Measurement of superconducting qubits and causality

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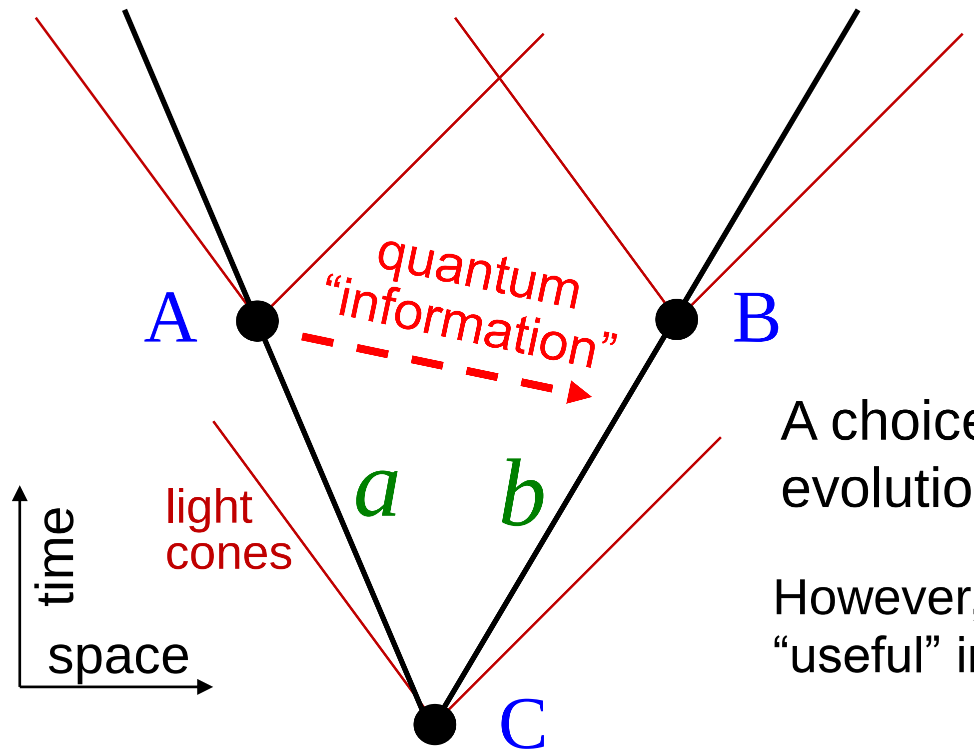
Outline:

- Introduction (causality, POVM measurement)
- Circuit QED setup for measurement of superconducting qubits (transmons, Xmons)
- Qubit evolution due to measurement in circuit QED (simple and better theories)
- Experiments

Causality principle in quantum mechanics



objects a and b

observers A and B (and C)

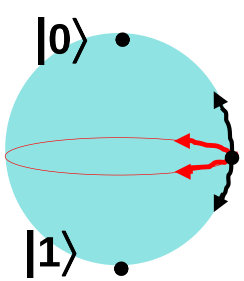
observers have "free will";
they can choose an action

A choice made by observer A <u>can</u> affect evolution of object b "back in time"

However, this retroactive control cannot pass "useful" information to B (no signaling)

Randomness saves causality (even C cannot predict result of A measurement)

Our focus: continuous collapse



<u>Ensemble-averaged</u> evolution of object b cannot depend on actions of observer A

Werner Heisenberg

Books:

Physics and Philosophy: The Revolution
in Modern Science

Philosophical Problems of Quantum Physics

The Physicist's Conception of Nature

Across the Frontiers

Niels Bohr

Immanuel Kant (1724-1804), German philosopher

Critique of pure reason (materialism, but not naive materialism)

Nature - "Thing-in-itself" (noumenon, not phenomenon)

Humans use "concepts (categories) of understanding";
make sense of phenomena, but never know noumena directly

A priori: space, time, causality

A naïve philosophy should not be a roadblock for good physics,
quantum mechanics requires a non-naïve philosophy

Wavefunction is not a reality, it is only our description of reality

Textbook ("orthodox") measurement

Operator of measured observable $A = \sum_k r_k |\psi_k\rangle\langle\psi_k|$

Measurement result r_k (eigenvalue of *A*)

with probability $P_{rk} = |\langle\psi_k|\psi_{in}\rangle|^2$

After measurement with result r_k $|\psi_{in}\rangle \rightarrow \dfrac{|\psi_k\rangle}{\| |\psi_k\rangle \|}$ (eigenstate of *A*)

Why the change (collapse) to the eigenstate?

Just common sense: "you get what you see"

(gives the same result for sequential measurements of a non-evolving object)

Simple ways to spoil "orthodox" measurement

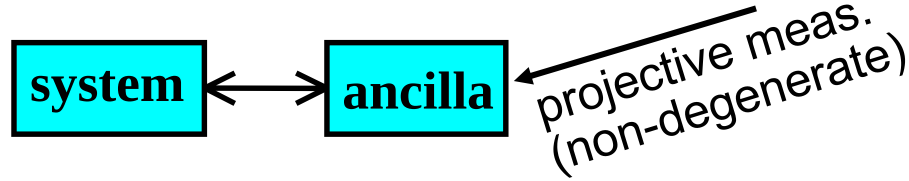
- rotation before measurement (possibly random)
- misreporting measurement result
- rotation after measurement (possibly depending on result)

"Orthodox" description assumes a "good" measurement

Generalized (POVM) measurement

Davies, Kraus, Holevo, 1970s-1980s

Physics: unitary interaction with another system (ancilla), then "orthodox" measurement of ancilla



Mathematics: measurement (Kraus) operators M_r

$$|\psi\rangle \to \frac{M_r\,|\psi\rangle}{||M_r\,|\psi\rangle||} \quad \text{or} \quad \rho \to \frac{M_r\,\rho M_r^\dagger}{\mathrm{Tr}(M_r\,\rho M_r^\dagger)}$$

$$\text{Probability:}\quad P_r = ||\, M_r\,|\psi\rangle\,||^2 \quad \text{or} \quad P_r = \mathrm{Tr}(M_r\,\rho M_r^\dagger)$$

$$\text{Completeness:}\quad \sum_r M_r^\dagger M_r = 1$$

Reduces to "orthodox" measurement when M_r are orthogonal projectors

This is how quantum information theorists think about the most general quantum measurement

Easy to introduce imperfections (incl. decoherence), e.g., via averaging over subsets of Kraus operators (information loss)

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Generalized (POVM) measurement (cont.)

$$|\psi\rangle \rightarrow \frac{M_r\,|\psi\rangle}{\|M_r\,|\psi\rangle\|} \qquad P_r = \|M_r|\psi\rangle\|^2 \qquad \sum_r M_r^\dagger M_r = 1$$

Easy to show that local generalized measurement obeys causality
(for ensemble-average state, i.e. averaged over results)

A useful way of thinking

polar decomposition: $M_r = U_r \underbrace{\sqrt{M_r^\dagger M_r}}_{\text{quantum Bayes rule}}$

unitary

Operator M_r defines a measurement basis (which diagonalizes $M_r^\dagger M_r$).
In this basis operator $\sqrt{M_r^+ M_r}$ acts as the quantum Bayes rule:

$$|\psi\rangle = \sum_i c_i\,|i\rangle \rightarrow \frac{\sum_i c_i \sqrt{P_r(i)}\,|i\rangle}{\text{Norm}}$$

so that probabilities $|c_i|^2$ follow the classical Bayes rule

Again, you get what you see

Often the same basis for all results

Bayes rule (1763, Laplace-1812)

$$\overbrace{P(i\,|\,\text{result})}^{\text{posterior probability}} = \frac{\overbrace{P(i)}^{\text{prior probab.}}\ \overbrace{P(\text{result}\,|\,i)}^{\text{likelihood}}}{\text{Norm}}$$

Continuous quantum measurement

POVM measurement is still instantaneous (as "orthodox" collapse).
In practice measurements are often continuous (noise, SNR).

gradual acquisition of information $\Rightarrow$ gradual collapse

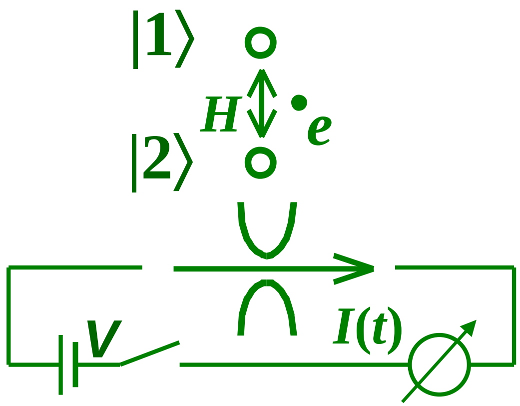
Continuous measurement can be viewed as a sequence of weak POVM measurements.

Still the same general principle: **you get what you see** (quantum evolution follows information obtained from measurement)

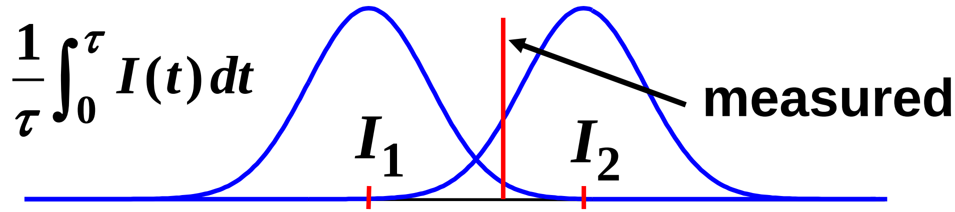
Many people contributed (different approaches): Davies, Kraus, Holevo, Mensky, Caves, <u>Carmichael, Milburn, Wiseman</u>, Aharonov, Gisin, Belavkin, etc. (very incomplete list)

Key words: POVM, restricted path integral, <u>quantum trajectories</u>, quantum filtering, quantum jumps, stochastic master equation, etc.

First solid-state example: DQD and QPC



$$\psi(t) = \alpha(t)\,|1\rangle + \beta(t)\,|2\rangle \quad \text{or} \quad \rho_{ij}(t)$$



1) $|\alpha(t)|^2$ and $|\beta(t)|^2$ evolve as probabilities,
   i.e. according to the Bayes rule (same for ρ_{ii})

2) phases of $\alpha(t)$ and $\beta(t)$ do not change
   (no dephasing), $\rho_{ij}/(\rho_{ii}\,\rho_{jj})^{1/2}$ = const

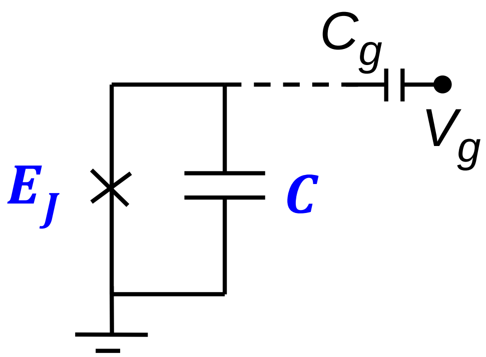
(A.K., 1999)

Can (indirectly) monitor wavefunction evolution in real time

However, experiments have been realized with superconducting qubits in circuit QED setup (microwave readout)

Narrowband (two signals): $I(t)\cos(\omega t) + Q(t)\sin(\omega t)$

Transmon (or Xmon): at present the most widely used superconducting qubit

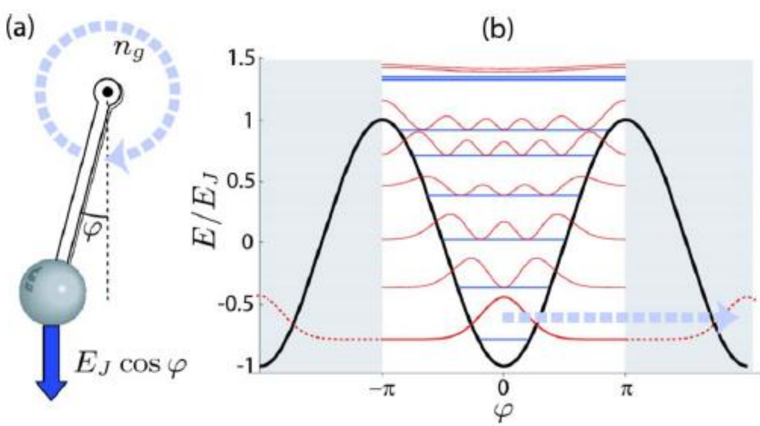


Josephson (tunnel) junction + capacitor

$$H = \frac{(2e)^2}{2C}(n-n_g)^2 - E_J\cos\phi$$

$$[\phi, n] = i \qquad n_g = C_g V_g / 2e$$

(superconducting phase ϕ, charge $2en$)



J. Koch et al., 2007 (Yale)

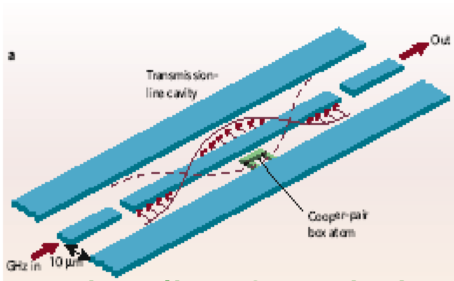
- single-electron box with E_J/E_C~100 (~5 charge states involved)
- almost insensitive to n_g
- E_J often tunable (two junctions)

A slightly nonlinear oscillator with two lowest levels used as $|0\rangle$ and $|1\rangle$.

$\delta \equiv \omega_{01} - \omega_{12}$ $\qquad$ $\delta/\omega_{01} \approx 3-6\%$

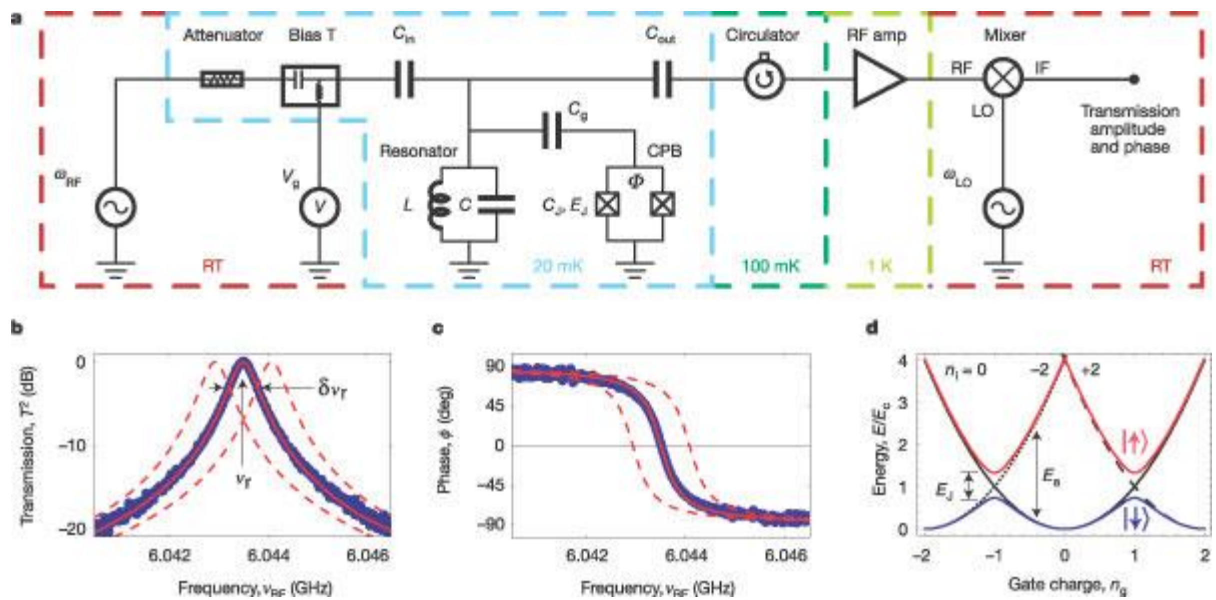
$\omega_{01}/2\pi \approx$ 4-6 GHz $\qquad$ $\delta/2\pi \approx$ 0.2-0.4 GHz

