*IWSSQC, Hangzhou, China, 05/26/18*

# Continuous measurement of qubits (and possible applications)

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Outline:

- Quantum Bayesian theory for continuous measurement of a qubit
- Roles of two quadratures in circuit QED setup
- Short review of first experiments
- Correlators in simultaneous measurement of non-commuting observables of a qubit
- Bacon-Shor code operating with continuous meas.
- Arrow of time in continuous measurement of a qubit

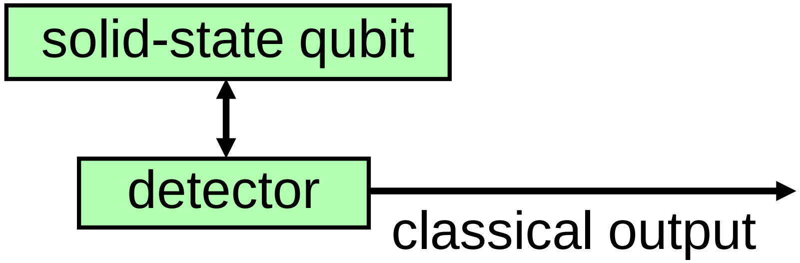
# What is “inside” collapse?
# What if collapse is stopped half-way?

Various approaches to non-projective (weak, continuous, partial, generalized, etc.) quantum measurements

**Names:** Davies, Kraus, Holevo, Mensky, Caves, Knight, Walls, Carmichael, Milburn, Wiseman, Aharonov, Molmer, Gisin, Percival, Belavkin, … (very incomplete list)

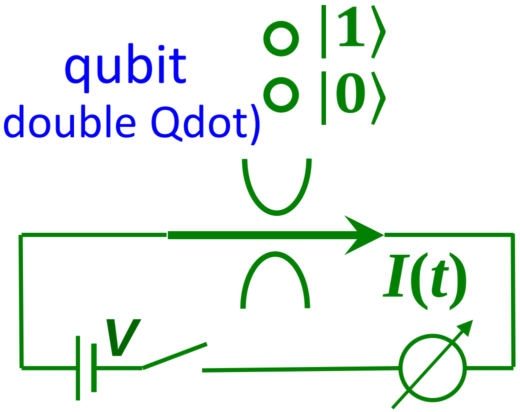
**Key words**: POVM, restricted path integral, quantum trajectories, quantum filtering, quantum jumps, stochastic master equation, etc.

We consider:



Quantum Bayesian approach

# Quantum Bayesian formalism for qubit meas.



qubit
(double Qdot)

detector
(quantum point contact)

Qubit evolution due to measurement
(<u>informational</u> back-action)

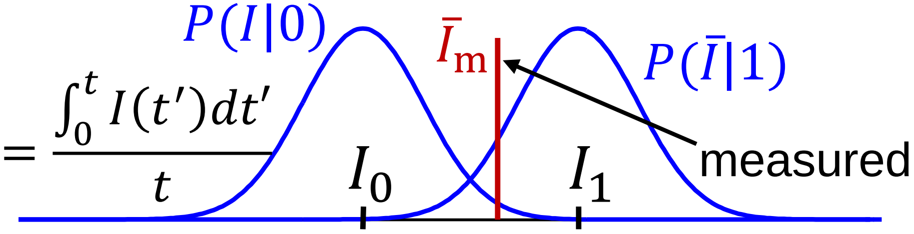
$|\psi(t)\rangle = \alpha(t)\,|0\rangle + \beta(t)\,|1\rangle$ or $\rho_{ij}(t)$

1) $|\alpha(t)|^2$ and $|\beta(t)|^2$ evolve as probabilities,
i.e. according to the Bayes rule (same for $\rho_{ii}$)

2) phases of $\alpha(t)$ and $\beta(t)$ do not change
(no dephasing!), $\rho_{ij}/\sqrt{\rho_{ii}\rho_{jj}} = \text{const}$

(A.K., 1998)

$$\bar{I}_{\rm m} = \frac{\int_0^t I(t')dt'}{t}$$



$$P(\bar{I}) = \rho_{00}(0)\,P(\bar{I}|0) + \rho_{11}(0)\,P(\bar{I}|1)$$

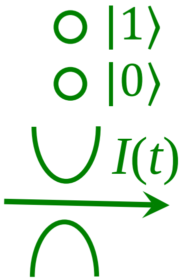
Bayes rule (1763, Laplace-1812):

$$\overbrace{P(A_i|\text{res})}^{\text{posterior probability}} = \frac{\overbrace{P(A_i)}^{\text{prior probab.}}\ \overbrace{P(\text{res}|A_i)}^{\text{likelihood}}}{\text{norm}}$$

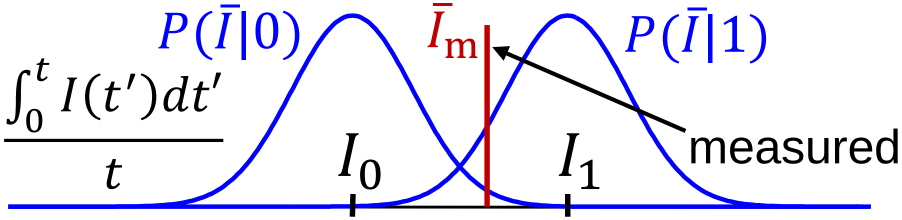
So simple because:

1) no entanglement at large QPC voltage
2) QPC is ideal detector
3) no other evolution of qubit

# Further steps in quantum Bayesian formalism



$$\bar{I}_\mathrm{m} = \frac{\int_0^t I(t')dt'}{t}$$



$$\alpha(t)\,|0\rangle + \beta(t)\,|1\rangle$$

$$\rho_{ij}(t)$$

1. Informational (“spooky”, quantum) back-action, $\times\sqrt{\text{likelihood}}$

$$|\psi(t)\rangle = \frac{\sqrt{P(\bar{I}_\mathrm{m}|0)}\,\alpha(0)\,|0\rangle + \sqrt{P(\bar{I}_\mathrm{m}|1)}\,\beta(0)\,|1\rangle}{\text{norm}}$$

2. Add unitary (phase) back-action, physical mechanisms for QPC and cQED

$$|\psi(t)\rangle = \frac{\sqrt{P(\bar{I}_\mathrm{m}|0)}\exp\left[iK\left(\bar{I}_\mathrm{m} - \frac{I_0+I_1}{2}\right)\right]\alpha(0)\,|0\rangle + \sqrt{P(\bar{I}_\mathrm{m}|1)}\,\beta(0)\,|1\rangle}{\text{norm}}$$

3. Add detector non-ideality (equivalent to dephasing) $\gamma = \Gamma - \frac{(\Delta I)^2}{4S_I} - \frac{K^2 S_I}{4}$

$$\rho_{ii}(t) = \frac{P(\bar{I}_\mathrm{m}|i)\,\rho_{ii}(0)}{\text{norm}}, \quad \frac{\rho_{01}(t)}{\sqrt{\rho_{00}(t)\,\rho_{11}(t)}} = \frac{e^{iK(\bar{I}_\mathrm{m} - \frac{I_0+I_1}{2})}\rho_{01}(0)}{\sqrt{\rho_{00}(0)\,\rho_{11}(0)}}\exp(-\gamma t)$$

# Further steps in quantum Bayesian formalism

4. Take derivative over time (if differential equation is desired)

Simple, but be careful about definition of derivative

$$\frac{df(t)}{dt} = \frac{f(t+dt/2) - f(t-dt/2)}{dt}$$

Stratonovich form
preserves usual calculus

$$\frac{df(t)}{dt} = \frac{f(t+dt) - f(t)}{dt}$$

Ito form

requires special calculus,
but keeps averages

5. Add Hamiltonian evolution (if any) and additional decoherence (if any)

Standard terms

Steps 1–5 form the quantum Bayesian approach to qubit measurement

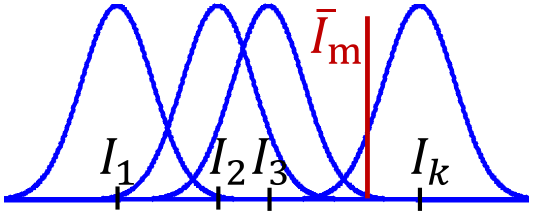
(A.K., 1998—2001)

# Generalization: measurement of operator *A*

"Informational" quantum Bayesian in differential (Ito) form:

$$\dot{\rho} = \frac{A\rho A - (A^2\rho + \rho A^2)/2}{2\eta S} + \frac{A\rho + \rho A - 2\rho \mathrm{Tr}\,(A\rho)}{\sqrt{2S}}\ \xi(t)$$

$I(t) = \mathrm{Tr}(A\rho) + \sqrt{S/2}\ \xi(t)$ noisy detector output



$S$: spectral density of the output noise

$\langle \xi(t)\, \xi(t')\rangle = \delta(t-t')$ normalized white noise

$\eta$: quantum efficiency

With additional unitary (Hamiltonian) back-action $B$ and additional evolution

$$\dot{\rho} = \mathcal{L}[\rho] + \frac{A\rho + \rho A - 2\rho \mathrm{Tr}\,(A\rho)}{\sqrt{2S}}\ \xi(t)\ - i[B,\rho]\ \frac{1}{\sqrt{2S}}\ \xi(t)$$

$\mathcal{L}[\rho]$: ensemble-averaged (Lindblad) evolution

The same as in the Quantum Trajectory theory (Wiseman, Milburn, …)

Nowadays "quantum trajectories" often mean Bayesian real-time monitoring

# Quantum trajectory theory

H. J. Carmichael, 1993
H. M. Wiseman and G. J. Milburn, 1993

optics

H.-S. Goan and G. J. Milburn, 2001
H.-S. Goan, G. J. Milburn, H. M. Wiseman, and H. B. Sun, 2001

solid-state, quantum point contact

J. Gambetta, A. Blais, M. Boissonneault, A. A. Houck, D. I. Schuster, and S. M. Girvin, 2008

circuit QED

Relation between Quantum Trajectory and Quantum Bayesian formalisms

Essentially the same thing, but look different

Quantum trajectory theory uses mathematical language (superoperators), quantum Bayesian theory uses simple physical approach (undergraduate-level)

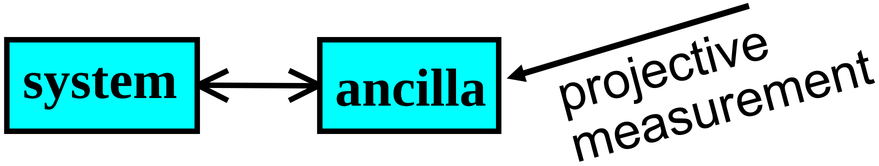
Computationally, Bayesian theory is usually better (more than first-order)

Another meaning of “quantum trajectories“: real-time monitoring of evolution (often done by quantum Bayesian theory)

# Quantum measurement in POVM formalism

Davies, Kraus, Holevo, etc.

