Probing "inside" quantum collapse with solid-state qubits

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Outline:

- What is "inside" collapse? Bayesian framework.
  - broadband meas. (double-dot qubit & QPC)
  - narrowband meas. (circuit QED setup)
- Realized experiments (partial collapse, uncollapse, persistent Rabi oscillations)
- Quantum feedback of Rabi oscillations

Quantum mechanics =

Schrödinger equation (evolution)

+

collapse postulate (measurement)

1) Probability of measurement result $p_r = |\langle \psi | \psi_r \rangle|^2$

2) Wavefunction after measurement $= \psi_r$

- State collapse follows from common sense
- Does not follow from Schrödinger Eq. (contradicts)

What is "inside" collapse?

What if collapse is stopped half-way?

What is the evolution due to measurement? (What is "inside" collapse?)

- controversial for last 80 years, many wrong answers, many correct answers
- solid-state systems are more natural to answer this question

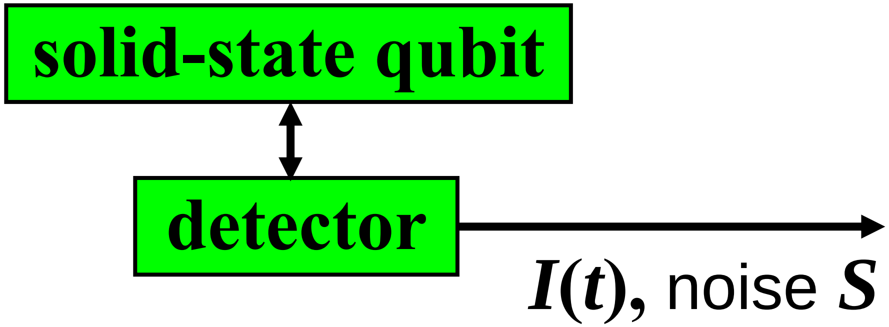
Various approaches to non-projective (weak, continuous, partial, generalized, etc.) quantum measurements

Names: Davies, Kraus, Holevo, Mensky, Caves, Knight, Walls, Carmichael, Milburn, Wiseman, Gisin, Percival, Belavkin, etc. (very incomplete list)

Key words: POVM, restricted path integral, quantum trajectories, quantum filtering, quantum jumps, stochastic master equation, etc.

Our limited scope:

(simplest system, experimental setups)



Quantum Bayesian framework

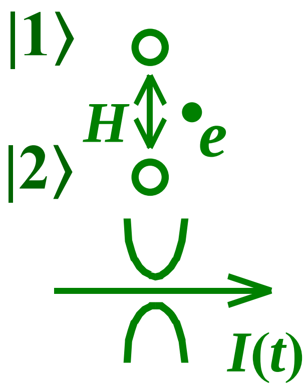
(slight technical extension of the collapse postulate)

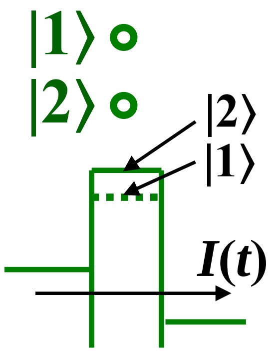
1) Quantum back-action (spooky, physically unexplainable)
   simple: update the state using **information** from measurement and probability concept (Bayes rule)
2) Add "classical" back-action if any (anything with a physical mechanism)
3) Add noise/decoherence if any
4) Add Hamiltonian (unitary) evolution if any

(Practically equivalent to many other approaches: POVM, quantum trajectory, quantum filtering, etc.)

"Typical" setup: double-quantum-dot qubit + quantum point contact (QPC) detector

Gurvitz, 1997





$$H = H_{QB} + H_{DET} + H_{INT}$$

$$H_{QB} = \frac{\varepsilon}{2}\sigma_Z + H\sigma_X$$

$$I(t) = I_0 + \frac{\Delta I}{2} z(t) + \xi(t)$$

const + signal + noise

Two levels of average detector current: I_1 for qubit state $|1\rangle$, I_2 for $|2\rangle$

Response: $\Delta I = I_1 - I_2$ Detector noise: white, spectral density S_I

For low-transparency QPC

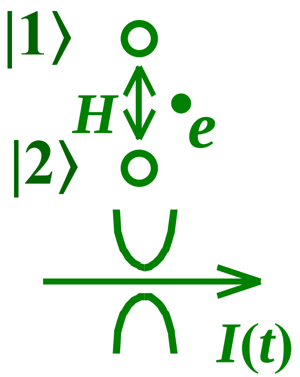
$$H_{DET} = \sum_l E_l a_l^\dagger a_l + \sum_r E_r a_r^\dagger a_r + \sum_{l,r} T(a_r^\dagger a_l + a_l^\dagger a_r)$$

$$H_{INT} = \sum_{l,r} \Delta T\, (c_1^\dagger c_1 - c_2^\dagger c_2)(a_r^\dagger a_l + a_l^\dagger a_r)$$

$$S_I = 2eI$$

Bayesian formalism for DQD-QPC system

$H_{QB} = 0$



Qubit evolution due to measurement (quantum back-action):

$$\psi(t) = \alpha(t)\,|1\rangle + \beta(t)\,|2\rangle \quad \text{or} \quad \rho_{ij}(t)$$

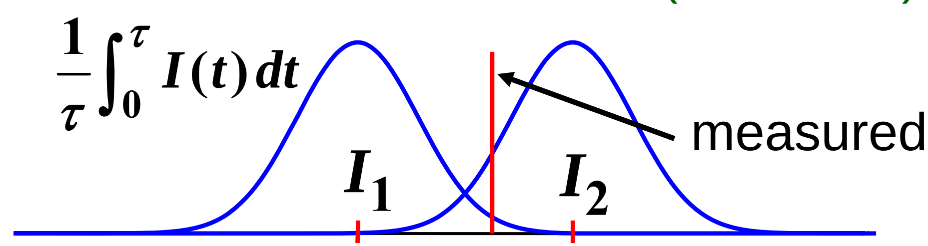
1) $|\alpha(t)|^2$ and $|\beta(t)|^2$ evolve as probabilities,
   i.e. according to the **Bayes rule** (same for ρ_{ii})

2) phases of $\alpha(t)$ and $\beta(t)$ do not change
   (**no dephasing!**), $\rho_{ij}/(\rho_{ii}\,\rho_{jj})^{1/2} = \text{const}$

(A.K., 1998)

Bayes rule (1763, Laplace-1812):

$$\underbrace{P(A_i\,|\,\text{res})}_{\text{posterior probability}} = \frac{\underbrace{P(A_i)}_{\text{prior probab.}}\ \underbrace{P(\text{res}\,|\,A_i)}_{\text{likelihood}}}{\sum_k P(A_k)\,P(\text{res}\,|\,A_k)}$$



So simple because:

1) no entaglement at large QPC voltage
   (classical detector; Markovian)
2) QPC happens to be an ideal detector
3) no Hamiltonian evolution of the qubit

Assumptions needed for the Bayesian formalism:

- Detector voltage is much larger than the qubit energies involved

$eV \gg \hbar\Omega$, $eV \gg \hbar\Gamma$, $\hbar/eV \ll (1/\Omega, 1/\Gamma)$, $\Omega=(4H^2+\varepsilon^2)^{1/2}$

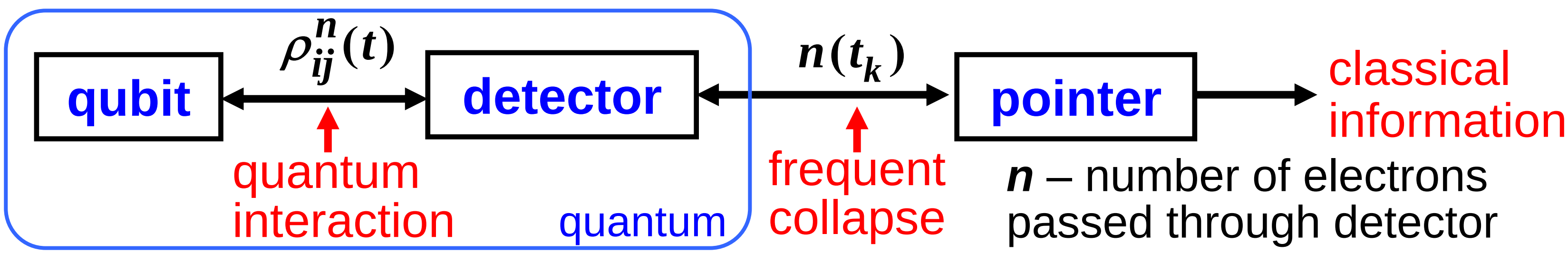
(no coherence in the detector, **classical output**, Markovian approximation)

- Simpler if weak response, $|\Delta I| \ll I_0$, (coupling $C \sim \Gamma/\Omega$ is arbitrary)

Derivations:

1) "logical": via correspondence principle and comparison with decoherence approach (A.K., 1998)

2) "microscopic": Schr. eq. + collapse of the detector (A.K., 2000)

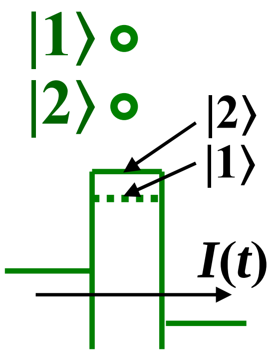


3) from "quantum trajectory" formalism developed for quantum optics (Goan-Milburn, 2001; also: Wiseman, Sun, Oxtoby, etc.)

4) from POVM formalism (Jordan-A.K., 2006)

5) from Keldysh formalism (Wei-Nazarov, 2007)

Now add classical back-action and decoherence

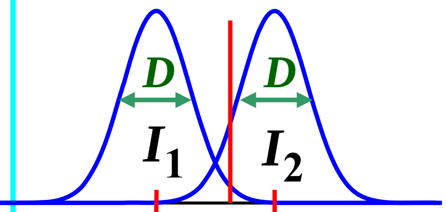


$\Delta I = I_1 - I_2$

noise S_I

$$I_m \equiv \frac{1}{\tau}\int_0^\tau I(t)\,dt$$

$$D = S_I / 2\tau$$



$$H_{qb} = 0$$

quantum backaction (non-unitary, "spooky", "unphysical")

$$\begin{cases} \dfrac{\rho_{11}(\tau)}{\rho_{22}(\tau)} = \dfrac{\rho_{11}(0)}{\rho_{22}(0)} \boxed{\dfrac{\exp[-(I_m - I_1)^2/2D]}{\exp[-(I_m - I_2)^2/2D]}} \\ \rho_{12}(\tau) = \rho_{12}(0) \boxed{\sqrt{\dfrac{\rho_{11}(\tau)\,\rho_{22}(\tau)}{\rho_{11}(0)\,\rho_{22}(0)}}} \exp(iKI_m\tau)\exp(-\gamma\tau) \end{cases}$$

no self-evolution of qubit assumed

decoherence

classical backaction (unitary)

Example of classical ("physical") backaction:

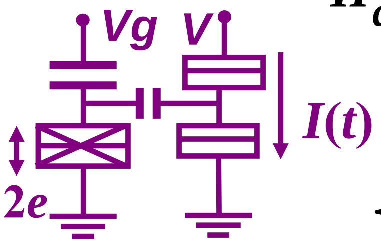
Each electron passed through QPC rotates qubit (sensitivity of tunneling phase for an asymmetric barrier)

$$\arg(T^*\Delta T) \neq 0$$

$$H_{DET} = \sum_l E_l a_l^\dagger a_l + \sum_r E_r a_r^\dagger a_r + \sum_{l,r} T(a_r^\dagger a_l + a_l^\dagger a_r)$$

$$H_{INT} = \sum_{l,r} \Delta T\,(c_1^\dagger c_1 - c_2^\dagger c_2)(a_r^\dagger a_l + a_l^\dagger a_r)$$

Another example of classical back-action



$H_{qb}=0$

$$\begin{cases} \dfrac{\rho_{11}(\tau)}{\rho_{22}(\tau)} = \dfrac{\rho_{11}(0)}{\rho_{22}(0)} \dfrac{\exp[-(I_m - I_1)^2/2D]}{\exp[-(I_m - I_2)^2/2D]} \\ \rho_{12}(\tau) = \rho_{12}(0) \sqrt{\dfrac{\rho_{11}(\tau)\,\rho_{22}(\tau)}{\rho_{11}(0)\,\rho_{22}(0)}} \exp(iKI_m\tau)\exp(-\gamma\tau) \end{cases}$$

quantum backaction (non-unitary, “spooky”, “unphysical”)

no self-evolution of qubit assumed

decoherence

classical backaction (unitary)

$\Delta I = I_1 - I_2$

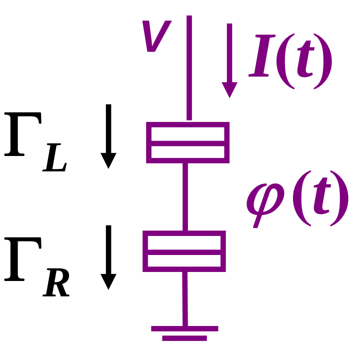
noise S_I

$\bar{I} \equiv \frac{1}{\tau}\int_0^\tau I(t)\,dt$

$D = S_I/2\tau$

Classical backaction:

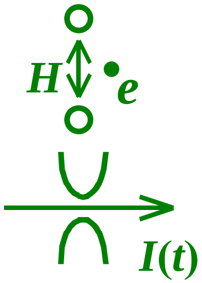
Correlation between voltage and current noises in SET

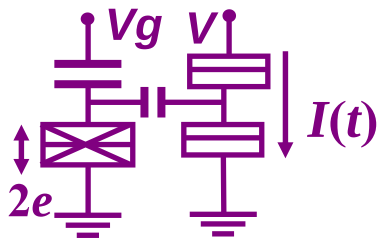


$$S_{I\varphi} \neq 0 \qquad \frac{S_{I\varphi}(0)}{\sqrt{S_{II}(0)S_{\varphi\varphi}(0)}} = \frac{\Gamma_L - \Gamma_R}{\sqrt{2(\Gamma_L^2+\Gamma_R^2)}}$$

