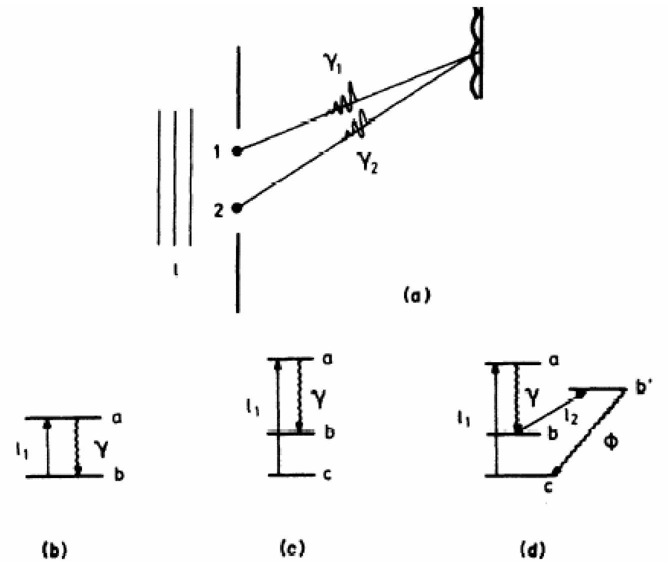


FIG. 1. (a) Figure depicting light impinging from left on atoms at sites 1 and 2. Scattered photons γ_1 and γ_2 produce interference pattern on screen. (b) Two-level atoms excited by laser pulse l_1, and emit γ photons in $a \rightarrow b$ transition. (c) Three-level atoms excited by pulse l_1 from $c \rightarrow a$ and emit photons in $a \rightarrow b$ transition. (d) Four-level system excited by pulse l_1 from $c \rightarrow a$ followed by emission of γ photons in $a \rightarrow b$ transition. Second pulse l_2 takes atoms from $b \rightarrow b'$. Decay from $b' \rightarrow c$ results in emission of ϕ photons.

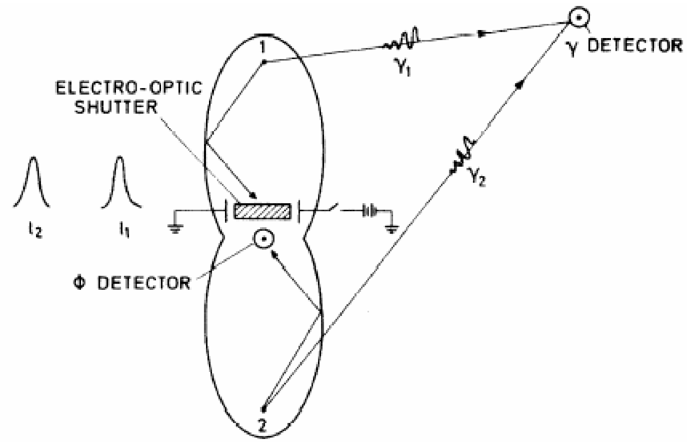


FIG. 2. Laser pulses l_1 and l_2 incident on atoms at sites 1 and 2. Scattered photons γ_1 and γ_2 result from $a \rightarrow b$ transition. Decay of atoms from $b' \rightarrow c$ results in ϕ photon emission. Elliptical cavities reflect ϕ photons onto common photodetector. Electro-optic shutter transmits ϕ photons only when switch is open. Choice of switch position determines whether we emphasize particle or wave nature of γ photons.

Interference fringes restored for two-detector correlations (since “which-path” information is erased)

Our idea of uncollapsing is quite different:
we really extract information and then erase it

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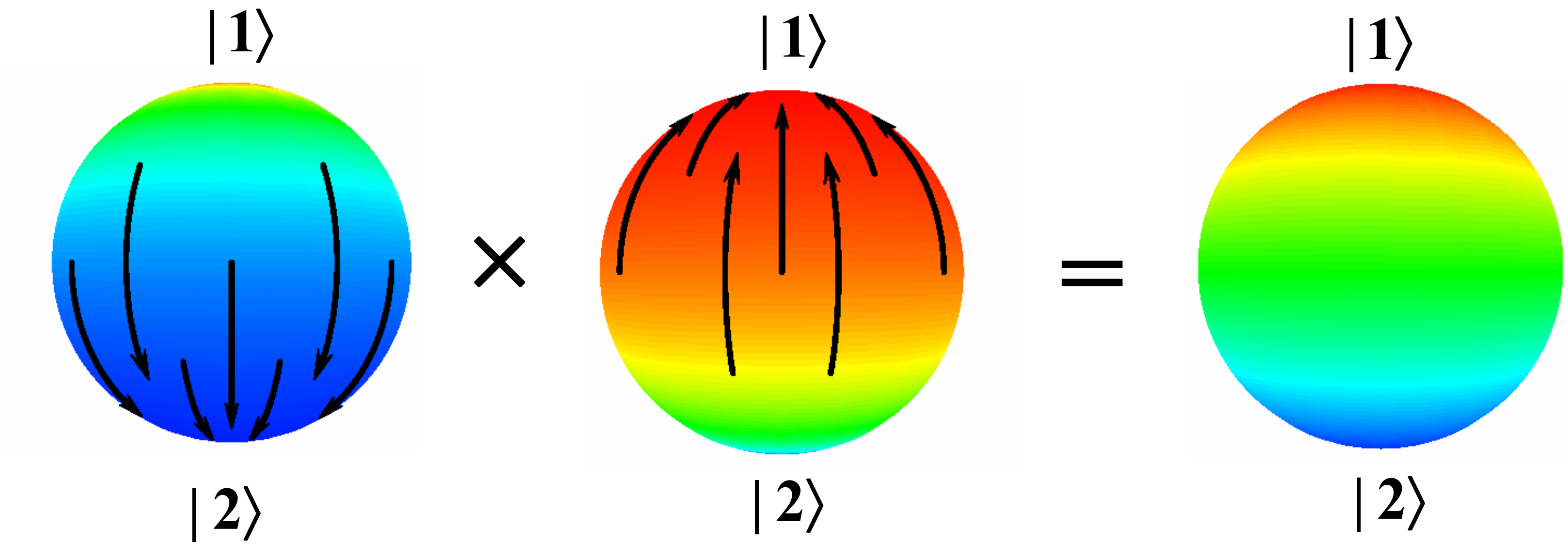
Uncollapsing of a qubit state

Evolution due to partial (weak, continuous, etc.) measurement is **non-unitary** (though coherent if detector is good!), therefore it is impossible to undo it by Hamiltonian dynamics.

How to undo? One more measurement!

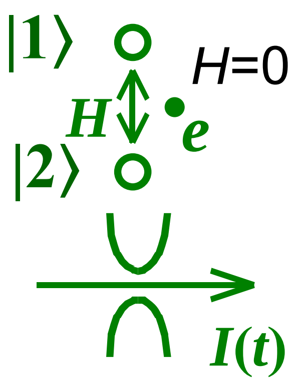


need ideal (quantum-limited) detector

(similar to Koashi-Ueda, PRL-1999)

(Figure partially adopted from Jordan-A.K.-Büttiker, PRL-06)

First example: DQD-QPC system



Qubit evolution due to measurement (quantum back-action):

$$\psi(t) = \alpha(t)\,|1\rangle + \beta(t)\,|2\rangle \quad \text{or} \quad \rho_{ij}(t)$$

1) $|\alpha(t)|^2$ and $|\beta(t)|^2$ evolve as probabilities,
i.e. according to the **Bayes rule** (same for ρ_{ii})

2) phases of $\alpha(t)$ and $\beta(t)$ do not change
(**no decoherence!**), $\rho_{ij}/(\rho_{ii}\,\rho_{jj})^{1/2}$ = const

(A.K., 1998)

Bayes rule (1763, Laplace-1812):

$$\underbrace{P(A_i\,|\,\text{res})}_{\text{posterior probability}} = \frac{\underbrace{P(A_i)}_{\text{prior probab.}}\ \underbrace{P(\text{res}\,|\,A_i)}_{\text{likelihood}}}{\sum_k P(A_k)\,P(\text{res}\,|\,A_k)}$$

So simple because:

1) QPC happens to be an ideal detector
2) no Hamiltonian evolution of the qubit

Similar formalisms developed earlier. Key words: Imprecise, weak, selective, or conditional measurements, POVM, Quantum trajectories, Quantum jumps, Restricted path integral, etc.

