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3 topics:

Increasing qubit lifetime by uncollapse

Y. P. Zhong, Z. L. Wang, J. M. Martinis, A. N. Cleland,
A. N. Korotkov, and H. Wang, *Nature Comm*. 5, 3135 (2014)

Purcell effect with microwave drive: suppression of qubit relaxation rate

E. A. Sete, J. M. Gambetta, and A. N. Korotkov, arXiv:1401.5545

Error matrices in quantum process tomography

A. N. Korotkov, arXiv:1309.6405

Increasing qubit lifetime by uncollapse

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A. N. Korotkov, and H. Wang, *Nature Comm*. 5, 3135 (2014)

- First experiment, showing increase of intrinsic lifetime of a superconducting qubit (by a factor of ~3) using a quantum algorithm
- Based on uncollapsing, realized with partial quantum measurement
- Caveat: selective procedure ("quantum error detection", not "error correction")

"Usefulness" of continuous/partial quantum measurement for solid-state qubits

- Quantum feedback

  Theory: Wiseman-Milburn (1993), Ruskov-Korotkov (2002)

  Expt: Haroche et al (2011), Siddiqi et al. (2012)

- Entanglement by measurement

  Theory: Ruskov-Korotkov (2003)

  Expt: DiCarlo et al. (2013), Siqqiqi et al. (2013-2014)

- Energy relaxation suppression by uncollapsing

  Theory: Korotkov-Keane (2010)

  Expt: Kim et al. (2011, 2012), Wang et al. (2013)

- State discrimination with "don't know" option

  Theory: now working with Todd Brun

  (usefulness is very questionable)

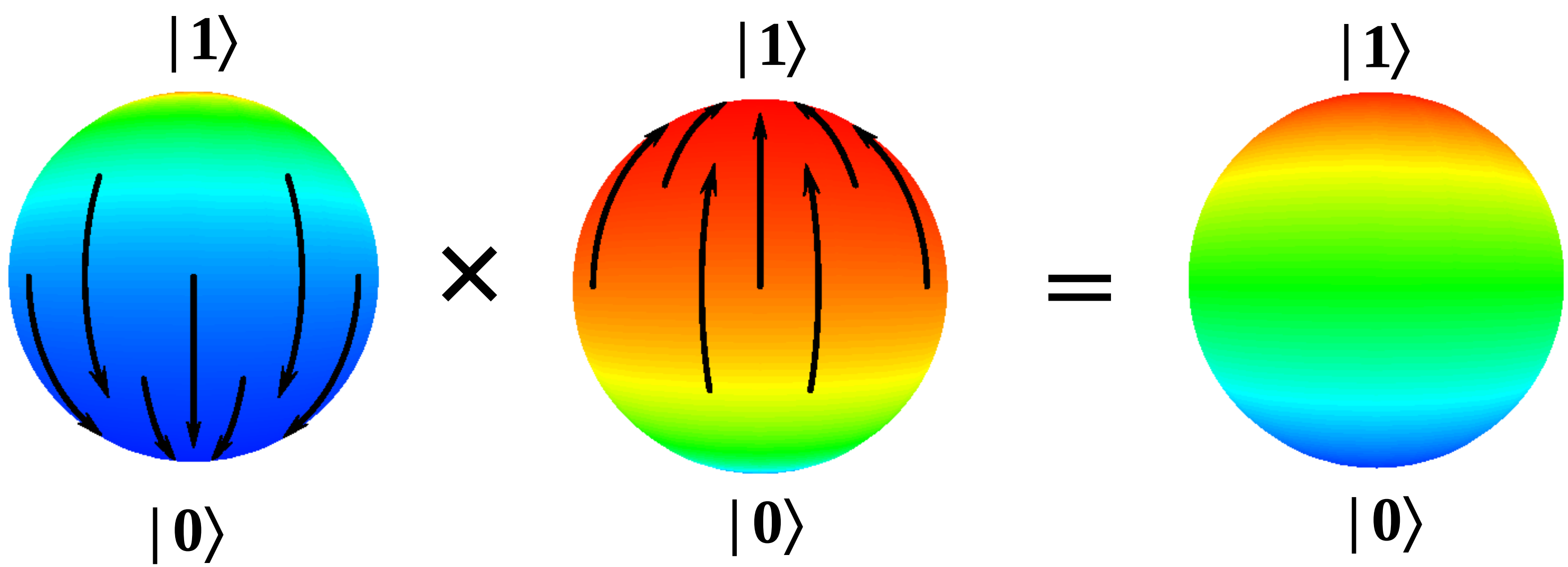
Other suggestions???

Uncollapse of a qubit state

Korotkov & Jordan, PRL-2006

Evolution due to partial (weak, continuous, etc.) measurement is **non-unitary** (though coherent if detector is good), therefore it is impossible to undo it by Hamiltonian dynamics.

How to uncollapse? One more measurement!



First experiment:
N. Katz et al., PRL-2008

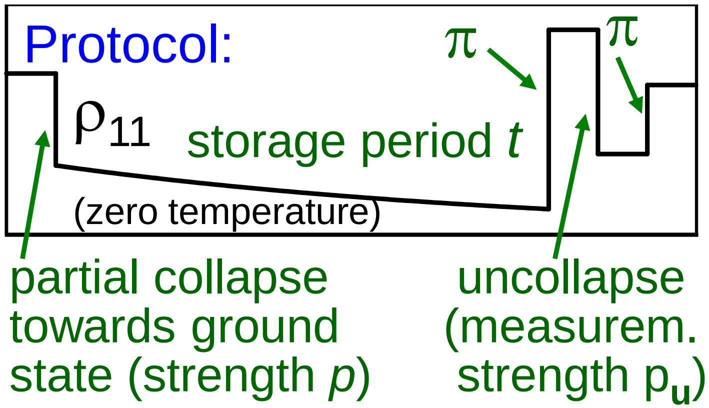
POVM language:

$$cM^{-1} \times M = c\mathbf{1}$$

(Figure partially adopted from A. Jordan, A. Korotkov, and M. Büttiker, PRL-2006)

Suppression of energy relaxation by uncollapse

A.K. & K. Keane, PRA-2010



Ideal case (T_1 during storage only)

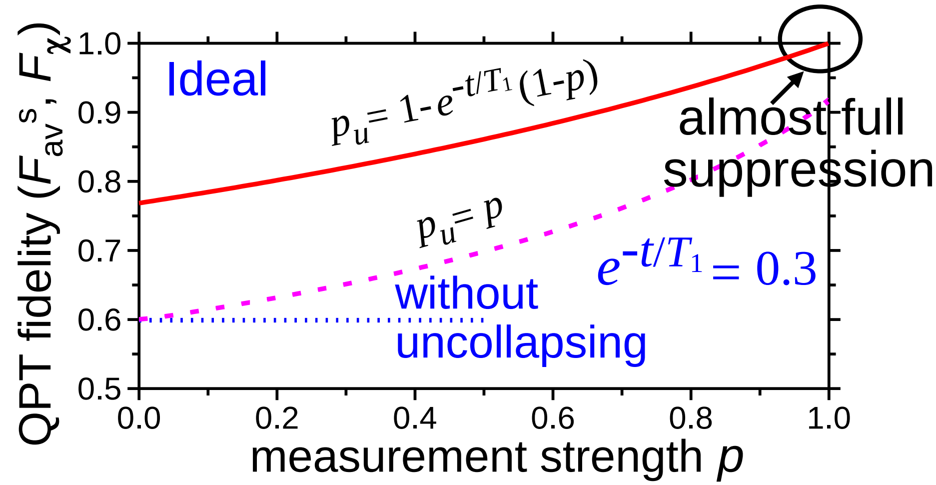
for initial state $|\psi_{in}\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$

