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Robust quantum state transfer using tunable couplers

Eyob A. Sete, Eric Mlinar, and Alexander N. Korotkov,
arXiv:1411.7103

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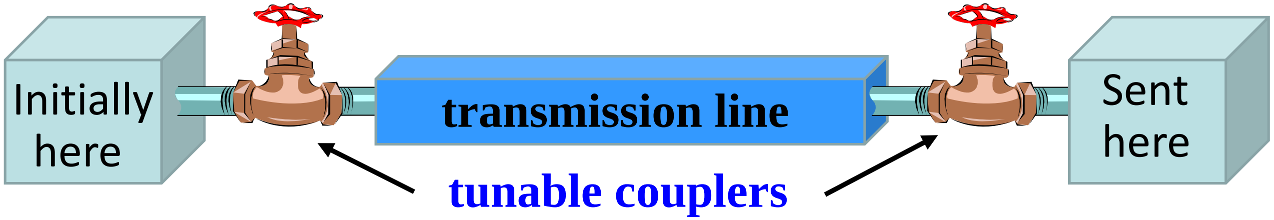

Kyle Keane, 2012

Outline:

- Main idea of the protocol
- Effect of the pulse shape variations
- Effect of multiple reflections
- Effect of frequency mismatch

Quantum State Transfer and Entanglement Distribution among Distant Nodes in a Quantum Network

J. I. Cirac,[1,2] P. Zoller,[1,2] H. J. Kimble,[1,3] and H. Mabuchi[1,3]
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(Received 12 November 1996)

We propose a scheme to utilize photons for ideal quantum transmission between atoms located at *spatially separated* nodes of a quantum network. The transmission protocol employs special laser pulses that excite an atom inside an optical cavity at the sending node so that its state is mapped into a *time-symmetric* photon wave packet that will enter a cavity at the receiving node and be absorbed by an atom there *with unit probability*. Implementation of our scheme would enable reliable transfer or sharing of entanglement among spatially distant atoms. [S0031-9007(97)02983-9]

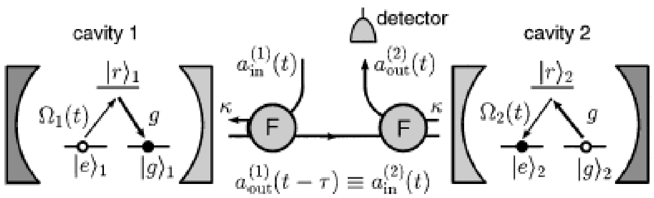

FIG. 1. Schematic representation of unidirectional quantum transmission between two atoms in optical cavities connected by a quantized transmission line (see text for explanation).

Time-symmetric photon wave packet

PHYSICAL REVIEW A **75**, 010301(R) (2007)

High-fidelity transfer of an arbitrary quantum state between harmonic oscillators

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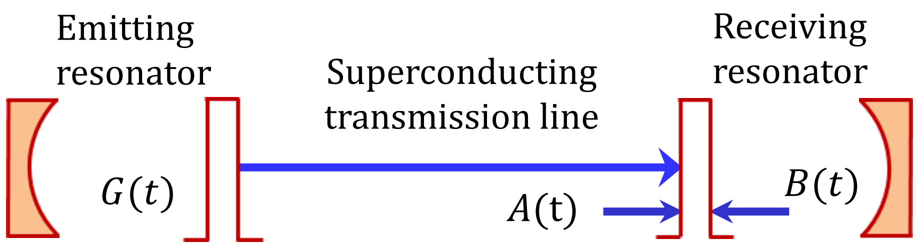
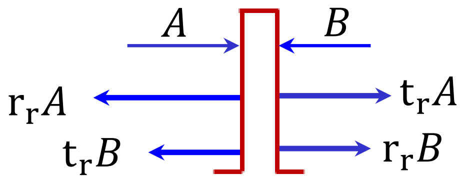
One tunable coupler (emitting)

Main idea: two couplers, destructive interference



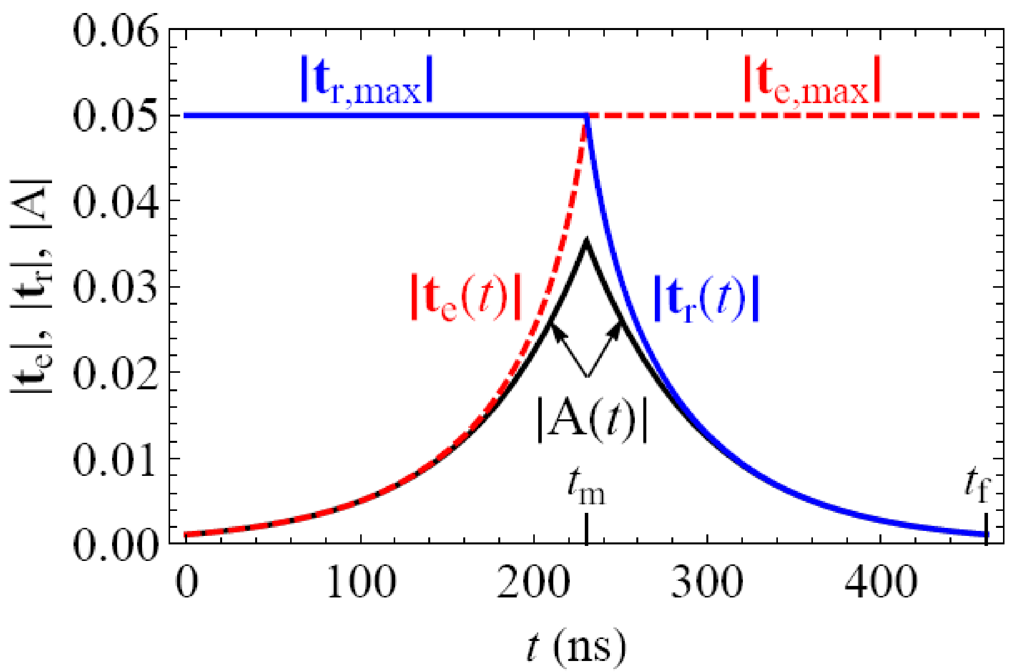


Tune the emitting or receiving coupling (t,r) so that

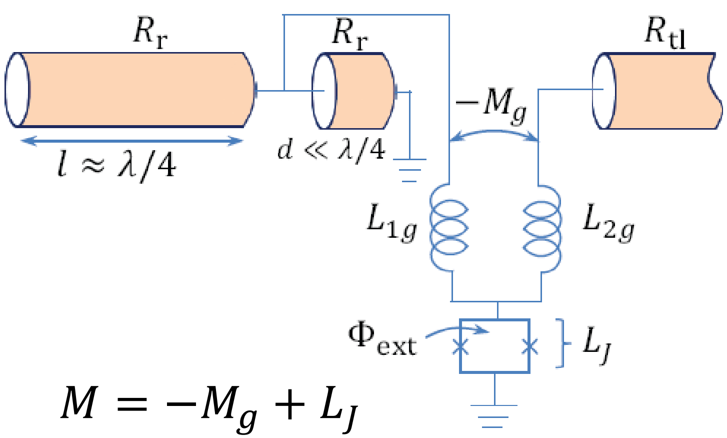
Korotkov PRA 84, 014510 (2011)

$$\mathbf{r}_r A + \mathbf{t}_r B = 0$$

Back-reflected field into the transmission line is cancelled (**destructive interference)**



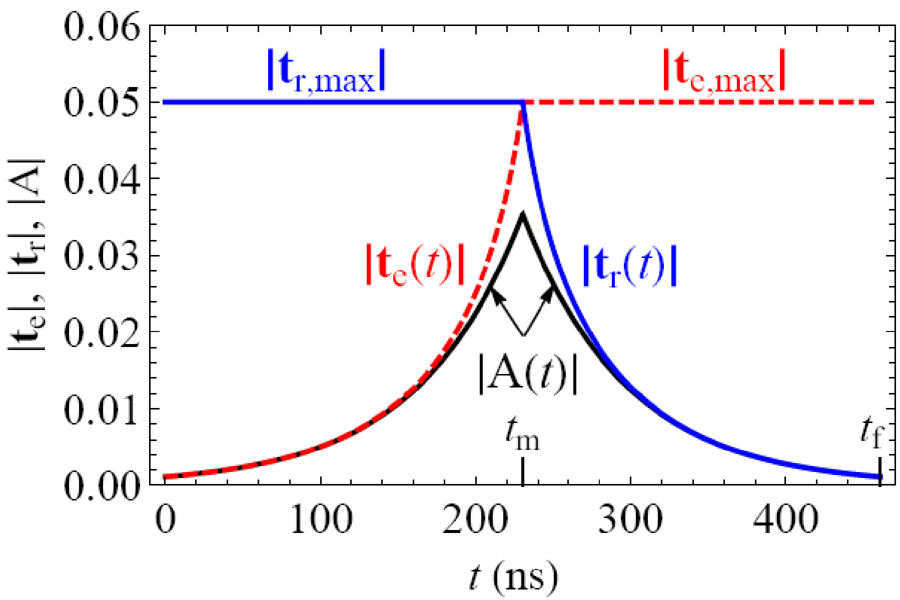
Tunable coupler



$$M = -M_g + L_J$$

Yin et al. PRL 110, 107001 (2013)

Tunable couplers' transmission amplitudes



- Emitting part realized (**Houck group**, PRA 2014)
- Receiving part realized with 99.4% fidelity (**Martinis group**, PRL 2014)

Loss: $1-\eta \approx \exp(-t_f/2\tau)$
(exponentially small)

Typical parameters:
$\omega/2\pi \approx 6$ GHz
$|t|_{\max} \approx 0.05 \Rightarrow \tau \approx 33$ ns
$t_f \approx 460$ ns $\Rightarrow \eta \approx 0.999$

Increasing part:
$A(t) \sim \exp(t/2\tau)$

requires

$$\mathbf{t}_e(t) = \begin{cases} \dfrac{\mathbf{t}_{e,\max}}{\sqrt{2\, e^{(t_m-t)/\tau_r} - 1}}, & t \le t_m \\ \mathbf{t}_{e,\max}, & t > t_m \end{cases}$$

Decreasing part:
$A(t) \sim \exp(-t/2\tau)$

requires

$$\mathbf{t}_r(t) = \begin{cases} \mathbf{t}_{r,\max}, & t \le t_m \\ \dfrac{\mathbf{t}_{r,\max}}{\sqrt{2\, e^{(t-t_m)/\tau_e} - 1}}, & t > t_m \end{cases}$$

