

Weak (continuous, partial, etc.) measurements in solid state systems: theory and experiments (what is “inside” collapse)

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- Outline:**
- Simple Bayesian formalism (+ time arrow)
 - Persistent Rabi oscillations (+ expt.)
 - Wavefunction uncollapse (+ expts.)
 - Some other experimental proposals

Ackn.:

Theory: R. Ruskov, A. Jordan

Expt.: UCSB (J. Martinis, N. Katz et al.),
Sacay (D. Esteve, P. Bertet et al.)

Funding:

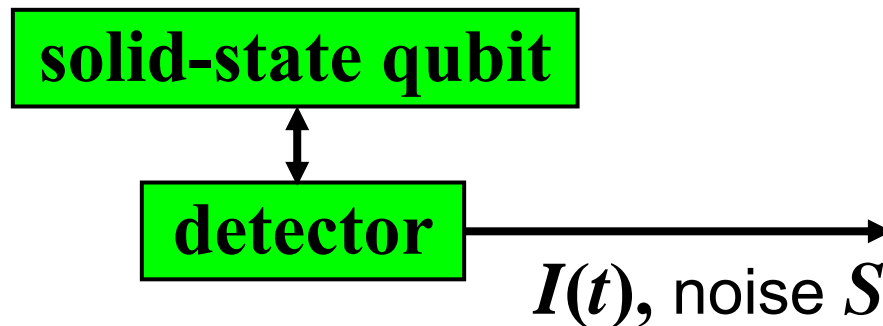


Many approaches to non-projective (weak, continuous, partial, generalized, etc.) quantum measurements

Names: Davies, Kraus, Holevo, Mensky, Caves, Aharonov, Vaidman, Knight, Plenio, Walls, Carmichael, Milburn, Wiseman, Gisin, Percival, Belavkin, etc. (very incomplete list)

Key words: POVM, restricted path integral, weak values, quantum jumps, quantum trajectories, quantum filtering, stochastic master equation, etc.

Our limited scope:
(simplest system,
experimental setups)



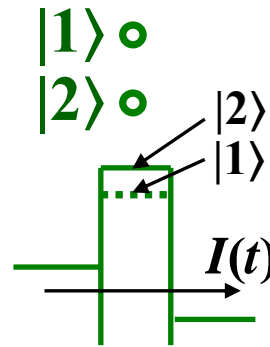
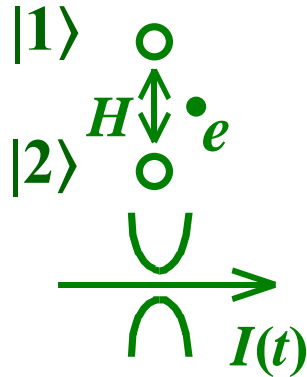
What is the evolution due to measurement?

(no post-selection, most standard quantum ideology)



“Typical” setup: double-quantum-dot (DQD) qubit + quantum point contact (QPC) detector

Gurvitz, 1997



$$H = H_{\text{QB}} + H_{\text{DET}} + H_{\text{INT}}$$

$$H_{\text{QB}} = \frac{\varepsilon}{2} \sigma_z + H \sigma_x$$

$$I(t) = I_0 + \frac{\Delta I}{2} z(t) + \xi(t)$$

const + signal + noise

Two levels of average detector current: I_1 for qubit state $|1\rangle$, I_2 for $|2\rangle$

Response: $\Delta I = I_1 - I_2$ Detector noise: white, spectral density S_I

For low-transparency QPC

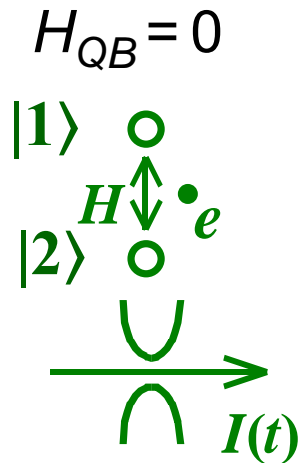
$$H_{\text{DET}} = \sum_l E_l a_l^\dagger a_l + \sum_r E_r a_r^\dagger a_r + \sum_{l,r} T (a_r^\dagger a_l + a_l^\dagger a_r)$$

$$H_{\text{INT}} = \sum_{l,r} \Delta T (c_1^\dagger c_1 - c_2^\dagger c_2) (a_r^\dagger a_l + a_l^\dagger a_r)$$

$$S_I = 2eI$$



Bayesian formalism for DQD-QPC system



Qubit evolution due to measurement (quantum back-action):

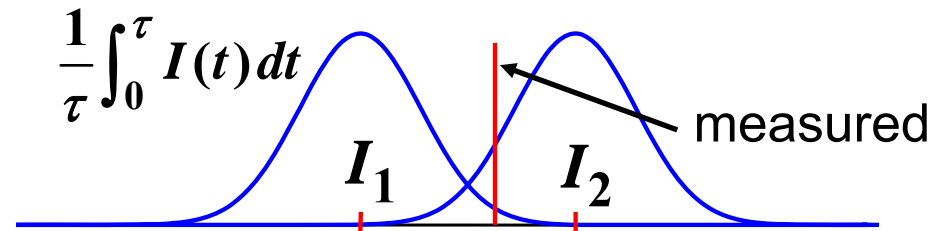
$$\psi(t) = \alpha(t) |1\rangle + \beta(t) |2\rangle \quad \text{or} \quad \rho_{ij}(t)$$

- 1) $|\alpha(t)|^2$ and $|\beta(t)|^2$ evolve as probabilities,
i.e. according to the **Bayes rule** (same for ρ_{ij})
- 2) phases of $\alpha(t)$ and $\beta(t)$ do not change
(no dephasing!), $\rho_{ij}/(\rho_{ii}\rho_{jj})^{1/2} = \text{const}$

(A.K., 1998)

Bayes rule (1763, Laplace-1812):

$$\underbrace{P(A_i | \text{res})}_{\text{posterior probability}} = \frac{\underbrace{P(A_i)}_{\text{prior probab.}} \underbrace{P(\text{res} | A_i)}_{\text{likelihood}}}{\sum_k P(A_k) P(\text{res} | A_k)}$$

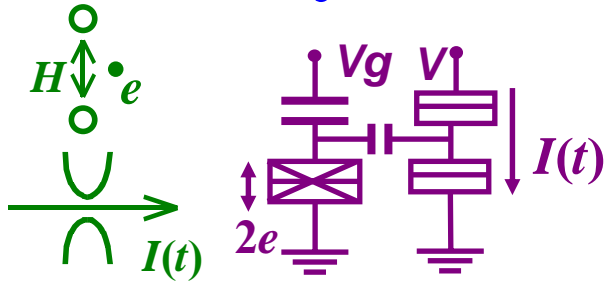


So simple because:

- 1) no entanglement at large QPC voltage
- 2) QPC happens to be an ideal detector
- 3) no Hamiltonian evolution of the qubit



Bayesian formalism for a single qubit



- Time derivative of the quantum Bayes rule
- Add unitary evolution of the qubit
- Add decoherence γ (if any)

$$\dot{\rho}_{11} = -\dot{\rho}_{22} = -2H \operatorname{Im} \rho_{12} + \rho_{11}\rho_{22} \frac{2\Delta I}{S_I} [\underline{I(t)} - I_0]$$

$$\dot{\rho}_{12} = i\varepsilon\rho_{12} + iH(\rho_{11} - \rho_{22}) + \rho_{12}(\rho_{11} - \rho_{22}) \frac{\Delta I}{S_I} [\underline{I(t)} - I_0] - \gamma\rho_{12}$$

(A.K., 1998)

$\gamma = \Gamma - (\Delta I)^2 / 4S_I$, Γ – ensemble decoherence

$\gamma = 0$ for QPC detector

Averaging over result $I(t)$ leads to conventional master equation with Γ

Evolution of qubit *wavefunction* can be monitored if $\gamma=0$ (quantum-limited)

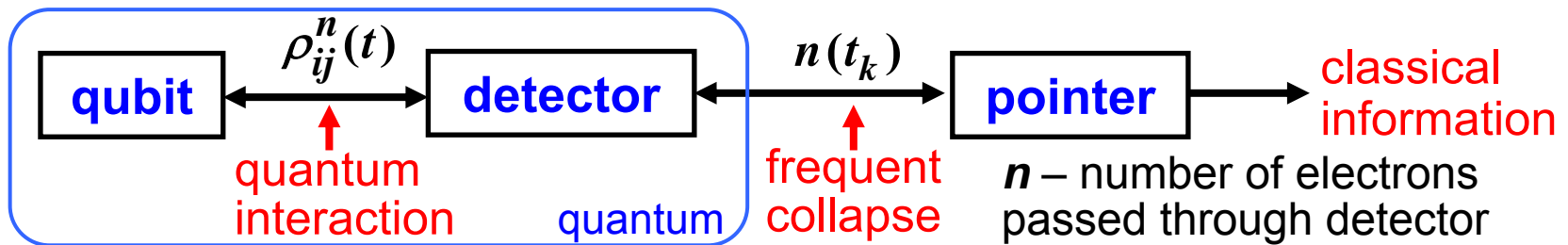


Assumptions needed for the Bayesian formalism:

- Detector voltage is much larger than the qubit energies involved
 $eV \gg \hbar\Omega, eV \gg \hbar\Gamma, \hbar/eV \ll (1/\Omega, 1/\Gamma), \Omega = (4H^2 + \varepsilon^2)^{1/2}$
(no coherence in the detector, **classical output**, Markovian approximation)
- Simpler if weak response, $|\Delta| \ll I_0$, (coupling $C \sim \Gamma/\Omega$ is arbitrary)

Derivations:

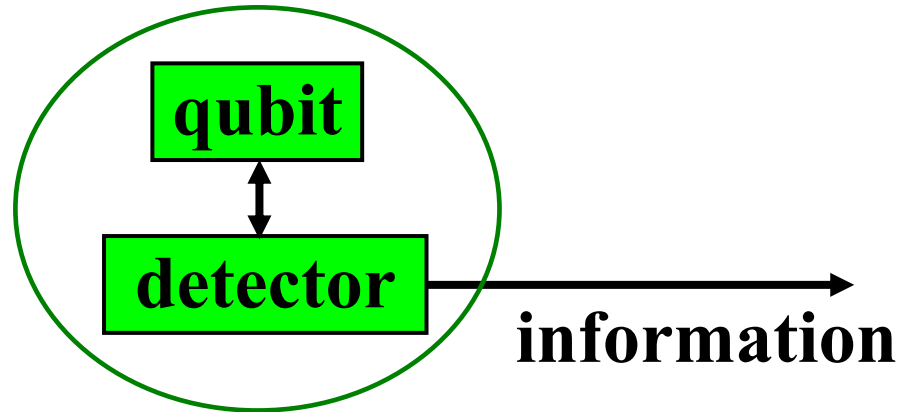
- 1) “logical”: via correspondence principle and comparison with decoherence approach (A.K., 1998)
- 2) “microscopic”: Schr. eq. + collapse of the detector (A.K., 2000)



- 3) from “quantum trajectory” formalism developed for quantum optics (Goan-Milburn, 2001; also: Wiseman, Sun, Oxtoby, etc.)
- 4) from POVM formalism (Jordan-A.K., 2006)
- 5) from Keldysh formalism (Wei-Nazarov, 2007)



Why not just use Schrödinger equation for the whole system?



Impossible in principle!

