# Flying microwave qubits with nearly perfect transfer efficiency

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# Abstract

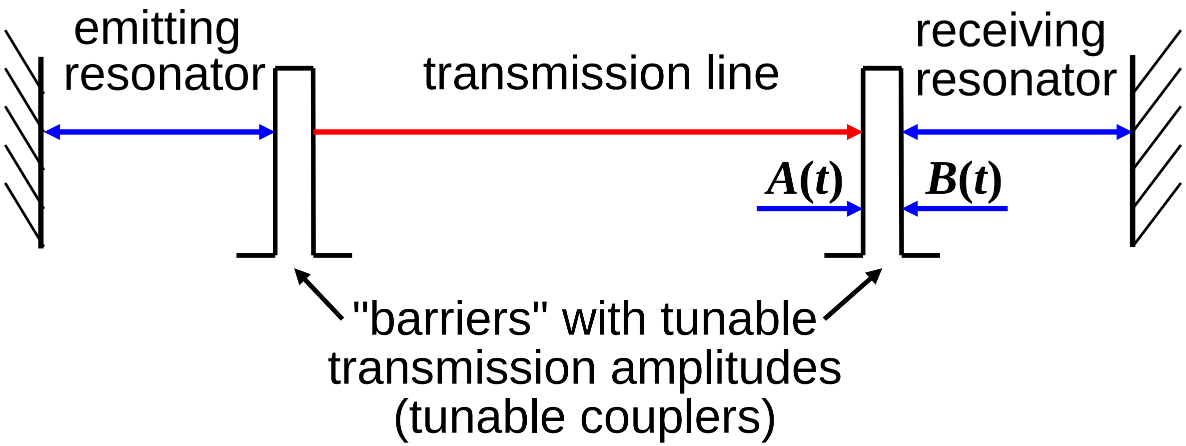
We propose a procedure for transferring the state a microwave qubit via a transmission line from one resonator to another resonator, with a theoretical efficiency arbitrarily close to 100%. The emission and capture of the microwave energy is performed using tunable couplers, whose transmission coefficients vary in time. Using the superconducting phase qubit technology and tunable couplers with maximum coupling of 100 MHz, the procedure with theoretical efficiency $\eta$=0.999 requires a duration of about 400 ns (excluding propagation time) and an ON/OFF ratio of 45. The procedure may also be used for a quantum state transfer with optical photons.

The author thanks Andrei Galiautdinov, Michael Geller, Andrew Cleland, and John Martinis for attracting interest to this problem and useful discussions.

arXiv:1103.0592

# The system we consider

**Goal: transfer a microwave qubit from a resonator to a resonator via a transmission line**



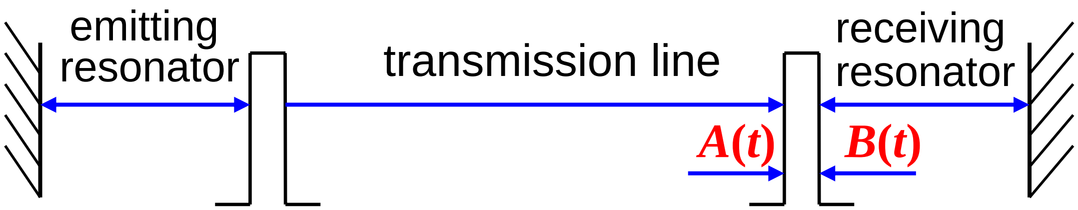

**Seems to be doable experimentally with phase qubit technology**

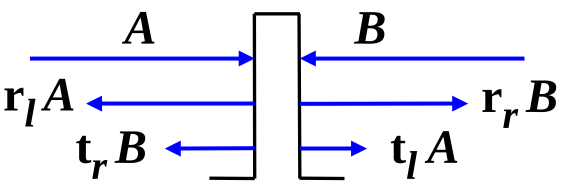
**Possibly can be done in optics as well**

Can be done if couplers are faster than $1/\omega$ (Galiautdinov, Geller, unpub.).

We consider large Q-factors, as in K. Jahne, B. Yurke, and U. Gavish, Phys. Rev. A 75, 010301 (2007).