Method of analysis

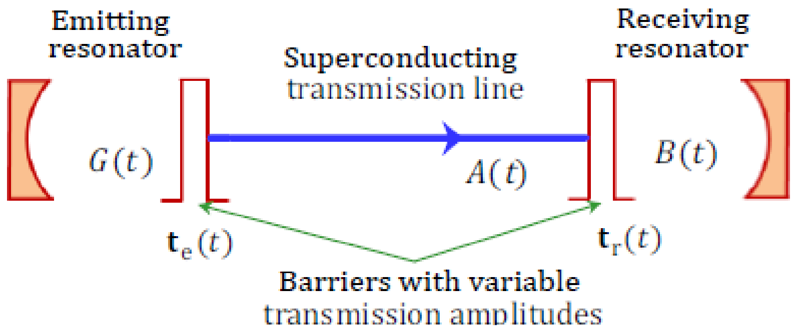
Classical field equations:

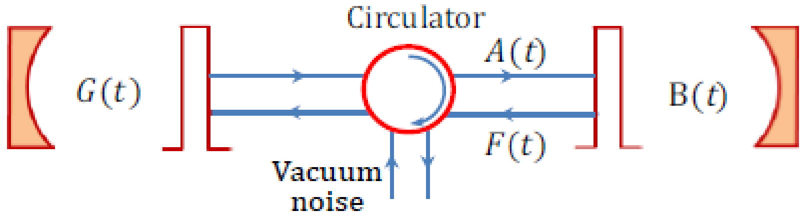
$$\dot{G} = -i\Delta\omega_e\, G - \left(\frac{1}{T_{1,e}} + \frac{|\mathbf{t}_e|^2}{2\tau_{\rm rt,e}}\right) G$$

$$\dot{B} = -i\Delta\omega_r\, B - \left(\frac{1}{T_{1,r}} + \frac{|\mathbf{t}_r|^2}{2\tau_{{\rm rt},r}}\right) B + \frac{\mathbf{t}_r}{\tau_{{\rm rt},r}}\, A(t)$$

$$A(t) = \sqrt{\eta_{tl}}\ \ t_e\, G(t)$$

η_{tl} is efficiency of the transmission line





- We characterize the performance of the protocol via energy transfer efficiency

$$\eta = \frac{|B(t_f)|^2}{|G(0)|^2}$$

- It is also sufficient for quantum case:

$$|\psi_{in}\rangle = (\alpha|0\rangle + \beta|1\rangle)|0\rangle_a \implies |\psi_{fin}\rangle = \alpha|0\rangle|0\rangle_a + \beta e^{i\varphi_f}\left(\sqrt{\eta}|1\rangle|0\rangle_a + \sqrt{1-\eta}|0\rangle|1\rangle_a\right)$$

$$\boxed{F_\chi = \frac{1}{4}\left(1 + \eta + 2\sqrt{\eta}\cos\varphi_f\right)}$$

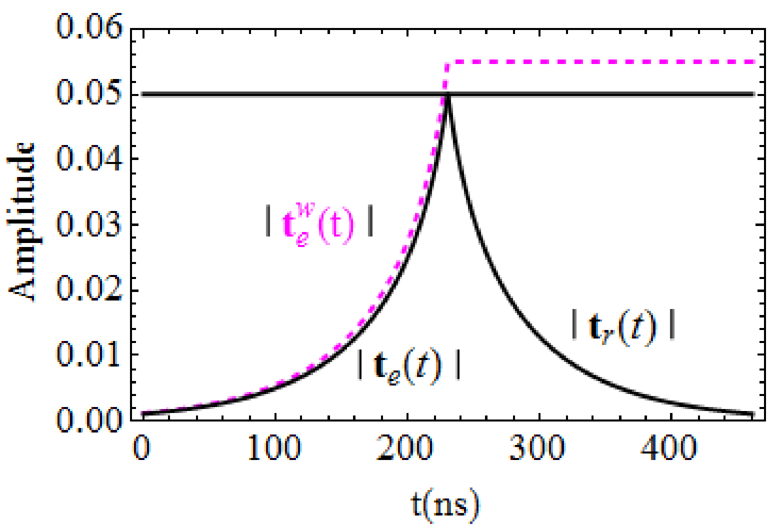
Quantum process fidelity

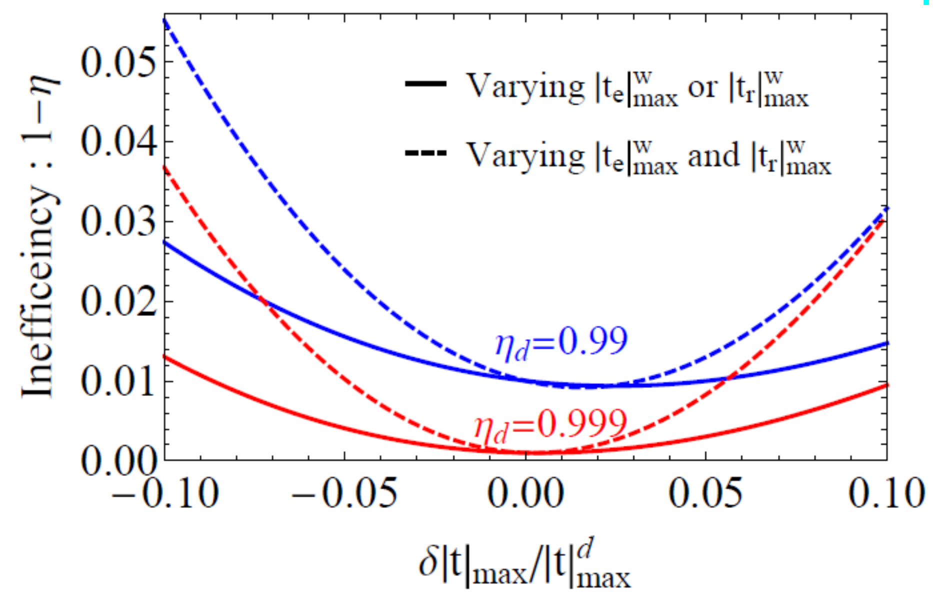
Imperfections of pulse shapes

A. Imperfection due to "wrong" max. transmission amplitude: $\left|\mathbf{t}_{\mathrm{e/r}}\right|_{max}$

$$\mathbf{t}_{\mathrm{e}}^{w}(t)=\frac{\mathbf{t}_{\mathrm{e,max}}^{w}}{\sqrt{2\,e^{(t_{m,e}^{d}-t)/\tau_{r}^{d}}-1}} \qquad \mathbf{t}_{\mathrm{r}}^{w}(t)=\frac{\mathbf{t}_{\mathrm{r,max}}^{w}}{\sqrt{2\,e^{(t-t_{m,r}^{d})/\tau_{e}^{d}}-1}}$$

$$|t|_{max}^{w}=|\mathbf{t}|_{max}^{d}+\delta|\mathbf{t}|_{\max}$$





±5% change in $\left|\mathbf{t}_{\mathrm{e/r}}\right|_{\max}$ leads to $-\delta\eta=0.006$

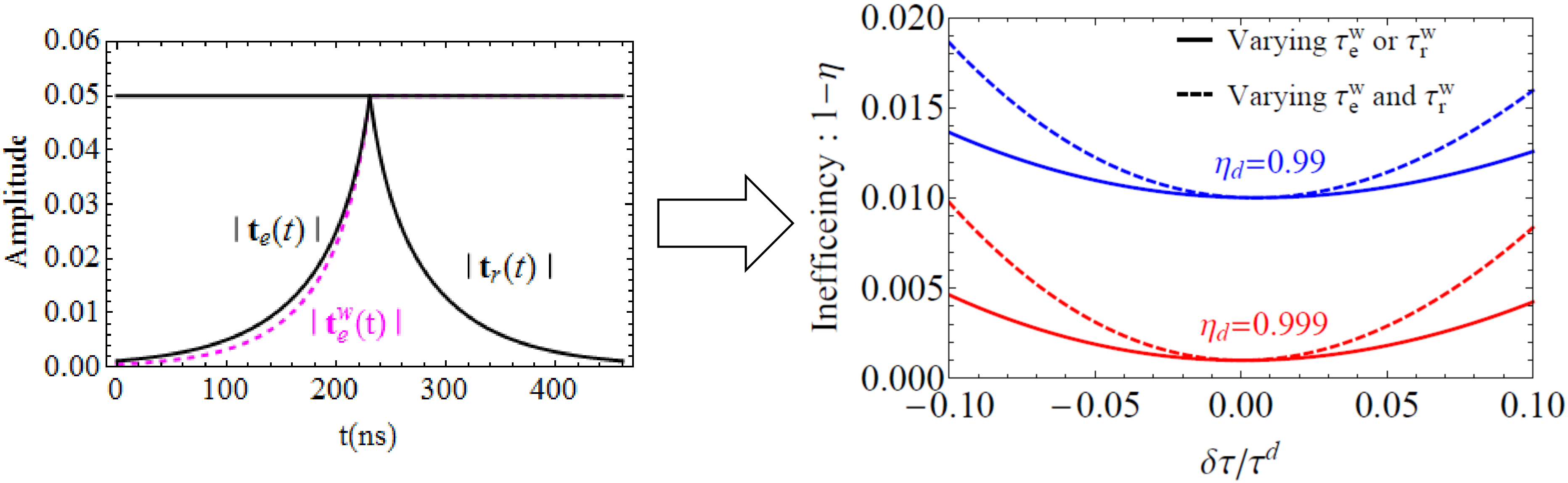
Change in Inefficiency:

$$\boxed{-\delta\eta\approx\frac{\delta|\mathbf{t}_e|_{\max}^2}{|\mathbf{t}_e|_{\max}^2}+\frac{\delta|\mathbf{t}_r|_{\max}^2}{|\mathbf{t}_r|_{\max}^2}+1.24\frac{\delta|t_r|_{\max}}{\delta|t_r|_{\max}}\frac{\delta|t_e|_{\max}}{\delta|t_e|_{\max}}}$$

B. “wrong” buildup/leakage time $\tau_{e/r}$

$$\mathbf{t}_{\rm e}^{w}(t) = \frac{\mathbf{t}_{\rm e,max}^{d}}{\sqrt{2\ e^{(t_{m,e}^{d}-t)/\tau_r^w}} - 1} \qquad \mathbf{t}_{\rm r}^{w}(t) = \frac{\mathbf{t}_{\rm r,max}^{d}}{\sqrt{2\ e^{(t-t_{m,r}^{d})/\tau_e^w}} - 1}$$

$$\tau^{w} = \tau^{d} + \delta\tau$$



±5 % change in τ leads to $-\delta\eta = 0.001$

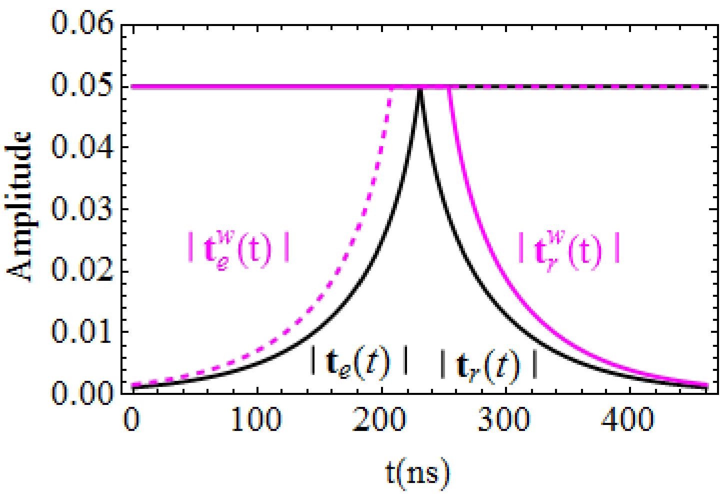
Change in Inefficiency:

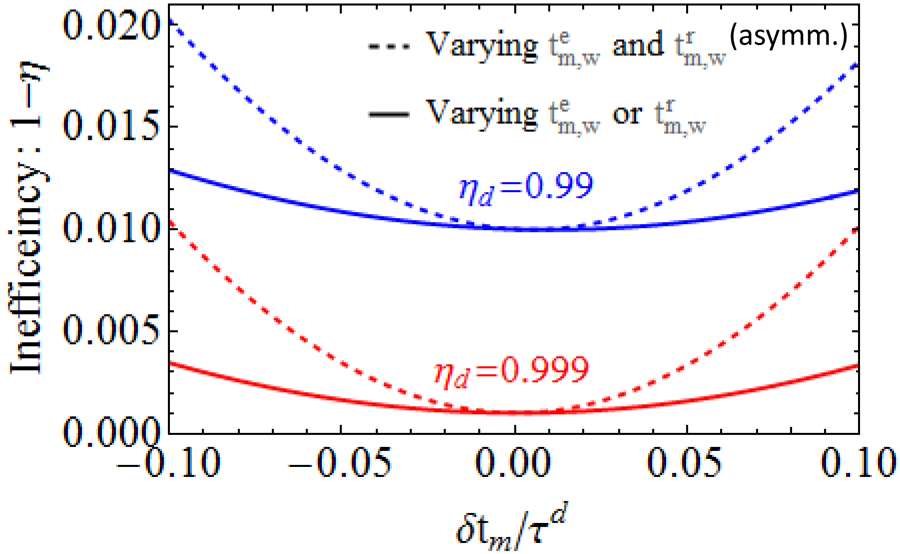
$$\boxed{-\delta\eta \approx 0.34\left[\left(\frac{\delta\tau_{\rm e}}{\tau^{\rm d}}\right)^2 + \left(\frac{\delta\tau_{\rm r}}{\tau^{\rm d}}\right)^2\right] + 0.12\frac{\delta\tau_{\rm e}}{\tau^{\rm d}}\frac{\delta\tau_{\rm r}}{\tau^{\rm d}}}$$

C. mismatched mid-time t_{m}

$$\mathbf{t}_{\mathrm{e}}^{w}(t) = \frac{\mathbf{t}_{\mathrm{e,max}}^{d}}{\sqrt{2\ e^{(t_{m,e}^{w}-t)/\tau_r^d}-1}} \qquad \mathbf{t}_{\mathrm{r}}^{w}(t) = \frac{\mathbf{t}_{\mathrm{r,max}}^{d}}{\sqrt{2\ e^{(t-t_{m,r}^{w})/\tau_e^d}-1}}$$

$$t_m^w = t_m^d + \delta t_m$$





For τ=33 ns mismatch of 3 ns leads to $-\delta\eta = 0.002$

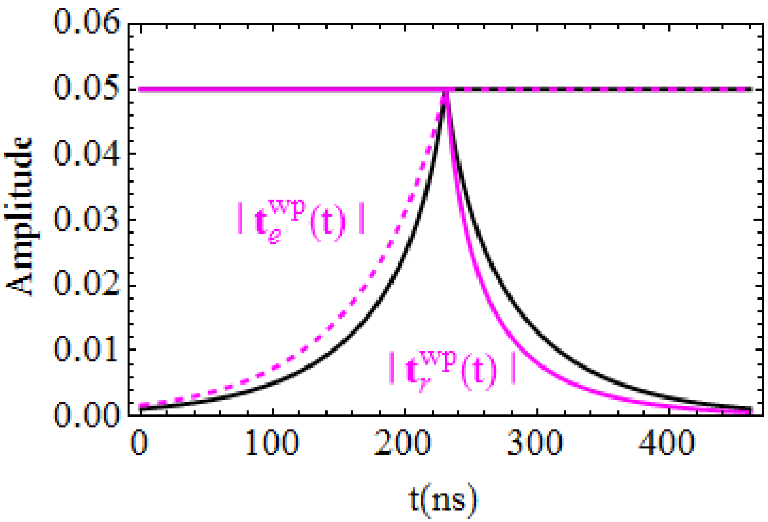
Change in inefficiency:

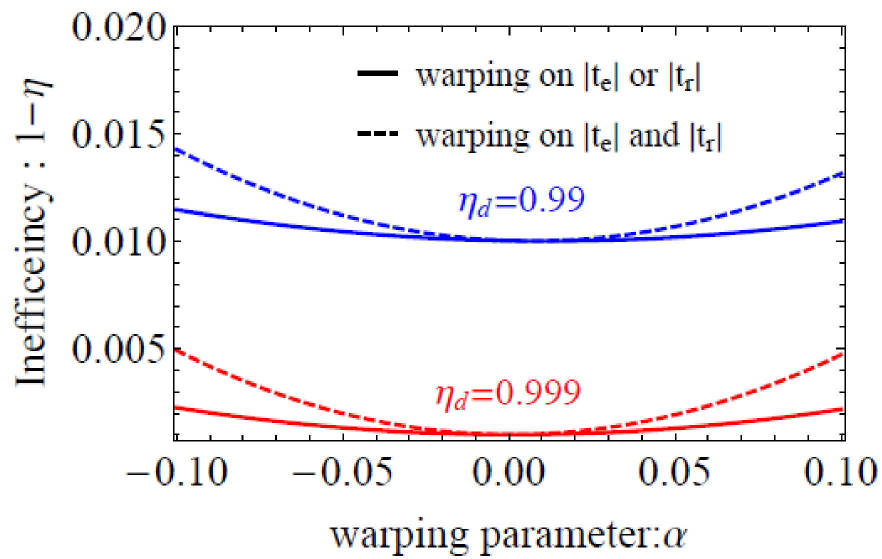
$$\boxed{-\delta\eta \approx 0.25\left(\frac{\delta t_{\mathrm{m}}^{\mathrm{e}} - \delta t_{\mathrm{m}}^{\mathrm{r}}}{\tau^{\mathrm{d}}}\right)^2}$$

D. Nonlinear shape distortion (“warping”)

Due to imperfect calibration of tunable couplers

$$\mathbf{t}^{wp}_{\mathrm{e/r}} = \mathbf{t}^{d}_{\mathrm{e/r}}[1 + \alpha_{\mathrm{e/r}}(\mathbf{t}^{d}_{\mathrm{e/r}} - \mathbf{t}^{d}_{\mathrm{e/r,max}})]$$





5% nonlinear distortaion leads to $-\delta\eta = 0.001$

Change in inefficiency:

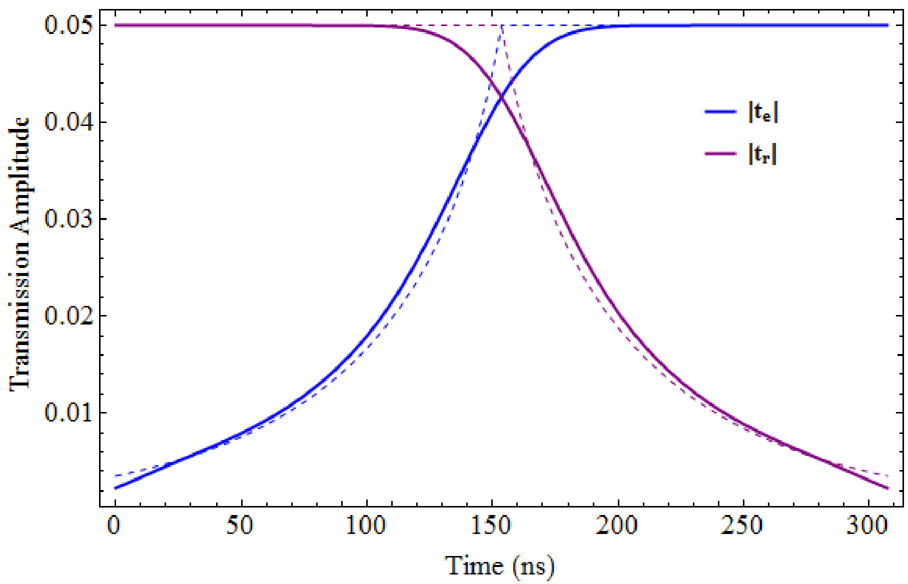
$$\boxed{-\delta\eta \approx 0.22(\alpha_e^2 + \alpha_r^2) + 0.12\alpha_e\alpha_r}$$

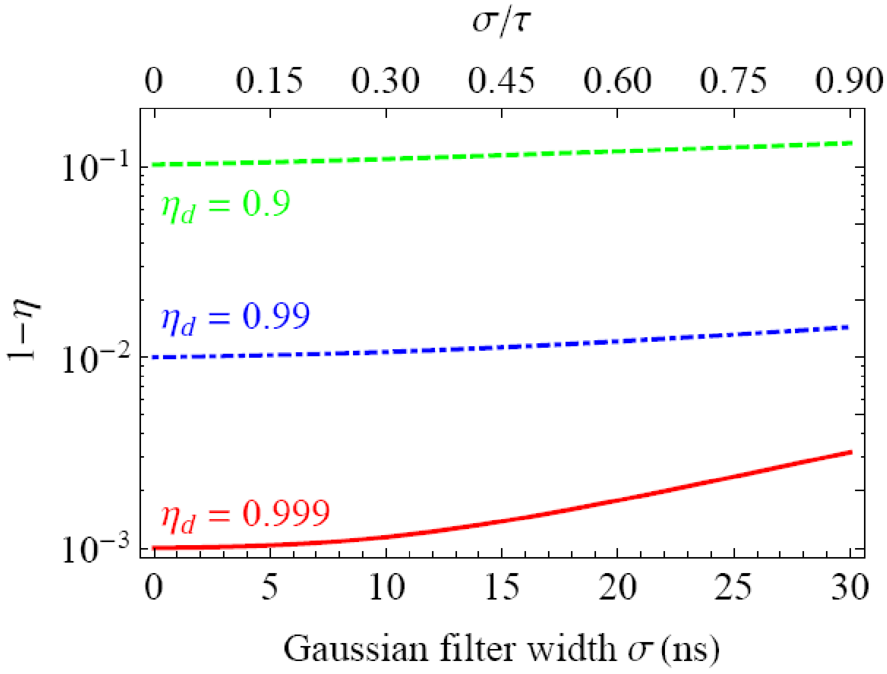
Gaussian filtering

Experimentally, desired pulse shapes pass through Gaussian filter. How will this affect efficiency?

