Persistent Rabi oscillations and quantum feedback in solid state

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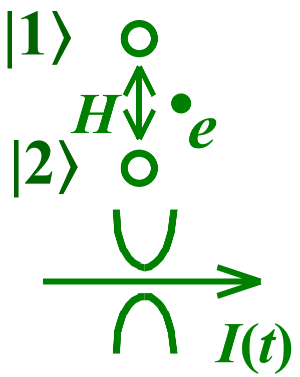
Outline:
- Persistent Rabi oscillations: theory
- Saclay experiment
- Quantum feedback of persistent Rabi osc.
- Persistent Rabi osc. revealed in noise

Acknowledgements

Theory: R. Ruskov, D. Averin, Q. Zhang
Experiment: P. Bertet, D. Esteve, et al.

Bayesian formalism for DQD-QPC system

$H_{QB} = 0$



Qubit evolution due to measurement (quantum back-action):

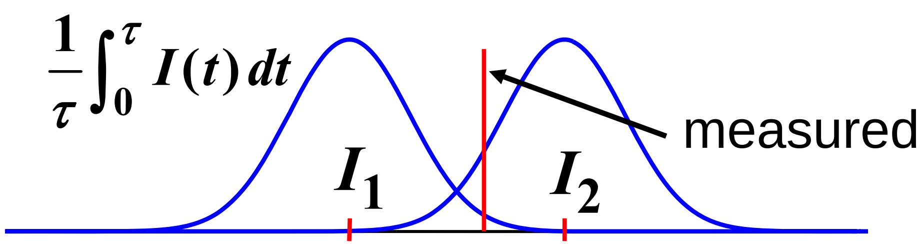
$$\psi(t) = \alpha(t)\,|1\rangle + \beta(t)\,|2\rangle \quad \text{or} \quad \rho_{ij}(t)$$

1) $|\alpha(t)|^2$ and $|\beta(t)|^2$ evolve as probabilities,
   i.e. according to the **Bayes rule** (same for ρ_{ii})

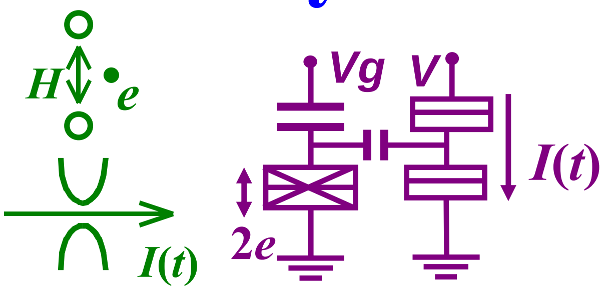
2) phases of $\alpha(t)$ and $\beta(t)$ do not change
   (**no dephasing!**), $\rho_{ij}/(\rho_{ii}\,\rho_{jj})^{1/2} = \text{const}$

Bayes rule (1763, Laplace-1812):

$$\underbrace{P(A_i\,|\,\text{res})}_{\text{posterior probability}} = \frac{\underbrace{P(A_i)}_{\text{prior probab.}}\ \underbrace{P(\text{res}\,|\,A_i)}_{\text{likelihood}}}{\sum_k P(A_k)\,P(\text{res}\,|\,A_k)}$$



Bayesian formalism for a single qubit



- Time derivative of the quantum Bayes rule
- Add unitary evolution of the qubit
- Add decoherence γ (if any)

$$\dot{\rho}_{11} = -\dot{\rho}_{22} = -2H\,\mathrm{Im}\,\rho_{12} + \rho_{11}\rho_{22}\frac{2\Delta I}{S_I}[\underline{\underline{I(t)}} - I_0]$$

$$\dot{\rho}_{12} = i\varepsilon\rho_{12} + iH(\rho_{11}-\rho_{22}) + \rho_{12}(\rho_{11}-\rho_{22})\frac{\Delta I}{S_I}[\underline{\underline{I(t)}} - I_0] - \gamma\rho_{12}$$

(A.K., 1998)

$\gamma = \Gamma - (\Delta I)^2/4S_I$, Γ – ensemble decoherence

$\gamma = 0$ for QPC detector

Averaging over result $I(t)$ leads to conventional master equation with Γ

Evolution of qubit *wavefunction* can be monitored if γ=0 (quantum-limited)

Can we verify the Bayesian formalism experimentally?

Direct way:

prepare → partial measur. → control (rotation) → projective measur.

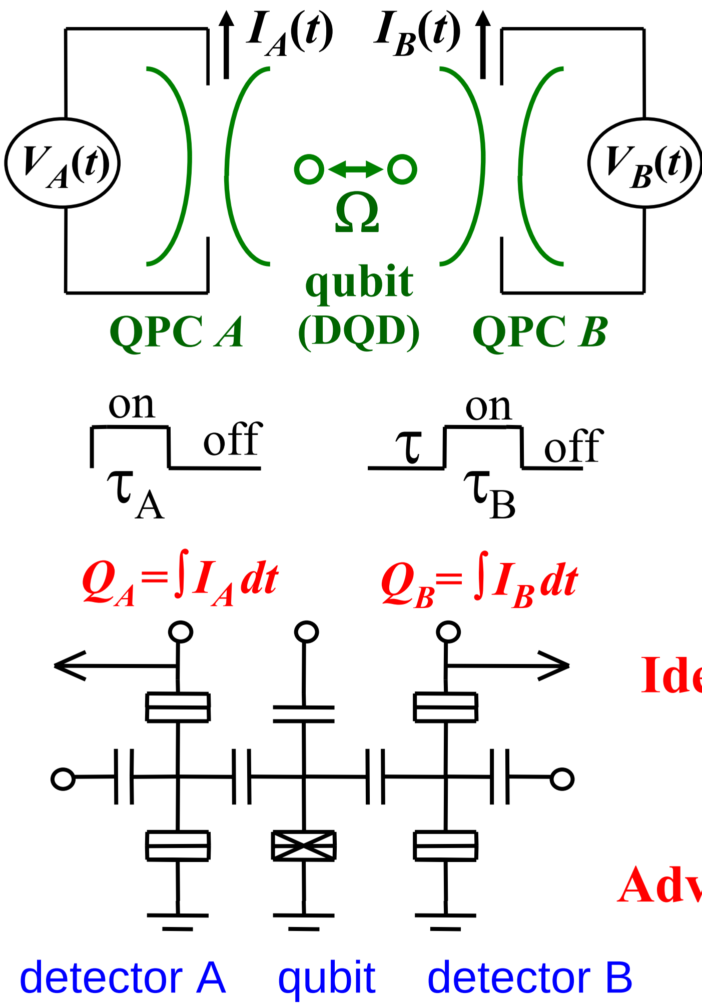
A.K.,1998

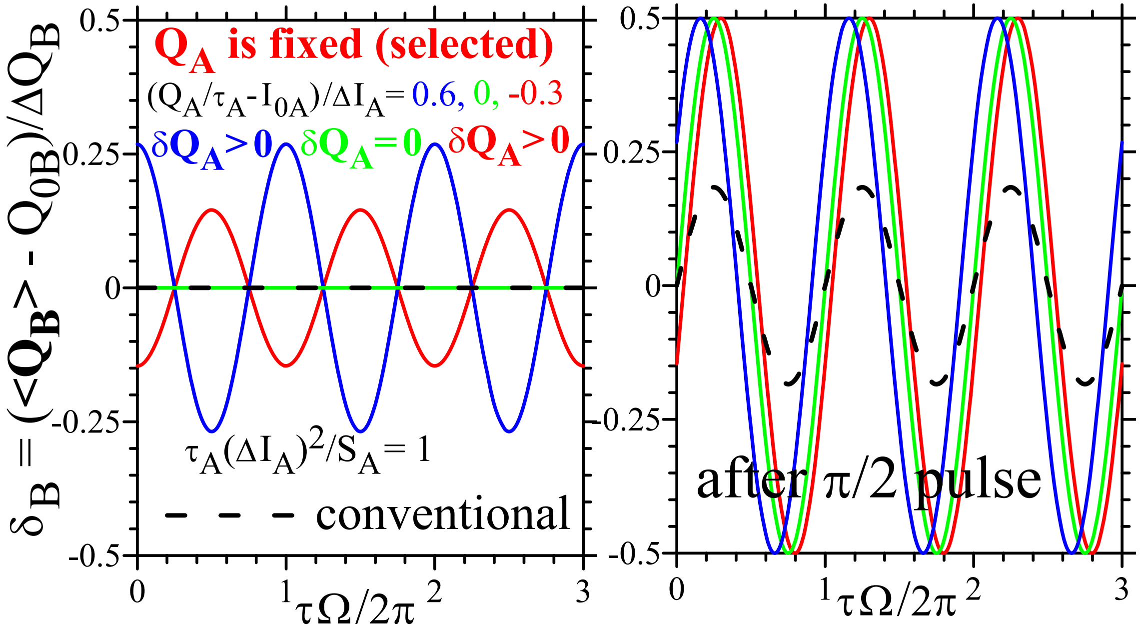
However, difficult: bandwidth, control, efficiency
(expt. realized only for supercond. phase qubits)

Tricks are needed for real experiments

Bell-type measurement correlation

(A.K., PRB-2001)





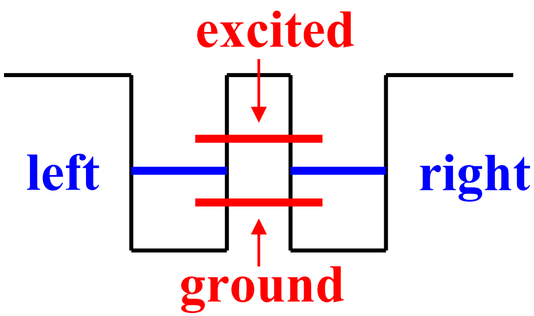
Idea: two consecutive measurements of a qubit by two detectors; probability distribution $\mathbf{P}(Q_A, Q_B, \tau)$ shows effect of the first measurement on the qubit state.

Advantage: solves the bandwidth problem

Same idea with another averaging → weak values

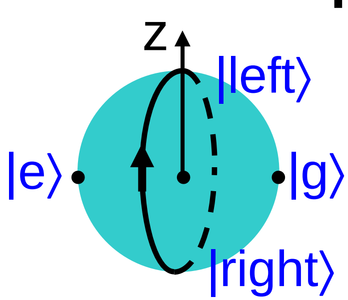
(Romito, Gefen, Blanter, PRL-2008)

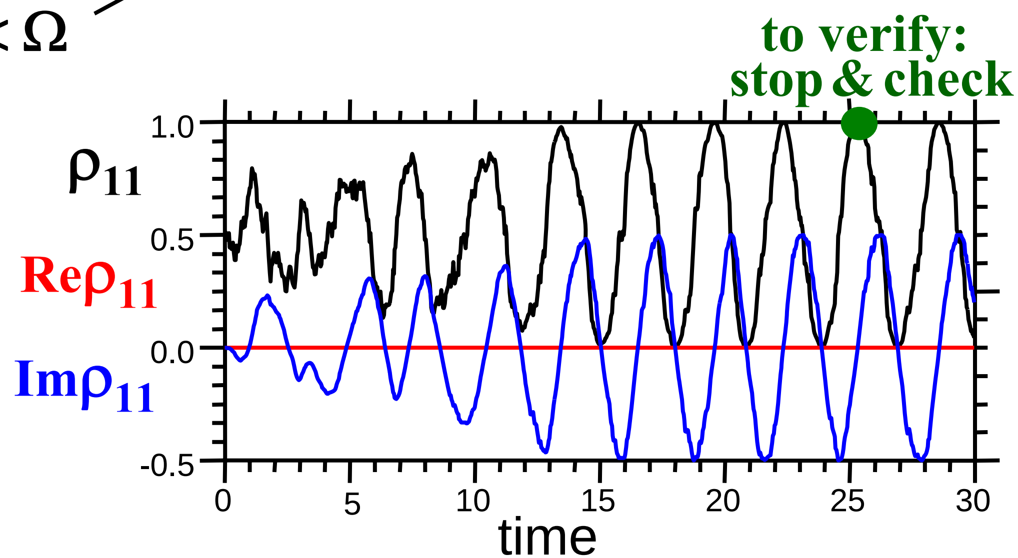
Non-decaying (persistent) Rabi oscillations



- Relaxes to the ground state if left alone (low-T)
- Becomes fully mixed if coupled to a high-T (non-equilibrium) environment
- Oscillates persistently between left and right if (weakly) measured continuously

$$\frac{(\Delta I)^2}{4S_I} \ll \Omega$$

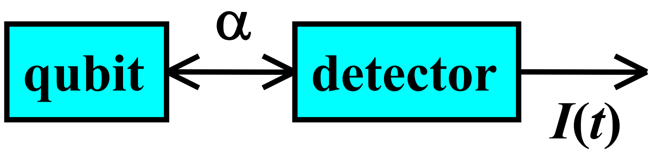


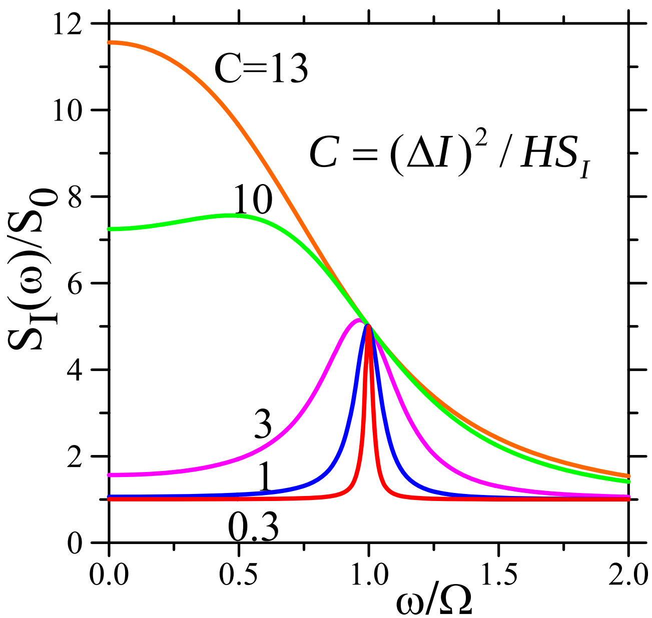


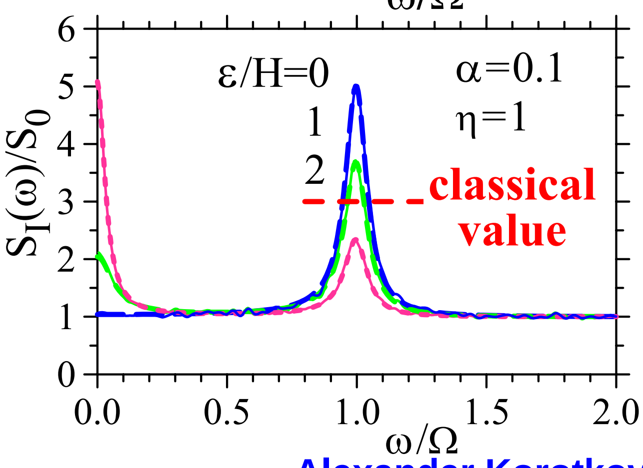
A.K., PRB-1999

Direct experiment is difficult

Measured spectrum of qubit coherent oscillations







What is the spectral density $S_I(\omega)$ of detector current?