(easy to understand when $\Gamma_L \ll \Gamma_R$)

Now add Hamiltonian evolution





- Time derivative of the quantum Bayes rule
- Add unitary evolution of the qubit

$$\dot{\rho}_{11} = -\dot{\rho}_{22} = -2\frac{H}{\hbar}\,\mathrm{Im}\,\rho_{12} + \rho_{11}\rho_{22}\frac{2\Delta I}{S_I}[\underline{\underline{I(t)}} - I_0]$$

$$\dot{\rho}_{12} = i\varepsilon\rho_{12} + i\frac{H}{\hbar}(\rho_{11} - \rho_{22}) + \rho_{12}(\rho_{11} - \rho_{22})\frac{\Delta I}{S_I}[\underline{\underline{I(t)}} - I_0] - \gamma\rho_{12}$$

$\Delta I = I_1 - I_2$, $I_0 = (I_1 + I_2)/2$, S_I – detector noise

(A.K., 1998)

$\gamma = 0$ for QPC

For simulations: $I = I_0 + \frac{\Delta I}{2}(\rho_{11} - \rho_{22}) + \xi$

noise $S_\xi = S_I$

Evolution of qubit *wavefunction* can be monitored if γ=0 (quantum-limited)

Relation to "conventional" master equation

$$\dot{\rho}_{11} = -\dot{\rho}_{22} = -2H\,\mathrm{Im}\,\rho_{12} + \rho_{11}\rho_{22}\frac{2\Delta I}{S_I}[I(t) - I_0]$$

$$\dot{\rho}_{12} = i\varepsilon\rho_{12} + iH(\rho_{11} - \rho_{22}) + \rho_{12}(\rho_{11} - \rho_{22})\frac{\Delta I}{S_I}[I(t) - I_0]$$
$$+ iK[I(t) - I_0]\rho_{12} - \gamma\rho_{12}$$

$\hbar = 1$

response ΔI

noise S_I

Averaging over measurement result $I(t)$ leads to usual master equation:

$$\dot{\rho}_{11} = -\dot{\rho}_{22}/dt = -2H\,\mathrm{Im}\,\rho_{12}$$

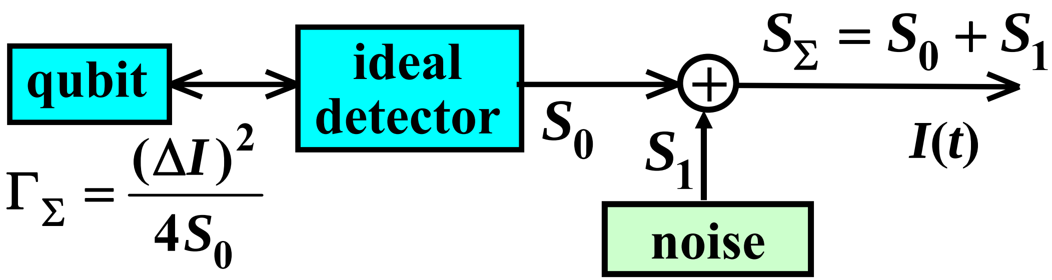
$$\dot{\rho}_{12} = i\varepsilon\rho_{12} + iH(\rho_{11} - \rho_{22}) - \Gamma\rho_{12}$$

Γ – ensemble decoherence, $\Gamma = (\Delta I)^2/4S_I + K^2 S_I/4 + \gamma$

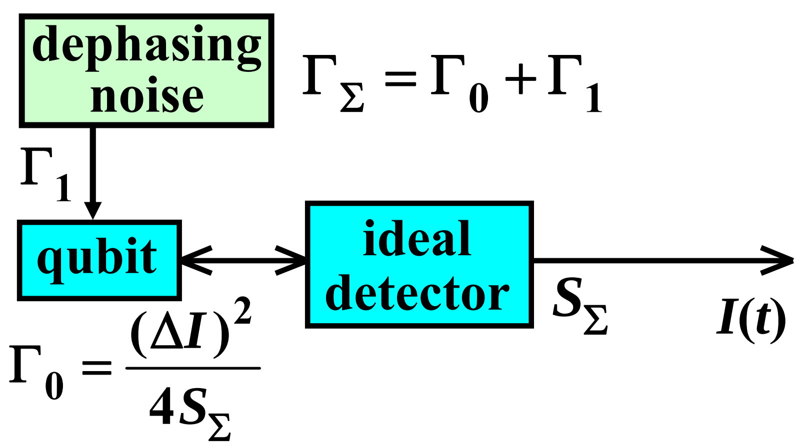
spooky physical dephasing

Quantum efficiency: $\eta = \dfrac{(\Delta I)^2/4S_I}{\Gamma}$ or $\tilde{\eta} = 1 - \dfrac{\gamma}{\Gamma}$

Two ways to think about a non-ideal detector (η<1)



$$\eta = \frac{(\Delta I)^2 / 4S_\Sigma}{\Gamma_\Sigma}$$



These ways are equivalent
(same results for any expt.)
⇒ **matter of convenience**

Stratonovich and Ito forms for nonlinear stochastic differential equations

Definitions of the derivative:

$$\frac{df(t)}{dt} = \lim_{\Delta t\to 0} \frac{f(t+\Delta t/2) - f(t-\Delta t/2)}{\Delta t} \qquad \text{(Stratonovich)}$$

$$\frac{df(t)}{dt} = \lim_{\Delta t\to 0} \frac{f(t+\Delta t) - f(t)}{\Delta t} \qquad \text{(Ito)}$$

Why matters? Usually $(f+df)^2 \approx f^2 + 2f\,df, \;\; (df)^2 \ll df$

But if $df = \xi\, dt$ (white noise ξ), then $(df)^2 = \xi^2 dt^2 \approx \frac{S_\xi}{2} dt$

Simple translation rule:

$$\frac{d}{dt} x_i(t) = G_i(\vec{x},t) + F_i(\vec{x},t)\,\xi(t) \qquad \text{(Stratonovich)}$$

$$\frac{d}{dt} x_i(t) = G_i(\vec{x},t) + F_i(\vec{x},t)\,\xi(t) + \frac{S_\xi}{4}\sum_k \frac{\partial F_i(\vec{x},t)}{dx_k} F_k(\vec{x},t) \qquad \text{(Ito)}$$

Advantage of Stratonovich: usual calculus rules (intuition)

Advantage of Ito: simple averaging

Methods for calculations

Monte Carlo

- "Ideologically" simplest
- In many cases most efficient

Idea:
- use finite time step Δt
- find probability distribution for $I_m(\Delta t)$
- pick a random number for $I_m(\Delta t)$
- do quantum Bayesian update

Analytics (or non-random numerics)

- Be very careful about Ito-Stratonovich issue
- Use Stratonovich form for derivations (derivatives, etc.)
- Convert into Ito for averaging over noise
- Very good idea to compare with Monte Carlo and/or check second order terms in dt

Quantum measurement in POVM formalism

Davies, Kraus, Holevo, etc.

(Nielsen-Chuang, pp. 85, 100)

$$\boxed{\text{system}} \longleftrightarrow \boxed{\text{ancilla}} \longleftarrow \text{projective measurement}$$

Measurement (Kraus) operator M_r (any linear operator in H.S.) : $\psi \to \dfrac{M_r\psi}{\| M_r\psi \|}$ or $\rho \to \dfrac{M_r \rho M_r^\dagger}{\mathrm{Tr}(M_r \rho M_r^\dagger)}$

Probability : $P_r = \| M_r \psi \|^2$ or $P_r = \mathrm{Tr}(M_r \rho M_r^\dagger)$

Completeness : $\sum_r M_r^\dagger M_r = 1$

(People often prefer linear evolution and non-normalized states)

Relation between POVM and quantum Bayesian formalism:

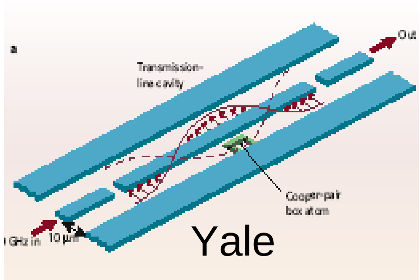
decomposition $M_r = \underset{\text{unitary}}{U_r} \underbrace{\sqrt{M_r^\dagger M_r}}_{\text{Bayes}}$

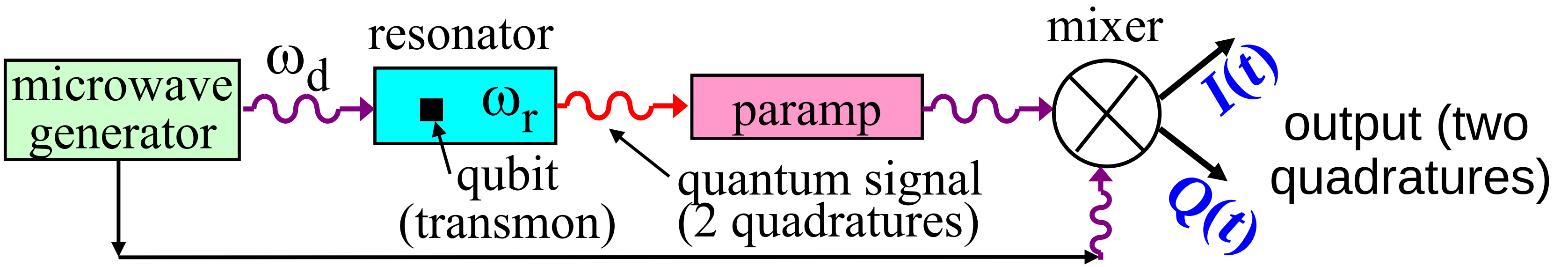
(almost equivalent)

Narrowband linear measurement

Difference from broadband: two quadratures

System: qubit in cQED setup + parametric amplifier





Paramp traditionally discussed in terms of noise temperature

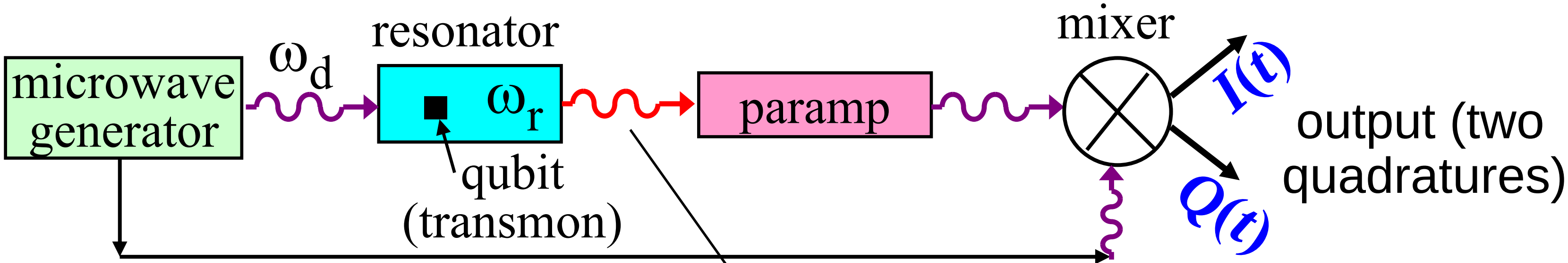
$\theta \geq 0$ for phase-sensitive (degenerate, homodyne) paramp

$\theta \geq \dfrac{\hbar\omega}{2}$ for phase-preserving (non-degenerate, heterodyne) paramp

Haus, Mullen, 1962
Giffard, 1976

Ackn.: Likharev, Devoret

We will discuss it in terms of qubit evolution due to measurement



Simplest case

$$H = \frac{\hbar\tilde{\omega}_{qb}}{2}\sigma_z + \hbar\omega_r a^\dagger a + \hbar\chi a^\dagger a\sigma_z \quad \text{(dispersive)}$$

$$\frac{\omega_r}{Q} = \kappa >> \max(\Gamma, \Omega_R) \quad \text{(Markovian, “bad cavity”)}$$

$\kappa_{out} = \kappa$ (everything collected; e.g. reflection)

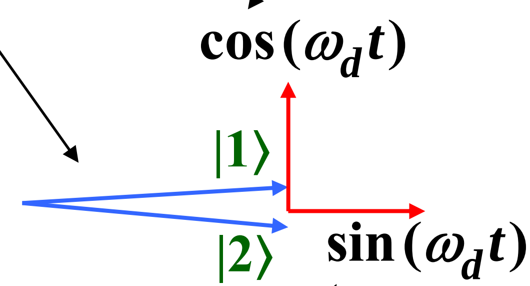
$\chi << \kappa$ (weak response)

$\omega_d = \omega_r$ (center of resonance, only phase change if transmission)

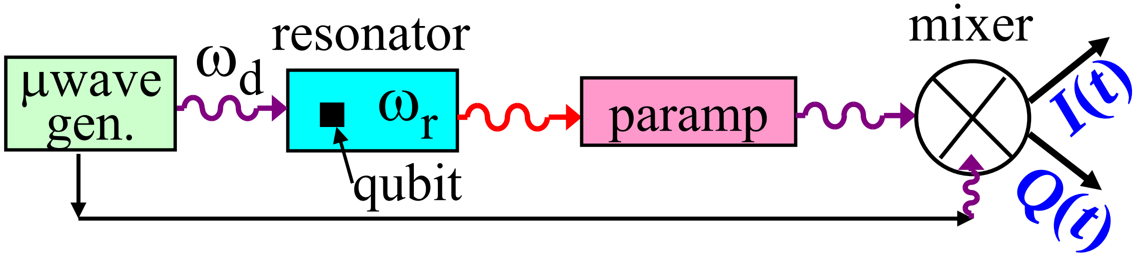
assume everything most ideal

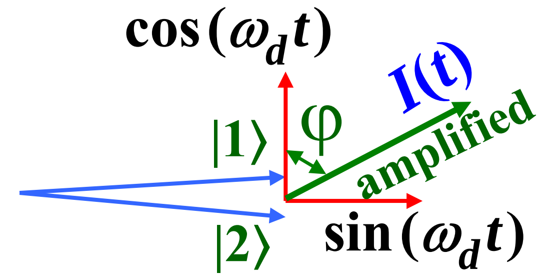
Blais et al., 2004
Gambetta et al., 2006, 2008

carries information about qubit (σ_z) (quantum back-action)



carries information about fluctuating photon number in the resonator (classical back-action)





