Microwave squeezing in transients

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Outline:

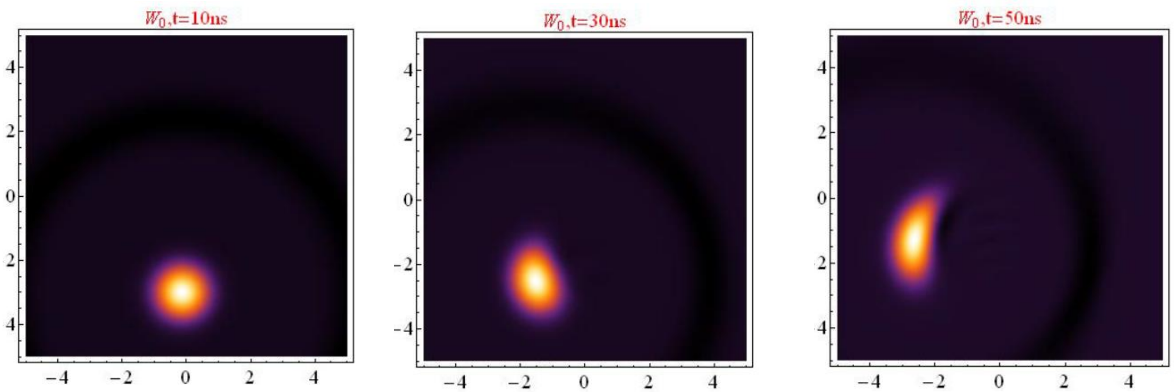
- Motivation
- How to think about intracavity and propagating (itinerant) squeezed microwave
- Squeezing by parametric drive: simplest case
- Semiclassical formalism for transients (linear case)
- Experimental proposal
- Generalization: nonlinear resonator, parametric and coherent drives

develop language (the second and third items)

Atalaya, Khezri, and Korotkov, arXiv:1804.08789

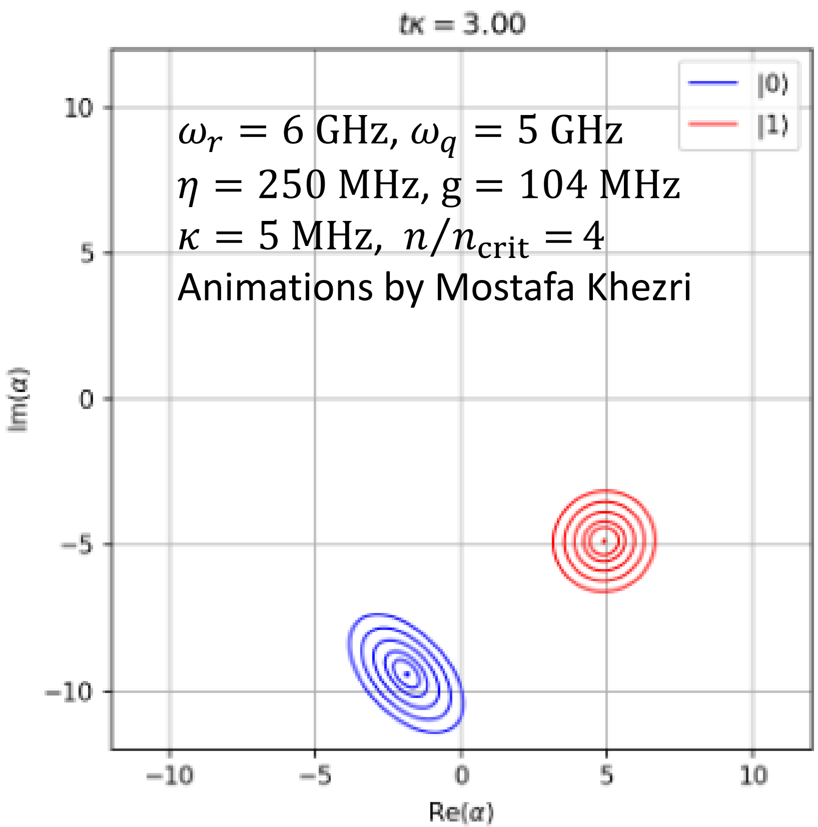
Feedback from experts in squeezing is appreciated

Motivation



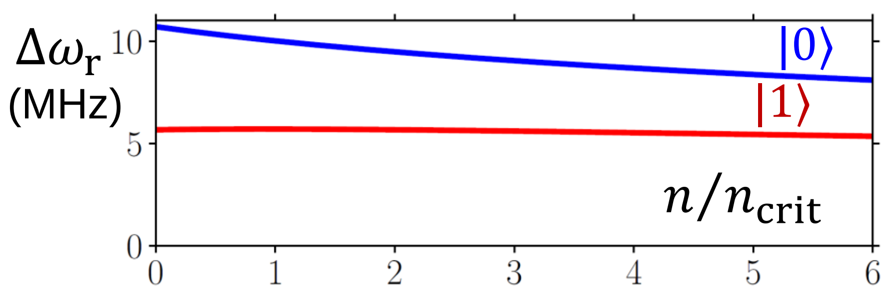
Self-generated squeezing in "Catch-Disperse-Release" qubit measurement