Assume classical output, eV » $\hbar\Omega$

$$\varepsilon = 0, \quad \Gamma = \eta^{-1}(\Delta I)^2/4S_0$$

$$S_I(\omega) = S_0 + \frac{\Omega^2(\Delta I)^2\Gamma}{(\omega^2-\Omega^2)^2+\Gamma^2\omega^2}$$

Spectral peak can be seen, but peak-to-pedestal ratio ≤ 4η ≤ 4

(result can be obtained using various methods, not only Bayesian method)

Weak coupling, $\alpha = C/8 \ll 1$

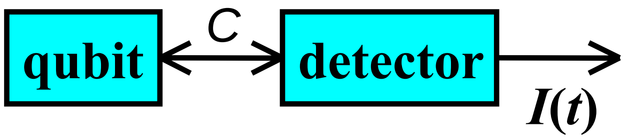
$$S_I(\omega) = S_0 + \frac{\eta S_0 \varepsilon^2/H^2}{1+(\omega\hbar^2\Omega^2/4H^2\Gamma)^2} + \frac{4\eta S_0(1+\varepsilon^2/2H^2)^{-1}}{1+[(\omega-\Omega)\Gamma(1-2H^2/\hbar^2\Omega^2)]^2}$$

A.K., LT'99
Averin-A.K., 2000
A.K., 2000
Averin, 2000
Goan-Milburn, 2001
Makhlin et al., 2001
Balatsky-Martin, 2001
Ruskov-A.K., 2002
Mozyrsky et al., 2002
Balatsky et al., 2002
Bulaevskii et al., 2002
Shnirman et al., 2002
Bulaevskii-Ortiz, 2003
Shnirman et al., 2003

Contrary:
Stace-Barrett, PRL'04

Derivations of the spectrum

A.K., 2001



$$I(t) = I_0 + \frac{\Delta I}{2} z(t) + \xi(t)$$

(const + signal + noise)

z is Bloch coordinate

$$S_I(\omega) = S_{\xi\xi} + \frac{\Delta I^2}{4} S_{zz}(\omega) + \frac{\Delta I}{2}[S_{\xi z}(\omega) + S_{\xi z}(\omega)]$$

Bayesian approach

$$z = \cos\phi, \;\; \phi = \Omega t + \varphi, \;\; \frac{d}{dt}\varphi = -\sin\phi \frac{\Delta I}{S_0}[\frac{\Delta I}{2}\cos\phi + \xi]$$

$$dz = \sin^2\phi\, [(\Delta I^2 / 2S_0)\cos\phi + (\Delta I / S_0)\,\xi]$$

$\Rightarrow$ correlation between noise ξ and qubit state *z* at later time, 4=2+2, *z* is shifted in the same direction as ξ (reality follows observation)

Ensemble approach

No ξ-z correlation, but treat *z* as operator

Technically: start with z=±1 (“collapse”), then usual master equation

Results of both approached are the same

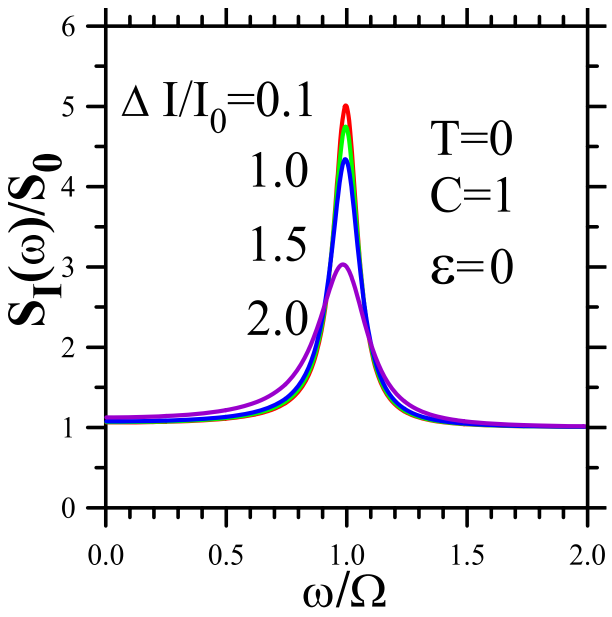
One more derivation (via MacDonald's formula, good for small-transpar. QPC)

Ruskov & A.K., 2002

MacDonald's formula (textbook):

$$S_I(\omega) = 2\omega \int_0^\infty \frac{d\langle Q^2(\tau)\rangle}{d\tau} \sin(\omega\tau)\, d\tau$$

$$I = dQ/dt, \;\; Q = en$$



Bloch equations (Gurvitz, 1997):

$$\frac{d}{dt}\rho_{11}^n = -I_1\rho_{11}^n + I_1\rho_{11}^{n-1} - 2H\,\mathrm{Im}\,\rho_{12}^n$$

$$\frac{d}{dt}\rho_{22}^n = -I_2\rho_{22}^n + I_2\rho_{22}^{n-1} + 2H\,\mathrm{Im}\,\rho_{12}^n$$

$$\frac{d}{dt}\rho_{12}^n = -\frac{1}{2}(I_1+I_2)\rho_{12}^n + \sqrt{I_1I_2}\,\rho_{12}^{n-1} + $$

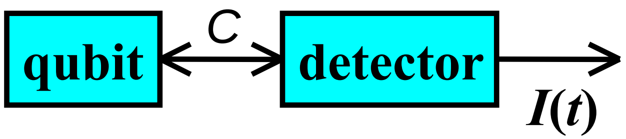
$$+ i\varepsilon\rho_{12}^n + iH(\rho_{11}^n - \rho_{22}^n)$$

$$\langle Q^2(\tau)\rangle = \sum n^2[\rho_{11}^n + \rho_{22}^n]$$

Peak-to-pedestal ratio:

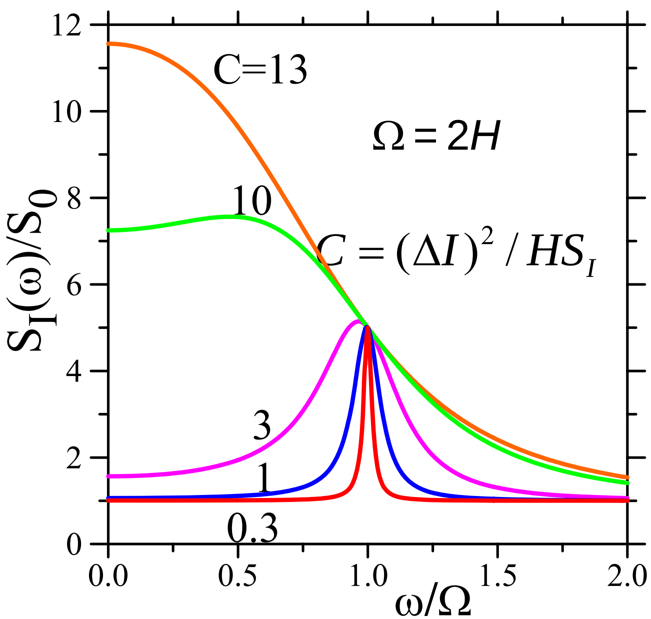
4 in weak-response case,
2 in strong-response case

Why measuring Rabi spectrum is an "easy" experiment



$$I(t) = I_0 + \frac{\Delta I}{2} z(t) + \xi(t)$$

(const + signal + noise)

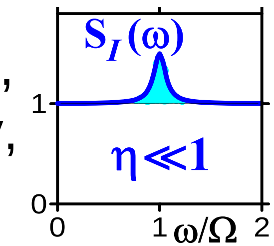


Ω - Rabi frequency

peak-to-pedestal ratio = $4\eta \leq 4$

$$S_I(\omega) = S_0 + \frac{\Omega^2 (\Delta I)^2 \Gamma}{(\omega^2 - \Omega^2)^2 + \Gamma^2 \omega^2}$$

amplifier noise ⇒ higher pedestal, poor quantum efficiency, but the peak is the same!!!



integral under the peak ⇔ variance $\langle z^2 \rangle$

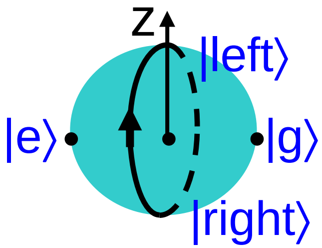
How to distinguish experimentally persistent from non-persistent? Easy!

perfect Rabi oscillations: $\langle z^2 \rangle = \langle \cos^2 \rangle = 1/2$

imperfect (non-persistent): $\langle z^2 \rangle \ll 1/2$

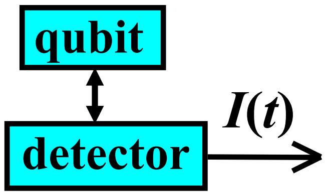
quantum (Bayesian) result: $\langle z^2 \rangle = 1$ (!!!)

(demonstrated in Saclay expt.)



How to understand $\langle z^2 \rangle = 1$?

$$I(t) = I_0 + \frac{\Delta I}{2} z(t) + \xi(t)$$



First way (mathematical)

We actually measure operator: $z \rightarrow \sigma_z$

$$z^2 \rightarrow \sigma_z{}^2 = 1$$

(What does it mean? Difficult to say…)

Second way (Bayesian)

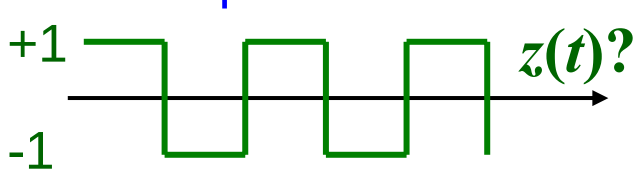
$$S_I(\omega) = S_{\xi\xi} + \frac{\Delta I^2}{4} S_{zz}(\omega) + \frac{\Delta I}{2} S_{\xi z}(\omega)$$

quantum back-action changes z in accordance with the noise ξ

Equal contributions (for weak coupling and η=1)

"what you see is what you get": observation becomes reality

Can we explain it in a more reasonable way (without spooks/ghosts)?



or some other $z(t)$?

No (under assumptions of macrorealism; Leggett-Garg, 1985)

Leggett-Garg inequalities (1985)

Assumptions of macrorealism:

1) $Q(t)$ is well-defined at all times

2) noninvasive measurability of Q(t)

(instantaneous "strong" measurement)

$$Q(t) = \pm 1, \;\; K_{ij} = \langle Q_i Q_j \rangle$$

$$1 + K_{12} + K_{23} + K_{13} \geq 0$$

$$K_{12} + K_{23} + K_{34} - K_{14} \leq 2$$

Violated by QM (-0.5, 2√2)

Quantum Mechanics versus Macroscopic Realism: Is the Flux There when Nobody Looks?

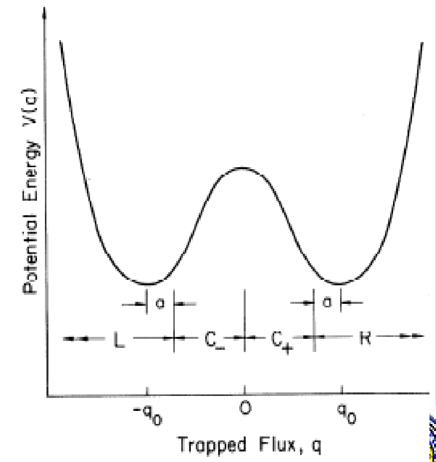
A. J. Leggett
Department of Physics,[a] University of Illinois at Urbana-Champaign, Urbana, Illinois 61801, and Department of Physics, Harvard University, Cambridge, Massachusetts 02138

and

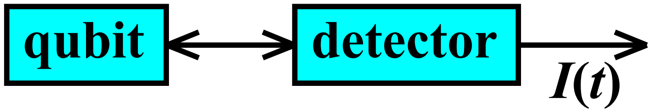
Anupam Garg
University of Illinois at Urbana-Champaign, Urbana, Illinois 61801

(Received 19 November 1984)

It is shown that, in the context of an idealized "macroscopic quantum coherence" experiment, the predictions of quantum mechanics are incompatible with the conjunction of two general assumptions which are designated "macroscopic realism" and "noninvasive measurability at the macroscopic level." The conditions under which quantum mechanics can be tested against these assumptions in a realistic experiment are discussed.



Leggett-Garg-type inequalities for continuous measurement of a qubit



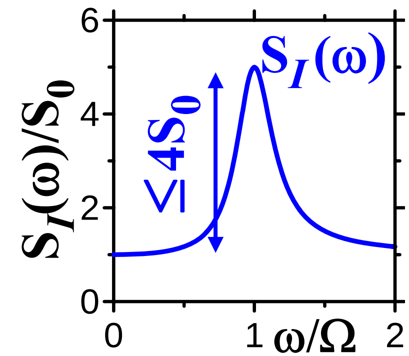
Ruskov-A.K.-Mizel, PRL-2006
Jordan-A.K.-Büttiker, PRL-2006

Assumptions of macrorealism (similar to Leggett-Garg'85):

$$I(t) = I_0 + (\Delta I/2)\, z(t) + \xi(t)$$

$$|z(t)| \le 1, \quad \langle \xi(t)\; z(t+\tau)\rangle = 0$$

Leggett-Garg,1985
$K_{ij} = \langle Q_i Q_j \rangle$
if $Q = \pm 1$, then
$1+K_{12}+K_{23}+K_{13} \ge 0$



Then for correlation function

$$K(\tau) = \langle I(t)\, I(t+\tau)\rangle$$

	quantum result	violation
$K(\tau_1) + K(\tau_2) - K(\tau_1+\tau_2) \le (\Delta I/2)^2$	$\frac{3}{2}(\Delta I/2)^2$	$\times \frac{3}{2}$

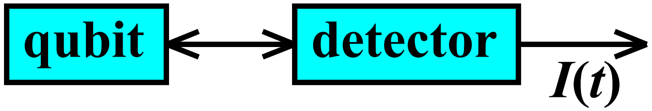
and for area under narrow spectral peak

$\int [S_I(f) - S_0]\, df \le (8/\pi^2)(\Delta I/2)^2$	$(\Delta I/2)^2$	$\times \frac{\pi^2}{8}$

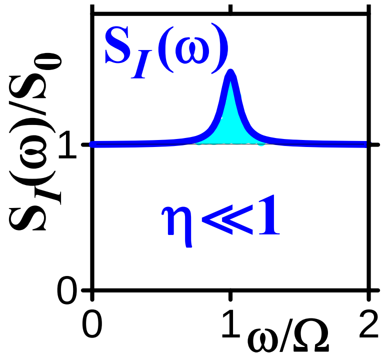
η is not important! **Experimentally measurable violation**

(Saclay experiment)

May be a physical (realistic) back-action?



$$I(t) = I_0 + \frac{\Delta I}{2} z(t) + \xi(t)$$



OK, cannot explain without back-action

$$\langle \xi(t)\, z(t+\tau) \rangle \neq 0$$

But may be there is a simple classical back-action from the noise?

In principle, classical explanation cannot be ruled out (e.g. computer-generated $I(t)$; no non-locality as in optics)

Try reasonable models: linear modulation of the qubit parameters (H and ε) by noise $\xi(t)$

No, does not work!

Our (spooky) back-action is quite peculiar: $\langle \xi(t)\, dz(t+0) \rangle > 0$

"what you see is what you get": observation becomes reality

Experiment on Rabi (Larmor) spectrum?