Technical reason: Outgoing information (measurement result) makes it an open system

Philosophical reason: Random measurement result, but deterministic Schrödinger equation

Einstein: God does not play dice

Heisenberg: unavoidable quantum-classical boundary



Fundamental limit for ensemble decoherence

$$\Gamma = (\Delta I)^2 / 4S_I + \gamma$$

ensemble decoherence rate \nearrow \nwarrow single-qubit decoherence

\sim information flow [bit/s]

“measurement time” (S/N=1)
 $\tau_m = 2S_I / (\Delta I)^2$

$$\Gamma \tau_m \geq \frac{1}{2}$$

$$\gamma \geq 0 \Rightarrow$$

$$\Gamma \geq (\Delta I)^2 / 4S_I$$

A.K., 1998, 2000
 S. Pilgram et al., 2002
 A. Clerk et al., 2002
 D. Averin, 2000, 2003

$$\eta = 1 - \frac{\gamma}{\Gamma} = \frac{(\Delta I)^2 / 4S_I}{\Gamma}$$

detector ideality (quantum efficiency)
 $\eta \leq 100\%$

Translated into energy sensitivity: $(\epsilon_O \epsilon_{BA})^{1/2} \geq \hbar/2$

Danilov, Likharev,
 Zorin, 1983

$\epsilon_O, \epsilon_{BA}$: sensitivities [J/Hz] limited by output noise and back-action

Known since 1980s (Caves, Clarke, Likharev, Zorin, Vorontsov, Khalili, etc.)

$$(\epsilon_O \epsilon_{BA} - \epsilon_{O,BA}^2)^{1/2} \geq \hbar/2 \Leftrightarrow \Gamma \geq (\Delta I)^2 / 4S_I + K^2 S_I / 4$$



Arrow of time in continuous measurement of a qubit

A.K., write-up available
similar to uncollapsing (A.K.-Jordan-06)

$$\begin{aligned}\dot{\rho}_{11} &= -\dot{\rho}_{22} = -2H \operatorname{Im} \rho_{12} + \rho_{11}\rho_{22}(2\Delta I / S_I)[I(t) - I_0] \\ \dot{\rho}_{12} &= i\varepsilon\rho_{12} + iH(\rho_{11} - \rho_{22}) + \rho_{12}(\rho_{11} - \rho_{22})(\Delta I / S_I)[I(t) - I_0]\end{aligned}\quad I(t) - I_0 = \frac{\Delta I}{2}z(t) + \xi(t)$$

Evolution is time-reversed if $H_{QB} \rightarrow -H_{QB}$, $(I - I_0) \rightarrow -(I - I_0)$

Reversed movie is correct, but can be distinguished probabilistically

$$\frac{P_{\text{backward}}}{P_{\text{forward}}} = \exp\left(-\frac{(\Delta I)^2}{S_I} \int z^2(t) dt\right)$$

In case of persistent Rabi oscillations (later)

$$P_{\text{backward}} / P_{\text{forward}} = \exp(-\tau / \tau_m) \quad \tau_m = 2S_I / (\Delta I)^2$$



POVM vs. Bayesian formalism

General quantum measurement (POVM formalism) (Nielsen-Chuang, p. 85,100):

Measurement (Kraus) operator M_r (any linear operator in H.S.): $\psi \rightarrow \frac{M_r \psi}{\|M_r \psi\|}$ or $\rho \rightarrow \frac{M_r \rho M_r^\dagger}{\text{Tr}(M_r \rho M_r^\dagger)}$

Probability: $P_r = \|M_r \psi\|^2$ or $P_r = \text{Tr}(M_r \rho M_r^\dagger)$

Completeness: $\sum_r M_r^\dagger M_r = 1$ (People often prefer linear evolution and non-normalized states)

- POVM is essentially a projective measurement in an extended Hilbert space
- **Easy to derive: interaction with ancilla + projective measurement of ancilla**
- For extra decoherence: incoherent sum over subsets of results

Relation between POVM and quantum Bayesian formalism: decomposition $M_r = U_r \underbrace{\sqrt{M_r^\dagger M_r}}_{\text{Bayes}}$
↑
unitary

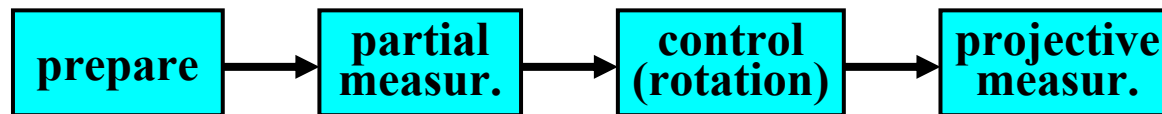
Mathematically POVM and quantum Bayes are almost equivalent

We focus not on the mathematical structures, but on particular setups and experimental consequences



Can we verify the Bayesian formalism experimentally?

Direct way:



A.K., 1998

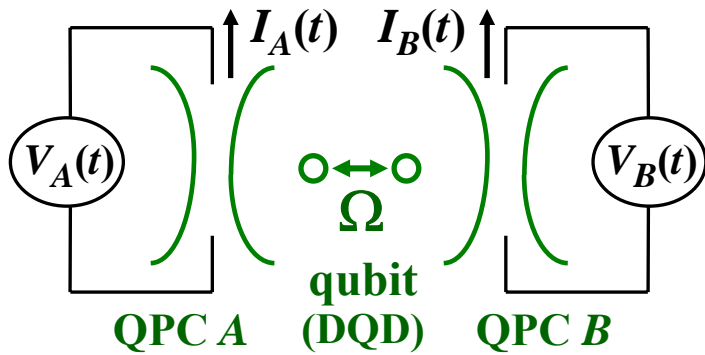
However, difficult: bandwidth, control, efficiency
(expt. realized only for supercond. phase qubits)

Tricks are needed for real experiments



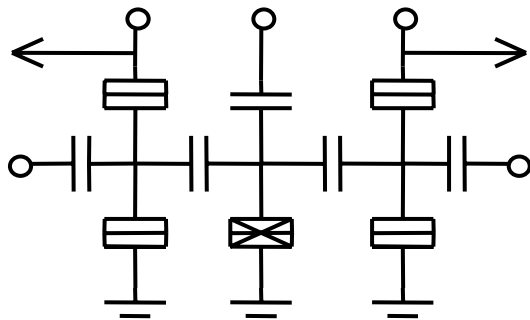
Bell-type measurement correlation

(A.K., PRB-2001)

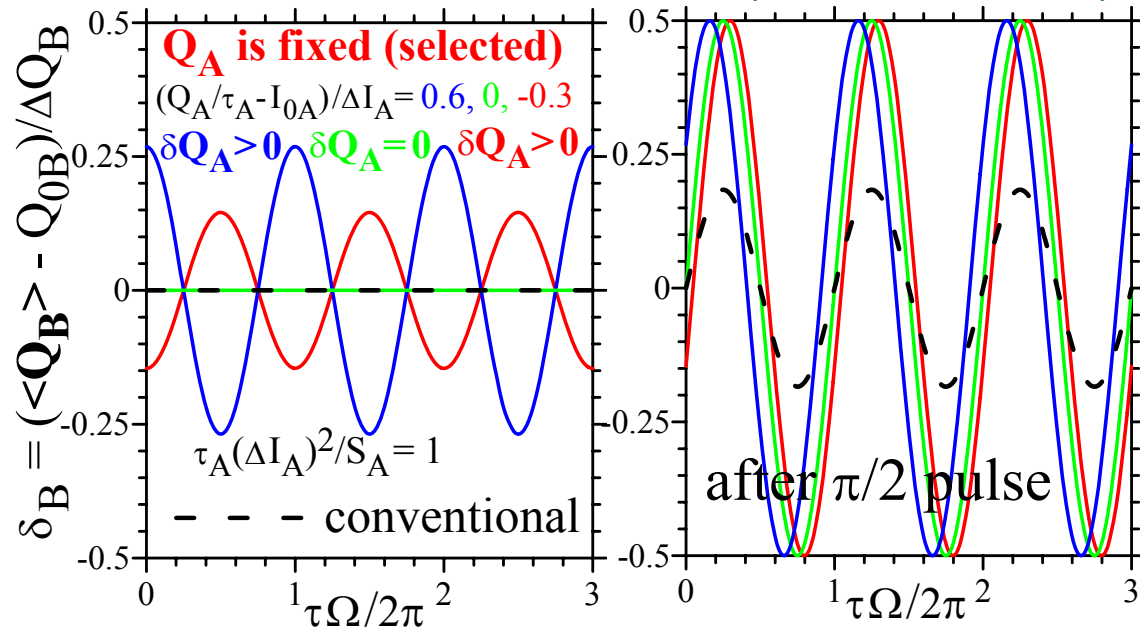


$$Q_A = \int I_A dt$$

$$Q_B = \int I_B dt$$



detector A qubit detector B



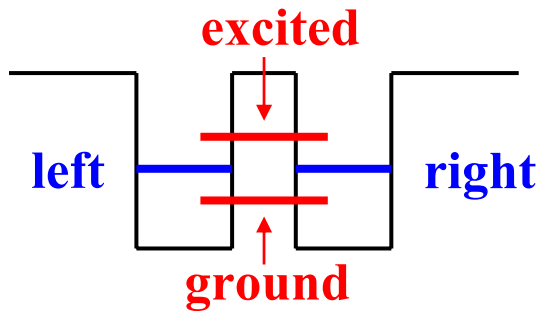
Idea: two consecutive measurements of a qubit by two detectors; probability distribution $\mathbf{P}(Q_A, Q_B, \tau)$ shows effect of the first measurement on the qubit state.

Advantage: solves the bandwidth problem

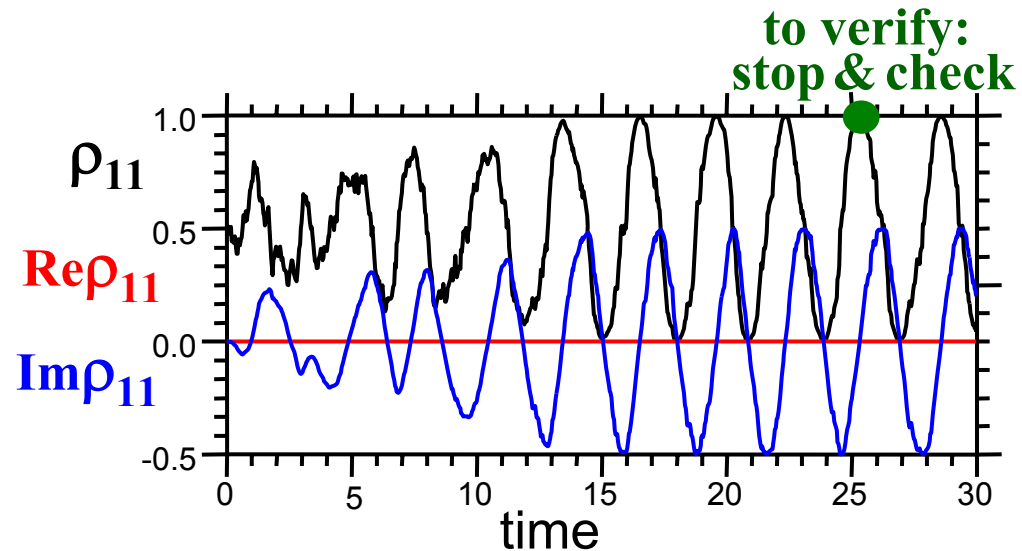
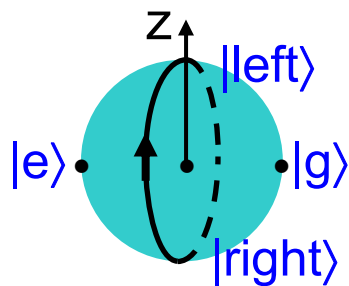
Same proposal with another averaging → weak values
(Romito, Gefen, Blanter, PRL-2008)



Persistent Rabi oscillations



- Relaxes to the ground state if left alone (low- T)
- Becomes fully mixed if coupled to a high- T (non-equilibrium) environment
- Oscillates persistently between left and right if (weakly) measured continuously



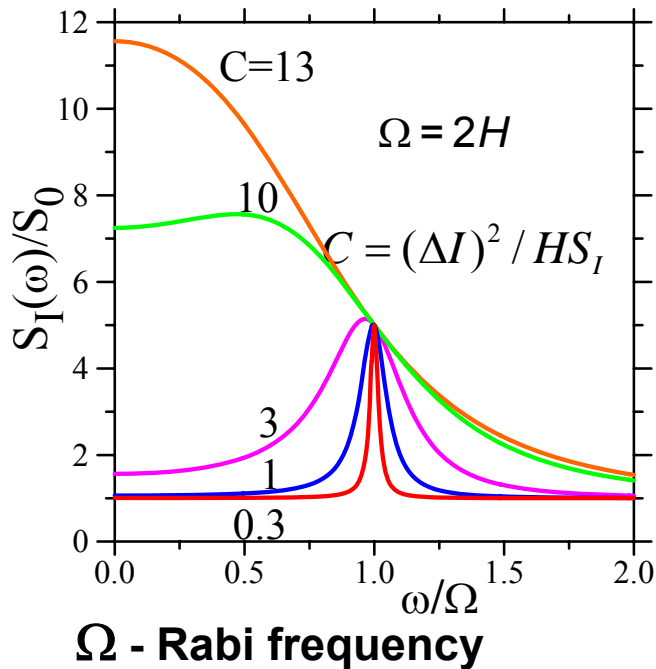
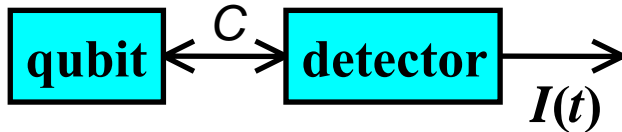
A.K., PRB-1999

Direct experiment is difficult



Indirect experiment: spectrum of persistent Rabi oscillations

A.K., LT'1999
A.K.-Averin, 2000



peak-to-pedestal ratio = $4\eta \leq 4$

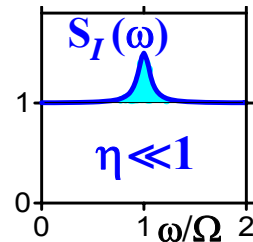
$$S_I(\omega) = S_0 + \frac{\Omega^2 (\Delta I)^2 \Gamma}{(\omega^2 - \Omega^2)^2 + \Gamma^2 \omega^2}$$

$$I(t) = I_0 + \frac{\Delta I}{2} z(t) + \xi(t)$$

(const + signal + noise)

z is Bloch coordinate

amplifier noise \Rightarrow higher pedestal,
poor quantum efficiency,
but the peak is the same!!!



integral under the peak \Leftrightarrow variance $\langle z^2 \rangle$

How to distinguish experimentally
persistent from non-persistent? **Easy!**

perfect Rabi oscillations: $\langle z^2 \rangle = \langle \cos^2 \rangle = 1/2$

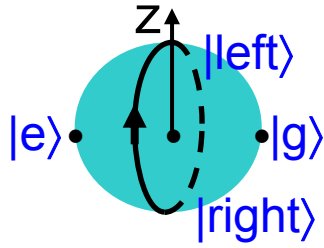
imperfect (non-persistent): $\langle z^2 \rangle \ll 1/2$

quantum (Bayesian) result: **$\langle z^2 \rangle = 1$ (!!!)**

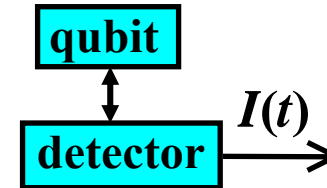
(demonstrated in Saclay expt.)