Names: Davies, Kraus, Holevo, Mensky, Caves, Gardiner, Carmichael, Plenio, Knight, Walls, Gisin, Percival, Milburn, Wiseman, Habib, etc. (very incomplete list)

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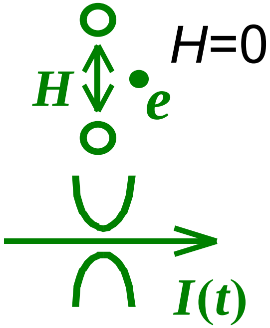
Graphical representation of the Bayesian evolution

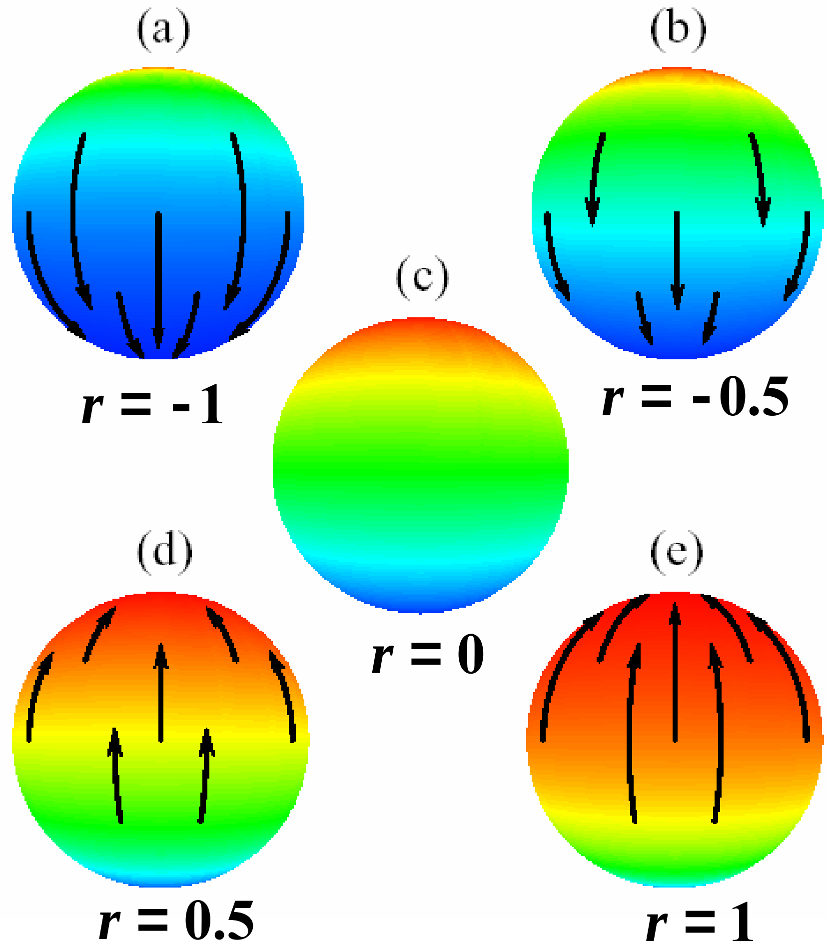


$$\frac{\rho_{11}(t)}{\rho_{22}(t)} = \frac{\rho_{11}(0)}{\rho_{22}(0)} \exp[2r(t)]$$

$$\frac{\rho_{12}(t)}{\sqrt{\rho_{11}(t)\rho_{22}(t)}} = \text{const}$$

where measurement result $r(t)$ is

$$r(t) = \frac{\Delta I}{S_I}\left[\int_0^t I(t')\,dt' - I_0 t\right]$$



Jordan-Korotkov-Büttiker, PRL-06

If $r = 0$, then no information and no evolution!

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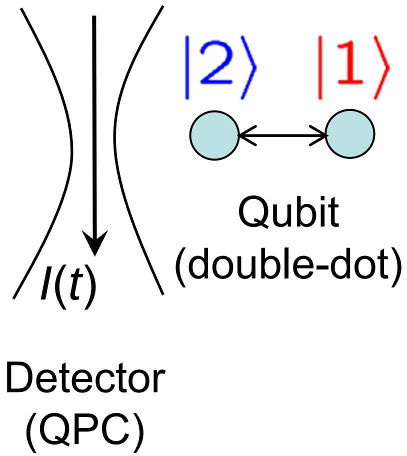
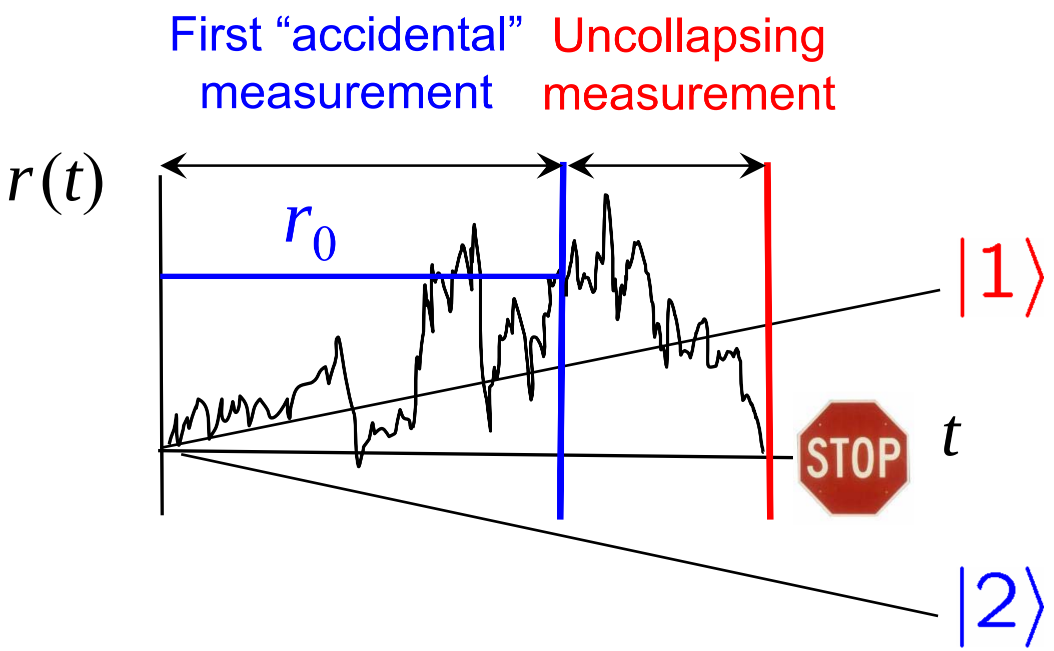
Uncollapsing for qubit-QPC system

A.K. & Jordan, PRL-2006





Simple strategy: continue measuring until $r(t)$ becomes zero!

Then any unknown initial state is fully restored.

(same for an entangled qubit)

It may happen though that $r=0$ never happens;
then undoing procedure is unsuccessful.

Probability of success

Trick: since non-diagonal matrix elements are not directly involved, we can analyze classical probabilities (as if qubit is in some certain, but unknown state); then simple diffusion with drift

Results:

Probability of successful uncollapsing

$$P_S = \frac{e^{-|r_0|}}{e^{|r_0|}\rho_{11}(0) + e^{-|r_0|}\rho_{22}(0)}$$

where r_0 is the result of the measurement to be undone, and $\rho(0)$ is initial state (traced over entangled qubits)

Larger $|r_0|$ ⇒ more information ⇒ less likely to uncollapse

Averaged probability of success (over result r_0)

$$P_{av} = 1 - \text{erf}[\sqrt{t/2T_m}\,]$$

(does not depend on initial state; cannot!)

where $T_m = 2S_I/(\Delta I)^2$ (“measurement time”)

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Uncollapse requires a quantum-limited detector

Fundamental limit for energy sensitivity

$$(\varepsilon_O \varepsilon_{BA} - \varepsilon_{O,BA}^2)^{1/2} \geq \hbar / 2$$

Danilov, Likharev, Zorin, 1983

where ε_O is output-noise-limited sensitivity [J/Hz], ε_{BA} is back-action-limited sensitivity [J/Hz], and $\varepsilon_{O,BA}$ is correlation

Also Clarke, Tesche, Caves, Likharev, etc. (1980s);
Averin-2000, Clerk et al.-2002, Pilgram et al.-2002, etc.