Circuit QED setup for qubit measurement



Schoelkopf & Girvin

A. Blais et al., 2004 (Yale)



A. Wallraff et al., 2004 (Yale)

Idea: 1) qubit is coupled with a (microwave) resonator (CPW or lumped);
2) qubit state $|0\rangle$ or $|1\rangle$ slightly changes the resonator frequency;
3) change of resonator frequency is sensed by a microwave transmission (or reflection), amplification, and mixing

$$H = \frac{1}{2}\omega_{qb}\sigma_z + \omega_r a^\dagger a + \chi a^\dagger a \sigma_z$$ (dispersive interaction)

Some details and complications

1. Dispersive interaction $\chi a^\dagger a \sigma_z$ follows from Janes-Cummings (or Tavis-Cummings) interaction at large qubit-resonator detuning

J-C $\quad H = \frac{\omega_q}{2}\sigma_z + \omega_r a^\dagger a + g(a^\dagger|0\rangle\langle 1| + a\,|1\rangle\langle 0|)$

$\chi \approx \dfrac{g^2}{\omega_q - \omega_r}$ only if $|\omega_q - \omega_r| \gg g, \quad n \ll n_{crit} = \dfrac{(\omega_q - \omega_r)^2}{4g^2}$

n is number of photons, $\bar{n} = \langle a^+ a\rangle$

In experiments usually $\bar{n} \ll n_{crit}$ or $\bar{n} < n_{crit}$ or $\bar{n} < 4n_{crit}$ (increase of $\bar{n}$ improves measurement, but something goes wrong at big $\bar{n}$)

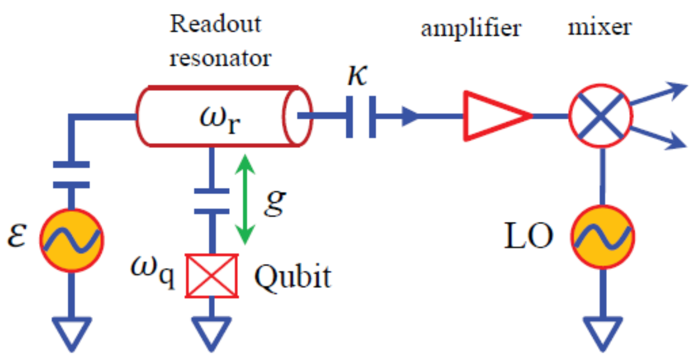
2. For a transmon at least 3 levels should be considered to find χ

$\chi \approx -\dfrac{g^2\delta_q}{(\omega_q - \omega_r)^2} \qquad \delta_q \equiv \omega_{01} - \omega_{12}$

(then still OK to use two-level approximation in eigenbasis)

Some details and complications (cont.)

3. Significant qubit energy relaxation due to coupling with decaying resonator (Purcell effect)



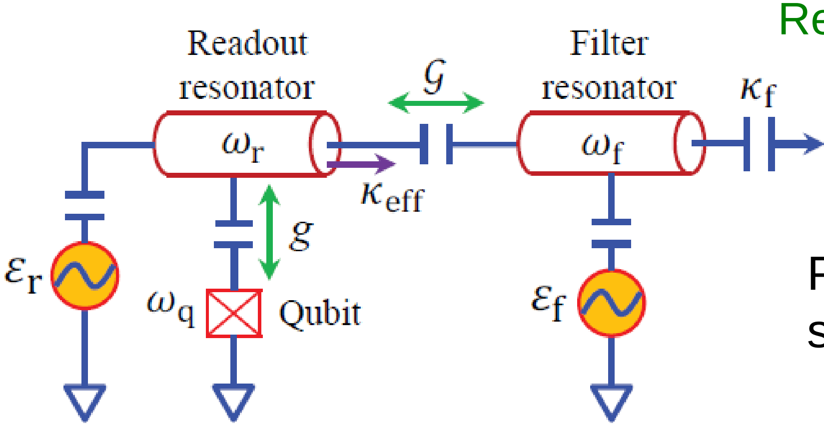
$$\Gamma_P = \kappa \frac{g^2}{(\omega_q - \omega_r)^2}$$

κ is decay rate for resonator

The same result from classical EE analysis (Esteve et al., 1986)

At present the best way to avoid Purcell decay is to use Purcell filter

Reed et al., 2010 (Yale), Jeffrey et al., 2014 (UCSB)



Sete et al., 2015

$$\kappa_{eff}(\omega) = \frac{4G^2}{\kappa_f} \frac{1}{1+[2(\omega_f - \omega)/\kappa_f]}$$

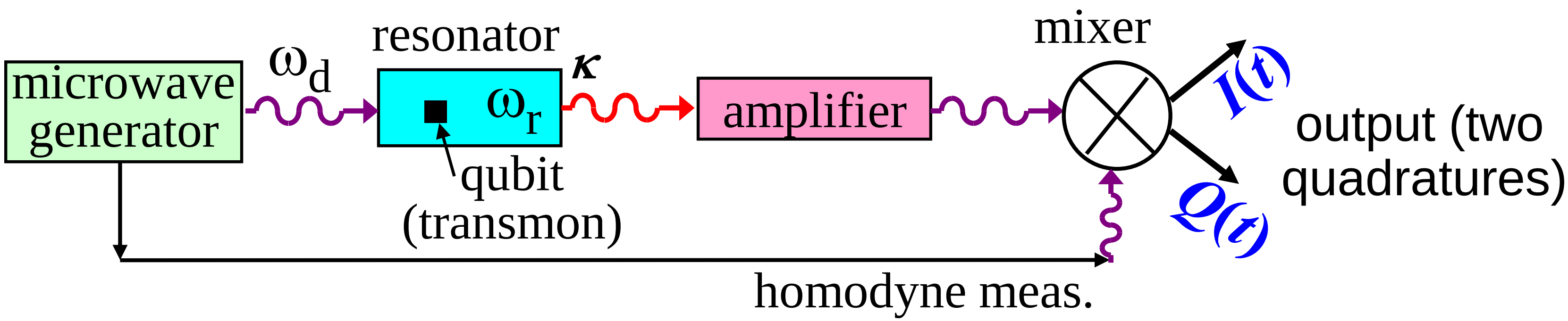
Purcell decay suppression

$$\frac{\kappa_q}{\kappa_r} = \frac{1+[2(\omega_r - \omega_f)/\kappa_f]}{1+[2(\omega_q - \omega_f)/\kappa_f]}$$

Classical analysis is easier than quantum

Circuit QED measurement

Our focus: qubit evolution due to measurement



$$H=\frac{1}{2}\omega_{qb}\sigma_z+\omega_r a^\dagger a+\chi a^\dagger a\sigma_z$$
$$+(\varepsilon e^{-i\omega_d t}a^\dagger+h.c.)+H_\kappa$$

qubit state changes resonator frequency;
number of photons affects qubit frequency (ac Stark shift),
$\Rightarrow$ fluctuations lead to dephasing

Blais et al., 2004
Gambetta et al., 2006, 2008

Qubit ensemble dephasing rate

$$\Gamma=\frac{8\chi^2\bar{n}}{\kappa}$$

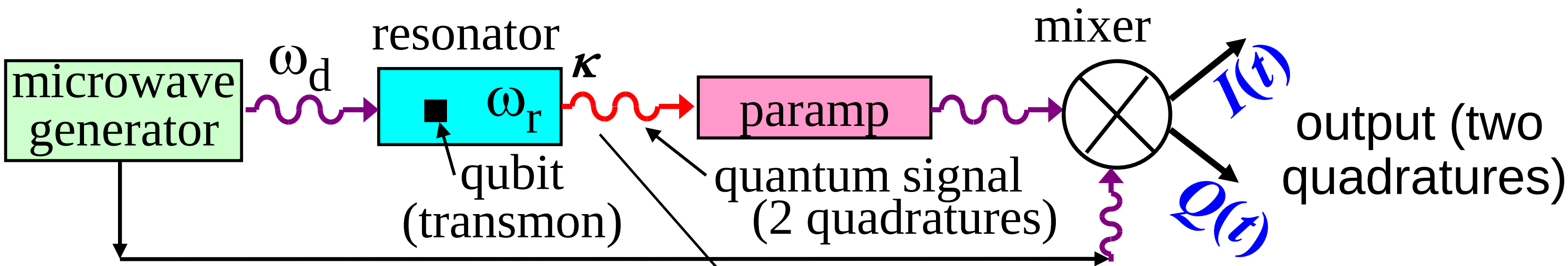
assuming $|\omega_r-\omega_d\pm\chi|\ll\kappa$

(easy to derive by tracing over emitted microwave field)

$$\alpha_\pm=\frac{-i\varepsilon}{\kappa/2+i(\omega_r-\omega_d\pm\chi)}$$

We are interested in non-averaged qubit evolution

Physical roles of two quadratures

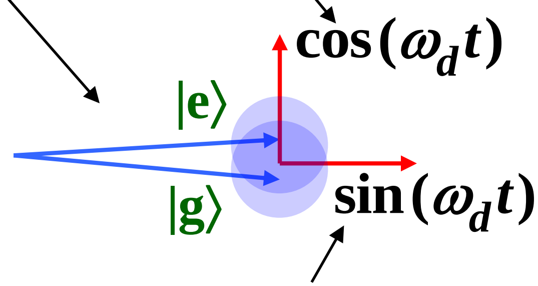


Two quadratures:
$A(t)\cos\omega_d t + B(t)\sin\omega_d t$

Assume "bad cavity" regime
(weak measurement):

$$\kappa \gg \Gamma = 8\chi^2\,\overline{n}/\kappa$$

carries information about qubit
(⇒ "quantum" back-action)



carries information about fluctuating
photon number in the resonator
(⇒ "classical" back-action)

With parametric amplifier we can choose
which quadrature to amplify (measure)

Phase-sensitive and phase-preserving parametric amplifiers (paramps)

Paramps are traditionally discussed in terms of noise temperature θ

$\theta \geq 0$ for phase-sensitive (degenerate, homodyne) paramp

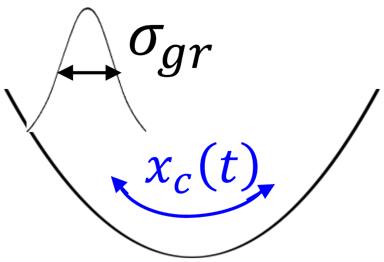
$\theta \geq \frac{\hbar\omega}{2}$ for phase-preserving (non-degenerate, heterodyne) paramp

(adds "half a quantum" of noise)

Haus, Mullen (1962), Giffard (1976), Caves (1982), Devyatov et al. (1986)

A way to understand

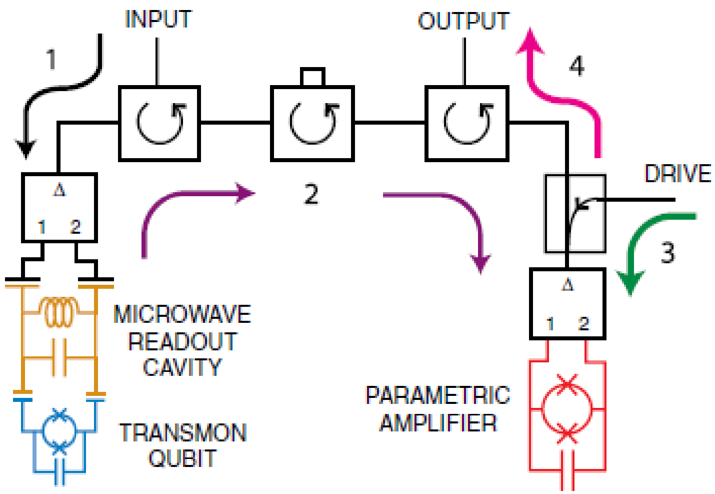
Game: Charlie prepares "coherent state" of an oscillator, $x_c(t) = A\cos\omega t + B\sin\omega t$, and gives it to David. David's goal is to find *A* (or both *A* and *B*).



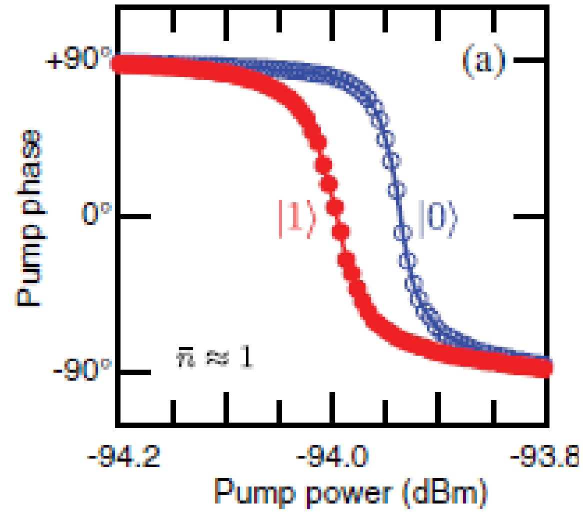
A can be measured with accuracy σ_{gr} ("orthodox" or stroboscopic QND at times $\pi n/\omega$)

Both *A* and *B* can be measured with accuracy $\sqrt{2}\sigma_{gr}$ each (adds "half a quantum" into each quadrature)

A phase-sensitive superconducting paramp



Vijay et al., 2011 (UCB)



Circulators, directional couplers, and hybrids

Theory based on Dykman, Krivoglaz (1971-1984, 1980)

- A (classical) nonlinear oscillator: frequency changes with amplitude
- A strong pump is added to the signal at the same frequency
- Signal in-phase with pump increases amplitude, changes frequency
- Frequency change leads to the phase change at the output

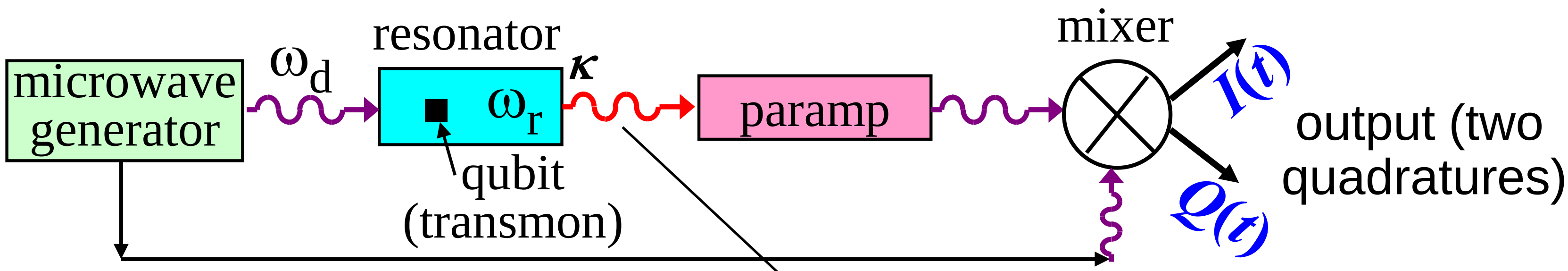
Other modes of operation: double-pump at $\omega \pm \delta$, parametric pump at 2ω

Important: one quadrature is amplified, the other quadrature is de-amplified

Same device operates as phase-preserving paramp
if pumped at shifted frequency

Continuous measurement of a qubit: simple quantum Bayesian approach

Korotkov, arXiv:1111.4061



Assume:

$$H = \omega_{qb}\sigma_z/2 + \omega_r a^\dagger a + \chi a^\dagger a \sigma_z$$ (dispersive)

$$\kappa >> \max(\Gamma, \Omega_R)$$ ("bad cavity", Markovian, almost no entanglement)

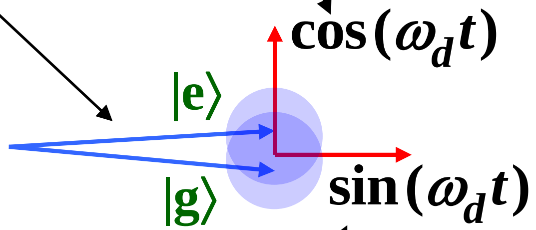
Also assume (for simplicity):

$\kappa_{out} = \kappa$ (everything collected)