Durkan and Welland, 2001 (STM-ESR experiment similar to Manassen-1989)

APPLIED PHYSICS LETTERS VOLUME 80, NUMBER 3 21 JANUARY 2002

Electronic spin detection in molecules using scanning-tunneling-microscopy-assisted electron-spin resonance

C. Durkan[a] and M. E. Welland
Nanoscale Science Laboratory, Department of Engineering, University of Cambridge, Trumpington Street, Cambridge CB2 1PZ, United Kingdom

(Received 8 May 2001; accepted for publication 8 November 2001)

By combining the spatial resolution of a scanning-tunneling microscope (STM) with the electronic spin sensitivity of electron-spin resonance, we show that it is possible to detect the presence of localized spins on surfaces. The principle is that a STM is operated in a magnetic field, and the resulting component of the tunnel current at the Larmor (precession) frequency is measured. This component is nonzero whenever there is tunneling into or out of a paramagnetic entity. We have

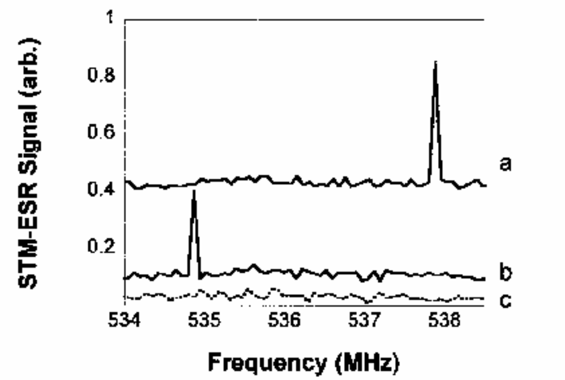


FIG. 3. STM-ESR spectra of (a), (b) two different areas (a few nm apart) of the molecule-covered sample and (c) bare HOPG. The graphs are shifted vertically for clarity.

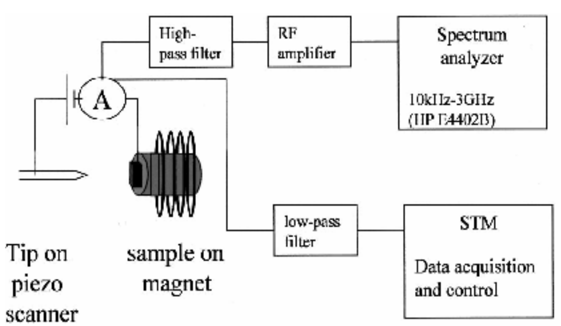


FIG. 1. Schematic of the electronics used in STM-ESR.

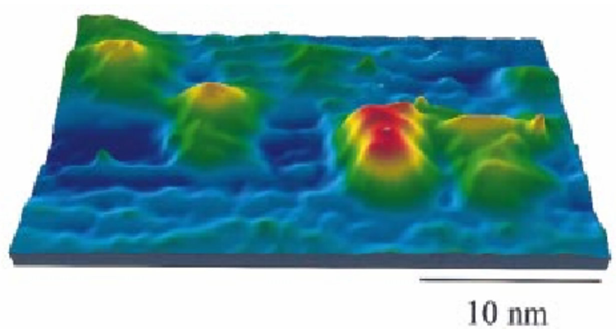


FIG. 2. (Color) STM image of a 250 Å×150 Å area of HOPG with four adsorbed BDPA molecules.

$$\frac{\text{peak}}{\text{noise}} \leq 3.5$$

(Colm Durkan, private comm.)

Recently reproduced:
Messina et al., JAP-2007

Questionable

Somewhat similar experiment

"Continuous monitoring of Rabi oscillations in a Josephson flux qubit"

$$H = -\frac{1}{2}(\Delta\sigma_x + \varepsilon\sigma_z) - W\sigma_z \cos\omega_{HF} t$$

$$(\omega_{HF} \approx \sqrt{\Delta^2 + \varepsilon^2};\ \ \varepsilon \neq 0)$$

E. Il'ichev *et al.*, PRL, 2003

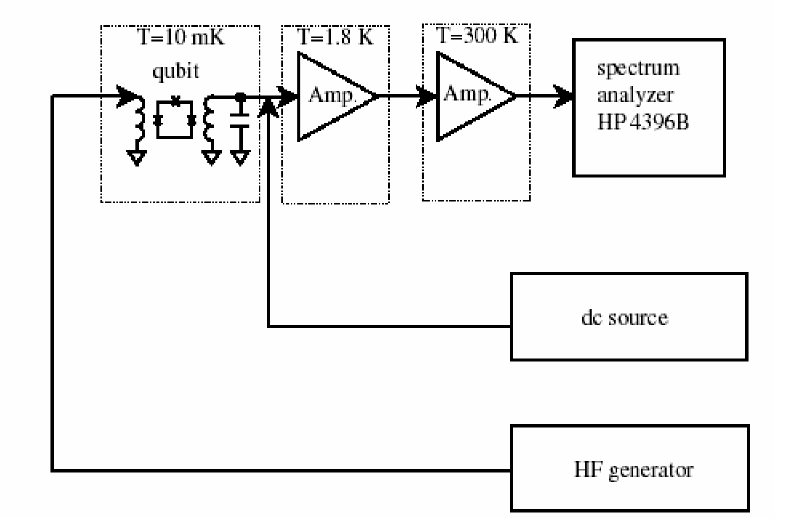


FIG. 1. Measurement setup. The flux qubit is inductively coupled to a tank circuit. The dc source applies a constant flux $\Phi_e \approx \frac{1}{2}\Phi_0$. The HF generator drives the qubit through a separate coil at a frequency close to the level separation $\Delta/h = 868$ MHz. The output voltage at the resonant frequency of the tank is measured as a function of HF power.

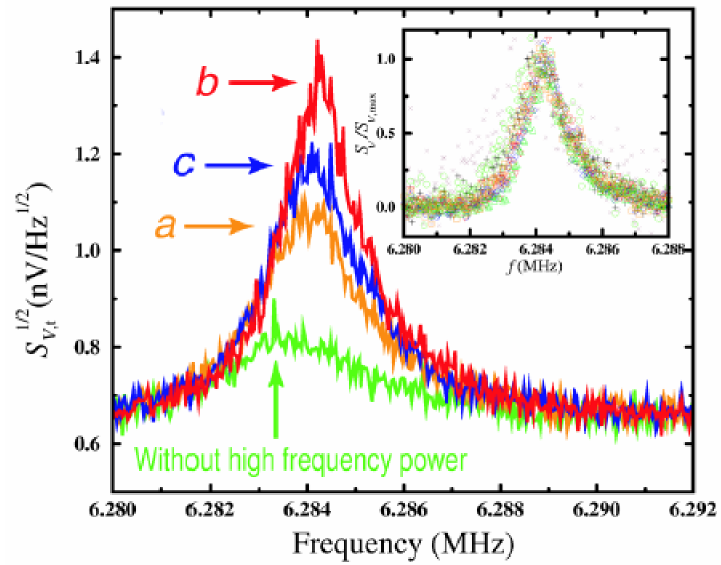
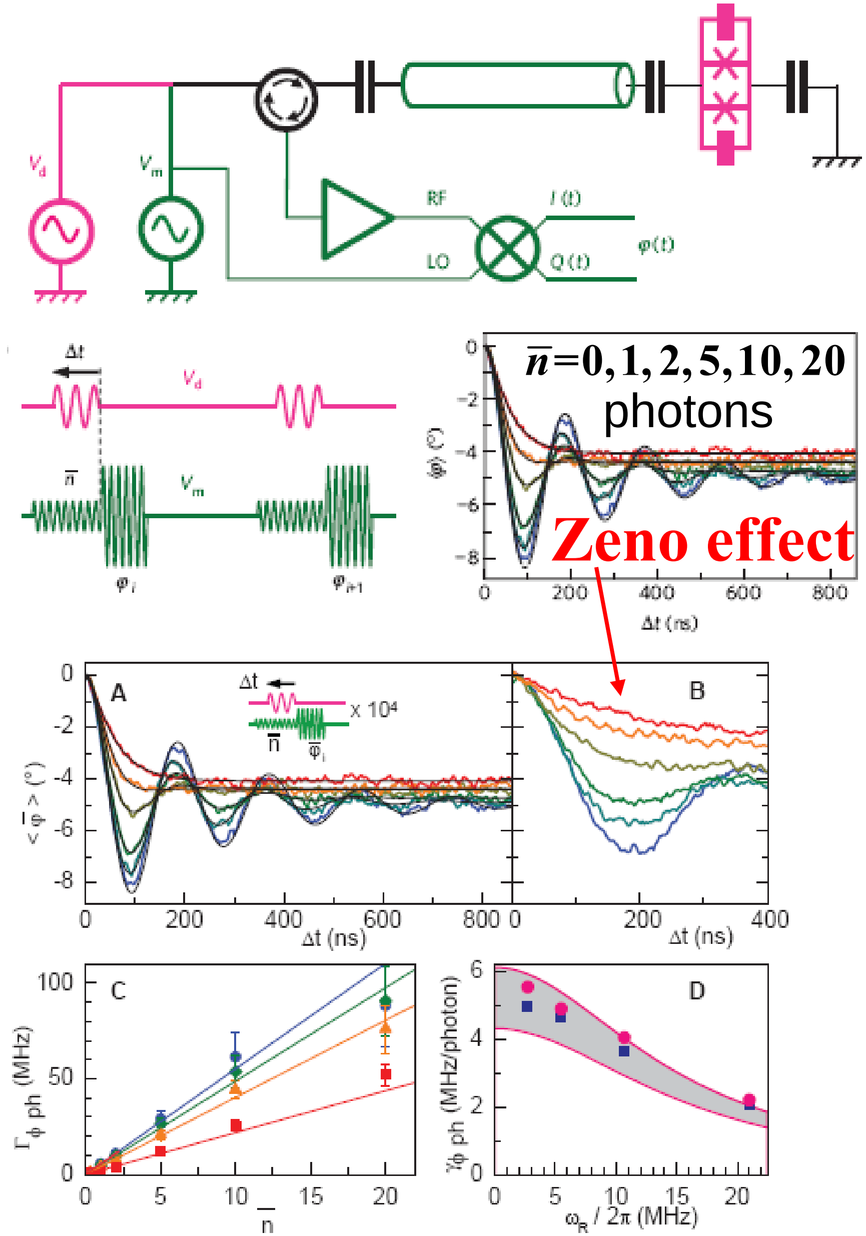


FIG. 3 (color online). The spectral amplitude of the tank voltage for HF powers $P_a < P_b < P_c$ at 868 MHz, detected using the setup of Fig. 1. The bottom curve corresponds to the background noise without an HF signal. The inset shows normalized voltage spectra for seven values of HF power, with background subtracted. The shape of the resonance, being determined by the tank circuit, is essentially the same in each

low-bandwidth tank ⇒ qubit monitoring is impossible

Saclay experiment



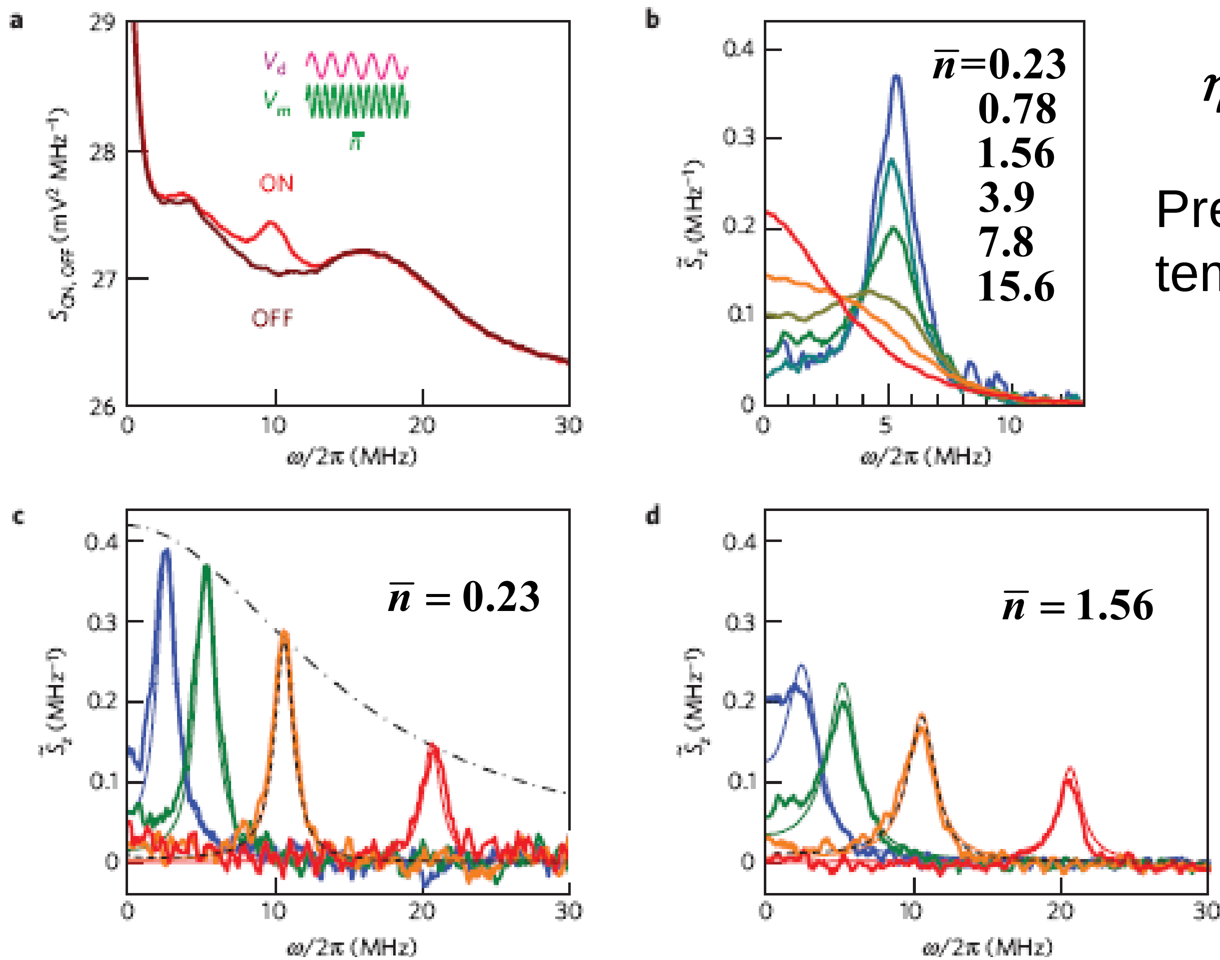
A.Palacios-Laloy, F.Mallet, F.Nguyen, P. Bertet, D. Vion, D. Esteve, A.K., Nature Phys., 2010

- superconducting charge qubit (transmon) in circuit QED setup
- microwave reflection from cavity: full collection, only phase modulation
- driven Rabi oscillations

Standard (not continuous) measurement here: ensemble-averaged Rabi starting from ground state