Phase-sensitive (degenerate) paramp

quadrature $\cos(\omega_d t+\varphi)$ is amplified,
quadrature $\sin(\omega_d t+\varphi)$ is suppressed

Assume $I(t)$ measures $\cos(\omega_d t+\varphi)$, then $Q(t)$ not needed

get some information ($\sim\cos^2\varphi$) about qubit state and some information ($\sim\sin^2\varphi$) about photon fluctuations

$$\begin{cases} \dfrac{\rho_{gg}(\tau)}{\rho_{ee}(\tau)} = \dfrac{\rho_{gg}(0)}{\rho_{ee}(0)} \dfrac{\exp[-(\bar{I}-I_g)^2/2D]}{\exp[-(\bar{I}-I_e)^2/2D]} \\ \rho_{ge}(\tau) = \rho_{ge}(0)\sqrt{\dfrac{\rho_{gg}(\tau)\,\rho_{ee}(\tau)}{\rho_{gg}(0)\,\rho_{ee}(0)}}\,\underbrace{\exp(iK\bar{I}\tau)}_{\text{unitary}} \end{cases}$$

(rotating frame)

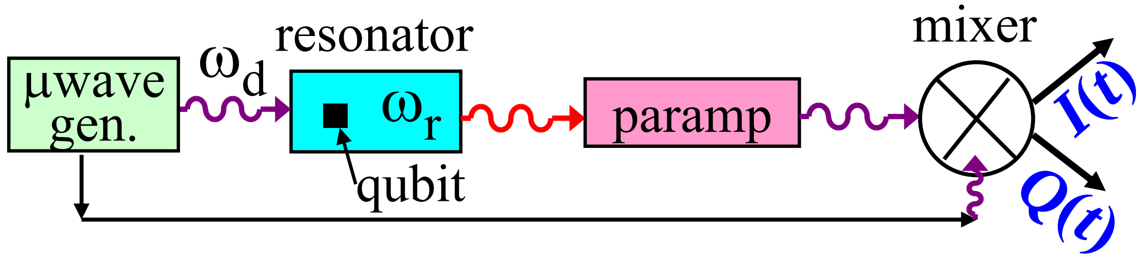
$$\bar{I} \equiv \frac{1}{\tau}\int_0^\tau I(t)\,dt \qquad D = S_I/2\tau$$

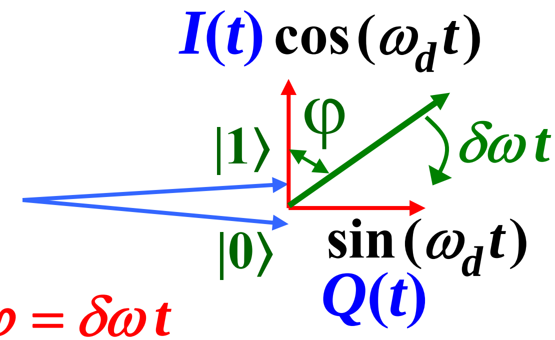
$$I_g - I_e = \Delta I \cos\varphi \qquad K = \frac{\Delta I}{S_I}\sin\varphi$$

$$\Gamma = \frac{(\Delta I\cos\varphi)^2}{4S_I} + K^2\frac{S_I}{4} = \frac{\Delta I^2}{4S_I} = \frac{8\chi^2\bar{n}}{\kappa}$$

Same as for QPC/SET, but trade-off (φ) between quantum & classical back-actions

A.K., arXiv:1111.4016





Phase-preserving (nondegenerate) paramp

$\varphi = \delta\omega\, t$

Now information in both $I(t)$ and $Q(t)$.

Choose
$I(t) \leftrightarrow \cos(\omega_d t)$ (qubit information)
$Q(t) \leftrightarrow \sin(\omega_d t)$ (photon fluct. info)

Small $\delta\omega \Rightarrow$ can follow $\varphi(t)$
Large $\delta\omega$ ($\gg\Gamma$) $\Rightarrow$ averaging over φ (phase-preserving)

$$\begin{cases} \dfrac{\rho_{gg}(\tau)}{\rho_{ee}(\tau)} = \dfrac{\rho_{gg}(0)}{\rho_{ee}(0)} \dfrac{\exp[-(\bar{I}-I_g)^2/2D]}{\exp[-(\bar{I}-I_e)^2/2D]} \\ \rho_{ge}(\tau) = \rho_{ge}(0)\sqrt{\dfrac{\rho_{gg}(\tau)\,\rho_{ee}(\tau)}{\rho_{gg}(0)\,\rho_{ee}(0)}}\exp(iK\bar{Q}\tau) \end{cases}$$

$$\bar{I} \equiv \frac{1}{\tau}\int_0^\tau I(t)\,dt \qquad \bar{Q} \equiv \frac{1}{\tau}\int_0^\tau Q(t)\,dt \qquad D = \frac{S_I}{2\tau}$$

$$I_g - I_e = \frac{\Delta I}{\sqrt{2}} \qquad K = \frac{\Delta I}{\sqrt{2}S_I}$$

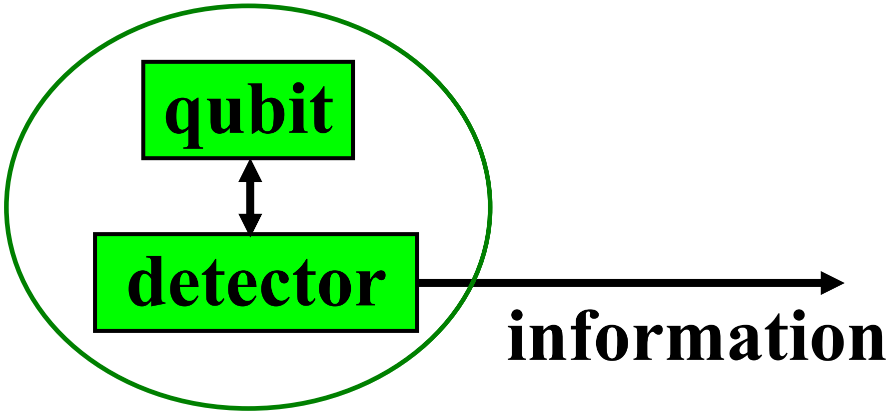
$$\Gamma = \frac{\Delta I^2}{8S_I} + \frac{\Delta I^2}{8S_I} = \frac{8\chi^2\bar{n}}{\kappa}$$

Understanding important
for quantum feedback

Equal contributions to ensemble dephasing
from quantum & classical back-actions

A.K., arXiv:1111.4016

Why not just use Schrödinger equation for the whole system?



Impossible in principle!

Technical reason: Outgoing information makes it an open system

Philosophical reason: Random measurement result, but deterministic Schrödinger equation

Einstein: God does not play dice (actually plays!)
Heisenberg: unavoidable quantum-classical boundary

Can we verify the Bayesian formalism experimentally?

Direct way:

prepare → **partial measur.** → **control (rotation)** → **projective measur.**

A.K.,1998

However, difficult: bandwidth, control, efficiency
(expt. realized only for supercond. phase qubits)

Tricks are needed for real experiments

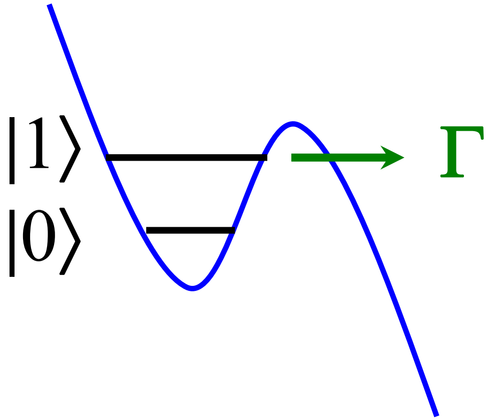
Experimental proposals

- Direct experimental verification (1998)
- Measured spectrum of Rabi oscillations (1999, 2000, 2002)
- Bell-type correlation experiment (2000)
- Quantum feedback of Rabi oscillations (2002, 2005)
- Entanglement by measurement (2002)
- Measurement by a quadratic detector (2003)
- Squeezing of a nanomechanical resonator (2004)
- Violation of Leggett-Garg inequality (2005, 2010)
- Partial collapse of a phase qubit (2005)
- Measurement reversal (2006, 2008, 2010)
- Decoherence suppression by uncollapsing (2010)
- Persistent Rabi oscillations probed via noise (2011)

Superconducting experiments "inside" quantum collapse

- UCSB-2006 Partial collapse
- UCSB-2008 Reversal of partial collapse (uncollapse)
- Saclay-2010 Continuous measurement of Rabi oscillations
  (+violation of Leggett-Garg inequality)
- Berkeley-2012 (coming soon)

Partial collapse of a Josephson phase qubit



N. Katz, M. Ansmann, R. Bialczak, E. Lucero, R. McDermott, M. Neeley, M. Steffen, E. Weig, A. Cleland, J. Martinis, A. Korotkov, Science-06

What happens if no tunneling?

Main idea:

$$\psi = \alpha|0\rangle + \beta|1\rangle \;\rightarrow\; \psi(t) = \begin{cases} |out\rangle, \text{ if tunneled} \\ \dfrac{\alpha|0\rangle + \beta e^{-\Gamma t/2} e^{i\varphi}|1\rangle}{\sqrt{|\alpha|^2 + |\beta|^2 e^{-\Gamma t}}}, \text{ if not tunneled} \end{cases}$$

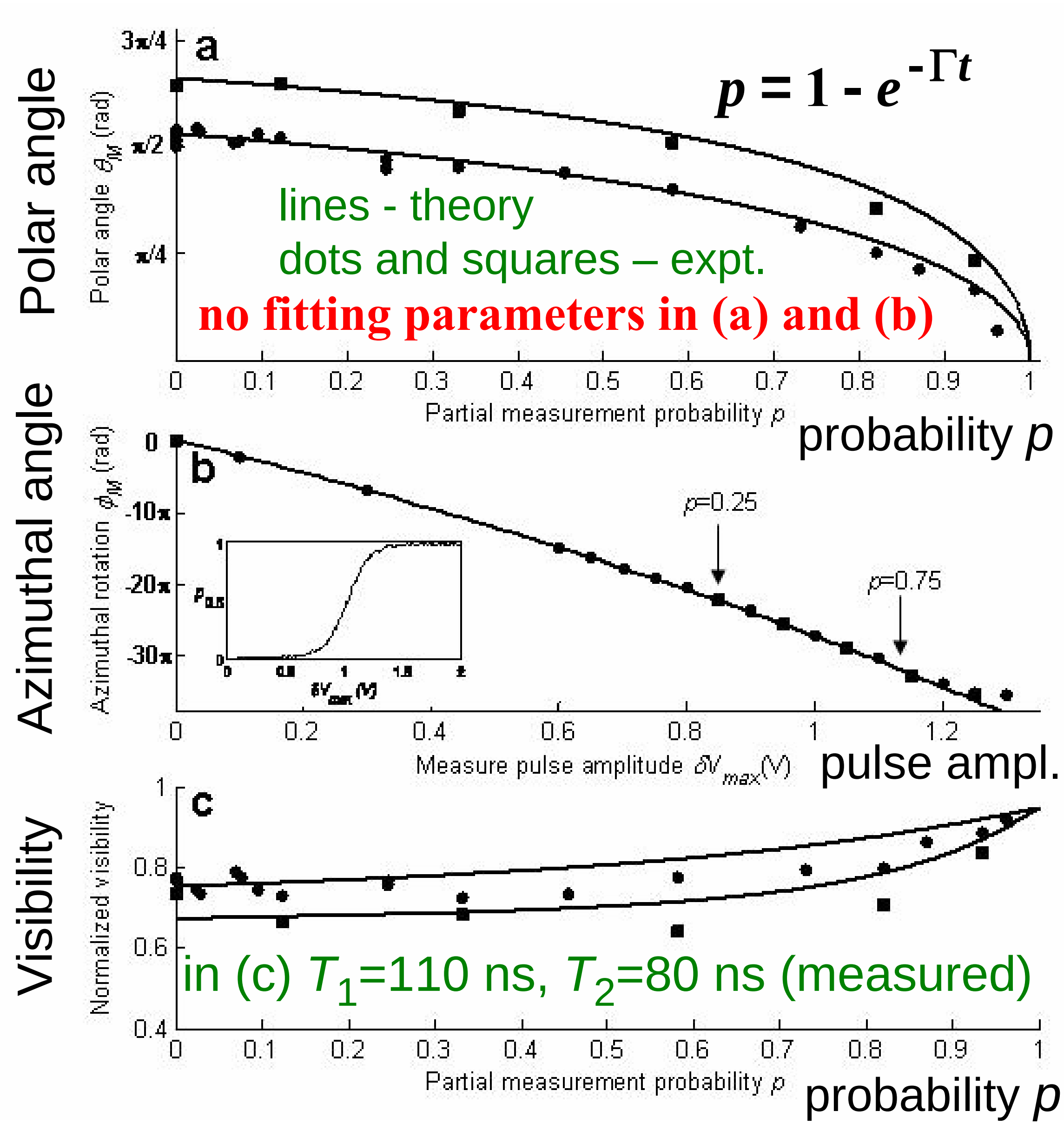
Non-trivial: • amplitude of state $|0\rangle$ grows without physical interaction
• finite linewidth only after tunneling

continuous null-result collapse

(idea similar to Dalibard-Castin-Molmer, PRL-1992)

Partial collapse: experimental results

N. Katz *et al.*, Science-06



- In case of no tunneling phase qubit evolves
- Evolution is described by the Bayesian theory without fitting parameters
- Phase qubit remains coherent in the process of continuous collapse (expt. ~80% raw data, ~96% corrected for T_1,T_2)

