Two-qubit decoherence mechanisms revealed via quantum process tomography

A. G. Kofman and A. N. Korotkov, arXiv:0903.0671

QPT basics

Quantum operation (completely positive **linear** map)

(i.e. positive Hermitian (d.m.) $\rightarrow$ positive Hermitian (d.m.); "completely" means even with ancilla)

$$\rho = L[\rho^0], \qquad \rho_{ij} = \sum_{k,l=0}^{d-1} L_{ij,kl}\,\rho^0_{kl}$$

Can be represented with Kraus operators (not uniquely): $L[\rho^0] = \sum_n K_n \rho^0 K_n^\dagger$

Unique representation: $$L[\rho^0] = \sum_{m,n=0}^{d^2-1} \chi_{mn} E_m \rho^0 E_n^\dagger$$

where E_m is a chosen (arbitrary) basis of operators

"Pauli basis": products of (I,X,Y,Z) for each qubit (jargon: X=σ_X, etc.)

Sometimes modified Pauli basis: ($I,X,-iY,Z$)

χ-matrix

$$L[\rho^0] = \sum_{m,n=0}^{d^2-1} \chi_{mn} E_m \rho^0 E_n^\dagger$$

- not quite intuitive
- Hermitian, $d^2\times d^2$, positive-semidefinite
- Tr χ=1 (usually)
- for a unitary operation U $\quad \chi_{mn} = k_m k_n^*, \quad U = \sum_{n=0}^{d^2-1} k_n E_n$

Markovian decoherence (λ-matrix)

$$\frac{d}{dt}\rho = M\rho \quad \text{then} \quad L = \exp(Mt) \quad \text{(if treat density matrix as a vector)}$$

$$M = M_{\text{coh}} + D \qquad \text{Can introduce } \lambda\text{-matrix:} \quad D[\rho] = \sum_{m,n=0}^{d^2-1} \lambda_{mn} E_m \rho E_n^\dagger$$

λ-matrix is similar to χ-matrix,
but Tr λ=0 and different dimension ([χ]=[λt])

If M_{coh}=0, then at weak decoherence $\quad \chi \approx \chi^I + \lambda t, \quad \chi^I_{mn} = \delta_{m0}\delta_{n0}$

Local vs. non-local decoherence

Uncoupled 2-qubit system and local decoherence: $\chi = \chi^{(1)} \otimes \chi^{(2)}$

Natural to introduce nonlocality parameter for decoherence:

$$\varepsilon_{NL} = \frac{\mathbf{Tr}\,|\,\chi - \tilde{\chi}\,|}{\mathbf{Tr}\,|\,\chi - \chi_{\mathbf{ideal}}\,|}, \quad \tilde{\chi} = \tilde{\chi}^{(1)} \otimes \tilde{\chi}^{(2)}, \;\; \tilde{\chi}^{(1)} = \mathbf{Tr}_2 \chi$$

$$|\,A\,| \equiv \sqrt{A^\dagger A},$$ Tr $|A|$ - “trace norm”

Not so easy for **coupled** system. However, in Markovian case can use λ-matrix:

$$\varepsilon'_{NL} = \frac{\mathbf{Tr}\,|\,\lambda - \tilde{\lambda}\,|}{\mathbf{Tr}\,|\,\lambda\,|}, \quad \tilde{\lambda} = \tilde{\lambda}^{(1)} \otimes \chi^{I(2)} + \chi^{I(1)} \otimes \tilde{\lambda}^{(2)}, \;\; \tilde{\lambda}^{(1)} = \mathbf{Tr}_2 \lambda$$

$\varepsilon'_{NL} \approx \varepsilon_{NL}$ for no coherent evolution and weak decoherence

Considered models of decoherence

Energy relaxation

$$\lambda_{00} = -2(1/T_1^{(1)} + 1/T_1^{(2)})$$
$$\lambda_{11} = \lambda_{22} = 1/T_1^{(2)},\ \lambda_{44} = \lambda_{88} = 1/T_1^{(1)}$$
$$\lambda_{03} = \lambda_{30} = 1/T_1^{(2)},\ \lambda_{0,12} = \lambda_{12,0} = 1/T_1^{(1)}$$
$$\lambda_{21} = -\lambda_{12} = i/T_1^{(2)},\ \lambda_{84} = -\lambda_{48} = i/T_1^{(1)}$$

Partially correlated (nonlocal) dephasing

$$\lambda_{00} = -(1/T_\varphi^{(1)} + 1/T_\varphi^{(2)})/2$$
$$\lambda_{33} = 1/2T_\varphi^{(2)},\ \lambda_{12,12} = 1/2T_\varphi^{(1)}$$
$$\lambda_{3,12} = \lambda_{12,3} = -\lambda_{0,15} = -\lambda_{15,0} = \kappa/2\sqrt{T_\varphi^{(1)}T_\varphi^{(2)}}$$