$|\psi_f\rangle = |\psi_{in}\rangle$ with probability $(1-p)\,e^{-t/T_1}$

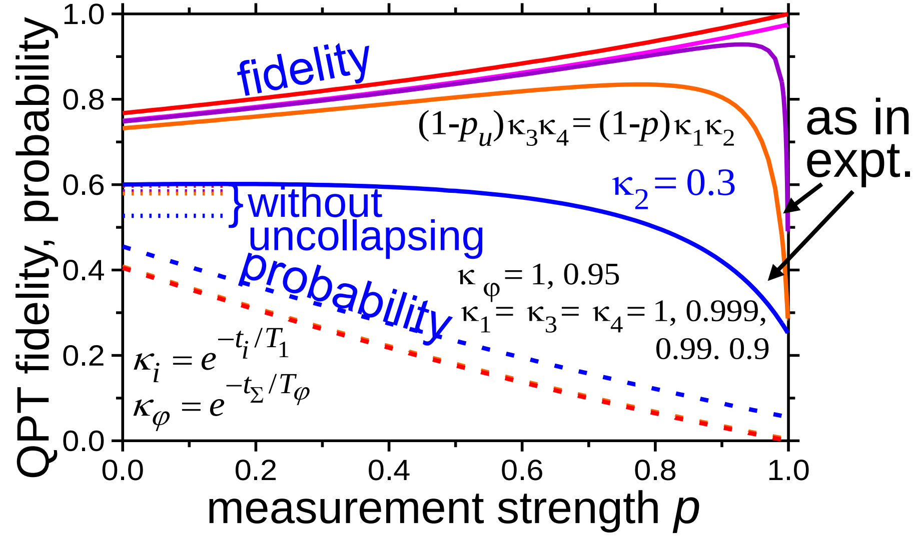
$|\psi_f\rangle = |0\rangle$ with $(1-p)^2|\beta|^2 e^{-t/T_1}(1-e^{-t/T_1})$

procedure preferentially selects events without energy decay

Uncollapse seems to be **the only way** to protect against energy relaxation without encoding in a larger Hilbert space (QEC, DFS)



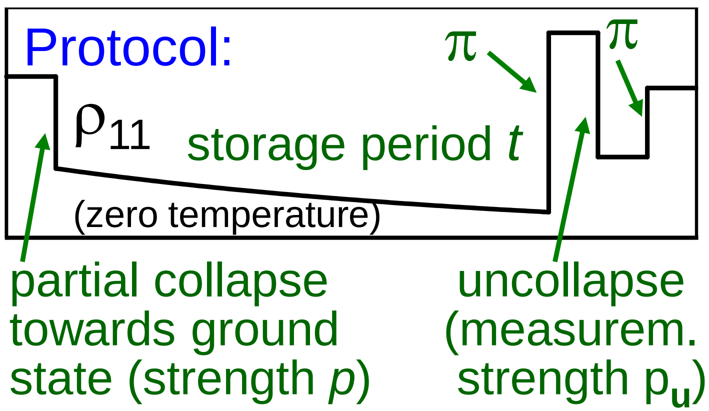
Realistic case (T_1 and T_φ at all stages)

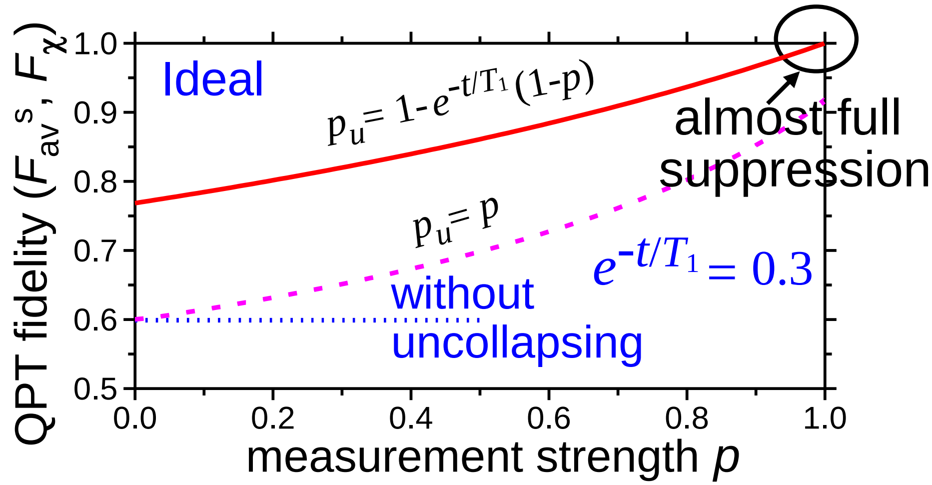


Trade-off: fidelity vs. probability

Suppression of energy relaxation by uncollapse

A.K. & K. Keane, PRA-2010





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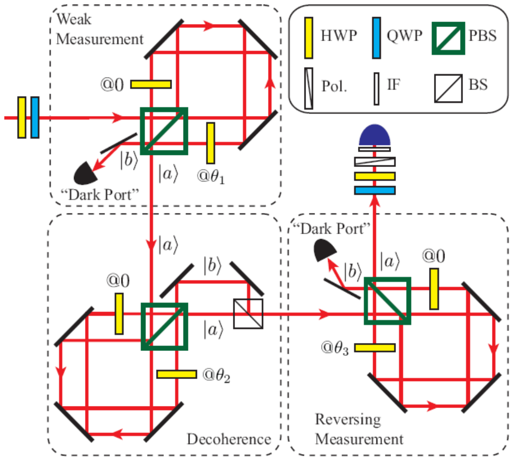
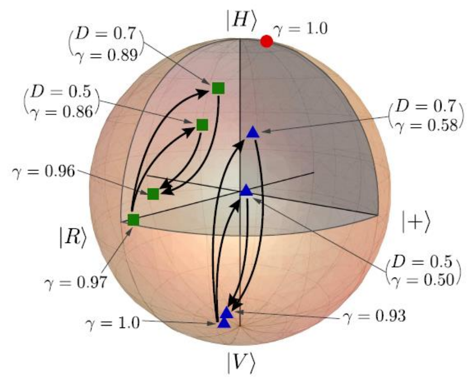
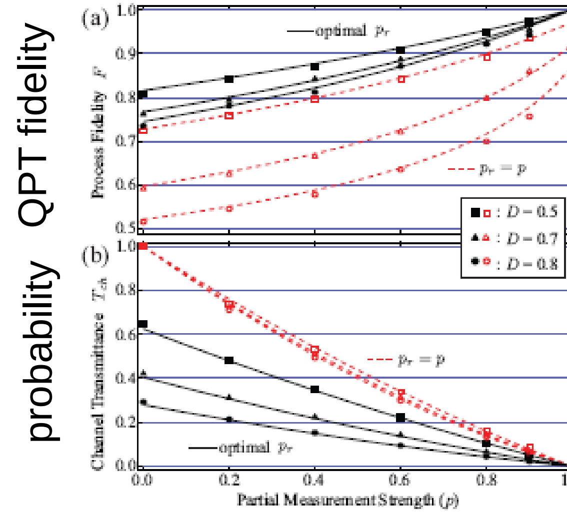
"Sleeping beauty" analogy