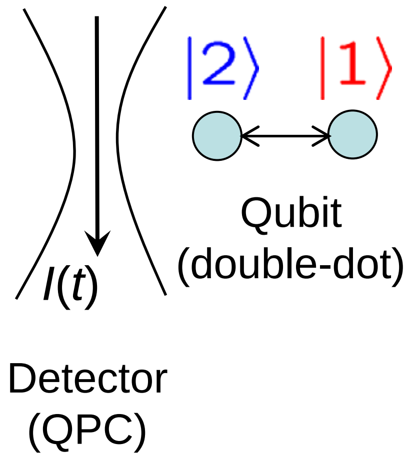
quantum efficiency

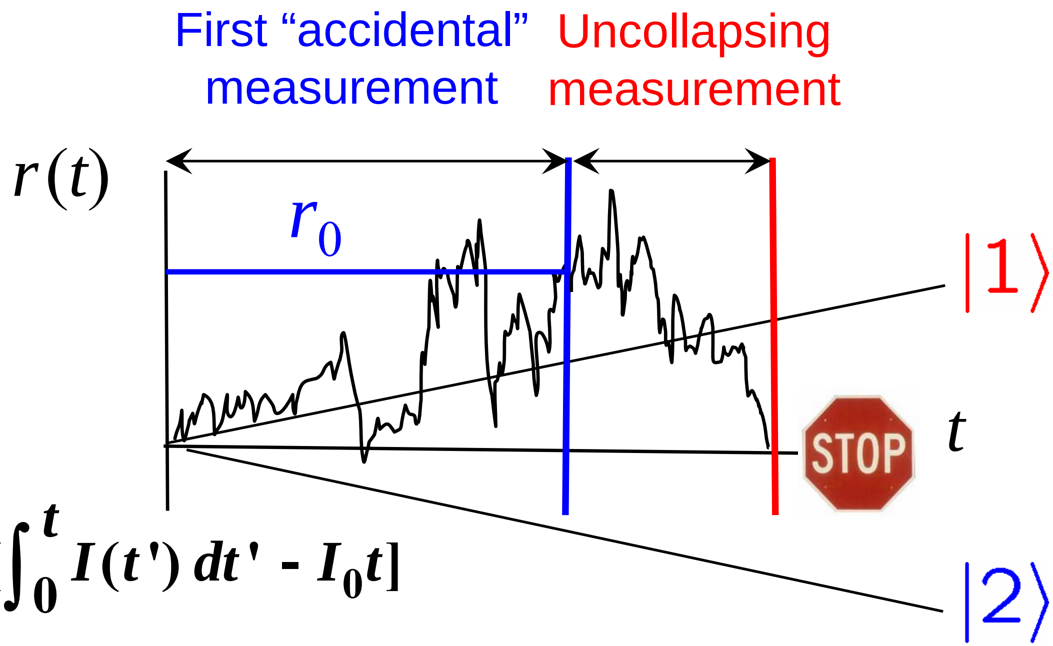
$\eta_0 > 0.8$

Good confirmation of the theory

Uncollapsing for qubit-QPC system (theory)

A.K. & Jordan, 2006





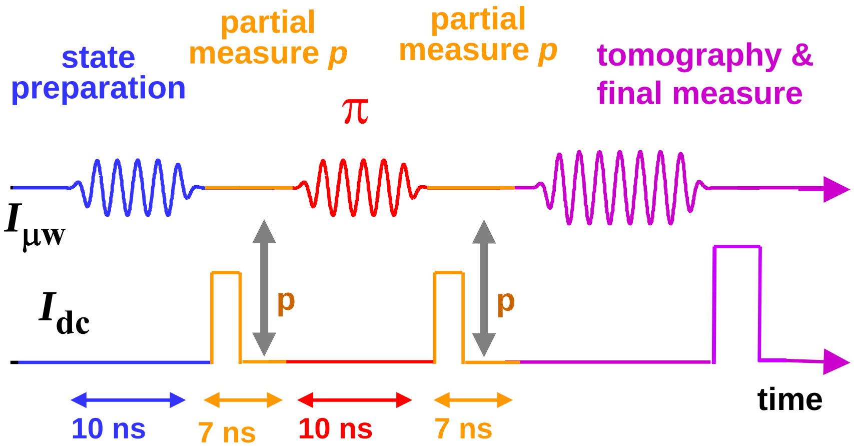
$$r(t) = \frac{\Delta I}{S_I}\left[\int_0^t I(t')\,dt' - I_0 t\right]$$

Simple strategy: continue measuring until *r*(*t*) becomes zero!
Then any unknown initial state is fully restored.

(same for an entangled qubit)

It may happen though that $r = 0$ never happens;
then undoing procedure is unsuccessful.

Experiment on wavefunction uncollapse

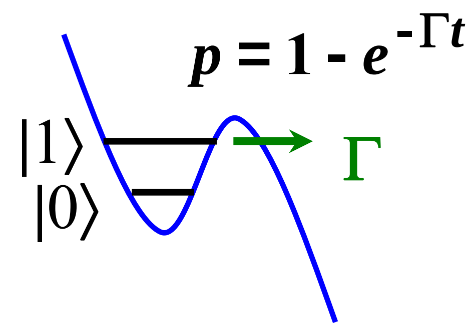


N. Katz, M. Neeley, M. Ansmann, R. Bialzak, E. Lucero, A. O'Connell, H. Wang, A. Cleland, J. Martinis, and A. Korotkov, PRL-2008

Uncollapse protocol:

- partial collapse
- π-pulse
- partial collapse (same strength)

If no tunneling for both measurements, then initial state is fully restored



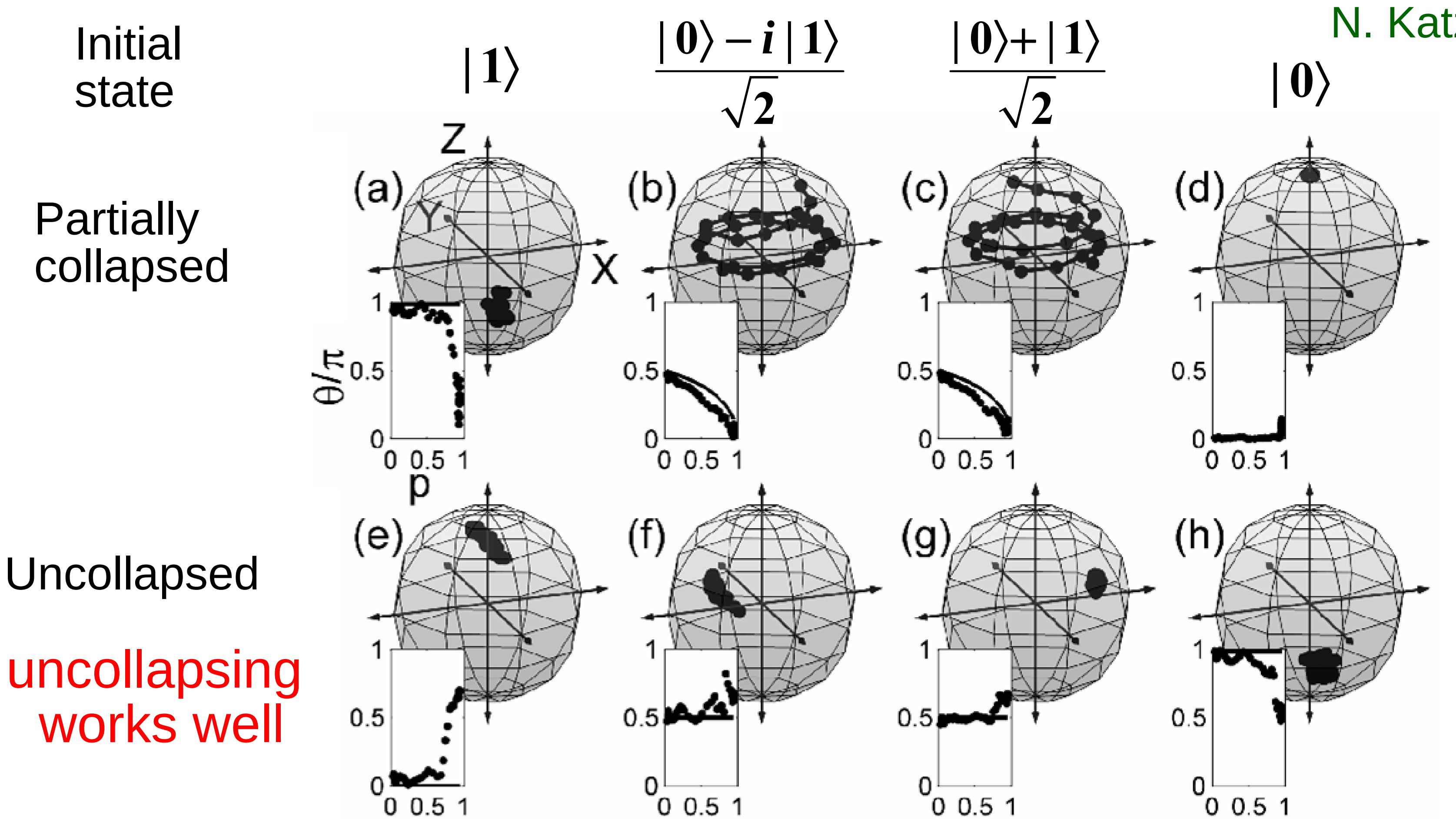
$$\alpha|0\rangle + \beta|1\rangle \rightarrow \frac{\alpha|0\rangle + e^{i\phi}\beta e^{-\Gamma t/2}|1\rangle}{\text{Norm}} \rightarrow$$

$$\frac{e^{i\phi}\alpha e^{-\Gamma t/2}|0\rangle + e^{i\phi}\beta e^{-\Gamma t/2}|1\rangle}{\text{Norm}} = e^{i\phi}(\alpha|0\rangle + \beta|1\rangle)$$

phase is also restored ("spin echo")

Experimental results on the Bloch sphere

N. Katz et al.



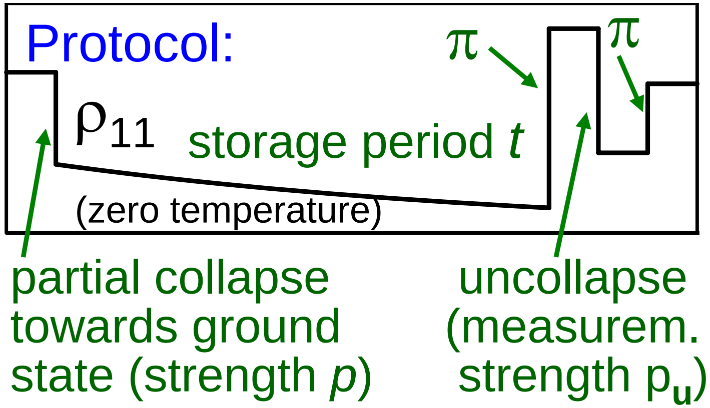
uncollapsing works well

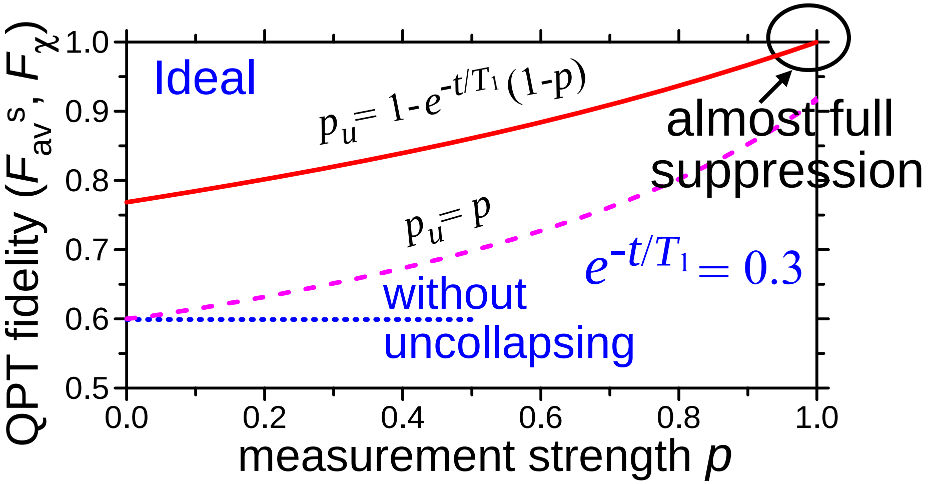
Both spin echo (azimuth) and uncollapsing (polar angle)

Difference: spin echo – undoing of an unknown unitary evolution, uncollapsing – undoing of a known, but non-unitary evolution

Suppression of T_1-decoherence by uncollapse

A.K. & Keane, PRA-2010





Ideal case (T_1 during storage only)

for initial state $|\psi_{in}\rangle=\alpha\,|0\rangle+\beta\,|1\rangle$

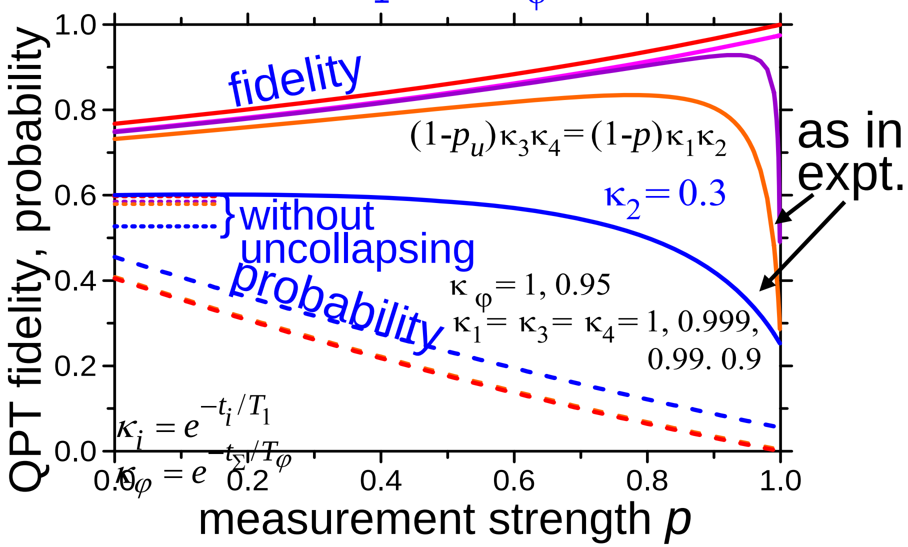
$|\psi_f\rangle=|\psi_{in}\rangle$ with probability $(1-p)\,e^{-t/T_1}$