# Main idea



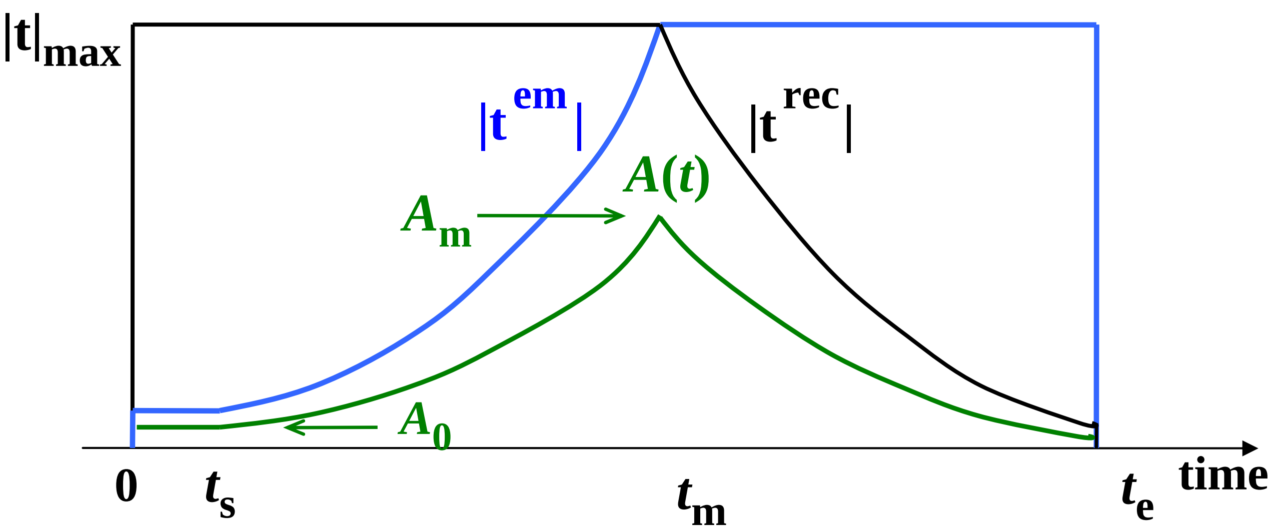




Tune receiving barrier ($t$, $r$) or emitting barrier ($A$) so that

$$r_l A + t B = 0$$

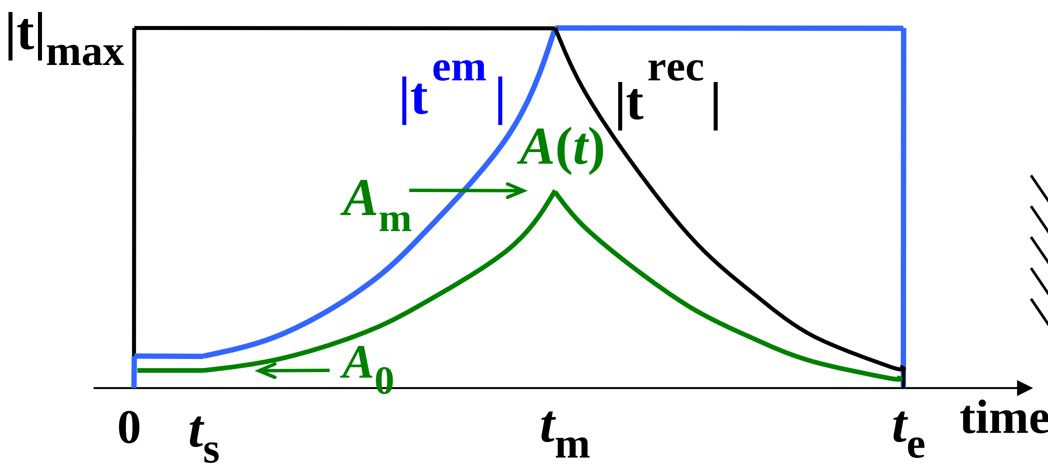
Then reflection back into the transmission line is cancelled.