Sete, Galiautdinov, Mlinar, Martinis, and Korotkov, PRL-2013



Self-generated squeezing in usual cQED qubit measurement at $\bar{n} \gtrsim n_{\rm crit}$

Khezri and Korotkov, PRA-2017

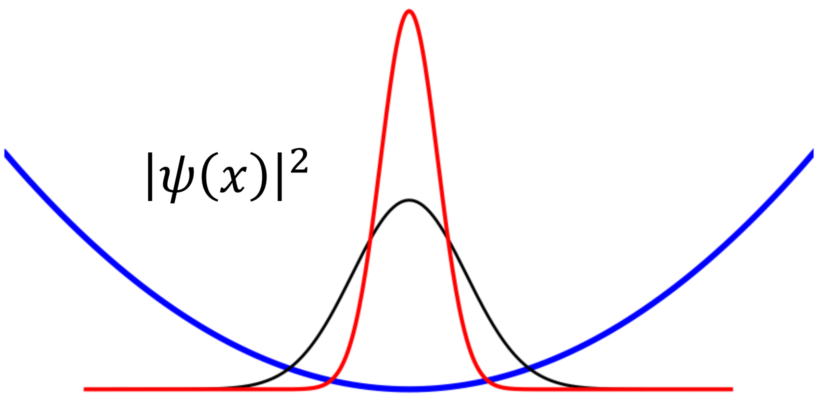


- How this affects fidelity of qubit readout? (**transients** are important)
- How to characterize propagating squeezed field in transients?

Use of squeezed microwaves in superconducting circuits

- R. Movshovich, B. Yurke, P. Kaminsky, et al., PRL 1990
- M. Castellanos-Beltran, … and K. Lehnert, Nature Phys. 2008
- N. Bergeal, … R. Schoelkopf, S. Girvin, and M. Devoret, Nature 2010
- N. Didier, A. Kamal, W. Oliver, A. Blais, and A. Clerk, PRL 2015
- A. Eddins, … A. Clerk, and I. Siddiqi, PRL 2018
- A. Bienfait, … D. Esteve, K. Mølmer, and P. Bertet, PRX 2017
- C. Eichler, … and A. Wallraff, PRL 2011
- E. Flurin, … M. Devoret, and B. Huard, PRL 2012
- D. Toyli, … W. Oliver, A. Blais, and I. Siddiqi, PRX 2016
- C. Wilson, … F. Nori, and P. Delsing, Nature 2011
- P. Lähteenmäki, G. Paraoanu, J. Hassel, and P. Hakonen, PNAS 2013
- F. Mallet, … and K. Lehnert, PRL 2011
- S. Kono, … and Nakamura, PRL 2017
- E. Wollman, … A. Clerk, and K. Schwab, Science 2015 (mechanical res.)
- J. Clark, Lecocq, Simmonds, Aumentado, J. Teufel, Nature Phys. 2016

Squeezing of intracavity field or mechanical resonator: undergraduate view



$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + \frac{m\omega^2 x^2}{2}\,\psi$$

Solve (Gaussian initial state)

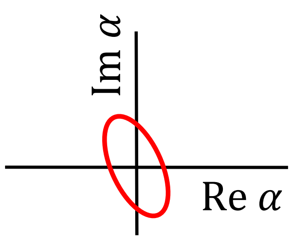
$$\psi = \frac{e^{-i\phi}}{\sqrt[4]{2\pi\sigma_{gr}^2 D}}\exp\left[-\frac{x^2(1+iB)}{4\sigma_{gr}^2\, D}\right]$$

$$D(t) = D_0 + \Delta D\cos(2\omega t + \theta)$$

$$B(t) = \Delta D\sin(2\omega t + \theta) \qquad \dot{\phi} = \omega/2D$$

$$D_{\max} D_{\min} = (D_0 + \Delta D)(D_0 - \Delta D) = 1$$

This is called “squeezed vacuum”

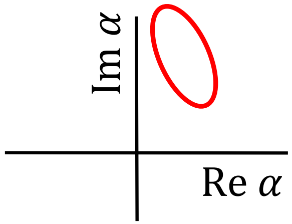


Variance oscillates with 2ω
(as for classical fluctuations)

Squeezing: undergraduate view (cont.)

Now "squeezed coherent state": just add oscillation of the center

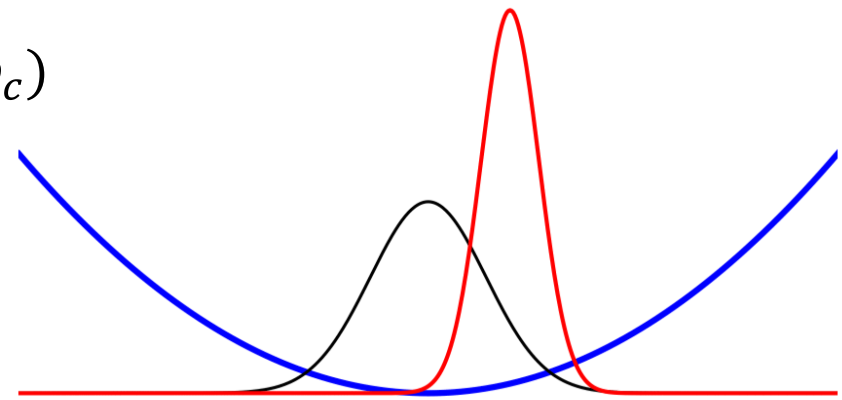
$$\psi = \frac{e^{-i\phi}}{\sqrt[4]{2\pi\sigma_{gr}^2 D}} \exp\left[-\frac{(x-x_c)^2(1+iB)}{4\sigma_{gr}^2\, D}\right] \exp(i\, p_c x/\hbar)$$



