Continuous quantum measurement of solid-state qubits

Alexander Korotkov

University of California, Riverside

Outline:

- Introduction (quantum measurement)
- Quantum Bayesian theory of qubit measurement
- Experiments on partial and continuous measurement of superconducting qubits
- Simultaneous measurement of non-commuting observables of a qubit
- Arrow of time in continuous qubit measurement

"Orthodox" (Copenhagen) quantum mechanics

Schrödinger equation
+
collapse postulate

1) Fundamentally random measurement result r
(out of allowed set of eigenvalues). Probability: $p_r = |\langle\psi|\psi_r\rangle|^2$

2) State after measurement corresponds to result: $|\psi_r\rangle$

- Instantaneous, single quantum system (not ensemble)
- Contradicts Schr. Eq., but follows from common sense
- Needs "observer" to **get information**

Why so strange (unobjective)?

- "Shut up and calculate"
- May be QM founders were stupid?
- Use proper philosophy?

Werner Heisenberg

Books:

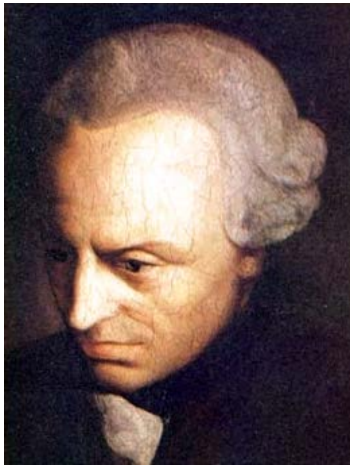
Physics and Philosophy: The Revolution in Modern Science

Philosophical Problems of Quantum Physics

The Physicist's Conception of Nature

Across the Frontiers

Niels Bohr



Immanuel Kant (1724-1804), German philosopher

Critique of pure reason (materialism, but not naive materialism)

Nature - “Thing-in-itself” (noumenon, not phenomenon)

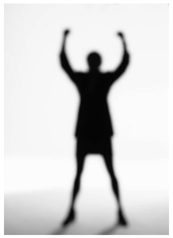
Humans use “concepts (categories) of understanding”;
make sense of phenomena, but never know noumena directly

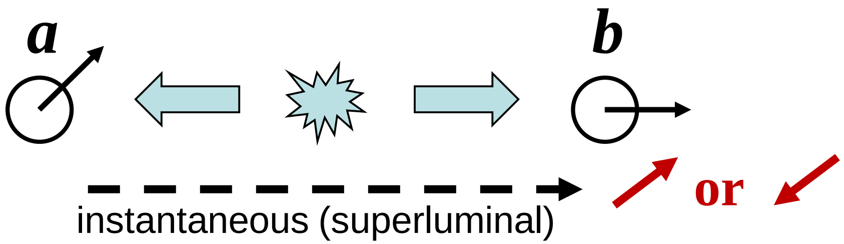
A priori: space, time, causality

A naïve philosophy should not be a roadblock for good physics, quantum mechanics requires a non-naïve philosophy

Wavefunction is not a reality, it is only our description of reality

Bell's inequality (EPR paradox, CHSH)





$$|\psi\rangle = \frac{\uparrow_1\downarrow_2 - \downarrow_1\uparrow_2}{\sqrt{2}}$$

rotationally invariant

Experiments (1982--present, photons): yes, "spooky action-at-a-distance"

What about causality?

Not too bad: only "useless" (quantum) information is transmitted faster than light, you cannot transmit "useful" (classical) information by choosing meas. direction $\boldsymbol{a}$

The other meas. result does not depend on $\boldsymbol{a}$ Randomness saves causality

Collapse is still instantaneous: not a "physical" process

Consequence of causality: No-cloning theorem (1982)