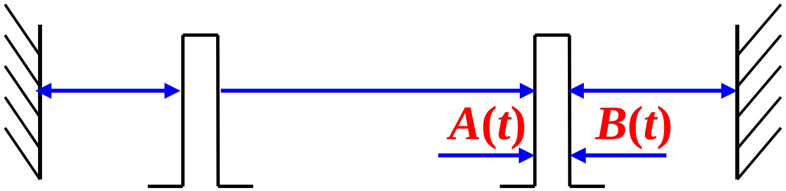
Automatic phasing if resonators have equal frequencies




- Assume high-Q resonators
- Need some time $t_s \sim t_{bu}$ to build up
- Energy loss during $t_s$ and after $t_e$

# Simple analysis




Increasing part:

$$A(t) \sim \exp(\omega t/2Q) = \exp(t/2t_{bu})$$

$$Q = \omega\tau_{rt}/|t|^2_{max}, \quad t_{bu} \equiv Q/\omega$$

(from time reversal)

round-trip time:

$\tau_{rt} = \pi/\omega$ for λ/4 resonator

$\tau_{rt} = 2\pi/\omega$ for λ/2 resonator

Build-up part:

$$t_s \approx t_{bu}$$

$$(t_s = 2t_{bu}\ln 2)$$

Decreasing part:

$$A(t) \sim \exp(-\omega t/2Q)$$
$$= \exp(-t/2t_{bu})$$

(now for emitting res.)

Losses:

$$1-\eta_{bu} \approx \exp\left(-\frac{t_m - t_s}{t_{bu}}\right)$$

$$1-\eta_e \approx \exp\left(-\frac{t_e - t_m}{t_{bu}}\right)$$

Results:

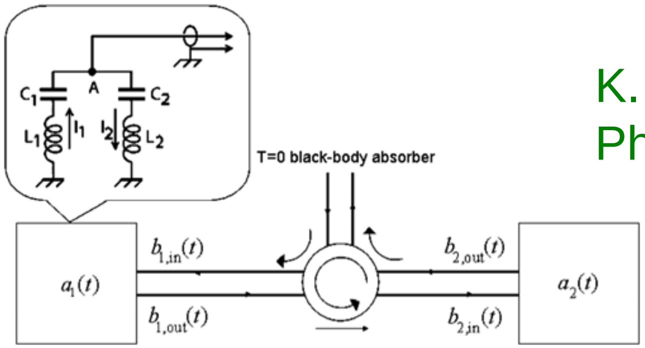
$$t_e \approx 2t_{bu}\ln\frac{1}{1-\eta}$$

$$\frac{\text{ON}}{\text{OFF}} \approx \frac{\sqrt{2}}{\sqrt{1-\eta}}$$

Estimates:

For $|t|_{max}$= 0.1 ($Q$=314)
ω/2π = 6 GHz, η=0.999,
we get
$t_e$ = 120 ns,
ON/OFF = 45

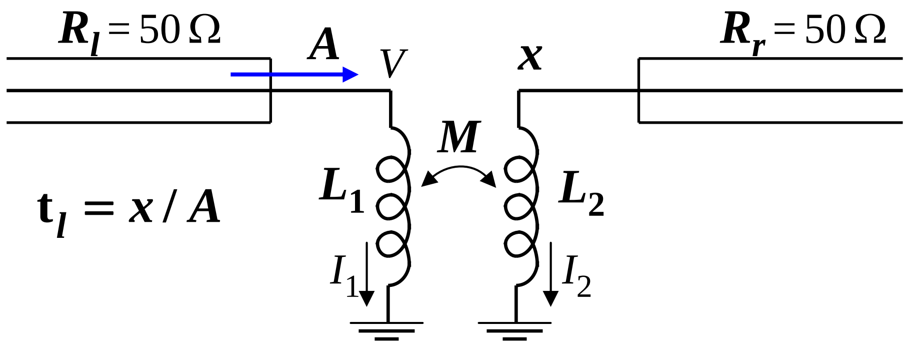
# Comparison with Jahne-Yurke-Gavish



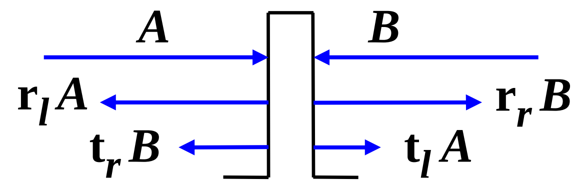

K. Jahne, B. Yurke, U. Gavish, Phys. Rev. A 75, 010301 (2007)

- They modulate only emitting coupler. This makes ON/OFF>>1/(1-$\eta$) and increases duration by ~1/(1-$\eta$) times. This makes their idea inapplicable to a real experiment.
- They obtain results by formal optimization, without a simple picture of reflection cancellation.
- They assume a circulator between resonators (not actually needed; without circulator the losses cannot increase more than twice).
- We analyze in detail transmission/reflection coefficients for a real tunable coupler (R. Bialczak et al., PRL-2011).