$|\psi_f\rangle=|0\rangle$ with $(1-p)^2|\beta|^2 e^{-t/T_1}(1-e^{-t/T_1})$

procedure preferentially selects events without energy decay

Uncollapse seems to be **the only way** to protect against T_1-decoherence without encoding in a larger Hilbert space (QEC, DFS)

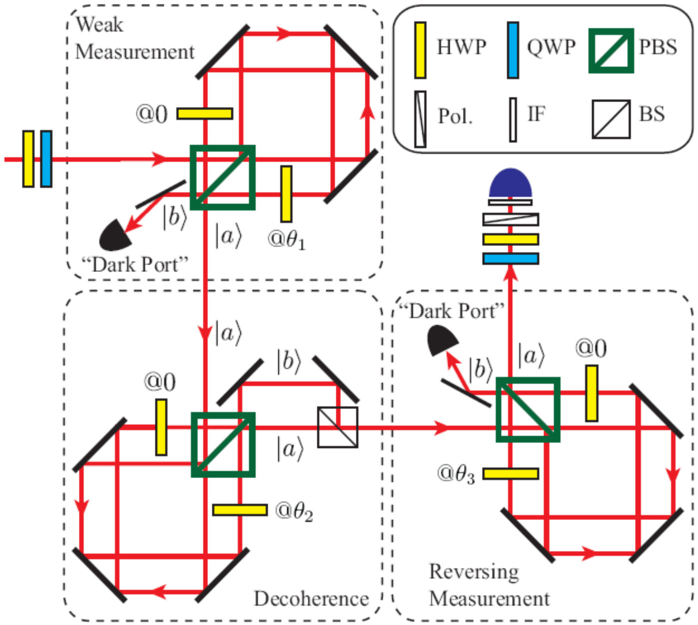
Realistic case (T_1 and T_φ at all stages)

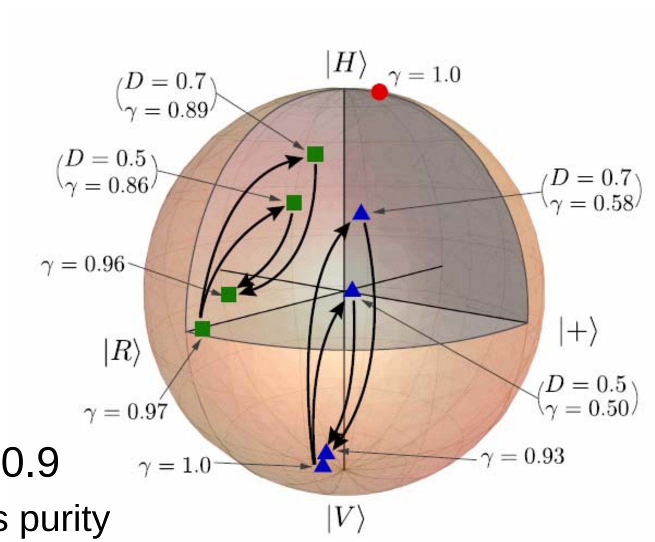


Trade-off: fidelity vs. probability

Realization with photons

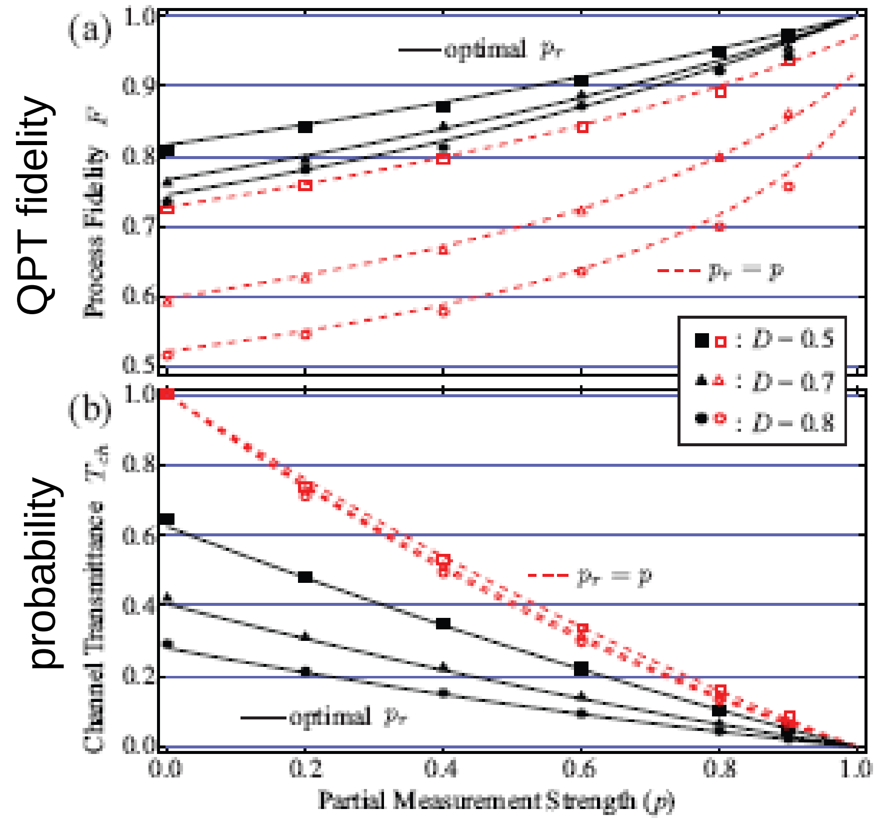
J.-C. Lee, Y.-C. Jeong, Y.-S. Kim, and Y.-H. Kim, Opt. Express-2011





p=0.9

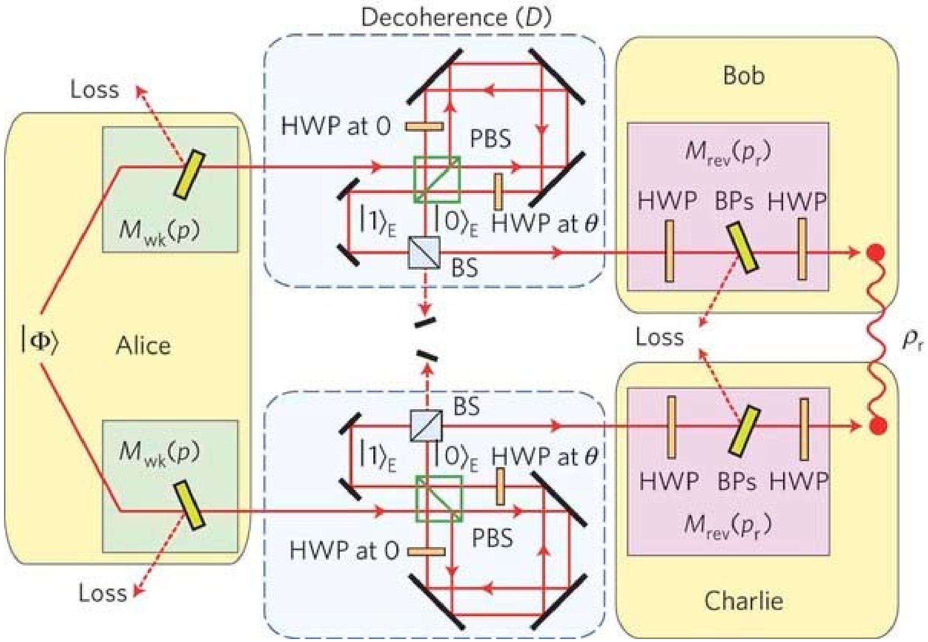
γ is purity

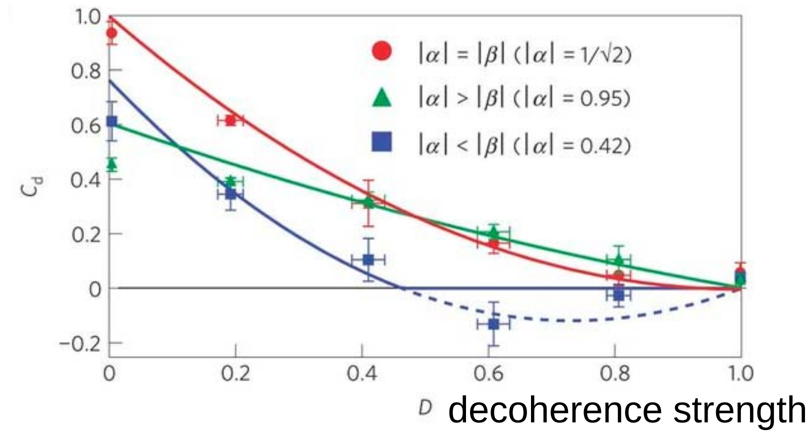


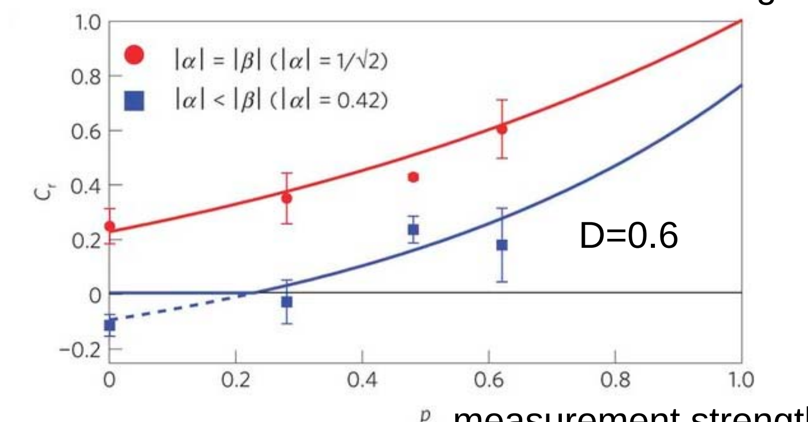
- Works perfectly (optics, not solid state!)
- Amplitude damping ("energy relaxation") decoherence is imitated in a clever way

Uncollapsing preserves entanglememt

Y.-S. Kim, J.-C. Lee, O. Kwon, and Y.-H. Kim, Nature Phys.-2012

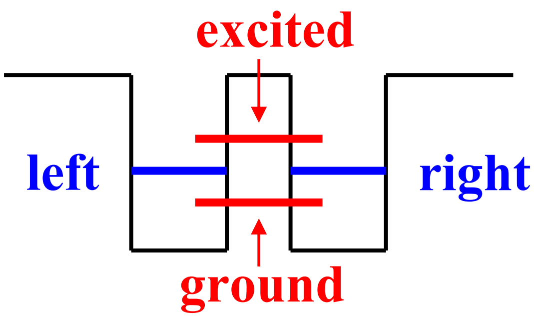






- Extension of 1-qubit experiment
- Revives entanglement even from "sudden death"

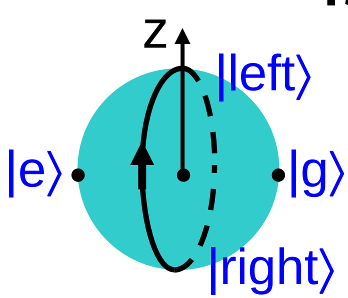
Non-decaying (persistent) Rabi oscillations

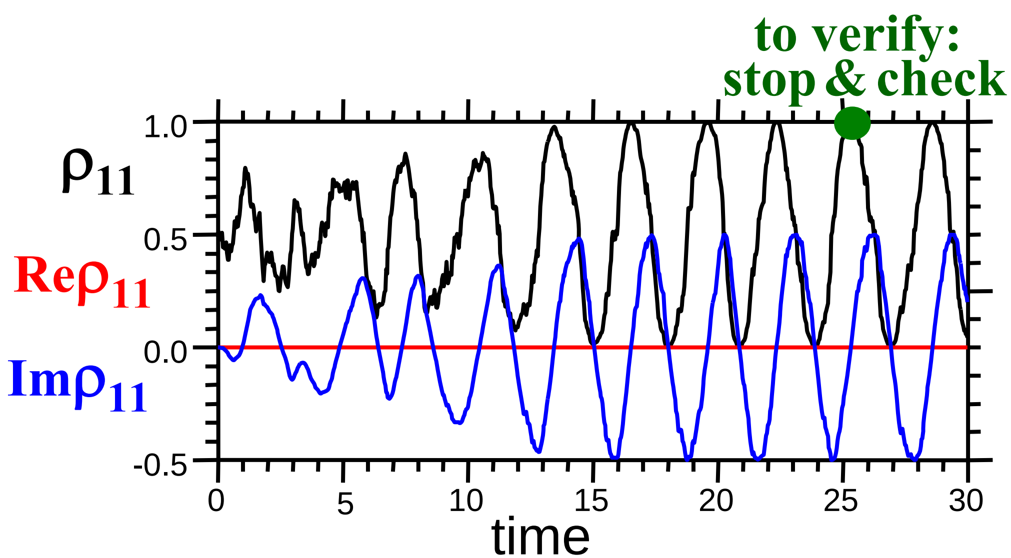


- Relaxes to the ground state if left alone (low-T)
- Becomes fully mixed if coupled to a high-T (non-equilibrium) environment
- Oscillates persistently between left and right if (weakly) measured continuously

$$\frac{(\Delta I)^2}{4S_I} \ll \Omega$$

("reason": attraction to left/right states)