$$x_c(t) = X_{\rm amp}\cos(\omega t + \varphi_c)$$

$$p_c(t) = m\dot{x}_c = -m\omega X_{\rm amp}\sin(\omega t + \varphi_c)$$

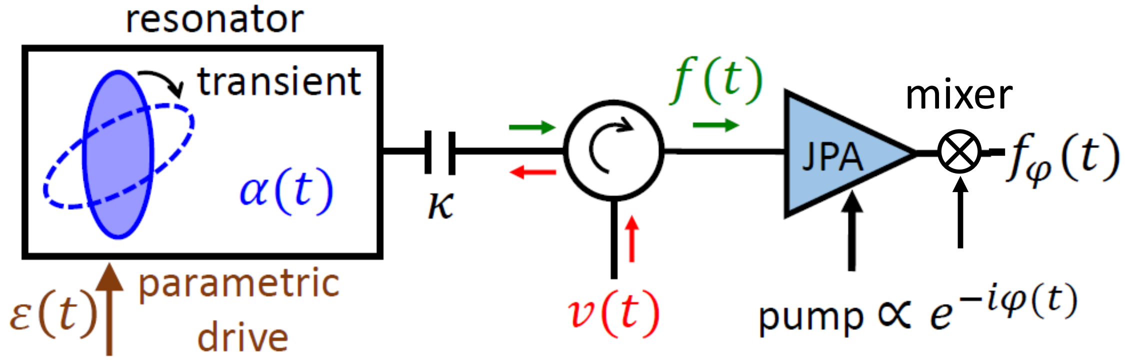
Center oscillates with ω,
variance oscillates with 2ω,
oscillation phases are not related



<u>General Gaussian state</u>: just construct density matrix $\rho(x,x')$ and Gaussian-average over the center; then $D_{\max}D_{\min} \geq 1$

Usual squeezing generation: parametric drive

(nonlinear resonator and coherent drive later)



$$H = \Omega\, a^\dagger a + \frac{i}{4}\,[\varepsilon^*(t)\; a^2 - \varepsilon(t)\; a^{\dagger 2}]$$

$$\Omega = \omega_r - \omega_d, \quad \varepsilon(t) = |\varepsilon(t)|\, e^{i\theta(t)}$$

Physically, resonator frequency modulation:

$$\omega_r(t) = \omega_r - |\varepsilon| \sin(2\omega_d t - \theta)$$

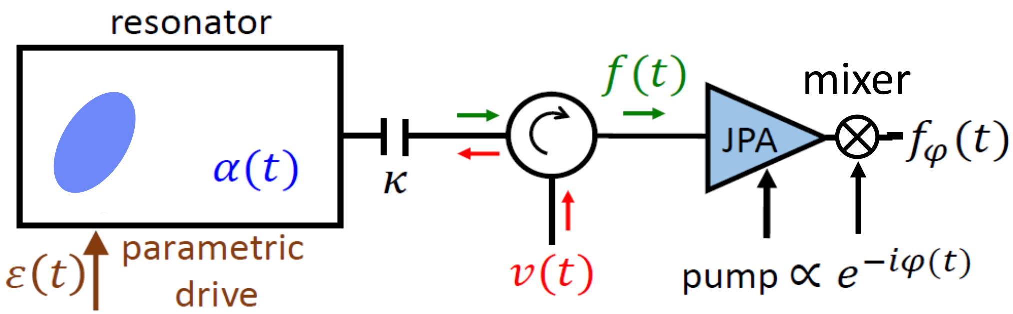
Easy to add "signal" via coherent drive (parametric amplifier)

Similar to child swing:
energy decay rate $\kappa + |\varepsilon|$ for quadrature phase $\theta/2$,
$\kappa - |\varepsilon|$ for phase $(\theta + \pi)/2$

Increased decay squeezes intracavity quadrature $\theta/2$

Do not consider instability: only "squeezed vacuum"

How to think about propagating squeezed field? Just anticorrelation of the amplifier noise!