# Tunable coupler in terms of *S*-parameters




$$\mathrm{t}_l = x / A$$




$$S = \begin{pmatrix} \mathrm{r}_l & \mathrm{t}_r \\ \mathrm{t}_l & \mathrm{r}_r \end{pmatrix}$$

In simple case ($R_l$=$R_r$=$R$) *S*-matrix is unitary,

$$\mathrm{t}_r = \mathrm{t}_l = \mathrm{t},\ \mathrm{r}_r = -\mathrm{r}_l^*(\mathrm{t}/\mathrm{t}^*)$$

$\mathrm{t}/\sqrt{\mathrm{r}_l \mathrm{r}_r}$ is imaginary

## Weak coupling, high-impedance:

$$\mathrm{t} = -i\frac{2MR}{\omega L_1 L_2},\ \mathrm{r}_l = \mathrm{r}_r \approx 1$$

## Weak coupling:

$$\mathrm{t} = \frac{2i\omega MR}{(i\omega L_1 + R)(i\omega L_2 + R)},\ \mathrm{r}_l \approx \frac{i\omega L_1 - R}{i\omega L_1 + R}$$

## General case (also non-unitary *S*):

$$\mathrm{t}_l = \frac{-2iMR_r}{\omega(L_1L_2 - M^2)} \frac{1}{1 - i\frac{R_rL_1 + R_lL_2}{\omega(L_1L_2 - M^2)} - \frac{R_lR_r}{\omega^2(L_1L_2 - M^2)}}$$

$$\mathrm{r}_l = \frac{1 - i\dfrac{R_rL_1 - R_lL_2}{\omega(L_1L_2 - M^2)} + \dfrac{R_lR_r}{\omega^2(L_1L_2 - M^2)}}{1 - i\dfrac{R_rL_1 + R_lL_2}{\omega(L_1L_2 - M^2)} - \dfrac{R_lR_r}{\omega^2(L_1L_2 - M^2)}}.$$

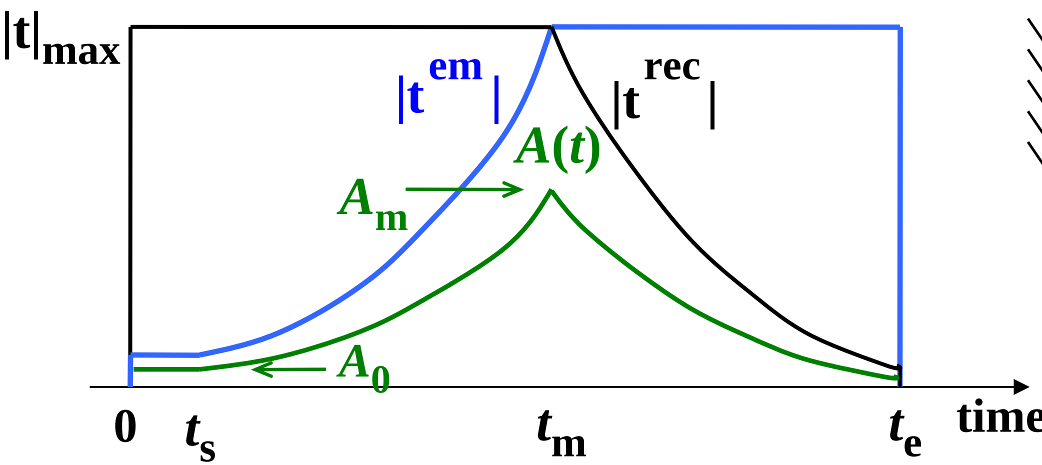
$$\mathrm{t}_r = \mathrm{t}_l (R_l / R_r),\ \mathrm{r}_r = -\mathrm{r}_l^* (\mathrm{t}_l / \mathrm{t}_l^*)$$

