Quantum limits in measurement of superconducting qubits

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Outline:
- Current limitations for phase qubits
- Quantum limits
  - binary-output detector
  - broadband linear detector
  - narrowband linear detector

Main current subjects of study

(PI: John Martinis)

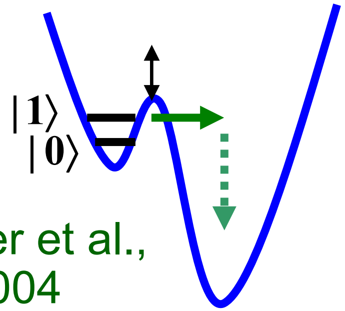
- Tunable coupling of phase qubits
- Quantum error detection/correction for phase qubits

(see posters by Kyle Keane
and by Ricardo Pinto)

Types of measurement limitations

- Technical limitations/problems
  (gradual improvement possible)
- Theoretical limitations for a particular measurement
  method (solution: use proper parameters and/or
  a better method)
- Fundamental quantum limits (no solution,
  but not much of practical limitations)

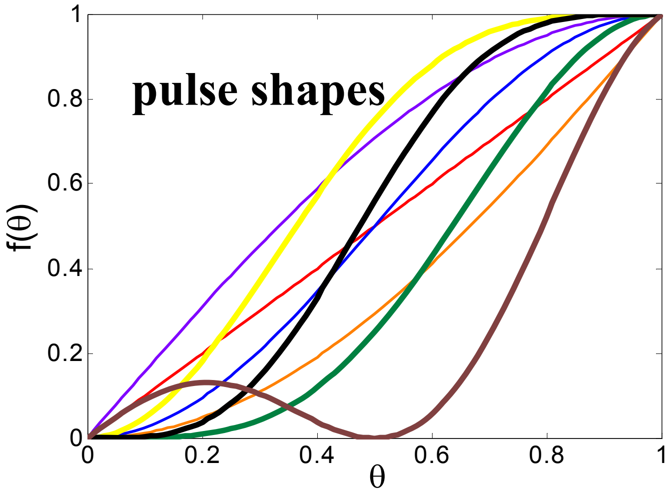
Example of limitations for a particular method (tunneling measurement of phase qubits)

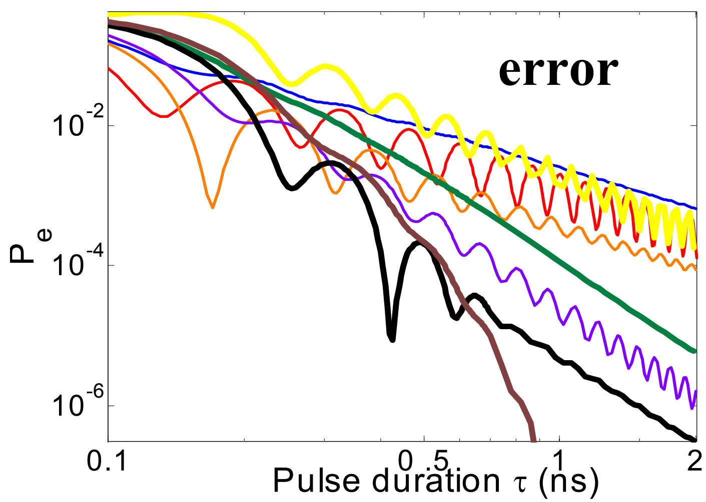


Cooper et al., 2004

Several sources for errors:

Non-adiabatic error

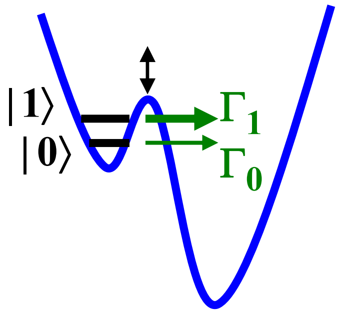


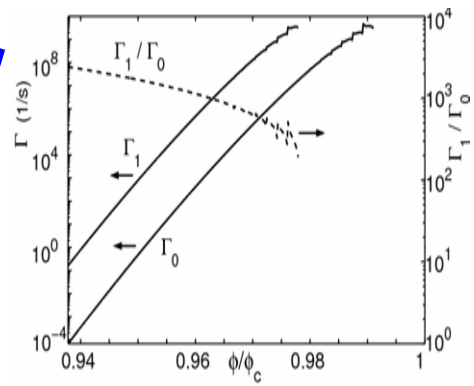


Zhang et al., 2006

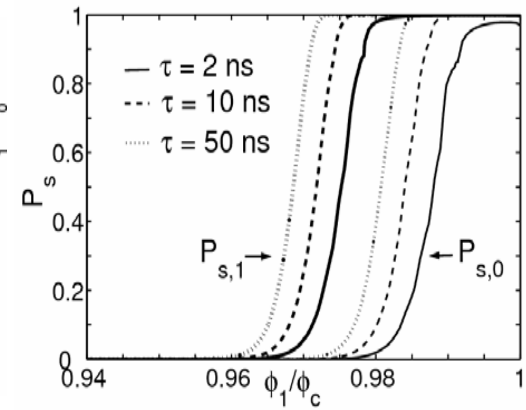
Solution: avoid too short pulses (<2 ns),
use proper pulse shape (currently Slepian)