$\omega_d = \omega_r$ (center of resonance)

assume everything most ideal

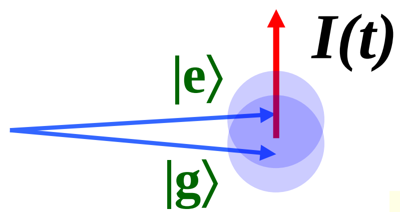
carries information about qubit (σ_z)



carries information about fluctuating photon number in the resonator

Equivalent to (simplified) quantum trajectory theory by Gambetta et al., PRA-2008

Amplify "informational" quadrature



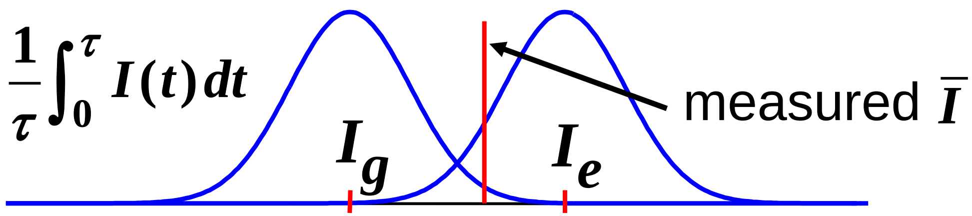
If qubit evolves only due to measurement, then the diagonal elements of its density matrix must evolve as probabilities (i.e., via classical Bayes rule).

qubit state $\begin{pmatrix} \rho_{ee} & \rho_{eg} \\ \rho_{ge} & \rho_{gg} \end{pmatrix}$

$$\frac{\rho_{gg}(\tau)}{\rho_{ee}(\tau)} = \frac{\rho_{gg}(0)}{\rho_{ee}(0)} \frac{\exp[-(\bar{I}-I_g)^2/2D]}{\exp[-(\bar{I}-I_e)^2/2D]}$$

$D = S_I/2\tau$ (noise)

$$\bar{I} = \frac{1}{\tau}\int_0^\tau I(t)\,dt$$



For quantum-limited phase-sensitive paramp

$$S_I = \frac{(I_e - I_g)^2 \kappa}{32\chi^2 \bar{n}}$$

Now average over measurement results $\bar{I}$

$$|\rho_{eg}(\tau)| \le \langle\sqrt{\rho_{ee}(\tau)\rho_{gg}(\tau)}\rangle = \sqrt{\rho_{ee}(0)\rho_{gg}(0)}\,\exp\left(-\frac{8\chi^2\bar{n}}{\kappa}\tau\right)$$

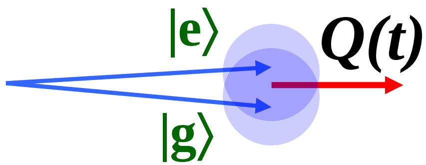
But $\Gamma = 8\chi^2\,\bar{n}/\kappa$. Therefore, no additional dephasing is possible.

$$\rho_{ge}(\tau) = \rho_{ge}(0)\sqrt{\frac{\rho_{gg}(\tau)\,\rho_{ee}(\tau)}{\rho_{gg}(0)\,\rho_{ee}(0)}}$$

(in rotating frame)

Number of photons in the resonator does not fluctuate (we do not measure it)

Now amplify orthogonal quadrature



No information about the qubits state

$$\rho_{ee}(\tau)=\rho_{ee}(0),\ \rho_{gg}(\tau)=\rho_{gg}(0)$$

From $\overline{Q}=\tau^{-1}\int_0^\tau Q(t)\,dt$ we can infer fluctuation of photon number in the resonator

Number of emitted photons: $N=\overline{n}\kappa\tau\pm\sqrt{\overline{n}\kappa\tau}.$ Therefore, $\delta N/\sqrt{\overline{n}\kappa\tau}=\delta\overline{Q}/\sigma_Q.$

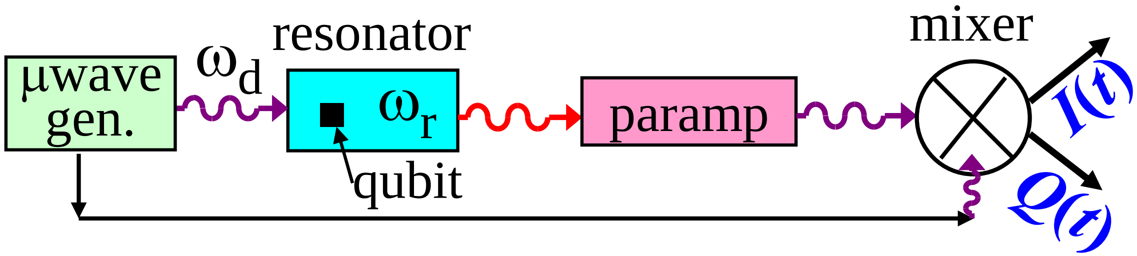
Each photon shifts qubit frequency by 2χ (ac Stark shift), rotates qubit phase by $2\chi\times(2/\kappa)$.

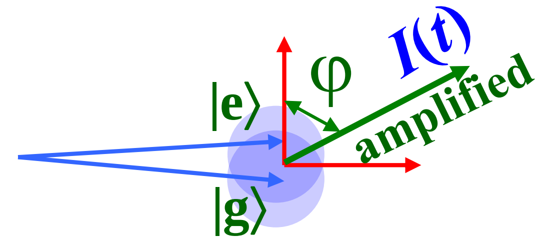
$$\rho_{ge}(\tau)=\rho_{ge}(0)\ \exp[i\,\frac{(I_e-I_g)_{\max}}{2D}\,\overline{Q}]$$

$$D=S/2\tau$$

Now the number of photons in the resonator fluctuates, but we can monitor how it fluctuates

Phase-sensitive paramp, amplify arbitrary phase φ





get some information (~cos$^2\varphi$) about qubit state and some information (~sin$^2\varphi$) about photon fluctuations

$$\begin{cases} \dfrac{\rho_{gg}(\tau)}{\rho_{ee}(\tau)} = \dfrac{\rho_{gg}(0)}{\rho_{ee}(0)} \dfrac{\exp[-(\bar{I}-I_g)^2/2D]}{\exp[-(\bar{I}-I_e)^2/2D]} \\ \rho_{ge}(\tau) = \rho_{ge}(0)\sqrt{\dfrac{\rho_{gg}(\tau)\,\rho_{ee}(\tau)}{\rho_{gg}(0)\,\rho_{ee}(0)}}\exp(iK\bar{I}\tau) \end{cases}$$

Bayes

unitary

$$\bar{I} \equiv \frac{1}{\tau}\int_0^\tau I(t)\,dt \qquad D = S_I/2\tau$$

$$I_g - I_e = \Delta I \cos\varphi \qquad K = \frac{\Delta I}{S_I}\sin\varphi$$

$$\Gamma = \frac{(\Delta I\cos\varphi)^2}{4S_I} + K^2\frac{S_I}{4} = \frac{\Delta I^2}{4S_I} = \frac{8\chi^2\bar{n}}{\kappa}$$

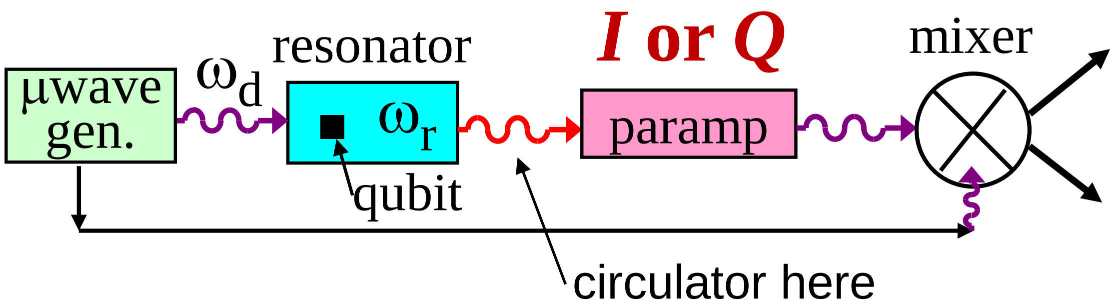
(rotating frame)

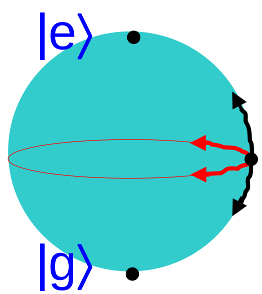
Can monitor wavefunction!

A.K., arXiv:1111.4016

Same as for QPC, but φ controls trade-off between "quantum" & "classical" back-actions (we choose if photon number fluctuates or not)

Choosing qubit evolution retroactively





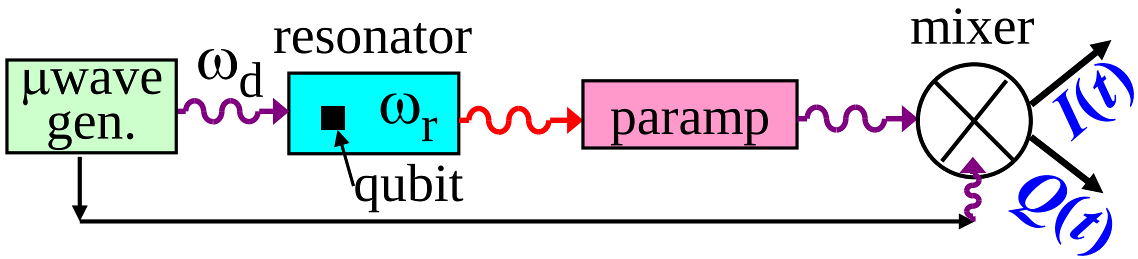
We can retroactively choose the qubit evolution to be either along meridian or along parallel or in between (delayed choice)

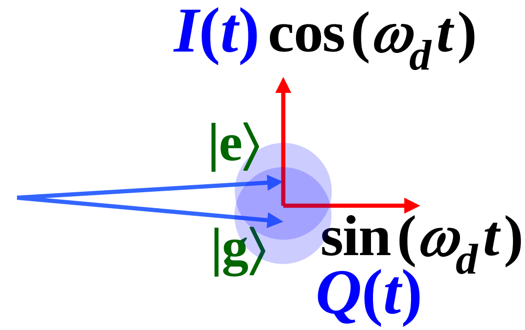
Does not violate causality because ensemble-averaged evolution is not affected (cannot send “useful” information)

A.K., arXiv:1111.4016

Phase-preserving paramp

Now information in both $I(t)$ and $Q(t)$





Ways to derive: - informational (with twice larger noise)
- phase-preserving with rotating φ (then average)
- split the signal, use two paramps for I and Q

$$\begin{cases} \dfrac{\rho_{gg}(\tau)}{\rho_{ee}(\tau)} = \dfrac{\rho_{gg}(0)}{\rho_{ee}(0)} \dfrac{\exp[-(\bar{I}-I_g)^2/2D]}{\exp[-(\bar{I}-I_e)^2/2D]} \\ \rho_{ge}(\tau) = \rho_{ge}(0)\sqrt{\dfrac{\rho_{gg}(\tau)\,\rho_{ee}(\tau)}{\rho_{gg}(0)\,\rho_{ee}(0)}}\,\exp(iK\bar{Q}\tau) \end{cases}$$

Bayes unitary

$$\bar{I} \equiv \frac{1}{\tau}\int_0^\tau I(t)\,dt \qquad \bar{Q} \equiv \frac{1}{\tau}\int_0^\tau Q(t)\,dt \qquad D = \frac{S_I}{2\tau}$$

$$I_g - I_e = \frac{\Delta I}{\sqrt{2}} \qquad K = \frac{\Delta I}{\sqrt{2}S_I}$$

$$\Gamma = \frac{\Delta I^2}{8S_I} + \frac{\Delta I^2}{8S_I} = \frac{8\chi^2\bar{n}}{\kappa}$$

Separated information in ***I*** and ***Q*** channels

Again, can monitor wavefunction

2-channel feedback, if needed (easy to undo Q)

Equal contributions to ensemble dephasing from “quantum” & “classical” back-actions

A.K., arXiv:1111.4016

Imperfect quantum efficiency

$$\eta = \eta_{\text{collection}}\ \eta_{\text{amplifier}}$$

$$\eta_{\text{collection}} = \frac{\kappa_{\text{amplified}}}{\kappa_{\text{total}}} \qquad \eta_{\text{amplifier}} = \frac{S_{\text{quantum-limited}}}{S_{\text{actual}}}$$

Experimentally measurable parameters:

qubit ensemble dephasing Γ, signal $\Delta I = (I_e - I_g)_{\max}$, and noise S

Imperfect efficiency is equivalent to qubit dephasing:

phase-sensitive:
$$\gamma = \Gamma - \frac{(\Delta I)^2}{4S} = (1-\eta)\Gamma$$

$$\rho_{ge}(\tau) = \rho_{ge}(0)\sqrt{\frac{\rho_{gg}(\tau)\,\rho_{ee}(\tau)}{\rho_{gg}(0)\,\rho_{ee}(0)}}\ \exp(iK\bar{I}\tau)\ e^{-\gamma\tau}$$

phase-preserving:
$$\gamma = \Gamma - \frac{(\Delta I)^2}{2S} = (1-\eta)\Gamma$$

$$\rho_{ge}(\tau) = \rho_{ge}(0)\sqrt{\frac{\rho_{gg}(\tau)\,\rho_{ee}(\tau)}{\rho_{gg}(0)\,\rho_{ee}(0)}}\ \exp(iK\bar{Q}\tau)\ e^{-\gamma\tau}$$

Evolving qubit

Easy to add qubit evolution:

- Time derivative of the above discussed (Bayesian) equations
- Add terms for the qubit evolution (Rabi osc., decoherence, etc.)