(Nielsen-Chuang, pp. 85, 100)



Measurement (Kraus) operator $M_r$ (any linear operator in H.S.) : $\psi \to \dfrac{M_r\psi}{||M_r\psi||}$ or $\rho \to \dfrac{M_r\rho M_r^\dagger}{\mathrm{Tr}(M_r^\dagger M_r\rho)}$

Probability : $P_r = ||M_r\psi||^2$ or $P_r = \mathrm{Tr}(M_r^\dagger M_r\rho)$

Completeness : $\sum_r M_r^\dagger M_r = 1$

(People often prefer linear evolution and non-normalized states)

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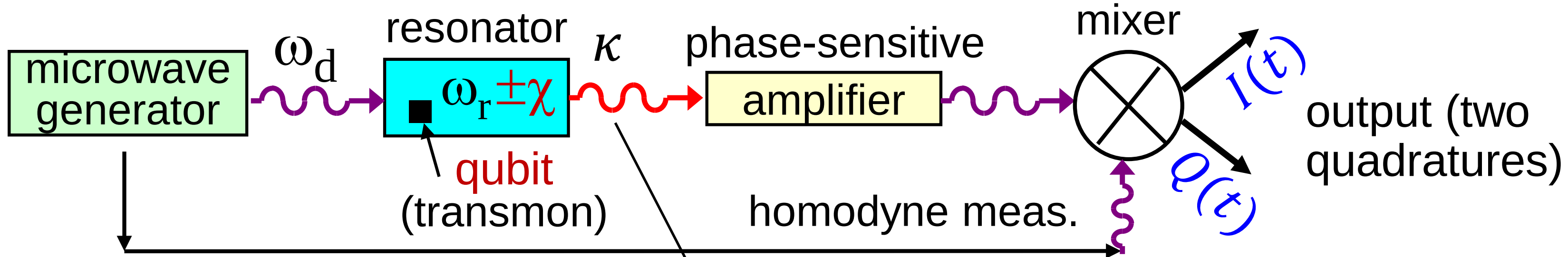
## Relation between POVM and quantum Bayesian formalism

polar decomposition: $M_r = \underset{\nearrow \ \text{unitary}}{U_r}\underbrace{\sqrt{M_r^\dagger M_r}}_{\text{Bayes}}$ (steps 1 and 2 above)

# Quantum Bayesian theory for circuit QED setup

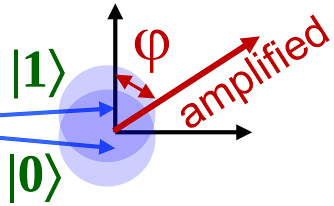


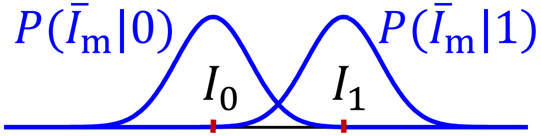
A. Blais et al., PRA 2004
A. Wallraff et al., Nature 2004
J. Gambetta et al., PRA 2008

Two quadratures:

1) information on qubit state
⇒ informational back-action

2) information on fluct. photon number
⇒ unitary (phase) back-action





$$\begin{cases} \dfrac{\rho_{11}(\tau)}{\rho_{00}(\tau)} = \boxed{\dfrac{\rho_{11}(0)\exp[-(\bar{I}_m - I_1)^2/2D]}{\rho_{00}(0)\exp[-(\bar{I}_m - I_0)^2/2D]}} \\ \rho_{01}(\tau) = \boxed{\rho_{01}(0)\sqrt{\dfrac{\rho_{00}(\tau)\rho_{11}(\tau)}{\rho_{00}(0)\rho_{11}(0)}}}\exp(iK\bar{I}_m\tau) \end{cases}$$

Bayes unitary

$$P(\bar{I}_m) = \rho_{00}(0)\,P(\bar{I}_m|0) + \rho_{11}(0)\,P(\bar{I}_m|1)$$

$$\bar{I}_m = \tau^{-1}\int_0^\tau I(t)dt \qquad D = S_I/2\tau$$

$$I_0 - I_1 = \Delta I\cos\varphi \qquad K = \Delta I\sin\varphi/S_I$$

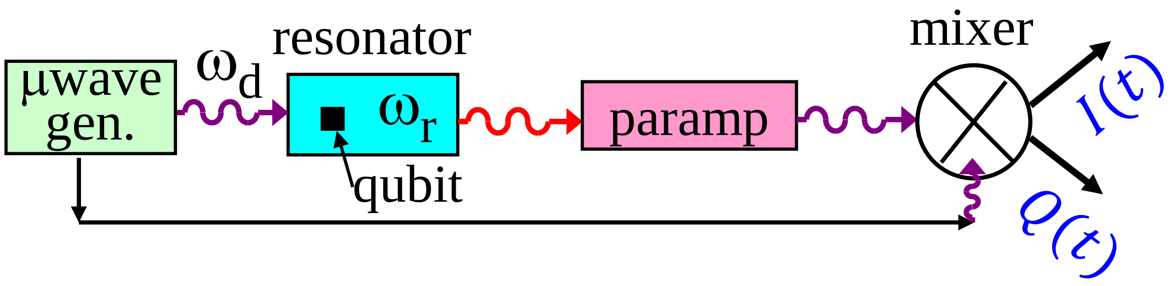
$$\Gamma = \frac{(\Delta I\cos\varphi)^2}{4S_I} + K^2\frac{S_I}{4} = \frac{\Delta I^2}{4S_I} = \frac{8\chi^2\bar{n}}{\kappa}$$

Amplified phase φ controls trade-off between informational and phase back-actions (we choose if photon number fluctuates or not)

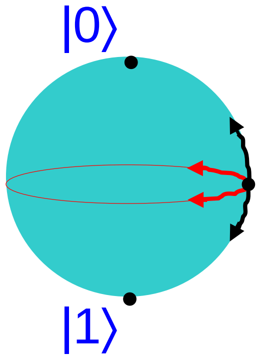
A.K., arXiv:1111.4016

# Causality in quantum mechanics

Ensemble-averaged evolution
cannot be affected back in time
(single realization can be affected)



We can choose direction of qubit evolution
to be either along parallel or along meridian
or in between (delayed choice)

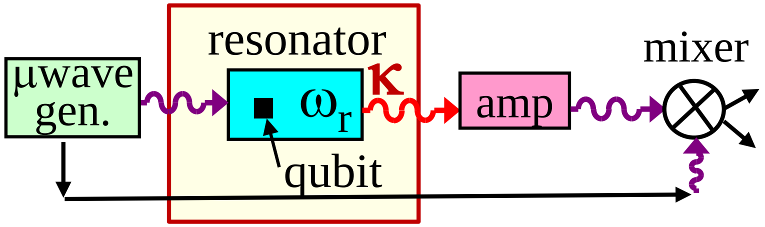


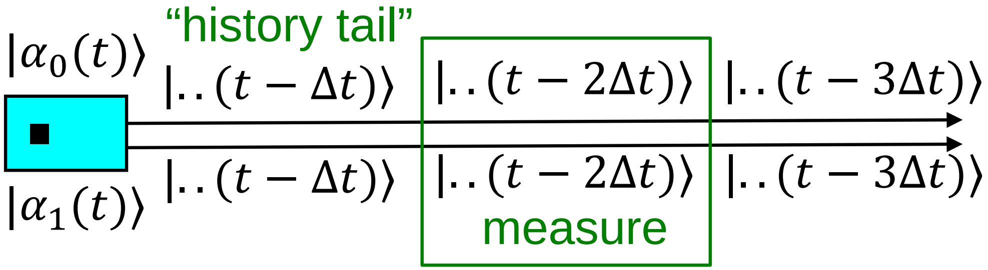
A.K., arXiv:1111.4016

Expt. confirmation: K. Murch et al., Nature 2013

# Beyond the "bad-cavity" limit

A.K., PRA 2016





The same quantum Bayesian approach, now applied to entangled qubit-resonator system (arbitrary $\kappa$, classical equations for $\alpha_j(t)$)

$$\hat{\rho}(t) = \sum_{j,k=0,1} \rho_{jk}(t)\, |j\rangle\langle k| \otimes |\alpha_j(t)\rangle\langle\alpha_k(t)|$$

$$\frac{\rho_{11}(t+\Delta t)}{\rho_{00}(t+\Delta t)} = \frac{\rho_{11}(t)}{\rho_{00}(t)} \exp(I_m \cos\phi_d\, \Delta I_{\max}/D)$$