Realization with photons



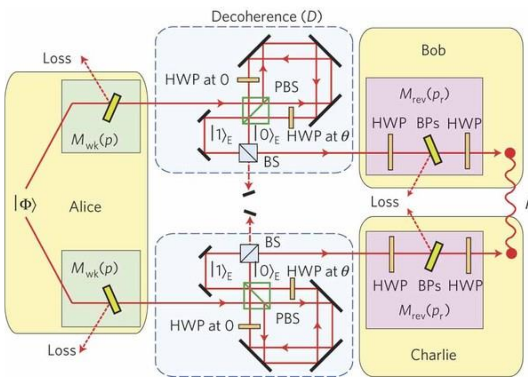
J.C. Lee, Y.C. Jeong, Y.S. Kim, & Y.H. Kim, Opt. Express-2011

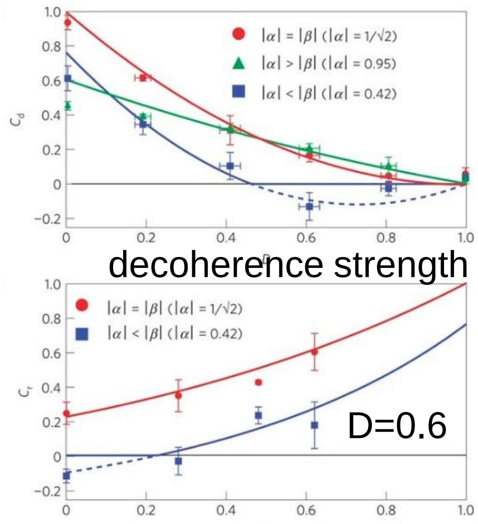




Entanglement preservation by uncollapsing

Y.S. Kim, J.C. Lee, O. Kwon, Y.H. Kim, Nature Phys.-2012





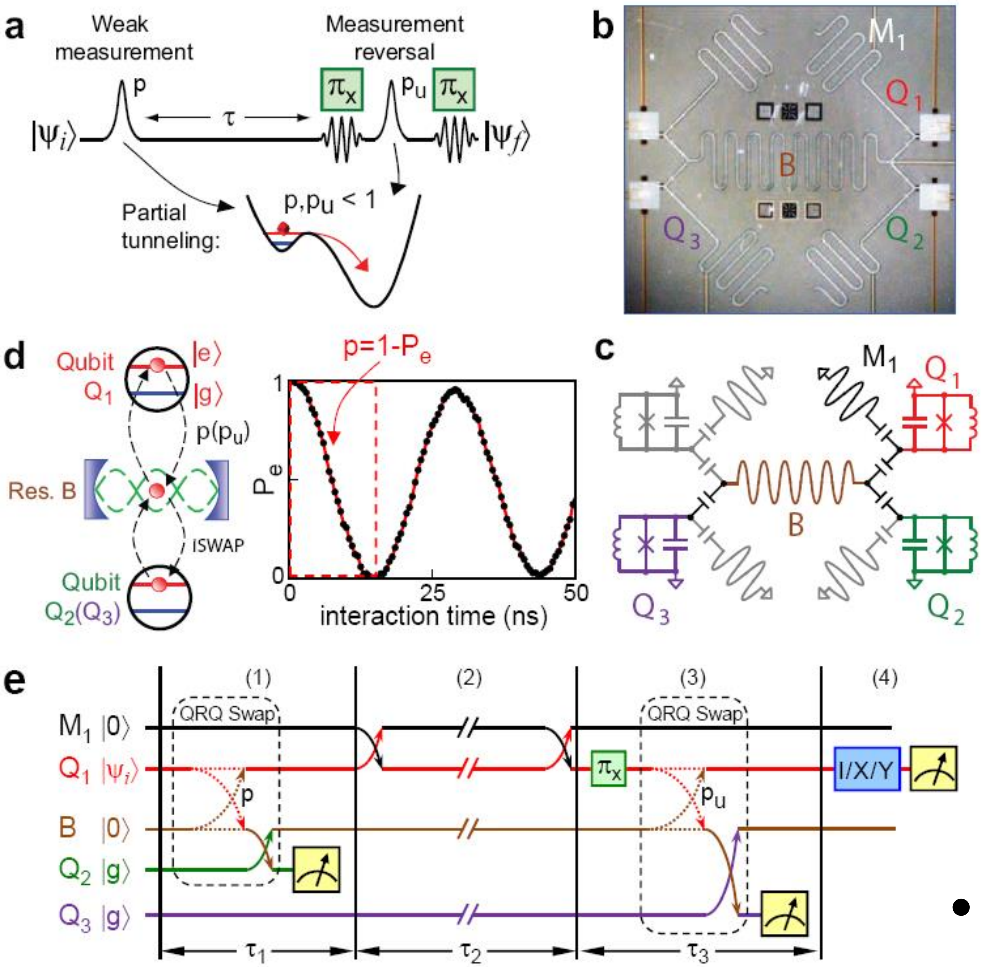
- Works perfectly (optics, not solid state!)
- Energy relaxation is imitated (amplitude damping)
- No real qubits with single-shot measurement

Revives entanglement from "sudden death"