Discrimination error

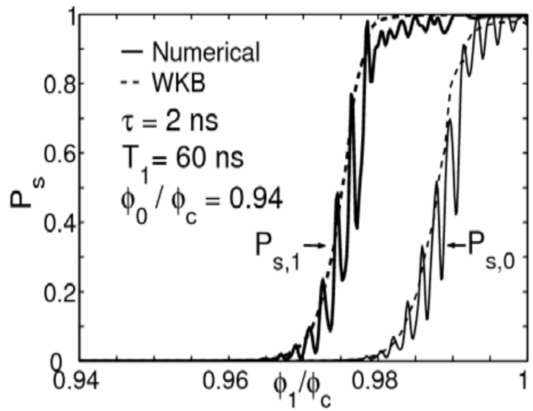




Theoretical S-curves



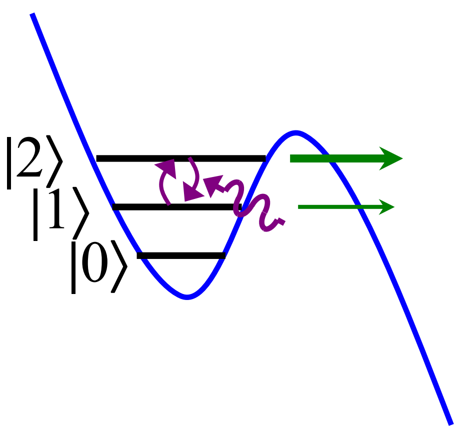
Zhang et al.



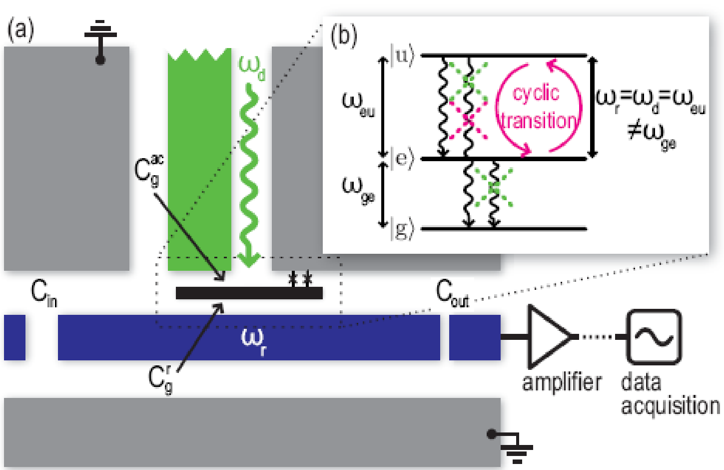
$$\text{Error} \simeq (\Gamma_0/\Gamma_1)[1+\ln(\Gamma_1/\Gamma_0)]$$

error ~3% at the optimal point

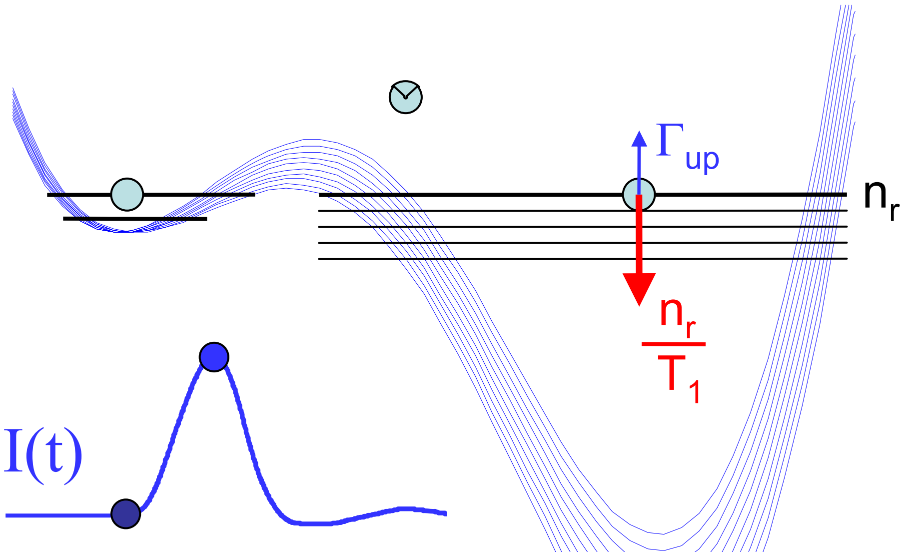
Solutions: use next level, use non-tunneling measurement (dispersive or resonant)