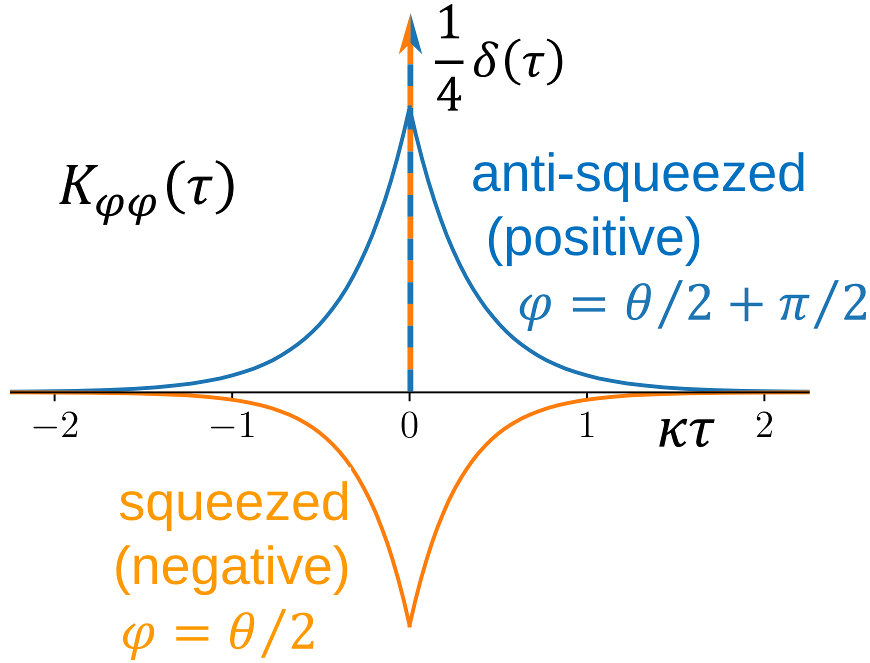
$$H = \Omega\, a^\dagger a + \frac{i}{4}\,[\varepsilon^* a^2 - \varepsilon\, a^{\dagger 2}]$$

Simplest case:

$$\Omega = 0,\ \ \varepsilon = \text{const}$$

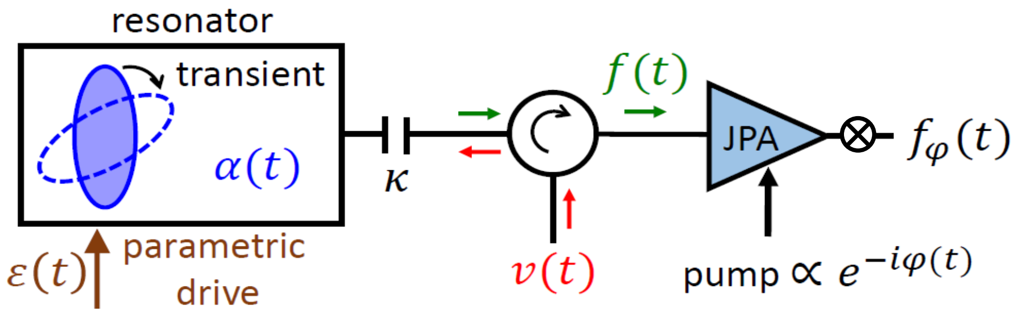
$f_\varphi(t)$ is measured φ-quadrature signal (after mixer)

$$K_{\varphi\varphi}(\tau) = \langle\, f_\varphi(t)\, f_\varphi(t+\tau)\,\rangle$$



- Always $\delta(\tau)/4$ noise at $\tau \approx 0$ (vacuum noise); more for an imperfect amplifier
- For quadrature $\varphi = \theta/2$, negative correlator for $\tau \neq 0$ (squeezing)
- For $\varphi = (\theta+\pi)/2$, positive correlator (anti-squeezing)

Why anticorrelation? Simple semiclassical theory



$$H = \Omega\, a^\dagger a + \frac{i}{4}\left[\varepsilon^* a^2 - \varepsilon\, a^{\dagger 2}\right]$$

$$\dot{\alpha} = -i\Omega\alpha - \frac{\kappa}{2}\alpha - \frac{\varepsilon}{2}\alpha^* + \sqrt{\kappa}\, v(t)$$

$$f = -v(t) + \sqrt{\kappa}\,\alpha$$

$$f_\varphi = \mathrm{Re}(\mathrm{e}^{-i\varphi} f)$$

energy decay $\kappa \pm |\varepsilon|$

transmission

reflection

$v(t)$ is classical complex noise

$$\langle v(t)\, v^*(t')\rangle = (\bar{n}_b + 1/2)\,\delta(t-t')$$

$$\langle v(t)\, v(t')\rangle = 0 \qquad \bar{n}_b = \frac{1}{\exp(\omega_r/T) - 1}$$

Alternatively, real noise $v_{qu}(t)$ for any quadrature

$$\langle v_{qu}(t)\, v_{qu}(t')\rangle = \frac{1+2\bar{n}_b}{4}\,\delta(t-t')$$

Similar to input-output theory

$$\alpha \to \hat{a},\;\; f \to \hat{f},\;\;\; v(t) \to \hat{v}(t)$$

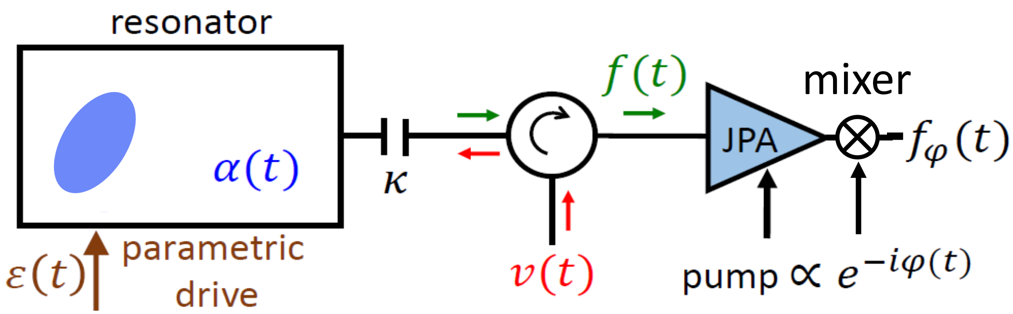
$$\left[\hat{v}(t), \hat{v}^\dagger(t')\right] = \delta(t-t')$$

$$\langle \hat{v}^\dagger(t')\, \hat{v}(t)\rangle = \bar{n}_b\, \delta(t-t')$$

Exact model for a linear resonator
(Wigner function can be interpreted as probability distribution, same eqs.)