How to understand $\langle z^2 \rangle = 1$?



$$I(t) = I_0 + \frac{\Delta I}{2} z(t) + \xi(t)$$



First way (mathematical)

We actually measure operator: $z \rightarrow \sigma_z$

$$z^2 \rightarrow \sigma_z^2 = 1$$

(What does it mean?
Difficult to say...)

Second way (Bayesian)

$$S_I(\omega) = S_{\xi\xi} + \frac{\Delta I^2}{4} S_{zz}(\omega) + \frac{\Delta I}{2} S_{\xi z}(\omega)$$

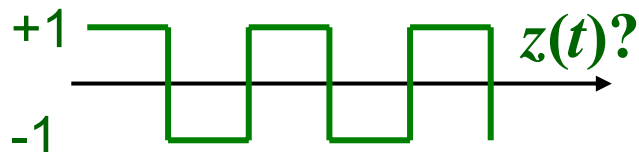


quantum back-action changes z
in accordance with the noise ξ

Equal contributions (for weak
coupling and $\eta=1$)

“what you see is what you get”: observation becomes reality

Can we explain it in a more reasonable way (without spooks/ghosts)?

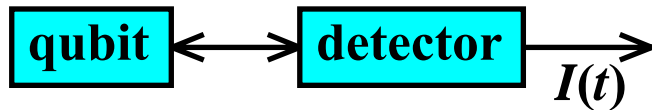


or some other $z(t)$?

No (under assumptions of macrorealism;
Leggett-Garg, 1985)



Leggett-Garg-type inequalities for continuous measurement of a qubit



Ruskov-A.K.-Mizel, PRL-2006
Jordan-A.K.-Büttiker, PRL-2006

Assumptions of macrorealism
(similar to Leggett-Garg'85):

$$I(t) = I_0 + (\Delta I / 2) z(t) + \xi(t)$$

$$|z(t)| \leq 1, \quad \langle \xi(t) z(t + \tau) \rangle = 0$$

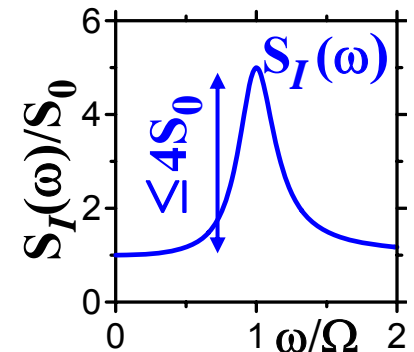
Leggett-Garg, 1985

$$K_{ij} = \langle Q_i Q_j \rangle$$

if $Q = \pm 1$, then

$$1 + K_{12} + K_{23} + K_{13} \geq 0$$

$$K_{12} + K_{23} + K_{34} - K_{14} \leq 2$$



Then for correlation function

$$K(\tau) = \langle I(t) I(t + \tau) \rangle$$

$$K(\tau_1) + K(\tau_2) - K(\tau_1 + \tau_2) \leq (\Delta I / 2)^2$$

and for area under narrow spectral peak

$$\int [S_I(f) - S_0] df \leq (8 / \pi^2) (\Delta I / 2)^2$$

quantum result

$$\frac{3}{2} (\Delta I / 2)^2$$

violation

$$\times \frac{3}{2}$$

$$(\Delta I / 2)^2$$

$$\times \frac{\pi^2}{8}$$

η is not important!

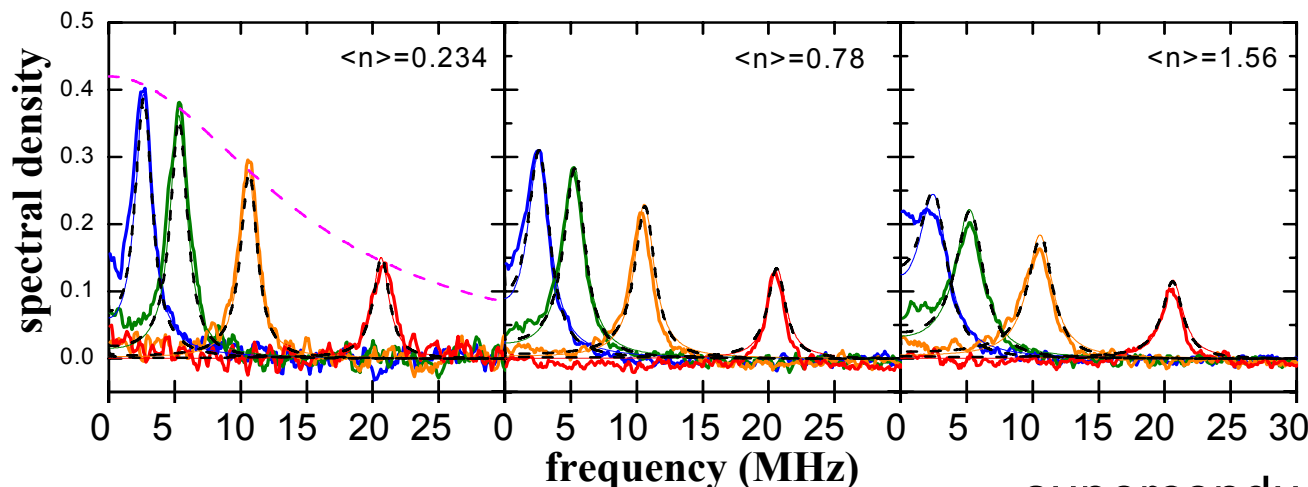
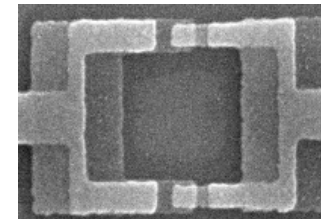
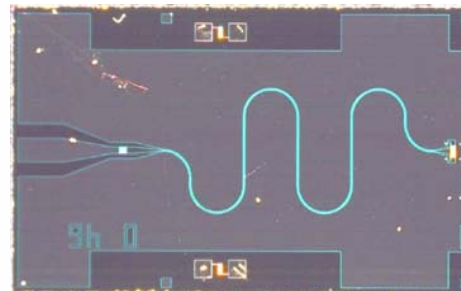
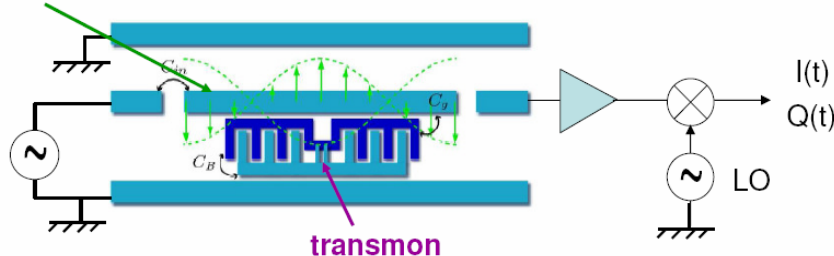
Experimentally measurable violation

(Saclay experiment)



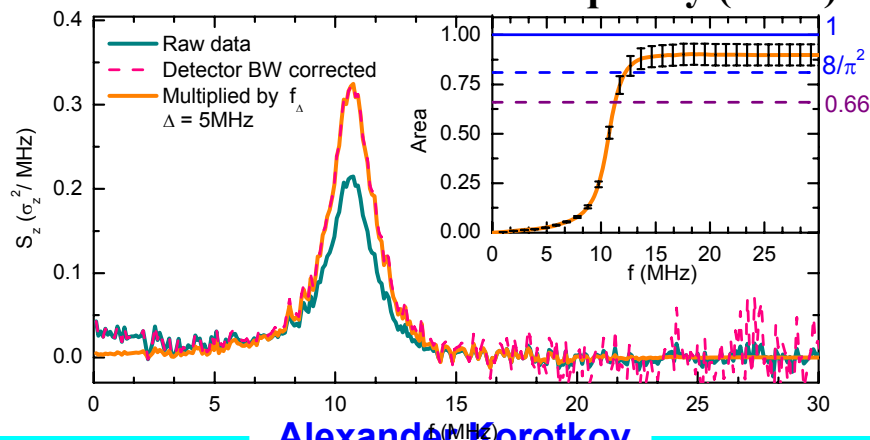
Recent experiment (Saclay group, unpub.)

Stripline resonator



A. Palacios-Laloy et al.
(unpublished)

courtesy of
Patrice Bertet



- superconducting charge qubit (transmon) in circuit QED setup (microwave reflection from cavity)
- driven Rabi oscillations
- perfect spectral peaks
- LGI violation (both K and S)



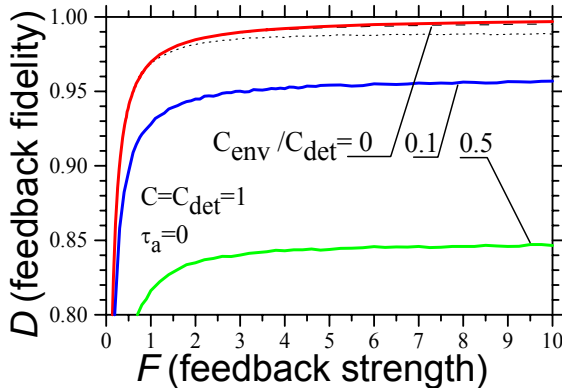
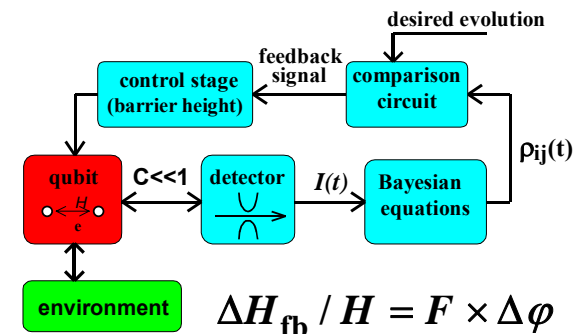
Next step: quantum feedback? Useful?

Goal: persistent Rabi oscillations with zero linewidth (synchronized)

Types of quantum feedback:

Bayesian

Best but very difficult
(monitor quantum state and control deviation)

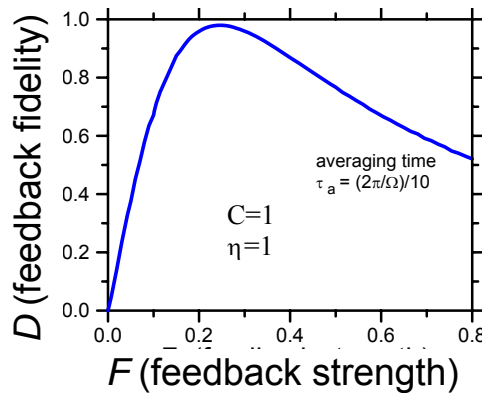


Ruskov & A.K., 2002

Direct

a la Wiseman-Milburn
(1993)
(apply measurement signal to control with minimal processing)

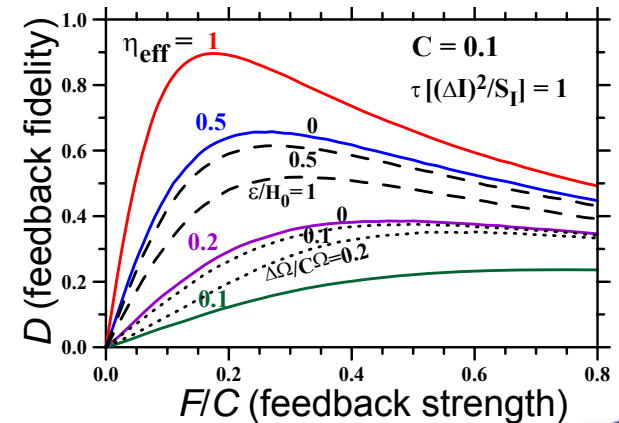
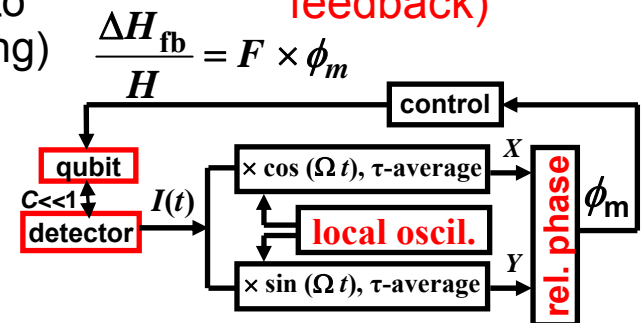
$$\frac{\Delta H_{\text{fb}}}{H} = F \sin(\Omega t) \times \left(\frac{I(t) - I_0}{\Delta I / 2} - \cos \Omega t \right)$$



Ruskov & A.K., 2002

“Simple”

Imperfect but simple
(do as in usual classical feedback)