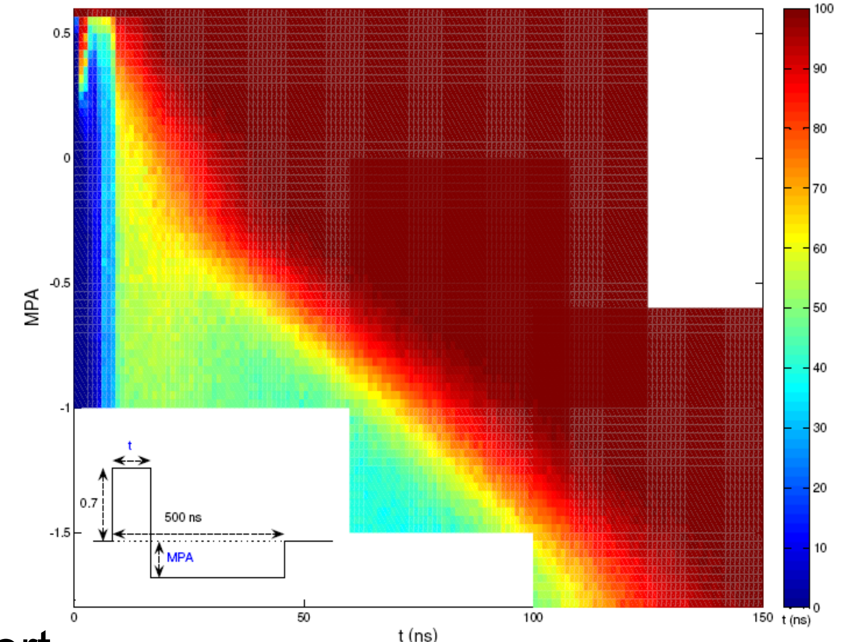
Mesoscopic shelving
Englert et al., arXiv:0904.1769



Repopulation error (Zhang et al.)



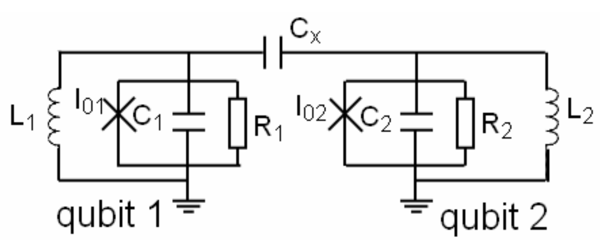
animation by Martinis

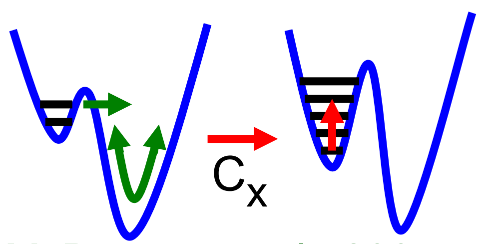


Solutions: proper pulse shape, not too short

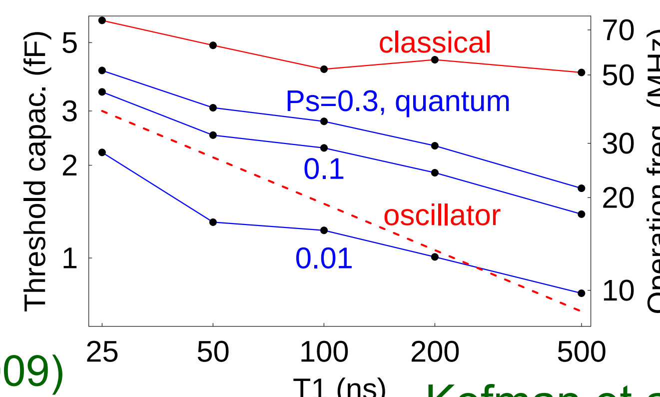
Haohua Wang et al. (unpub)

Cross-talk error





McDermott et al., 2005



Kofman et al.

Solutions: resonator in between (Ansmann et al., 2009)

tunable coupling (Bialczak et al., 2010)

No unsolvable problems in measurement of phase qubits

Quantum limits for qubit measurement

quantum limits = limits due to quantum back-action

quantum backaction = informational back-action

(unexplainable, no mechanism, Bayes, non-unitary)

QM allows fast projective measurement of a qubit
$\Rightarrow$ no quantum limits fundamentally hurting QC

However, quantum limits are within reach in SC qubit measurement (we start seeing informational back-action). They have importance for QC (secondary, not primary), surely important for more general QI and other fields, and very interesting fundamentally.