$\Delta I_{\rm max}$: max response

$D$: noise variance

$\phi_{\rm d}$: angle from optimal quadrature

$$\frac{\rho_{10}(t+\Delta t)}{\rho_{10}(t)} = \frac{\sqrt{\rho_{11}(t+\Delta t)\rho_{00}(t+\Delta t)}}{\sqrt{\rho_{11}(t)\rho_{00}(t)}} \exp(-\gamma\Delta t)$$
$$\times \exp(-i\delta\omega_{\rm ac\,Stark}\Delta t)\exp(-iI_m \sin\phi_d\, \Delta I_{\max}/2D)$$

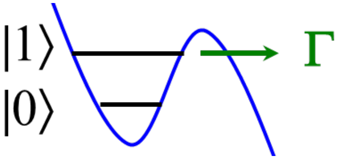
$$\Gamma = (\kappa/2)\,|\alpha_1-\alpha_0|^2$$
$$\gamma = \Gamma - \Delta I_{\max}^2/8D\Delta t$$
$$\eta = (\Gamma-\gamma)/\Gamma$$

$$\delta\omega_{\rm ac\,Stark} = \kappa\,{\rm Im}(\alpha_1^*\alpha_0) + {\rm Re}[\varepsilon^*(\alpha_1-\alpha_0)] = 2\chi{\rm Re}(\alpha_1^*\alpha_0) - \frac{d}{dt}{\rm Im}(\alpha_1^*\alpha_0)$$

Equivalent to "polaron" approach in quantum trajectories, but undergraduate-level derivation and possibly faster computationally

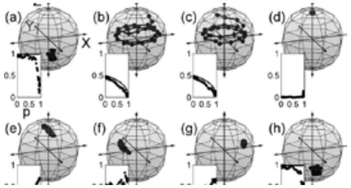
# First experiments (superconducting qubits)

**1.** N. Katz, M. Ansmann, R. Bialczak, E. Lucero, R. McDermott, M. Neeley, M. Steffen, E. Weig, A. Cleland, J. Martinis, and A. Korotkov, Science 2006



Partial collapse of phase qubit: the state remains pure, but evolves in accordance with acquired information

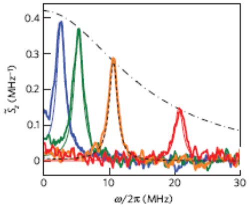
**2.** N. Katz, M. Neeley, M. Ansmann, R. Bialzak, E. Lucero, A. O’Connell, H. Wang, A. Cleland, J. Martinis, and A. Korotkov, PRL 2008



Uncollapse: qubit state is restored if classical information is erased (two POVMs cancel each other). Phase qubit

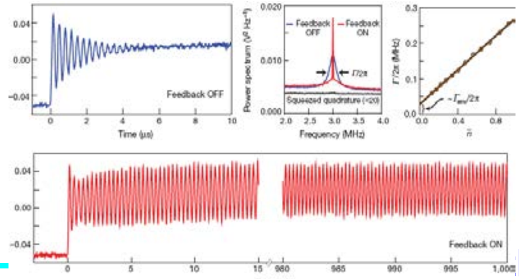
further develop. Y. Zhong, … H. Wang, Nature Comm. 2014 $T_1$ increased 3x
Z. Minev, … M. Devoret, arXiv 2018 q. jump mid-flight

**3.** A. Palacios-Laloy, F. Mallet, F. Nguyen, P. Bertet, D. Vion, D. Esteve, and A. Korotkov, Nature Phys. 2010



Continuous monitoring of Rabi oscillations (Rabi oscillations do not decay in time). Transmon, circuit QED

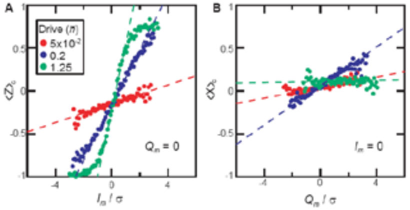
**4.** R. Vijay, C. Macklin, D. Slichter, S. Weber, K. Murch, R. Naik, A. Korotkov, and I. Siddiqi, Nature 2012



Quantum feedback of Rabi oscillations: maintaining desired phase forever. Transmon, phase-sensitive amp.

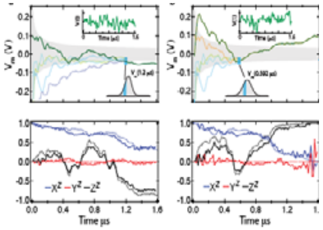
# First experiments (cont.)

**5.** M. Hatridge, S. Shankar, M. Mirrahimi, F. Schackert, K. Geerlings, T. Brecht, K. Sliwa, B. Abdo, L. Frunzio, S. Girvin, R. Schoelkopf, M. Devoret, Science 2013



Direct check of quantum back-action for measurement of a qubit. Phase-preserving amplifier.

**6.** K. Murch, S. Weber, C. Macklin, and I. Siddiqi, Nature 2013



Direct check of individual quantum trajectories against quantum Bayesian theory.
Phase-sensitive amplifier.

Many more experiments since then, including 2-qubit entanglement by continuous measurement (in one resonator and in remote resonators), qubit lifetime increase by uncollapse, phase feedback, and simultaneous measurement of non-commuting observables

Practicaly all our proposals have been realized

Still no experiments with semiconductors. Who will be the first?

# Possible applications of continuous quantum measurement

- Quantum feedback
- Continuous quantum error correction
- Better readout fidelity (continuous cQED measurement)
- Understanding of actual measurement (neighbors, etc.)
- Entanglement (even remote) by measurement
- Parameter monitoring
- Less disturbance from strong on/off controls

# Simultaneous measurement of non-commuting observables of a qubit

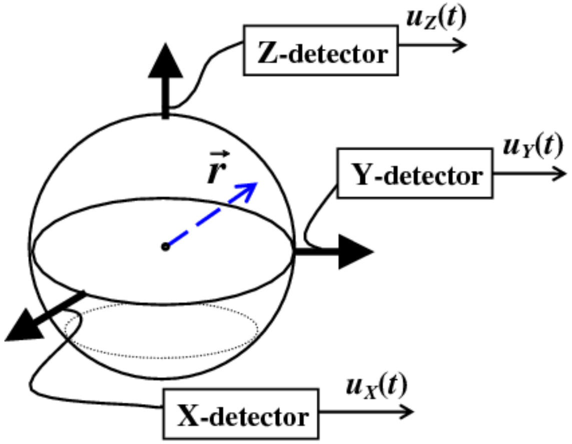
Nothing forbids simultaneous continuous measurement of non-commuting observables

Very simple quantum Bayesian description: just add terms for evolution

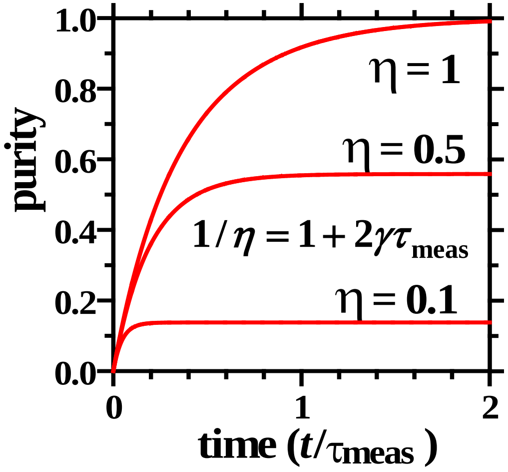
Measurement of three complementary observables for a qubit

Ruskov, A.K., Molmer, PRL 2010

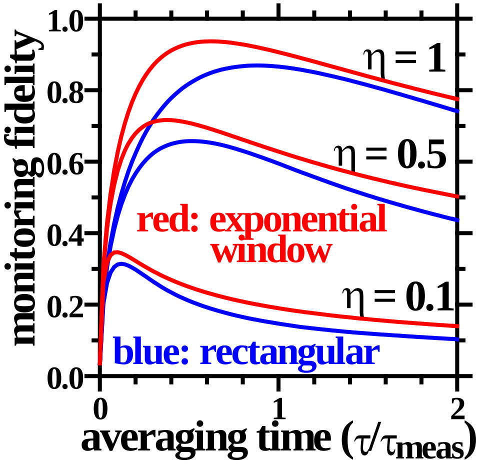
Evolution: $$\frac{d\vec{r}}{dt} = -2\gamma\vec{r} + a\{\vec{u}(t)(1-r^2) - [\vec{r}\times[\vec{r}\times\vec{u}(t)]]\}$$ diffusion over Bloch sphere