Easy to generalize to nonlinear case

Simplest case: no detuning, steady state



$$H = \frac{i}{4}\,[\varepsilon^* a^2 - \varepsilon\, a^{\dagger 2}]$$

quadrature $\varphi = \theta/2$ (everything is real)

$$\dot{\alpha} = -\frac{\kappa + |\varepsilon|}{2}\alpha + \sqrt{\kappa}\, v_{qu}(t)$$

$$f_\varphi = -v_{qu}(t) + \sqrt{\kappa}\,\alpha$$

$$\langle v_{qu}(t)\, v_{qu}(t')\rangle = \frac{1}{4}\,\delta(t-t')$$

$$\alpha(t) = \sqrt{\kappa}\int_{-\infty}^{t} v_{qu}(t')\, e^{-(\kappa+|\varepsilon|)(t-t')/2}\, dt'$$

$$\langle \alpha(0)\,\alpha(\tau)\rangle = \frac{\kappa}{4(\kappa+|\varepsilon|)}\, e^{-(\kappa+|\varepsilon|)\tau/2}$$

$$\langle v_{qu}(0)\,\alpha(\tau)\rangle = \frac{\sqrt{\kappa}}{4}\, e^{-(\kappa+|\varepsilon|)\tau/2}$$

$$\langle f_\varphi(0)\, f_\varphi(\tau)\rangle = -\frac{\kappa\,|\varepsilon|}{4(\kappa+|\varepsilon|)} e^{-(\kappa+|\varepsilon|)\tau/2}$$

$$\langle f_\varphi(0)\, f_\varphi(\tau)\rangle = \kappa\,\langle \alpha(0)\,\alpha(\tau)\rangle - \sqrt{\kappa}\,\langle v_{qu}(0)\,\alpha(\tau)\rangle$$

$\tau > 0$

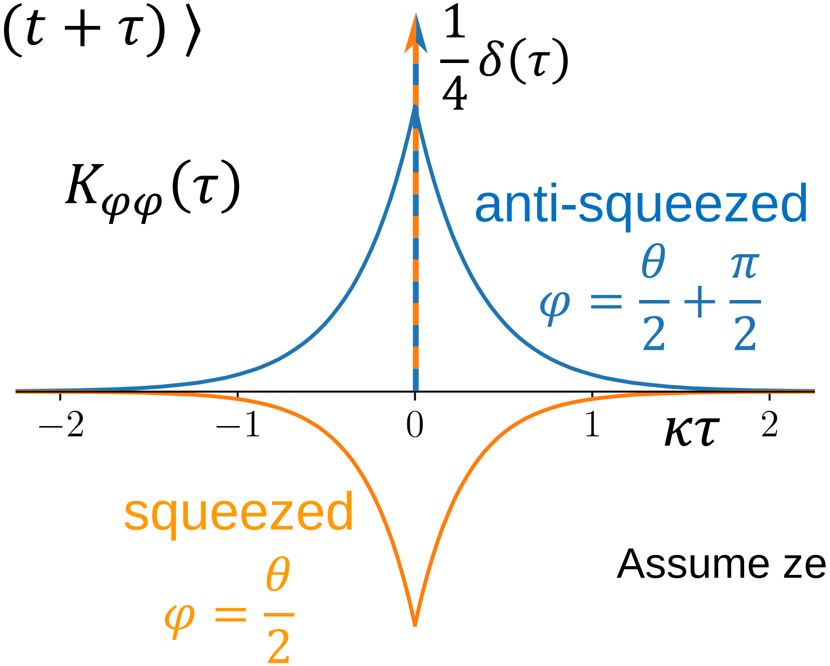
anticorrelation

Anticorrelation (squeezing)

Similar for orthogonal (unsqueezed) quadrature, just $|\varepsilon| \to -|\varepsilon|$

Simplest case: no detuning, steady state (cont.)

$$K_{\varphi\varphi}(\tau) = \langle\, f_\varphi(t)\, f_\varphi(t+\tau)\,\rangle$$



Assume zero temperature

$$\kappa_\pm = \kappa \pm |\varepsilon|$$

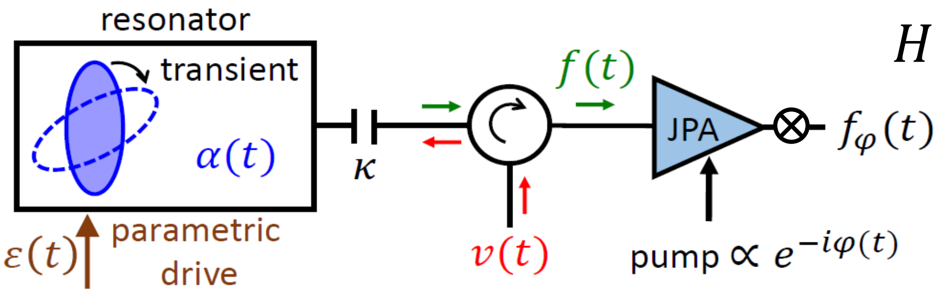
$$K_{\varphi\varphi}(\tau) = \frac{\delta(\tau)}{4} - \frac{\kappa\,|\varepsilon|}{4\kappa_+}\, e^{-\kappa_+|\tau|/2}\cos^2(\varphi-\theta/2) + \frac{\kappa\,|\varepsilon|}{4\kappa_-}\, e^{-\kappa_-|\tau|/2}\sin^2(\varphi-\theta/2)$$