In a different language

$$\Gamma = (\Delta I)^2/4S_I + \gamma$$

- Γ: ensemble decoherence rate
- $(\Delta I)^2/4S_I$: ~ information flow [bit/s]
- γ: single-qubit decoherence

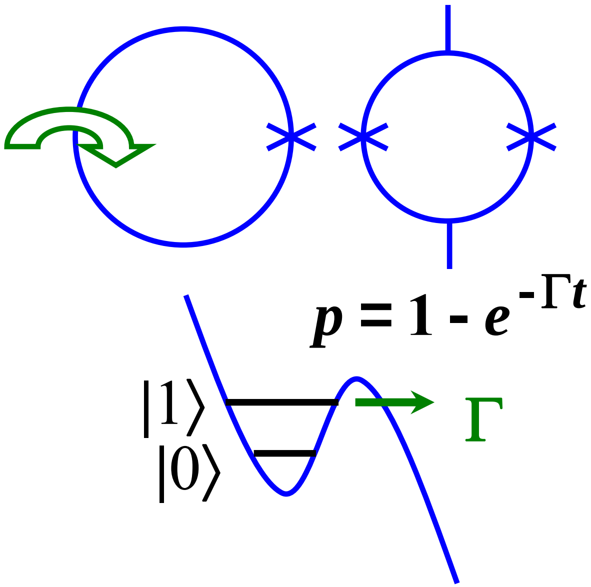
A.K., 1998, 2000
D. Averin, 2000, 2003
S. Pilgram et al., 2002
A. Clerk et al., 2002

A definition: ideal (quantum-limited) detector does not decohere a single qubit

$$\eta = \frac{(\Delta I)^2}{4S_I\Gamma} = \frac{\hbar^2/4}{\varepsilon_O \varepsilon_{BA}} = \eta_{\text{opt}}$$

Second example: uncollapsing of a superconducting phase qubit

1) Start with an unknown state
2) Partial measurement of strength *p*
3) π-pulse (exchange |0> ↔ |1>)
4) One more measurement with the **same strength *p***
5) π-pulse



N. Katz et al., Science-2006 PRL-2008

This is what was demonstrated experimentally (in more detail later)

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General theory of uncollapsing

Measurement operator M_r (any linear operator in H.S.): $$\rho \rightarrow \frac{M_r \rho M_r^\dagger}{\mathrm{Tr}(M_r \rho M_r^\dagger)}$$

(POVM formalism for an ideal detector)
Nielsen-Chuang, p.100

Completeness: $\sum_r M_r^\dagger M_r = 1$ Probability: $P_r = \mathrm{Tr}(M_r \rho M_r^\dagger)$

Undoing measurement operator: $C \times M_r^{-1}$ (to satisfy completeness, eigenvalues cannot be >1)

$$\max(C) = \min_i \sqrt{p_i}\,, \quad p_i = \mathrm{Tr}(M_r^\dagger M_r |i\rangle\langle i|)$$

p_i – probability of the measurement result r for initial state $|i\rangle$

Probability of success: $$P_S \leq \frac{\min_i p_i}{\sum_i p_i \rho_{ii}(0)} = \frac{\min P_r}{P_r[\rho(0)]}$$

$P_r[\rho(0)]$ – probability of result r for initial state $\rho(0)$,

$\min P_r$ – probability of result r minimized over all possible initial states

(similar to Koashi-Ueda, PRL, 1999)

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General theory of uncollapsing (cont.)

Overall probability: result *r* and successful uncollapsing

$$\tilde{P}_S = P_r[\rho(0)] \times P_S$$

It cannot depend on initial state

(otherwise we learn something after uncollapsing)

Exact upper bound: $\tilde{P}_S \leq \min P_r$

(probability of result *r* minimized over initial states)

Averaged (over *r*) overall probability of uncollapsing:

$$P_{S,av} \leq \sum_r \min P_r$$

(independent of initial state as well)

Characterization of (irrecoverable) collapse strength:

$$1 - P_{S,av} = 1 - \sum_r \min P_r$$

Comparison of the general bound for uncollapsing success with two examples

General bound: $$P_S \le \frac{\min P_r}{P_r[\rho(0)]}$$

First example (DQD+QPC)

$$P_S \le \frac{\min(p_1, p_2)}{p_1\rho_{11}(0) + p_2\rho_{22}(0)}$$

where $p_i = (\pi S_I / t)^{-1/2} \exp[-(\bar{I} - I_i)^2 t / S_I]\, d\bar{I}$

Coincides with the actual result, so the upper bound is reached,

therefore uncollapsing strategy is optimal

Second example (phase qubit)

Probabilities of no-tunneling are 1 and exp(-Γt)=1-p

$$P_S \le \frac{1-p}{\rho_{00}(0) + (1-p)\rho_{11}(0)}$$

uncollapsing for phase qubit is also optimal

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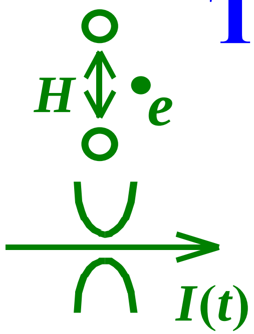

Third example: evolving charge qubit

$$\hat{H}_{QB} = (\varepsilon/2)(c_1^\dagger c_1 - c_2^\dagger c_2) + H(c_1^\dagger c_2 + c_2^\dagger c_1)$$

(now non-zero *H* and ε, qubit evolves during measurement)

1) Bayesian equations to calculate measurement operator
2) unitary operation, measurement by QPC, unitary operation

Fourth example: general uncollapsing for *N* entangled charge qubits

1) unitary transformation of *N* qubits
2) null-result measurement of a certain strength by a strongly nonlinear QPC (tunneling only for state $|11..1\rangle$)
3) repeat 2^N times, sequentially transforming the basis vectors of the diagonalized measurement operator into $|11..1\rangle$

(also reaches the upper bound for success probability)

Jordan & A.K., arXiv:0906.3468

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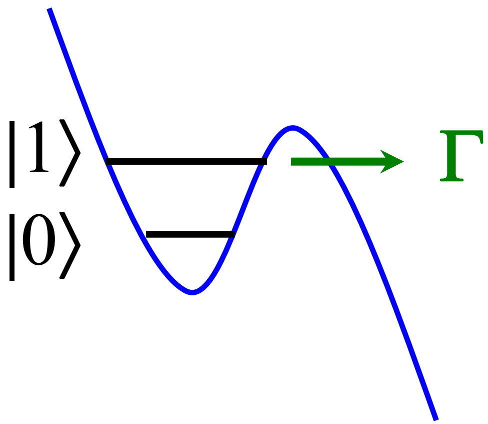
Partial collapse of a phase qubit

N. Katz, M. Ansmann, R. Bialczak, E. Lucero,
R. McDermott, M. Neeley, M. Steffen, E. Weig,
A. Cleland, J. Martinis, A. Korotkov, Science-06



How does a coherent state evolve in time before tunneling event?

(What happens when nothing happens?)

Qubit "ages" in contrast to a radioactive atom!

Main idea:

$$\psi = \alpha\,|0\rangle + \beta\,|1\rangle \;\rightarrow\; \psi(t) = \begin{cases} |out\rangle, \text{ if tunneled} \\ \dfrac{\alpha\,|0\rangle + \beta\, e^{-\Gamma t/2} e^{i\varphi}\,|1\rangle}{\sqrt{|\alpha|^2 + |\beta|^2 e^{-\Gamma t}}}, \text{ if not tunneled} \end{cases}$$

(better theory: Leonid Pryadko & A.K., 2007)

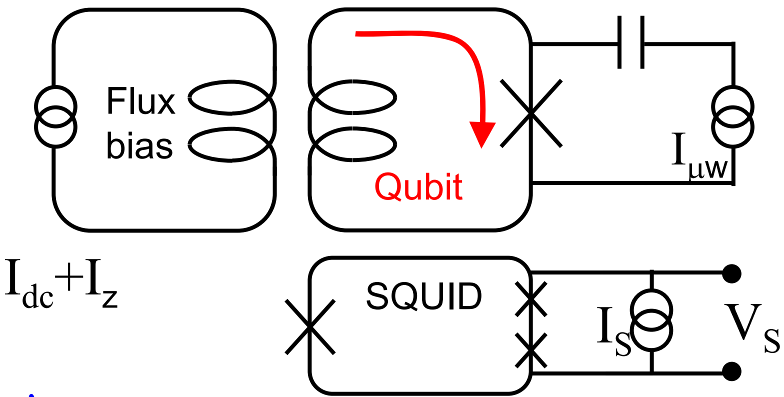
amplitude of state |0> grows without physical interaction