state purification



simple monitoring

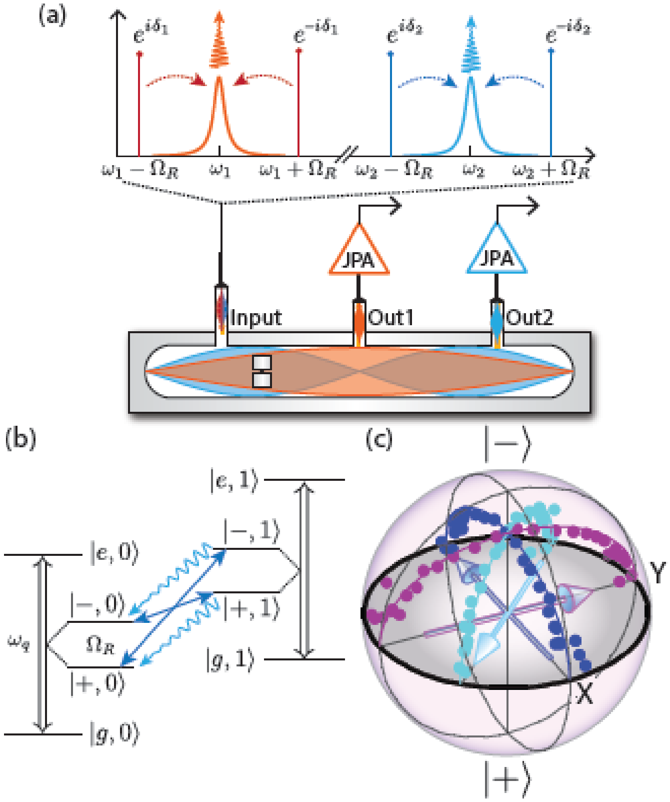


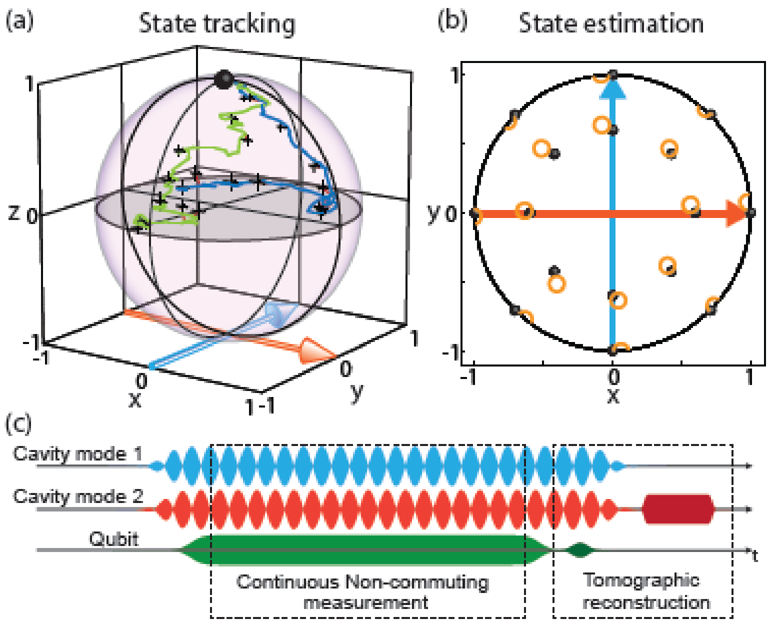
Until very recently it was unclear how to realize experimentally

# Simultaneous measurement of $\boldsymbol{\sigma_x}$ and $\boldsymbol{\sigma_z}$

Actually, any $\sigma_z \cos\varphi + \sigma_x \sin\varphi$

S. Hacohen-Gourgy, L. Martin, E. Flurin, V. Ramasesh, B. Whaley, and I. Siddiqi, Nature 2016





- Measurement in rotating frame of fast Rabi oscillations (40 MHz)
- Double-sideband rf wave modulation with the same frequency
- Two resonator modes for two channels

quantum trajectory theory for simulations

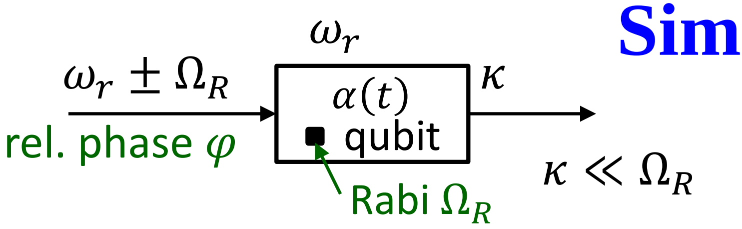
$$\Omega_{\text{Rabi}} = \Omega_{\text{SB}} = 2\pi \times 40 \text{ MHz}$$

$$\kappa/2\pi = 4.3 \text{ and } 7.2 \text{ MHz}$$

$$\Gamma_1^{-1} = \Gamma_2^{-1} = 1.3 \ \mu\text{s}$$

$$\boldsymbol{\Gamma \ll \kappa \ll \Omega_{\text{Rabi}}}$$

# Simple physical picture



Physical qubit (Rabi $\Omega_R$)

$$z_{\rm ph}(t) = r_0 \cos(\Omega_R t + \phi_0)$$

$$x_{\rm ph}(t) = r_0 \sin(\Omega_R t + \phi_0)$$

$$y_{\rm ph}(t) = y_0$$

This modulates resonator frequency

$$\omega_r(t) = \omega_r^b + \chi r_0 \cos(\Omega_R t + \phi_0)$$

Drive with modulated amplitude

$$A(t) = \varepsilon \sin(\Omega_R t + \varphi)$$

Then evolution of field $\alpha(t)$ is

$$\dot{\alpha} = -i\chi r_0 \cos(\Omega_R t + \phi_0)\,\alpha$$
$$-i\varepsilon \sin(\Omega_R t + \varphi) - \frac{\kappa}{2}\alpha$$

Now solve this differential equation

Fast oscillations (neglect $\kappa$)

$$\Delta\alpha(t) = i\,\frac{\varepsilon}{\Omega_R}\cos(\Omega_R t + \varphi)$$

Insert, then slow evolution is

$$\dot{\alpha}_s = \frac{\chi\varepsilon}{2\Omega_R}\underbrace{r_0 \cos(\phi_0 - \varphi)}_{} - \frac{\kappa}{2}\alpha_s$$

Thus, slow evolution is determined by effective qubit (in rotating frame),

$$z = r_0 \cos(\phi_0),\; x = r_0 \sin(\phi_0),\; y = y_0,$$

measured along axis $\varphi$ (basis $|1_\varphi\rangle$, $|0_\varphi\rangle$)

$$r_0 \cos(\phi_0 - \varphi) = \mathrm{Tr}[\sigma_\varphi \rho]$$

$$\sigma_\varphi = \sigma_z \cos\varphi + \sigma_x \sin\varphi$$

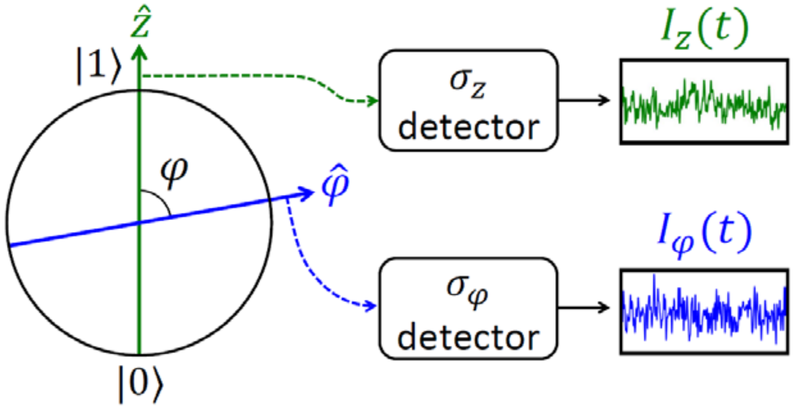
Stationary state $\alpha_{\rm st,1} = -\alpha_{\rm st,0} = \dfrac{\chi\varepsilon}{\Omega_R \kappa}$

From this point, usual Bayesian theory

More accurately, $\varphi \to \varphi + \kappa/2\Omega_R$

J. Atalaya, S. Hacohen-Gourgy, L. Martin,
I. Siddiqi, and A.K., arXiv:1702.08077

# Correlators in simultaneous measurement of non-commuting qubit observables



J. Atalaya, S. Hacohen-Gourgy, L. Martin, I. Siddiqi, and A.K., arXiv:1702.08077

$$I_z(t) = \mathrm{Tr}[\sigma_z \rho(t)] + \sqrt{\tau_z}\, \xi_z(t)$$

$$I_\varphi(t) = \mathrm{Tr}\left[\sigma_\varphi \rho(t)\right] + \sqrt{\tau_\varphi}\, \xi_\varphi(t)$$

$$\sigma_\varphi = \sigma_z \cos\varphi + \sigma_x \sin\varphi$$

$$K_{ij}(\tau) = \langle I_j(t+\tau)\, I_i(t)\rangle$$

$\tau_{z,\varphi}$: “measurement time” (SNR=1)

“Collapse recipe” (no phase back-action): replace continuous meas. with projective meas. at time moments $t$ and $t+\tau$, use ensemble-averaged evolution in between

(proof via Bayesian equations)

self-correlator

$$K_{zz}(\tau) = \frac{1}{2}\left[1 + \frac{\Gamma_z + \cos(2\varphi)\Gamma_\varphi}{\Gamma_+ - \Gamma_-}\right] e^{-\Gamma_- \tau} + \frac{1}{2}\left[1 - \frac{\Gamma_z + \cos(2\varphi)\Gamma_\varphi}{\Gamma_+ - \Gamma_-}\right] e^{-\Gamma_+ \tau}$$