A.K., 2005



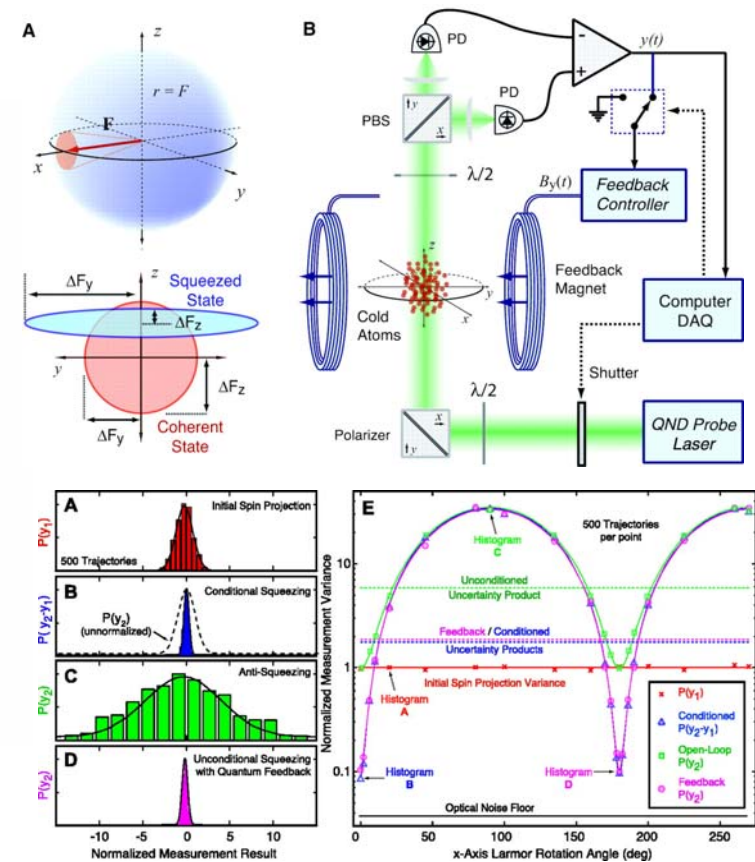
Quantum feedback in optics

First experiment: Science 304, 270 (2004)

Real-Time Quantum Feedback Control of Atomic Spin-Squeezing

JM Geremia,* John K. Stockton, Hideo Mabuchi

Real-time feedback performed during a quantum nondemolition measurement of atomic spin-angular momentum allowed us to influence the quantum statistics of the measurement outcome. We showed that it is possible to harness measurement backaction as a form of actuation in quantum control, and thus we describe a valuable tool for quantum information science. Our feedback-mediated procedure generates spin-squeezing, for which the reduction in quantum uncertainty and resulting atomic entanglement are not conditioned on the measurement outcome.



First detailed theory:

H.M. Wiseman and G. J. Milburn,
Phys. Rev. Lett. 70, 548 (1993)



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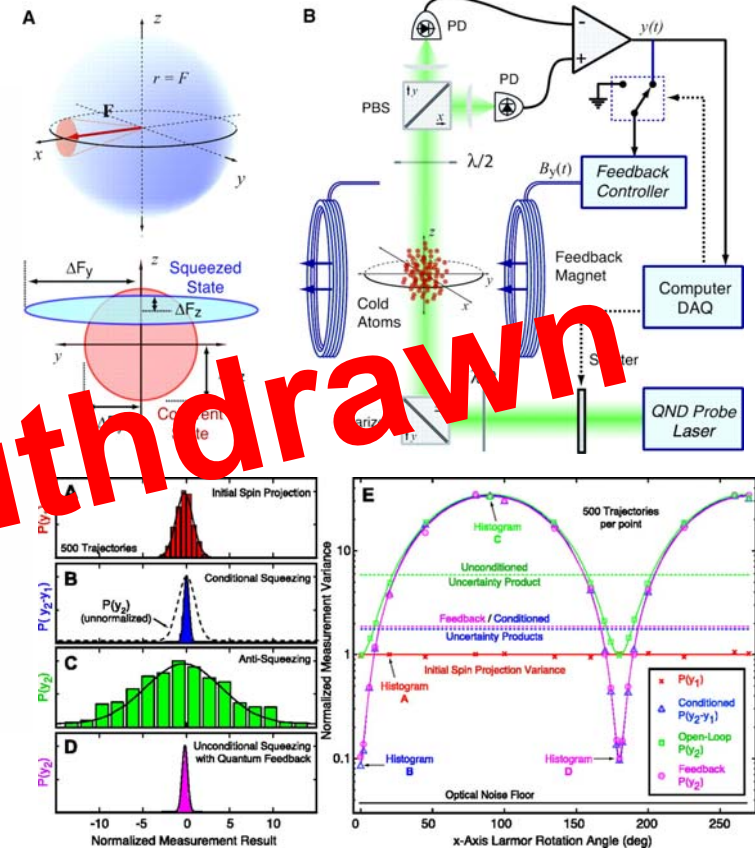
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PRL 94, 203002 (2005) also withdrawn

First detailed theory:

H.M. Wiseman and G. J. Milburn,
Phys. Rev. Lett. 70, 548 (1993)



Recent experiment:

Cook, Martin, Geremia,
Nature 446, 774 (2007)
(coherent state discrimination)



Undoing a weak measurement of a qubit ("uncollapse")

A.K. & Jordan, PRL-2006

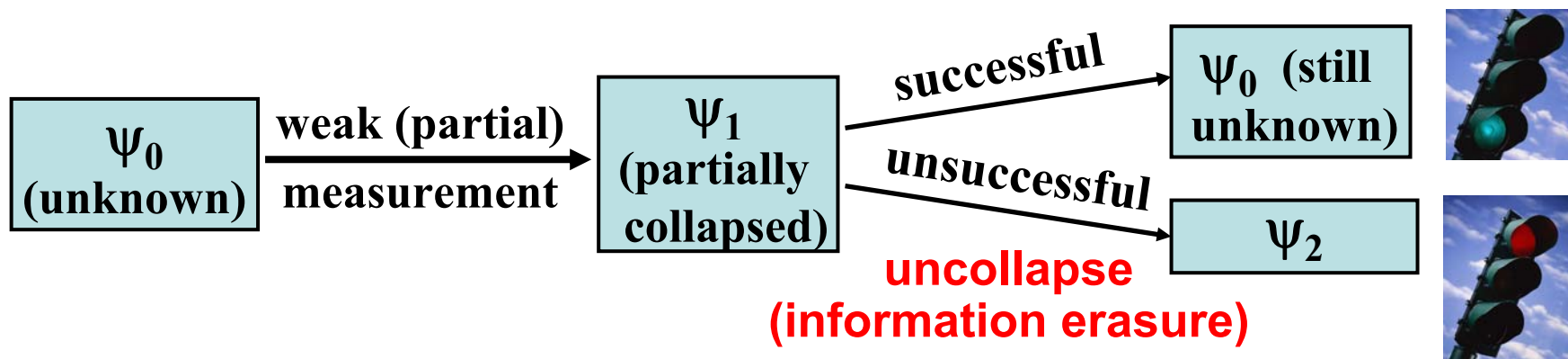


It is impossible to undo "orthodox" quantum measurement (for an unknown initial state)

Is it possible to undo partial quantum measurement?
(To restore a "precious" qubit accidentally measured)

Yes! (but with a finite probability)

If undoing is successful, an unknown state is **fully** restored



Quantum erasers in optics

Quantum eraser proposal by Scully and Drühl, PRA (1982)

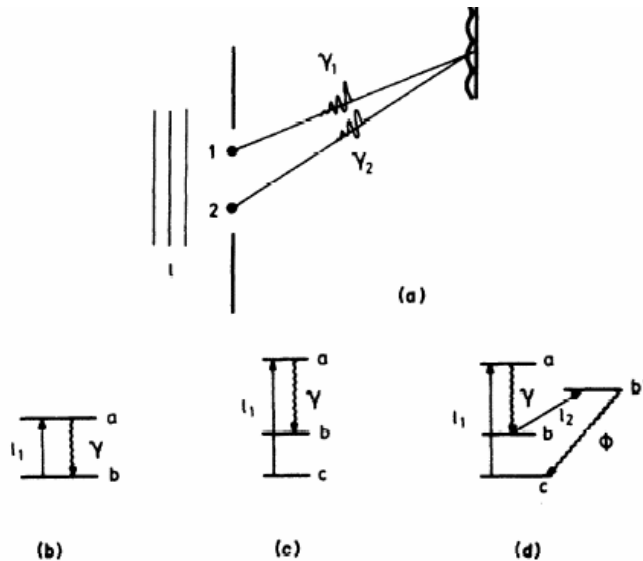


FIG. 1. (a) Figure depicting light impinging from left on atoms at sites 1 and 2. Scattered photons γ_1 and γ_2 produce interference pattern on screen. (b) Two-level atoms excited by laser pulse l_1 , and emit γ photons in $a \rightarrow b$ transition. (c) Three-level atoms excited by pulse l_1 from $c \rightarrow a$ and emit photons in $a \rightarrow b$ transition. (d) Four-level system excited by pulse l_1 from $c \rightarrow a$ followed by emission of γ photons in $a \rightarrow b$ transition. Second pulse l_2 takes atoms from $b \rightarrow b'$. Decay from $b' \rightarrow c$ results in emission of ϕ photons.

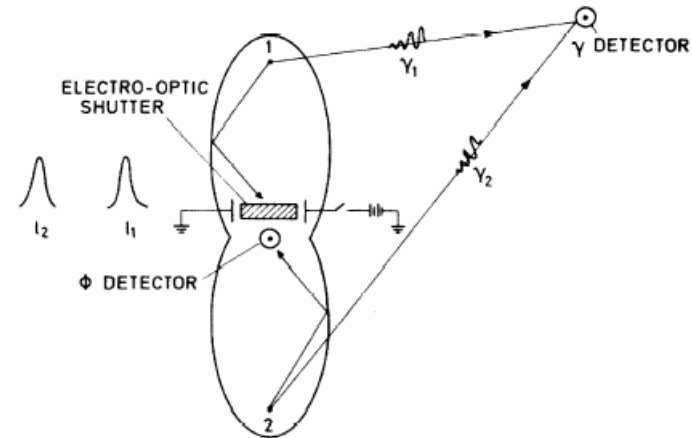


FIG. 2. Laser pulses l_1 and l_2 incident on atoms at sites 1 and 2. Scattered photons γ_1 and γ_2 result from $a \rightarrow b$ transition. Decay of atoms from $b' \rightarrow c$ results in ϕ photon emission. Elliptical cavities reflect ϕ photons onto common photodetector. Electro-optic shutter transmits ϕ photons only when switch is open. Choice of switch position determines whether we emphasize particle or wave nature of γ photons.

Interference fringes restored for two-detector correlations (since "which-path" information is erased)

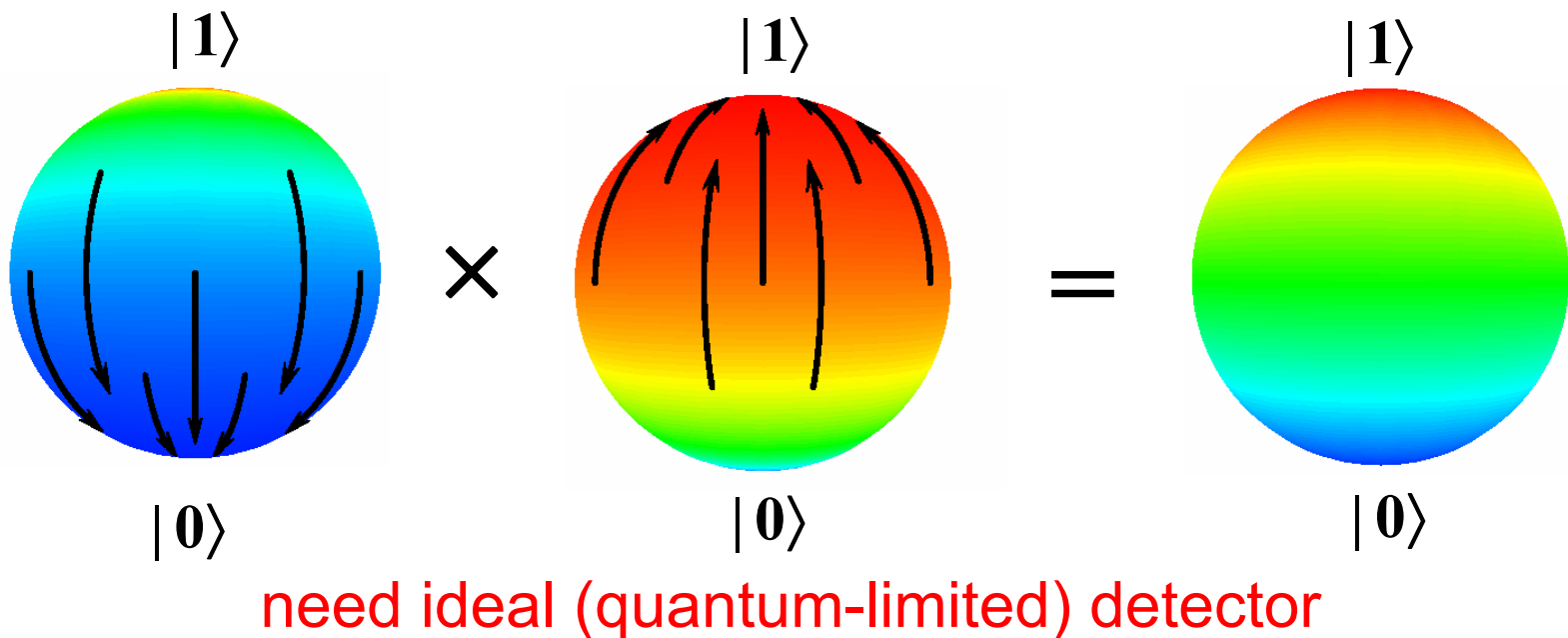
Our idea of uncollapsing is quite different:
 we really extract quantum information and then erase it



Uncollapse of a qubit state

Evolution due to partial (weak, continuous, etc.) measurement is **non-unitary**, so impossible to undo it by Hamiltonian dynamics.