Types of measurement (terminology)

- destructive, non-destructive, half-destructive
- invasive, non-invasive (confusing?)
- linear, switching/bifurcation
- QND, non-QND
  $|1\rangle \to |1\rangle$, $|0\rangle \to |0\rangle$ (repeatability of results)
  commutes with Hamiltonian $\Rightarrow$ measurement of energy
  (so comp. basis is energy) or a trick (stroboscopic, etc.)
  why care? easier (longer), useful for reinitialization
- single-shot or not (single-shot required for a QC)
  switching or linear with reasonable SNR

Characterization

- fidelity: $F_0 = p$(“0”, if $|0\rangle$), $F_1 = p$(“1”, if $|1\rangle$); most important for QC
- quantum efficiency (var. defs.; related to post-measured state)
- many other characteristics

General characterization of a non-destructive binary-output (single-shot) qubit detector

general POVM (superoperator) for each result:

16 +16 – 4 = **28** real parameters to describe (too many!)

28 = 2 (meas. axis) + 2 (fidelity) + 2×3 (unitary) + 2×9 (decoherence)

Simplifications:

1) Textbook projective only 2 parameters (meas. axis)

2) Perfect fidelity $F_0=F_1=1$; then only meas. axis is interesting (6 more parameters affect only reinitialization)

3) QND $|0\rangle\rightarrow|0\rangle$, $|1\rangle\rightarrow|1\rangle$; then 6 parameters

QND binary-output detector

A.K., 2008

6 parameters: fidelity (F_0, F_1), decoherence (D_0, D_1), and phases (ϕ_0, ϕ_1)

result 0: $$\begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix} \to \frac{1}{P_0} \begin{pmatrix} F_0 \rho_{00} & \sqrt{F_0(1-F_1)}\, e^{-D_0} e^{i\phi_0} \rho_{01} \\ c.c. & (1-F_1)\rho_{11} \end{pmatrix}$$

result 1: $$\begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix} \to \frac{1}{P_1} \begin{pmatrix} (1-F_0) \rho_{00} & \sqrt{(1-F_0)F_1}\, e^{-D_1} e^{i\phi_1} \rho_{01} \\ c.c. & F_1\rho_{11} \end{pmatrix}$$

(simple Bayes)

$$P_0 = F_0\rho_{00} + (1-F_1)\rho_{11}, \;\; P_1 = (1-F_0)\rho_{00} + F_1\rho_{11}$$

Corresponding quantum limits

result 0: $$\frac{|\rho_{01}^{\text{after}}|}{|\rho_{01}|} \le \frac{1}{P_0}\sqrt{F_0(1-F_1)}$$ **result 1:** $$\frac{|\rho_{01}^{\text{after}}|}{|\rho_{01}|} \le \frac{1}{P_1}\sqrt{F_1(1-F_0)}$$

ensemble decoherence: $$\frac{|\rho_{01}^{\text{after}}|}{|\rho_{01}|} \le \sqrt{F_0(1-F_1)} + \sqrt{(1-F_0)F_1}$$

natural to introduce quantum efficiencies by comparing with quantum limits

(easy to realize η_0=1, but difficult η_0=η_1=1)

Quantum limits for linear qubit detectors

Broadband (QPC, SET, etc.) or narrowband (cQED)

broadband, "QND"

quantum backaction (non-unitary)

$$\begin{cases} \dfrac{\rho_{11}(\tau)}{\rho_{00}(\tau)} = \dfrac{\rho_{11}(0)}{\rho_{00}(0)} \dfrac{\exp[-(\bar{I}-I_1)^2/2D]}{\exp[-(\bar{I}-I_0)^2/2D]} \\ \rho_{10}(\tau) = \rho_{10}(0) \sqrt{\dfrac{\rho_{11}(\tau)\,\rho_{00}(\tau)}{\rho_{11}(0)\,\rho_{00}(0)}} \exp(iK\bar{I}\tau)\exp(-\gamma\tau) \end{cases}$$

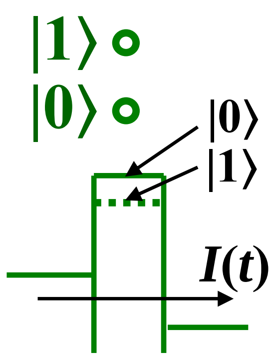
$$\bar{I} \equiv \frac{1}{\tau}\int_0^\tau I(t)\,dt$$

$$D = S_I/2\tau$$

decoherence

classical backaction (unitary)