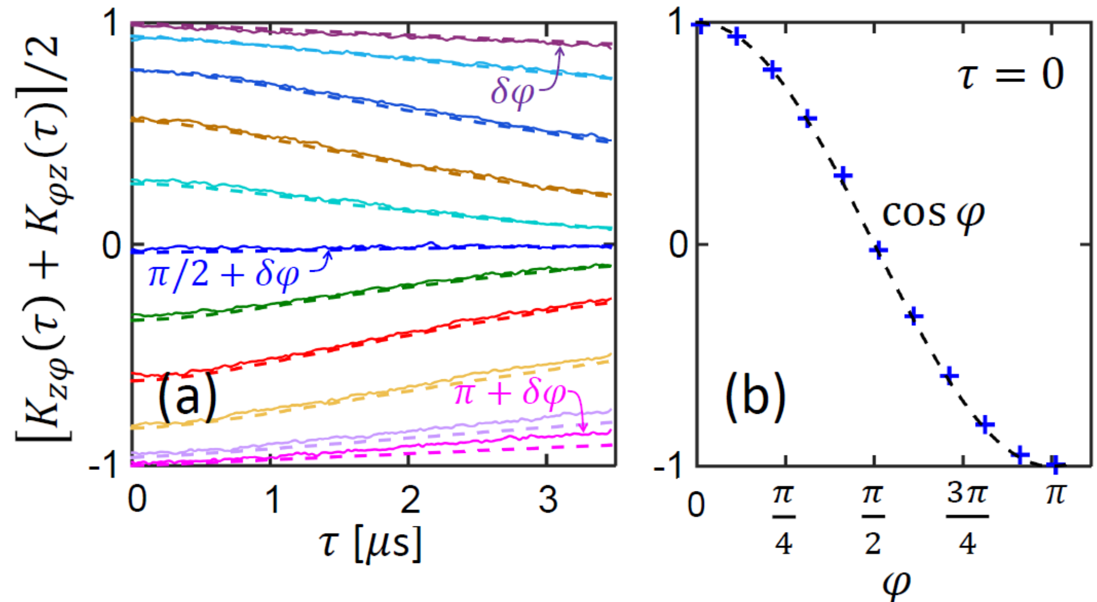
no dependence on initial state

cross-correlator

$$K_{z\varphi}(\tau) = \frac{\left(\Gamma_z + \Gamma_\varphi\right)\cos\varphi + 2\tilde{\Omega}_R \sin\varphi}{\Gamma_+ - \Gamma_-}\left(e^{-\Gamma_- \tau} - e^{-\Gamma_+ \tau}\right) + \frac{\cos\varphi}{2}\left(e^{-\Gamma_- \tau} + e^{-\Gamma_+ \tau}\right)$$

$$\Gamma_\pm = \frac{1}{2}\left(\Gamma_z + \Gamma_\varphi \pm \left[\Gamma_z^2 + \Gamma_\varphi^2 + 2\Gamma_z\Gamma_\varphi \cos(2\varphi) - 4\tilde{\Omega}_R^2\right]^{1/2}\right) + 1/2T_1 + 1/2T_2$$

# Comparison with experiment

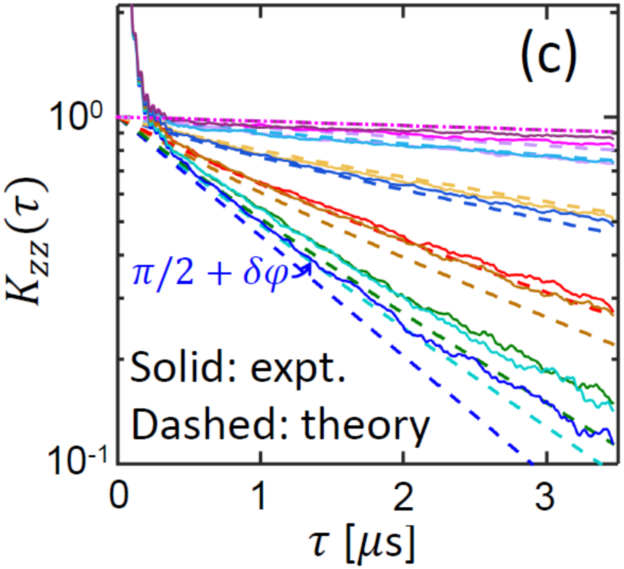


Cross-correlators
for 11 values of $\varphi$
between 0 and $\pi$

Maximally non-commuting:
$\varphi = \pi/2$

Correction to angle:

$$\delta\varphi = \frac{\kappa_\varphi - \kappa_z}{2\Omega_R}$$



Self-correlators

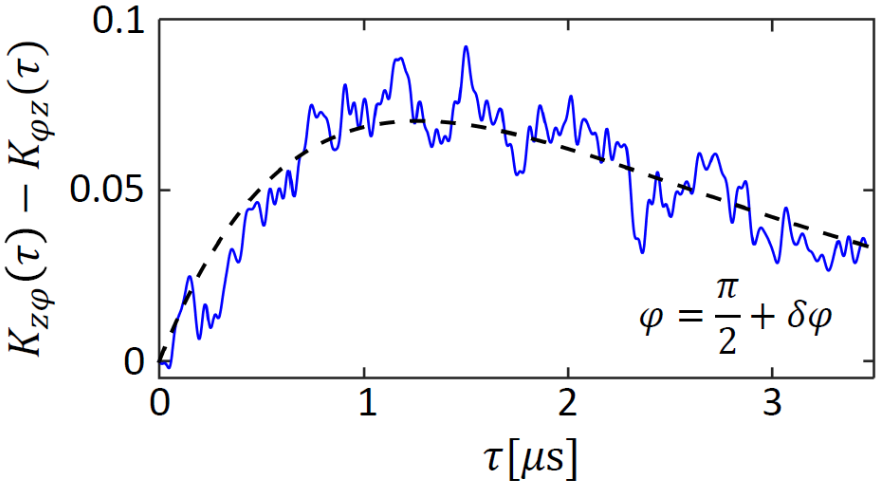
200,000 experimental traces

Good agreement

# Parameter estimation via correlators

Rabi frequency mismatch: $\widetilde{\Omega}_R = \Omega_R - \Omega_{\text{sideband}}$

$$K_{z\varphi}(\tau) - K_{\varphi z}(\tau) = \frac{\widetilde{\Omega}_R \sin\varphi}{\Gamma_+ - \Gamma_-}\left(e^{-\Gamma_+\tau} - e^{-\Gamma_-\tau}\right)$$



Fitting: $\widetilde{\Omega}_\mathrm{R} = \Omega_R - \Omega_{\text{sideband}} \approx 2\pi \times 12\ \text{kHz}$

Very sensitive technique

$(\Omega_R/2\pi = 40\ \text{MHz})$

J. Atalaya, S. Hacohen-Gourgy, L. Martin,
I. Siddiqi, and A.K., arXiv:1702.08077

# Generalization to *N*-time correlators

J. Atalaya, S. Hacohen-Gourgy, L. Martin, I. Siddiqi, and A.K., PRA-2018

Many detectors, $N$ time moments

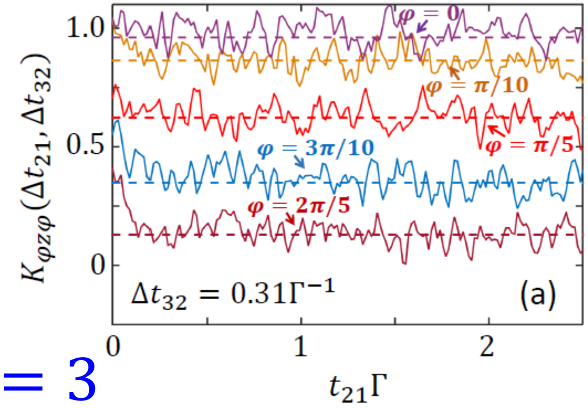
$$K_{l_1\dots l_N}(t_1,\dots t_N)=\langle I_{l_N}(t_N)\dots I_{l_2}(t_2)\, I_{l_1}(t_1)\rangle$$

The same collapse recipe works OK

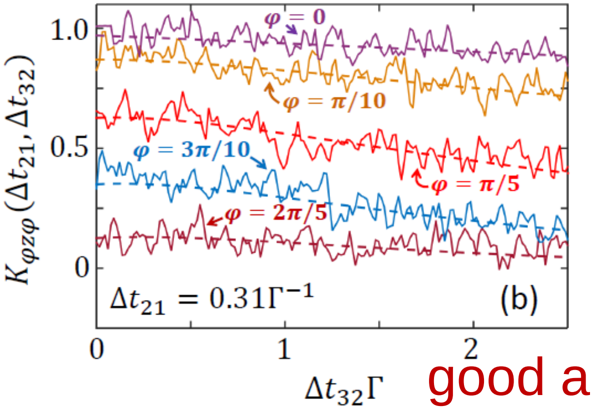
Surprising factorization: $\langle I_{l_3}(t_3)\, I_{l_2}(t_2)\, I_{l_1}(t_1)\rangle=\langle I_{l_3}(t_3)\, I_{l_2}(t_2\,)\rangle\times\langle I_{l_1}(t_1)\rangle,$

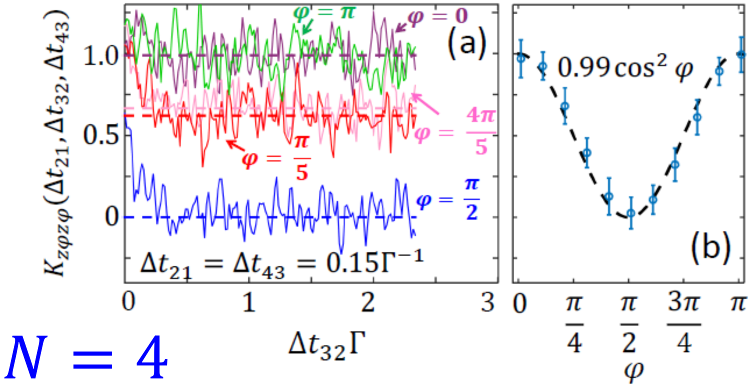
(unital case)

$$\langle I_{l_4}(t_4) I_{l_3}(t_3)\, I_{l_2}(t_2)\, I(t_1)\rangle=\langle I_{l_4}(t_4)\, I_{l_3}(t_3)\rangle\times\langle I_{l_2}(t_2)\, I_{l_1}(t_1)\rangle,$$ etc.