How to undo? One more measurement!



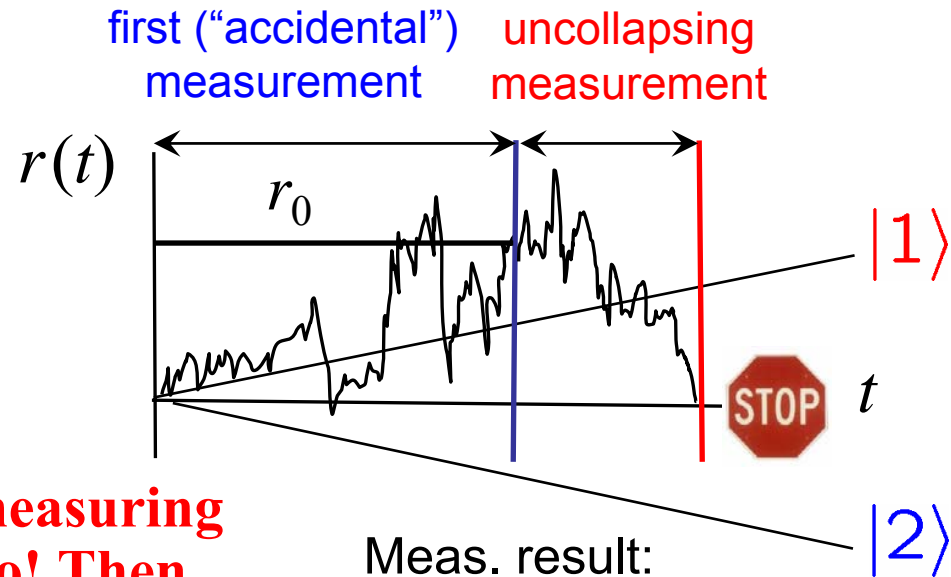
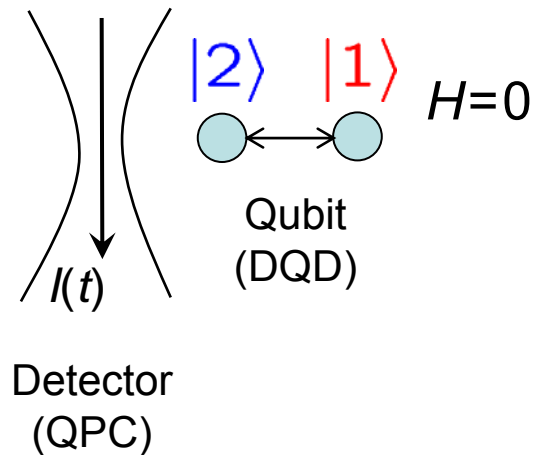
(similar to Koashi-Ueda, PRL-1999,
also Nielsen-Caves-1997, Royer-1994, etc.)

(Figure partially adopted from
Jordan-A.K.-Büttiker, PRL-06)



Uncollapsing for DQD-QPC system

A.K. & Jordan, 2006



Simple strategy: continue measuring until result $r(t)$ becomes zero! Then any initial state is fully restored.

(same for an entangled qubit)

However, if $r = 0$ never happens, then uncollapsing procedure is unsuccessful.

Meas. result:

$$r(t) = \frac{\Delta I}{S_I} \left[\int_0^t I(t') dt' - I_0 t \right]$$

If $r = 0$, then no information and no evolution!

Probability of success:

$$P_S = \frac{e^{-|r_0|}}{e^{|r_0|} \rho_{11}(0) + e^{-|r_0|} \rho_{22}(0)}$$



General theory of uncollapsing

POVM formalism
(Nielsen-Chuang, p.100)

Measurement operator M_r : $\rho \rightarrow \frac{M_r \rho M_r^\dagger}{\text{Tr}(M_r \rho M_r^\dagger)}$

Probability: $P_r = \text{Tr}(M_r \rho M_r^\dagger)$ Completeness: $\sum_r M_r^\dagger M_r = 1$

Uncollapsing operator: $C \times M_r^{-1}$ (to satisfy completeness, eigenvalues cannot be >1)

$\max(C) = \min_i \sqrt{p_i}$, p_i – eigenvalues of $M_r^\dagger M_r$

Probability of success:

$$P_s \leq \frac{\min P_r}{P_r(\rho_{\text{in}})}$$

A.K. & Jordan, 2006

$P_r(\rho_{\text{in}})$ – probability of result r for initial state ρ_{in} ,

$\min P_r$ – probability of result r minimized over all possible initial states

Averaged (over r) probability of success: $P_{av} \leq \sum_r \min P_r$

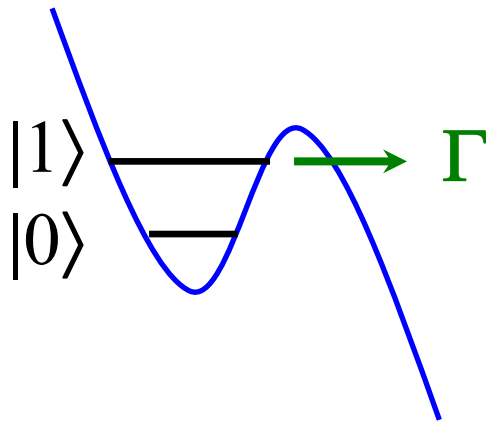
(cannot depend on initial state, otherwise get information)

(similar to Koashi-Ueda, 1999)



Partial collapse of a Josephson phase qubit

N. Katz, M. Ansmann, R. Bialczak, E. Lucero,
R. McDermott, M. Neeley, M. Steffen, E. Weig,
A. Cleland, J. Martinis, A. Korotkov, Science-06



**How does a qubit state evolve
in time before tunneling event?**

(What happens when nothing happens?)

Main idea:

$$\psi = \alpha |0\rangle + \beta |1\rangle \rightarrow \psi(t) = \begin{cases} |out\rangle, & \text{if tunneled} \\ \frac{\alpha |0\rangle + \beta e^{-\Gamma t/2} e^{i\varphi} |1\rangle}{\sqrt{|\alpha|^2 + |\beta|^2} e^{-\Gamma t}}, & \text{if not tunneled} \end{cases}$$

(better theory: Pryadko & A.K., 2007)

amplitude of state $|0\rangle$ grows without physical interaction

finite linewidth only after tunneling

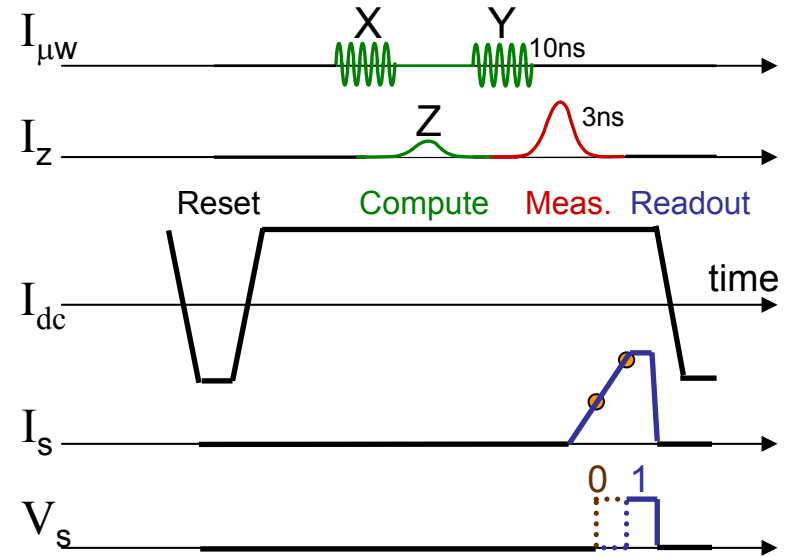
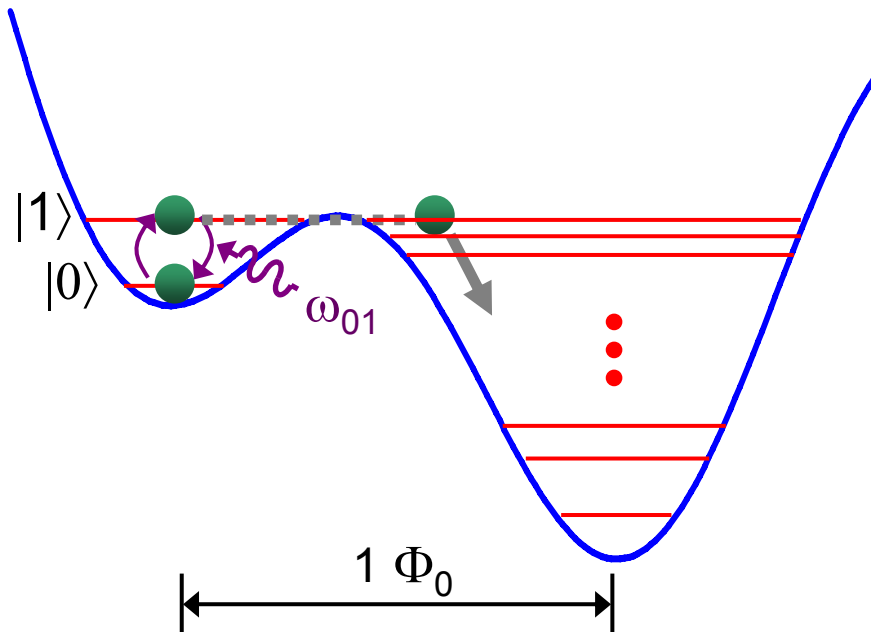
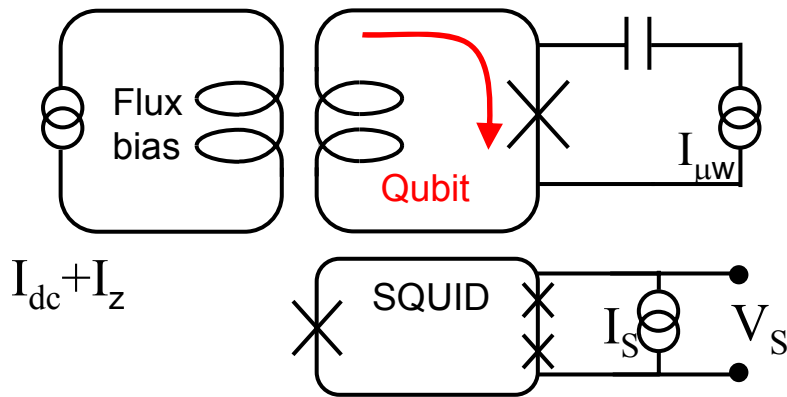
continuous null-result collapse

(similar to optics, Dalibard-Castin-Molmer, PRL-1992)

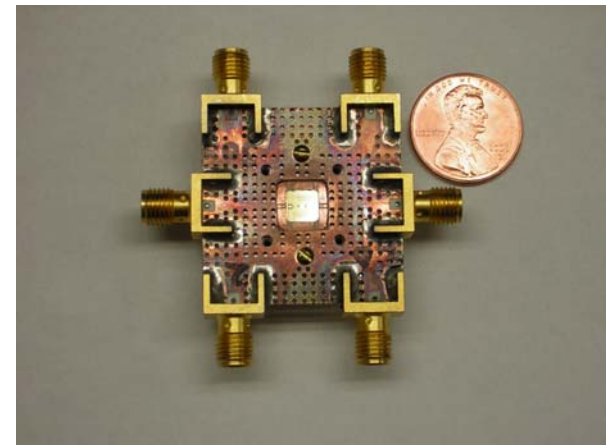


Superconducting phase qubit at UCSB

Courtesy of Nadav Katz (UCSB)



Repeat 1000x
prob. 0,1



Experimental technique for partial collapse

Nadav Katz *et al.*
(John Martinis group)

Protocol:

- 1) State preparation
(via Rabi oscillations)
- 2) Partial measurement by
lowering barrier for time t
- 3) State tomography (micro-
wave + full measurement)