Now continuous measurement

Palacios-Laloy et al., 2010



$$\eta = \frac{\Delta S}{4S} \simeq 10^{-2}$$

Pre-amplifier noise temperature T_N= 4 K

$$\frac{1}{1+\frac{2T_N}{\hbar\omega}} \approx 0.03$$

Theory by dashed lines, very good agreement

Violation of Leggett-Garg inequalities

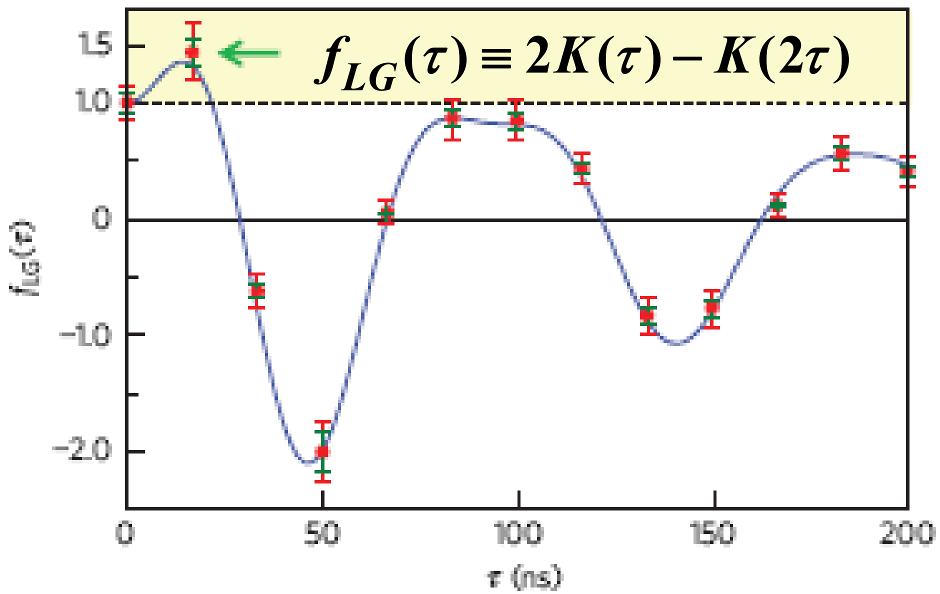
Palacios-Laloy et al., 2010

In time domain

Rescaled to qubit z-coordinate $K(\tau) \equiv \langle z(t)\, z(t+\tau) \rangle$

$$K(\tau_1) + K(\tau_2) - K(\tau_1 + \tau_2) \le 1 \quad \Rightarrow 2K(\tau) - K(2\tau) \le 1$$

$$f_{LG}(0) = K(0) = \langle z^2 \rangle \qquad \langle z^2 \rangle = 1.01 \pm 0.15$$



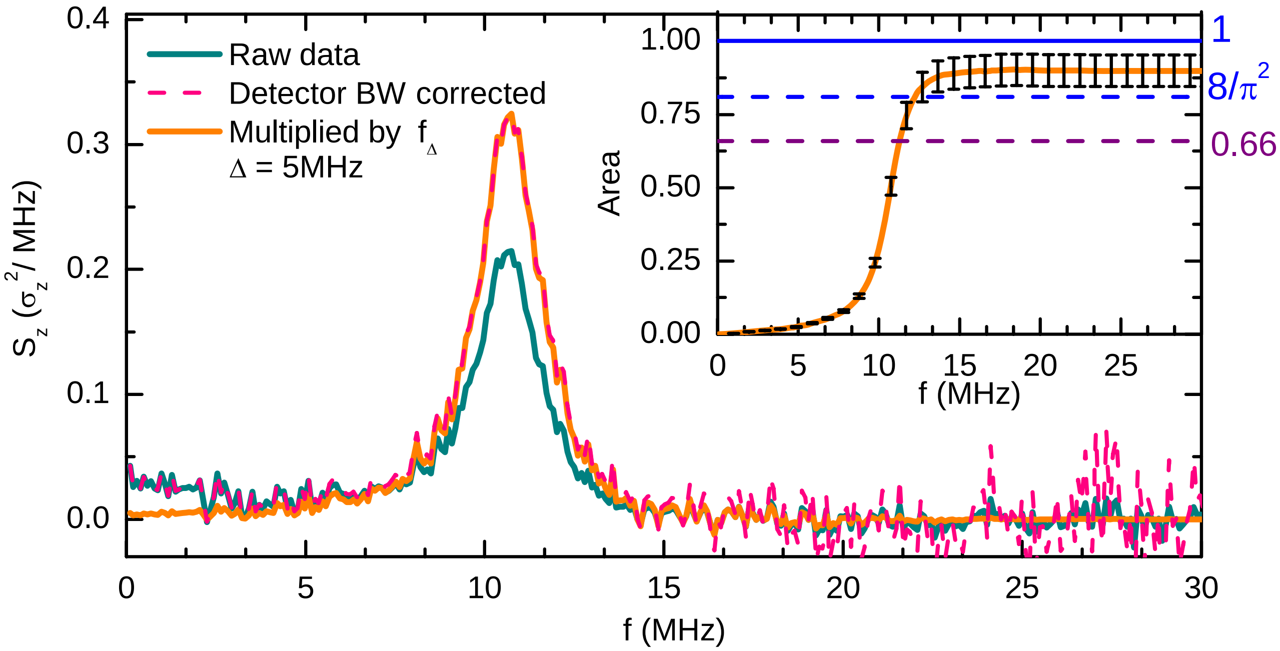
$f_{LG}(17\,\text{ns}) = 1.44 \pm 0.12$ Ideal $f_{LG,max}$=1.5

Standard deviation $\sigma = 0.065 \Rightarrow$ violation by 5σ

Violation of Leggett-Garg inequalities

In frequency domain

courtesy of
Patrice Bertet
(unpublished)



Also violated, but not so well as in time domain

Next topic: Quantum feedback for persistent Rabi oscillations

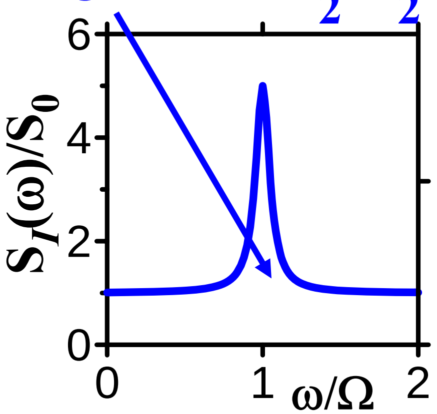
In simple monitoring the phase of persistent Rabi oscillations fluctuates randomly:

$$z(t) = \cos[\Omega t + \varphi(t)] \qquad \text{for } \eta=1$$

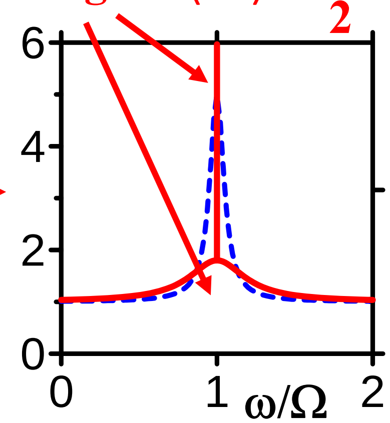
phase noise $\Rightarrow$ finite linewidth of the spectrum

Goal: produce persistent Rabi oscillations without phase noise by synchronizing with a classical signal $z_{desired}(t) = \cos(\Omega t)$

integral $\langle z^2 \rangle = \frac{1}{2} + \frac{1}{2} = 1$



integral $\langle z^2 \rangle = \frac{1}{2}$



$$I(t) = I_0 + \frac{\Delta I}{2} z(t) + \xi(t)$$

$$S_I = S_0 + \frac{\Delta I^2}{4} S_{zz} + \frac{\Delta I}{2} S_{\xi z}$$

synchronized

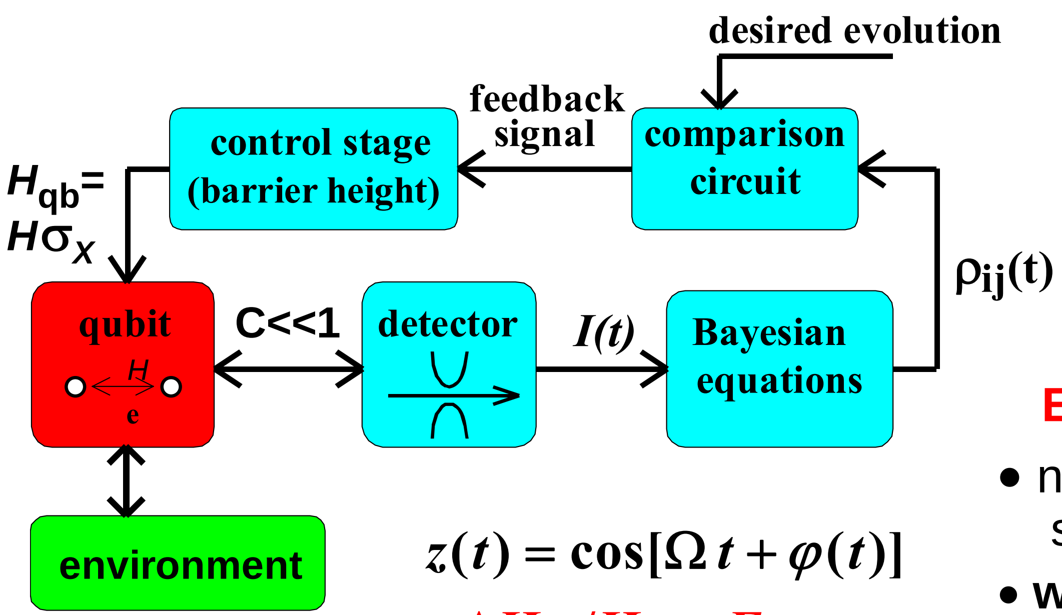
cannot synchronize

Several ways to organize quantum feedback

First idea: Bayesian feedback

(most straightforward but most difficult experimentally)

The wavefunction is monitored via Bayesian equations, and then usual (linear) feedback of the Rabi phase



$$z(t) = \cos[\Omega t + \varphi(t)]$$

$$\Delta H_{FB}/H = -F\times\varphi$$

$$\Delta\Omega/\Omega = -F\times\varphi$$

Ruskov & A.K., 2002

How to characterize feedback efficiency/fidelity?

D = average scalar product of desired and actual vectors on Bloch sphere

$$D = 2\langle \mathrm{Tr}\,\rho_{\text{desired}}\,\rho\rangle - 1$$

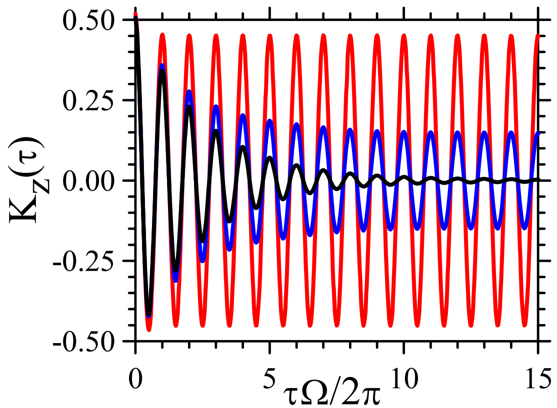
Experimental difficulties:

- necessity of **very fast** real-time solution of Bayesian equations
- **wide bandwidth** ($\gg\Omega$, GHz-range) of the line delivering noisy signal $I(t)$ to the “processor”

Performance of Bayesian quantum feedback

(no extra environment)

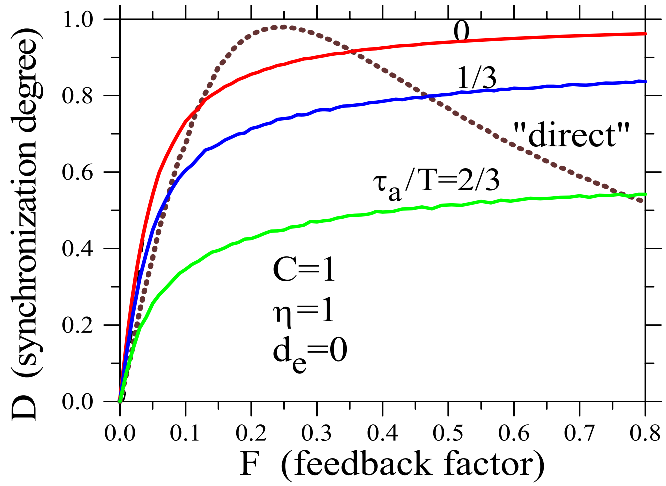
Qubit correlation function



$$K_z(\tau) = \frac{\cos \Omega t}{2} \exp\left[\frac{C}{16F}\left(e^{-2FH\tau/\hbar} - 1\right)\right]$$

(for weak coupling and good fidelity)

Fidelity (synchronization degree)



$C = \hbar(\Delta I)^2 / S_I H$ – coupling

τ_a^{-1} – available bandwidth

F – feedback strength

$$D = 2\langle \mathrm{Tr}\rho\rho_{desir}\rangle - 1$$

Detector current correlation function

$$K_I(\tau) = \frac{(\Delta I)^2}{4} \frac{\cos \Omega t}{2} \left(1 + e^{-2FH\tau/\hbar}\right) \times \exp\left[\frac{C}{16F}\left(e^{-2FH\tau/\hbar} - 1\right)\right] + \frac{S_I}{2}\delta(\tau)$$

For ideal detector and wide bandwidth, fidelity can be arbitrarily close to 100%

$$D = \exp(-C/32F)$$

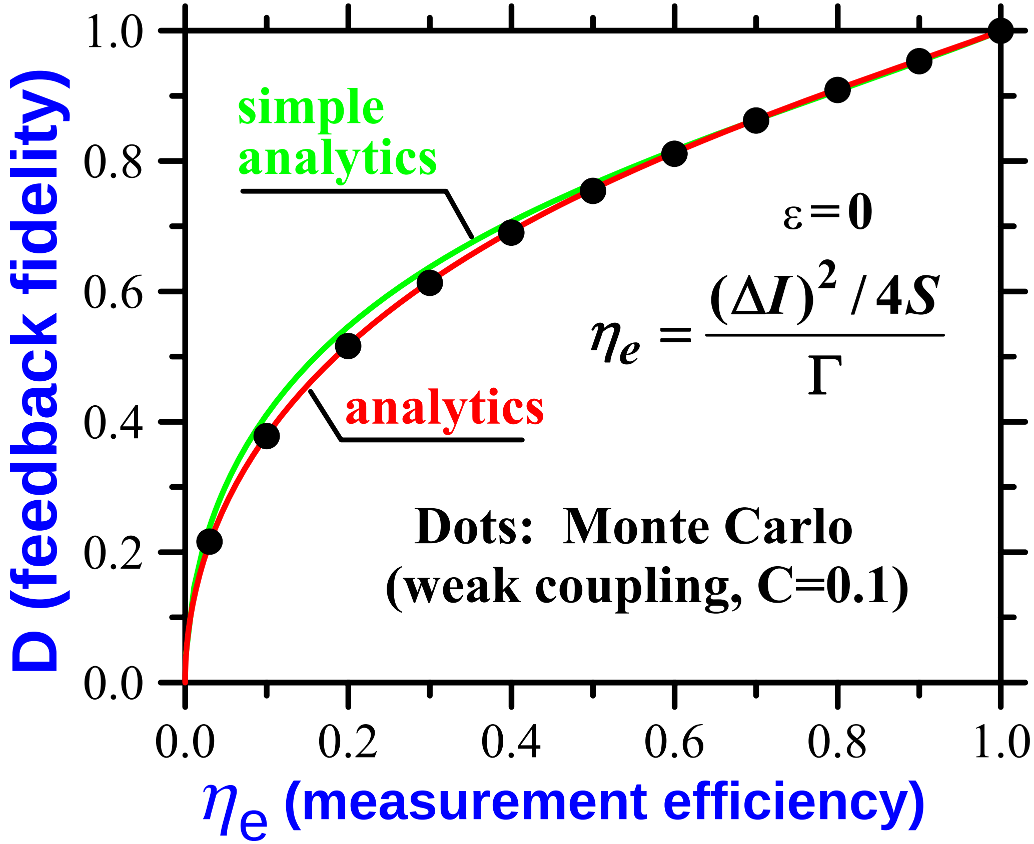
Ruskov-A.K., PRB 66, 041401(R) (2002)