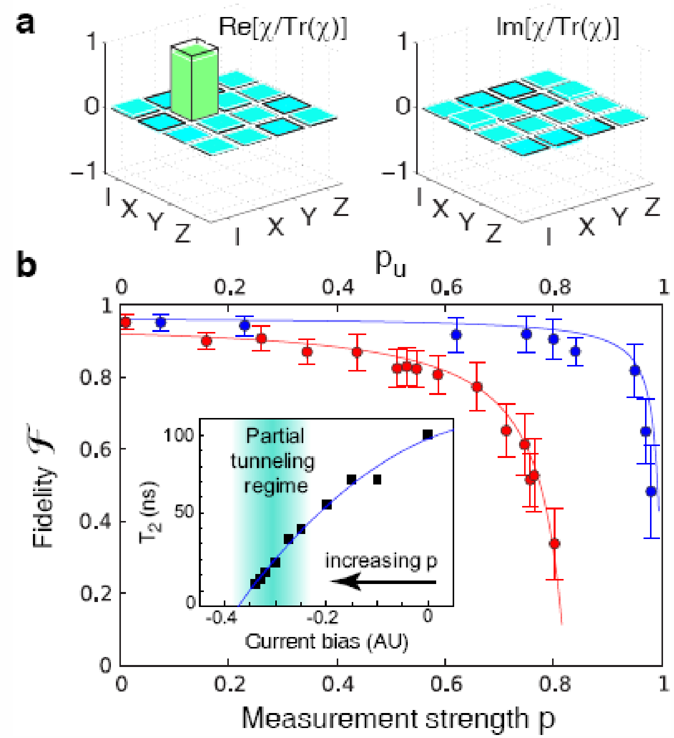
Realization with s/c phase qubits

Y. Zhong, Z. Wang, J. Martinis, A. Cleland,
A. Korotkov, and H. Wang, Nature Comm. (2014)

Quantum circuit and algorithm



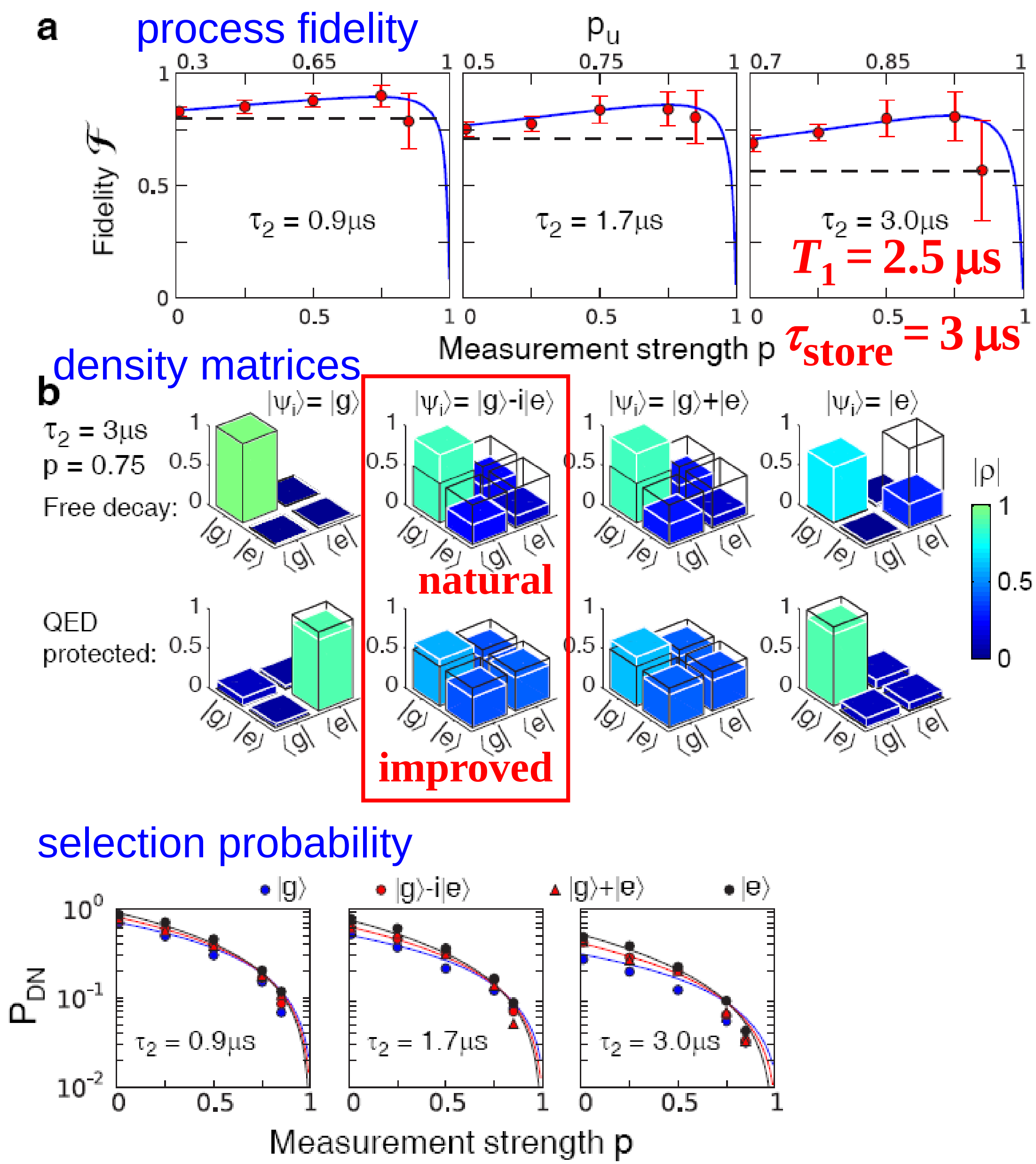
Basic uncollapse results



Device with 4 phase qubits and 5 resonators,
3 qubits and 2 resonators used in the algorithm

- Quantum state stored in resonator
- Weak measurement is implemented with ancilla qubit (better than partial)

Lifetime increase by uncollapse



Y. Zhong et al. (2014)

Uncollapse increases effective T_1 by ~3x

- "Quantum error detection" (not correction)
- First demonstration of real improvement (suppression of natural decoherence)

Conclusions (part 1)

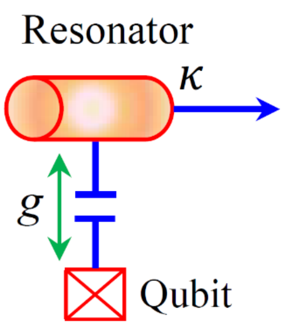
- Uncollapse may be useful
- 3x qubit lifetime increase demonstrated
- “Error detection”, not “error correction”

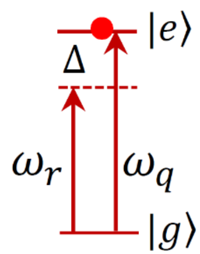
Open questions

- Other “useful” procedures based on continuous or partial (generalized) quantum measurements
- Relation between uncollapse and quantum feedback; both can suppress decoherence but produce either known (desired) or unknown (preserved) state. May be some combination is useful?

Purcell effect with microwave drive: suppression of qubit relaxation rate

E. A. Sete, J. M. Gambetta, and A. N. Korotkov, arXiv:1401.5545





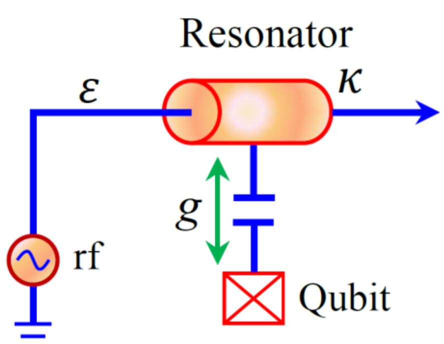
"Usual" Purcell effect: energy relaxation of a qubit via coupling with a leaking resonator

$$\Gamma_0 = \kappa \frac{g^2}{\Delta^2}$$

κ – resonator bandwidth
g – qubit-resonator coupling
Δ – detuning (Δ>>g)

simple interpretation: $(g/\Delta)^2$ is "tail" probability

Now with drive: what is the change?



n photons in resonator on average (coh. state)

Naïve hypotheses:

1) $$\Gamma = \kappa \left(\frac{\sqrt{n}\, g}{\Delta} \right)^2$$ because coupling increases as $\sqrt{n}\, g$

2) $$\Gamma = \Gamma_0$$ no change because the system is linear

Surprising answer: relaxation rate <u>decreases</u> with n

Simple formula for Purcell rate with drive

"Unraveling" of resonator decay:
either "jump" or "no-jump" evolution.

The "jump" mixing JC eigenstates
leads to qubit relaxation:

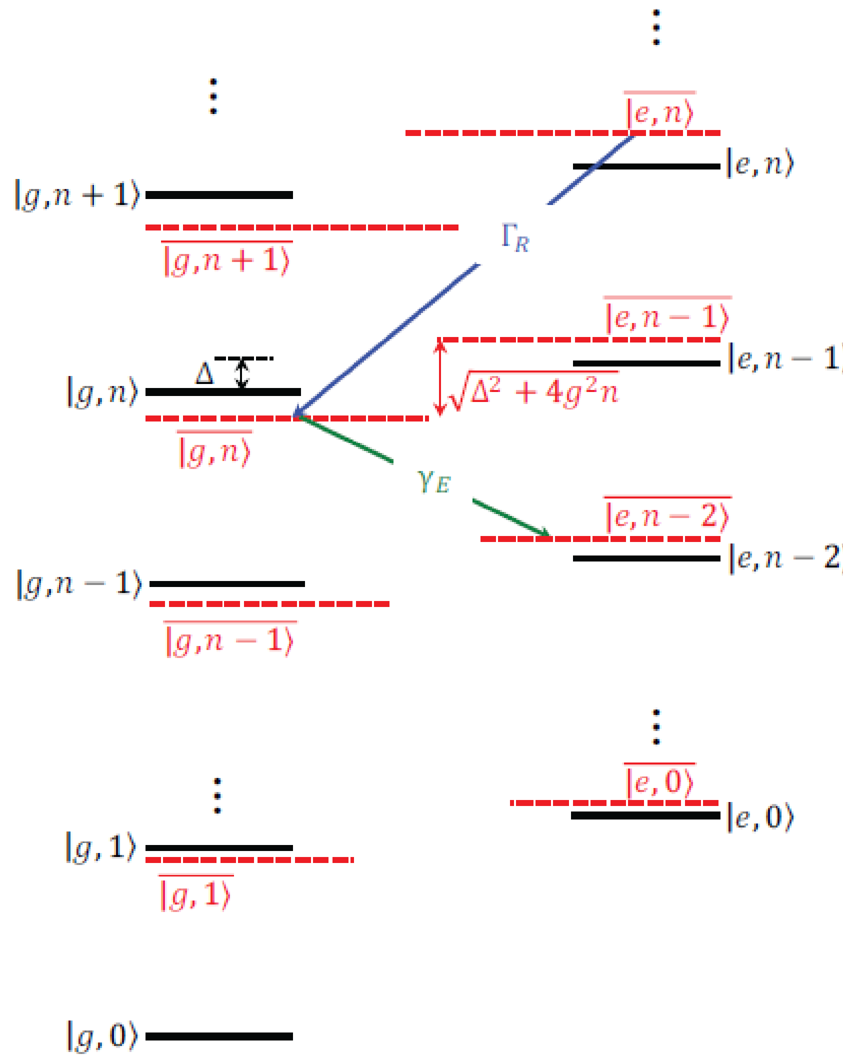
$$\Gamma = \kappa \left| \overline{\langle g,n} \,|\, a \,|\, \overline{e,n\rangle} \right|^2$$

(+trivial averaging over *n*)

In particular, for *n*>>1 (strong drive)

$$\Gamma = \frac{\kappa g^2}{\Delta^2} \left(\frac{1}{1+\bar{n}/n_c} + \frac{1}{\sqrt{1+\bar{n}/n_c}} \right)^2$$

$n_c = \Delta^2 / 4g^2$ "critical" number of photons

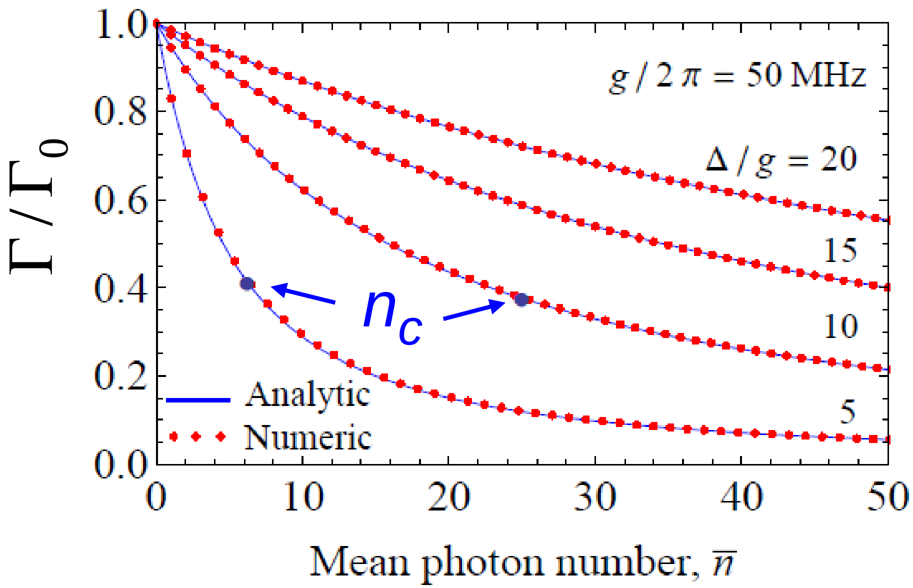


JC ladder

Is this simple formula correct?

$$\Gamma = \kappa \left| \overline{\langle g,n} \,|\, a \,|\, \overline{e,n\rangle} \right|^2 \qquad \Gamma = \frac{\kappa g^2}{\Delta^2}\left(\frac{1}{1+\bar{n}/n_c} + \frac{1}{\sqrt{1+\bar{n}/n_c}} \right)^2 \qquad n_c = \Delta^2 / 4g^2$$

Good agreement with numerics



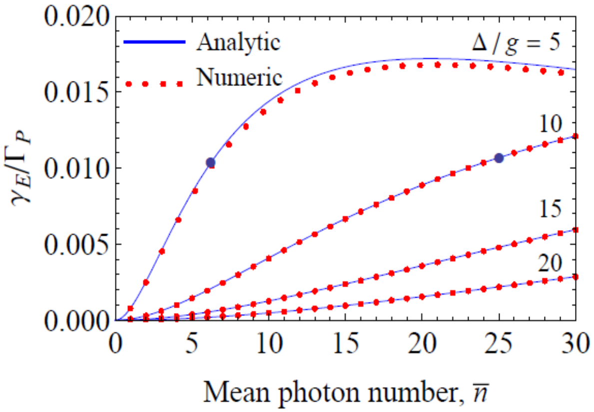
- Significant suppression of qubit relaxation rate when approaching nonlinear regime, $n \sim n_c$
- Strong suppression in strongly nonlinear regime, $n \gg n_c$

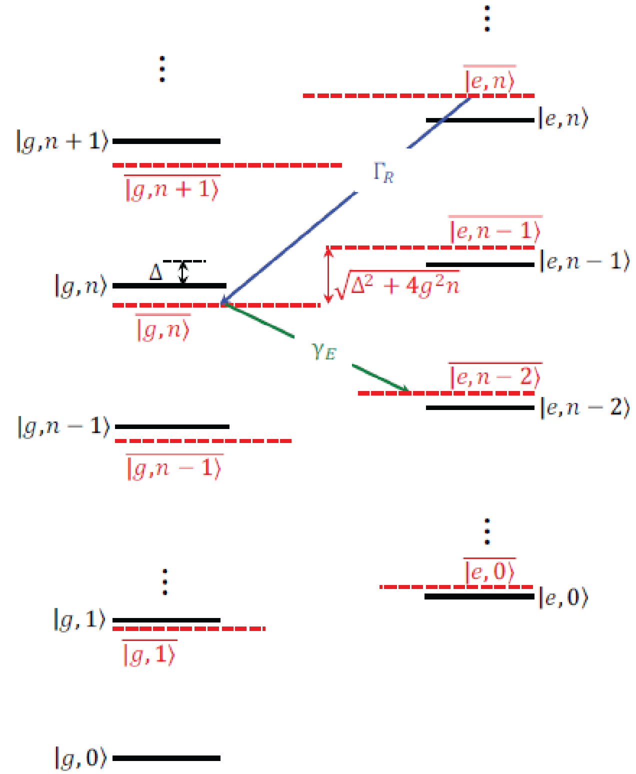
We also checked the simple formula using formal approach (very cumbersome) in weakly nonlinear regime, $n \ll n_c$

Also weak qubit excitation

$$\gamma_E = \kappa \left| \overline{\langle e,n-2 \,|\, a \,|\, \overline{g,n}\rangle} \right|^2$$ (similar derivation)

$$\gamma_E = \frac{\kappa g^2}{\Delta^2}\left(\frac{1}{1+\bar{n}/n_c} - \frac{1}{\sqrt{1+\bar{n}/n_c}} \right)^2 \quad \text{for } \bar{n} \gg 1$$





Physical interpretation of relaxation suppression

Microwave drive causes ac Stark shift of the qubit frequency, which increases effective detuning, thus decreasing Purcell rate.
However, no quantitative agreement with actual result.