Need to be careful with definition of the derivative:

$$\frac{df(t)}{dt} = \lim_{\Delta t \to 0} \frac{f(t+\Delta t/2) - f(t-\Delta t/2)}{\Delta t}$$

(Stratonovich)
usual calculus

$$\frac{df(t)}{dt} = \lim_{\Delta t \to 0} \frac{f(t+\Delta t) - f(t)}{\Delta t}$$

(Ito)
simple averaging

Quantum trajectory theory uses Ito definition,
I usually use Stratonovich definition

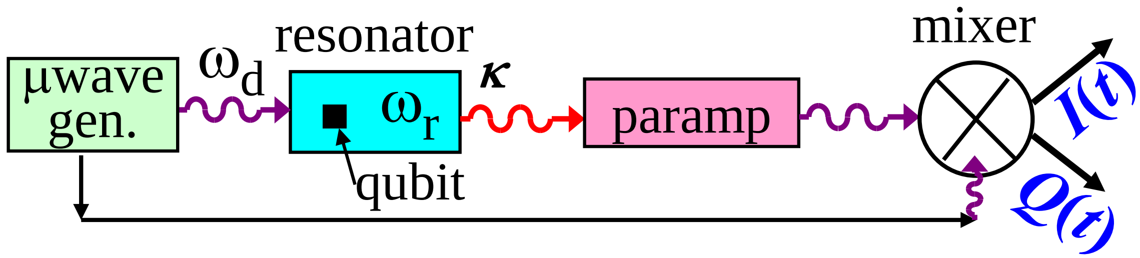
A better theory

A.K., in preparation

Remove the “bad cavity” assumption
(now significant qubit-cavity entanglement is OK)

Easy for a non-evolving qubit; then the theory can be based on coherent states for the resonator (equiv. to “polaron” transformation approach)

Just an elementary quantum mechanics and common sense



Idea:
- Consider evolution for the qubit in the state $|0\rangle$, then in $|1\rangle$
- Combine as superposition
- Simple “orthodox” model for homodyne measurement of field

Also equivalent to quantum trajectory theory
by Gambetta et al., PRA-2008

Preliminaries (optical coherent states)

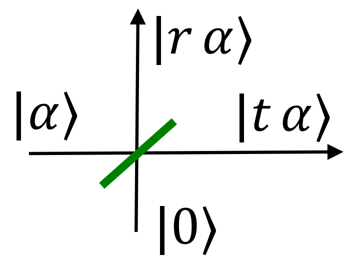
$$|0\rangle = \psi_{\rm gr}(x) = \left(\frac{m\omega_{\rm r}}{\pi\hbar}\right)^{1/4} \exp\left(-\frac{m\omega_{\rm r}}{2\hbar}\,x^2\right)$$

$$|\alpha\rangle \equiv \; \psi_{\rm gr}(x-x_{\rm c})\, \exp(ip_{\rm c}x/\hbar)\, \exp(-ip_{\rm c}x_{\rm c}/2\hbar)$$

$$\alpha \equiv \frac{x_{\rm c}}{2\sigma_x} + i\,\frac{p_{\rm c}}{2\sigma_p} = x_{\rm c}\sqrt{\frac{m\omega_{\rm r}}{2\hbar}} + ip_{\rm c}\,\frac{1}{\sqrt{2\hbar m\omega_{\rm r}}},$$

$$|\alpha\rangle = e^{-\frac{1}{2}|\alpha|^2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}}\,|n\rangle \qquad \bar{n} = |\alpha|^2 \qquad \langle\alpha|\beta\rangle = e^{-\frac{1}{2}|\alpha-\beta|^2}\, e^{-i\,{\rm Im}(\alpha\beta^*)}$$

Passing through a beamsplitter



no entanglement, classical

Driven resonator with leakage κ

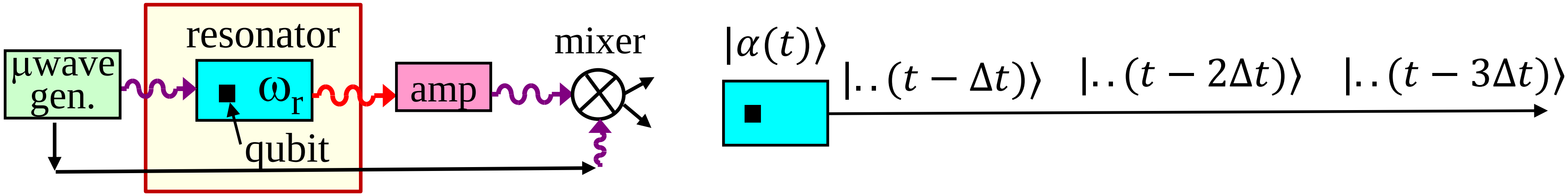
$$H = \hbar\omega_{\rm r}\hat{a}^\dagger\hat{a} + \hbar\varepsilon e^{-i\omega_d t}\hat{a}^\dagger + \hbar\varepsilon^* e^{i\omega_d t}\hat{a}$$

$$|\psi(t)\rangle = \underline{e^{-i\varphi(t)}}\,|\alpha(t)\rangle \qquad \dot{\tilde{\alpha}} = -i(\omega_{\rm r}-\omega_{\rm d})\,\tilde{\alpha} - \frac{\kappa}{2}\,\tilde{\alpha} - i\varepsilon$$

$$\underline{\dot{\varphi} = {\rm Re}(\varepsilon^*\tilde{\alpha}).}$$

remains pure in spite of dissipation (explanation via beamsplitter or jump – no jump Lindblad)

Measurement and "history tail"



Assume a non-evolving qubit in state $|0\rangle$, then resonator frequency $\omega_r - \chi$

$$|\Psi(t)\rangle = e^{-i\varphi_0(t)}|0\rangle\,|\alpha_0(t)\rangle \prod_m |\alpha_0(t-m\,\Delta t)\sqrt{\kappa\,\Delta t}\rangle$$

$$\dot{\alpha}_0 = \ldots$$
$$\dot{\varphi}_0 = \ldots$$

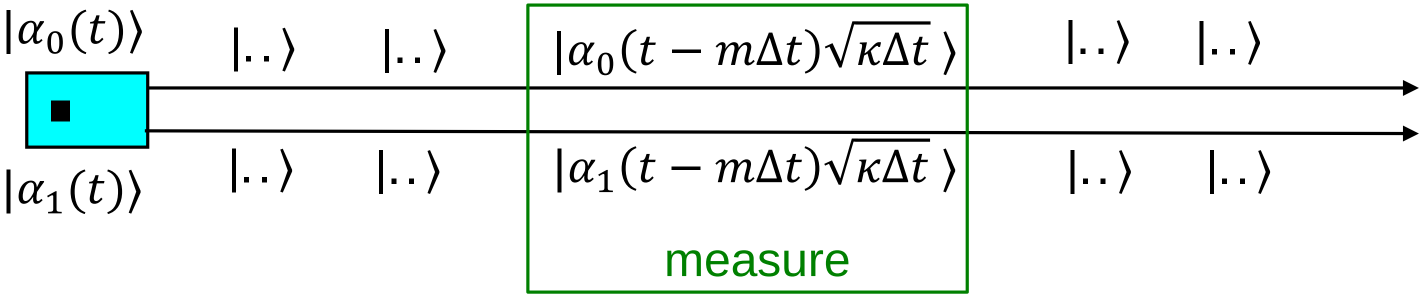
Similar for qubit in state $|1\rangle$, then frequency $\omega_r + \chi$

Now non-evolving superposition of $|0\rangle$ and $|1\rangle$ (as in "many worlds")

$$|\Psi(t)\rangle = c_0 e^{-i\varphi_0(t)}|0\rangle\,|\alpha_0(t)\rangle \prod_m |\alpha_0(t-m\,\Delta t)\sqrt{\kappa\,\Delta t}\rangle$$
$$+ c_1 e^{-i\varphi_1(t)}|1\rangle\,|\alpha_1(t)\rangle \prod_m |\alpha_1(t-m\,\Delta t)\sqrt{\kappa\,\Delta t}\rangle$$

Measure pieces of the "history tail" in "orthodox" way (all coherent states)

Homodyne measurement of "history tail" pieces



Simple model of homodyne measurement

- Add a large coherent state (pump)
- Measure number of photons *n*
- "Orthodox" collapse onto obtained random *n*

This changes superposition coefficients c_0 and c_1:

$$\tilde{c}_0 = \frac{c_0\,\exp[-(n-\bar{n}_0)^2/4\sigma^2]}{\text{Norm}},$$

$$\tilde{c}_1 = \frac{c_1\,\exp[-(n-\bar{n}_1)^2/4\sigma^2]}{\text{Norm}}\,e^{-i\,\Delta\varphi},$$

(Gaussian approximation,
σ is noise variance,
α_p is added pump)

$$\Delta\varphi = \frac{n-\frac{\bar{n}_0+\bar{n}_1}{2}}{\sigma}\sqrt{\kappa\Delta t}\,\text{Im}\{[\alpha_0(t_m)-\alpha_1(t_m)]\,e^{-i\phi_p}\}$$
$$+\kappa\Delta t\,\text{Im}[\alpha_1^*(t_m)\,\alpha_0(t_m)],\quad \phi_p=\arg(\alpha_p),$$

Resulting evolution (phase-sensitive case)

$$\hat{\rho}(t) = \sum_{j,j'=0,1} \rho_{jj'}(t)\, |j\rangle\langle j'| \otimes |\alpha_j(t)\rangle\langle\alpha'_j(t)|$$

$$P(\tilde{I}_{\rm m}) = \rho_{00}(t)\, \frac{\exp\{-[\tilde{I}_{\rm m} + (\Delta I_{\rm max}\cos\phi_{\rm d})/2]^2/2D\}}{\sqrt{2\pi D}}$$
$$+\rho_{11}(t)\, \frac{\exp\{-[\tilde{I}_{\rm m} - (\Delta I_{\rm max}\cos\phi_{\rm d})/2]^2/2D\}}{\sqrt{2\pi D}},$$

$$\frac{\rho_{11}(t+\Delta t)}{\rho_{00}(t+\Delta t)} = \frac{\rho_{11}(t)}{\rho_{00}(t)}\, \exp[\tilde{I}_{\rm m}\cos(\phi_{\rm d})\, \Delta I_{\rm max}/D],$$
$$\frac{\rho_{10}(t+\Delta t)}{\rho_{10}(t)} = \frac{\sqrt{\rho_{11}(t+\Delta t)\, \rho_{00}(t+\Delta t)}}{\sqrt{\rho_{11}(t)\, \rho_{00}(t)}}\, \exp(-\gamma\Delta t)$$
$$\times \exp(-i\delta\omega_{{\rm q},1+2}\Delta t)\exp[-i\tilde{I}_{\rm m}\sin(\phi_{\rm d})\Delta I_{\rm max}/2D],$$
$$\gamma = \Gamma_{\rm d} - (\Delta I_{\rm max})^2/4S_I, \quad \eta = (\Gamma_{\rm d} - \gamma)/\Gamma_{\rm d},$$

- Practically the same as in simple quantum Bayesian approach
- Now applied to entangled qubit-resonator system
- Allows arbitrary κ, transient evolution
- Qubit evolves only due to measurement (no Rabi)
- Equivalent to "polaron" approach in quantum trajectories, but very simple derivation

(similar for phase-preserving case,
just *I* for diagonal and *Q* for off-diagonal)

How to include qubit evolution properly?

- Similar approach of the “history tail”, but cannot use coherent states, use density matrices in Fock space
- Much more difficult computationally
- Equivalent to “full” quantum trajectory theory, but different numerical procedure with finite time steps

Quantum trajectory equations

$$\dot{\varrho}_J(t) = \mathcal{L}_{\rm tot}\varrho_J(t) + i\sqrt{\kappa\eta}[Q_\phi, \varrho_J][J(t) - \sqrt{\kappa\eta}\langle 2I_\phi\rangle_t]$$
$$+ \sqrt{\kappa\eta}\mathcal{M}[2I_\phi]\varrho_J(t)[J(t) - \sqrt{\kappa\eta}\langle 2I_\phi\rangle_t]$$

$$-\frac{i}{\hbar}[H_{\rm eff}, \varrho(t)] + \kappa\mathcal{D}[a]\varrho(t) + \gamma_1\mathcal{D}[\sigma]\varrho(t)$$

$$+ \gamma_\phi\mathcal{D}[\sigma_z]\varrho(t)/2 = \mathcal{L}_{\rm tot}\varrho(t), \qquad \mathcal{D}[A]\varrho = A\varrho A^\dagger - A^\dagger A\varrho/2 - \varrho A^\dagger A/2$$

$$\mathcal{M}[c]\varrho = (c - \langle c\rangle_t)\varrho/2 + \varrho(c - \langle c\rangle_t)/2 \qquad \langle c\rangle_t = \mathrm{Tr}[c\varrho_J(t)]$$

$$2I_\phi = ae^{-i\phi} + a^\dagger e^{i\phi} \qquad 2Q_\phi = -iae^{-i\phi} + ia^\dagger e^{i\phi}$$

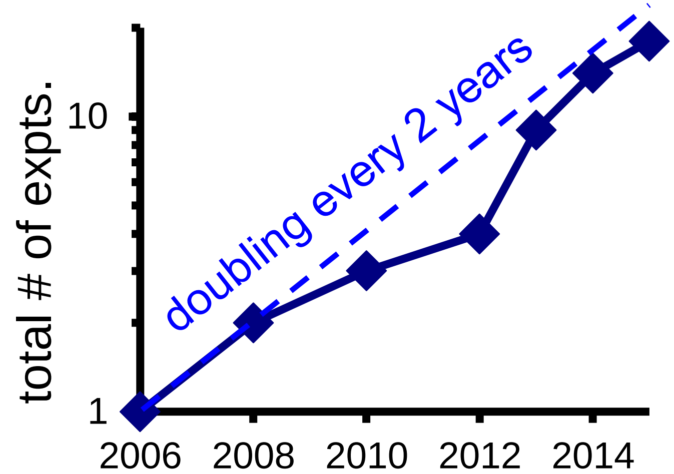
$$J(t) = \sqrt{\kappa\eta}\langle 2I_\phi\rangle_t + \xi(t) \qquad E[\xi(t)] = 0, \;\; E[\xi(t)\xi(t')] = \delta(t - t')$$

$J(t)$ is homodyne (phase-sensitive) measurement result

J. Gambetta et al., PRA-2008

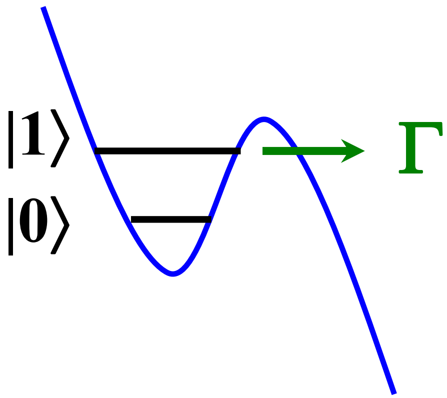
Similar to Wiseman, Milburn (1993)

Experiments on partial and continuous measurement of superconducting qubits



1. N. Katz et al. (UCSB), Science 312, 1498 (2006); partial collapse
2. N. Katz et al. (UCSB), PRL 101, 200401 (2008); uncollapse
3. A. Palacios-Laloy et al. (Saclay), Nature Phys. 6, 442 (2010); continuous Rabi + weak Leggett-Garg
4. R. Vijay et al. (Berkeley), Nature 490, 77 (2012); quantum feedback of Rabi oscillations
5. M. Hatridge et al. (Yale), Science 339, 178 (2013); partial meas. in cQED (phase-preserving)
6. K. Murch et al. (Berkeley), Nature 502, 211 (2013); quantum trajectories (phase-sensitive)
7. Campagne-Ibarcq et al. (Paris), PRX 3, 021008 (2013); stroboscopic meas. and feedback
8. D. Riste et al. (Delft), Nature 502, 350 (2013); entanglement by measurement
9. J. Groen et al. (Delft), PRL 111, 090506 (2013); partial meas., weak values, LG (via ancilla)
10. J. Zhong et al. (Zhejiang U., UCSB), Nature Comm. 5, 3135 (2014); T1 increase (3x) by uncollapse
11. N. Roch et al. (Berkeley), PRL 112, 170501 (2014); entanglement of remote qubit by meas.
12. S. Weber et al. (Berkeley), Nature 511, 570 (2014); trajectories with Rabi, most likely path
13. G. De Lange et al. (Delft), PRL 112, 080501 (2014); dephasing suppression by feedback
14. Campagne-Ibarcq et al. (Paris), PRL 112, 180402 (2014); interference between past and future
15. D. Tan et al. (Wash.U.), PRL 114, 090403 (2015); prediction/retrodicton
16. N. Foroozani et a. (Wash. U), arXiv:1508.01185; state-signal correlations
17. T. White et al. (UCSB), arXiv:1504.02707; Bell-Leggett-Garg with weak meas. (via ancilla)
18. Y. Liu et al. (Yale), arXiv:1509.00860; entanglement by measurement and by dissipation

Early experiments on <u>partial</u> collapse with superconducting phase qubits



Partial collapse N. Katz et al., Science-2006 (UCSB)

$$\psi = \alpha\,|0\rangle + \beta\,|1\rangle \;\rightarrow\; \psi(t) = \begin{cases} |out\rangle, \text{ if tunneled} \\ \dfrac{\alpha\,|0\rangle + \beta\, e^{-\Gamma t/2}\,|1\rangle}{\sqrt{|\alpha|^2 + |\beta|^2\, e^{-\Gamma t}}}, \text{ if not tunneled} \end{cases}$$

Reversal of partial collapse (uncollapse) N. Katz et al., PRL-2008 (UCSB)

$$\alpha\,|0\rangle + \beta|1\rangle \rightarrow \frac{\alpha\,|0\rangle + \beta\, e^{-\Gamma t/2}\,|1\rangle}{\text{Norm}} \rightarrow \frac{\alpha\, e^{-\Gamma t/2}\,|0\rangle + \beta\, e^{-\Gamma t/2}\,|1\rangle}{\text{Norm}} = \alpha\,|0\rangle + \beta|1\rangle$$