Easy to include detuning (longer formulas)

Fourier transform of this correlator is called “squeezing spectrum”

In transients, Fourier transform of $K_{\varphi\varphi}(t_1, t_2)$ does not make much sense

General case (transients)



$$H = \Omega(t)\, a^\dagger a + \frac{i}{4}\,[\varepsilon^*(t)\; a^2 - \varepsilon(t)\; a^{\dagger 2}]$$

Amplified/measured quadrature may change in time

$$K_{\varphi_1\varphi_2}(t_1,t_2) = \langle\, f_{\varphi_1}(t_1)\; f_{\varphi_2}(t_2)\,\rangle$$

This is what we need for noise of integrated weighted signal

The same simple semiclassical theory

$$\dot\alpha = -i\Omega\alpha - \frac{\kappa}{2}\alpha - \frac{\varepsilon}{2}\alpha^* + \sqrt{\kappa}\, v$$

$$f = -v + \sqrt{\kappa}\,\alpha$$

$$f_\varphi = \mathrm{Re}(\mathrm{e}^{-i\varphi} f)$$

$$K_{ff}(t_1,t_2) = \langle f(t_1)\, f(t_2)\rangle$$

$$K_{ff^*}(t_1,t_2) = \langle f(t_1)\, f^*(t_2)\rangle$$

Dependence on quadratures: 4 real parameters

$$K_{\varphi_1\varphi_2}(t_1,t_2) = \mathrm{Re}[K_{ff}(t_1,t_2)\; e^{-i(\varphi_1+\varphi_2)} + K_{ff^*}(t_1,t_2)\; e^{-i(\varphi_1-\varphi_2)}]/2$$

(only 3 parameters in a steady state, as for ellipse in phase space)

Main result for squeezing in transients

$$H = \Omega(t)\, a^\dagger a + \frac{i}{4}\left[\varepsilon^*(t)\; a^2 - \varepsilon(t)\; a^{\dagger 2}\right]$$

After some algebra we obtain

$$\frac{d}{dt_2}\begin{pmatrix} K_{ff}(t_1,t_2) \\ K_{ff^*}(t_1,t_2) \end{pmatrix} = \begin{pmatrix} -i\Omega - \kappa/2 & -\varepsilon\,(t_2)/2 \\ -\varepsilon^*(t_2)/2 & i\Omega - \kappa/2 \end{pmatrix}\begin{pmatrix} K_{ff}(t_1,t_2) \\ K_{ff^*}(t_1,t_2) \end{pmatrix}$$

Initial condition: from intracavity squeezing

$$\begin{pmatrix} K_{ff}(t_1,t_1) \\ K_{ff^*}(t_1,t_1) \end{pmatrix} = \kappa \begin{pmatrix} \langle \alpha^2(t_1)\rangle \\ \langle |\alpha(t_1)|^2\rangle - (1/2 + \bar{n}_b) \end{pmatrix}$$

Evolution of intracavity squeezing (equiv. to Khezri and Korotkov, 2017):

$$\frac{d}{dt}\langle \alpha^2\rangle = (-2i\Omega\; - \kappa)\,\langle \alpha^2\rangle - \varepsilon\langle |\alpha|^2\rangle$$

$$\frac{d}{dt}\langle |\alpha|^2\rangle = -\kappa\langle |\alpha|^2\rangle - \mathrm{Re}(\varepsilon^*\langle \alpha^2\rangle) + \kappa(\bar{n}_b + 1/2)$$

Easy-to-solve (at least numerically) differential equations

Alternative approaches

$$H = \Omega(t)\, a^\dagger a + \frac{i}{4}\, [\varepsilon^*(t)\; a^2 - \varepsilon(t)\; a^{\dagger 2}]$$

1. Input-output formalism

The same result

Actually, we did not check, just proved exact equivalence

The proof shows that intracavity state remains Gaussian
(if vacuum in a distant past; would not work for a "cat" state)

2. Quantum Bayesian formalism

The same result

Very different approach: no vacuum noise, small random "kicks" of intracavity Gaussian state due to information from continuous (noisy) measurement