Conclusions (part 2)

- Purcell relaxation decreases with drive
- Relaxation suppression may be strong in nonlinear regime
- Also qubit excitation (weak)

Open questions

- Good physical interpretation
- Relation between quantum and classical
- Now work on theory of Purcell filter; unclear which approach is more accurate: simple classical or quantum (RWA, many levels, “black box quantization”)

Error matrices in quantum process tomography

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Outline:
- Basics of χ-matrix
- Error matrices χ^{err} and $\tilde{\chi}^{err}$
- Some properties (incl. interpretation)
- Composition of gates
- Unitary corrections
- Error from Lindblad-form decoherence
- SPAM identification and subtraction

Basics of the QPT matrix χ

Definition $$\rho_{\rm fin} = \sum_{m,n} \chi_{mn} E_m \rho_{\rm in} E_n^\dagger,$$

Pauli basis $$I \equiv 1\!\!1, \;\; X \equiv \sigma_x, \;\; Y \equiv \sigma_y, \;\; Z \equiv \sigma_z,$$

two qubits: ***II, IX, IY, IZ, XI, XX, … ZZ***

Pauli basis is orthogonal (almost orthonormal) $$\langle E_m | E_n \rangle \equiv {\rm Tr}(E_m^\dagger E_n) = \delta_{mn} d, \quad d = 2^N$$

χ-matrix for unitary *U* $$\chi_{mn} = u_m u_n^*, \; U = \sum u_n E_n, \; u_n = \frac{1}{d} {\rm Tr}(U E_n^\dagger)$$

Fidelity (unitary desired, trace-preserving actual) $$F_\chi = {\rm Tr}(\chi^{\rm des} \chi)$$

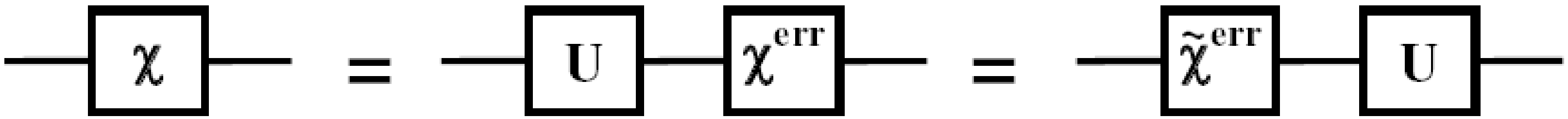
Relation to average state fidelity (IBM term.: process fid. vs gate fid.)

$$1 - F_\chi = (1 - F_{\rm av}) \, \frac{d+1}{d}$$

$$F_{\rm av} = \overline{{\rm Tr}(\rho_{\rm fin} \rho_{\rm fin}^{\rm des})} \qquad F_{\rm av} \geq F_\chi$$

Fidelity when compare with a non-unitary process $$F_\chi = \left({\rm Tr} \sqrt{\sqrt{\chi}\, \chi^{\rm des} \sqrt{\chi}} \right)^2$$

Definition of error matrices



U is the desired unitary, the rest is "error"

$$\rho_{\text{fin}} = \sum\nolimits_{m,n} \chi^{\text{err}}_{mn} E_m U \rho_{\text{in}} U^\dagger E_n^\dagger,$$

$$\rho_{\text{fin}} = \sum\nolimits_{m,n} \tilde{\chi}^{\text{err}}_{mn} U E_m \rho_{\text{in}} E_n^\dagger U^\dagger.$$

Equivalent to the χ-matrix and to each other (two languages)

$$\chi^{\text{err}} = V\chi V^\dagger, \;\; V_{mn} = \text{Tr}(E_m^\dagger E_n U^\dagger)/d,$$

$$\tilde{\chi}^{\text{err}} = \tilde{V}\chi \tilde{V}^\dagger, \;\; \tilde{V}_{mn} = \text{Tr}(E_m^\dagger U^\dagger E_n)/d.$$

Same math. properties as for χ-matrix (Hermitian, positive, trace-one, etc.)

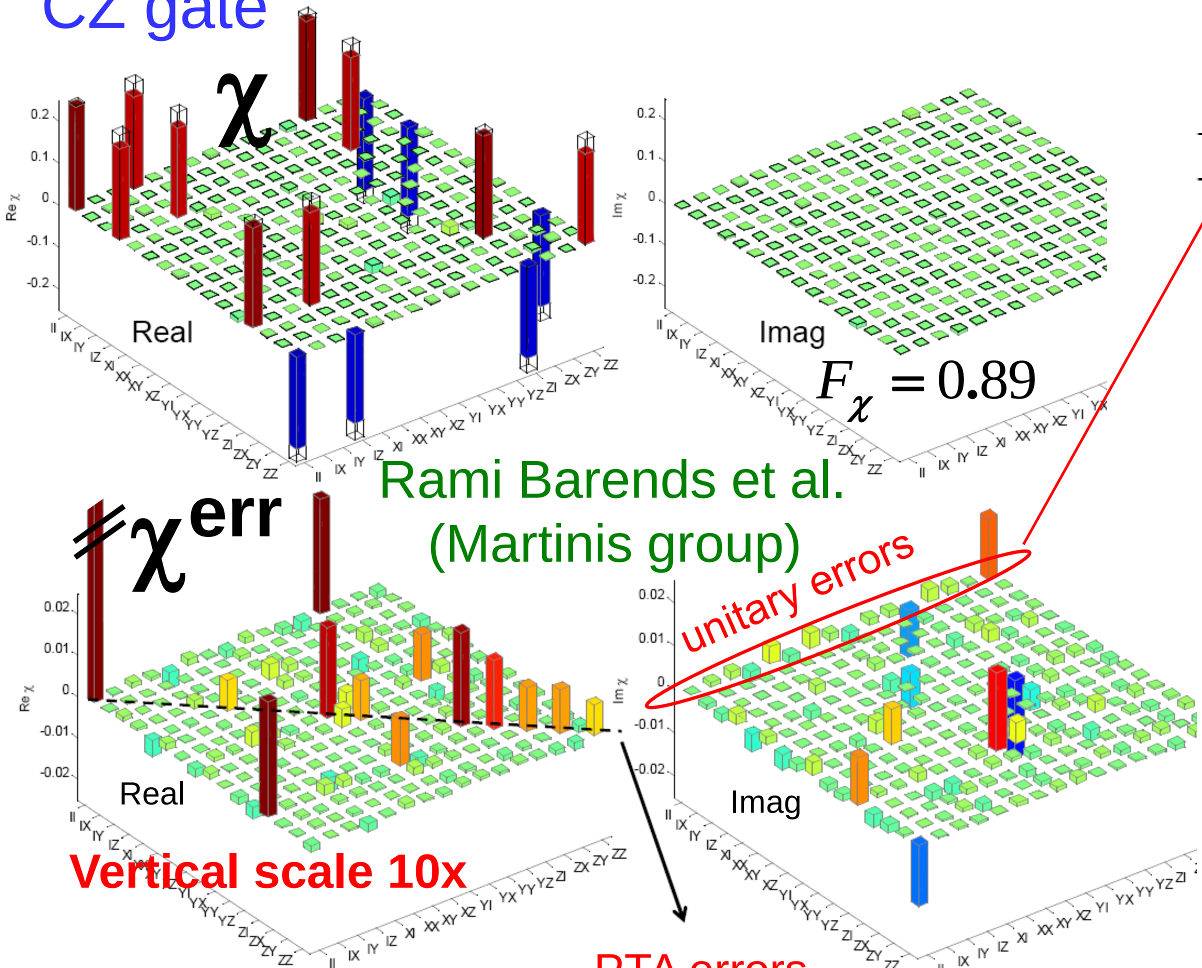
Convenience: only one big element at the top left corner, other non-zero elements indicate imperfections

$$F_\chi = \chi^{\text{err}}_{00} = \tilde{\chi}^{\text{err}}_{00},$$

$$0 \equiv I, II, \ldots$$

Experimental example

CZ gate



Rami Barends et al. (Martinis group)

Unitary imperfection:

$$\mathrm{Im}(\chi^{err}_{n0}) \approx -iF_\chi u^{err}_n$$

$$U^{err} = \sum_n u^{err}_n E_n$$

Why imaginary part?