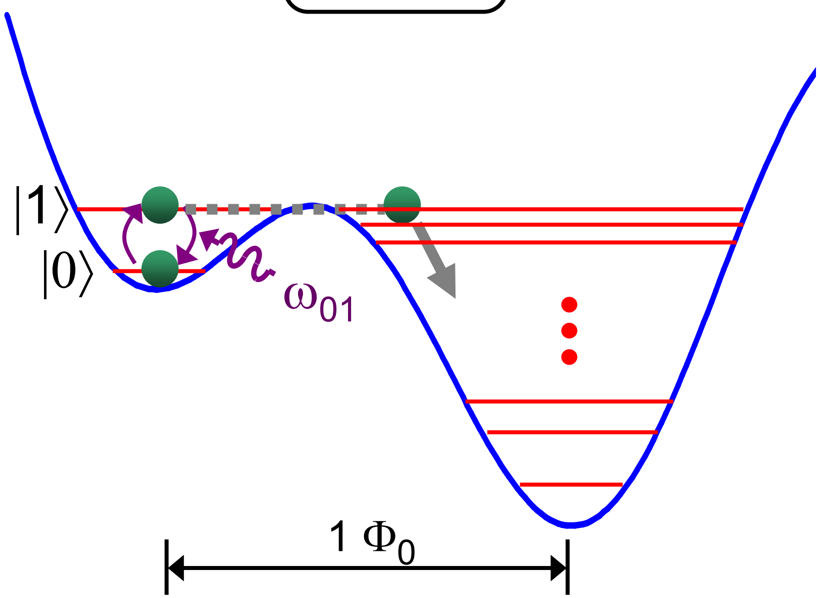
continuous null-result collapse

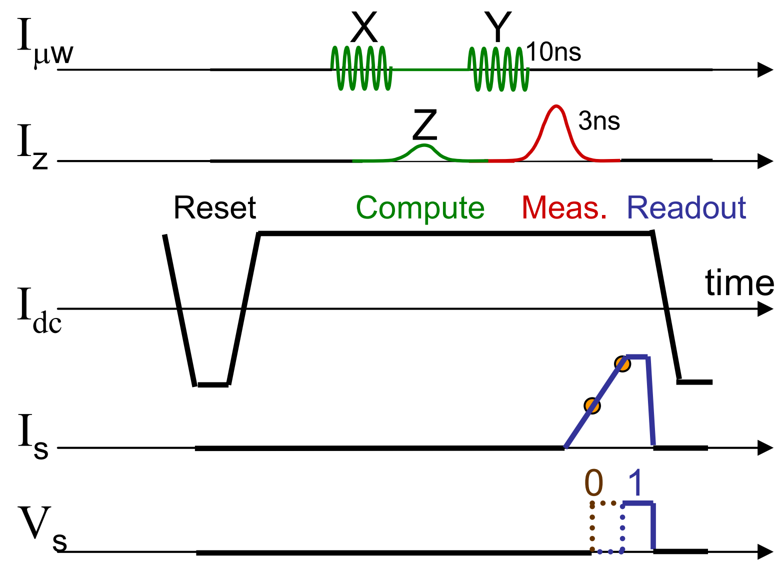
(similar to optics, Dalibard-Castin-Molmer, PRL-1992)

Superconducting phase qubit at UCSB

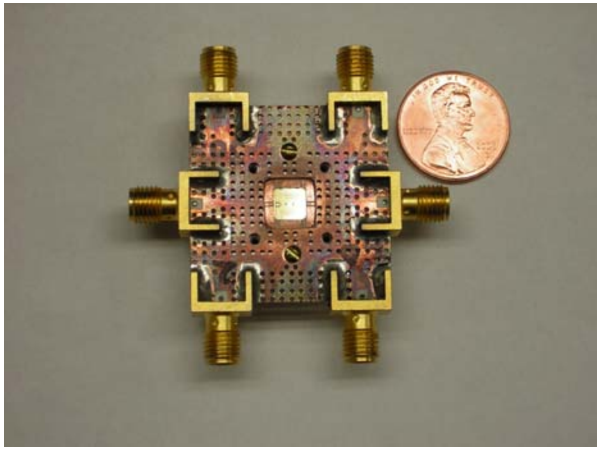
Courtesy of Nadav Katz (UCSB)





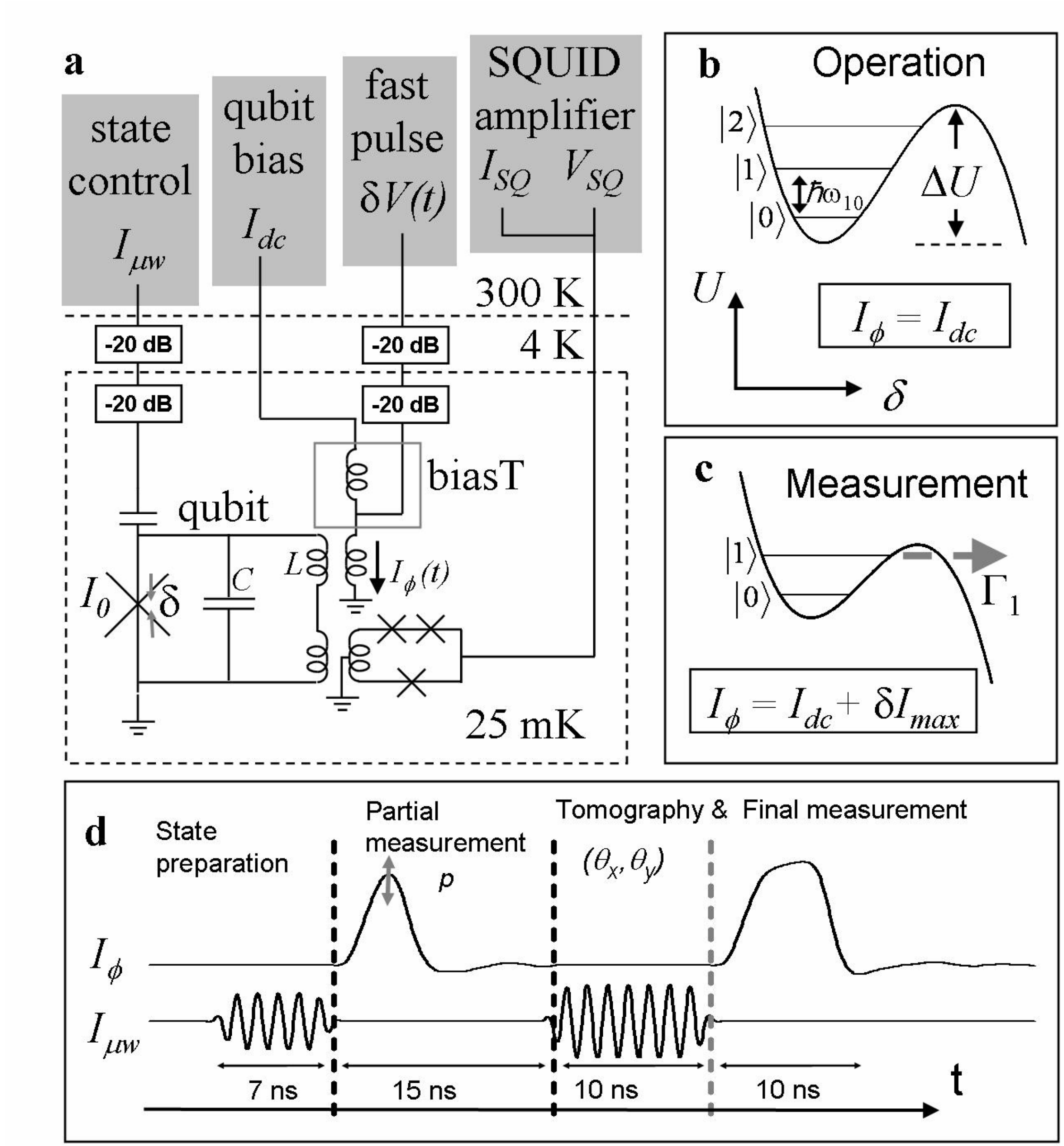


Repeat 1000x
prob. 0,1



Schematic similar to the flux qubit (Friedman et al., 2000), but both qubit states in the same well

Experimental technique for partial collapse



Nadav Katz *et al.*
(John Martinis' group)

Protocol:

1) **State preparation by applying microwave pulse (via Rabi oscillations)**

2) **Partial measurement by lowering barrier for time *t***

3) **State tomography (micro-wave + full measurement)**

Measurement strength

$p = 1 - \exp(-\Gamma t)$

is actually controlled by Γ, not by *t*

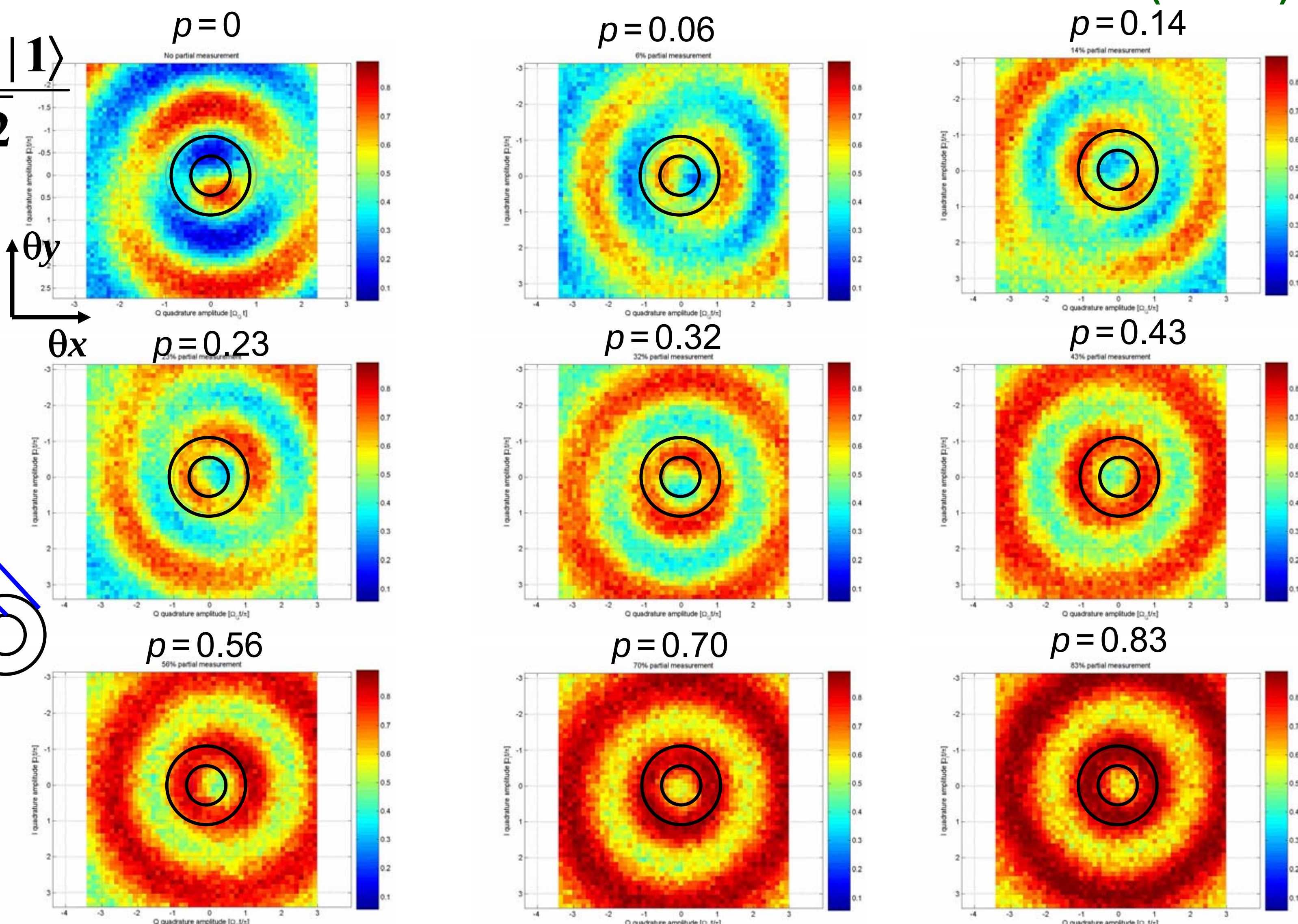
***p*=0: no measurement**

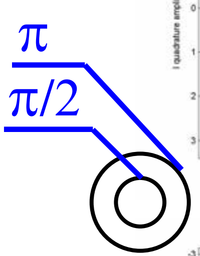
***p*=1: orthodox collapse**

Experimental tomography data

$$\psi_{in} = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

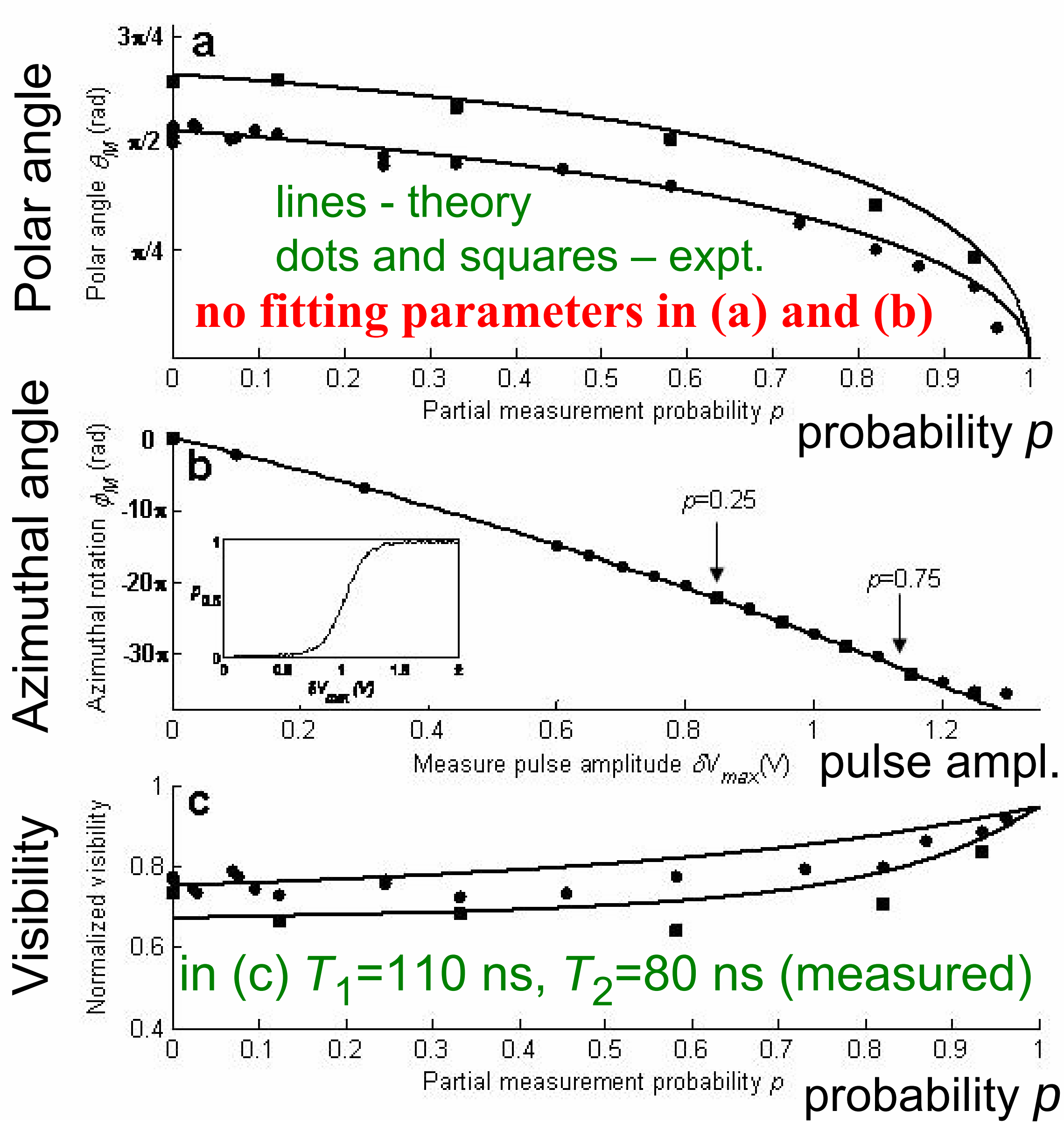
Nadav Katz *et al.* (UCSB)





now only 3 points are used

Partial collapse: experimental results



N. Katz *et al.*, Science-06

- In case of no tunneling (null-result measurement) phase qubit evolves
- This evolution is well described by a simple Bayesian theory, without fitting parameters
- Phase qubit remains fully coherent in the process of continuous collapse (experimentally ~80% raw data, ~96% after account for T1 and T2)

quantum efficiency
$\eta_0 > 0.8$

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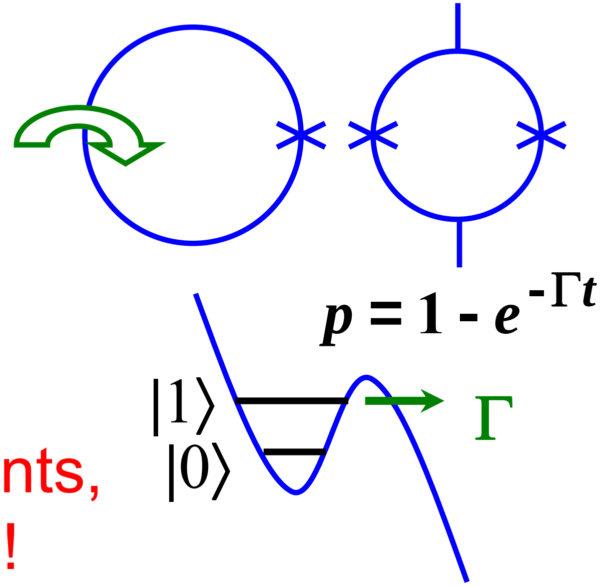
Uncollapsing of a phase qubit state

A.K. & Jordan, 2006

1) Start with an unknown state
2) Partial measurement of strength *p*
3) π-pulse (exchange $|0\rangle \leftrightarrow |1\rangle$)
4) One more measurement with the **same strength** *p*
5) π-pulse



If no tunneling for both measurements, then initial state is fully restored!