$$\mathbf{t}_j(t) \;\rightarrow\; \mathbf{t}_{g,j}(t) = \frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{\infty} dx\, e^{-t^2/2\sigma^2}\, \mathbf{t}_j(x-t), \qquad j = e, r$$

$\eta_{design} = 0.99$
$\sigma_{filter} = 10ns$





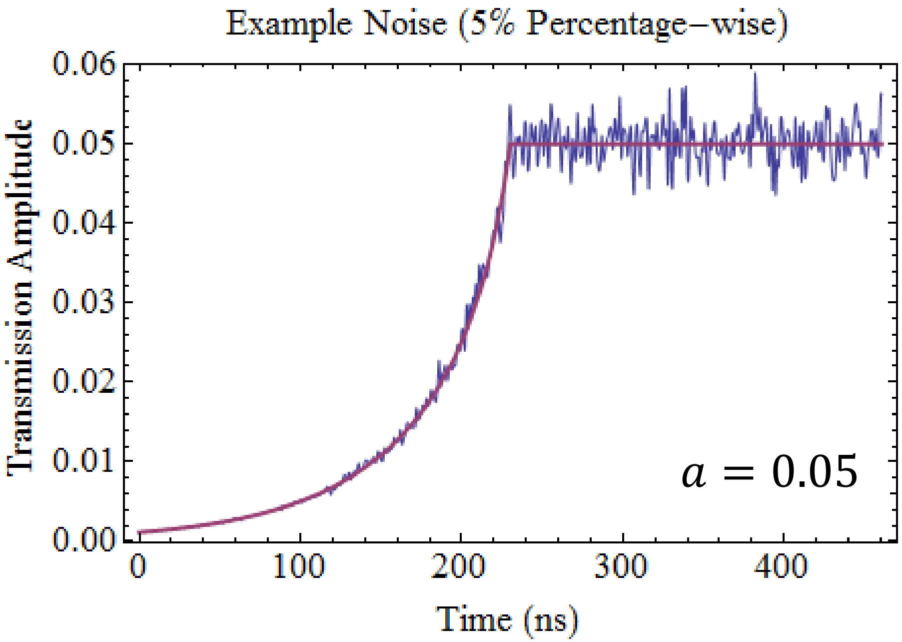
procedure is almost immune to this effect, even with a filtering width of 10ns (or higher).

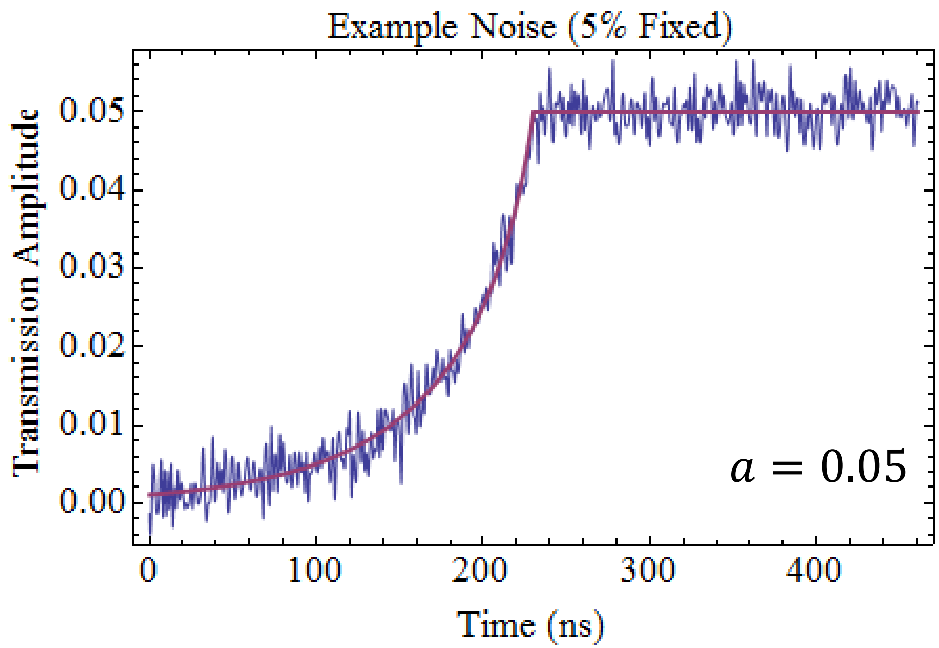
Noisy transmission amplitudes

Experimentally, desired pulse shapes can acquire some unavoidable noise.

Model ($j = e, r$):

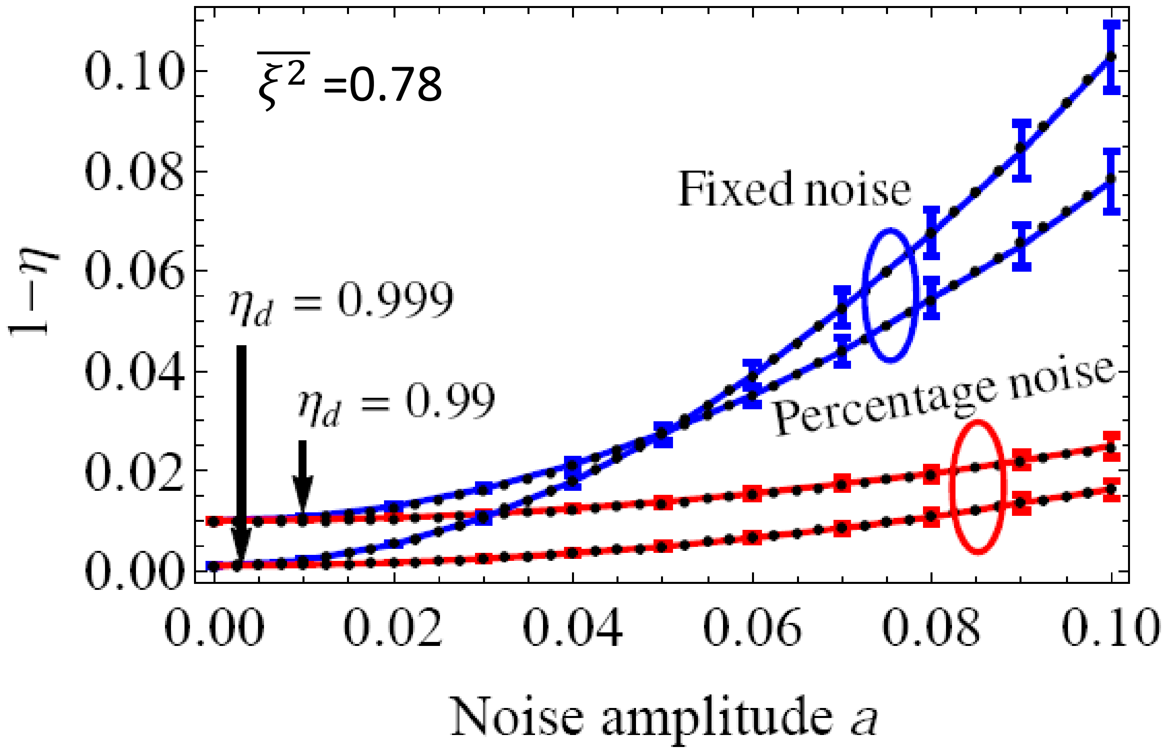
$$\mathbf{t}_j(t) \;\rightarrow\; \mathbf{t}_{p,j}(t) = \mathbf{t}_j(t) + a\, \mathbf{t}_j(t)\, \xi(t), \qquad \mathbf{t}_j(t) \;\rightarrow\; \mathbf{t}_{f,j}(t) = \mathbf{t}_j(t) + a\, |\mathbf{t}_j|_{max}\, \xi(t)$$





$\xi(t)$: Gaussian white noise, zero mean, unit std. dev.

Noisy transmission amplitudes



- Average inefficiency over 100 trials
- Noise of up to about 2% is tolerable
- Fixed noise can be problematic

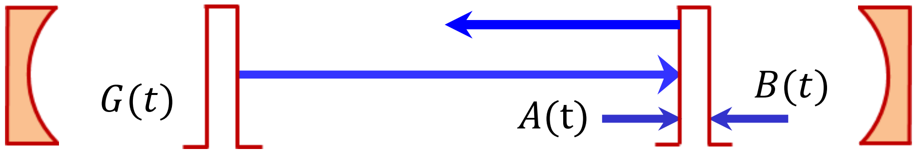
Additional inefficiency is approximately

$$-\delta\eta = c_n a^2 \overline{\xi^2} \quad \begin{cases} c_n = 2 & \text{Percentage noise} \\ c_n = 2\ln\dfrac{1}{1-\eta_d} & \text{Fixed noise} \end{cases}$$