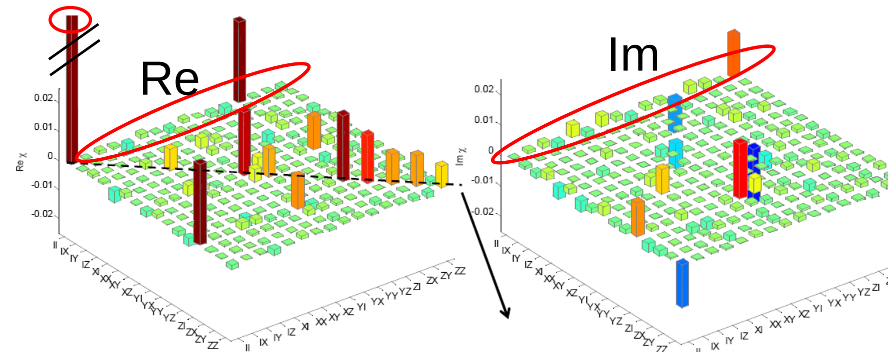
$$U^{err} = \cancel{e^{i\phi}} \exp(iH^{err})$$

$$\approx \mathbb{1} + \sum_{n\neq 0} i h^{err}_n E_n$$

real

Meaning of some elements

$$\chi^{\text{err}}_{mn}$$



χ^{err}_{00} - fidelity (top left)

$\mathbf{Im}(\chi^{\text{err}}_{0n}) = -\mathbf{Im}(\chi^{\text{err}}_{n0})$ - unitary imperfection (top row & left column); may be the biggest elements in χ^{err}

$$|\chi^{err}_{0n}| \leq \sqrt{\chi^{err}_{nn}} \leq \sqrt{1-F_\chi}, \qquad |\chi^{err}_{mn}| \leq \sqrt{\chi^{err}_{mm}\chi^{err}_{nn}} \leq (1-F_\chi)/2$$

$\mathbf{Re}(\chi^{\text{err}}_{0n}) = \mathbf{Re}(\chi^{\text{err}}_{n0})$ - non-unitary “Bayesian” evolution in the absence of “jumps” due to decoherence

Other elements (with $m\neq 0$, $n\neq 0$) originate from “strong jumps” due to decoherence

Diagonal elements ($n\neq 0$) have two contributions: from the “jumps” due to decoherence and second-order unitary imperfection, $\approx (\mathbf{Im}\,\chi^{\text{err}}_{0n})^2 / F_\chi$

The same applies to $\tilde{\chi}^{\text{err}}$

Decomposition into Kraus operators

Formal procedure: diagonalize χ^{err}

$$\chi^{\rm err} = TDT^{-1} \qquad D = {\rm diag}(\lambda_0, \lambda_1, ...) \qquad \lambda_0 \geq \lambda_1 \geq ...$$

One main eigenvalue λ_0 (≈1), other λ are small

$$F_\chi \leq \lambda_0 \leq 1 \qquad \sum_n \lambda_n = 1$$

Decomposition

$$\rho_{fin} = \sum_{k=0}^{d^2-1} \lambda_k A_k (U\rho_{in} U^\dagger) A_k^\dagger, \quad \sum_k \lambda_k A_k^\dagger A_k = 1\!\!1$$

$$A_k = \sum_n a_n^{(k)} E_n, \quad a_n^{(k)} = T_{nk}, \quad \chi^{\rm err}_{mn} = \sum_k \lambda_k a_m^{(k)} (a_n^{(k)})^*$$

Kraus operators A_k form orthonormal basis

Interpretation: "apply A_k with probability λ_k" (there are caveats)

A_0≈**1** describes "coherent" (gradual) evolution, others are "strong jumps"

Actually $|\psi_{\rm in}\rangle \to \frac{A_k U |\psi_{\rm in}\rangle}{\rm Norm}$ with probability $P_k = {\rm Tr}(\lambda_k A_k^\dagger A_k U \rho_{\rm in} U^\dagger)$,

so λ_k is average probability, $\lambda_k = \overline{P_k}$

Intuitive (approximate) way to think

$$F_\chi \approx 1 - \mathcal{E}_U - \mathcal{E}_D$$

$\mathcal{E}_D \equiv 1 - \lambda_0$ (average) probability of "strong decoherence jump"

$\mathcal{E}_U \equiv \sum_{n>0} |u_n^{\rm err}|^2$ unitary error in the case of no jump

no-jump scenario: $\sqrt{\lambda_0}\, A_0 \approx U^{\rm err} \underbrace{\left(1\!\!1 - \frac{1}{2} \sum_{k>0} \lambda_k A_k^\dagger A_k \right)}$ ("Bayesian update")

$U^{\rm err} \rightarrow \mathrm{Im}(\chi_{n0}^{\rm err})$; Bayesian update $\rightarrow \mathrm{Re}(\chi_{n0}^{\rm err})$

$\chi^{\rm err} = \chi^{\rm coh} + \chi^{\rm dec}$

"coherent" contribution to $\chi^{\rm err}$ is of the second order (except top row and left column), not important unless big unitary imperfection

Example: one-qubit T_1 and T_φ decoherence

$$\chi^{\rm err} = \tilde{\chi}^{\rm err} = \chi$$ (no unitary evolution)

Energy relaxation

$$\begin{pmatrix} F_\chi & 0 & 0 & t/4T_1 \\ & t/4T_1 & -it/4T_1 & 0 \\ & & t/4T_1 & 0 \\ & & & \sim t^2 \end{pmatrix}$$

$$A = \sqrt{t/4T_1}\,(X+iY)$$

$$A^\dagger A = (t/2T_1)(I-Z)$$

("jump" incl. prob.)