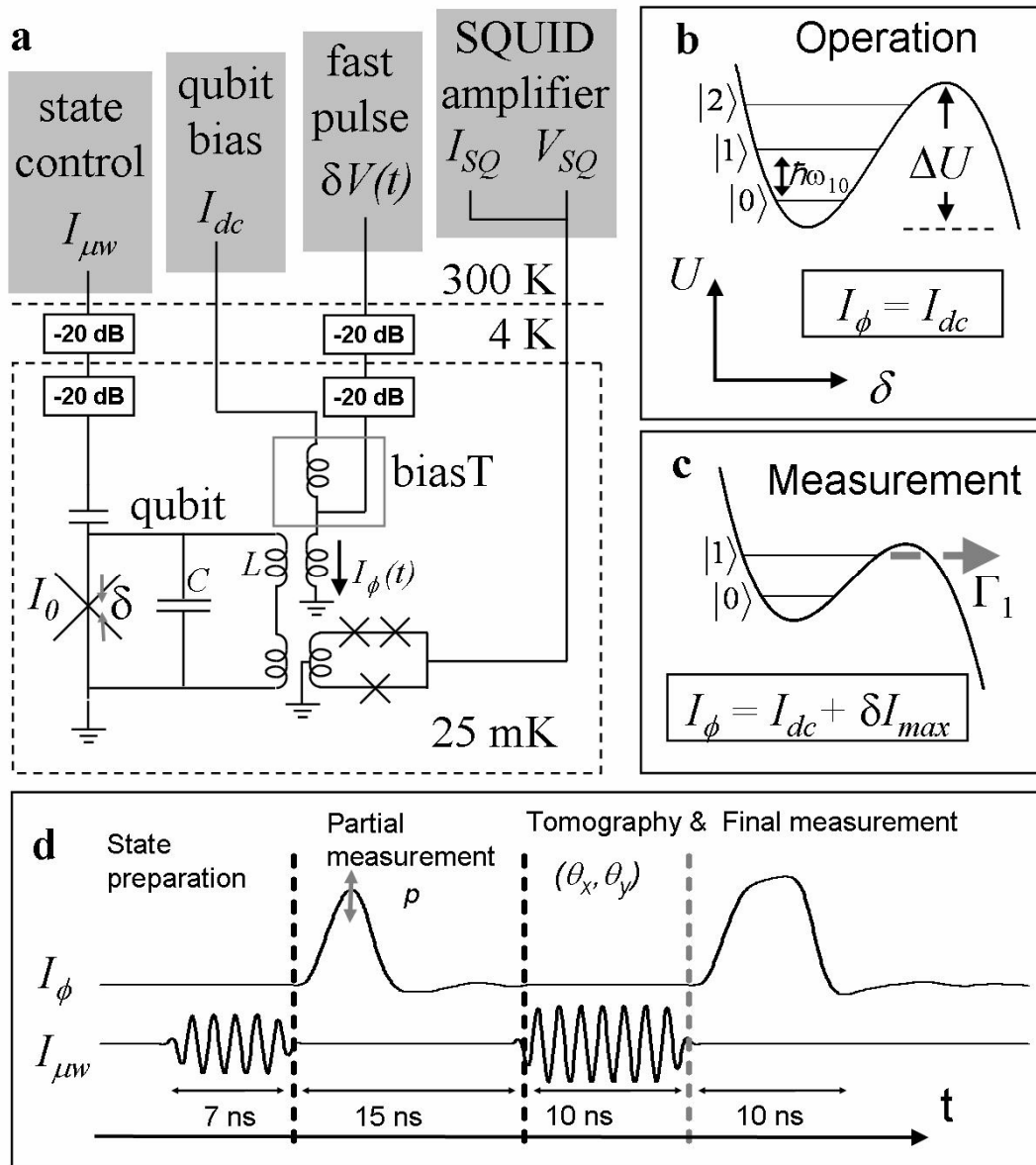
Measurement strength

$$p = 1 - \exp(-\Gamma t)$$

is actually controlled
by Γ , not by t

$p=0$: no measurement

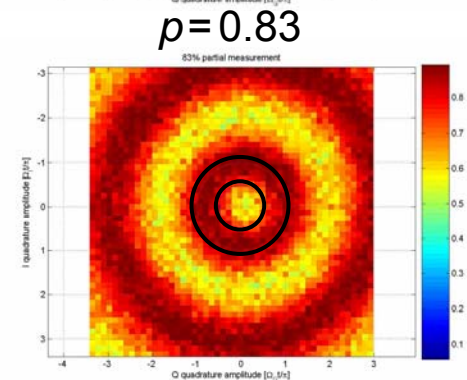
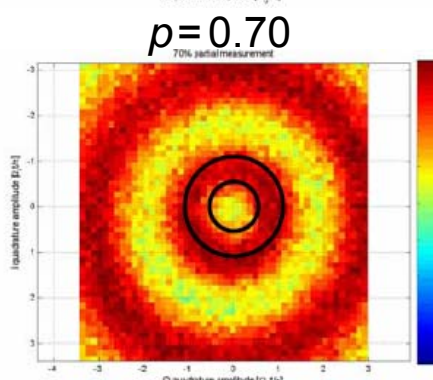
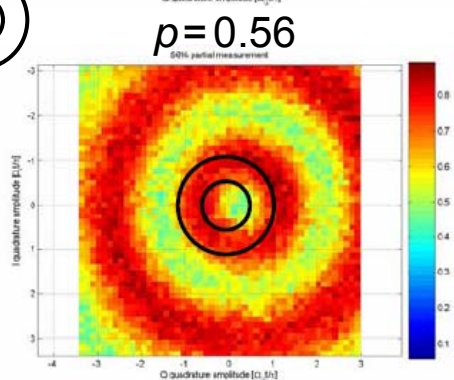
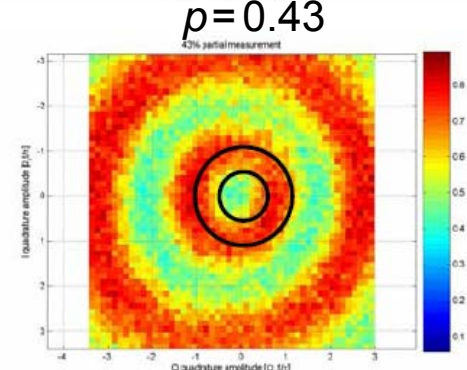
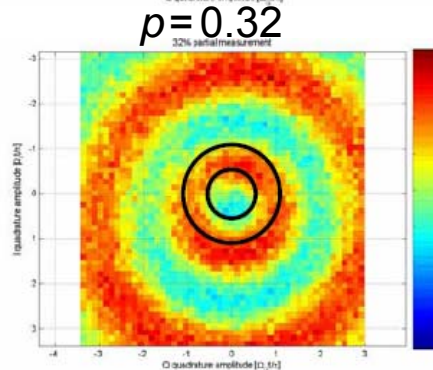
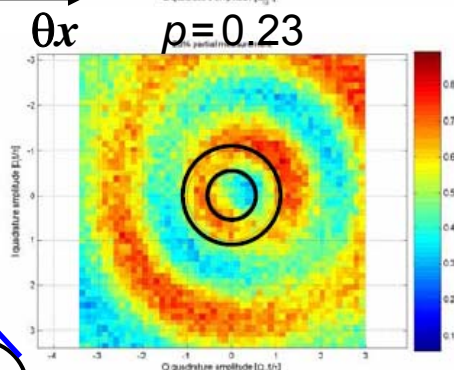
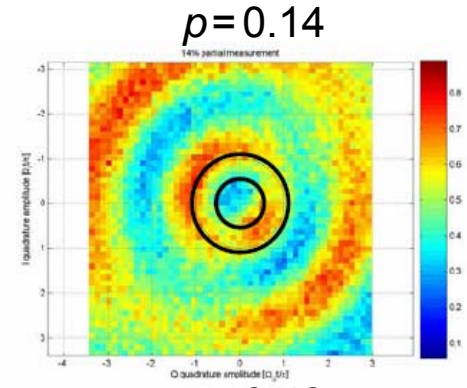
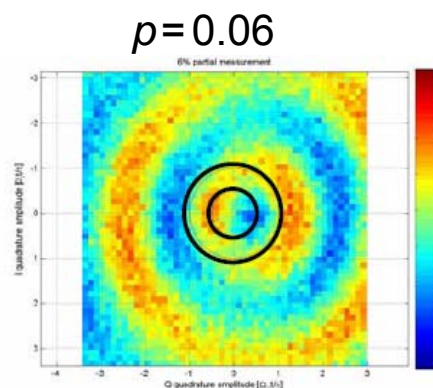
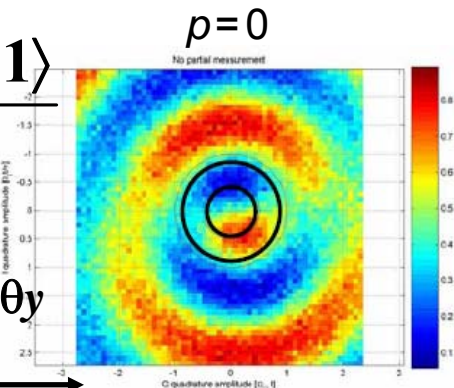
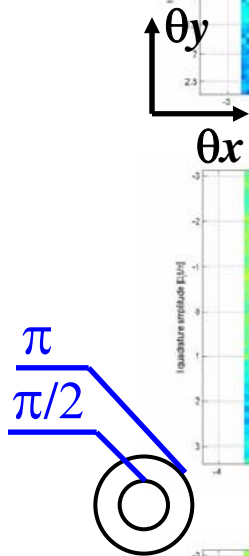
$p=1$: orthodox collapse



Experimental tomography data

Nadav Katz *et al.* (UCSB, 2005)

$$\psi_{in} = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$



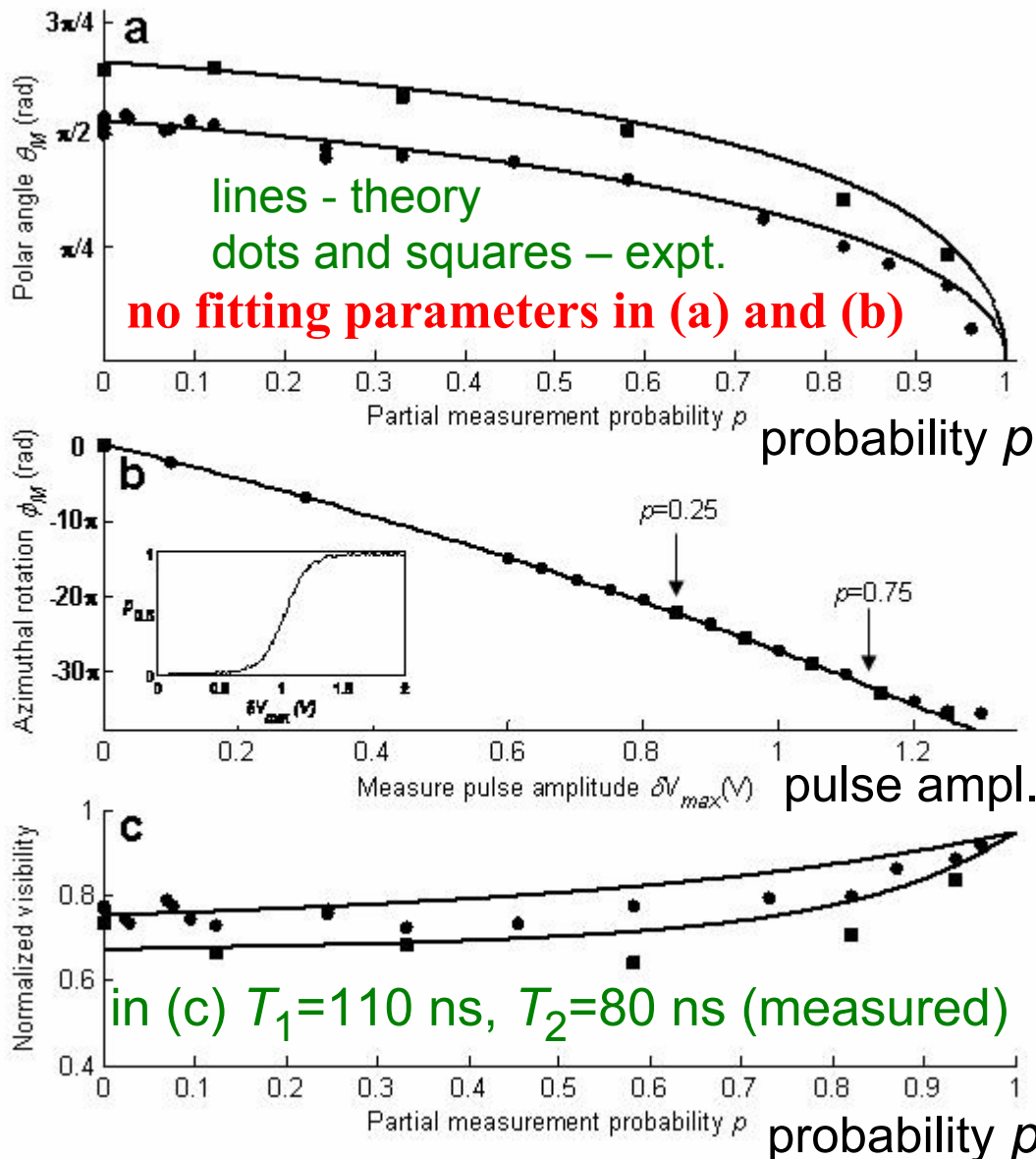
Partial collapse: experimental results

N. Katz *et al.*, Science-06

Polar angle

Azimuthal angle

Visibility



- In case of no tunneling phase qubit evolves
- Evolution is described by the Bayesian theory without fitting parameters
- Phase qubit remains coherent in the process of continuous collapse (expt. ~80% raw data, ~96% corrected for T_1, T_2)

quantum efficiency
 $\eta_0 > 0.8$

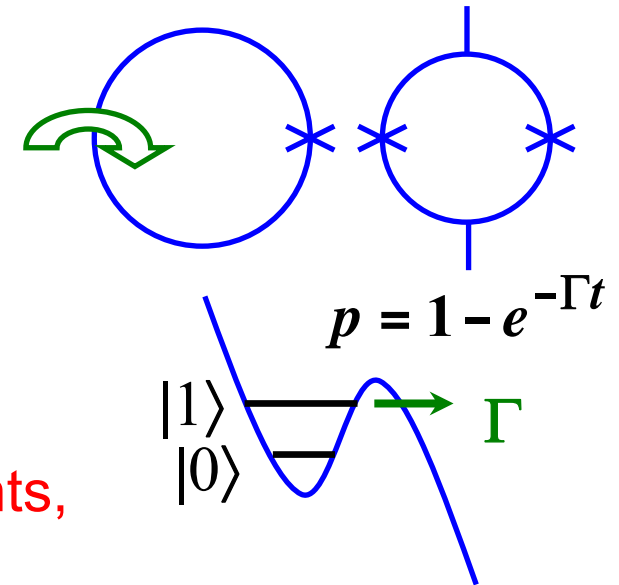


Uncollapse of a phase qubit state

A.K. & Jordan, 2006

- 1) Start with an unknown state
- 2) Partial measurement of strength p
- 3) π -pulse (exchange $|0\rangle \leftrightarrow |1\rangle$)
- 4) One more measurement with the **same strength p**
- 5) π -pulse