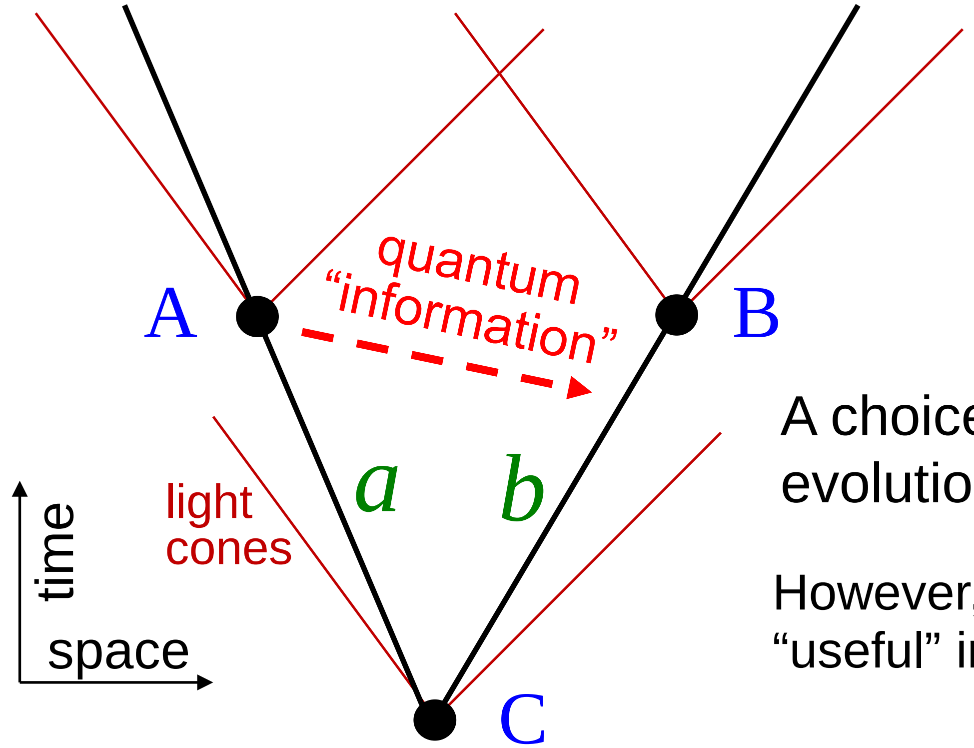
You cannot copy an unknown quantum state

Proof: Otherwise get information on direction $\boldsymbol{a}$ (and causality is violated)

Application: quantum cryptography

Information is an important concept in quantum mechanics

Causality principle in quantum mechanics



objects a and b

observers A and B (and C)

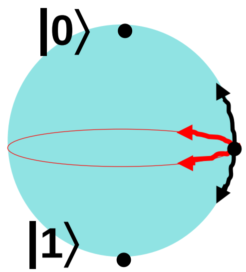
observers have "free will";
they can choose an action

A choice made by observer A can affect evolution of object b "back in time"

However, this retroactive control cannot pass "useful" information to B (no signaling)

Randomness saves causality (even C cannot predict result of A measurement)

Our focus: continuous collapse



Ensemble-averaged evolution of object b cannot depend on actions of observer A

What is "inside" collapse?
What if collapse is stopped half-way?

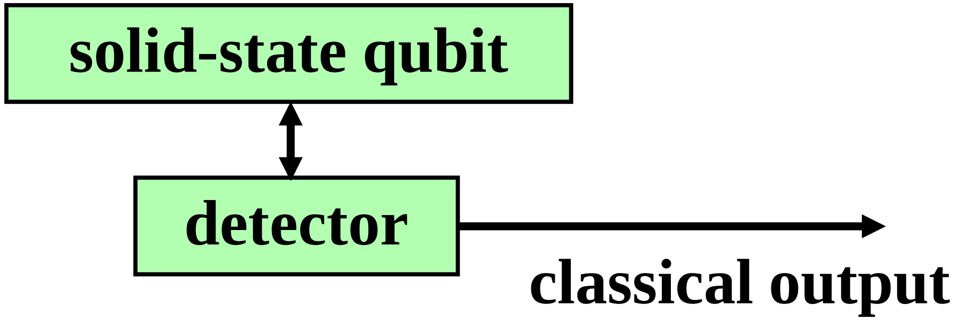
Various approaches to non-projective (weak, continuous, partial, generalized, etc.) quantum measurements

Names: Davies, Kraus, Holevo, Mensky, Caves, Knight, Walls, Carmichael, Milburn, Wiseman, Aharonov, Molmer, Gisin, Percival, Belavkin, … (very incomplete list)

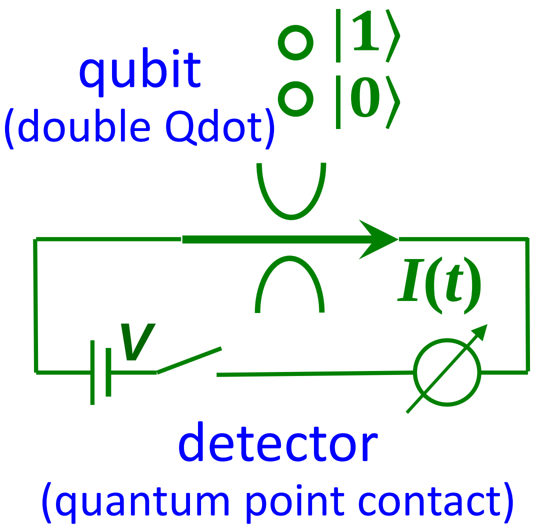
Key words: POVM, restricted path integral, <u>quantum trajectories</u>, quantum filtering, quantum jumps, stochastic master equation, etc.

Limited scope:

(simplest system,
experimental setups)



Quantum Bayesian formalism for qubit meas.