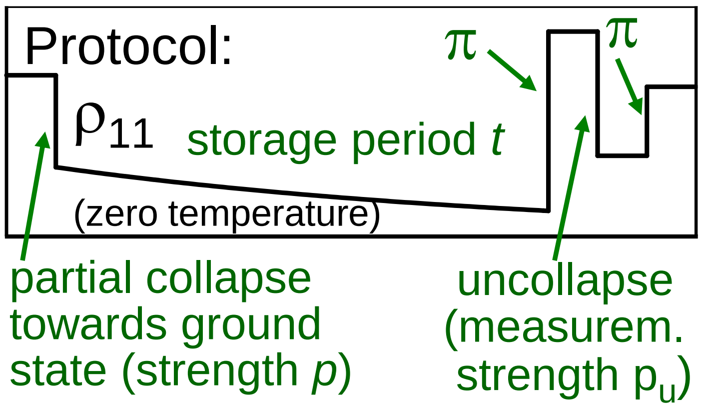
ψ_0 (unknown) $\xrightarrow{\text{partial meas.}}$ ψ_1 (partially collapsed)

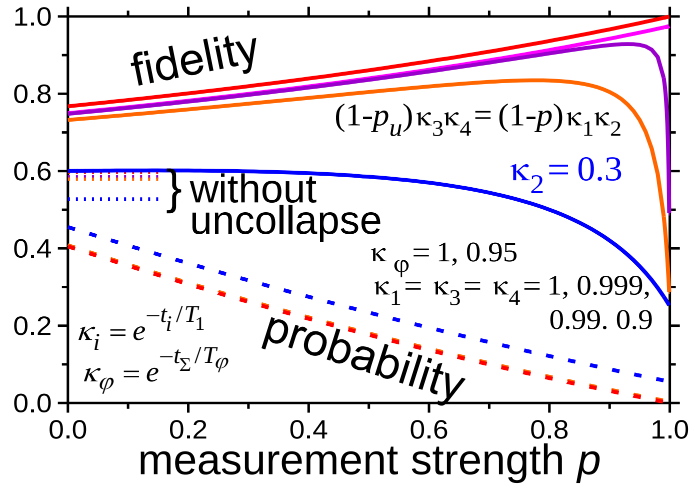
- successful → ψ_0 (still unknown)
- unsuccessful → ψ_2

uncollapse (information erasure)

Theory: A.K. & Jordan, PRL-2006

Decoherence suppression by uncollapse





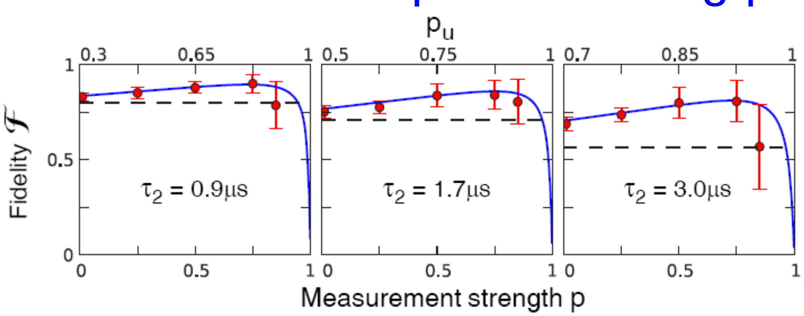
Theory: A.K & Keane, PRA-2010

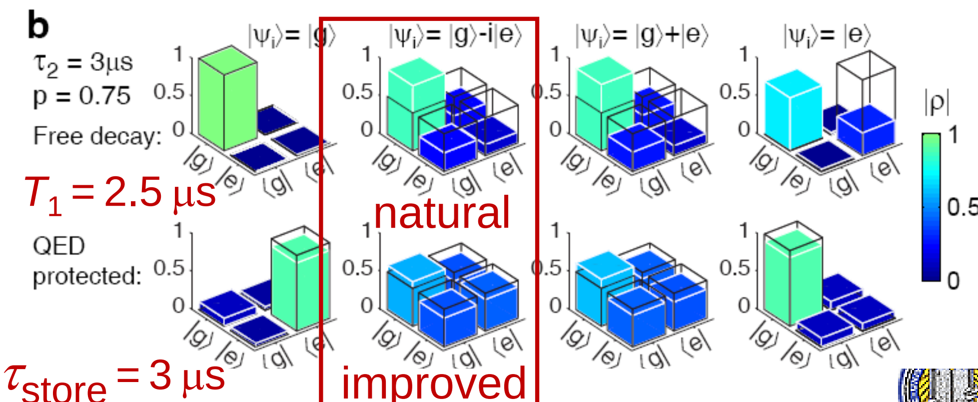
"Sleeping beauty" analogy

First realized in optics Lee et al., Opt. Expr.-2011

Also used for entanglement preservation Kim et al., Nature Phys.-2012

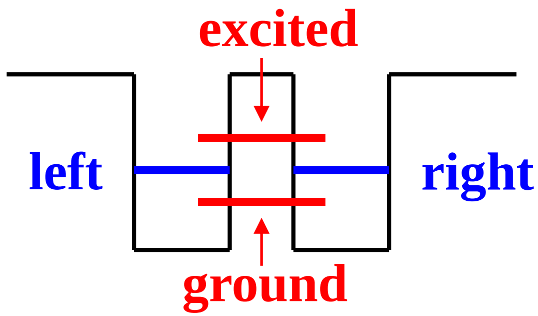
Realization with superconducting phase qubits Zhong et al., Nature Comm.-2014



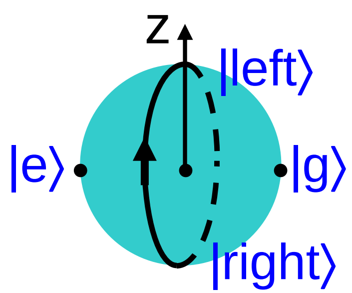


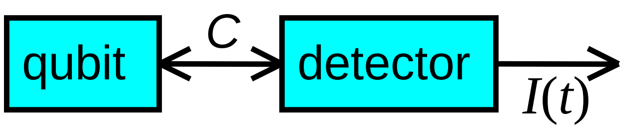
Increases effective T_1 by 3x

Non-decaying (persistent) Rabi oscillations

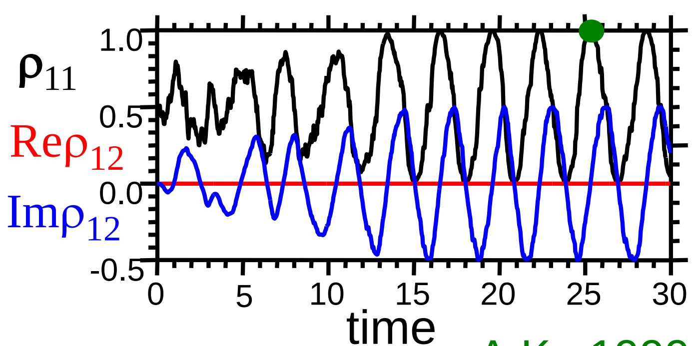


- Relaxes to the ground state if left alone (low-T)
- Becomes fully mixed if coupled to a high-T envir.
- Oscillates persistently between left and right if (weakly) measured continuously ("reason": attraction to left/right states)

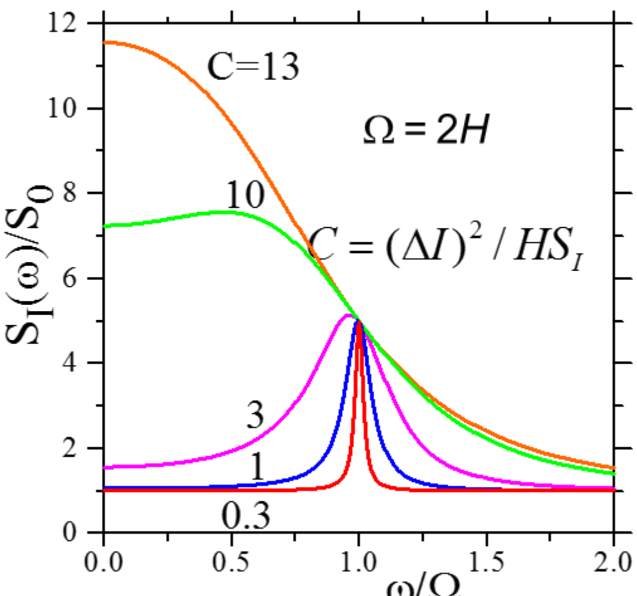




A.K., LT-1999, 2001
A.K.-Averin, 2001



A.K., 1999



Rabi frequency Ω

Integral under peak $\Leftrightarrow \langle z^2\rangle$ (Bloch sph.)

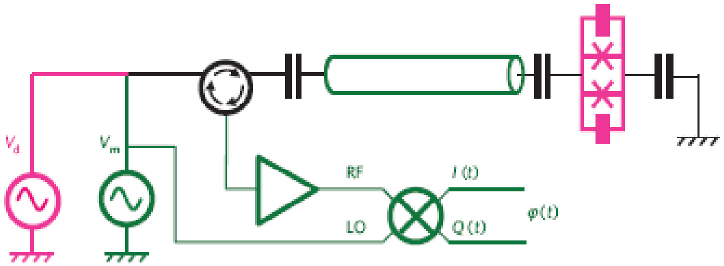
perfect Rabi: $\langle z^2\rangle = \langle \cos^2\rangle = 1/2$

quantum: $\langle z^2\rangle = 1$

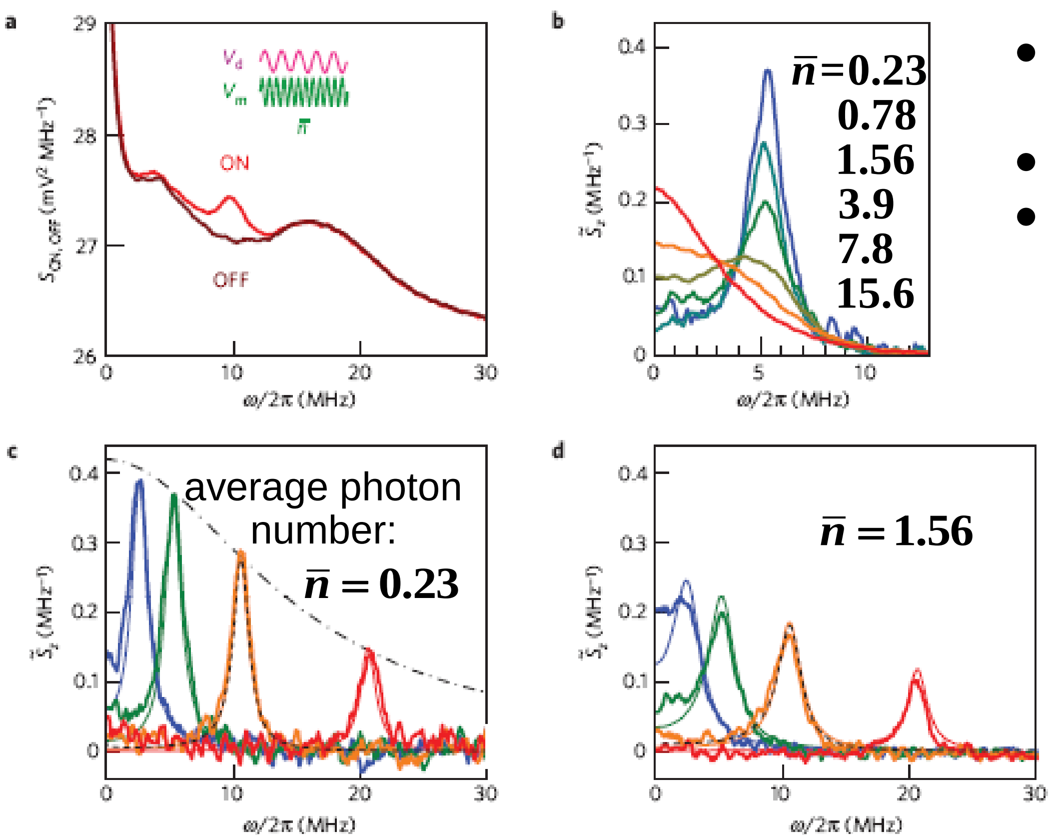
classical limit: $\int S_{\text{peak}} df < 8/\pi^2$

Ruskov, A.K., Mizel (2006)

Continuous monitoring of Rabi oscillations



A. Palacios-Laloy, F.Mallet, F.Nguyen, P. Bertet, D. Vion, D. Esteve, and A. Korotkov, Nature Phys. (2010)



- superconducting qubit (transmon) in circuit QED setup
- microwave reflection from cavity
- driven Rabi oscillations (|g> ↔|e>)

Theory by dashed lines, very good agreement

Pre-amplifier noise temperature T_N= 4 K

$$\frac{1}{1+\dfrac{2T_N}{\hbar\omega}} \approx 0.03$$

quantum efficiency $\eta = \dfrac{\Delta S}{4S} \sim 10^{-2}$

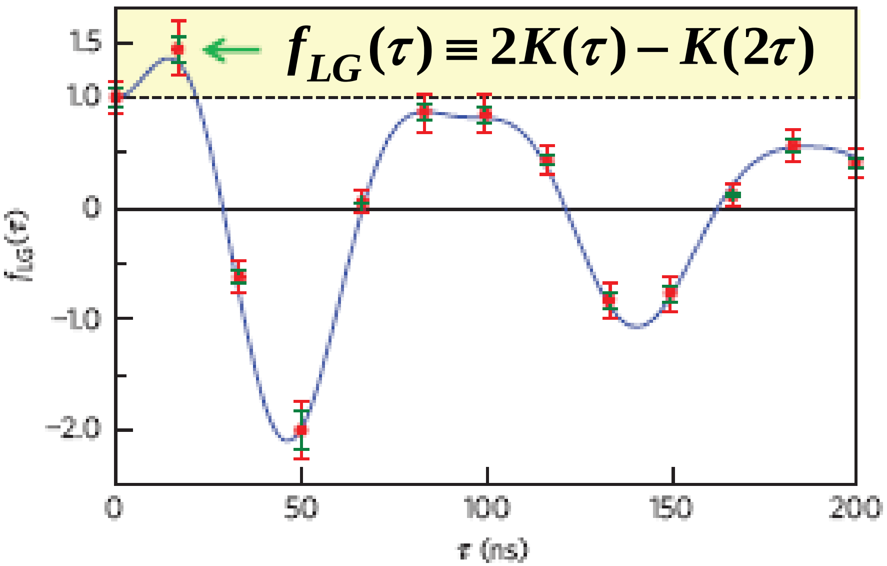
Violation of Leggett-Garg inequalities

A. Palacios-Laloy et al., 2010

In time domain

Rescaled to qubit *z*-coordinate $K(\tau) \equiv \langle z(t)\, z(t+\tau)\rangle$

$$K(\tau_1) + K(\tau_2) - K(\tau_1 + \tau_2) \le 1 \quad \Rightarrow 2K(\tau) - K(2\tau) \le 1$$



$$f_{LG}(0) = K(0) = \langle z^2 \rangle$$

$$\langle z^2 \rangle = 1.01 \pm 0.15$$

$$f_{LG}(17\,\text{ns}) = 1.44 \pm 0.12$$

Ideal $f_{\text{LG,max}}$=1.5

Standard deviation $\sigma = 0.065$

$\Rightarrow$ violation by 5σ

Many later experiments on Leggett-Garg ineq. violation, incl. optics and NMR

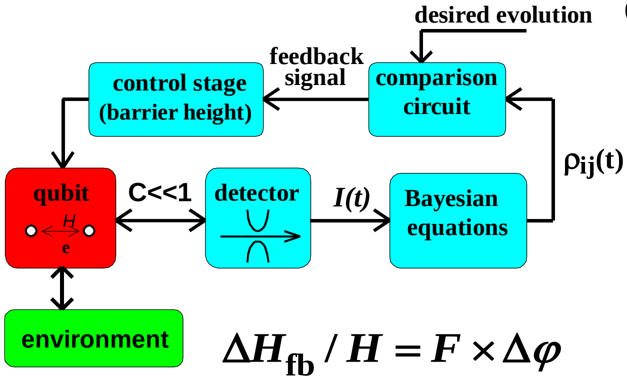
M. Goggin et al., PNAS-2011
J. Dressel et al., PRL-2011
G. Walhder et al., PRL-2011
V. Athalye et al., PRL-2011
A. Souza et al., NJP-2011
G. Knee et al., Nat. Comm.-2011
J. Groen et al., PRL-2013 (s/c, DiCarlo's group)