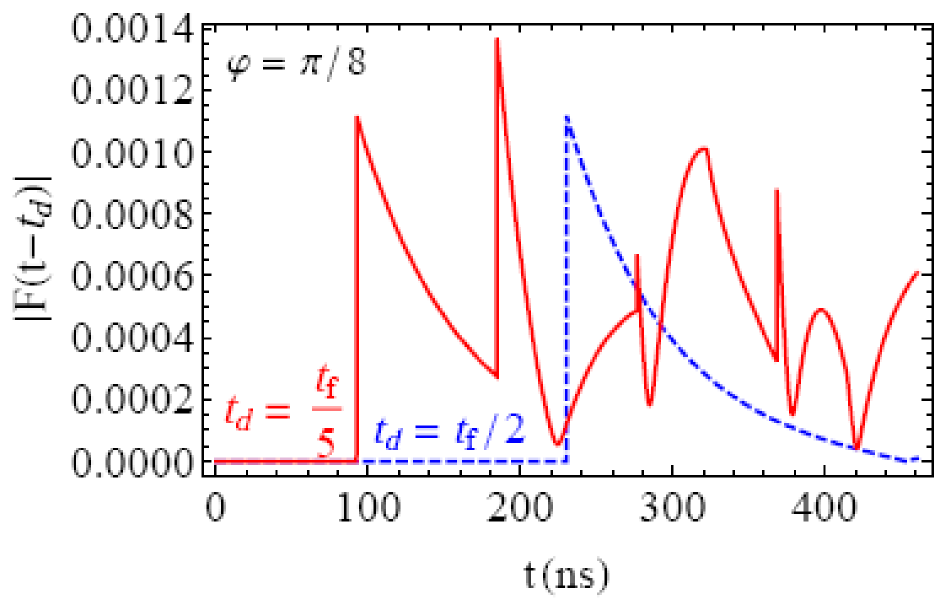
Effect of multiple reflections

- So far no multiple reflections considered
- When the resonators are close, multiple reflections becomes important

$$F = \mathbf{r}_{\mathrm{r}}\, A(t) + \ \mathbf{t}_{\mathrm{r}}\, B(t)$$

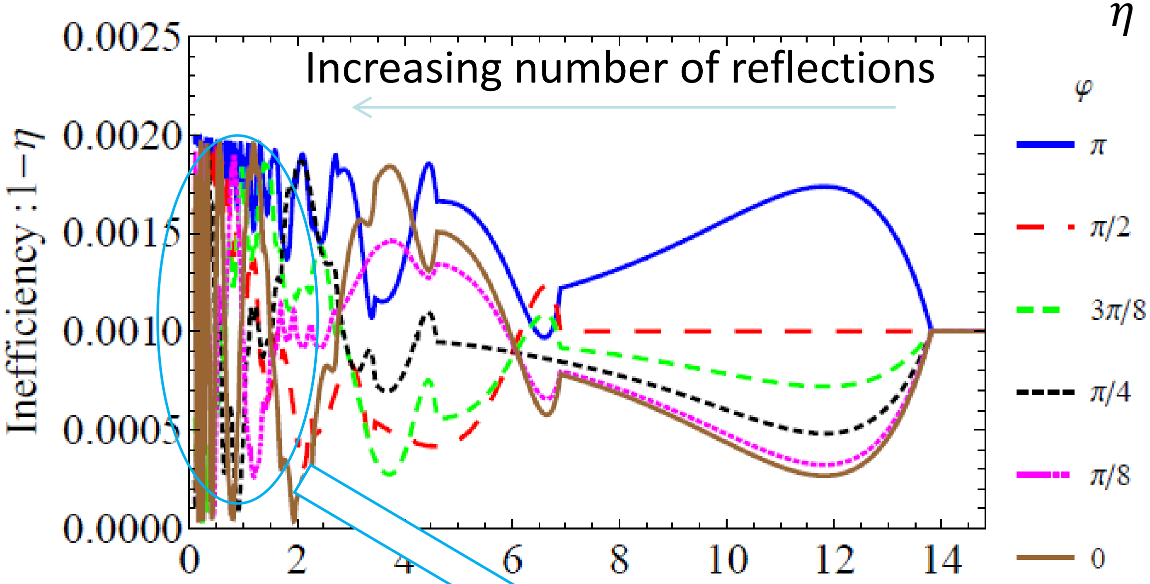


$t_d = 2l_{tl}/v$ — round-trip delay
$\varphi = \omega\, t_d$—accumulated phase of *F*

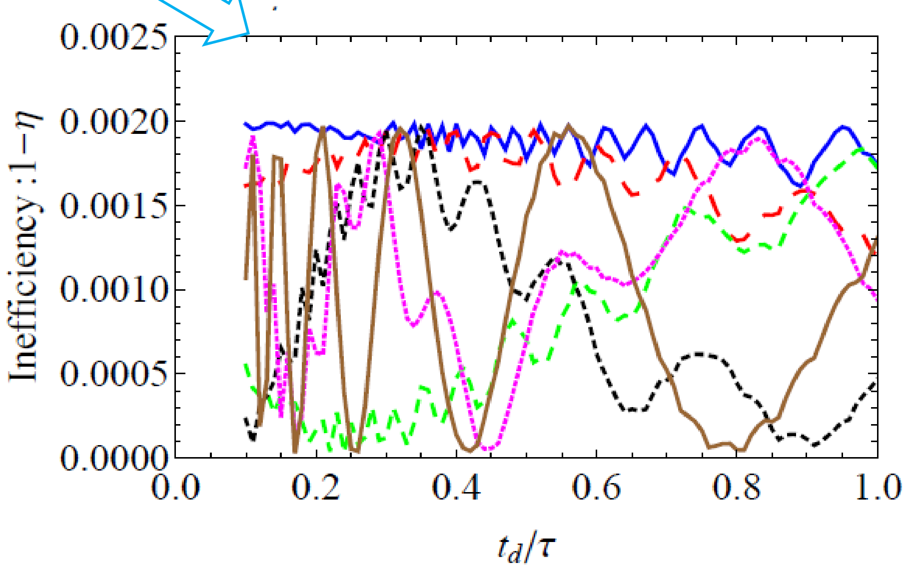


Effect of multiple reflections

$\eta = 0.999$ and $|\mathbf{t}|_{\max} = 0.05$



- Efficiency is robust to multiple reflections
- Inefficiency changes by up to factor of 2



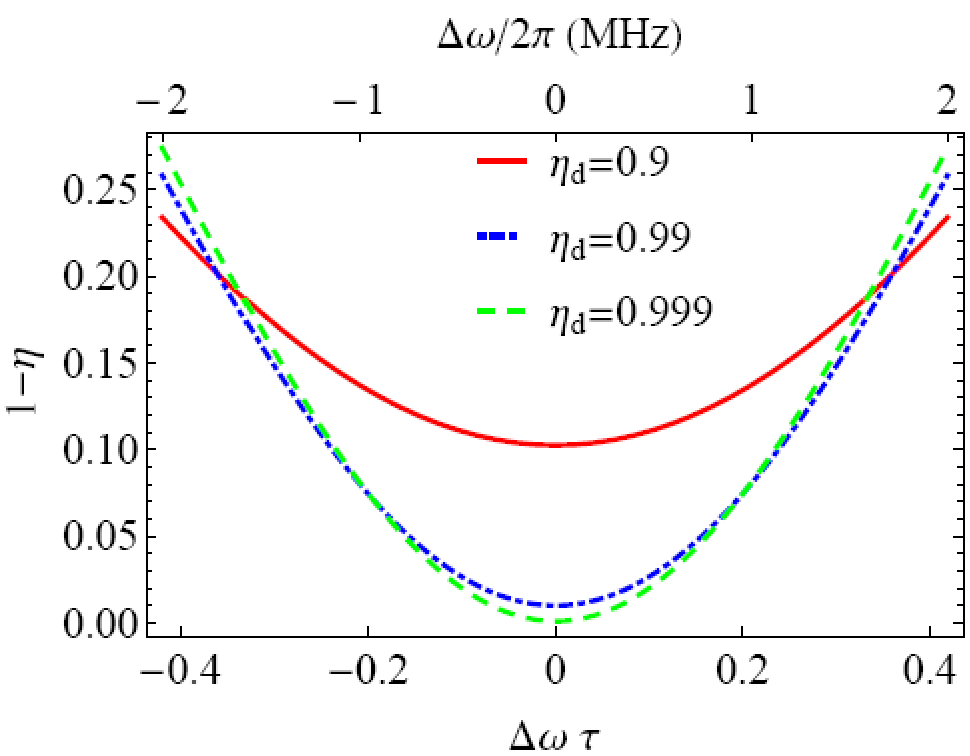
- For small round-trip delay the inefficiency saturates for $\varphi = \pi, \frac{\pi}{2}$
- $\varphi = 0$ is problematic due to resonance with the resonators

So far the state transfer protocol is (surprisingly) quite robust

Effect of frequency mismatch

1. Constant frequency mismatch—due to design imperfections
2. Time-dependent frequency mismatch—due to variable transmission amplitudes

Constant frequency mismatch



$$1-\eta \approx 2\left(\frac{\delta\omega}{\kappa_{\max}}\right)^2$$

Since our protocol is based on interference, maintaining equal frequencies is main requirement

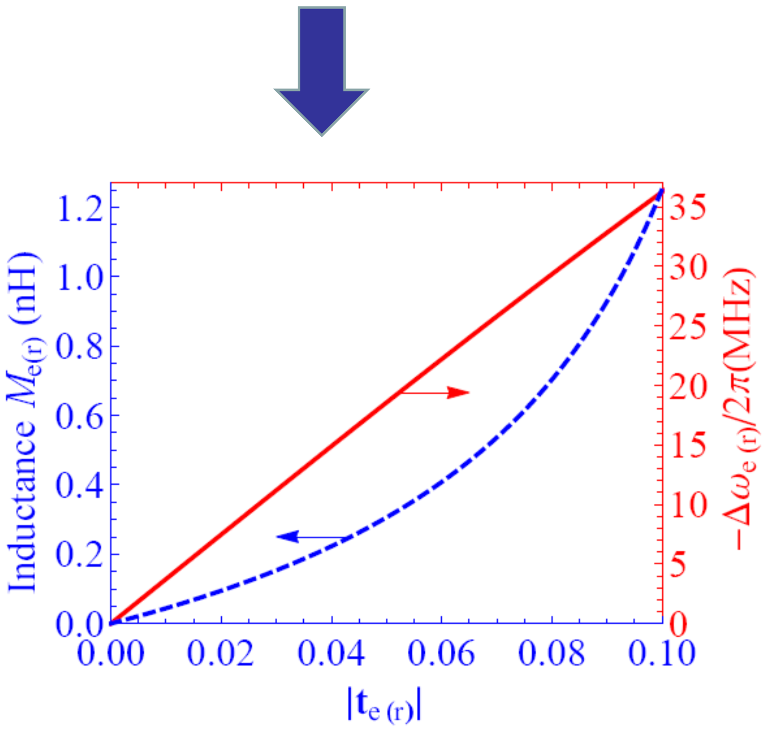
Tolerable (-δη<0.01) up to

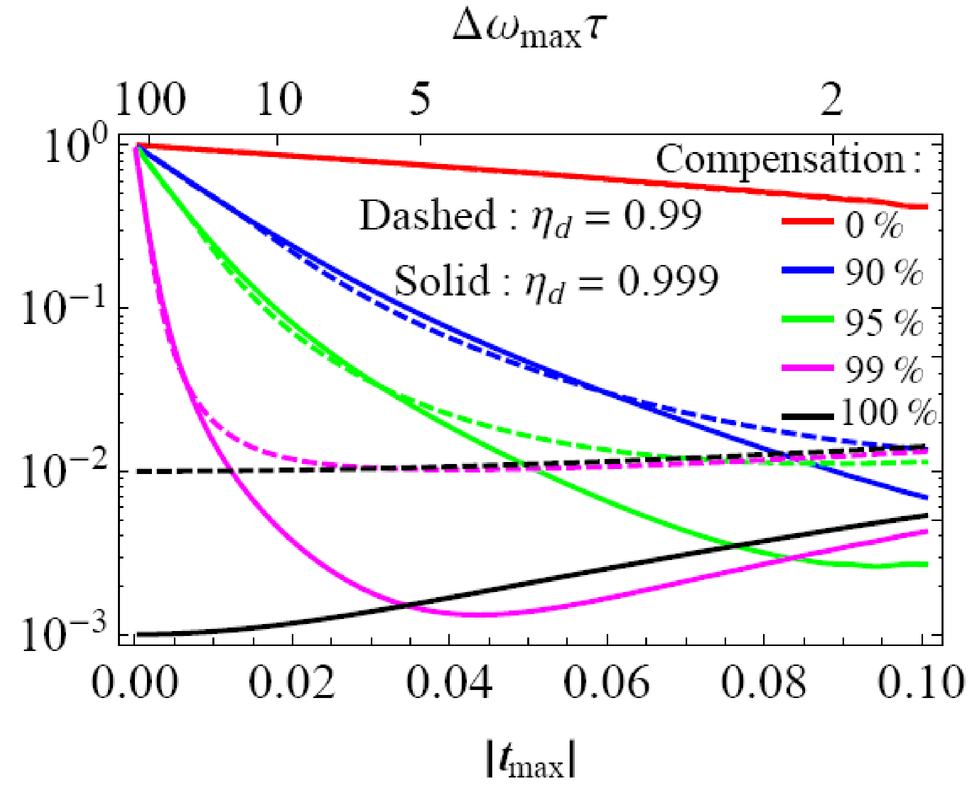
$\delta\omega/2\pi = 0.4$ MHz

Time-dependent frequency mismatch

Frequency change due to varied coupling (experiment)