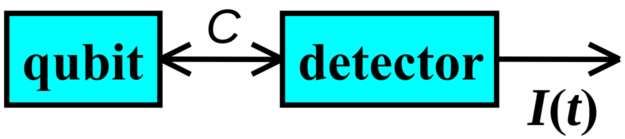


Direct experiment is difficult

A.K., PRB-1999

Indirect experiment: spectrum of persistent Rabi oscillations

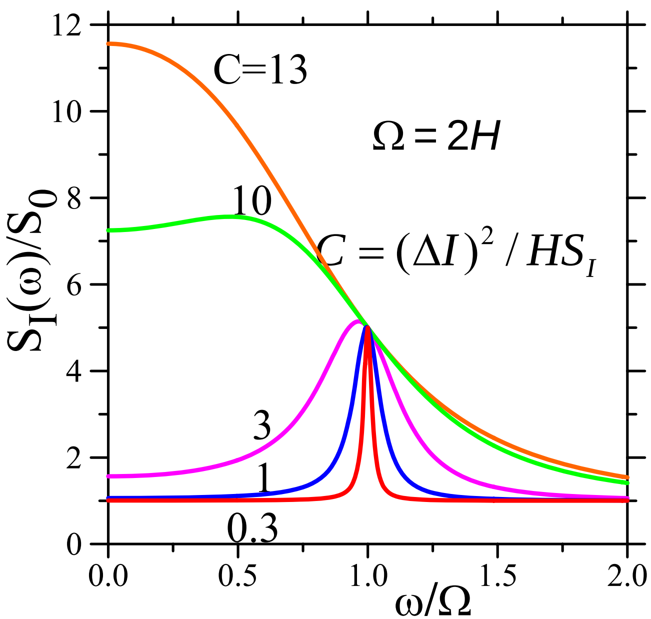
A.K., LT'1999
A.K.-Averin, 2000



$$I(t) = I_0 + \frac{\Delta I}{2} z(t) + \xi(t)$$

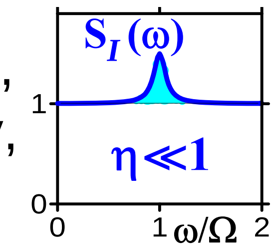
(const + signal + noise)

z is Bloch coordinate



Ω - Rabi frequency

amplifier noise ⇒ higher pedestal, poor quantum efficiency, but the peak is the same!!!



integral under the peak ⇔ variance $\langle z^2 \rangle$

peak-to-pedestal ratio = 4η ≤ 4

$$S_I(\omega) = S_0 + \frac{\Omega^2 (\Delta I)^2 \Gamma}{(\omega^2 - \Omega^2)^2 + \Gamma^2 \omega^2}$$

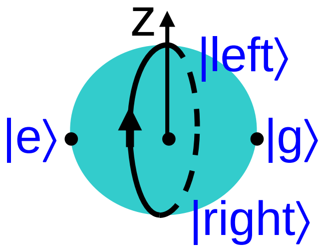
How to distinguish experimentally persistent from non-persistent? Easy!

perfect Rabi oscillations: $\langle z^2 \rangle = \langle \cos^2 \rangle = 1/2$

imperfect (non-persistent): $\langle z^2 \rangle \ll 1/2$

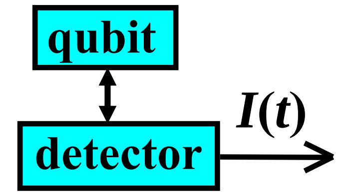
quantum (Bayesian) result: $\langle z^2 \rangle = 1$ (!!!)

(demonstrated in Saclay expt.)



How to understand $\langle z^2 \rangle = 1$?

$$I(t) = I_0 + \frac{\Delta I}{2} z(t) + \xi(t)$$



First way (mathematical)

We actually measure operator: $z \rightarrow \sigma_z$

$$z^2 \rightarrow \sigma_z{}^2 = 1$$

(What does it mean? Difficult to say…)

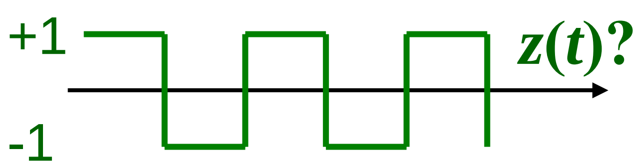
Second way (Bayesian)

$$S_I(\omega) = S_{\xi\xi} + \frac{\Delta I^2}{4} S_{zz}(\omega) + \frac{\Delta I}{2} S_{\xi z}(\omega)$$

quantum back-action changes z in accordance with the noise ξ (what you see becomes reality)

Equal contributions (for weak coupling and η=1)

Can we explain it in a more reasonable way (without spooks/ghosts)?



or some other $z(t)$?

No (under assumptions of macrorealism; Leggett-Garg, 1985)

Leggett-Garg-type inequalities for continuous measurement of a qubit

qubit ⟷ **detector** ⟶ $I(t)$

Ruskov-A.K.-Mizel, PRL-2006
Jordan-A.K.-Büttiker, PRL-2006

Assumptions of macrorealism (similar to Leggett-Garg'85):

$$I(t) = I_0 + (\Delta I/2)\, z(t) + \xi(t)$$

$$|z(t)| \le 1, \quad \langle \xi(t)\, z(t+\tau)\rangle = 0$$

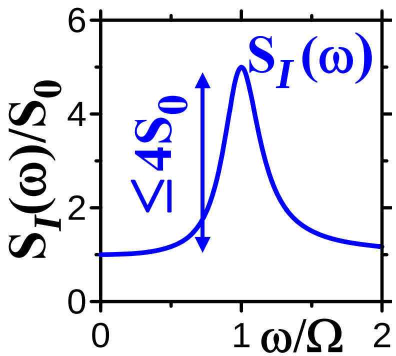
Leggett-Garg,1985

$K_{ij} = \langle Q_i Q_j \rangle$

if $Q = \pm 1$, then

$1+K_{12}+K_{23}+K_{13} \ge 0$

$K_{12}+K_{23}+K_{34} - K_{14} \le 2$



Then for correlation function

$$K(\tau) = \langle I(t)\, I(t+\tau)\rangle$$

	quantum result	violation
$K(\tau_1)+K(\tau_2)-K(\tau_1+\tau_2) \le (\Delta I/2)^2$	$\frac{3}{2}(\Delta I/2)^2$	$\times\frac{3}{2}$

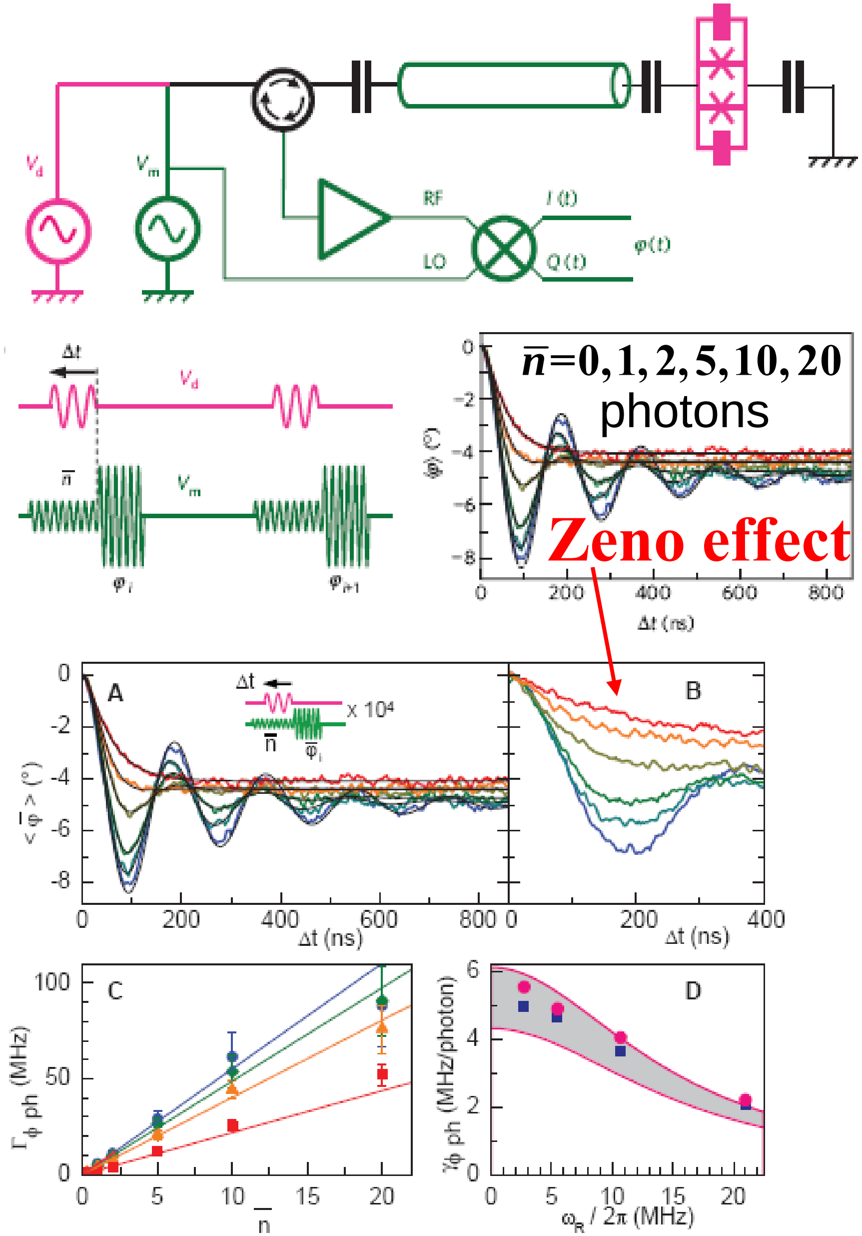
and for area under narrow spectral peak

$\int [S_I(f)-S_0]\, df \le (8/\pi^2)(\Delta I/2)^2$	$(\Delta I/2)^2$	$\times\frac{\pi^2}{8}$

η is not important! **Experimentally measurable violation**

(Saclay experiment)

Saclay experiment



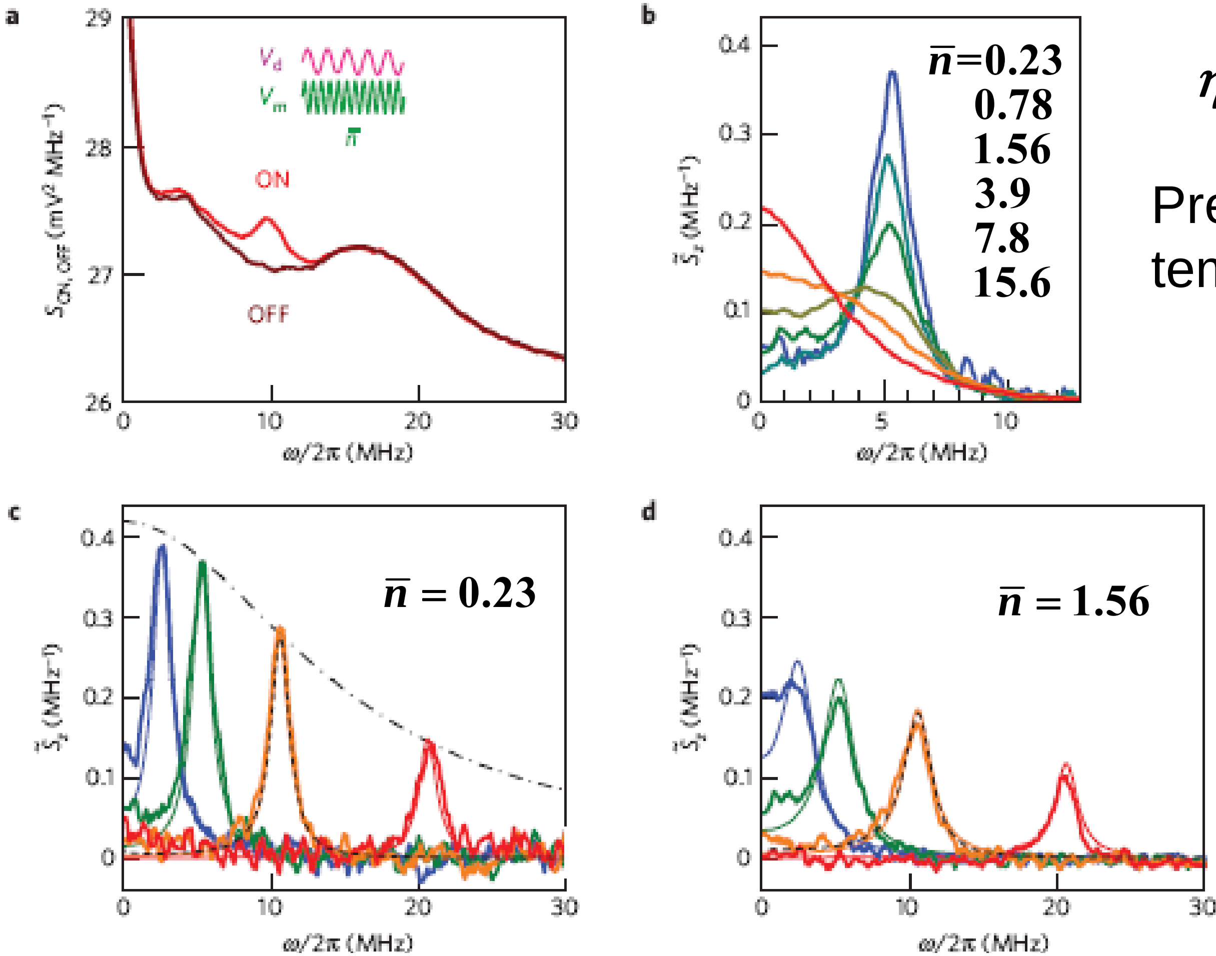
A.Palacios-Laloy, F.Mallet, F.Nguyen, P. Bertet, D. Vion, D. Esteve, and A. Korotkov, Nature Phys., 2010