If no tunneling for both measurements,
then initial state is fully restored!



$$\alpha |0\rangle + \beta |1\rangle \rightarrow \frac{\alpha |0\rangle + e^{i\phi} \beta e^{-\Gamma t/2} |1\rangle}{\text{Norm}} \rightarrow$$

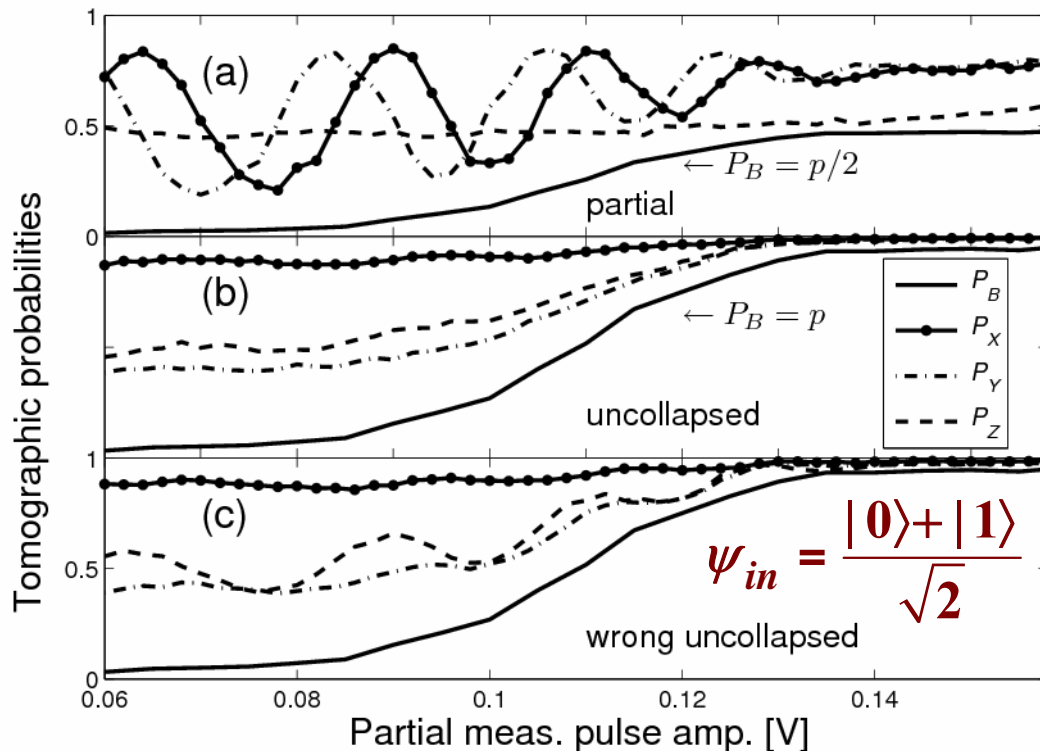
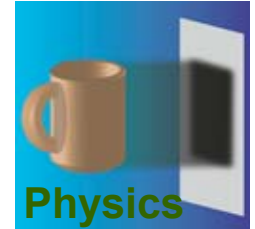
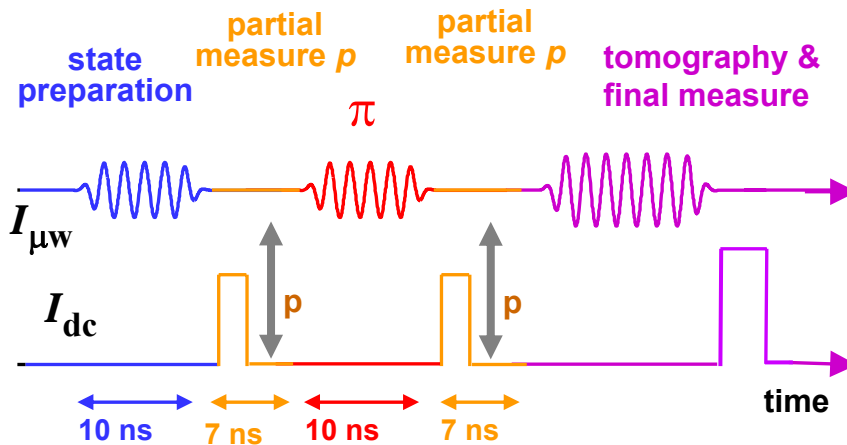
$$\frac{e^{i\phi} \alpha e^{-\Gamma t/2} |0\rangle + e^{i\phi} \beta e^{-\Gamma t/2} |1\rangle}{\text{Norm}} = e^{i\phi} (\alpha |0\rangle + \beta |1\rangle)$$

phase is also restored (spin echo)



Experiment on wavefunction uncollapse

N. Katz, M. Neeley, M. Ansmann, R. Bialzak, E. Lucero, A. O'Connell, H. Wang, A. Cleland, J. Martinis, and A. Korotkov, PRL-2008



Uncollapse protocol:

- partial collapse
- π -pulse
- partial collapse (same strength)

State tomography with X , Y , and no pulses

Background P_B should be subtracted to find qubit density matrix

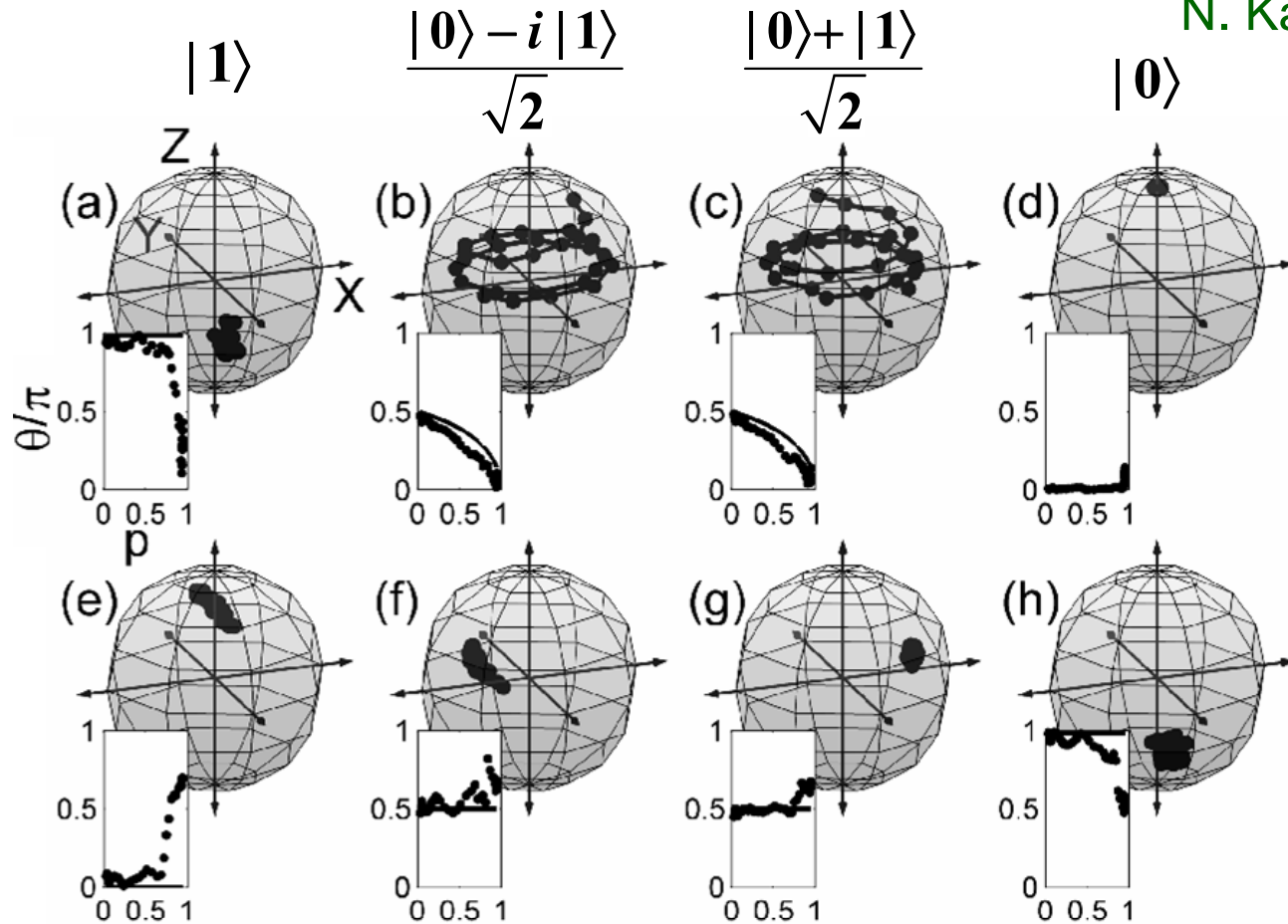


Experimental results on the Bloch sphere

N. Katz et al.

Initial
state

Partially
collapsed



Uncollapsed

uncollapsing
works well!

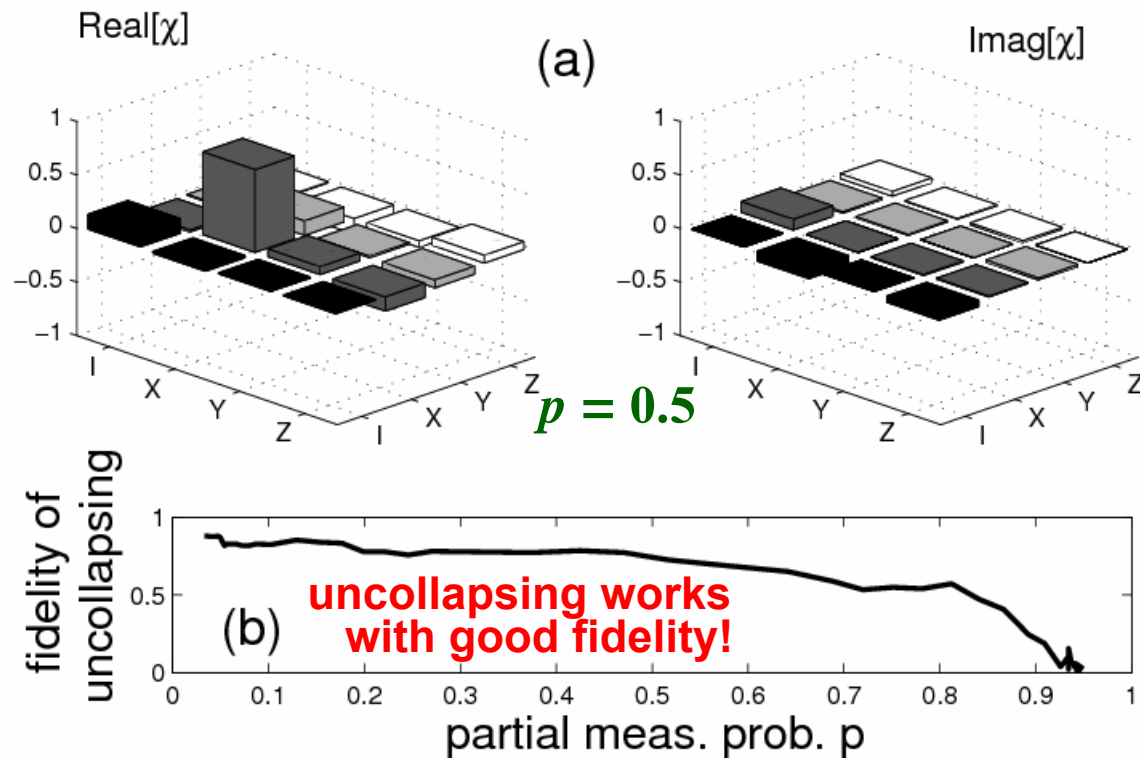
Both spin echo (azimuth) and uncollapsing (polar angle)

Difference: spin echo – undoing of an unknown unitary evolution,
uncollapsing – undoing of a known, but non-unitary evolution



Quantum process tomography

N. Katz et al.
(Martinis group)



Why getting worse at $p > 0.6$?