A.K., 1998, 2000



$\Delta I = I_1 - I_0$

noise S_I

$K=\gamma=0$ for symmetric QPC

$$\Gamma = (\Delta I)^2/4S_I + \gamma'$$

ensemble decoherence rate

~ information flow [bit/s]

single-qubit decoherence

"measurement time" (S/N=1)

$$\tau_m = 2S_I/(\Delta I)^2$$

$$\Gamma\tau_m \geq \frac{1}{2}$$

A.K., 1998, 2000
Pilgram et al., 2002
Clerk et al., 2002
Averin, 2000,2003

$$\gamma' \geq 0 \;\Rightarrow\; \boxed{\Gamma \geq (\Delta I)^2/4S_I}$$

For non-zero K (classical backaction)

$$\Gamma = (\Delta I)^2/4S_I + K^2 S_I/4 + \gamma \quad \Rightarrow \quad \Gamma \geq (\Delta I)^2/4S_I + K^2 S_I/4$$

Translate into energy sensitivity (for SET; A.K.-2000)

$$\Gamma \geq (\Delta I)^2/4S_I \quad \Leftrightarrow \quad (\varepsilon_O\, \varepsilon_{BA})^{1/2} \geq \hbar/2$$

Danilov, Likharev, Zorin, 1983

$$\Gamma \geq (\Delta I)^2/4S_I + K^2 S_I/4 \quad \Leftrightarrow \quad (\varepsilon_O\, \varepsilon_{BA} - \varepsilon_{O,BA}{}^2)^{1/2} \geq \hbar/2$$

ε_O, ε_{BA}: sensitivities [J/Hz] limited by output noise and back-action

Known since 1980s (Caves, Clarke, Likharev, Zorin, Vorontsov, Khalili, etc.)

For qubit measurement these long-known quantum limits are related to the informational (Bayesian) back-action

Describing a qubit evolution due to measurement is a more appropriate language for QC

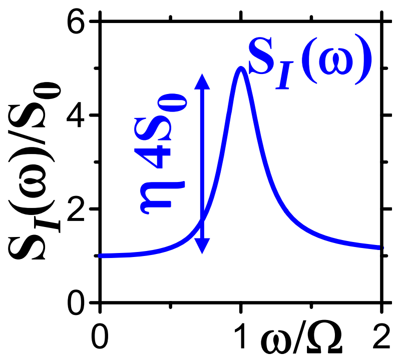
Quantum efficiency for linear qubit detection

quantum efficiency: comparison with quantum limit

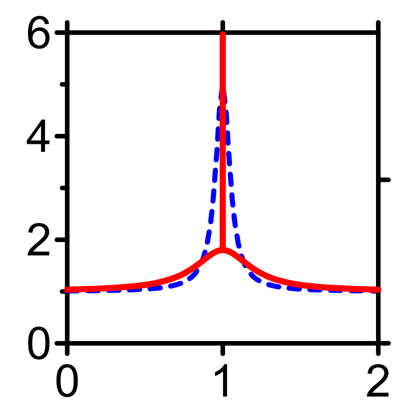
$$\eta = \frac{\hbar^2/4}{\varepsilon_O \varepsilon_{BA}} = \frac{1}{2\Gamma\tau_m} = 1 - \frac{\gamma'}{\Gamma}$$

(same meaning as in optics)

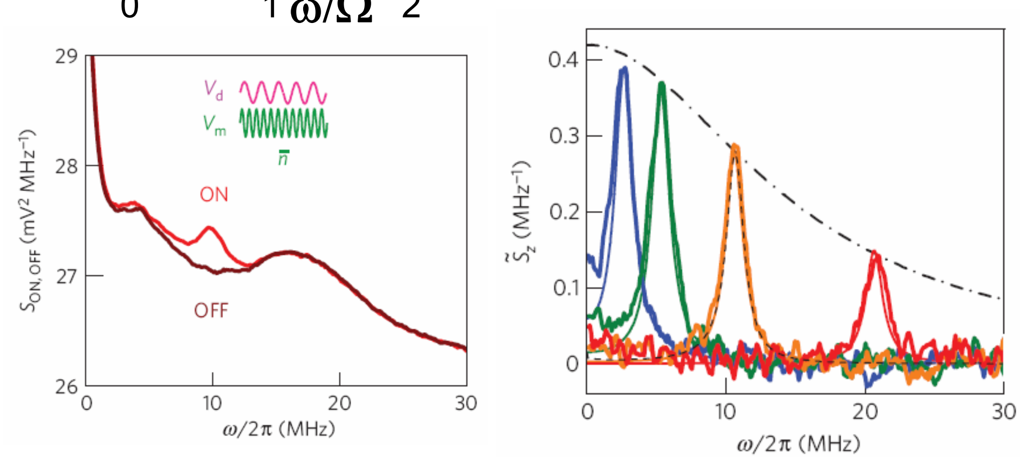
relevant quantities



peak height of persistent Rabi oscillations is limited by 4 times noise pedestal (<u>quantum limit</u>); state of the art: 2%



Quantum efficiency limits fidelity of quantum feedback (synchronization of persistent Rabi oscillations)