κ - correlation factor (κ=0 if local)

Noisy coupling

$$\lambda_{00} = -\Gamma_S$$
$$\lambda_{55} = \lambda_{10,10} = \lambda_{5,10} = \lambda_{10,5} =$$
$$= \lambda_{0,15} = \lambda_{15,0} = \Gamma_S/2$$

Specific pattern for each model!
(usually different elements)

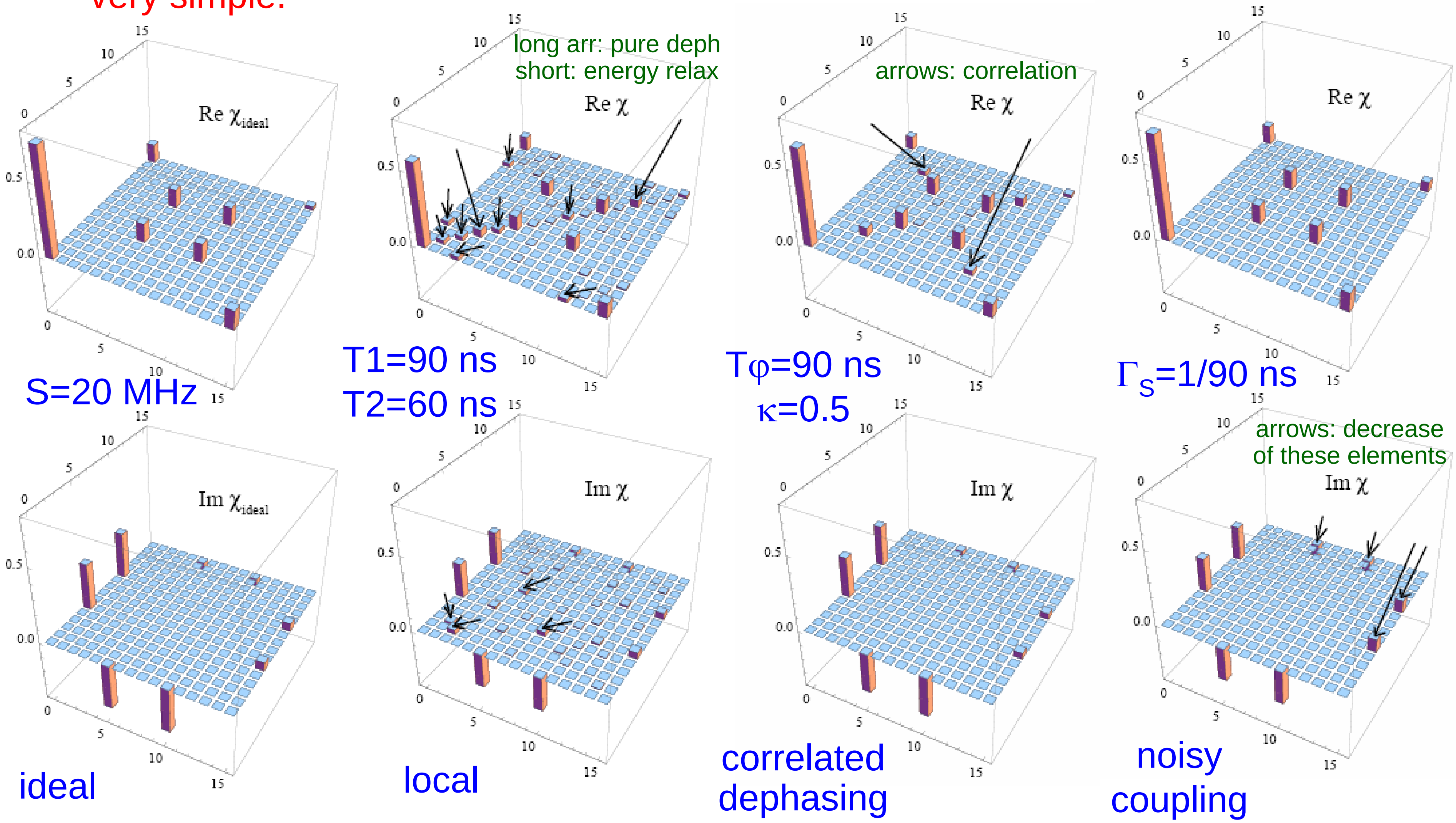
If no evolution (memory), then easy:

$$\chi \approx \chi^I + \lambda t, \quad \chi_{mn}^I = \delta_{m0}\delta_{n0}$$
$$\lambda = \lambda_{enrel} + \lambda_{corr\,deph} + \lambda_{noisy\,coupl}$$