$\delta\omega(t)/2\pi \sim 15$ MHz   Yin et al. **110**, 107001 (2013)





➢ At least 90-95% compensation is needed

Conclusions

- The state transfer protocol is (surprisingly) very robust to
  - pulse shape parameter deviations
  - pulse shape distortion
  - Gaussian filtering
  - noise of the pulse shapes
  - multiple reflections

- The protocol is very sensitive to frequency mismatch

- Active compensation is needed for the frequency change due to changing coupling. At least 90-95% compensation is needed.

- Emitting and receiving parts of the protocol has been demonstrated in separate experiments. Demonstration of a complete quantum state transfer is expected in 1-3 years.

Circuit QED qubit readout error from leakage to a neighboring qubit

Mostafa Khezri, Justin Dressel, Alexander N. Korotkov

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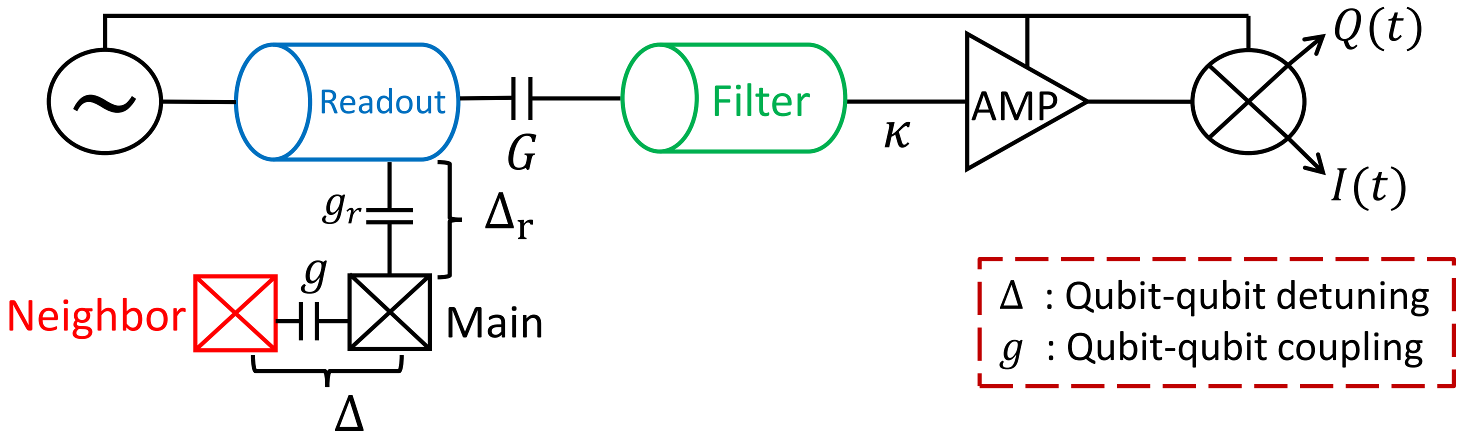
University of California, Riverside

Outline

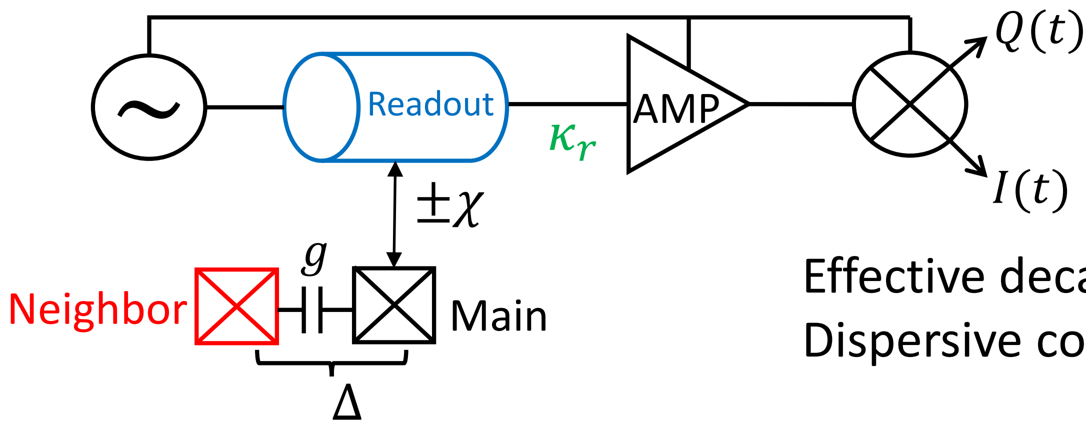
- Setup: cQED with neighboring qubit
- Switching in eigenbasis
- Misidentification error

cQED setup with neighboring qubit

- Measured qubit coupled to a pumped resonator <u>and</u> a detuned neighboring qubit



- Simplified dispersive model, filter removed
- Effect of filter captured via effective κ_r and dispersive approximation



Effective decay rate: $\kappa_r = 4G^2/\kappa$
Dispersive coupling: $\chi \simeq g_r^2/\Delta_r$

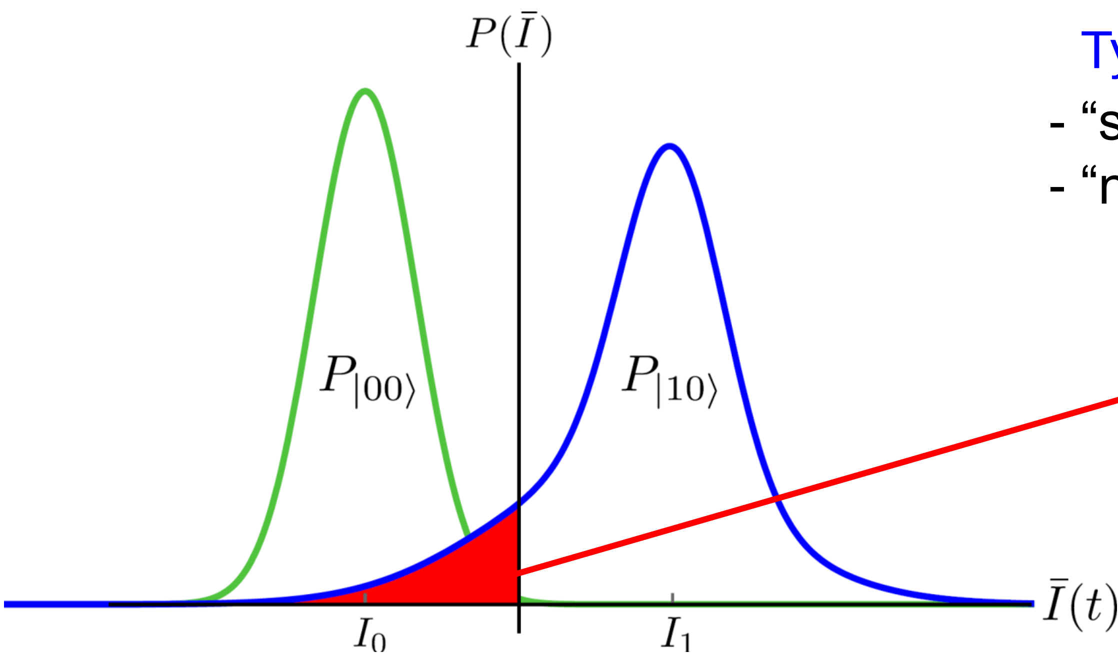
Measurement Error

Goal: distinguish $|00\rangle$ and $|10\rangle$ $(|\text{main,neighor}\rangle)$

Excitation oscillates (or jumps) between qubits, $|10\rangle \leftrightarrow |01\rangle$

Excited main qubit state can be **misidentified** as its ground state

Question: What is this misidentification error if (a) the bare basis or (b) the eigenbasis is used for encoding?



Types of error:

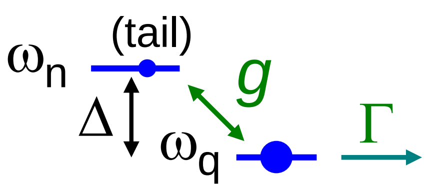
- "separation" error
- "non-QND" error (tails)

$$P_{\text{misID}} = \int_{-\infty}^{0} P_{|10\rangle}\, \mathrm{d}\bar{I}$$

Previous Work on Phase Qubits

A. Galiatdinov, A.N. Korotkov, J.M. Martinis, Phys. Rev. A 85, 042321 (2012)

Simple model:



$$H = \begin{bmatrix} \omega_n & g \\ g & \omega_q - i\Gamma/2 \end{bmatrix}$$

Assume $\Gamma^{-1} \ll t_{\mathrm{meas}} \ll (\Delta/g)^2\Gamma^{-1}$

then

$$\mathrm{error}_{\mathrm{bare}} \approx \frac{g^2}{\Delta^2 + (\Gamma/2)^2} \qquad \mathrm{error}_{\mathrm{eigen}} \approx \frac{g^2}{\Delta^2 + (\Gamma/2)^2}\left(\frac{\Gamma}{2\Delta}\right)^2$$

$\Rightarrow$ much smaller error for eigenstate if $\Gamma \ll \Delta$

In circuit QED measurement instead of tunneling rate Γ we have two parameters: measurement (dephasing) rate Γ and resonator leakage rate κ. Both of them happen to be important.