$$\alpha|0\rangle + \beta|1\rangle \to \frac{\alpha|0\rangle + e^{i\phi}\beta e^{-\Gamma t/2}|1\rangle}{\text{Norm}} \to$$

$$\frac{e^{i\phi}\alpha e^{-\Gamma t/2}|0\rangle + e^{i\phi}\beta e^{-\Gamma t/2}|1\rangle}{\text{Norm}} = e^{i\phi}(\alpha|0\rangle + \beta|1\rangle)$$

phase is also restored (spin echo)

Probability of success

Success probability if no tunneling during first measurement:

$$P_S = \frac{e^{-\Gamma t}}{\rho_{00}(0) + e^{-\Gamma t}\rho_{11}(0)} = \frac{1 - p}{\rho_{00}(0) + (1 - p)\rho_{11}(0)}$$

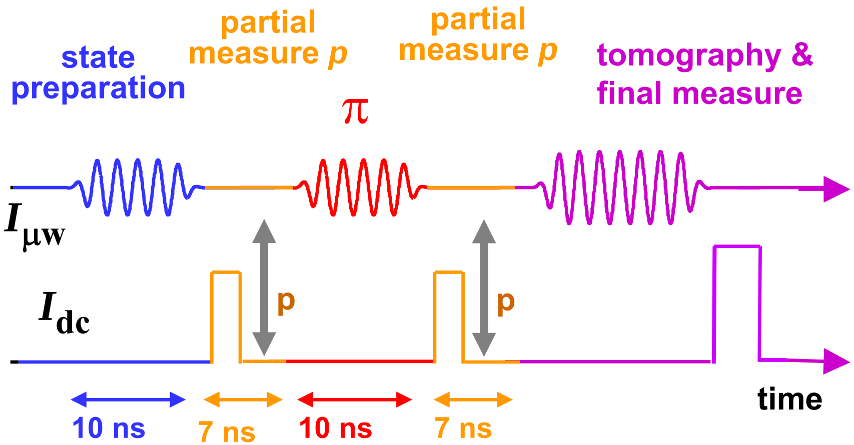
where ρ(0) is the density matrix of the initial state (either averaged unknown state or an entangled state traced over all other qubits)

Total (averaged) success probability: $P_{av} = 1 - p$

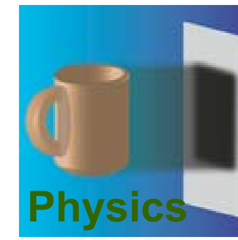
For measurement strength *p* increasing to 1, success probability decreases to zero (orthodox collapse), but still exact uncollapsing

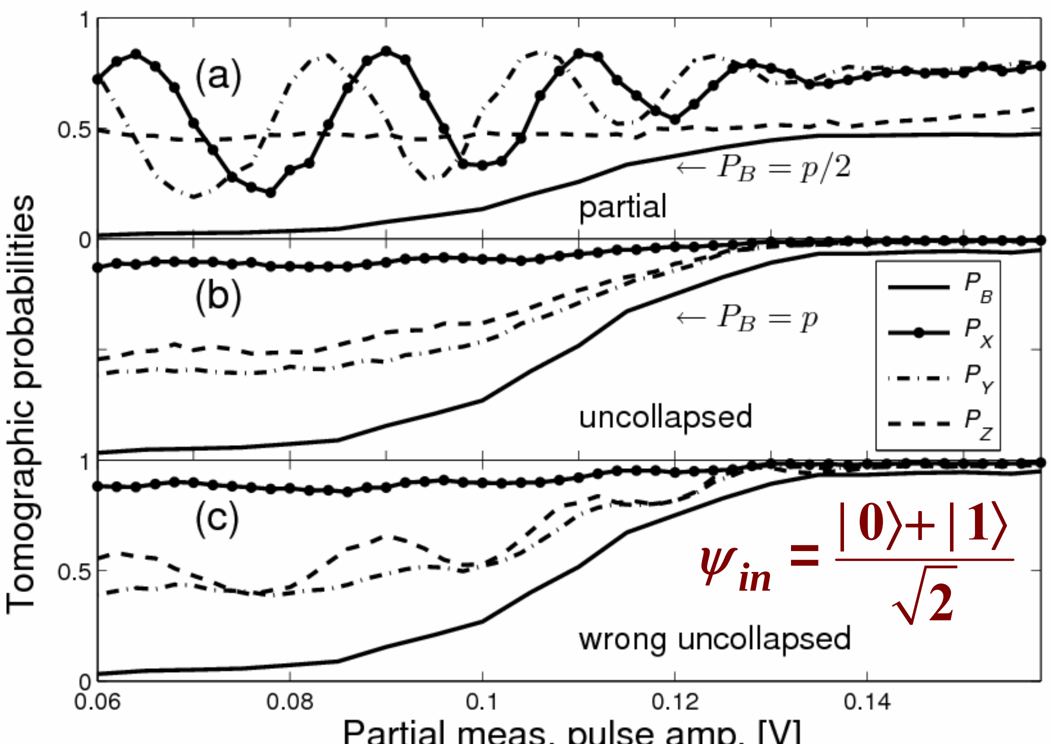
Optimal uncollapsing (reaches the upper bound)

Experiment on wavefunction uncollapsing



N. Katz, M. Neeley, M. Ansmann, R. Bialzak, E. Lucero, A. O'Connell, H. Wang, A. Cleland, J. Martinis, and A. Korotkov, PRL-2008





Uncollapse protocol:

- partial collapse
- π-pulse
- partial collapse (same strength)

State tomography with *X*, *Y*, and no pulses

Background P_B should be subtracted to find qubit density matrix

Experimental results on Bloch sphere

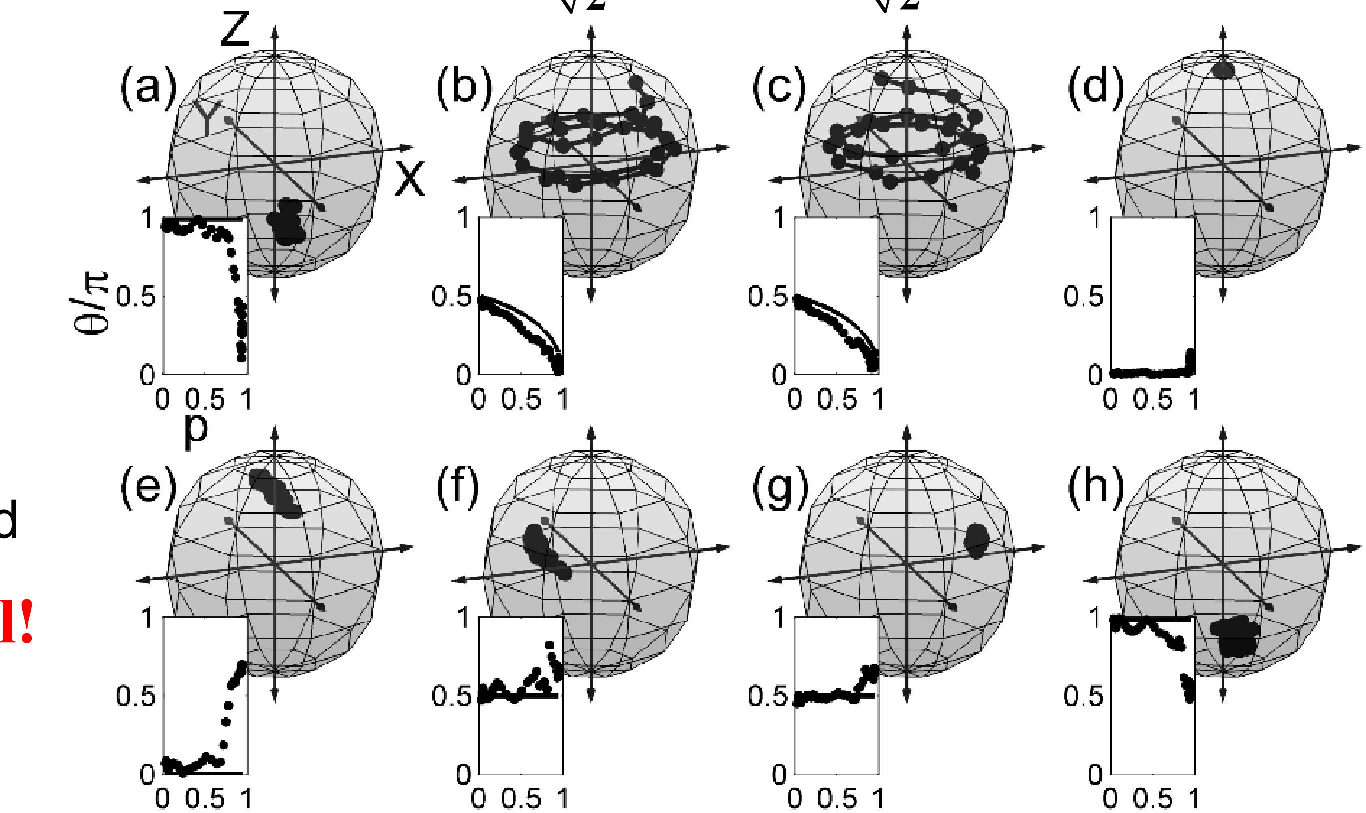
N. Katz et al.