- superconducting charge qubit (transmon) in circuit QED setup
- microwave reflection from cavity: full collection, only phase modulation
- driven Rabi oscillations (z-basis is |g>&|e>)

Standard (not continuous) measurement here: ensemble-averaged Rabi starting from ground state

Now continuous measurement

Palacios-Laloy et al., 2010



$$\eta = \frac{\Delta S}{4S} \sim 10^{-2}$$

Pre-amplifier noise temperature T_N= 4 K

$$\frac{1}{1+\dfrac{2T_N}{\hbar\omega}} \approx 0.03$$

Theory by dashed lines, very good agreement

Violation of Leggett-Garg inequalities

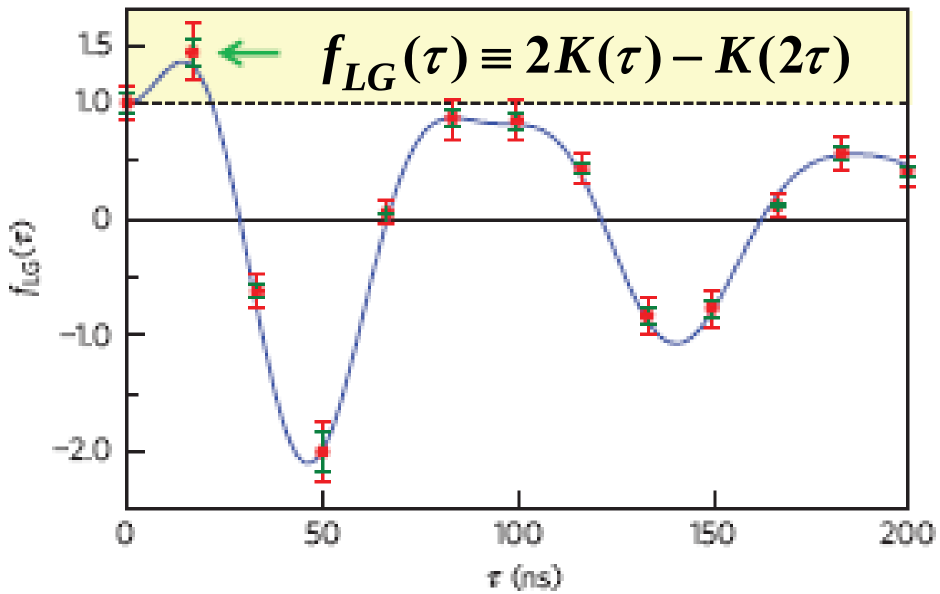
Palacios-Laloy et al., 2010

In time domain

Rescaled to qubit *z*-coordinate $K(\tau) \equiv \langle z(t)\, z(t+\tau) \rangle$

$$K(\tau_1) + K(\tau_2) - K(\tau_1+\tau_2) \le 1 \quad \Rightarrow 2K(\tau) - K(2\tau) \le 1$$

$$f_{LG}(0) = K(0) = \langle z^2 \rangle \qquad \langle z^2 \rangle = 1.01 \pm 0.15$$



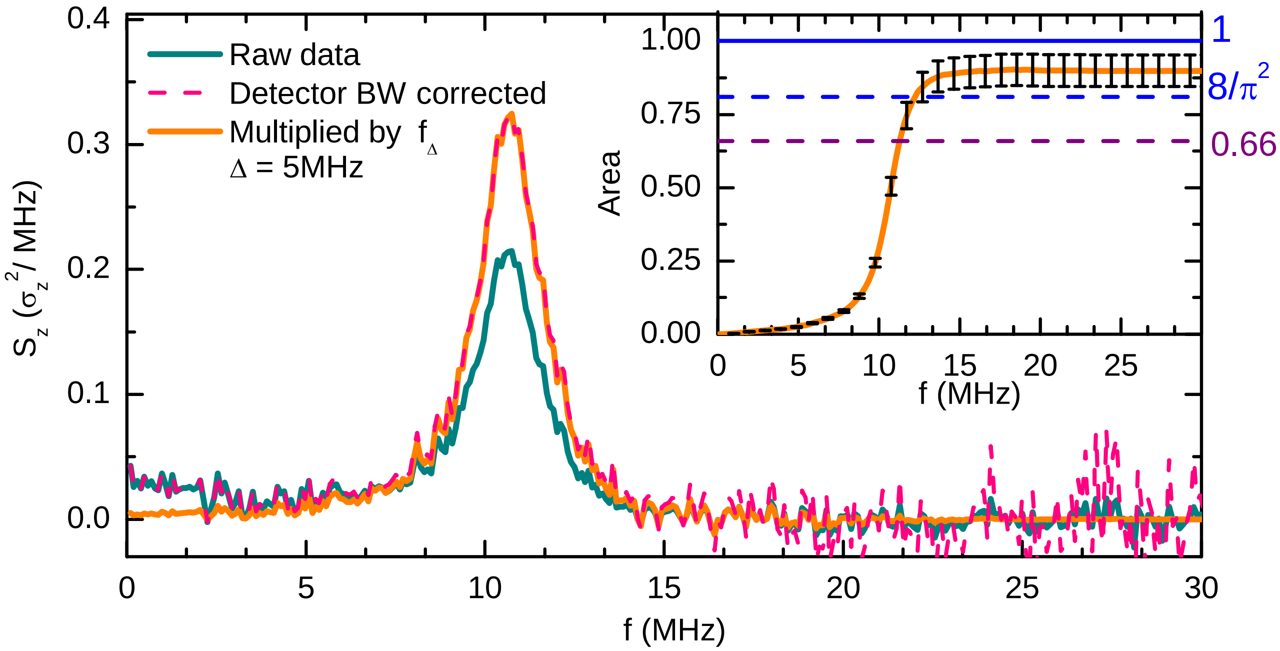
$f_{LG}(17\,\text{ns}) = 1.44 \pm 0.12$ Ideal $f_{\text{LG,max}}$=1.5

Standard deviation $\sigma = 0.065 \Rightarrow$ violation by 5σ

Violation of Leggett-Garg inequalities

In frequency domain

courtesy of
Patrice Bertet
(unpublished)



Also violated, but not so well as in time domain

Natural next step: quantum feedback control of persistent Rabi oscillations

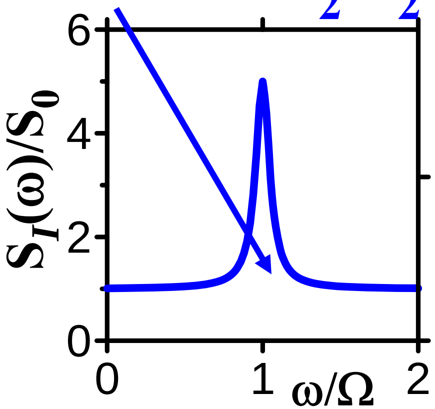
In simple monitoring the phase of persistent Rabi oscillations fluctuates randomly:

$$z(t) = \cos[\Omega t + \varphi(t)] \qquad \text{for } \eta=1$$

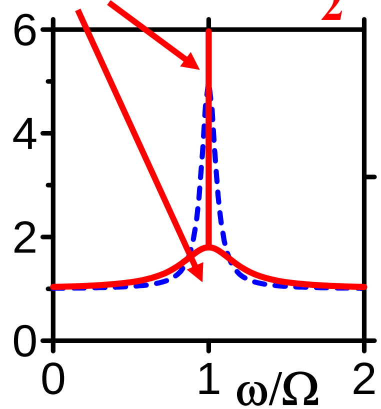
phase noise $\Rightarrow$ finite linewidth of the spectrum

Goal: produce persistent Rabi oscillations without phase noise by synchronizing with a classical signal $z_{\text{desired}}(t) = \cos(\Omega t)$

integral $\langle z^2 \rangle = \frac{1}{2} + \frac{1}{2} = 1$



integral $\langle z^2 \rangle = \frac{1}{2}$



$$I(t) = I_0 + \frac{\Delta I}{2} z(t) + \xi(t)$$

$$S_I = S_0 + \frac{\Delta I^2}{4} S_{zz} + \frac{\Delta I}{2} S_{\xi z}$$

synchronized

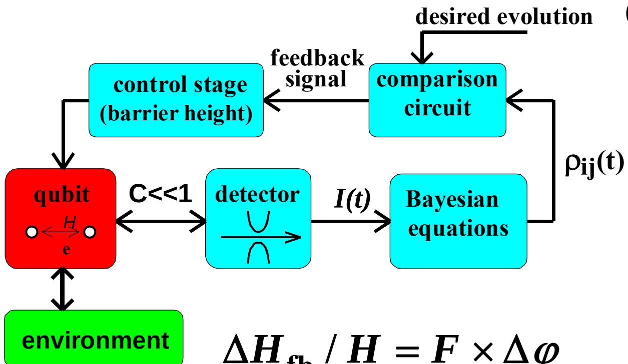
cannot synchronize

Several types of quantum feedback

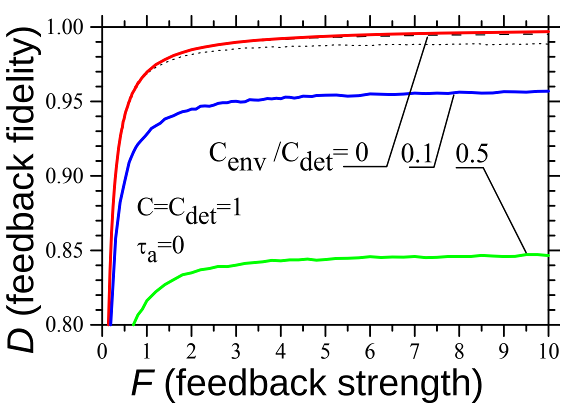
Bayesian

Best but very difficult

(monitor quantum state and control deviation)



$$\Delta H_{fb} / H = F \times \Delta\varphi$$



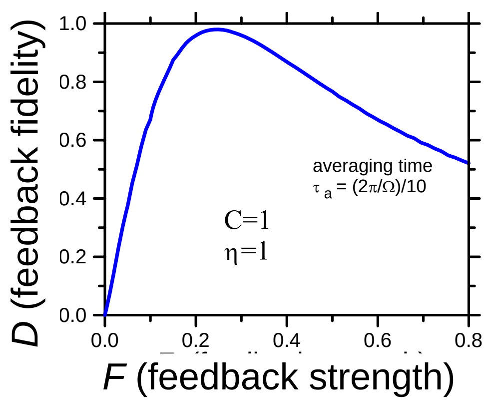
Ruskov & A.K., 2002

Direct

as in Wiseman-Milburn (1993)

(apply measurement signal to control with minimal processing)

$$\Delta H_{fb} / H = F \sin(\Omega t) \times \left(\frac{I(t) - I_0}{\Delta I / 2} - \cos\Omega t \right)$$



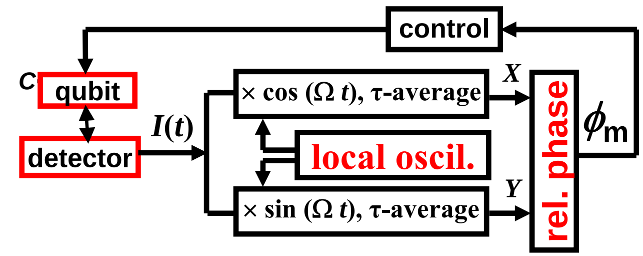
Ruskov & A.K., 2002

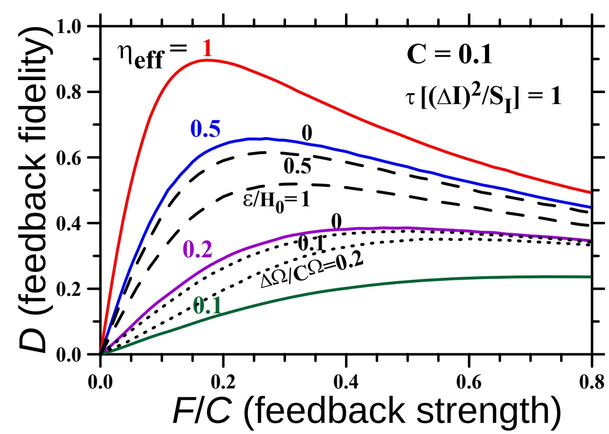
"Simple"

Imperfect but simple

(do as in usual classical feedback)

$$\frac{\Delta H_{fb}}{H} = F \times \phi_m$$





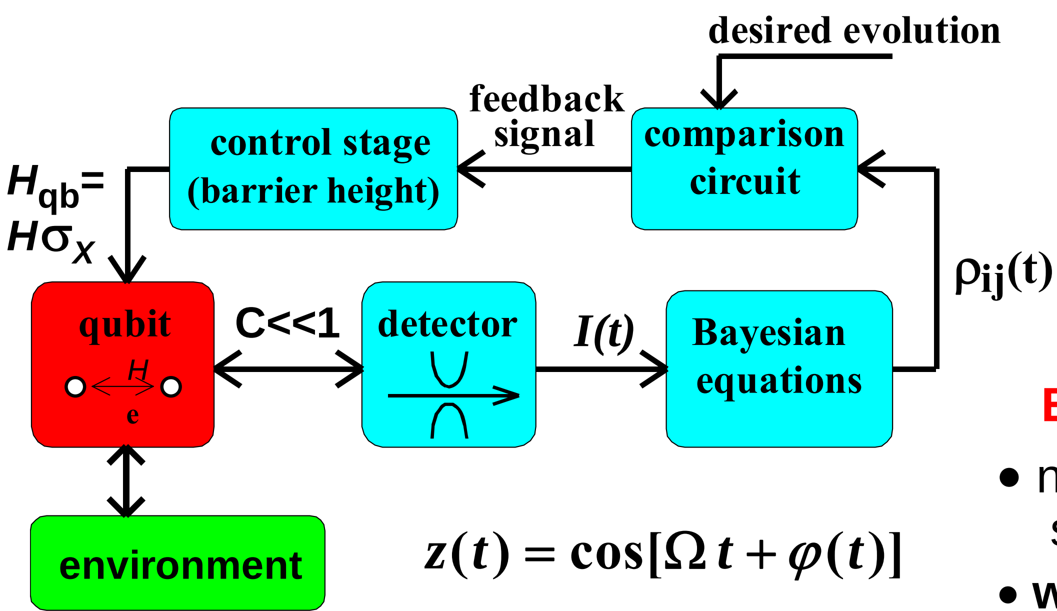
A.K., 2005

Several ways to organize quantum feedback

First idea: Bayesian feedback

(most straightforward but most difficult experimentally)

The wavefunction is monitored via Bayesian equations, and then usual (linear) feedback of the Rabi phase



$$z(t) = \cos[\Omega t + \varphi(t)]$$

$$\Delta H_{FB}/H = -F\times\varphi$$

$$\Delta\Omega/\Omega = -F\times\varphi$$

Ruskov & A.K., 2002

How to characterize feedback efficiency/fidelity?