Modeling Resonator + Two Qubits

1. Treat resonator field classically: replace photon number operator with a stochastic classical field

$$a^\dagger a \to n(t) = \bar{n} + \delta n(t)$$
$$\langle \delta n(t) \delta n(t') \rangle = \bar{n} e^{-\kappa_r |t-t'|/2}$$

2. Write a fluctuating Hamiltonian: Hamiltonian for effective qubit (one excitation subspace) in the dispersive regime

$$H = H_0 + V(t) = \left(\frac{\Delta_0}{2} + \chi \bar{n} \right) \sigma_z + g(\sigma_+ + \sigma_-) + V(t)$$

$$V(t) = \chi \delta n(t) \sigma_z$$

$$\Delta_0 = \omega_1 - \omega_2 \qquad \sigma_z = |e\rangle\langle e| - |g\rangle\langle g|$$

$$\chi = \frac{g_r^2}{\sqrt{\Delta_1^2 + 4 g_r^2 \bar{n}}} \qquad \sigma_+ = |e\rangle\langle g| \quad \sigma_- = |g\rangle\langle e| \qquad |e\rangle = |10\rangle \quad |g\rangle = |01\rangle$$

Hamiltonian of the effective qubit

$$H_0 = \begin{pmatrix} \Delta/2 & g \\ g & -\Delta/2 \end{pmatrix}$$

$$\Delta = \Delta_0 + 2\chi\bar{n}$$

Eigenbasis
Eigenenergy

$$|+\rangle = \sin\theta\,|g\rangle + \cos\theta|e\rangle$$
$$|-\rangle = \cos\theta\,|g\rangle - \sin\theta|e\rangle$$
$$\tan 2\theta = 2g/\Delta$$
$$\Omega = \sqrt{\Delta^2 + 4g^2}$$

$$\bar{n} \downarrow \bar{n} + \delta n(t)$$

$$\Delta \to \Delta + \delta\Delta(\mathrm{t})$$
$$\delta\Delta = 2\chi\delta n(t)$$

Effective
Coupling

$$H = \begin{pmatrix} \Omega/2 & \delta g \\ \delta g & -\Omega/2 \end{pmatrix}$$

H in eigenbasis

Switching rate in eigenbasis

1. Initially start at $|+\rangle$
2. Find population of the wrong state $|-\rangle$ after some time τ

$$\mathrm{Err}(\tau) = \left| \int_0^\tau \delta g(t) e^{i\Omega t} \mathrm{d}t \right|^2$$

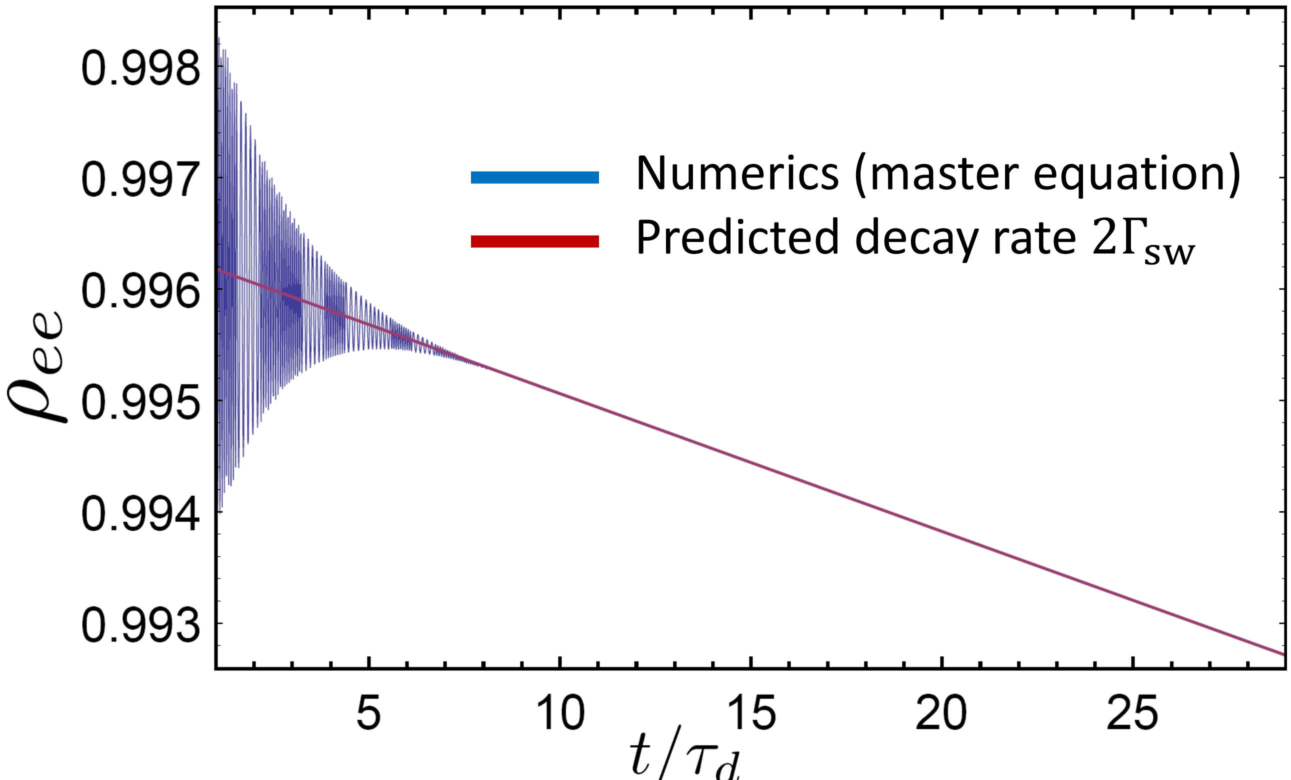
3. Define switching rate as $\Gamma_{\mathrm{sw}} = \lim_{\tau\to\infty} \mathrm{Err}(\tau)/\tau$
4. Use spectral density of fluctuation δg to derive switching rate

$$\boxed{\Gamma_{\mathrm{sw}} = \Gamma \frac{2g^2}{\Omega^2} \frac{\kappa_r^2}{\kappa_r^2 + 4\Omega^2}}$$

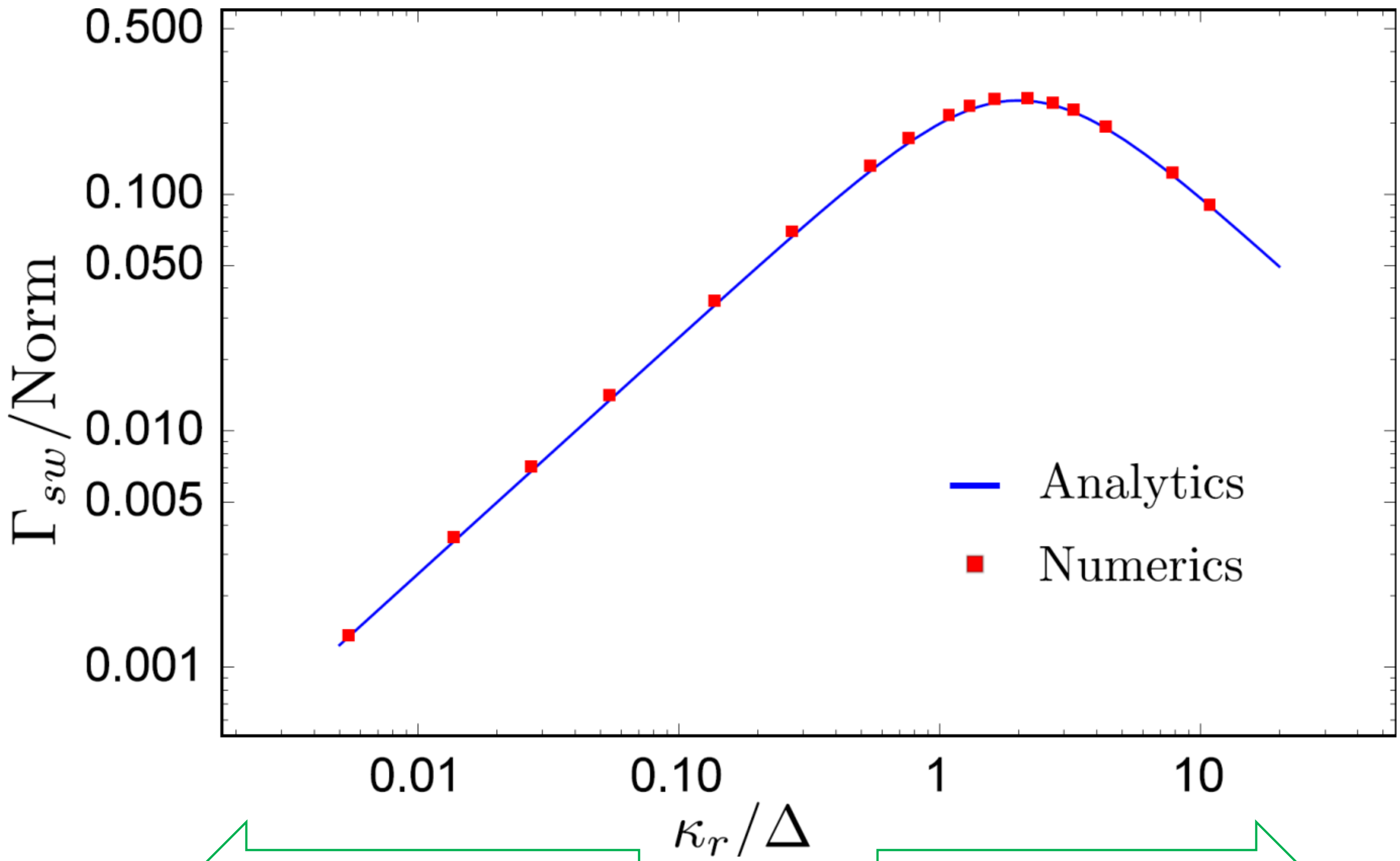
Dephasing rate: $\Gamma = \frac{8\chi^2 \bar{n}}{\kappa_r} = 1/2\tau_d$

Simulation of the switching

- Switching produces ensemble dephasing in the effective qubit with rate $2\Gamma_{sw}$
- Dephasing can be simulated using master (Lindblad) equation of the total system (qubits, resonator, pump, and their interactions)