Effect of non-ideal detector and extra environment

Zhang-Ruskov-A.K., 2005

Effective ideality of measurement η_e

$$\frac{1}{\eta_e} - 1 = \frac{\text{decoherence coupling}}{\text{measurement coupling}}$$



$$\eta_e = \frac{(\Delta I)^2/4S}{\Gamma}$$

Simple analytics:

$$D = \sqrt{1 + \frac{1}{2\eta_e} - \sqrt{\left(1+\frac{1}{2\eta_e}\right)^2 - 2}}$$

Full analytics:

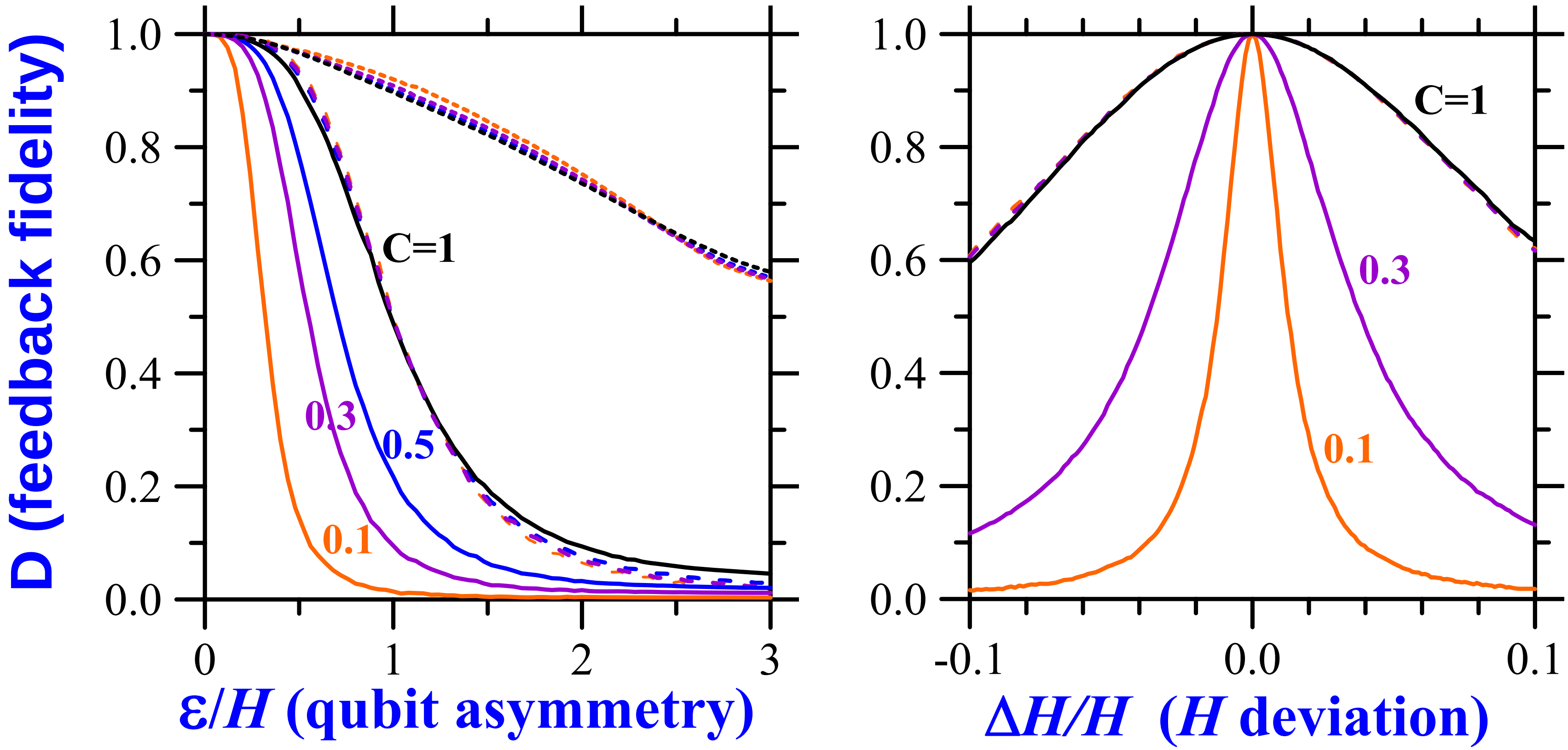
$$D = \frac{\int_0^1 P^2 G(P^2)\, dP}{\int_0^1 P\, G(P^2)\, dP}$$

$$G(P^2) = (1 - P^2)^{-5/2} \times$$
$$\times \exp[-(\eta_e^{-1} - 1)/2(1 - P^2)]$$

Example:

η_e=0.1, then fidelity 0.4 (still quite good!)

Effect of qubit parameter deviations (ε, *H*)

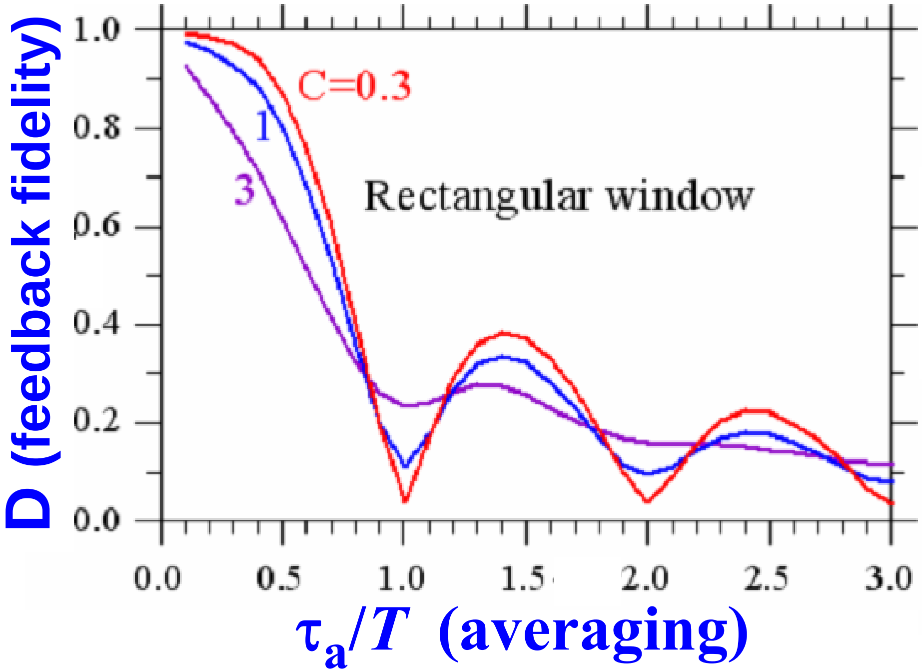


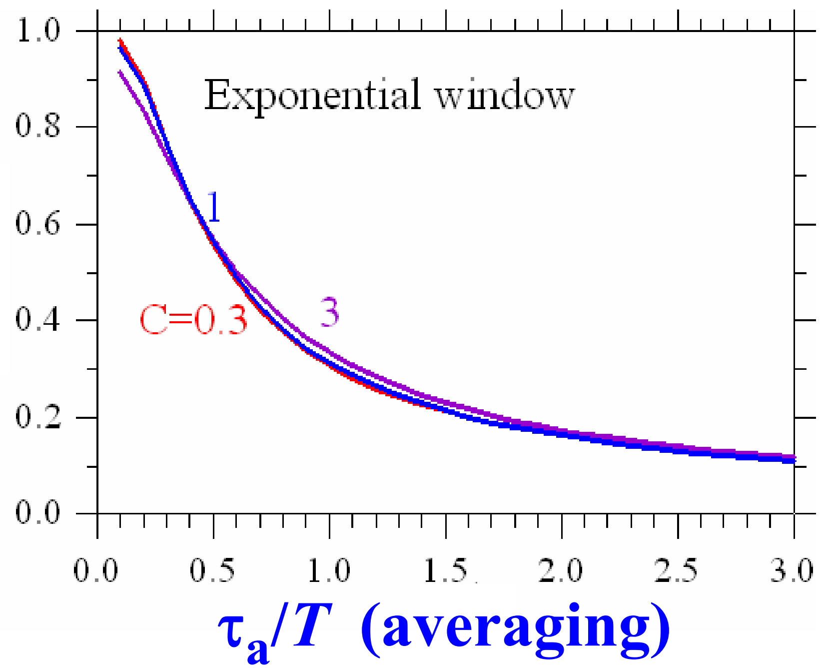
Feedback operation is robust against small unknown deviations of qubit parameters

Zhang-Ruskov-A.K., 2005

Effect of finite bandwidth

Averaging of the detector signal over time τ_a (using rectangular or exponential window) leads to information loss and therefore to the decrease of feedback fidelity D.



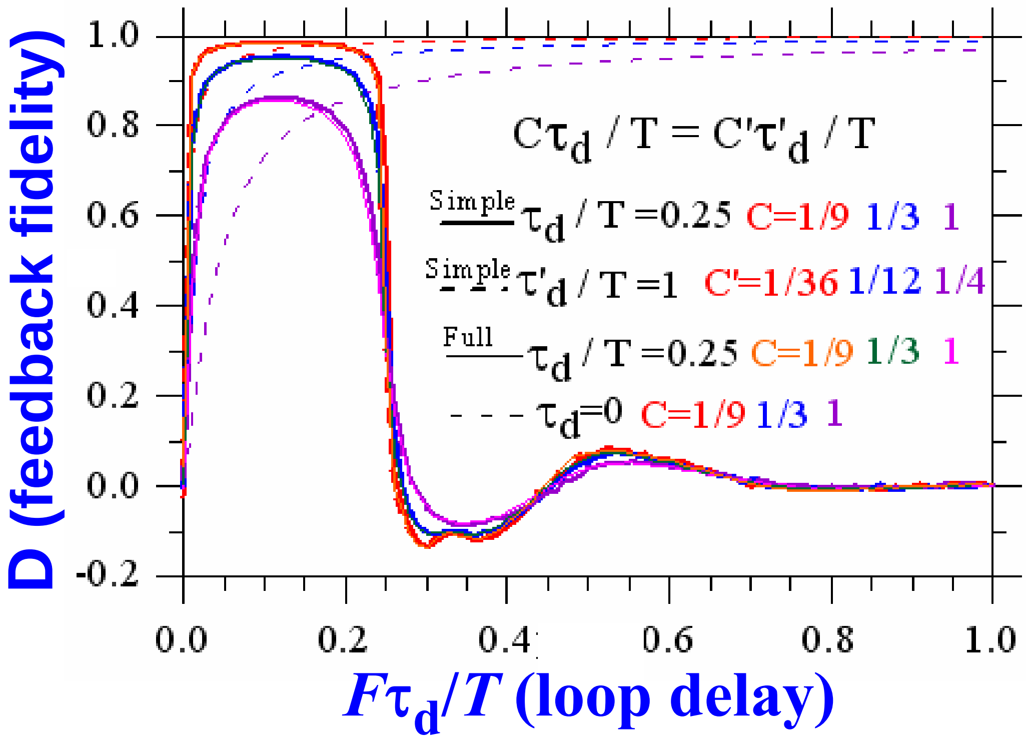


For good feedback performance the averaging time τ_a should be much smaller than Rabi period $T=2\pi/\Omega$

(**signal bandwidth >> Rabi frequency**)

Zhang-Ruskov-A.K., 2005

Effect of feedback loop delay



F – feedback strength

τ_d – loop delay

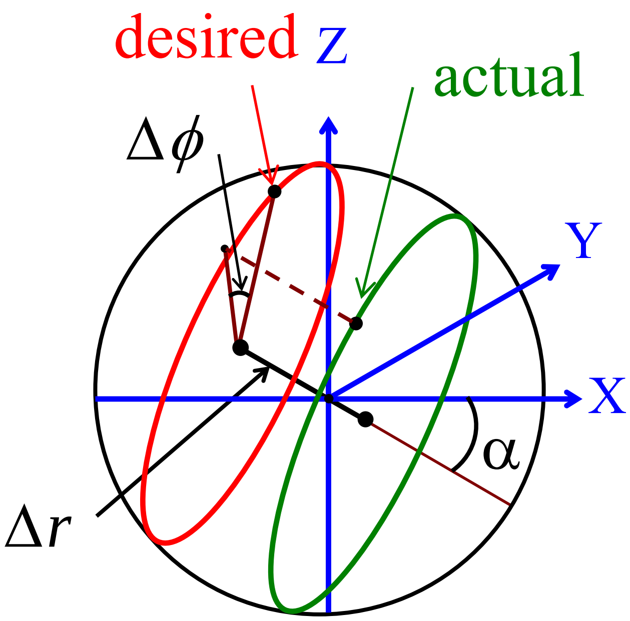
$T=2\pi/\Omega$ – Rabi period

C – detector coupling

Feedback loop becomes unstable ("oversteering") at $F\tau_d/T>1/4$

Zhang-Ruskov-A.K., 2005

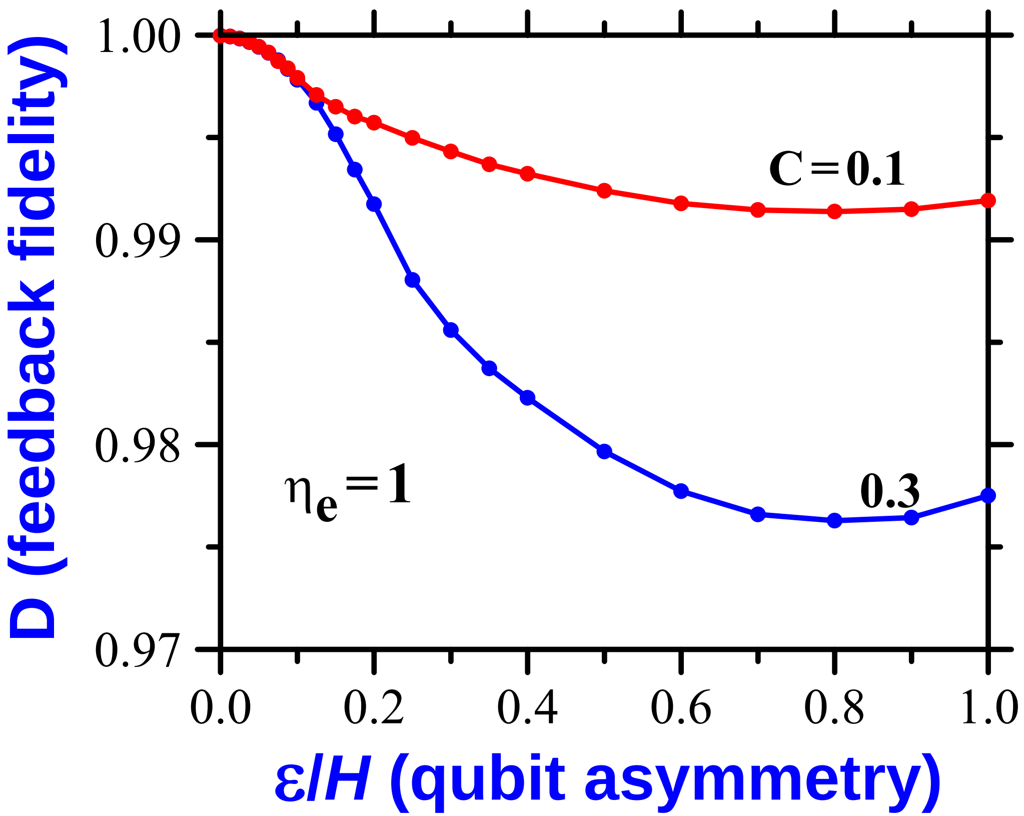
Control of energy-asymmetric qubit ($\varepsilon \neq 0$)