Quantum feedback to stabilize Rabi oscillations

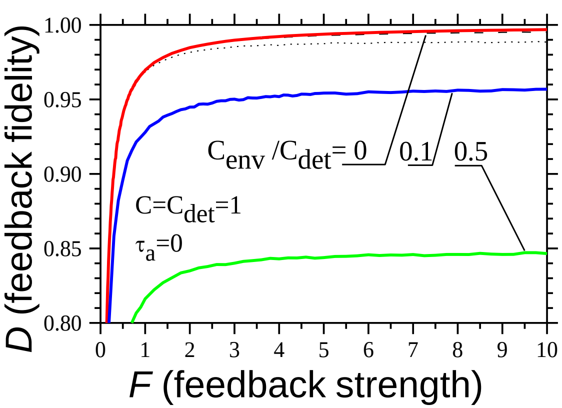
Bayesian

Best but very difficult

(monitor quantum state and control deviation)



$$\Delta H_{\rm fb}/H = F \times \Delta\varphi$$



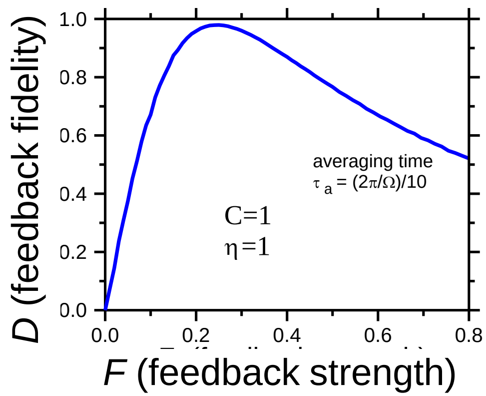
R. Ruskov & A.K., 2002

"Direct"

Similar to Wiseman-Milburn (1993)

(apply measurement signal to control with minimal processing)

$$\Delta H_{\rm fb}/H = F\sin(\Omega t) \times \left(\frac{I(t)-I_0}{\Delta I/2} - \cos\Omega t\right)$$



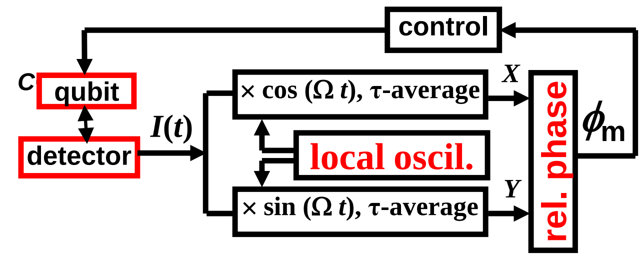
R. Ruskov & A.K., 2002

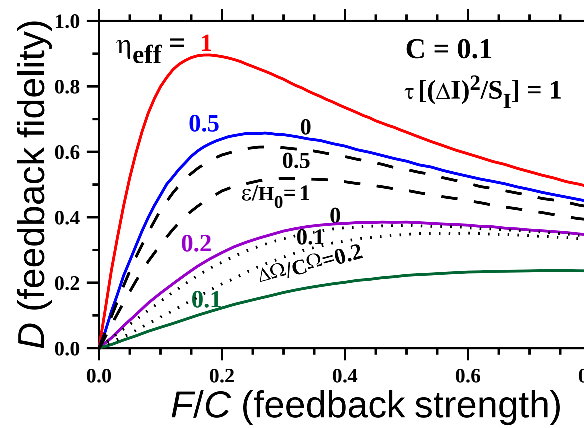
"Simple"

Imperfect but simple

(do as in usual classical feedback)

$$\frac{\Delta H_{\rm fb}}{H} = F \times \phi_m$$





A.K., 2005

Berkeley-2012 experiment: "direct" and "simple"

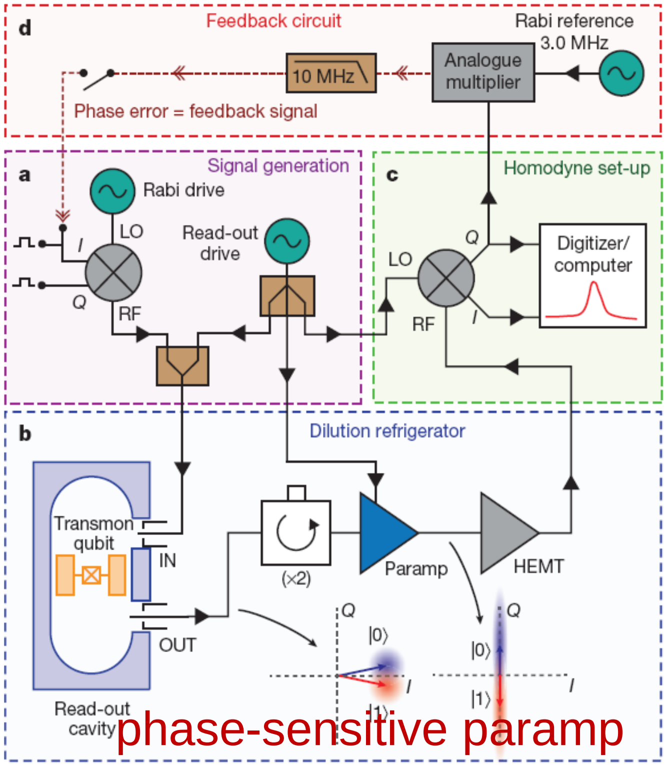
Quantum feedback of Rabi oscillations

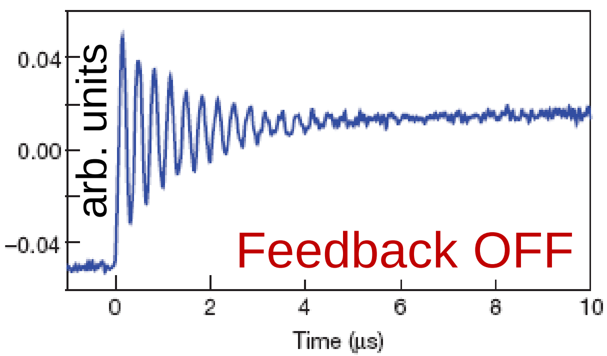
R. Vijay, C. Macklin, D. Slichter, S. Weber, K. Murch, R. Naik, A. Korotkov, and Irfan Siddiqi, Nature-2012

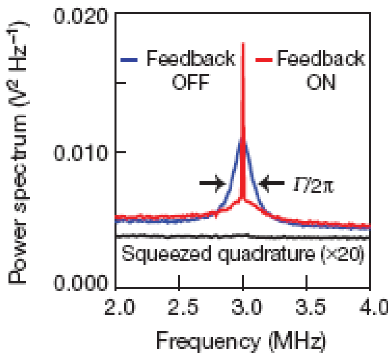
(quantum feedback with atoms, stabilizing photon number: C. Sayrin, … S. Haroche, Nature-2011)

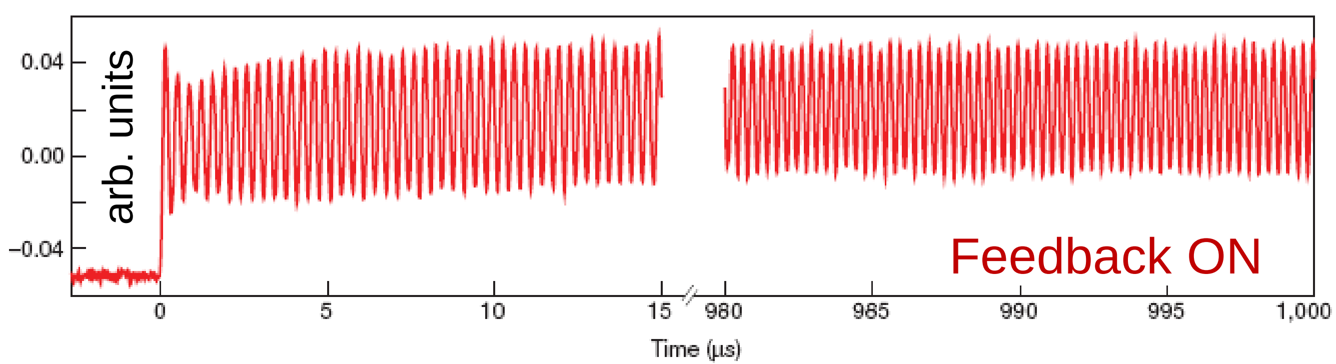
Simple idea: $I(t) \sim \cos(\Omega_R t - \theta_{ERR}) + \text{noise}$

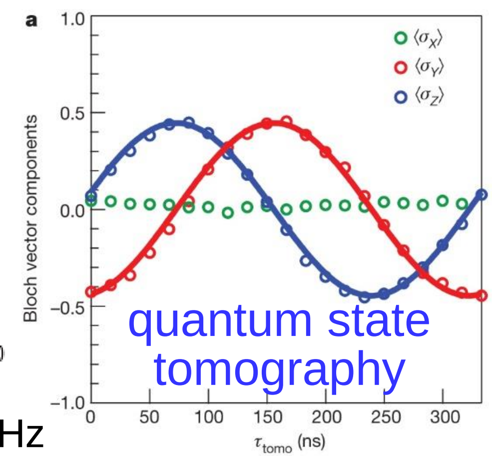
$$\Delta\Omega_R / \Omega_R = -F \sin(\theta_{ERR}), \quad \sin(\theta_{ERR}) \sim \overline{I(t)\sin(\Omega_R t)}$$