Initial state: $|1\rangle$, $\frac{|0\rangle - i|1\rangle}{\sqrt{2}}$, $\frac{|0\rangle + |1\rangle}{\sqrt{2}}$, $|0\rangle$

Partially collapsed

Uncollapsed

works well!



Both spin echo (azimuth) and uncollapsing (polar angle)

Difference: spin echo – undoing of an unknown unitary evolution, uncollapsing – undoing of a known, but non-unitary evolution

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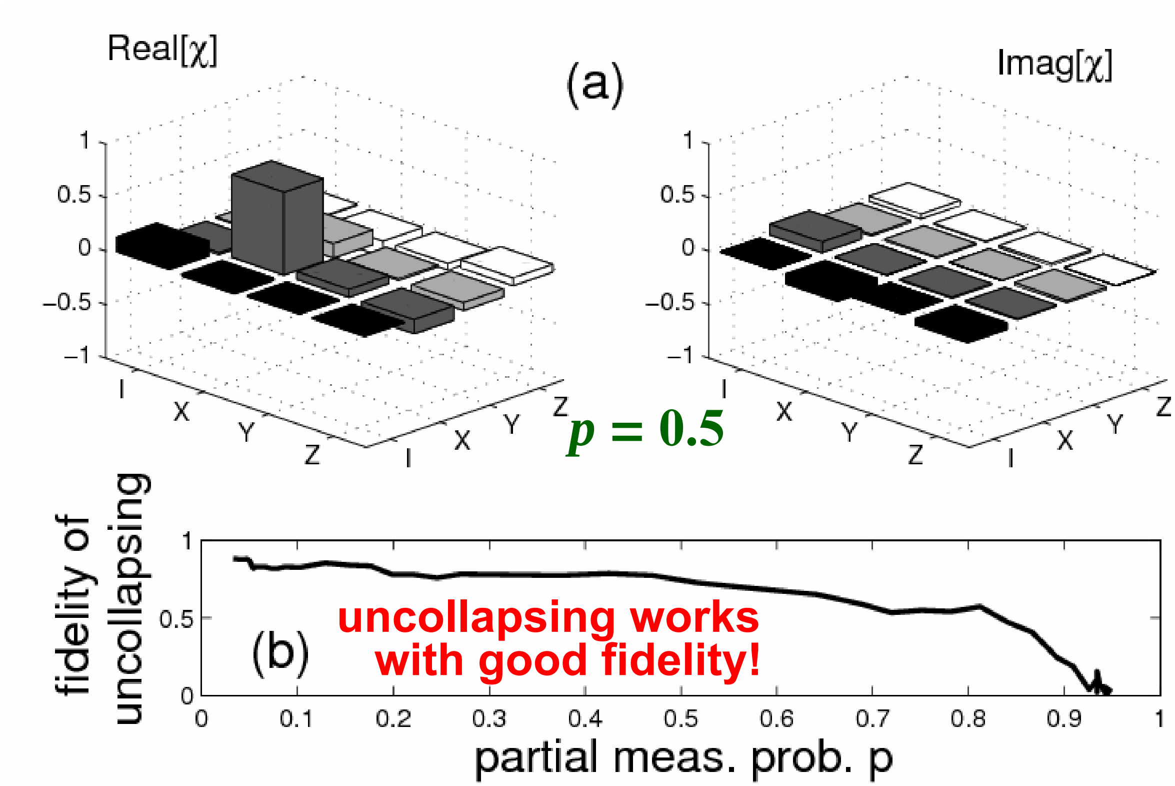
Quantum process tomography

N. Katz et al. (Martinis group)



Why getting worse at p>0.6?

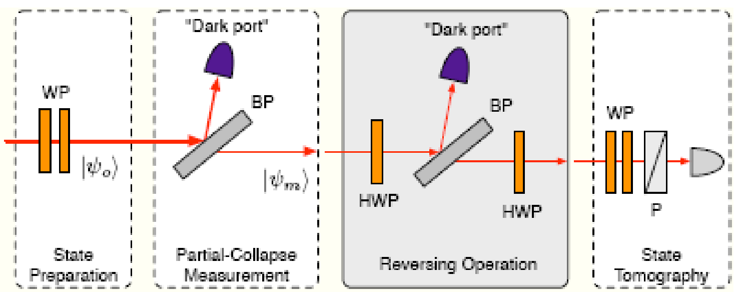
Energy relaxation $p_r = t/T_1 = 45\text{ns}/450\text{ns} = 0.1$

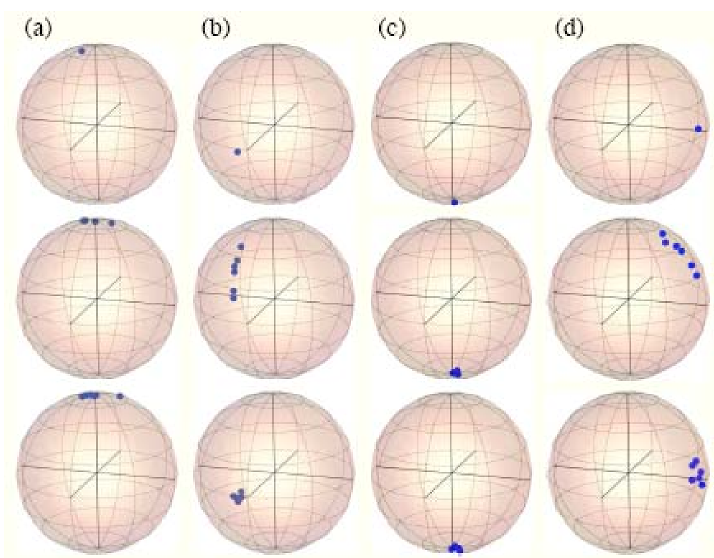
Selection affected when $1-p \sim p_r$

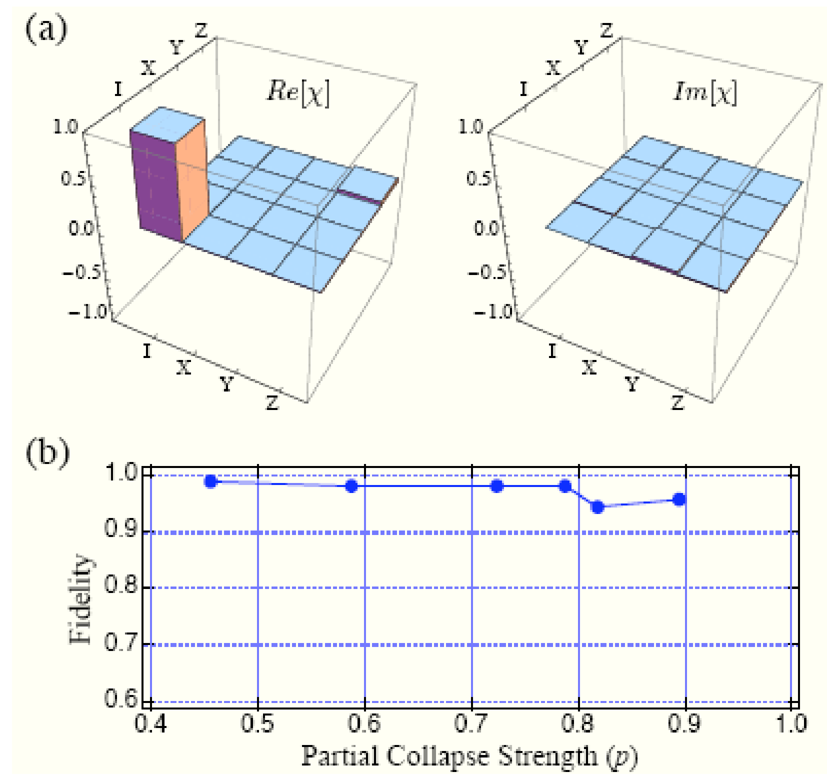
Overall: uncollapsing is well-confirmed experimentally