## *Q*-factors ($t_{bu}$=$Q/\omega$):

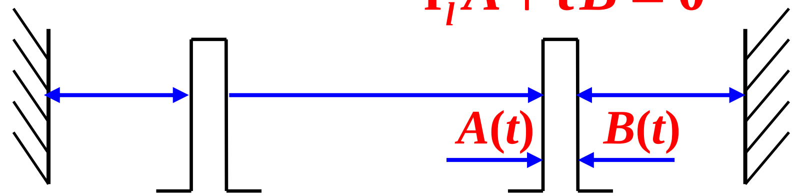
$$Q = \frac{\omega \tau_{rt}}{|\mathrm{t}|_l^2},\ \tau_{rt} = \frac{2R_lE}{|A|^2} \qquad \text{(resonator)}$$

$$Q = \frac{C_{qb}\omega^3 L_1^2 L_2^2 [1 + (R_r/\omega L_2)^2]}{M^2 R_r} \qquad \text{(qubit)}$$

# Capturing a wave




$$\mathrm{r}_l A + \mathrm{t} B = 0$$




$$\frac{dB}{dt} = \frac{\mathrm{r}_r e^{i\varphi} - 1}{\tau_{rt}} B + \frac{e^{i\varphi}}{\tau_{rt}} A\mathrm{t}$$

Assume weak coupling, high $Q$

Increasing part:

$$A(t) = A_0 \exp[(t - t_s)/2t_{bu}^{rec})$$

$$t_{bu} = Q_{\min}/\omega = \tau_{rt}/|\,\mathrm{t}\,|_{\max}^2$$

$$t_s = 2t_{bu} \ln 2$$

Decreasing part:

$$A(t) = A_m \exp[-(t - t_m)/2t_{bu}^{em})]$$

$$A_m = A_0 \exp[(t_m - t_s)/2t_{bu}^{rec})]$$

Losses at start and end:

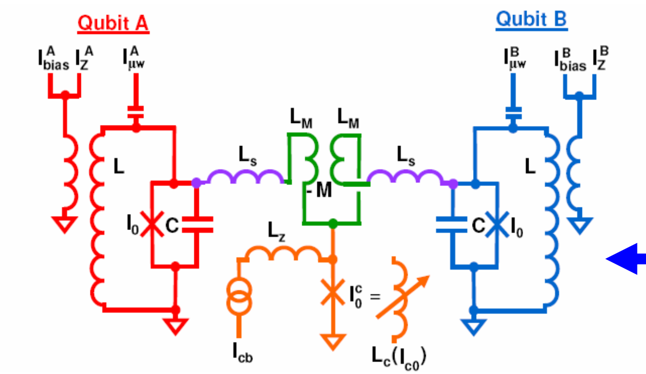
$$1 - \eta = \frac{A_0^2 t_{bu}(2\ln 2 - 1) + A_m^2 \exp[-(t_e - t_m)/t_{bu}]t_{bu}}{A_0^2 t_s + A_0^2[\exp[-(t_m - t_s)/t_{bu}] - 1]t_{bu} + A_m^2 t_{bu}}$$

Optimize for shortest duration $t_e$ at fixed $\eta$:

$$t_e = 2t_{bu} \ln \frac{2\sqrt{2\ln 2 - 1}}{1 - \eta}$$

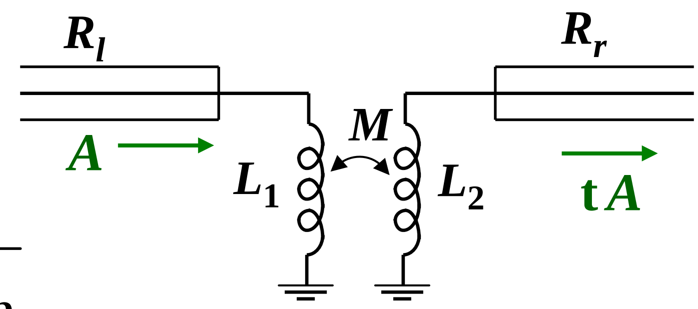
$$\frac{|\,\mathrm{t}\,|_{\max}^{rec}}{|\,\mathrm{t}\,|_{\min}^{rec}} = \frac{\sqrt{2}}{\sqrt{1-\eta}}, \quad \frac{|\,\mathrm{t}\,|_{\max}^{em}}{|\,\mathrm{t}\,|_{\min}^{em}} = \frac{\sqrt{2}\sqrt{2\ln 2 - 1}}{\sqrt{1-\eta}}$$