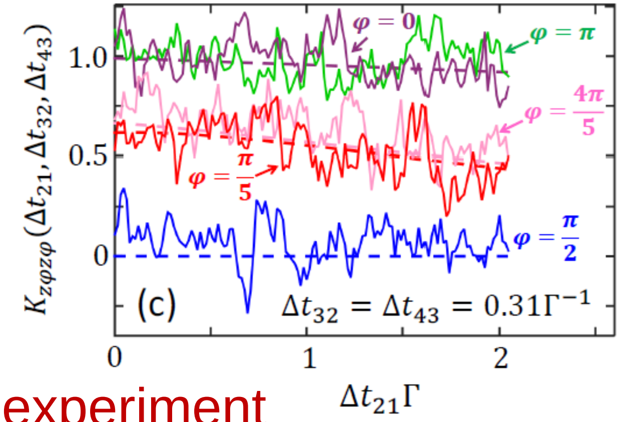
$N=3$



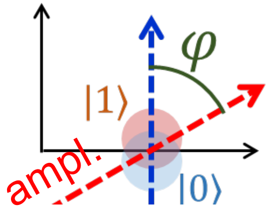


$N=4$

non-commuting observables



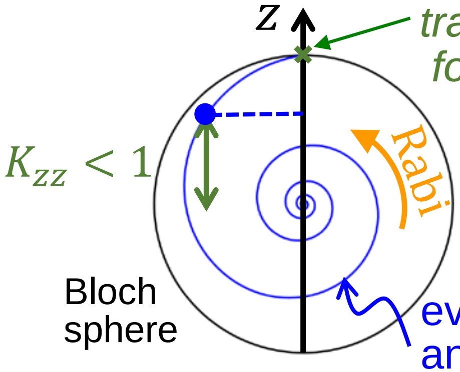
good agreement with experiment



# Correlators with phase backaction

J. Atalaya, S. Hacohen-Gourgy, I. Siddiqi, and A.K., in preparation

Only informational backaction ($\varphi = 0$)



*trajectory starts at $z = 1$ for any initial state*

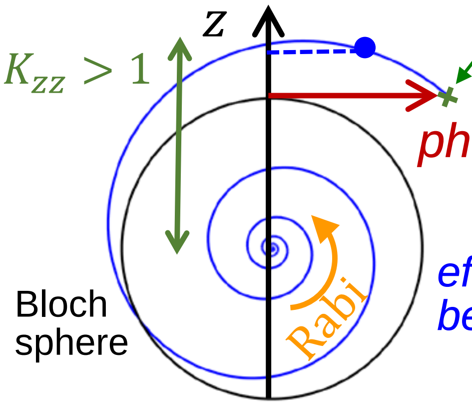
*effective trajectory is always inside Bloch sphere*

evolution due to Rabi and dephasing

$$K_{zz}(\tau) = \langle I_z(\tau)\, I_z(0)\rangle$$

$$I(t) = \mathrm{Tr}[\sigma_z \rho(t)] + \xi(t)$$

With phase backaction ($\varphi \neq 0$)



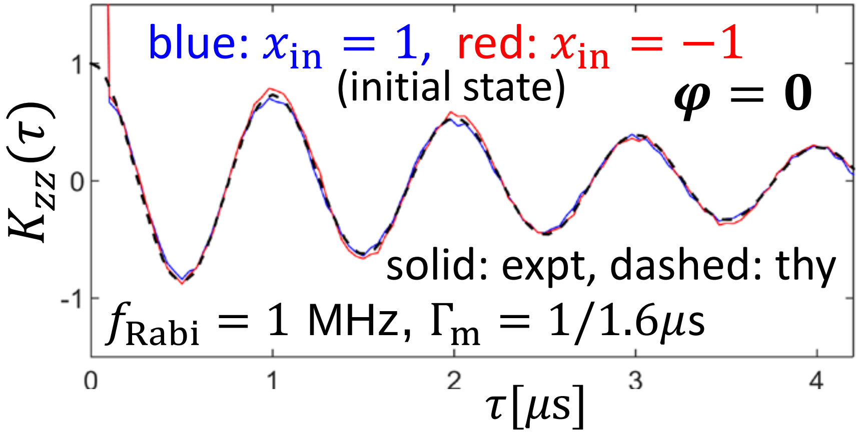
*trajectory starts* ***outside Bloch sphere***

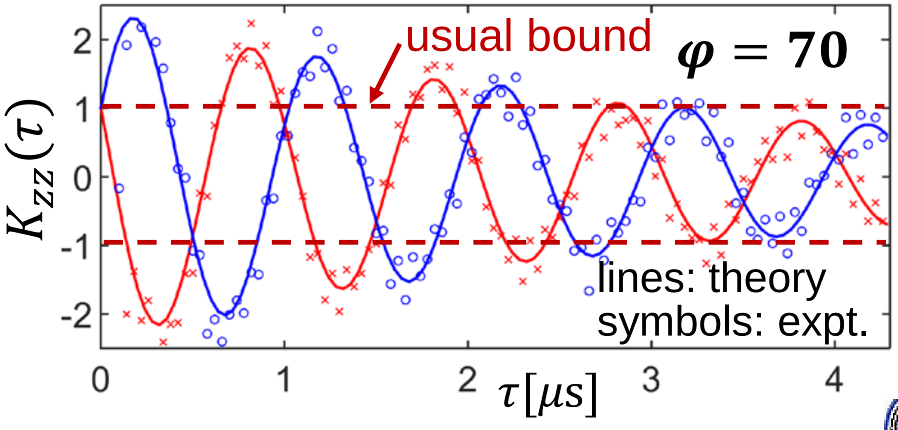
*phase-backaction kick:*

$(\hat{z} \times \boldsymbol{r}_{\text{in}}) \tan\varphi$

*effective trajectory can be outside Bloch sphere*

With phase backaction and Rabi oscillations, correlators may exceed 1

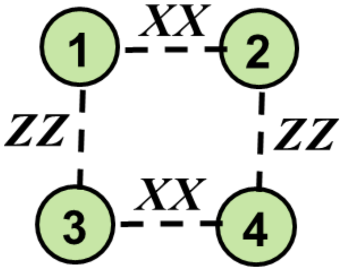


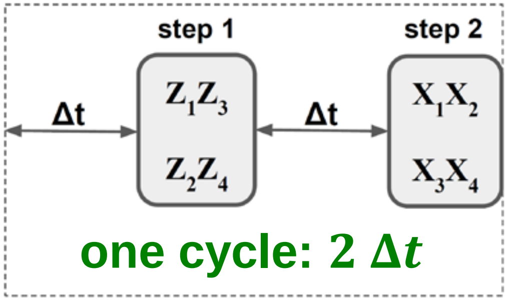


# Bacon-Shor code operating with continuous measurement of non-commuting operators

J. Atalaya, M. Bahrami, L. Pryadko, and A.K., PRA 2017

Conventional Bacon-Shor (subsystem) codes



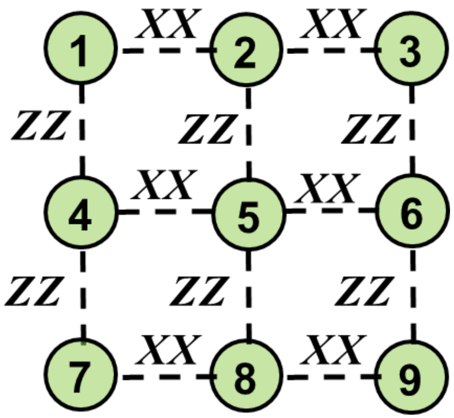


**one cycle: 2 $\Delta t$**

results ++ and - - are “good”

results + - and - + are “bad”

quantum error detection



Step 1: $Z_1Z_4,\ Z_2Z_5,\ Z_3Z_6,\ Z_4Z_7,\ Z_5Z_8,\ Z_6Z_9$

Step 2: $X_1X_2,\ X_4X_5,\ X_7X_8,\ X_2X_3,\ X_5X_6,\ X_8X_9$

q. error correction

D. Poulin, PRL 95, 230504 (2005)
D. Bacon, PRA 73, 012340 (2006)
Aliferis, Cross, PRL 98, 220502 (2007)

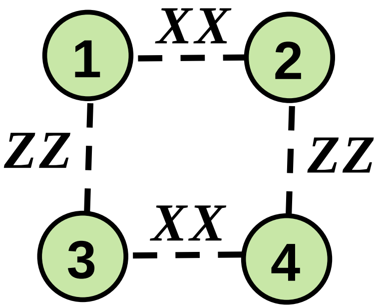
Conventional Bacon-Shor codes operate by sequential measurement of non-commuting operators

Advantage: only two-qubit operators

Can they operate with simultaneous continuous measurement of these non-commuting operators?