Recent experiment on uncollapsing using single photons

Y. Kim et al.,Opt. Expr.-09



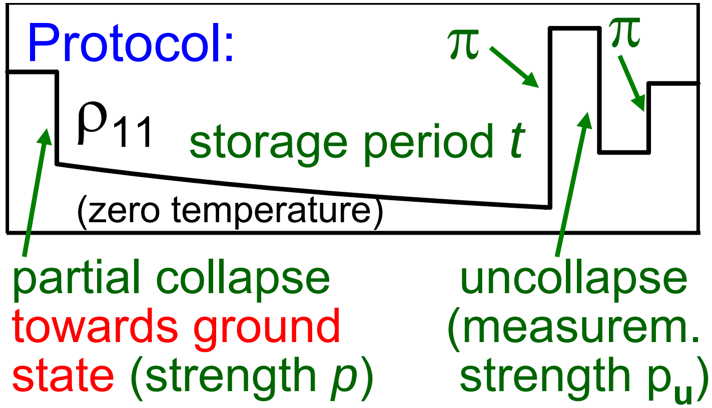




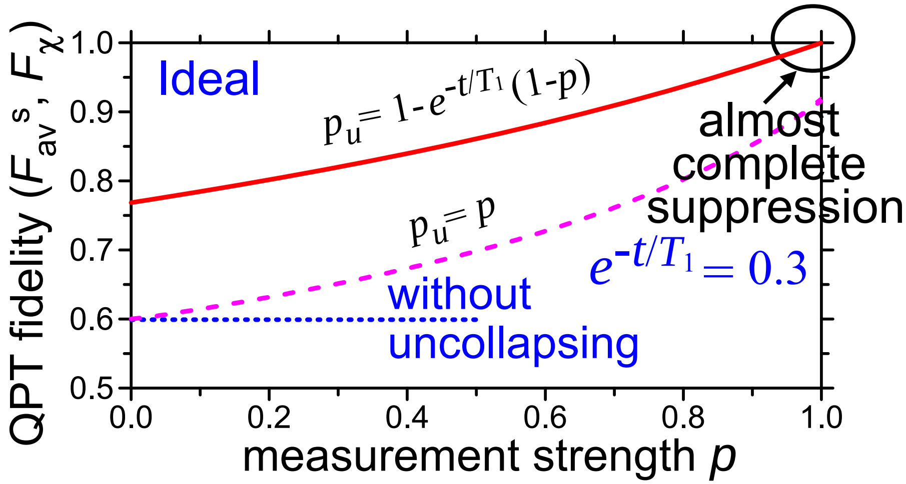
- very good fidelity of uncollapsing (>94%)
- measurement fidelity is probably not good (normalization by coincidence counts)

Suppression of T_1-decoherence by uncollapsing

Korotkov & Keane, arXiv:0908.1134



(almost same as existing experiment!)



Ideal case (T_1 during storage only, T=0)

for initial state $|\psi_{in}\rangle = \alpha|0\rangle + \beta|1\rangle$

$|\psi_f\rangle = |\psi_{in}\rangle$ with probability $(1-p)e^{-t/T_1}$

$|\psi_f\rangle = |0\rangle$ with $(1-p)^2|\beta|^2 e^{-t/T_1}(1-e^{-t/T_1})$

procedure preferentially selects events without energy decay

Trade-off: fidelity vs. selection probability

Unraveling of energy relaxation

$$\begin{pmatrix} |\beta|^2 e^{-t/T_1} & \alpha\beta^* e^{-t/2T_1} \\ \alpha^*\beta e^{-t/2T_1} & 1-|\beta|^2 e^{-t/T_1} \end{pmatrix} =$$

$$= p_t\,|0\rangle\langle 0| + (1-p_t)\,|\tilde\psi\rangle\langle\tilde\psi|$$

where $\quad p_t = |\beta|^2 (1-e^{-t/T_1})$

$$|\tilde\psi\rangle = (\alpha|0\rangle + \beta e^{-t/2T_1}|1\rangle)/Norm$$

⇒ optimum: $\quad 1 - p_u = e^{-t/T_1}(1-p)$

An issue with quantum process tomography (QPT)

QPT fidelity is usually $F_\chi = \mathrm{Tr}(\chi_{desired}\chi)$
where χ is the QPT matrix.

However, QPT is developed for a linear quantum process, while uncollapsing (after renormalization) is non-linear.

A better way: average state fidelity

$$F_{av} = \mathrm{Tr}(\rho_f U_0 |\psi_{in}\rangle\langle\psi_{in}|)\, d|\psi_{in}\rangle$$

Without selection

$$F_\chi = F_{av}^s = \frac{(d+1)F_{av} - 1}{d}, \;\; d = 2$$

Another way: “naïve” QPT fidelity
(via 4 standard initial states)

The two ways practically coincide
(within line thickness)

Analytics for the ideal case

Average state fidelity

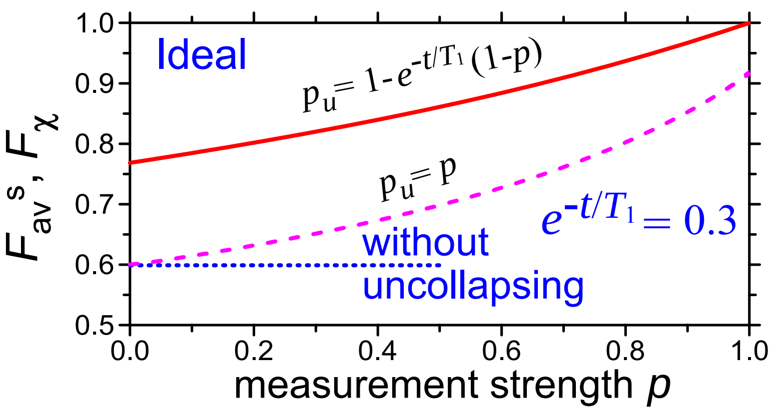
$$F_{av} = \frac{1}{2} + \frac{1}{C} + \frac{\ln(1+C)}{C^2}$$

“Naïve” QPT fidelity

$$F_\chi = -\frac{1}{4} + \frac{1}{4(1+C)} + \frac{4+C}{2(2+C)}$$

where $C = (1-p)(1-e^{-\Gamma t})$

$$p_u = 1 - e^{-\Gamma t}(1-p)$$



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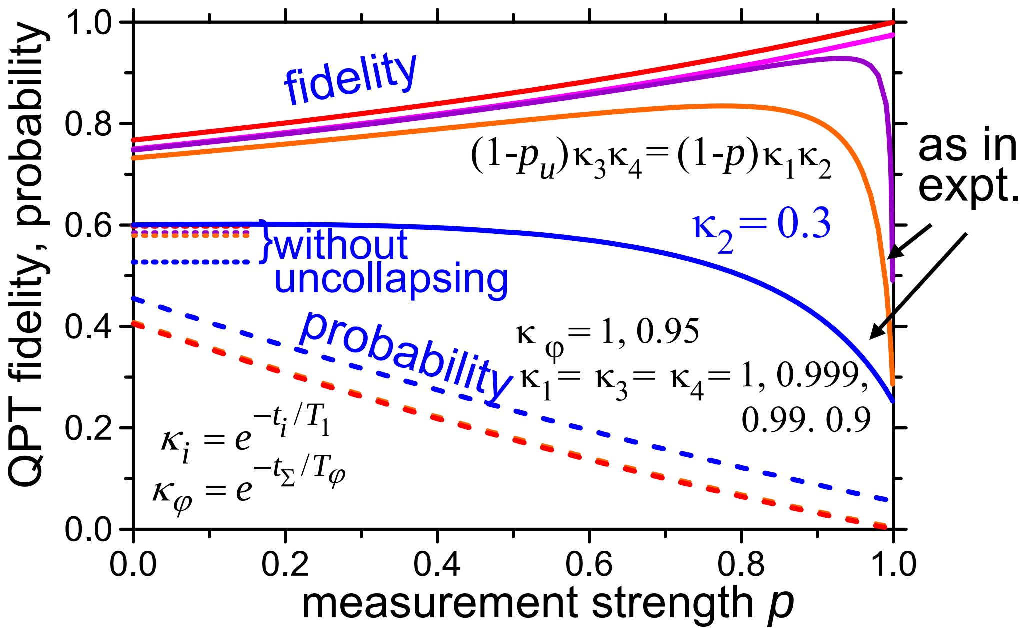
Realistic case (T_1 and T_φ at all stages)



- Easy to realize experimentally (similar to existing experiment)
- Increase of fidelity with *p* can be observed experimentally
- Improved fidelity can be observed with just one partial measurement

Uncollapse seems to be **the only way** to protect against T_1-decoherence without encoding in a larger Hilbert space (QEC, DFS)

- decoherence due to pure dephasing is not affected
- T_1-decoherence between first π-pulse and second measurement causes decrease of fidelity at *p* close to 1

A.K. & Keane, arXiv:0908.1134

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Conclusions

- Partial (weak, etc.) quantum measurement can be undone, though with a finite probability P_S, which decreases with increasing strength of measurement ($P_S = 0$ for orthodox case)
- Arbitrary initial state is uncollapsed exactly in the case of success (need a detector with perfect quantum efficiency)
- Uncollapsing is different from the quantum eraser
- Uncollapsing for a superconducting phase qubit and for a single-photon qubit has been demonstrated; would be very interesting to demonstrate also for a charge qubit
- Uncollapsing can suppress decoherence due to energy relaxation at low temperature

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