Now two degrees of freedom
for deviation: $\Delta\phi$ and Δr

New controller:

$$\Delta H_{FB} = -F\, H\, \Delta\phi - F_r\, H \sin\phi\, \Delta r$$



Even an asymmetric qubit can be efficiently feedback-controlled using only *H*-modulation

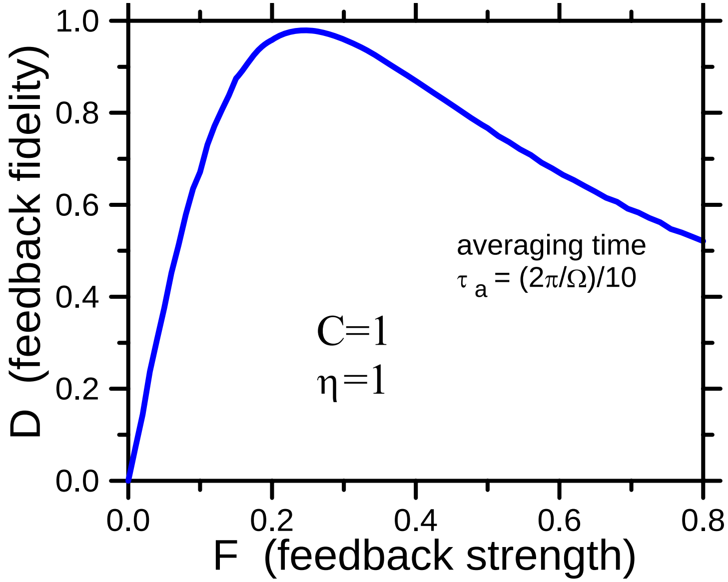
Zhang-Ruskov-A.K., 2005

Second idea: direct feedback

(similar to Wiseman-Milburn, 1993)

Squeezing of an optical cavity field by feedback of the homodyne detection signal (Wiseman-Milburn, 1993) **feedback ~ $I(t)$-I_0**

Our controller: $$\frac{\Delta H_{fb}}{H} = \frac{\Delta\Omega}{\Omega} = F \times \left(\frac{I(t) - I_0}{\Delta I / 2} - \cos(\Omega t) \right) \times \sin(\Omega t)$$

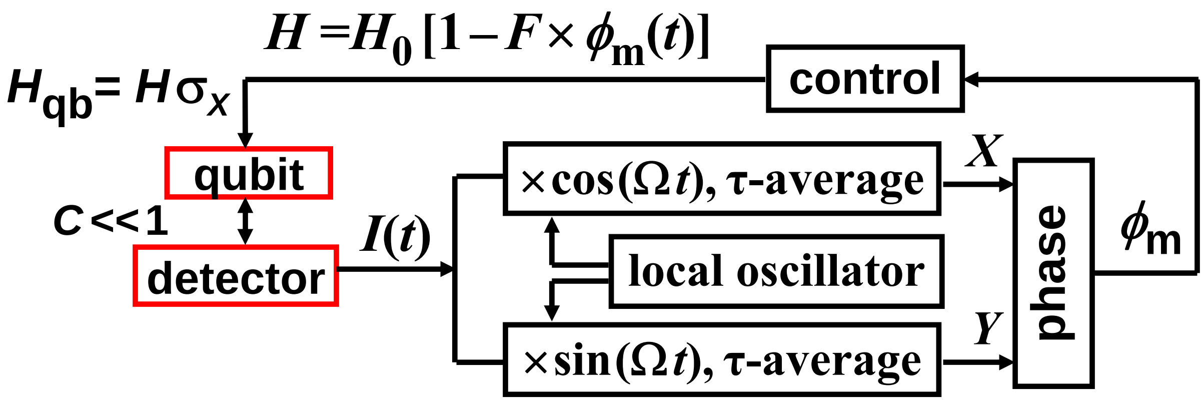


requires optimal feedback strength

Ruskov & A.K., 2002

Third idea: “Simple” quantum feedback

(A.K., 2005)



Idea: **use two quadrature components of the detector current *I*(*t*)**
to monitor approximately the phase of qubit oscillations
(a very natural way for usual classical feedback!)

$$X(t) = \int_{-\infty}^{t} [I(t') - I_0]\,\cos(\Omega t')\,\exp[-(t-t')/\tau]\,dt'$$
$$Y(t) = \int_{-\infty}^{t} [I(t') - I_0]\,\sin(\Omega t')\,\exp[-(t-t')/\tau]\,dt'$$

$$\phi_m = -\arctan(Y/X)$$

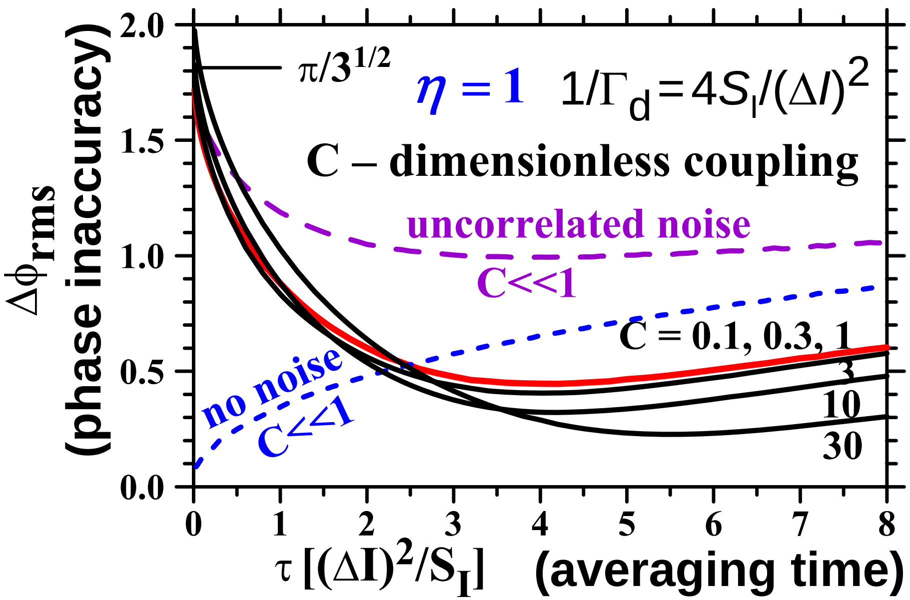
(similar formulas for a tank circuit instead of mixing with local oscillator)

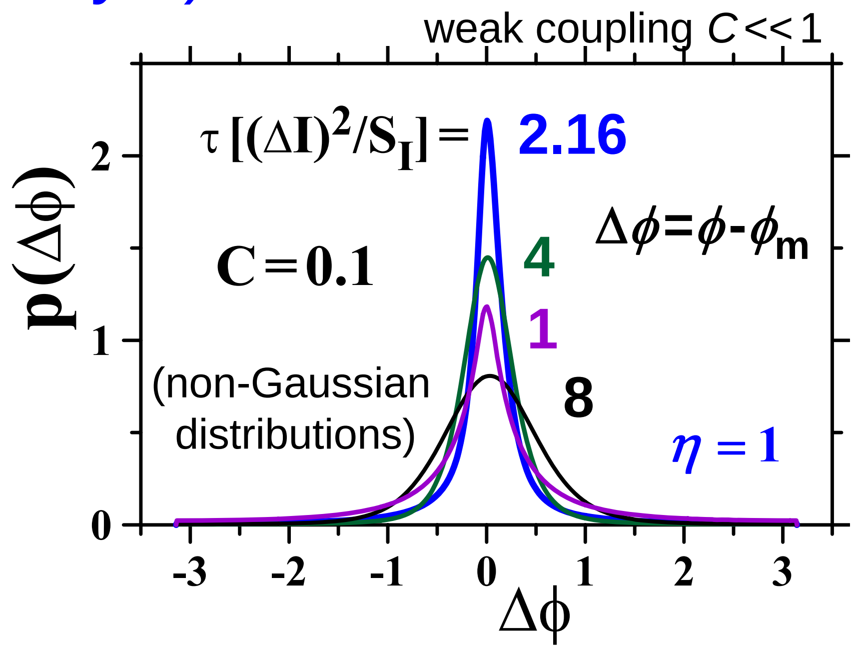
Advantage: **simplicity and relatively narrow bandwidth** $(1/\tau \sim \Gamma_d \ll \Omega)$

Essentially classical feedback. Does it really work?
(Anticipated problem: SNR<4 $\Rightarrow$ not much info in quadratures.)

Alexander Korotkov University of California, Riverside

Accuracy of phase monitoring via quadratures
(no feedback yet)





Noise improves the monitoring accuracy!

(purely quantum effect, "reality follows observations")

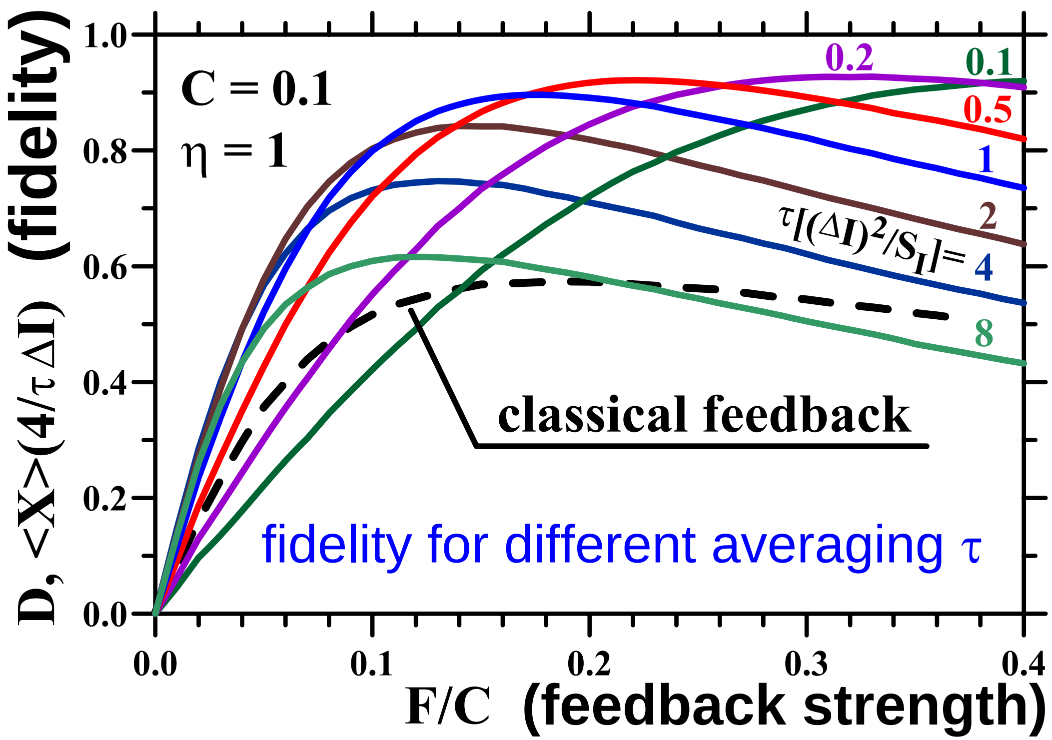
$$d\phi / dt = -[I(t) - I_0]\sin(\Omega t + \phi)(\Delta I / S_I)$$ (actual phase shift, ideal detector)

$$d\phi_m / dt = -[I(t) - I_0]\sin(\Omega t + \phi_m)/(X^2 + Y^2)^{1/2}$$ (observed phase shift)

Noise enters the actual and observed phase evolution in a similar way

Quite accurate monitoring! $\cos(0.44) \approx 0.9$

Simple quantum feedback



weak coupling C

D – feedback efficiency

$$D \equiv 2F_Q - 1$$

$$F_Q \equiv \langle \mathrm{Tr}\, \rho(t)\, \rho_{des}(t) \rangle$$

$D_{max} \approx 90\%$

How to verify feedback operation experimentally?

Simple: just check that in-phase quadrature $\langle X \rangle$ of the detector current is positive

$$D = \langle X \rangle (4/\tau \Delta I)$$

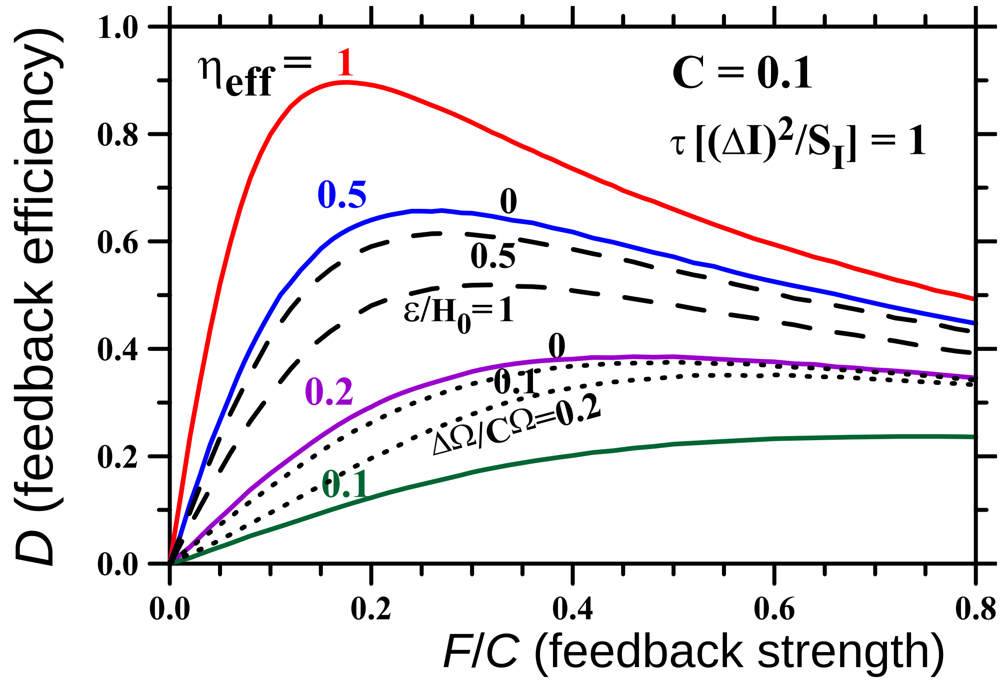
$\langle X \rangle$=0 for *any* non-feedback Hamiltonian control of the qubit

Effect of nonidealities

- nonideal detectors (finite quantum efficiency η)
- qubit energy asymmetry ε
- frequency mismatch $\Delta\Omega$

Quantum feedback still works quite well

(feedback loop must be faster than decoherence)



Main features:

- Fidelity D up to ~90% achievable (for η=1)
- Natural, practically classical feedback setup
- Averaging $\tau \sim 1/\Gamma \gg 1/\Omega$ (narrow bandwidth!)
- Detector efficiency (ideality) $\eta \sim 0.1$ still OK
- Robust to asymmetry ε and frequency shift $\Delta\Omega$
- Simple verification: positive in-phase quadrature $\langle X \rangle$

Simple enough experiment?!