Special case: steady state

$$H = \Omega\, a^\dagger a + \frac{i}{4}\, [\varepsilon^*\ a^2 - \varepsilon\, a^{\dagger 2}]$$

$$\frac{K_{ff}(0,\tau)}{1+2\bar{n}_b} = -\frac{\kappa\varepsilon}{4}\left[\left(1-\frac{2i\Omega}{\epsilon}\right)\frac{e^{-\kappa_-|\tau|/2}}{\kappa_-} + \left(1+\frac{2i\Omega}{\epsilon}\right)\frac{e^{-\kappa_+|\tau|/2}}{\kappa_+}\right]$$

$$\frac{K_{ff^*}(0,\tau)}{1+2\bar{n}_b} = \frac{\delta(\tau)}{2} + \frac{\kappa|\varepsilon|^2}{4\epsilon}\left(\frac{e^{-\kappa_-|\tau|/2}}{\kappa_-} - \frac{e^{-\kappa_+|\tau|/2}}{\kappa_+}\right)$$

$\kappa_\pm = \kappa \pm \epsilon,\qquad \epsilon = \sqrt{|\varepsilon|^2 - 4\Omega^2}$ if $|\Omega| < |\varepsilon|/2$ (overdamped case)

$\epsilon = i\,\sqrt{4\Omega^2 - |\varepsilon|^2}$ if $|\Omega| > |\varepsilon|/2$ (underdamped)

To remind,

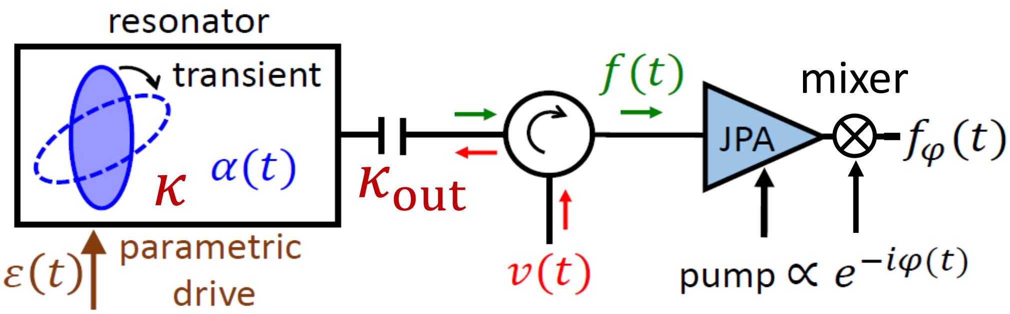
$$K_{\varphi_1\varphi_2}(0,\tau) = \frac{1}{2}\mathrm{Re}[K_{ff}(0,\tau\,)\ e^{-i(\varphi_1+\varphi_2)} + K_{ff^*}(0,\tau\,)\ e^{-i(\varphi_1-\varphi_2)}]$$

We see that in steady state, $K_{ff^*}(0,\tau)$ is always real (3 parameters instead of 4)

Therefore, same-quadrature description (ellipse) $K_{\varphi\varphi}(0,\tau)$ is sufficient,
in contrast to the transient regime.

The same relation (3 vs. 4 parameters) for integrated signal or Fourier transform.

Simple generalizations



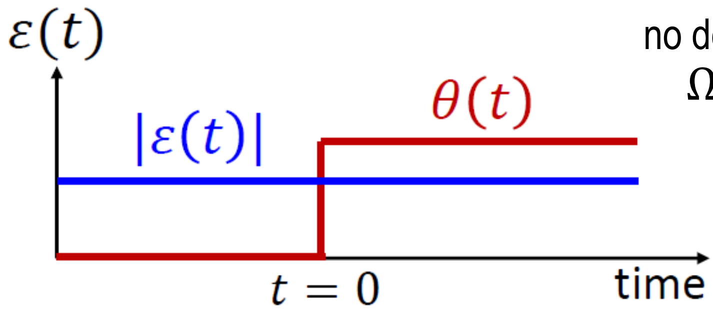
1. $\kappa_{out} < \kappa$ (extra decay in the resonator)

Then correlators $K_{\varphi_1\varphi_2}(t_1,t_2)$ are multiplied by κ_{out}/κ, except singularity at $t_1 \approx t_2$, which stays the same

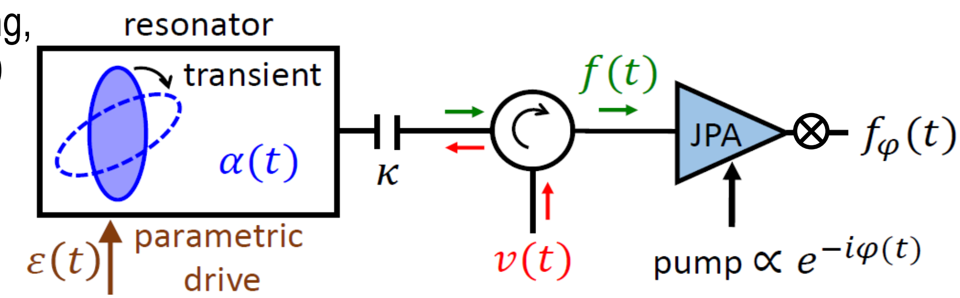
2. Phase-preserving amplifier (instead of phase-sensitive ampl.)