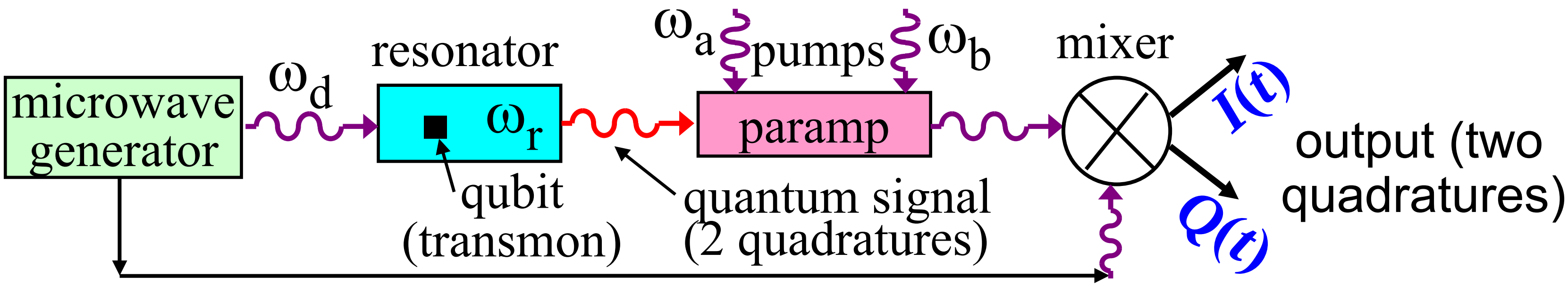


Palacios-Laloy et al., Nature Phys. (2010)

Narrowband linear measurement

Difference from broadband: two quadratures

System: qubit in cQED setup + parametric amplifier



Paramp traditionally discussed in terms of noise temperature

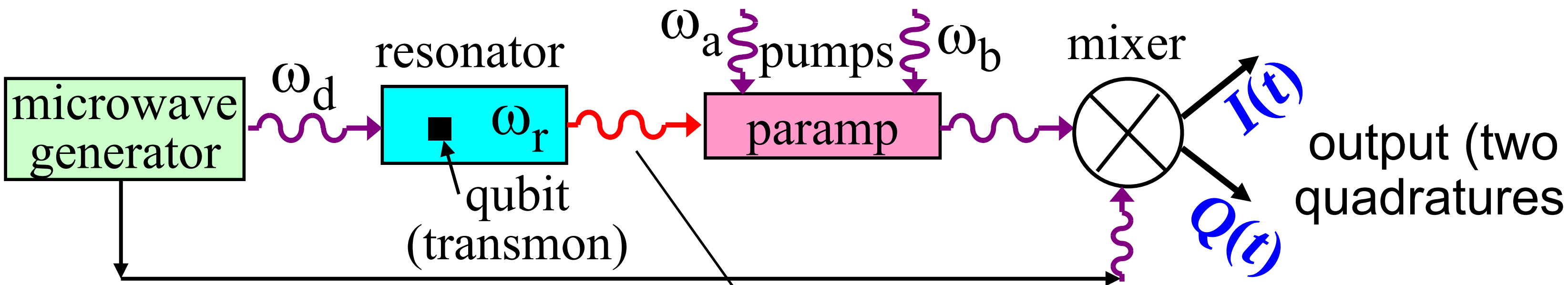
$\theta \geq 0$ for phase-sensitive (degenerate, homodyne) paramp

$\theta \geq \dfrac{\hbar\omega}{2}$ for phase-preserving (non-degenerate, heterodyne) paramp

Haus, Mullen, 1962
Giffard, 1976

We will discuss it in terms of qubit evolution due to measurement

Likharev, private comm.
Devoret, private comm.



Simplest case

$$H = \frac{\hbar\tilde{\omega}_{qb}}{2}\sigma_z + \hbar\omega_r a^\dagger a + \hbar\chi a^\dagger a \sigma_z$$ (dispersive)

$$\frac{\omega_r}{Q} = \kappa \gg \max(\Gamma, \Omega_R)$$ (Markovian, “bad cavity”)

$\kappa_{out} = \kappa$ (everything collected; i.e. reflection)

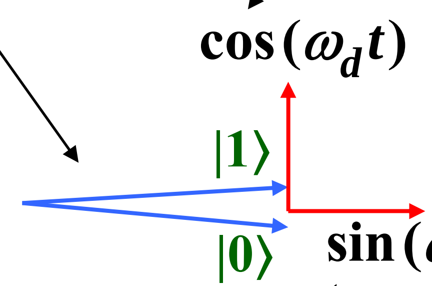
$\chi \ll \kappa$ (weak response)

$\omega_d = \omega_r$ (center of resonance, only phase change if transmission)