Quantum feedback in cQED setup

We have to undo both effects: disturbance of qubit phase ("classical")
and disturbance of Rabi phase ("spooky")

$\Rightarrow$ have to control both qubit parameters

Phase-sensitive case

$$\begin{cases} \dfrac{\rho_{gg}(\tau)}{\rho_{ee}(\tau)} = \dfrac{\rho_{gg}(0)}{\rho_{ee}(0)} \dfrac{\exp[-(\bar{I} - I_g)^2/2D]}{\exp[-(\bar{I} - I_e)^2/2D]} \\ \rho_{ge}(\tau) = \rho_{ge}(0) \sqrt{\dfrac{\rho_{gg}(\tau)\,\rho_{ee}(\tau)}{\rho_{gg}(0)\,\rho_{ee}(0)}} \exp(iK\bar{I}\tau) \end{cases}$$

Use the same signal for both,
direct feedback for qubit energy,
+some feedback for μwave amplitude

Phase-preserving case

$$\begin{cases} \dfrac{\rho_{gg}(\tau)}{\rho_{ee}(\tau)} = \dfrac{\rho_{gg}(0)}{\rho_{ee}(0)} \dfrac{\exp[-(\bar{I} - I_g)^2/2D]}{\exp[-(\bar{I} - I_e)^2/2D]} \\ \rho_{ge}(\tau) = \rho_{ge}(0) \sqrt{\dfrac{\rho_{gg}(\tau)\,\rho_{ee}(\tau)}{\rho_{gg}(0)\,\rho_{ee}(0)}} \exp(iK\bar{Q}\tau) \end{cases}$$

Use different quadratures,
for two feedback channels

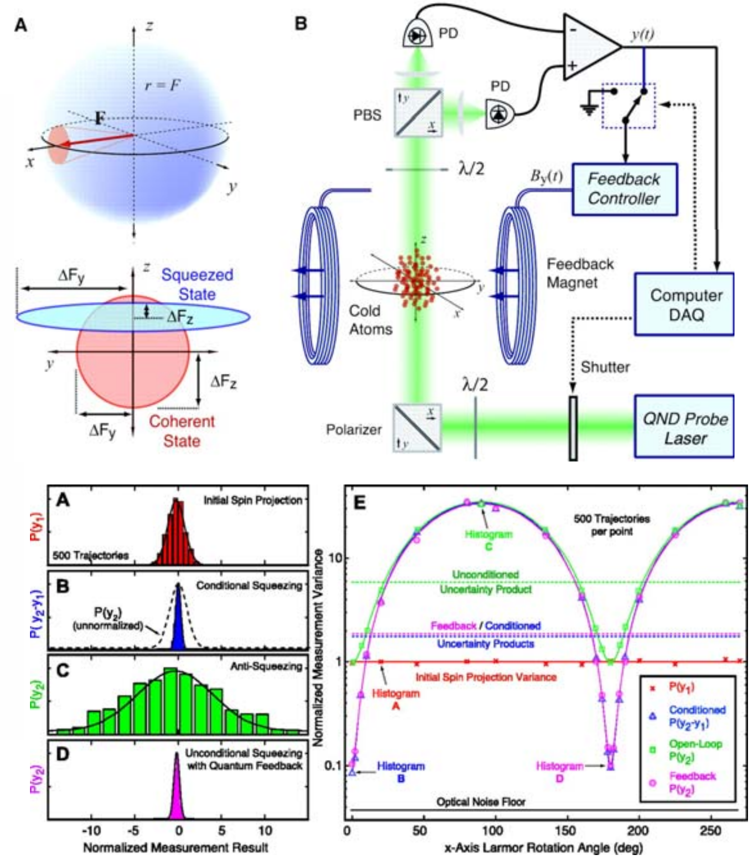
Quantum feedback in optics

First experiment: Science 304, 270 (2004)

Real-Time Quantum Feedback Control of Atomic Spin-Squeezing

JM Geremia,* John K. Stockton, Hideo Mabuchi

Real-time feedback performed during a quantum nondemolition measurement of atomic spin-angular momentum allowed us to influence the quantum statistics of the measurement outcome. We showed that it is possible to harness measurement backaction as a form of actuation in quantum control, and thus we describe a valuable tool for quantum information science. Our feedback-mediated procedure generates spin-squeezing, for which the reduction in quantum uncertainty and resulting atomic entanglement are not conditioned on the measurement outcome.



First detailed theory:

H.M. Wiseman and G. J. Milburn,
Phys. Rev. Lett. 70, 548 (**1993**)

Quantum feedback in optics

First experiment: Science 304, 270 (2004)

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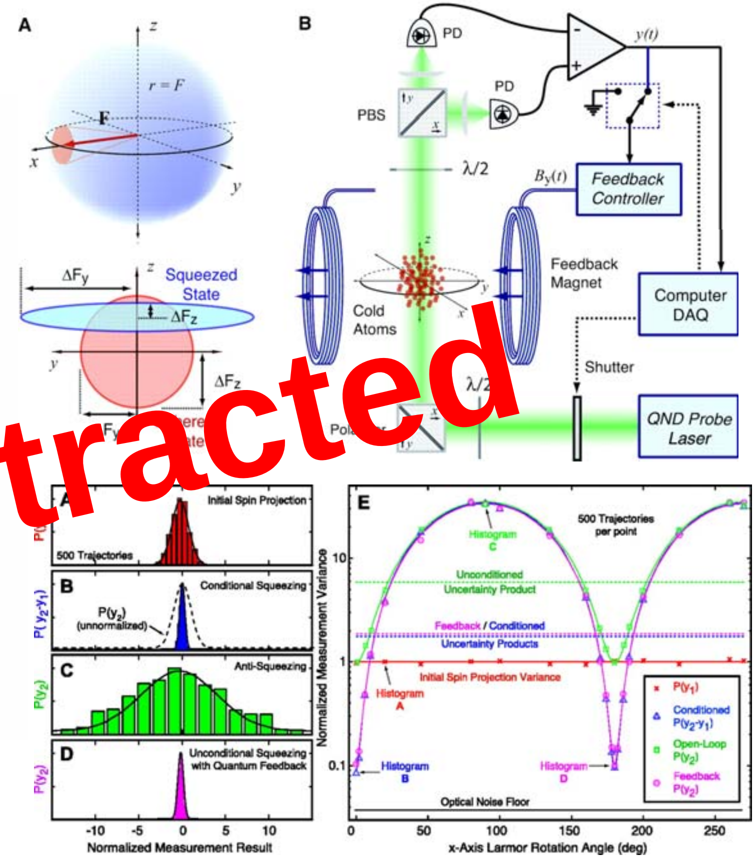
paper retracted

PRL 94, 203002 (2005) also withdrawn



More recent experiment:
Cook, Martin, Geremia, Nature 446, 774 (2007)
(coherent state discrimination)

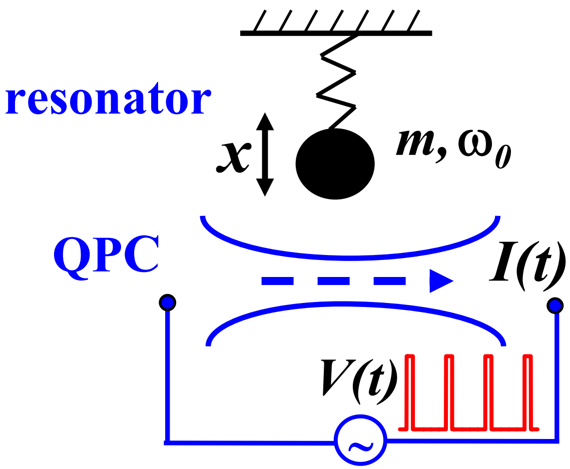
Also:
Reiner, Smith, Orozco, Wiseman, Gambetta, PRA 70, 023819 (2004), etc.

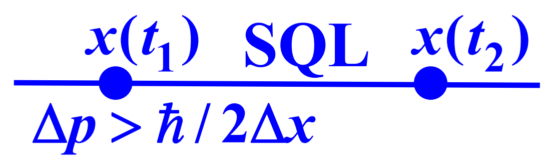
Stroboscopic QND squeezing of a nanoresonator

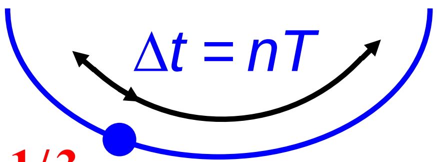
NEMS review: Blencowe, 2004

Ruskov, Schwab, A.K., 2005

Based on old (1978) Braginsky-Khalili-Thorne idea
Difference: weak measurement, quantum feedback

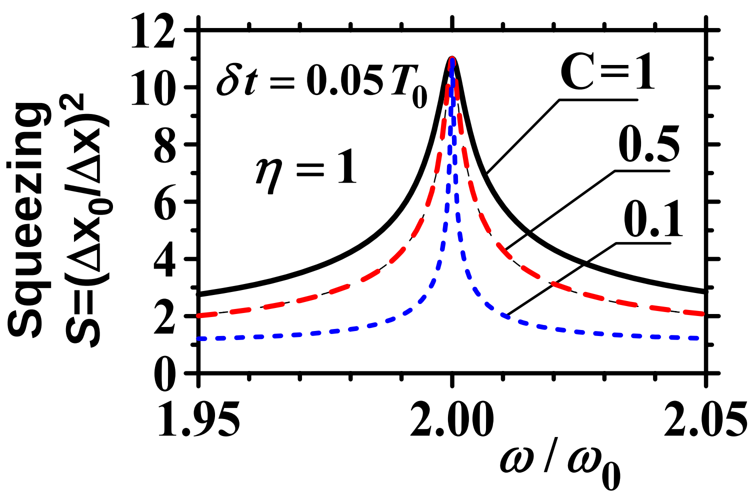


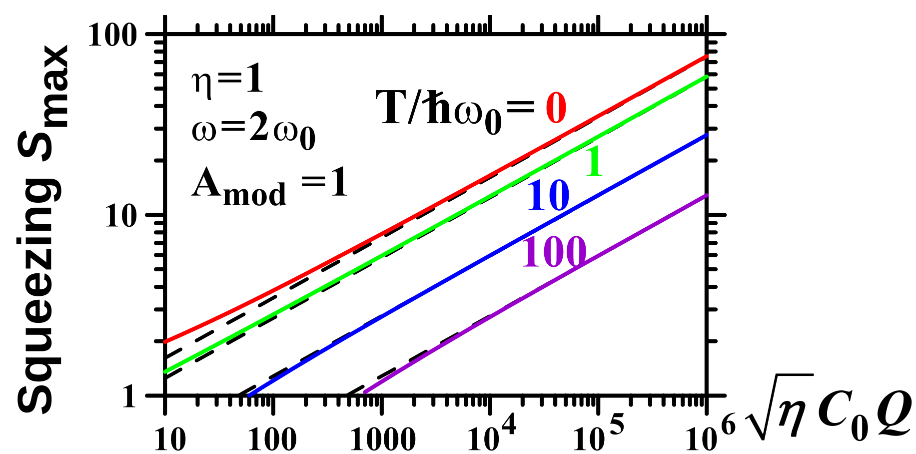




$$S_{\max} = \frac{3}{4}\left[\frac{\sqrt{\eta}\,C_0 Q}{\coth(\hbar\omega_0/2T)}\right]^{1/3}$$

C_0 – coupling with detector, η – detector efficiency, T – temperature, Q – resonator Q-factor





Beats the Standard Quantum Limit

In experiment (2005) $\eta^{1/2}C_0Q \sim 0.1$

Potential application: ultrasensitive force measurements

One more experimental proposal:

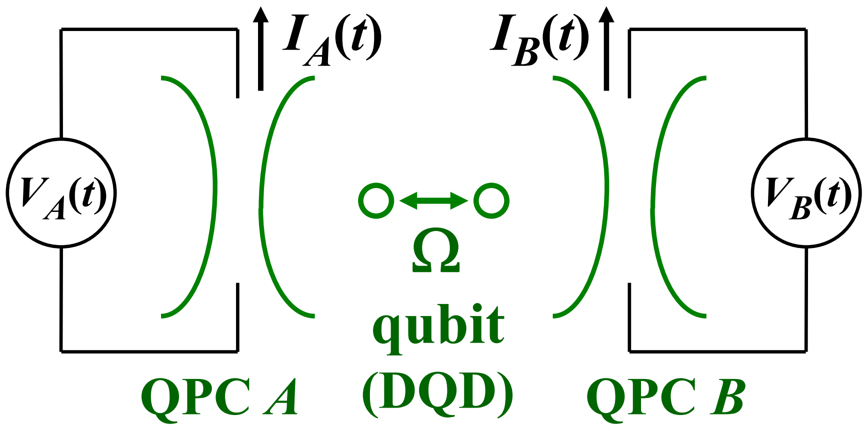
Persistent Rabi oscillations revealed in low-frequency noise

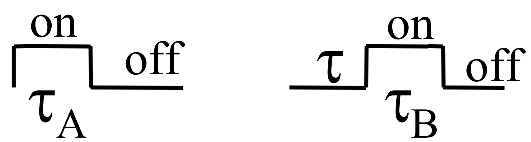
Hopefully, simple enough for semiconductor qubits

Goal: something easy for experiment, but still with a non-trivial measurement effect

A.K., arXiv:1004.0220, PRB-2011

Setup: one qubit & two detectors





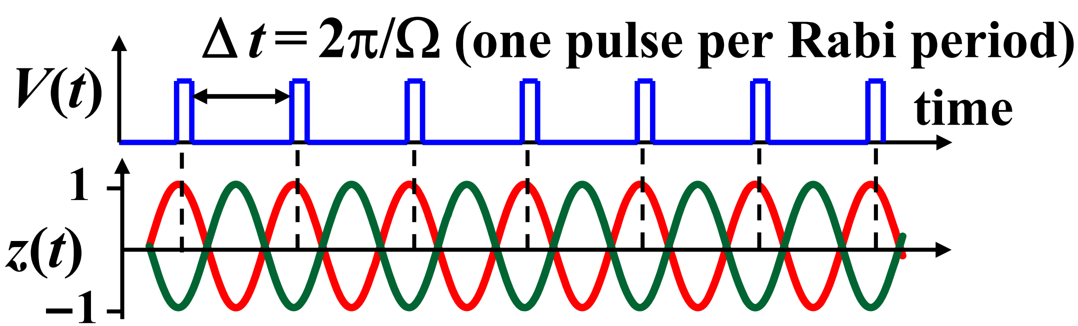
For single-shot measurements partial collapse can be revealed via **correlations** of $\int I_A$ and $\int I_B$.