# Compare with 2-qubit experiment




R. Bialczak et al., PRL-2011

$$t = -i\frac{2MR}{\omega L_1 L_2}$$




$$\Omega_{XX} = -\frac{M}{L_1 L_2 \omega C_{qb}}$$

Needs some “language translation”

## Transfer duration in terms of $\Omega_{XX}$

resonator $\to$ resonator

$$t_e = \frac{\tau_{rt}[1+(\frac{R}{\omega L_1})^2][1+(\frac{R}{\omega L_2})^2]}{2\Omega_{XX}R^2C_{qb}^2}\ln\frac{2\sqrt{2\ln 2-1}}{1-\eta}$$

qubit $\to$ qubit

$$t_e = 2\frac{1+(R/\omega L_1)^2}{\Omega_{XX}^2 RC_{qb}}\ln\frac{2\sqrt{2\ln 2-1}}{1-\eta}$$

## Estimates for flying qubits

Assume Bialczak’s parameters,
$\Omega_{XX}/2\pi = 100$ MHz, $\eta$=0.999

### State transfer duration

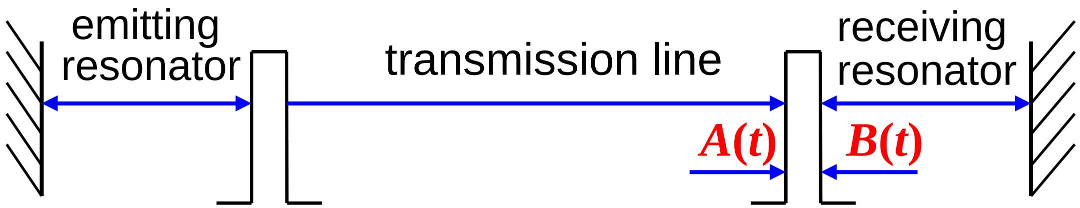
two $\lambda/4$ resonators: $t_e = 420\,\mathrm{ns}$

two phase qubits
(or $\lambda/2$ resonators): $t_e = 850\,\mathrm{ns}$

### Needed ON/OFF ratio

ON/OFF = 45 (100 MHZ -- 2.2 MHz)

# Tolerable detuning of resonators
## (crude estimate)




We need perfect interference for the whole duration

Assume exact cancellation of reflection at $t_m$ (maximum)

Then reflected wave amplitude $A_m \exp(-|t-t_m|/2t_{bu})\,\delta\omega\,|t-t_m|$

So the relative energy loss $1-\eta_{\delta\omega} \approx 2(\delta\omega\, t_{bu})^2$

So for required efficiency $\eta$ we can tolerate detuning

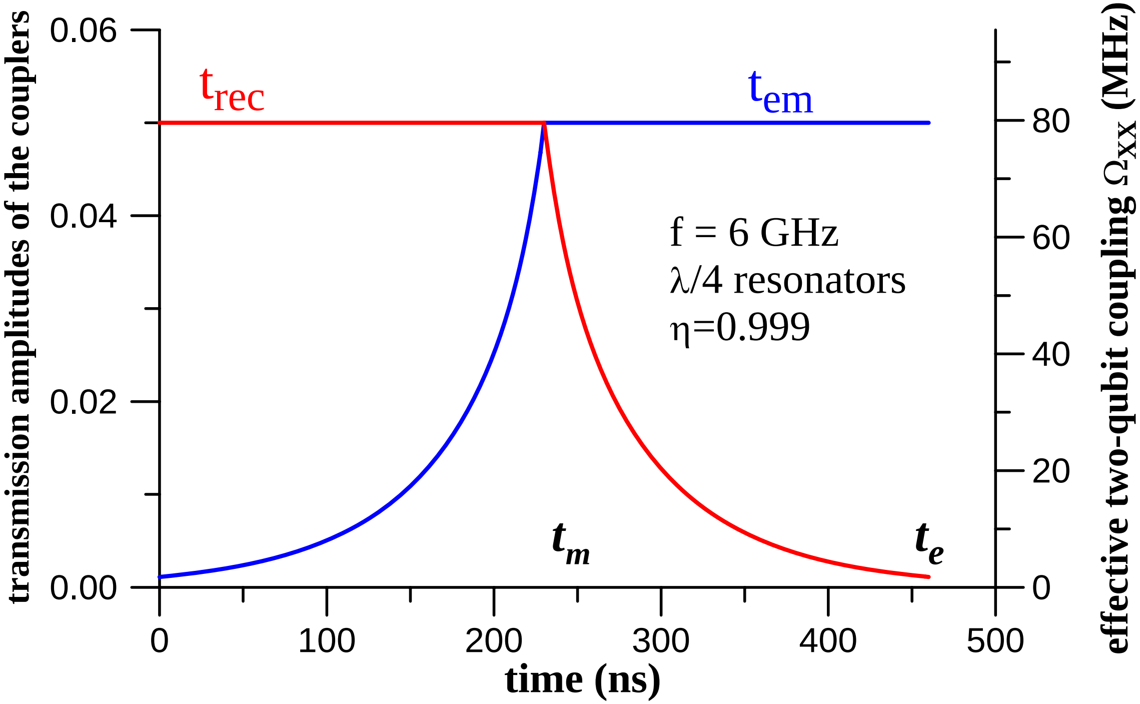
$$\frac{|\delta\omega|}{2\pi} \approx \frac{\sqrt{1-\eta}}{10\, t_{bu}}$$