Additional advantage: passive monitoring of errors

# 4-qubit Bacon-Shor code with continuous meas.



All four operators are measured at the same time

$$I_{X1X2}(t) = \mathrm{Tr}[X_1X_2\rho(t)] + \xi_1(t) \qquad I_{X3X4}(t) = \ldots$$

$$I_{Z1Z3}(t) = \mathrm{Tr}[Z_1Z_3\rho(t)] + \xi_2(t) \qquad I_{Z2Z4}(t) = \ldots$$

So far we do not know how to realize,
but X&Z and 2-qubit ZZ already realized

Error syndromes are indicated by correlators (instead of ++/- - “good”, + -/- + “bad”)

$\langle I_{X1X2}(t)\, I_{X3X4}(t)\rangle$ and $\langle I_{Z1Z3}(t)\, I_{Z2Z4}(t)\rangle$ +1 “good”, -1 “bad” (error)

Need to monitor correlators in real time, so time-averaging to reduce noise

$$C_X(t) = \int_{-\infty}^{t} dt' \frac{1}{T_c} e^{-(t-t')/T_c} \int_{-\infty}^{t'} dt'' \frac{1}{2\tau_c} e^{-(t'-t'')/\tau_c} \left[I_{X1X2}(t')\, I_{X3X4}(t'') + \mathrm{sym}\right]$$

$$C_Z(t) = \int_{-\infty}^{t} dt' \frac{1}{T_c} e^{-(t-t')/T_c} \int_{-\infty}^{t'} dt'' \frac{1}{2\tau_c} e^{-(t'-t'')/\tau_c} \left[I_{Z1Z3}(t')\, I_{Z2Z4}(t'') + \mathrm{sym}\right]$$

Error criterion: either $C_X(t)$ or $C_Z(t)$ become negative

Fluctuations of $C_X(t)$ and $C_Z(t)$ beyond threshold produce false alarms

# Use quantum Bayesian formalism

$$\dot{\rho} = \underbrace{\sum_k \frac{1}{2}\Gamma_k (G_k \rho G_k - \rho) + \frac{1}{2\sqrt{\tau_k}}\,(G_k\rho + \rho G_k - 2\rho \mathrm{Tr}[G_k \rho])\,\xi_k}_{\text{measurement}} + \underbrace{\sum_{i,E} \Gamma_i^{(E)} \mathcal{L}[E_i]\rho}_{\text{qubit errors (flips)}}$$

(Ito form)

$G_k$: measured operators ($k = 1$–$4$), $\tau_k$: “measurement” time, $\Gamma_k$: ensemble dephasing

$\eta_k = 1/2\Gamma_k \tau_k$: quantum efficiency, $\Gamma_i^{(E)}$: rate of error $E$ in $i$th qubit

Without qubit errors, logical qubit does not evolve, while gauge qubit evolves diffusively, exactly as in 1-qubit measurement of $\sigma_Z$ and $\sigma_X$ ⇒ same correlators

## Logical error rates

$$\gamma_X = 2T_R[(\Gamma_1^X + \Gamma_2^X)(\Gamma_3^X + \Gamma_4^X) + \Gamma_1^Y\Gamma_3^Y + \Gamma_2^Y\Gamma_4^Y]$$

$$\gamma_Z = 2T_R[(\Gamma_1^Z + \Gamma_3^Z)(\Gamma_2^Z + \Gamma_4^Z) + \Gamma_1^Y\Gamma_2^Y + \Gamma_3^Y\Gamma_4^Y]$$

$$\gamma_Y = 2T_R[\Gamma_1^Y\Gamma_4^Y + \Gamma_2^Y\Gamma_3^Y]$$

$T_R = T_c \ln 2$ (integration time $T_c$)

Crudely, response time $T_R$ replaces cycle time $\Delta t$

## False alarm rate

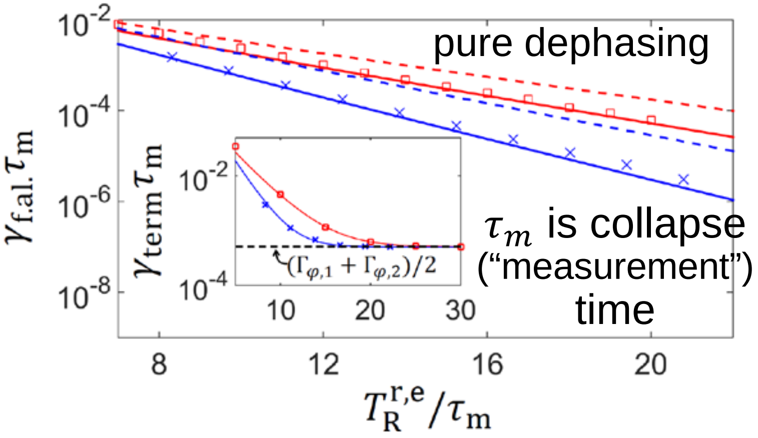
$$\gamma_{\text{false alarm}} = \frac{C \ln 2}{\sqrt{\pi T_R/\tau_m}} \exp\left(-\frac{C^2 T_R}{\tau_m}\right)$$

$C = 0.61$ for $\eta = 1$
$C = 0.54$ for $\eta = 0.5$
$\eta$ is quantum efficiency

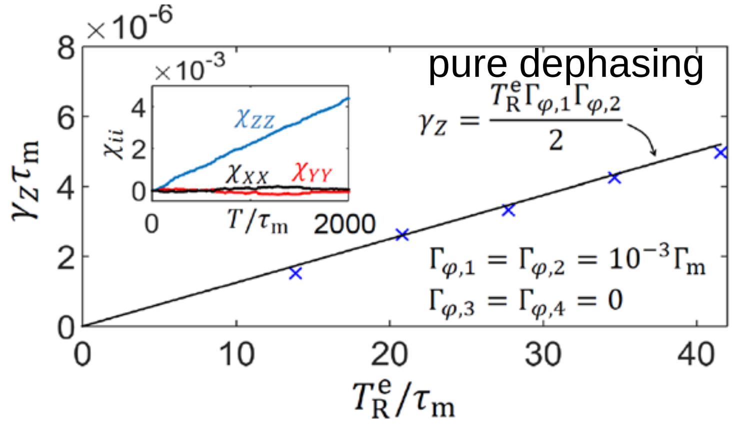
Trade-off: longer response time $T_R$ decreases false alarm rate (exponentially), but increases logical error rate (linearly)

# Monte Carlo simulation results

## False alarm rate vs. response time

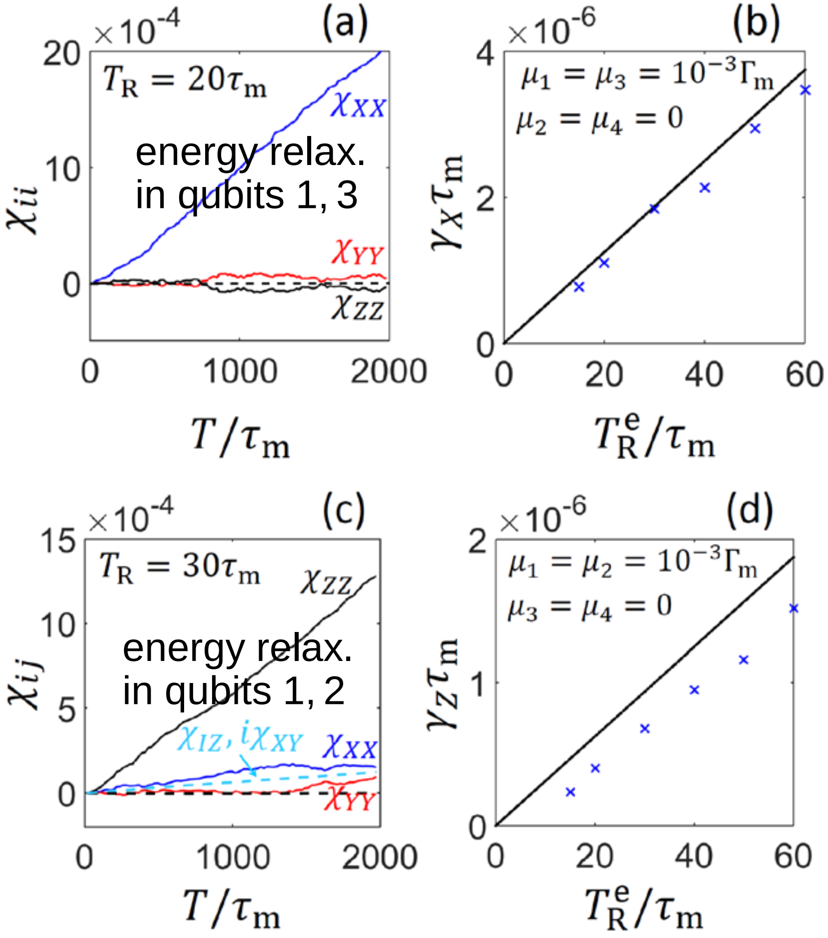


## Logical error rate vs. response time



## Process matrix $\chi$ vs. time

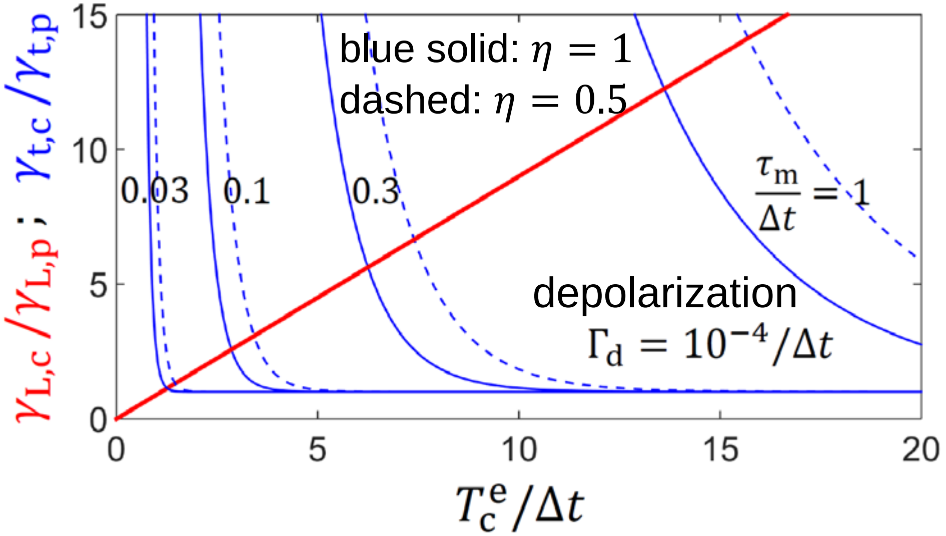
## Error rate vs. response time



Good agreement between (semi)analytical theory and numerical results

However, corrections (~30%) due to non-Gaussian noise of correlators

# Comparison with conventional projective case



four-qubit
Bacon-Shor
code

Red line: ratio of logical error rates (contin./proj.), Blue: ratio of termination rates

$\Delta t$ is cycle time for projective operation, $\tau_m$ is collapse ("measur.") time for continuous measurement, $T_c^e$ is integration time for correlators

Comparable operation for continuous and projective cases if $\tau_m \sim \Delta t/20$

Hidden assumption in projective case: $5\tau_m \ll \Delta t$, so crudely the same requirement.