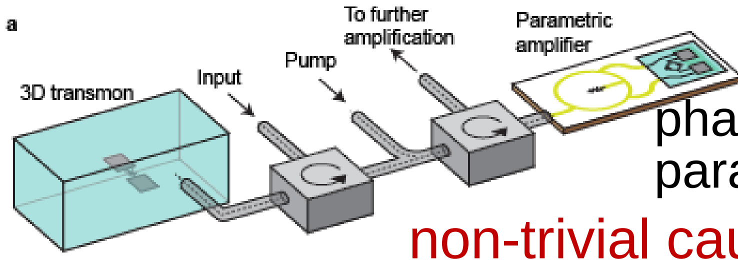




Rabi freq. 3 MHz, paramp BW 10 MHz, cavity LW 8 MHz, env. deph. 0.05 MHz

Single quantum trajectories of transmon qubit

K. Murch, S. Weber, C. Macklin, & I. Siddiqi, Nature-2013

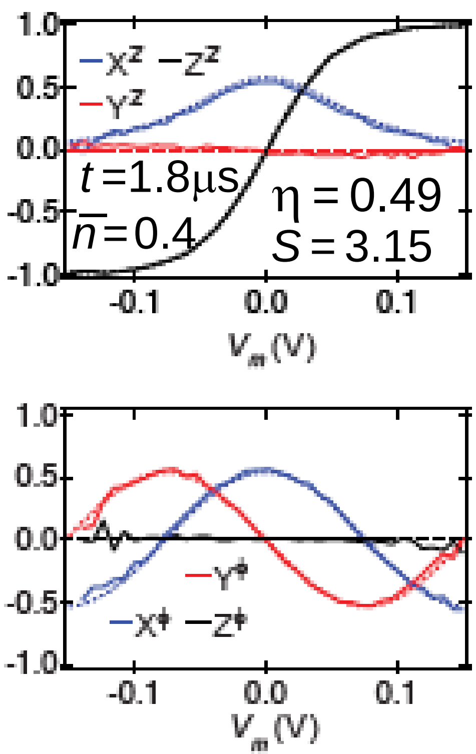


phase-sensitive paramp $\eta = 0.49$

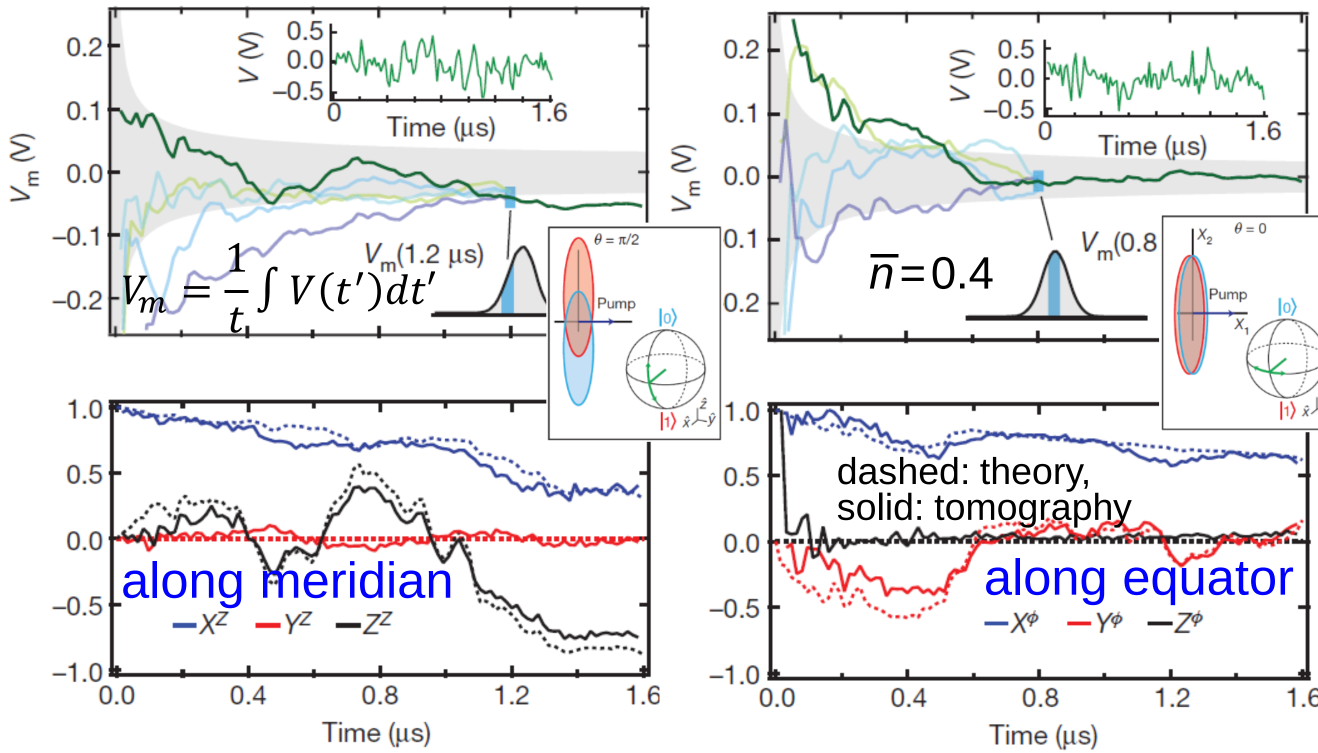
non-trivial causality

Coupling 0.52 MHz
Cavity LW 10.8 MHz
Paramp BW 20 MHz

Partial measurement: expt. vs. Bayesian theory



Individual quantum trajectories: experiment vs. Bayesian theory

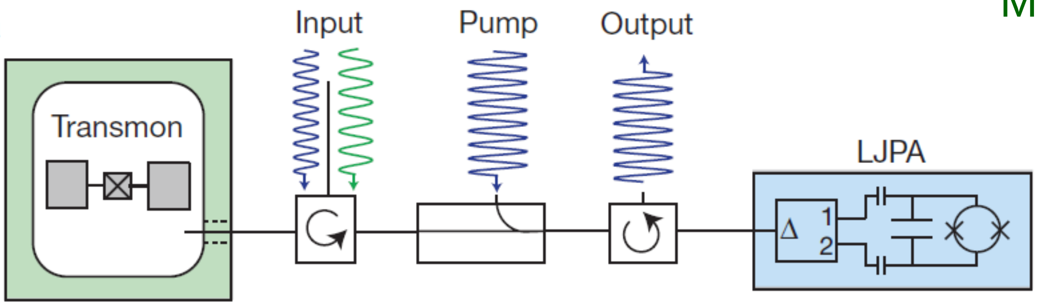


$$V_m = \frac{1}{t}\int V(t')dt'$$

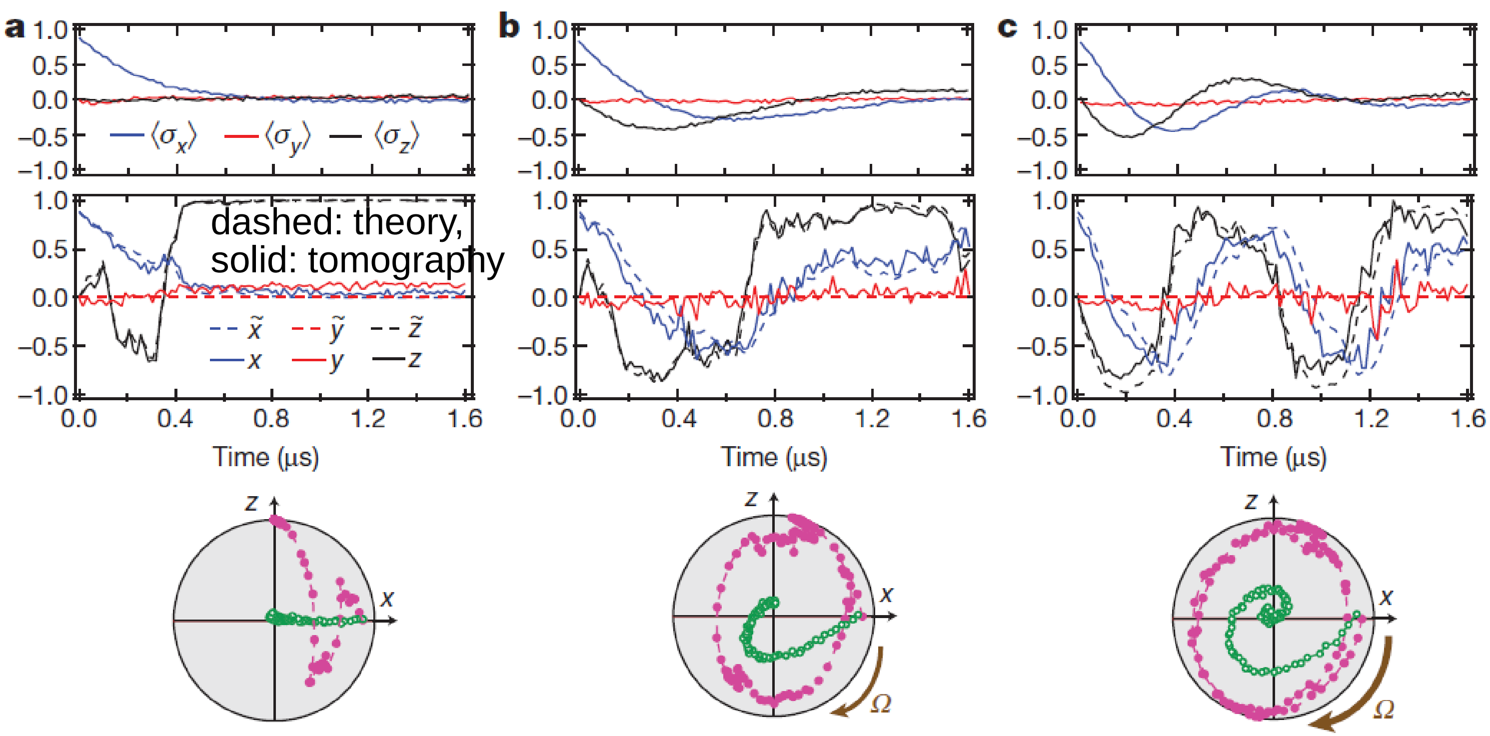
Good agreement with simple Bayesian theory (dashed)

Quantum trajectories with Rabi drive

S. Weber, Chantasri, Dressel, Jordan, Murch, and I. Siddiqi, Nature-2014



$\eta_{tot} = 0.4$



Ensemble-averaged

Individual

Good agreement with simple Bayesian theory

Most likely path with Rabi and post-selection

S. Weber et al. Nature-2014

Only $|\psi_{\rm in}\rangle$ is fixed   Both $|\psi_{\rm in}\rangle$ and $|\psi_{\rm fin}\rangle$ are fixed

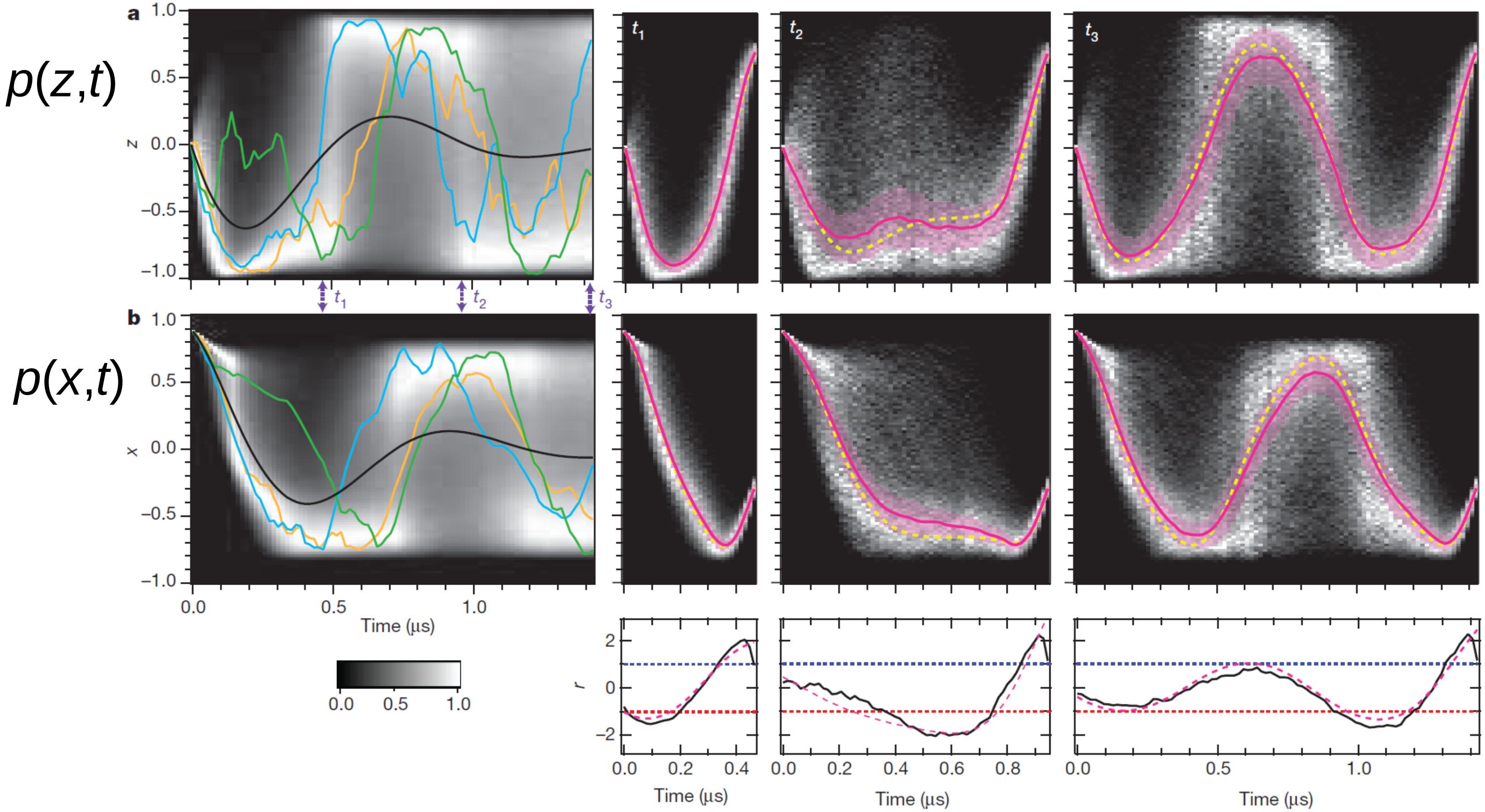
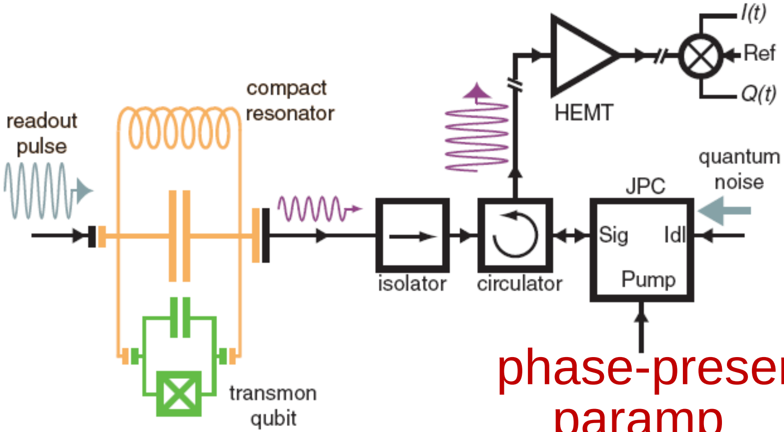


Figure 4 | Greyscale histograms of quantum trajectories in the driven case. The measurements begin at state ($x_I = 0.88, z_I = 0$). Here $\tau = 315$ ns, $\Gamma = 3.85 \times 10^6\,\mathrm{s}^{-1}$, $\Omega/2\pi = 1.08$ MHz. **a**, **b**, Histograms for z (**a**) and x (**b**) with representative trajectories plotted in colour and with the average trajectory shown in black. In the other panels in **a** and **b**, we post-select on the final state ($z_F = 0.7, x_F = -0.29$), with a post-selection window of ± 0.08. Solid magenta curves are the most likely trajectories for the experimental data, and the yellow dashed curves are from the theory. The standard deviations of the experimentally determined most likely paths are shown by magenta bands. As the time duration between the boundary conditions is increased from $t_1 = 0.464$ μs to $t_2 = 0.944$ μs and then to $t_3 = 1.424$ μs, the most likely trajectory connecting the initial and final states changes drastically but is well described by the theory (dashed line). The bottom panels compare the optimal detector signals (r; dashed lines) with the conditioned average signal (weak functions; black lines).

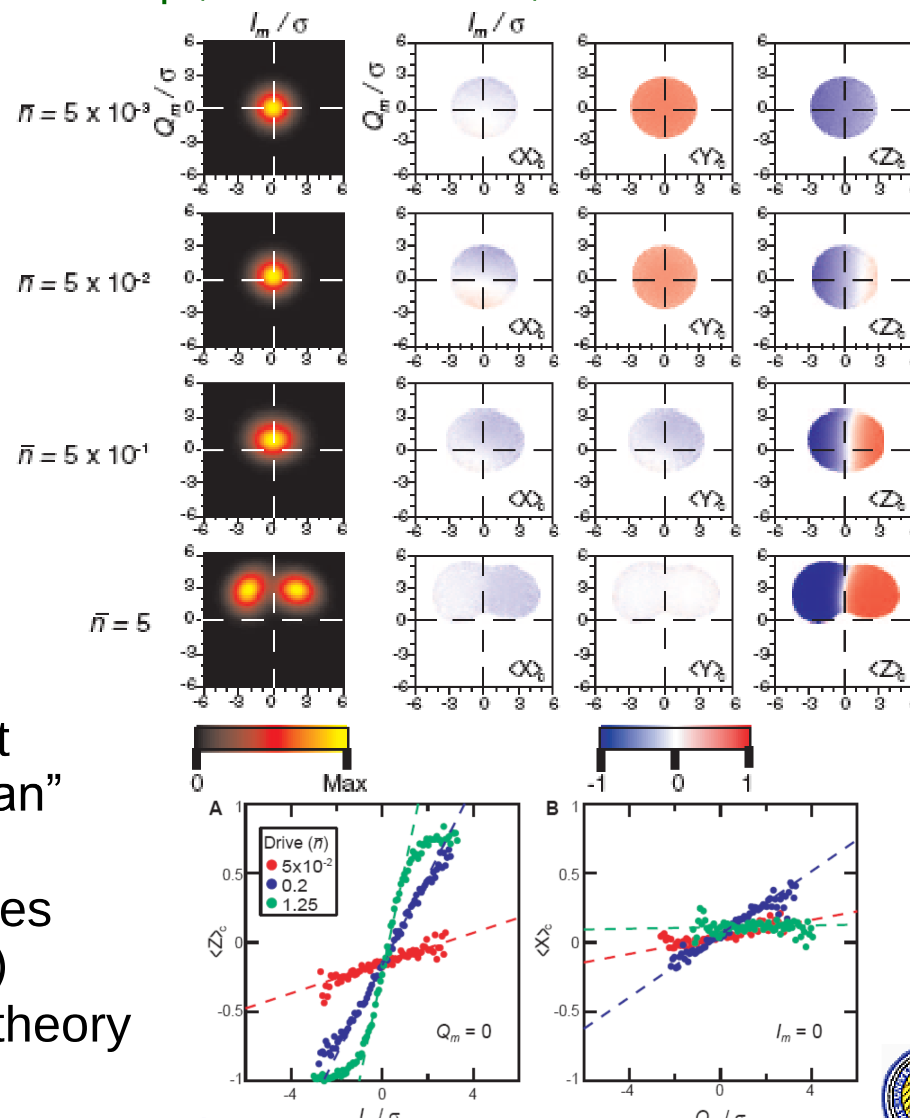
Good agreement with theory (dashed)

Phase-preserving continuous measurement



phase-preserving
paramp
$\eta = 0.2$

M. Hatridge, Shankar, Mirrahimi, Schackert, Geerlings, Brecht, Sliwa, Abdo, Frunzio, Girvin, Schoelkopf, and M. Devoret, Science-2013



Protocol:

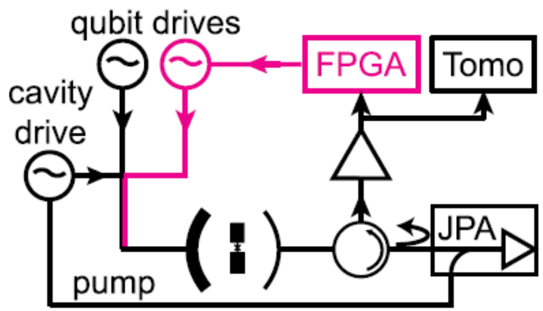
1) Start with |0>+|1>
2) Measure with controlled strength
3) Tomography of resulting state

Experimental findings:

- Result of *I*-quadrature measurement determines state shift along "meridian" of the Bloch sphere
- *Q*-quadrature meas. result determines shift along "parallel" (within equator)
- Agrees well with simple (Bayesian) theory

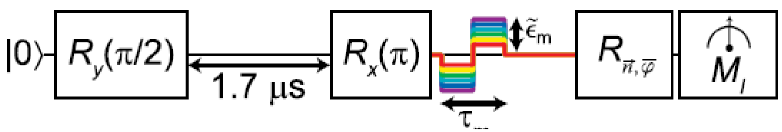
Suppression of measurement-induced dephasing by feedback (undoing motion along equator)