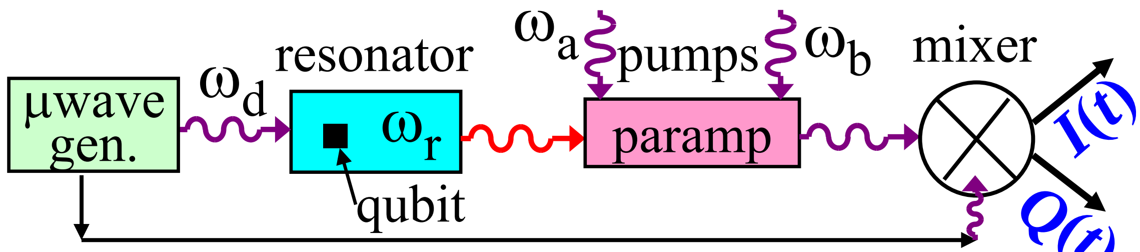
assume everything most ideal

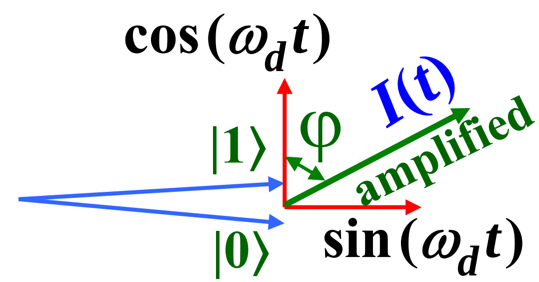
Blais et al., 2004
Gambetta et al., 2006, 2008

carries information about qubit (σ_z) (quantum back-action)



carries information about fluctuating photon number in the resonator (classical back-action)





Phase-sensitive (degenerate) paramp

pumps $\omega_a + \omega_b = 2\omega_d$

$\cos(\omega_a t + \varphi + \delta)$, $\cos((2\omega_a - \omega_d)t + \varphi - \delta)$

quadrature $\cos(\omega_d t + \varphi)$ is amplified,
quadrature $\sin(\omega_d t + \varphi)$ is suppressed

Assume $I(t)$ measures $\cos(\omega_d t + \varphi)$, then $Q(t)$ not needed

get some information (~$\cos^2\varphi$) about qubit state and
some information (~$\sin^2\varphi$) about photon fluctuations

$$\begin{cases} \dfrac{\rho_{11}(\tau)}{\rho_{00}(\tau)} = \dfrac{\rho_{11}(0)}{\rho_{00}(0)} \dfrac{\exp[-(\bar{I} - I_1)^2/2D]}{\exp[-(\bar{I} - I_0)^2/2D]} \\ \rho_{10}(\tau) = \rho_{10}(0) \sqrt{\dfrac{\rho_{11}(\tau)\,\rho_{00}(\tau)}{\rho_{11}(0)\,\rho_{00}(0)}}\, \exp(iK\bar{I}\tau) \end{cases}$$

unitary

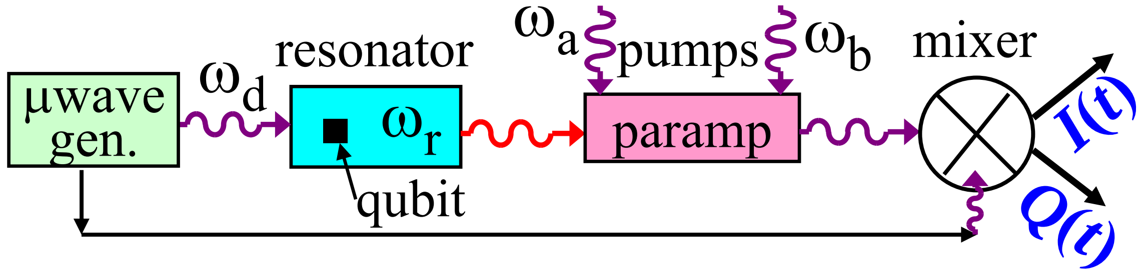
(rotating frame)

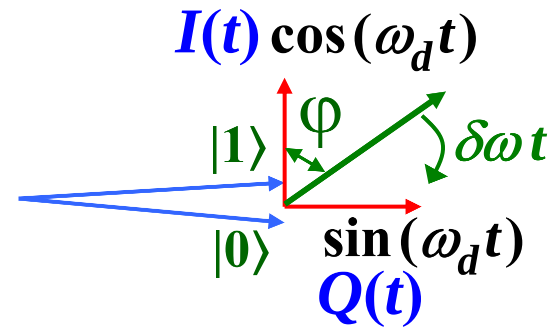
$$\bar{I} \equiv \frac{1}{\tau}\int_0^\tau I(t)\,dt \qquad D = S_I/2\tau$$

$$I_1 - I_0 = \Delta I \cos\varphi \qquad K = \frac{\Delta I}{S_I}\sin\varphi$$

$$\Gamma = \frac{(\Delta I \cos\varphi)^2}{4S_I} + K^2 \frac{S_I}{4} = \frac{\Delta I^2}{4S_I} = \frac{8\chi^2 \bar{n}}{\kappa}$$

Same as for QPC/SET, but trade-off (φ)
between quantum & classical back-actions





Phase-preserving (nondegenerate) paramp

pumps $\omega_a+\omega_b=2(\omega_d+\delta\omega)$ $\varphi=\delta\omega t$

Now information in both $I(t)$ and $Q(t)$.