Then correlators $K_{\varphi_1\varphi_2}(t_1,t_2)$ do not change at $t_1 \neq t_2$, while singularity $\delta(t_1-t_2)/4$ is doubled, $\rightarrow \delta(t_1-t_2)/2$ (even more, $\delta(t_1-t_2)/4\eta$ for inefficient amplifier)

Example (experimental proposal)



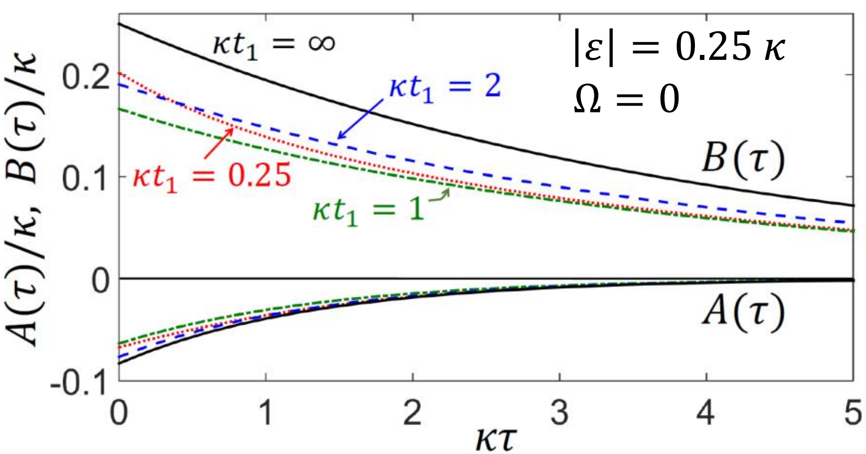
no detuning, $\Omega = 0$

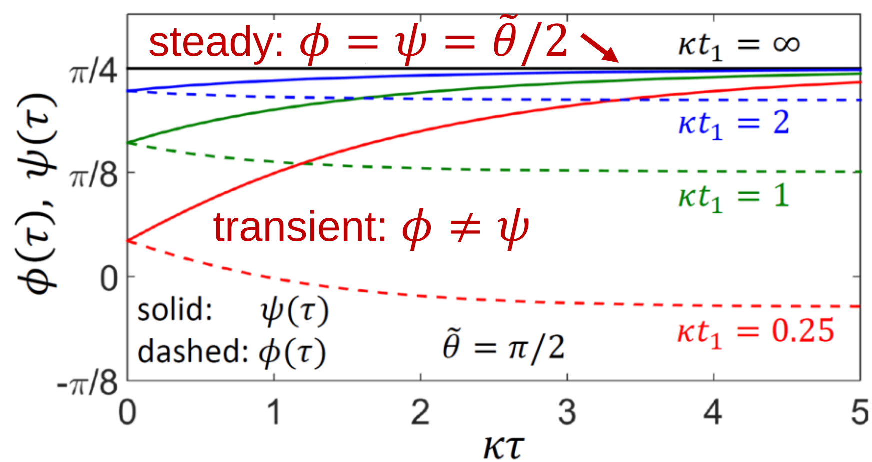


OK to use phase-preserving amplifier

Another form for output-signal correlator:

$$\langle f_{\varphi_1}(t_1)\, f_{\varphi_2}(t_2)\rangle = A\cos(\varphi_1-\phi)\cos(\varphi_2-\psi) + B\sin(\varphi_1-\phi)\sin(\varphi_2-\psi)$$





In the steady state $\phi = \psi$, only 3 real parameters (instead of 4)

Generalization: nonlinear resonator, parametric and coherent drives

$$H = \sum_n E(n)|n\rangle\langle n| + \frac{i}{4}\left[\varepsilon^*(t)\; a^2 - \varepsilon(t)\; a^{\dagger 2}\right] + \varepsilon_c^*(t)\, a + \varepsilon_c(t)\, a^\dagger$$

$$E(n) = \sum_{k=0}^{n-1} \left[\omega_r(n) - \omega_{\rm rf}\right]$$

Idea: separate "center" and fluctuations, **linearize fluctuations** near the center

$$\alpha(t) = \alpha_c(t) + \delta\alpha(t) \qquad f(t) = f_c(t) + \delta f(t)$$

$$\dot{\alpha}_c = -i[\omega_r(|\alpha_c|^2) - \omega_{\rm rf}]\,\alpha_c - \frac{\kappa}{2}\alpha_c - \frac{\varepsilon}{2}\,\alpha_c^* - i\varepsilon_c$$

For fluctuations, the same equations as in the linear case with substitutions:

$$\Omega \;\to\; \omega_r(|\alpha_c|^2) \;-\omega_{\rm rf} + \frac{d\omega_r(n)}{dn}\bigg|_{n=|\alpha_c|^2} |\alpha_c|^2$$

$$\varepsilon \;\to\; \varepsilon \;+ 2i\,\frac{d\omega_r(n)}{dn}\bigg|_{n=|\alpha_c|^2} \alpha_c^2$$

This is what is needed for cQED qubit readout and parametric amplifier

Now the semiclassical theory is not exact, needs Gaussian approximation (linearization) (not clear how to write quantum theory with arbitrary nonlinearity)

Conclusions

- Proper way to characterize a propagating squeezed microwave is via two-time correlators of measured quadrature signals with changing quadrature phases
- Simple semiclassical formalism to calculate correlators in transients (exact for linear resonator, Gaussian approximation for a nonlinear resonator)
- Important for qubit measurement, simple way to analyze parametric amplifiers
- 4 parameters in transients, instead of 3 parameters (traditional ellipse) in the steady state
- Simple to check experimentally

Atalaya, Khezri, and Korotkov, arXiv:1804.08789