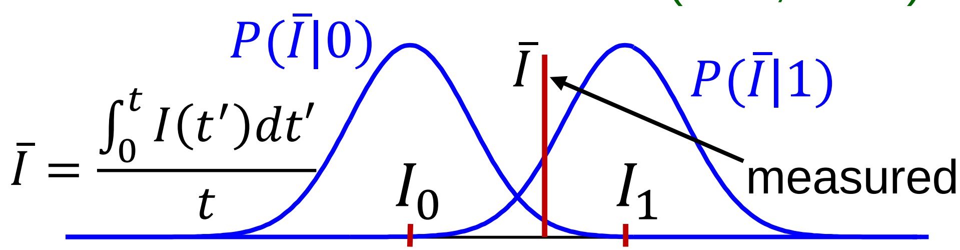
Qubit evolution due to measurement
(informational back-action)

$|\psi(t)\rangle = \alpha(t)\,|0\rangle + \beta(t)\,|1\rangle$ or $\rho_{ij}(t)$

1) $|\alpha(t)|^2$ and $|\beta(t)|^2$ evolve as probabilities,
i.e. according to the Bayes rule (same for ρ_{ii})

2) phases of $\alpha(t)$ and $\beta(t)$ do not change
(no dephasing!), $\rho_{ij}/\sqrt{\rho_{ii}\rho_{jj}} = \text{const}$

(A.K., 1998)



$$P(\bar{I}) = \rho_{00}(0)\,P(\bar{I}|0) + \rho_{11}(0)\,P(\bar{I}|1)$$

Bayes rule (1763, Laplace-1812):

$$\underbrace{P(A_i|\text{res})}_{\text{posterior probability}} = \frac{\underbrace{P(A_i)}_{\text{prior probab.}}\;\underbrace{P(\text{res}|A_i)}_{\text{likelihood}}}{\text{norm}}$$

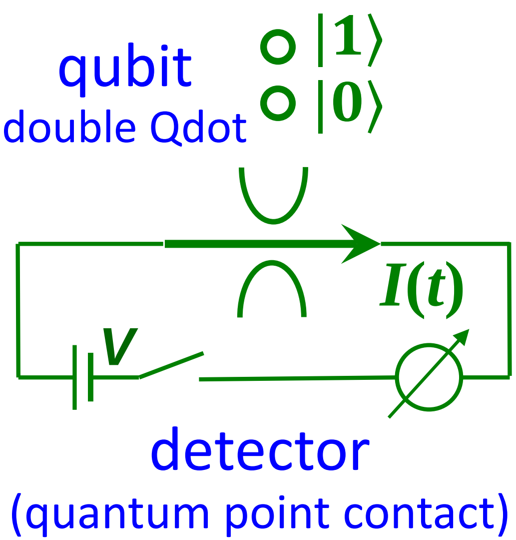
So simple because:

1) no entanglement at large QPC voltage
2) QPC is ideal detector
3) no other evolution of qubit

Two derivations

1. "Logical" derivation
 - Probabilities must evolve classically (quantum-classical correspondence)
 - Lower bound for ensemble dephasing since $|\rho_{01}| \leq \sqrt{\rho_{00}\rho_{11}}$
 - Comparison with ensemble-averaged evolution shows $\rho_{01} = \sqrt{\rho_{00}\rho_{11}}$

2. "Microscopic" derivation
 - Solve combined quantum evolution, qubit+detector
 - Apply textbook collapse to <u>detector</u>

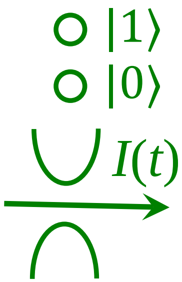


"Informational" quantum back-action:

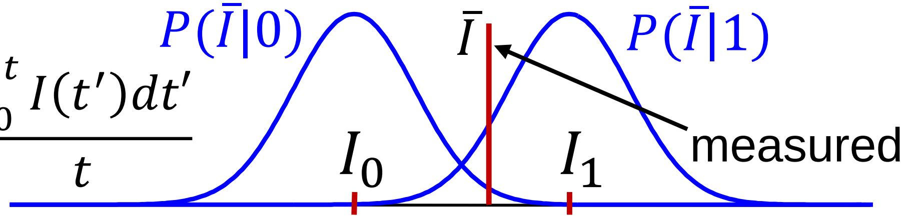
$\text{amplitude} \times \sqrt{\text{likelihood}}$

$$|\psi(t)\rangle = \frac{\sqrt{P(\bar{I}|0)}\,\alpha(0)\,|0\rangle + \sqrt{P(\bar{I}|1)}\,\beta(0)\,|1\rangle}{\text{norm}}$$