Energy relaxation $p_r = t/T_1 = 45\text{ns}/450\text{ns} = 0.1$

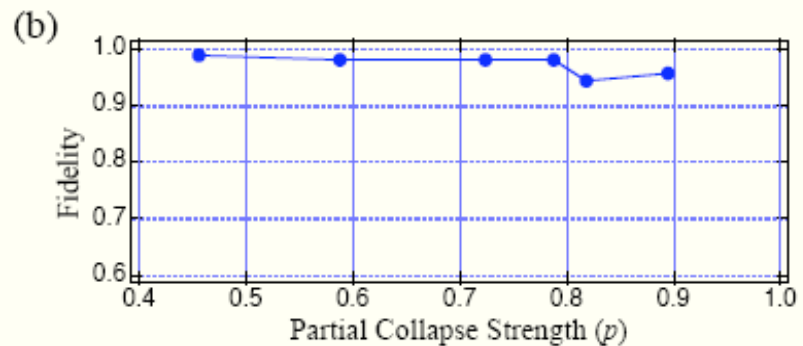
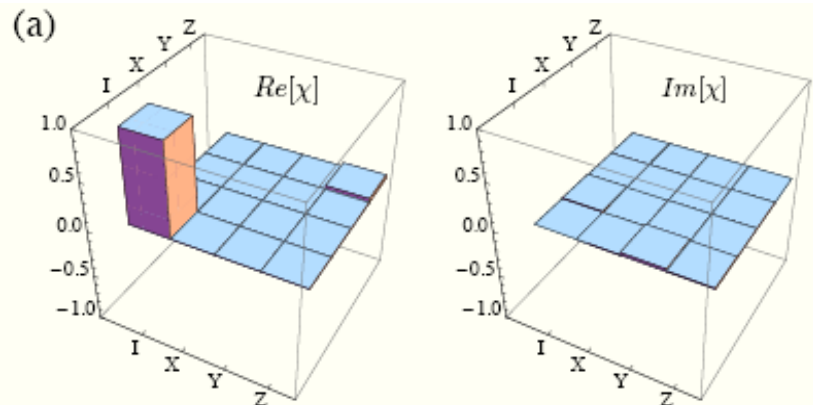
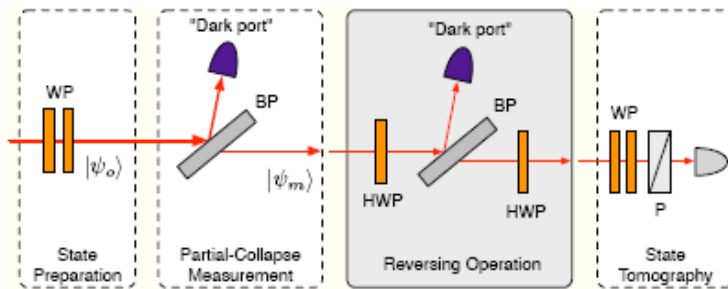
Selection affected when $1-p \sim p_r$

Overall: uncollapsing is well-confirmed experimentally



Recent experiment on uncollapsing using single photons

Kim et al., Opt. Expr.-2009

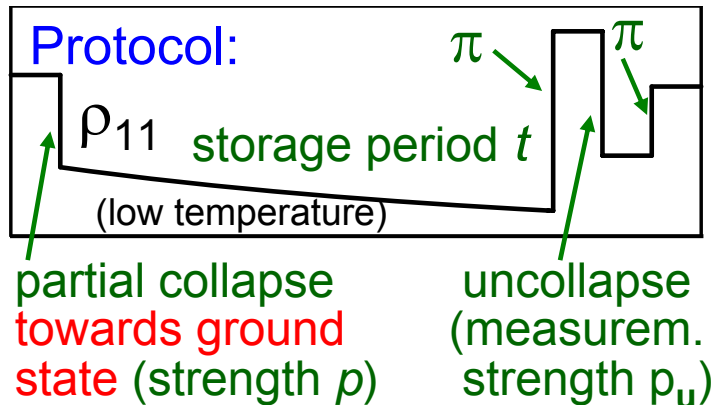


- very good fidelity of uncollapsing ($>94\%$)
- measurement fidelity is probably not good (normalization by coincidence counts)



Suppression of T_1 -decoherence by uncollapsing

Korotkov & Keane,
arXiv:0908.1134



best for $1 - p_u = (1 - p) \exp(-t/T_1)$

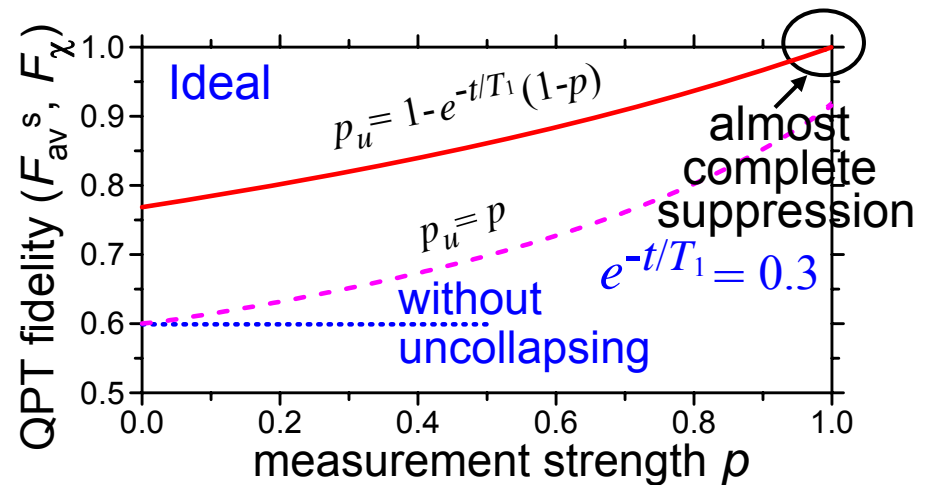
Ideal case (T_1 during storage only)

for initial state $|\psi_{\text{in}}\rangle = \alpha |0\rangle + \beta |1\rangle$

$|\psi_f\rangle = |\psi_{\text{in}}\rangle$ with probability $(1-p) e^{-t/T_1}$

$|\psi_f\rangle = |0\rangle$ with $(1-p)^2 |\beta|^2 e^{-t/T_1} (1 - e^{-t/T_1})$

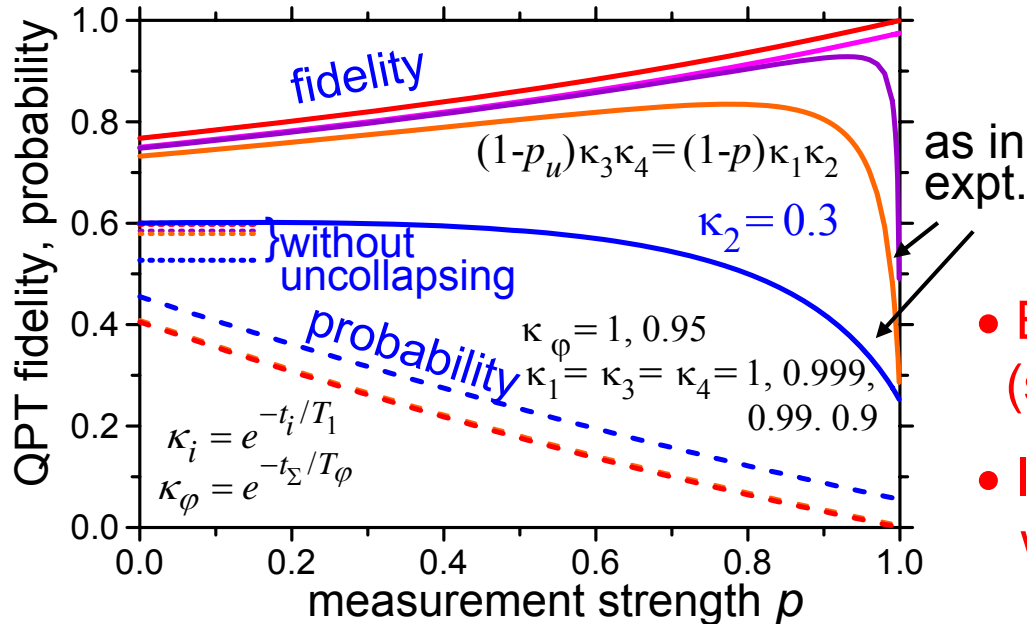
procedure preferentially selects
events without energy decay



Trade-off: fidelity vs. probability



Realistic case (T_1 and T_ϕ at all stages)



- Easy to realize experimentally (similar to existing experiment)
- Improved fidelity can be observed with just one partial measurement

- T_ϕ -decoherence is not affected
- fidelity decreases at $p \rightarrow 1$ due to T_1 between 1st π -pulse and 2nd meas.

Uncollapse seems **the only way** to protect against T_1 -decoherence without quantum error correction

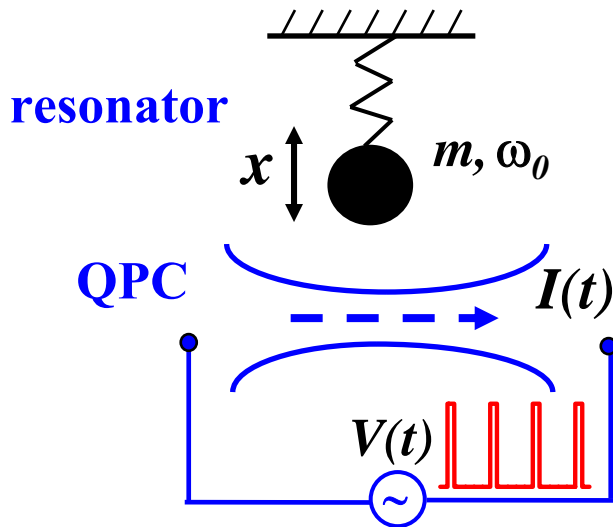
Trade-off: fidelity vs. selection probability

A.K. & Keane,
arXiv:0908.1134



Stroboscopic QND squeezing of a nanoresonator

Ruskov, Schwab, A.K., 2005

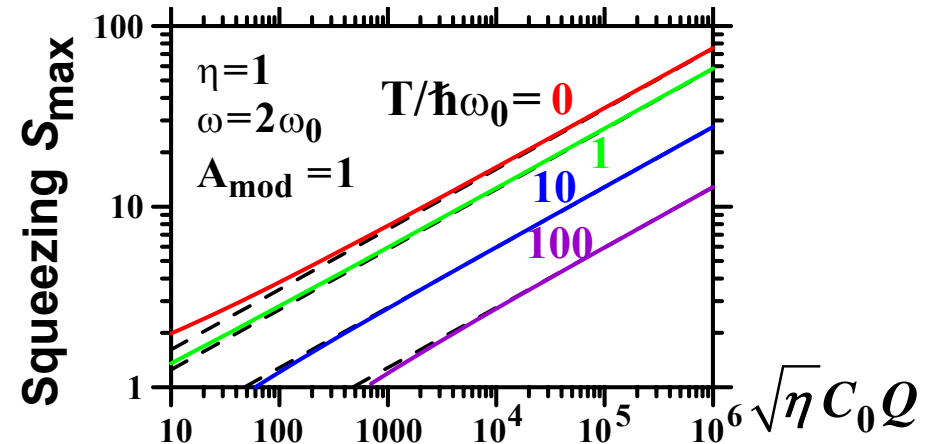
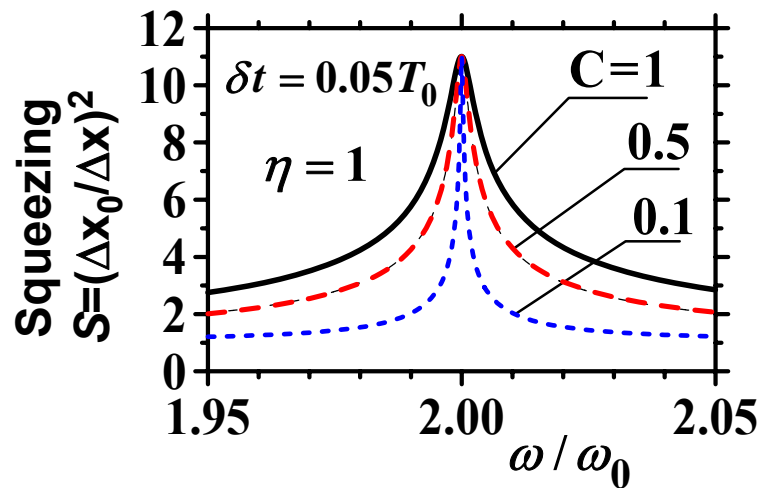


Based on old (1978) Braginsky-Khalili-Thorne idea
Difference: weak measurement, quantum feedback

$$\frac{x(t_1) \text{ SQL } x(t_2)}{\Delta p > \hbar / 2 \Delta x}$$

$$S_{\max} = \frac{3}{4} \left[\frac{\sqrt{\eta} C_0 Q}{\coth(\hbar \omega_0 / 2T)} \right]^{1/3}$$

C_0 – coupling with detector, η – detector efficiency,
 T – temperature, Q – resonator Q-factor



Beats the Standard Quantum Limit

Potential application: ultrasensitive force measurements



Conclusions

- It is easy to see what is “inside” collapse: simple Bayesian formalism works for many solid-state setups
- Evolution is time-reversible (need to flip H_{QB} and meas. result), but backward evolution is less probable
- Rabi oscillations are persistent if weakly measured
- Collapse can sometimes be undone (uncollapsing)
- Three direct solid-state experiments have been realized, many interesting experimental proposals are still waiting
- Weak (continuous, partial) measurement may be useful