0,1,2, …15 = *II, IX, IY, IZ,*
XI, XX, XY, XZ, … *ZZ*

If coupled qubits, then should not be easy $\chi \neq \chi_{ideal} + \lambda t$

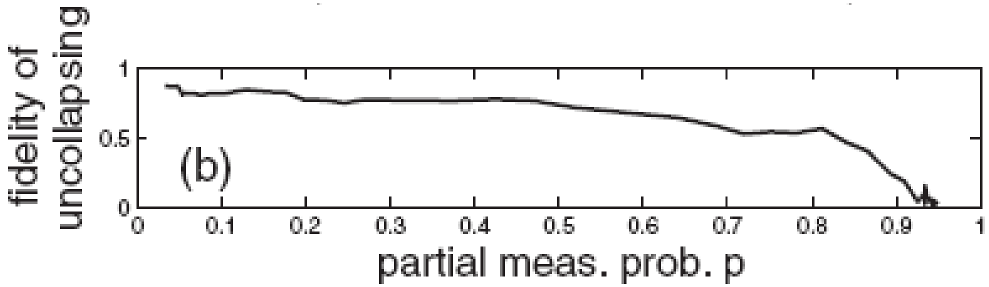
However, miraculously, for sqrt{iswap} and for largest extra elements of χ, it is still a simple addition, that makes distinguishing decoherence models very simple.



QPT for one-qubit uncollapsing

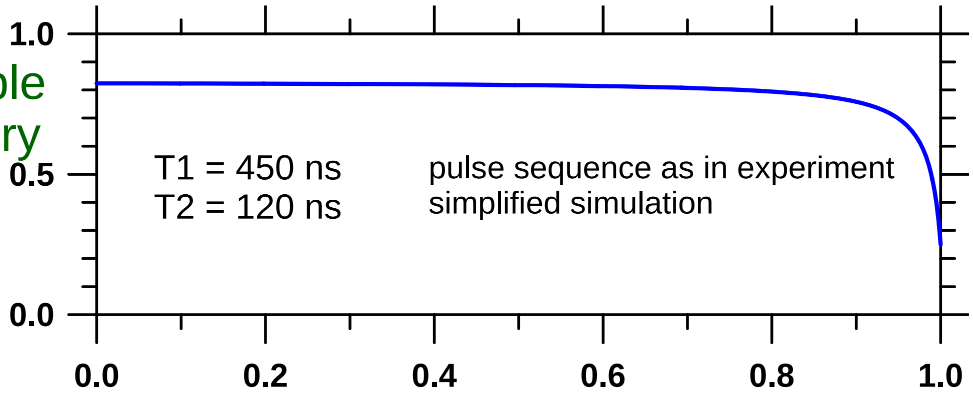
Kyle Keane (started recently)

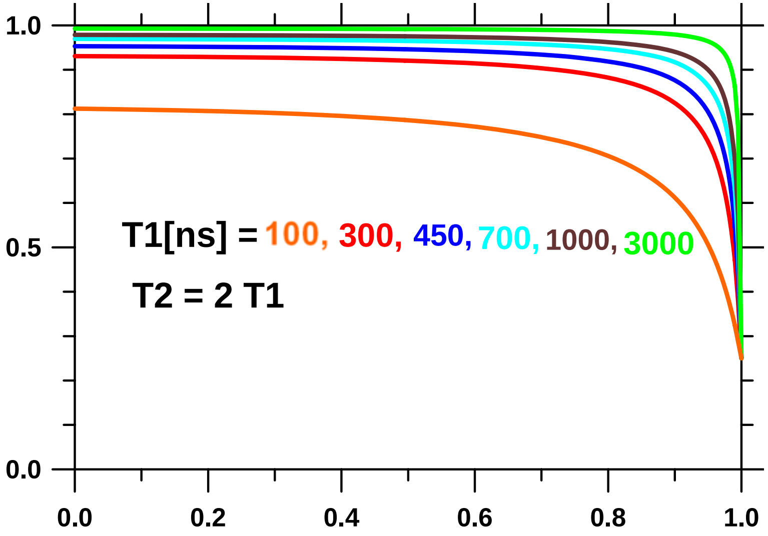
Nadav Katz et al., PRL-08



Fidelity decreases at large p due to T1 and selection

simple theory





Theoretically fidelity goes to 0.25 at p =1

Actually, problem with definition of χ-matrix, because probability depends on initial state