Using $t_{bu}$ = 30 ns (from estimate) we get

0.1 MHz for $\eta$=0.999 and 0.1 MHz for $\eta$=0.9

Seems to be the hardest requirement for an experiment
(most likely needs active frequency tuning)

# A modified (simpler) procedure




$$\tau_{bu} = \frac{\tau_{rt}}{|\,t_{\max}\,|^2}$$

$$t_m = \tau_{bu} \ln \frac{1}{1-\eta}$$

$$t_e = 2t_m$$

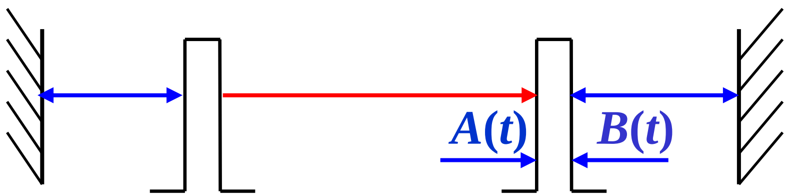
No initial build-up period $t_S$. Just pretend the needed amplitude is already in the receiving resonator. Then a fully symmetric procedure.

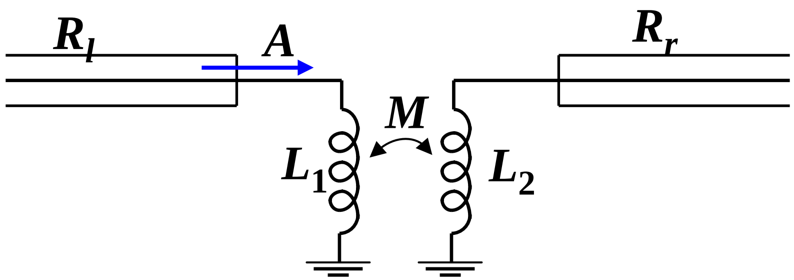
Time-dependence of the emitting and receiving couplings:

$$t_{em}(t) = \begin{cases} \dfrac{t_{\max}}{\sqrt{2e^{-(t-t_m)/\tau_{bu}} - 1}}, & t < t_m \\ t_{\max}, & t > t_m \end{cases}$$

$$t_{rec}(t) = \begin{cases} t_{\max}, & t < t_m \\ \dfrac{t_{\max}}{\sqrt{2e^{(t-t_m)/\tau_{bu}} - 1}}, & t > t_m \end{cases}$$

# Summary




- Simple idea: cancel reflection back into transmission line
- Seems to be doable with the phase qubit technology:
  ~400 ns, ON/OFF ~ 50 (between 100 MHz and 2 MHz)
  for $\eta$=0.999 excluding microwave absorption
- Tolerable detuning of resonators is quite small (0.1 - 1 MHz),
  a difficult requirement for an experiment

A. Korotkov, arXiv:1103.0592

Further things to do:

- Numerical simulations (effects of moderate coupling,
  tolerance to parameter deviations, shape distortions,
  timing imperfections, decoherence)
- Apply analysis to the new tunable coupler (Yi Yin et al.)
- Check applicability of the idea to other realizations (optics, etc.)