D = average scalar product of desired and actual vectors on Bloch sphere

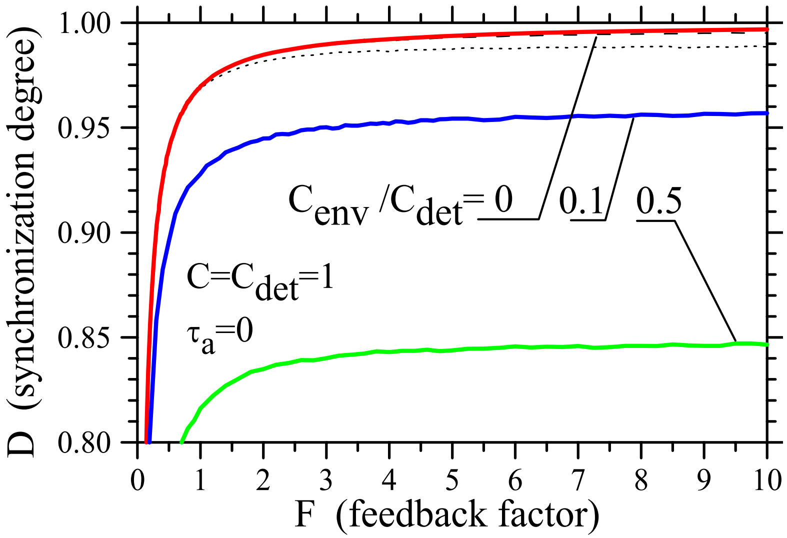
$$D = 2\langle \mathrm{Tr}\rho_{\text{desired}}\,\rho\rangle - 1$$

Experimental difficulties:

- necessity of **very fast** real-time solution of Bayesian equations
- **wide bandwidth** ($\gg\Omega$, GHz-range) of the line delivering noisy signal $I(t)$ to the "processor"

Performance of Bayesian feedback

Feedback fidelity vs. feedback strength



$C = \hbar(\Delta I)^2 / S_I H$ – coupling

F – feedback strength

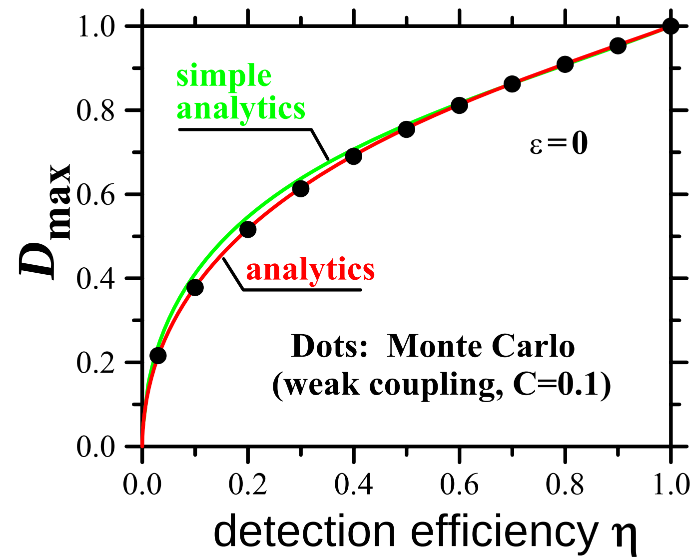
$$D = 2\langle \mathrm{Tr}\,\rho_{desired}\,\rho\rangle - 1$$

For ideal detector and wide bandwidth, feedback fidelity can be close to 100%

$$D = \exp(-C/32F)$$

Ruskov & A.K., 2002

Feedback fidelity vs. detector efficiency



$$\eta \ll 1 \;\Rightarrow\; D_{max} \approx 1.25\sqrt{\eta}$$

$$\eta \approx 1 \;\Rightarrow\; D_{max} \approx (1+\eta)/2$$

Zhang, Ruskov, A.K., 2005

other detrimental effects:

- parameter deviations
- finite bandwidth
- feedback loop delay

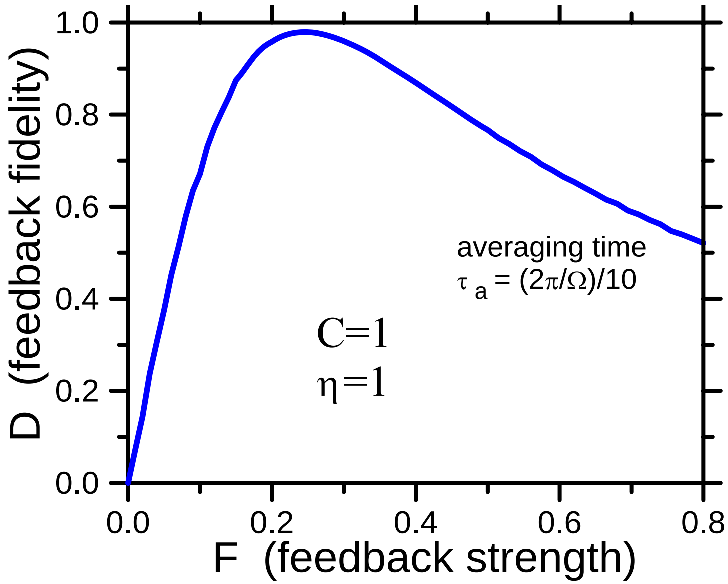
Second idea: direct feedback

(similar to Wiseman-Milburn, 1993)

Idea: apply measurement signal to control with minimal processing

feedback ~ $I(t)$-I_0

Our controller:
$$\frac{\Delta H_{\rm fb}}{H} = \frac{\Delta\Omega}{\Omega} = F \times \left(\frac{I(t)-I_0}{\Delta I/2} - \cos(\Omega t) \right) \times \sin(\Omega t)$$

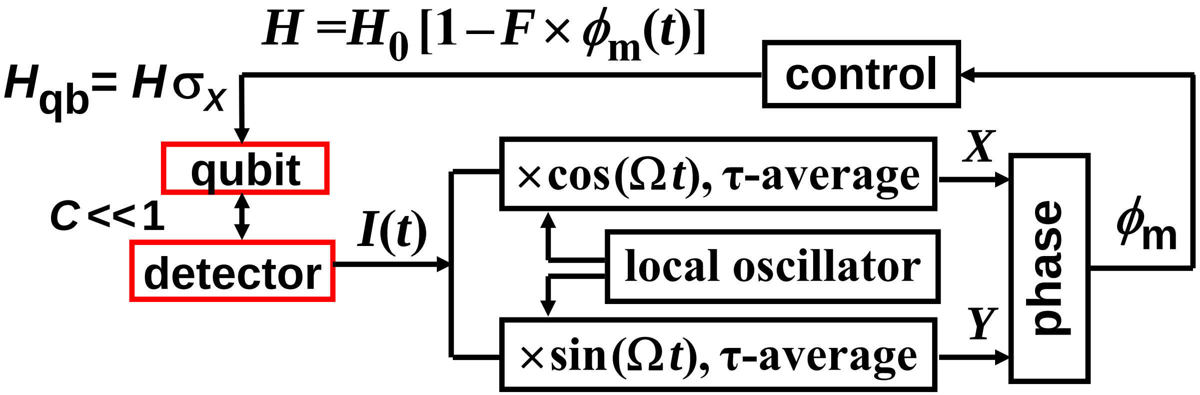


requires optimal feedback strength

Ruskov & A.K., 2002

Third idea: “Simple” quantum feedback

(A.K., 2005)



Goal: maintain coherent (Rabi) oscillations for arbitrarily long time

Idea: use two quadrature components of the detector current $I(t)$ to monitor approximately the phase of qubit oscillations

(a very natural way for usual classical feedback!)

$$X(t) = \int_{-\infty}^{t} [I(t') - I_0] \cos(\Omega t') \exp[-(t-t')/\tau]\, dt'$$

$$Y(t) = \int_{-\infty}^{t} [I(t') - I_0] \sin(\Omega t') \exp[-(t-t')/\tau]\, dt'$$

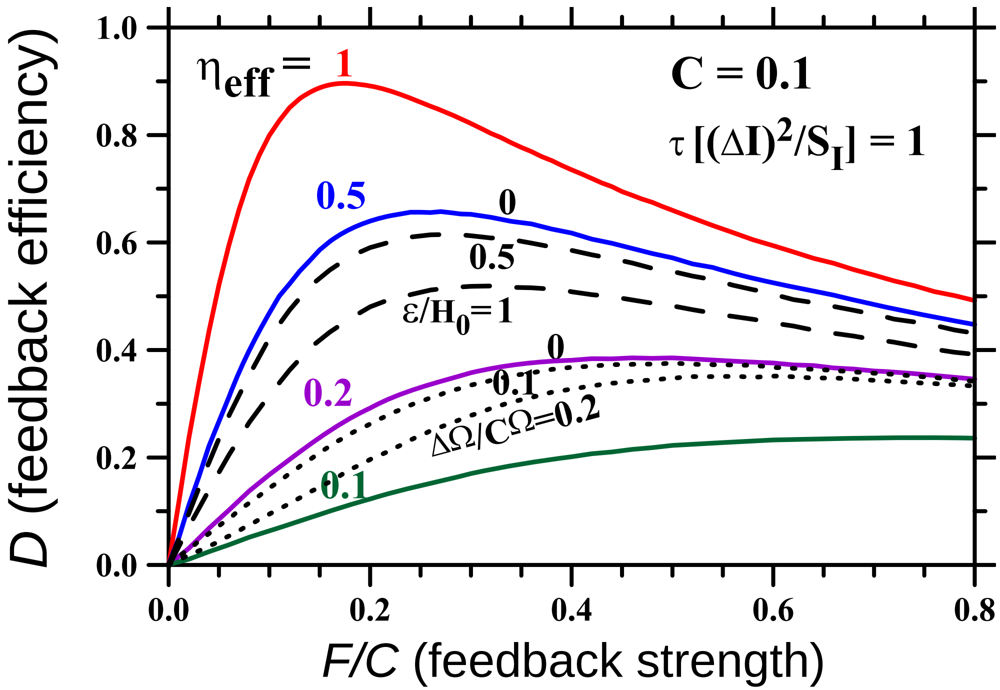
$$\phi_m = -\arctan(Y/X)$$

(similar formulas for a tank circuit instead of mixing with local oscillator)

Advantage: simplicity and relatively narrow bandwidth $(1/\tau \sim \Gamma_d \ll \Omega)$

Essentially classical feedback. Does it really work?

Fidelity of simple quantum feedback



$D_{max} \approx 90\%$

$$D \equiv 2F_Q - 1$$

$$F_Q \equiv \langle \mathrm{Tr}\, \rho(t)\, \rho_{des}(t) \rangle$$

Robust to imperfections
(inefficient detector, frequency mismatch, qubit asymmetry)

How to verify feedback operation experimentally?

Simple: just check that in-phase quadrature $\langle X \rangle$ of the detector current is positive $D = \langle X \rangle (4/\tau \Delta I)$

$\langle X \rangle$=0 for *any* non-feedback Hamiltonian control of the qubit

Simple enough for real experiment!

Quantum feedback in cQED setup

We have to undo both effects: disturbance of qubit phase ("classical")
and disturbance of Rabi phase ("spooky")

$\Rightarrow$ have to control both qubit parameters (except for phase-sens., φ=0)

Phase-preserving case

$$\begin{cases} \dfrac{\rho_{gg}(\tau)}{\rho_{ee}(\tau)} = \dfrac{\rho_{gg}(0)}{\rho_{ee}(0)} \dfrac{\exp[-(\bar{I} - I_g)^2/2D]}{\exp[-(\bar{I} - I_e)^2/2D]} \\ \rho_{ge}(\tau) = \rho_{ge}(0)\sqrt{\dfrac{\rho_{gg}(\tau)\,\rho_{ee}(\tau)}{\rho_{gg}(0)\,\rho_{ee}(0)}}\exp(iK\bar{Q}\tau) \end{cases}$$

Use different quadratures for two feedback channels

Use direct feedback for qubit energy +some feedback for μwave amplitude

Phase-sensitive case

$$\begin{cases} \dfrac{\rho_{gg}(\tau)}{\rho_{ee}(\tau)} = \dfrac{\rho_{gg}(0)}{\rho_{ee}(0)} \dfrac{\exp[-(\bar{I} - I_g)^2/2D]}{\exp[-(\bar{I} - I_e)^2/2D]} \\ \rho_{ge}(\tau) = \rho_{ge}(0)\sqrt{\dfrac{\rho_{gg}(\tau)\,\rho_{ee}(\tau)}{\rho_{gg}(0)\,\rho_{ee}(0)}}\exp(iK\bar{I}\tau) \end{cases}$$

Use the same signal for both

If φ=0 (K=0), then only feedback for μwave amplitude

Conclusions

- It is easy to see what is “inside” collapse: simple Bayesian framework works for many solid-state setups

- Measurement backaction necessarily has a “spooky” part (informational, without a physical mechanism); it may also have a “classical” part (with a physically understandable mechanism)

- Three superconducting experiments so far: partial collapse, uncollapse, monitoring of non-decaying Rabi oscillations

- Many other proposals. Hopefully other experiments are coming soon. Quantum feedback is one of most interesting.