Analytics vs. numerics



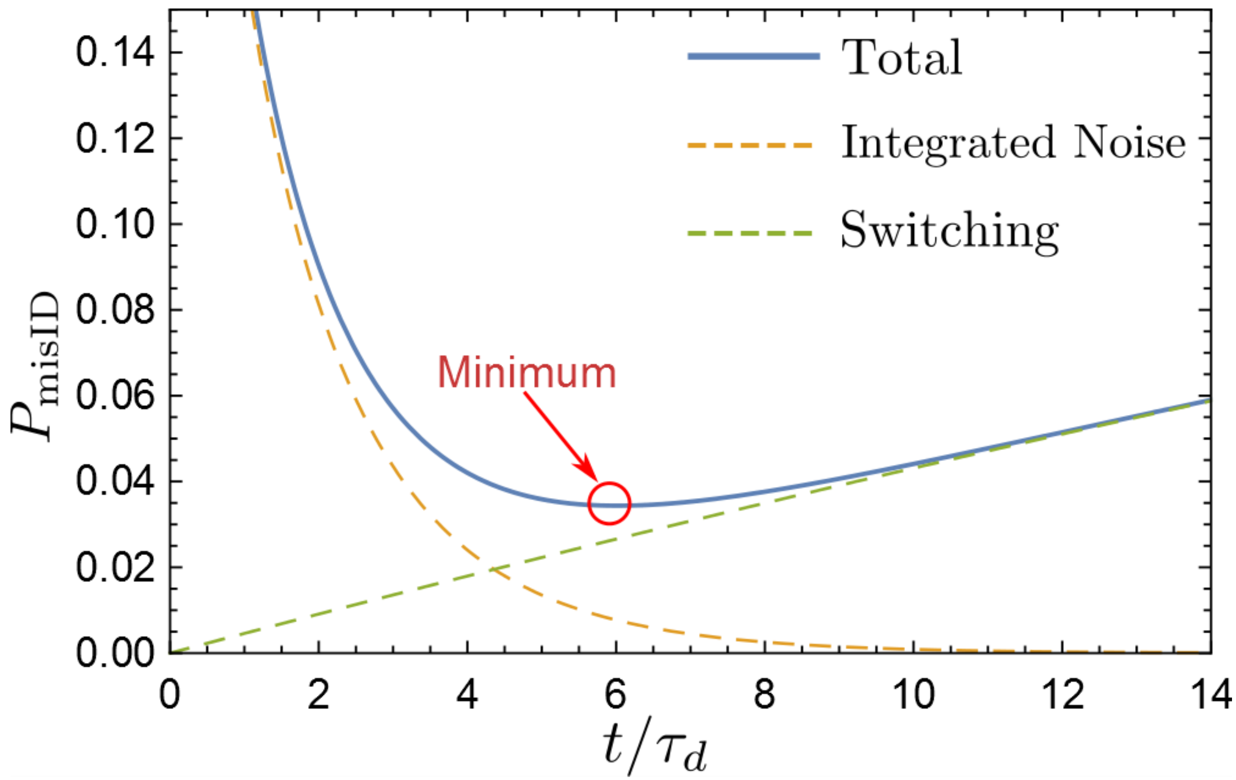
Narrow Linewidth

Wide Linewidth

$$\mathrm{Norm} = 16\chi^2\bar{n}g_q^2/(\Delta\Omega^2)$$

Misidentification error due to switching

$$P_{\mathrm{misID}} = \underbrace{\frac{e^{-\Gamma_{\mathrm{sw}} t}}{2}\left(1 - \mathrm{erf}\left[\sqrt{\frac{t}{2\tau_d}}\cos(2\theta)\right]\right)}_{\text{Integrated Noise Term}} + \frac{1 - e^{-\Gamma_{\mathrm{sw}} t}}{2}$$
$$+ \left(\mathrm{erf}\left[\sqrt{\frac{t}{2\tau_d}}\cos(2\theta)\right]\left(\frac{\tau_d}{2t\cos^2(2\theta)} - \frac{1}{2}\right) - \frac{\exp\left[\frac{-t\cos^2(2\theta)}{2\tau_d}\right]}{\cos(2\theta)}\sqrt{\frac{\tau_d}{2\pi t}}\right)\frac{(\Gamma_{\mathrm{sw}} t)^2}{4} e^{-\Gamma_{\mathrm{sw}} t}$$



Minimum misidentification error

- Minimum misidentification error when starting in the eigenbasis

$$\boxed{P_{\text{eigen}} \simeq \left(\frac{g_q}{\Omega}\right)^2 \frac{\kappa_r^2}{\kappa_r^2 + 4\Omega^2} \ln\left[\left(\frac{\Omega}{g_q}\right)^2 \frac{\kappa_r^2 + 4\Omega^2}{3\kappa_r^2}\right]}$$

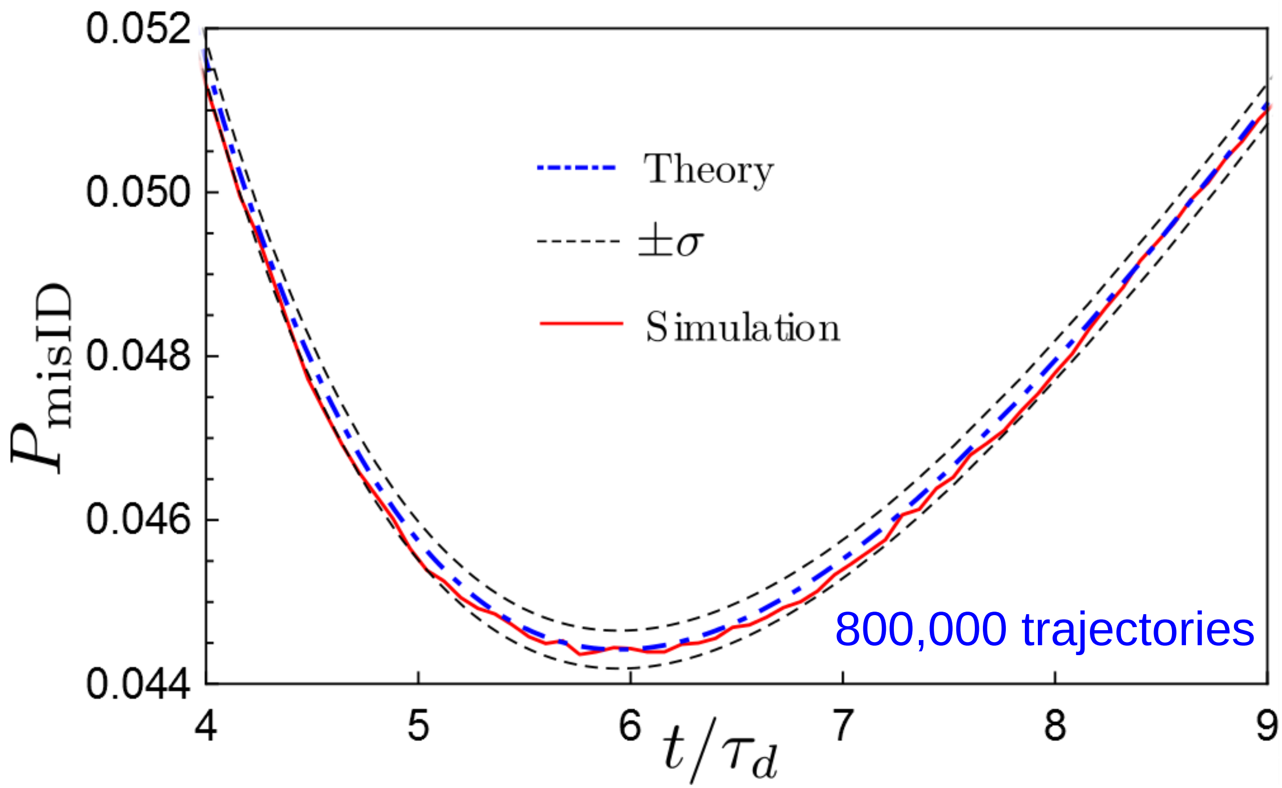
- Minimum misidentification error when starting in the bare basis

$$\boxed{P_{\text{bare}} \simeq P_{\text{eigen}} + \left(\frac{g}{\Omega}\right)^2}$$

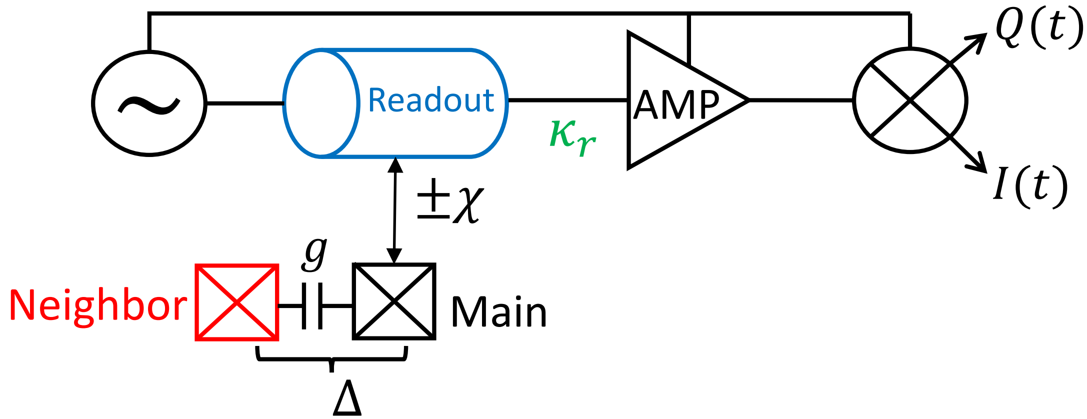
$\Omega = \sqrt{\Delta^2 + 4g^2}$ Always assume $g \ll \Delta, \Gamma_{meas} \ll \Delta$

Quantum Bayesian (trajectory) simulations for $\kappa \gg \Delta \gg \Gamma$

- For a wide linewidth resonator, P_{misID} can be simulated using quantum Bayesian update of states
- Simulation result shows excellent agreement with telegraph model



What is effective measurement basis?



Regime	Measured Basis	Error comparison
$(\Gamma, \kappa_r) \ll \Delta$	Eigenbasis	$P_{\text{eigen}} \ll P_{\text{bare}} \sim \left(\frac{g}{\Delta}\right)^2$
$(\Gamma, \kappa_r) \gg \Delta$	Bare basis (textbook)	$P_{\text{bare}} \ll P_{\text{eigen}} \sim \left(\frac{g}{\Delta}\right)^2$
$\Gamma \ll \Delta \ll \kappa_r$	Neither bare basis nor eigenbasis	$P_{\text{eigen}} \sim P_{\text{bare}} \sim \left(\frac{g}{\Delta}\right)^2$
$\kappa_r \ll \Delta \ll \Gamma$	Not experimental	N/A

Conclusions

- Coupling between neighboring detuned qubits causes jumps (switching) of the excitation in the eigenbasis due to measurement, this leads to measurement error
- Fortunately, the switching rate is small if $\kappa \ll \Delta$; then the measurement error can be much less than the "tail" $(g/\Delta)^2$
- Experimentally, using eigenbasis for encoding is much better than using bare basis; error difference is $(g/\Delta)^2$

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Thank you