G. de Lange, Riste, Tiggelman, Eichler, Tornberg, Johansson, Wallraff, Schouten, and L. DiCarlo, PRL-2014

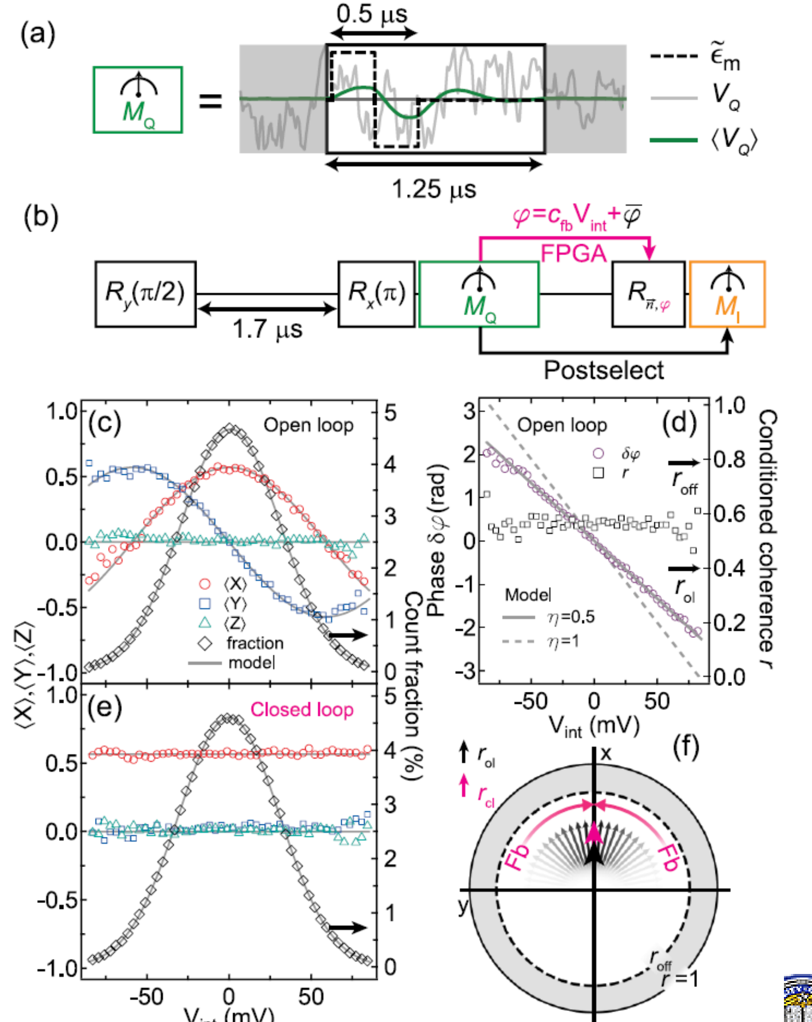


$\eta = 0.5$

Phase-sensitive amplifier, measure non-informational quadrature (back-action is along parallels)



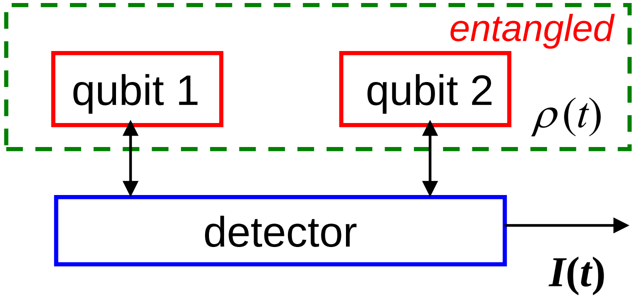
Echo sequence to analyze dephasing

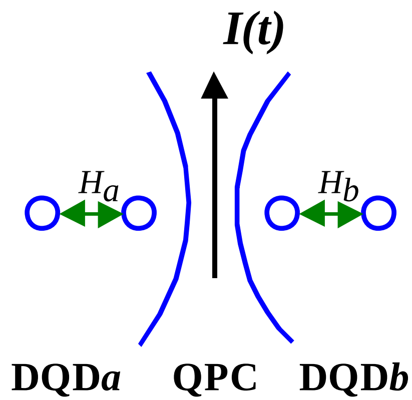


Idea: collect measurement signal (with weight function) to find back-action; then undo

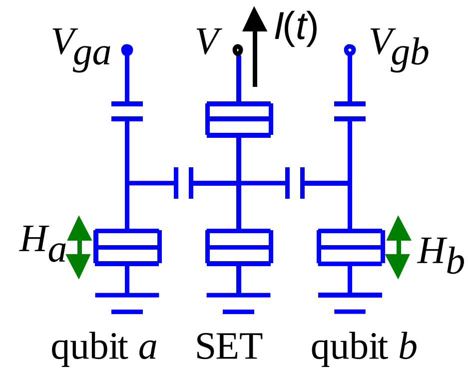
"refocusing" (feedback) increases qubit coherence $2|\rho_{01}|$ from 0.40 to 0.56

Entanglement by measurement (theory)





R. Ruskov & A.K., 2003



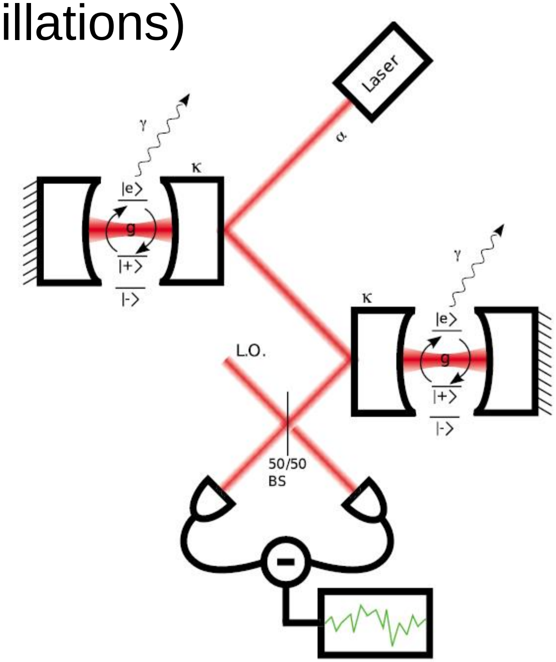
same current for |01> and |10>
⇒ entangles gradually

$$\frac{|0\rangle+|1\rangle}{\sqrt{2}}\otimes\frac{|0\rangle-|1\rangle}{\sqrt{2}}\to\frac{|10\rangle-|01\rangle}{\sqrt{2}}$$

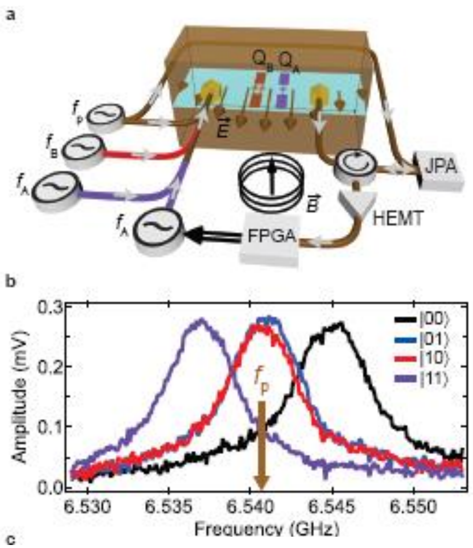
(probabilistically, even with Rabi oscillations)

Similar proposal in optics

J. Kerckhoff, L. Bouten, A. Silberfarb, and H. Mabuchi, 2009

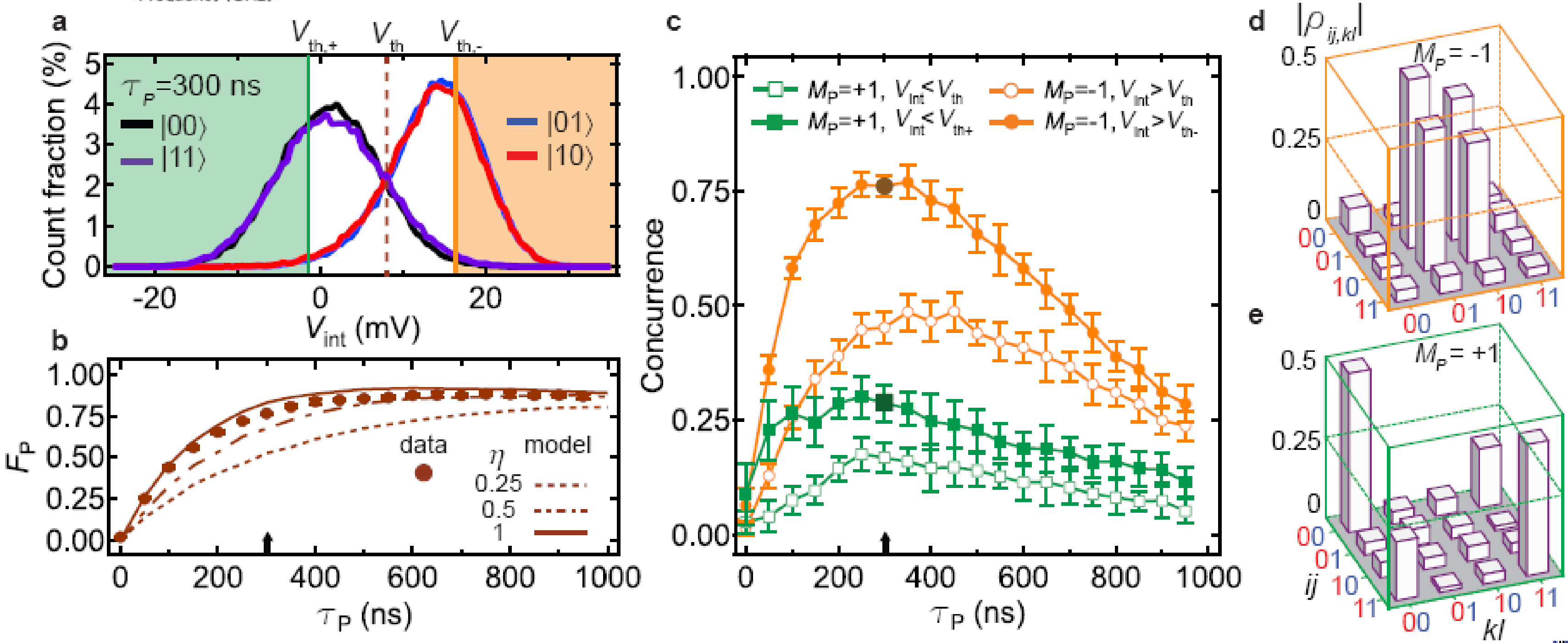


Entanglement by measurement (expt.)



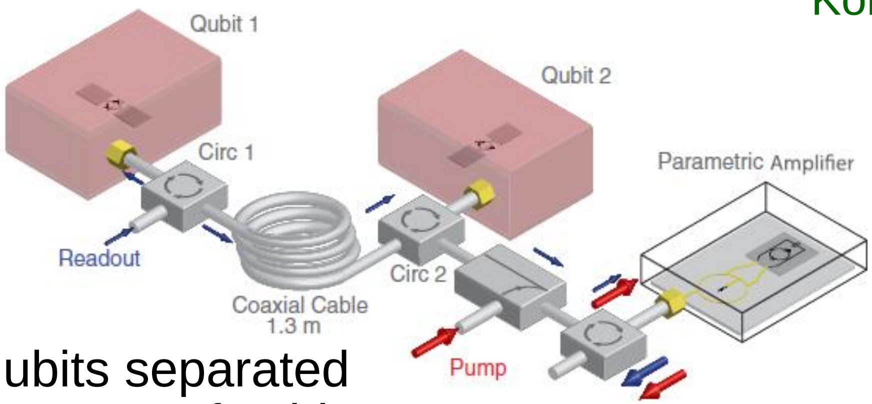
D. Riste, Dukalski, Watson, de Lange, Tiggelman, Blanter, Lehnert, Schouten, and L. DiCarlo, Nature-2013

- Two superconducting qubits in the same resonator, indistinguishable $|01\rangle$ and $|10\rangle$
- Max. concurrence 0.77
- Trick: $|00\rangle$ and $|11\rangle$ are only slightly distinguishable
- Max. deterministic concurrence 0.34
- Race against decoherence (η is not very important)

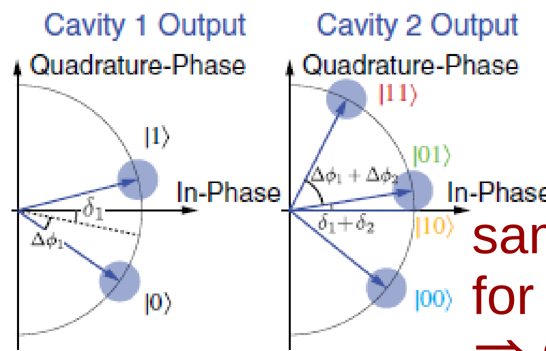


Measurement-induced entanglement of remote qubits

N. Roch, Schwartz, Motzoi, Macklin, Vijay, Eddins, Korotkov, Whaley, Sarovar, I. Siddiqi, PRL- 2014

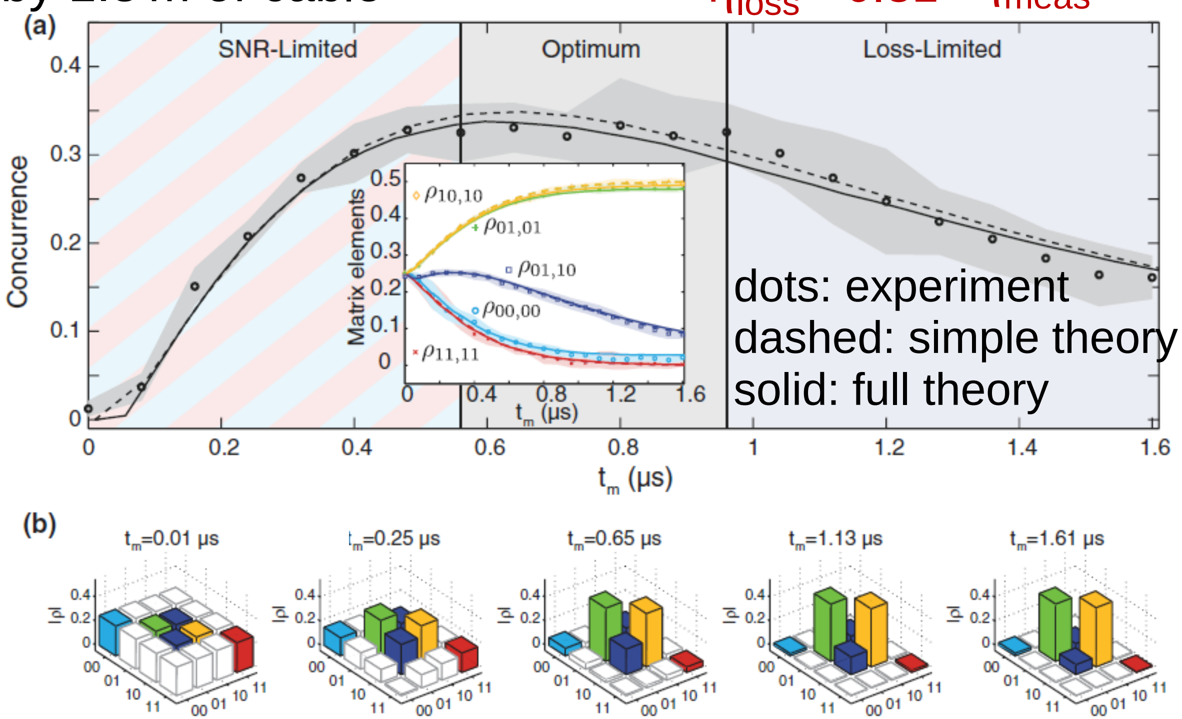


qubits separated by 1.3 m of cable



same output signal for $|01\rangle$ and $|10\rangle$
$\Rightarrow$ **entanglement**

$\eta_{loss} = 0.81$ $\eta_{meas} = 0.4$



dots: experiment
dashed: simple theory
solid: full theory

- First demonstration of remote entanglement of superconducting qubits
- Remote entanglement is much more difficult than local (η_{loss} is very important)
- Simple theory is close to full quantum trajectory, fits well experimental data.

Conclusions

- A very simple quantum Bayesian theory works well for continuous cQED measurement of superconducting qubits in the “bad cavity” regime (large κ)
- A better theory (for arbitrary κ) is still easy to understand
- Most of experimental proposals have been realized (incl. monitoring of trajectories, quantum feedback, and entanglement by measurement), though quantum efficiency is still low, $\eta \approx 0.5$
- Causality principle in quantum mechanics applies only to ensemble-averaged states (individual trajectories can be affected retroactively)