Markovian pure dephasing

$$\begin{pmatrix} F_\chi & 0 & 0 & 0 \\ & 0 & 0 & 0 \\ & & 0 & 0 \\ & & & t/2T_\varphi \end{pmatrix}$$

$$A = \sqrt{t/2T_\varphi}\; Z$$

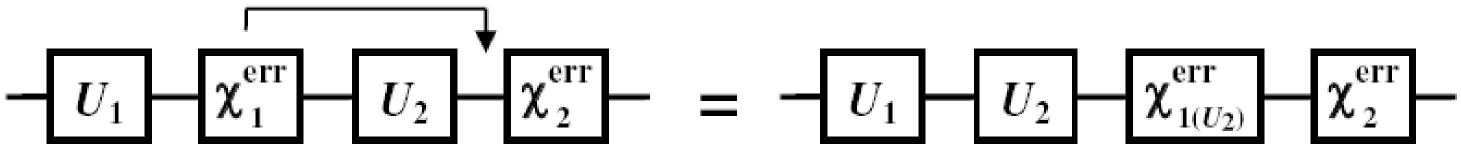
$$A^\dagger A = (t/2T_\varphi)\, I$$ (state-indep., no Bayes)

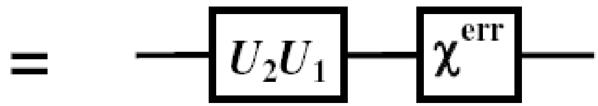
Non-Markovian pure dephasing

$$\chi_{ZZ} = \frac{1-\langle\cos\varphi\rangle}{2}$$ same as in Ramsey $$P_R = \frac{1}{2} + \frac{1}{2}\, e^{-t/2T_1}\langle\cos\varphi\rangle\,\cos\phi_R$$

Very slow fluctuations (Gaussian Ramsey): $\chi_{ZZ} = t^2/2T_\varphi^2$

Composition of error processes





Small elements in χ^{err} ⇒ first order is sufficient

If no *U*, then very simple: just add errors

$$\chi^{err} \approx \chi_1^{err} + \chi_2^{err} - \chi^{\mathbf{I}}$$

ideal

A little more accurate: $\chi_{mn}^{err} \approx F_2 \chi_{1,mn}^{err} + F_1 \chi_{2,mn}^{err}$

Even more accurate for diagonal elements:

$$\chi_{nn}^{err} \approx F_2 \chi_{1,nn}^{err} + F_1 \chi_{2,nn}^{err} + 2\,\mathrm{Im}(\chi_{1,0n}^{err})\,\mathrm{Im}(\chi_{2,0n}^{err})$$

With *U* two steps: “jump over unitary”, then add

$$\chi_{1(U_2)}^{err} = W_{(U_2)} \chi_1^{err} W_{(U_2)}^{\dagger}$$

$$W_{(U),mn} = \mathrm{Tr}(E_m^{\dagger} U E_n U^{\dagger})/d$$

Same for jumping Krauses:

$$A_k \to U A_k U^{\dagger}$$

Unitary corrections

$\mathbf{Im}(\chi_{0n}^{\text{err}})$ shows unitary imperfections

Can correct (at least some elements) by applying U^{corr}, then increase fidelity (only in the second order)

$$U^{\text{corr}} = \sum_n u_n^{\text{corr}} E_n \approx \mathbb{1}$$

$$\chi_{n0}^{\text{err}} \rightarrow \chi_{n0}^{\text{err}} + F_\chi u_n^{\text{corr}}$$

choose $\mathrm{Im}(u_n^{\text{corr}}) \approx -\mathrm{Im}(\chi_{n0}^{\text{err}})/F_\chi$

fidelity improvement $\Delta F_\chi \approx \sum_{n\neq 0} (\mathrm{Im}\,\chi_{n0}^{\text{err}})^2 / F_\chi$

χ^{err} from Lindblad-form decoherence

$$\dot{\rho} = -\frac{i}{\hbar}[H,\rho] + \sum_j \Gamma_j (B_j \rho B_j^\dagger - \frac{1}{2} B_j^\dagger B_j \rho - \frac{1}{2} \rho B_j^\dagger B_j)$$

Contribution during Δt

$$\frac{\chi^{err}_{mn}(t,\Delta t) - \chi^{\mathbf{I}}_{mn}}{\Gamma\, \Delta t} \equiv \mathcal{B}_{mn} = b_m b_n^* - \frac{c_m \delta_{n0} + c_n^* \delta_{m0}}{2},$$

$$b_n = \mathrm{Tr}(B E_n^\dagger)/d,$$

$$c_n = \mathrm{Tr}(B^\dagger B E_n^\dagger)/d = \sum_{p,q} b_p^* b_q \mathrm{Tr}(E_p^\dagger E_q E_n^\dagger)/d.$$

Jump over the unitary, then add

$$\tilde{\chi}^{err} \approx \chi^{\mathbf{I}} + \int_0^{t_G} \Gamma\, W^\dagger(t)\, \mathcal{B}\, W(t)\, dt,$$

$$W_{mn}(t) = \mathrm{Tr}[E_m^\dagger U(t) E_n U^\dagger(t)]/d,$$

$$U(t) = \exp[\frac{-i}{\hbar} \int_0^t H(t)\, dt],$$

Equivalently, jump Kraus *B* over *U*

$$\tilde{\chi}^{err} \approx \chi^{\mathbf{I}} + \int \Gamma \tilde{\mathcal{B}}(t)\, dt$$

$$\tilde{B}(t) \equiv U^\dagger(t) B U(t)$$

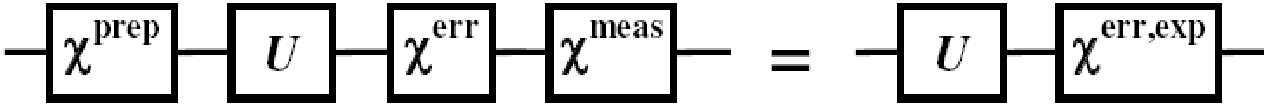
$\chi^{err} \propto \Gamma$, “pattern” depends on $U(t)$

Infidelity accumulates

$$F_\chi \approx 1 - \int_0^{t_G} \Gamma \sum_{n\neq 0} |b_n|^2\, dt$$

$$1 - F_\chi = \frac{t_G}{2T_1^{(a)}} + \frac{t_G}{2T_1^{(b)}} + \frac{t_G}{2T_\varphi^{(a)}} + \frac{t_G}{2T_\varphi^{(b)}}$$

SPAM identification and subtraction



Representation by error channels is an assumption

A way to check: $F_\chi \approx F_\chi^{\text{exp}} / F_\chi^I$ (compare with randomized benchmarking)

SPAM contributions depend on *U*

$$\chi^{\text{err,exp}} \approx \chi^{\text{err}} + W_{(U)}(\chi^{\text{prep}} - \chi^{\mathbf{I}})W_{(U)}^\dagger + (\chi^{\text{meas}} - \chi^{\mathbf{I}})$$

Simple if SPAM is dominated by one component, then compare with no gate

mainly meas. error, then

$$\chi^{\text{err}} \approx \chi^{\text{err,exp}} - (\chi^{\text{err},I} - \chi^{\mathbf{I}})$$

no gate

mainly prep. error, then

$$\tilde{\chi}^{\text{err}} \approx \tilde{\chi}^{\text{err,exp}} - (\chi^{\text{err},I} - \chi^{\mathbf{I}})$$

In general, need to know χ^{prep} and χ^{meas} separately.

Idea: use high-fidelity single-qubit gates to separate the contributions.

X and *Y* gates flip the sign of some off-diagonal elements, $\sqrt{X}$ and $\sqrt{Y}$ exchange some diagonal elements. Lengthy procedure, but possible. Need it only for significant elements of $\chi^{\text{err},I}$.

Conclusions (part 3)

- Error matrices χ^{err} and $\tilde{\chi}^{err}$ are more convenient to use than χ, easy conversion between them
- More intuitive understanding of some elements, natural separation into “coherent” and “jump” contributions
- Since all elements are small (except one), the first-order calculations may be sufficient for composition of gates and accumulation of Lindblad-form decoherence
- Unitary imperfections are easily seen, simple analysis of unitary corrections
- SPAM is a serious problem, but there is (hopefully) a way to identify and subtract it. If SPAM is dominated by one type of error, then rather simple, otherwise quite lengthy.

Open questions (part 3)

- How to deal with SPAM in QPT?
- Is it possible to think about SPAM as an error channel?
- Particular experimental procedures for QPT, taking (at least some) care of the SPAM

Thank you