(A.K., 2001)

Single-shot measurements are not yet available

⇒ use train (comb) of meas. pulses in QND regime

One-detector stroboscopic QND measurement



Stroboscopic QND:

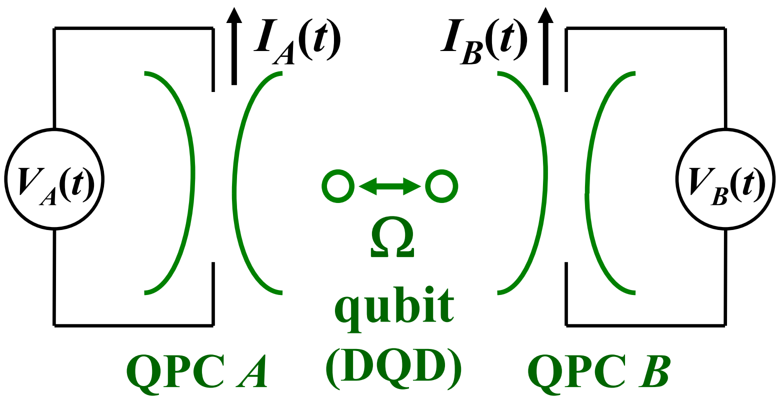
Braginsky, Vorontsov, Khalili, 1978

Jordan, Buttiker, 2005

Jordan, Korotkov, 2006

Stroboscopic QND measurement synchronizes (!) phase of persistent Rabi oscillations (attracts to either 0 or π)

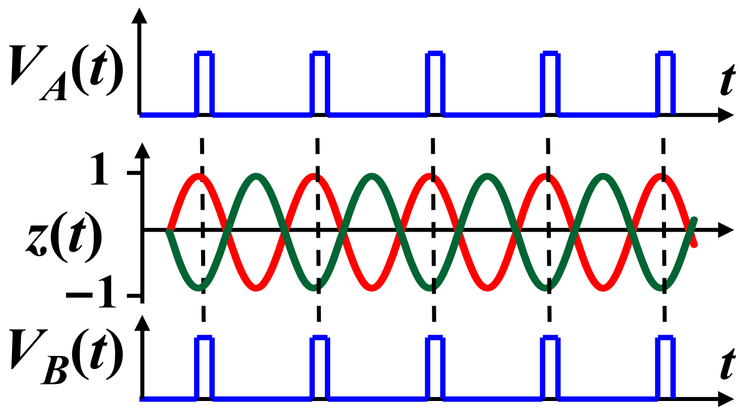
Idea of experiment



Perfect QND ⇒ correlation/anticorr. between currents in two detectors

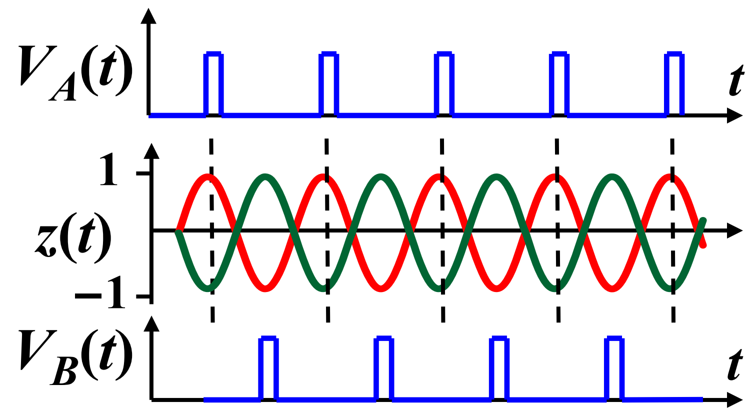
Imperfect QND ⇒ random switching between two Rabi phases (0 and π) ⇒ low-frequency telegraph noise

same combs on V_A and V_B



anticorrelation between I_A and I_B

π-shifted combs on V_A and V_B

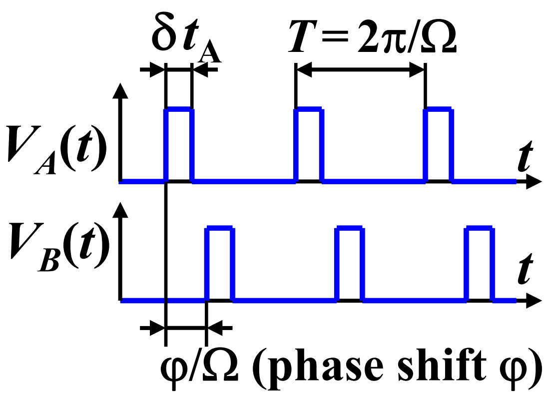


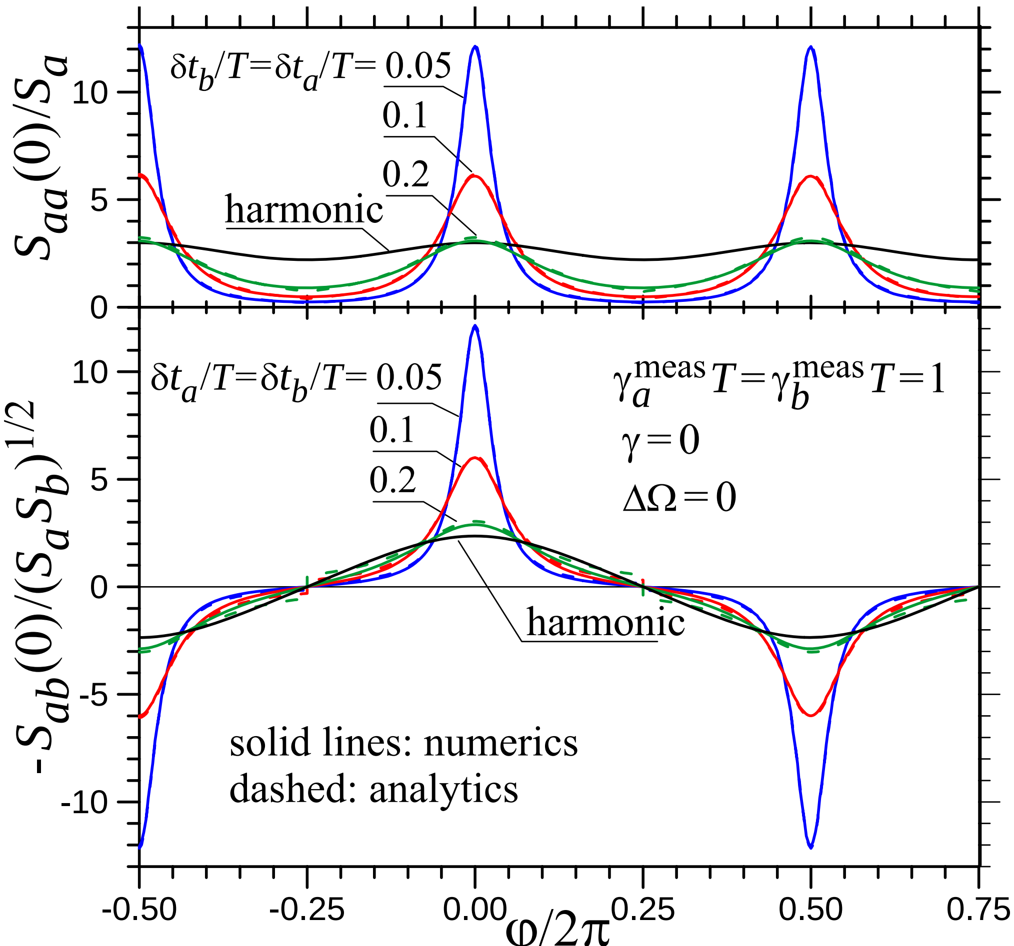
correlation (still QND!)

correlation/anticorrelation between low-frequency (telegraph) noises indicates presence of persistent Rabi oscillations

Noise dependence on phase shift between measurement combs

(indicates self-synchronized persistent Rabi oscillations)





Analytics:

$$S_A(\omega) = \underbrace{S_A \frac{\delta t_A}{T}}_{\text{shot noise}} + \underbrace{\left(\frac{\delta t_A}{T}\right)^2 \frac{(\Delta I_A)^2/2\Gamma_S}{1+(\omega/2\Gamma_S)^2}}_{\text{telegraph noise}}$$

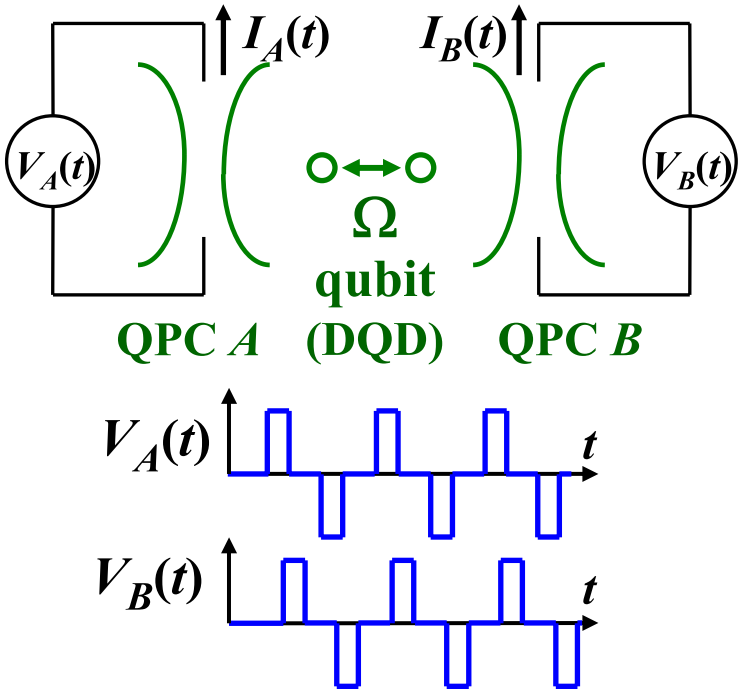
$$S_{AB}(\omega) = \pm \frac{\delta t_A \delta t_B}{T^2} \, \frac{\Delta I_A \Delta I_B / 2\Gamma_S}{1+(\omega/2\Gamma_S)^2}$$

(fully correlated/anticorrelated noise)

$$\Gamma_S = \frac{\varphi^2 M_A M_B + (\Delta\Omega T)^2}{2T(M_A+M_B)} + \frac{\delta t_A^2 M_A + \delta t_B^2 M_B}{6\,T^2/\pi^2} + \frac{1}{4T_2}$$

(switching rate)

$$M_{A,B} = \delta t_{A,B} (\Delta I_{A,B})^2 / 4S_{A,B}$$



Estimates

Assume:

QPC current $I = 100$ nA

response $\Delta I/I = 0.1$

duty cycle δt/T=0.1 (symmetric)

Rabi frequency ~ 2 GHz

Then: "attraction" (collapse) time 2 ns (few Rabi periods)

switching rate $\Gamma_S \simeq \frac{1}{4T_2} + \frac{1}{120\,\text{ns}} + \frac{\varphi^2}{15\,\text{ns}}$ (many Rabi periods)

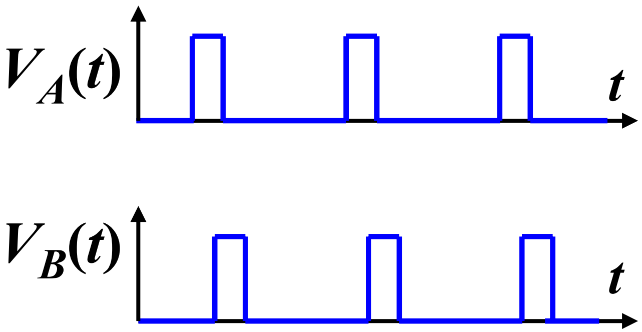
need $T_2 > 3\,\text{ns}$

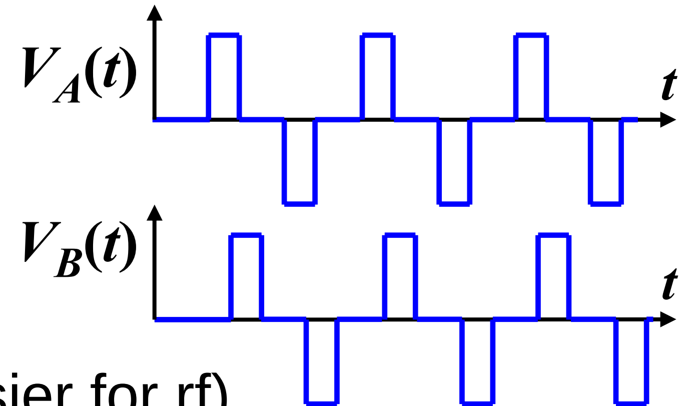
$$\frac{S_{\text{telegraph}}}{S_{\text{shot}}} \simeq \min(60, \frac{T_2}{0.5\,\text{ns}})$$ (relatively large noise signal)

seems to be doable

A.K., arXiv:1004.0220

Useful modification

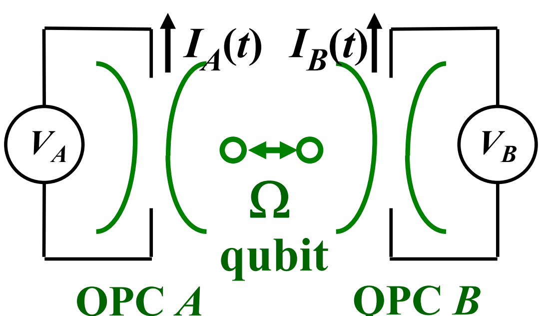


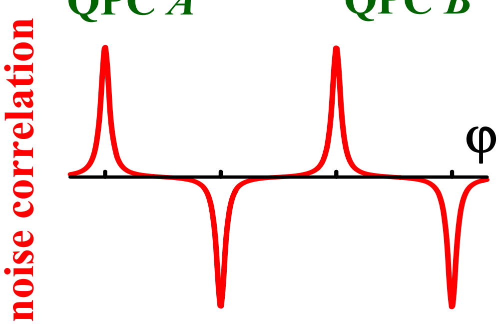


(zero average, easier for rf)

(harmonic rf is also OK)

Any alternative explanation?





1) no oscillations – then no corr./anticorr.

2) unsynchronized Rabi oscillations – then different dependence on φ (cos φ instead of φ^{-2}); also $\int S_{telegr}(f)\, df$ at least twice smaller

3) resonant frequency - driven Rabi?
Then oscillations between |g⟩ and |e⟩ (both do not give a signal) with different frequency. Driven Rabi <u>decreases</u> corr./anticorr. (not an alternative explanation, but should be avoided)

Good news: both phases insensitive to driven Rabi

Conclusions

- Rabi oscillations are persistent (non-decaying) if weakly measured continuously
- Spectral density of non-decaying Rabi oscillations has been measured in a superconducting qubit, also Leggett-Garg inequalities violated
- Persistent Rabi oscillations may be synchronized via quantum feedback; several types of feedback are possible (e.g., Bayesian, direct, simple)
- Hopefully, more experiments on persistent Rabi oscillations will be realized (e.g., quantum feedback in superconducting qubits and simple experiments in semiconductor qubits)