Advantage of continuous measurement: a time-dependent protocol is not needed, only passive monitoring of error syndrome

So far we considered only QED code (2x2), similar results are expected for QEC codes (3x3 and higher)

J. Atalaya et al., PRA-2017

# Arrow of time for continuous measurement

J. Dressel. A. Chantasri, A. Jordan, and A. Korotkov, PRL 2017

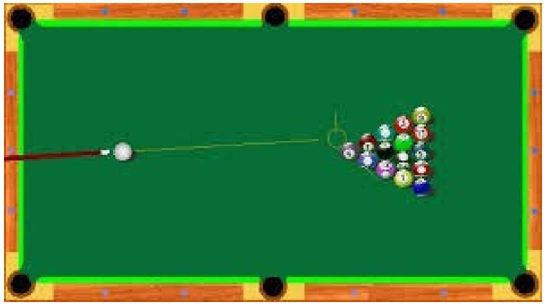
Unitary evolution is time-reversible.

Is continuous quantum measurement time-reversible?

If yes, can we distinguish forward and backward evolutions?

## Classical mechanics

Dynamics is time-reversible. However, for more than a few degrees of freedom, one time direction is much more probable than the other.



## Posing of quantum problem: a game

We are given a “movie”, showing quantum evolution $|\psi(t)\rangle$ of a qubit due to continuous measurement and Hamiltonian, together with “soundtrack”, representing noisy measurement record. We need to tell if the movie is played forward of backward.

# Reversing qubit evolution

Hamiltonian: $H = \hbar\Omega\sigma_y/2$

Measurement output: $r(t) = z(t) + \sqrt{\tau}\,\xi(t)$,

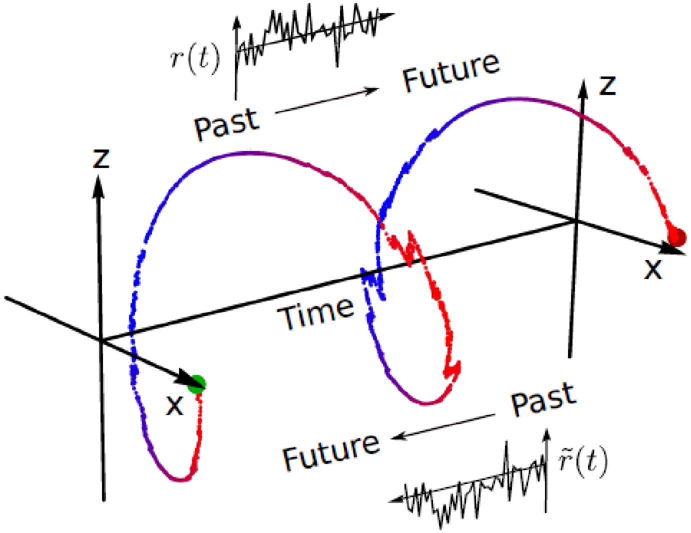
"measurement" (collapse) time $\tau$, white noise $\langle\xi(t)\,\xi(0)\rangle = \delta(t)$

Quantum Bayesian equations (Stratonovich form, quantum-limited detector)

$$\dot{x} = -\Omega z - xzr/\tau, \quad \dot{y} = -yz\,r/\tau, \quad \dot{z} = \Omega x + (1-z^2)\,r/\tau$$

Time-reversal symmetry: $t \to -t, \;\; \Omega \to -\Omega, \;\; r \to -r$

(so, need to flip Rabi direction and measurement record)



This quantum movie, played backwards, is fully legitimate (soundtrack is flipped)

Is there a way to distinguish forward from backward?

# Emergence of an arrow of time

Use classical Bayes rule to distinguish forward from backward movie

$$R = \frac{P_{\text{Forward}}[r(t)]}{P_{\text{Backward}}[r(t)]}$$

Since the measurement record ("soundtrack" ) is flipped, the particular noise realization becomes less probable (usually)

$$\left.\begin{aligned} r(t) &= z(t) + \sqrt{\tau}\,\xi(t) \\ -r(t) &= z(t) + \sqrt{\tau}\,\xi_B(t) \end{aligned}\right\} \Rightarrow \quad \xi_B(t) = -\xi(t) - \frac{2z(t)}{\sqrt{\tau}}$$

$\xi_B(t)$ is less probable than $\xi(t)$

$$\boxed{\ln R = \frac{2}{\tau}\int_0^T r(t)\,z(t)\,dt}$$

Relative log-likelihood, distinguishing time running forward or backward

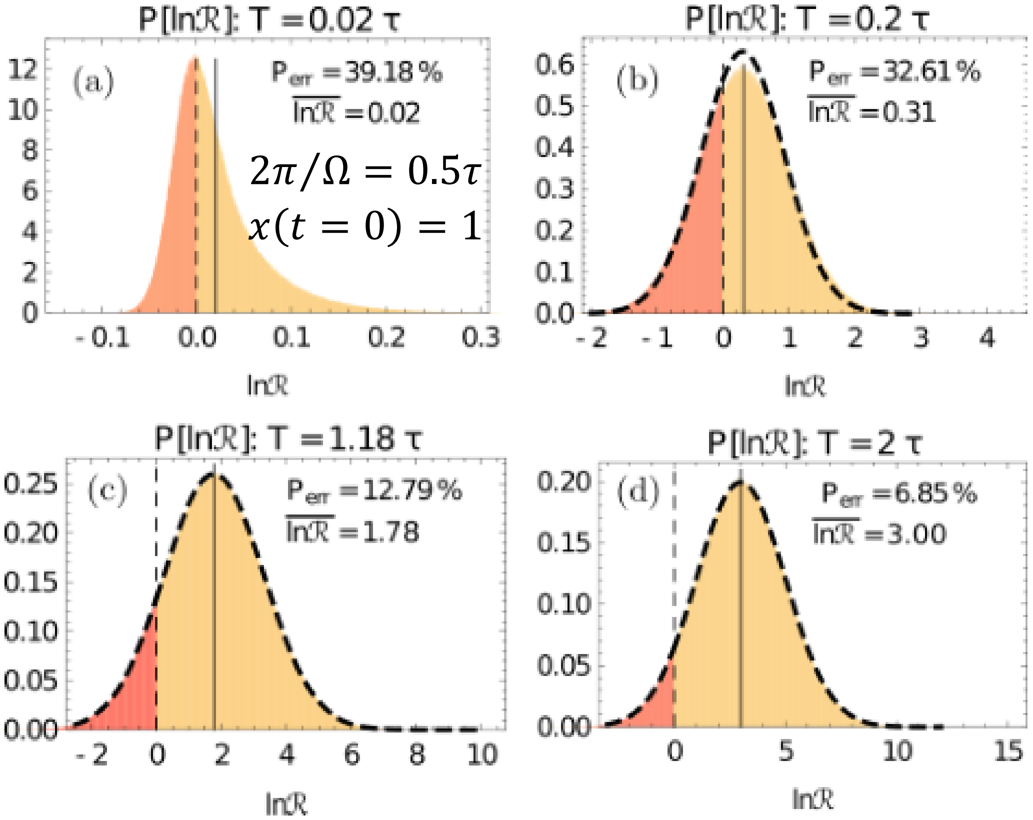
For a long movie time $T$, almost certainly $\ln R > 0$, so we will know the direction of time. For a short $T$, we will often make a mistake in guessing the time direction.

# Numerical results

$$R = \frac{P_F[r(t)]}{P_B[r(t)]}$$

$$\ln R = \frac{2}{\tau}\int_0^T r(t)\, z(t)\, dt$$

Probability distribution for $\ln R$



$2\pi/\Omega = 0.5\tau$

$x(t=0) = 1$

Asymptotic behavior (long *T*)

$$R \approx \frac{3T}{2\tau} \pm \sqrt{\frac{2T}{\tau}}$$

Probability of guessing the direction of time incorrectly:

$$P_{\rm err} \approx \frac{2}{3}\sqrt{\frac{\tau}{\pi T}}\, \exp\left(-\frac{9\,T}{16\,\tau}\right)$$

(decreases exponentially with the ratio $T/\tau$)

Statistical arrow of time emerges at timescale of “measurement time” $\tau$

Similar to classical entropy increase, but opposite direction: from more to less random

J. Dressel. A. Chantasri, A. Jordan, and A. Korotkov, PRL 2017

# Conclusions

- Quantum Bayesian approach is based on common sense and simple (undergraduate-level) physics; it is similar to Quantum Trajectory theory, though looks different
- Measurement back-action necessarily has “spooky” part (informational, without physical mechanism); it may also have unitary part (with physical mechanism)
- Many experiments demonstrated evolution “inside” collapse (most of our proposals already realized)
- Possibly useful (especially quantum feedback)
- Simultaneous measurement of non-commuting observables has become possible experimentally
- Bacon-Shor code can operate with continuous measurement of non-commuting gauge operators
- Continuous measurement of a qubit is time-reversible (with flipped record), but the arrow of time emerges statistically

# Thank you!