Choose
$I(t) \leftrightarrow \cos(\omega_d t)$ (qubit information)
$Q(t) \leftrightarrow \sin(\omega_d t)$ (photon fluct. info)

Small $\delta\omega \Rightarrow$ can follow $\varphi(t)$
Large $\delta\omega$ ($>>\Gamma$) $\Rightarrow$ averaging over φ (phase-preserving)

$$\begin{cases} \dfrac{\rho_{11}(\tau)}{\rho_{00}(\tau)} = \dfrac{\rho_{11}(0)}{\rho_{00}(0)} \dfrac{\exp[-(\bar{I}-I_1)^2/2D]}{\exp[-(\bar{I}-I_0)^2/2D]} \\ \rho_{10}(\tau) = \rho_{10}(0)\sqrt{\dfrac{\rho_{11}(\tau)\,\rho_{00}(\tau)}{\rho_{11}(0)\,\rho_{00}(0)}}\exp(iK\bar{Q}\tau) \end{cases}$$

(rotating frame)

$$\bar{I} \equiv \frac{1}{\tau}\int_0^\tau I(t)\,dt \qquad \bar{Q} \equiv \frac{1}{\tau}\int_0^\tau Q(t)\,dt \qquad D=\frac{S_I}{2\tau}$$

$$I_1 - I_0 = \frac{\Delta I}{\sqrt{2}} \qquad K=\frac{\Delta I}{\sqrt{2S_I}}$$

$$\Gamma = \frac{\Delta I^2}{8S_I} + \frac{\Delta I^2}{8S_I} = \frac{8\chi^2\bar{n}}{\kappa}$$

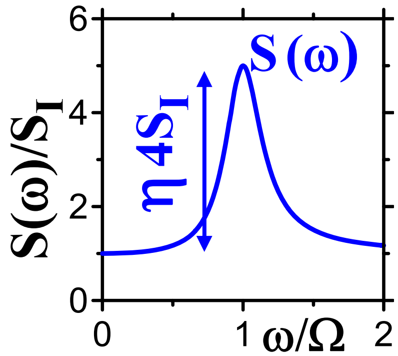
Equal contributions to ensemble dephasing from quantum & classical back-actions

Quantum limits for narrowband detection

If no information loss (good signal collection, good amplifiers, and no extra decoherence), then we can monitor (and feedback) qubit exactly

The limit $\theta \geq \hbar\omega/2$ for phase-preserving amplifiers does not hurt; just says that we need to use both quadratures in a smart way

Quantum efficiency in terms of persistent Rabi peak



Phase-sensitive (degenerate)

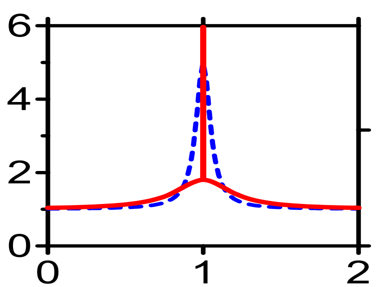
Simple: $S_{\text{peak}}/S_I \leq 4\cos^2\varphi \;\Rightarrow\; \eta \leq \cos^2\varphi$

After undoing classical back-action (by feedback): $\dfrac{S_{\text{peak}}}{S_I} \leq 4 \;\Rightarrow\; \eta \leq 1$

Phase-preserving (non-degenerate)

Simple: $S_{\text{peak}}/S_I \leq 2 \;\Rightarrow\; \eta \leq 1/2$

After undoing classical back-action (by feedback): $\dfrac{S_{\text{peak}}}{S_I} \leq 4 \;\Rightarrow\; \eta \leq 1$



In both cases if quantum feedback on both classical and quantum back-action, then half of the peak shrinks to line

Conclusions

- There are many theoretical limitations for measurement of phase qubits, but all of them have practical solutions
- Quantum limits are not of primary importance for QC, but still have importance and are within experimental reach
- Quantum limits are related to information acquisition
- Discussed some quantum limits for binary-output and linear (broadband and narrowband) detection of a qubit
- If use any good (quantum-limited) amplifier (including phase-preserving), then can monitor and feedback a qubit precisely. No quantum limitations in this sense.

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