Further steps in quantum Bayesian formalism



$$\bar{I} = \frac{\int_0^t I(t')dt'}{t}$$



$$\alpha(t)\,|0\rangle + \beta(t)\,|1\rangle$$

$$\rho_{ij}(t)$$

1. Informational ("quantum") back-action, $\times\sqrt{\text{likelihood}}$

$$|\psi(t)\rangle = \frac{\sqrt{P(\bar{I}|0)}\,\alpha(0)\,|0\rangle + \sqrt{P(\bar{I}|1)}\,\beta(0)\,|1\rangle}{\text{norm}}$$

2. Add unitary (phase) back-action, physical mechanisms for QPC and cQED

$$|\psi(t)\rangle = \frac{\sqrt{P(\bar{I}|0)}\exp\left[iK\left(\bar{I} - \frac{I_0+I_1}{2}\right)\right]\alpha(0)\,|0\rangle + \sqrt{P(\bar{I}|1)}\,\beta(0)\,|1\rangle}{\text{norm}}$$

3. Add detector non-ideality (equivalent to dephasing) $\gamma = \Gamma - \frac{(\Delta I)^2}{4S_I} - \frac{K^2 S_I}{4}$

$$\rho_{ii}(t) = \frac{P(\bar{I}|i)\,\rho_{ii}(0)}{\text{norm}}, \qquad \frac{\rho_{01}(t)}{\sqrt{\rho_{00}(t)\,\rho_{11}(t)}} = \frac{e^{iK(\bar{I}-\frac{I_0+I_1}{2})}\rho_{01}(0)}{\sqrt{\rho_{00}(0)\,\rho_{11}(0)}}\exp(-\gamma t)$$

Further steps in quantum Bayesian formalism

4. Take derivative over time (if differential equation is desired)

Simple, but be careful about definition of derivative

$$\frac{df(t)}{dt} = \frac{f(t+dt/2) - f(t-dt/2)}{dt}$$

Stratonovich form preserves usual calculus

$$\frac{df(t)}{dt} = \frac{f(t+dt) - f(t)}{dt}$$

Ito form

requires special calculus, but keeps averages

5. Add Hamiltonian evolution (if any) and additional decoherence (if any)

Standard terms

Steps 1–5 form the quantum Bayesian approach to qubit measurement

(A.K., 1998—2001)

Generalization: measurement of operator *A*

"Informational" quantum Bayesian in differential (Ito) form:

$$\dot{\rho} = \frac{A\rho A - (A^2\rho + \rho A^2)/2}{2\eta S} + \frac{A\rho + \rho A - 2\rho \mathrm{Tr}\,(A\rho)}{\sqrt{2S}}\ \xi(t)$$

$I(t) = \mathrm{Tr}(A\rho) + \sqrt{S/2}\ \xi(t)$ noisy detector output

S: spectral density of the output noise

$\langle \xi(t)\, \xi(t')\rangle = \delta(t-t')$ normalized white noise

η: quantum efficiency

With additional unitary (Hamiltonian) back-action B and additional evolution

$$\dot{\rho} = \mathcal{L}[\rho] + \frac{A\rho + \rho A - 2\rho \mathrm{Tr}\,(A\rho)}{\sqrt{2S}}\ \xi(t) - i[B,\rho]\,\frac{1}{\sqrt{2S}}\,\xi(t)$$

$\mathcal{L}[\rho]$: ensemble-averaged (Lindblad) evolution

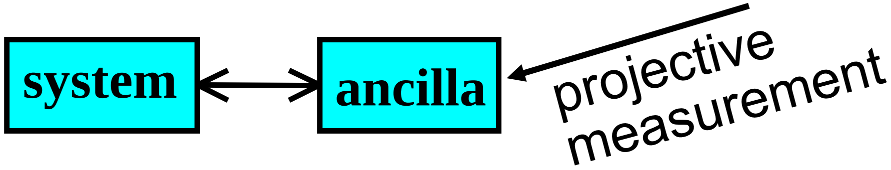
The same as in the Quantum Trajectory theory (Wiseman, Milburn, …)

Nowadays "quantum trajectories" often mean Bayesian real-time monitoring

Quantum measurement in POVM formalism

Davies, Kraus, Holevo, etc.

(Nielsen-Chuang, pp. 85, 100)



Measurement (Kraus) operator M_r (any linear operator in H.S.) : $\psi \rightarrow \dfrac{M_r \psi}{\|M_r \psi\|}$ or $\rho \rightarrow \dfrac{M_r \rho M_r^\dagger}{\mathrm{Tr}(M_r \rho M_r^\dagger)}$

Probability : $P_r = \| M_r \psi \|^2$ or $P_r = \mathrm{Tr}(M_r \rho M_r^\dagger)$

Completeness : $\sum_r M_r^\dagger M_r = 1$

(People often prefer linear evolution and non-normalized states)

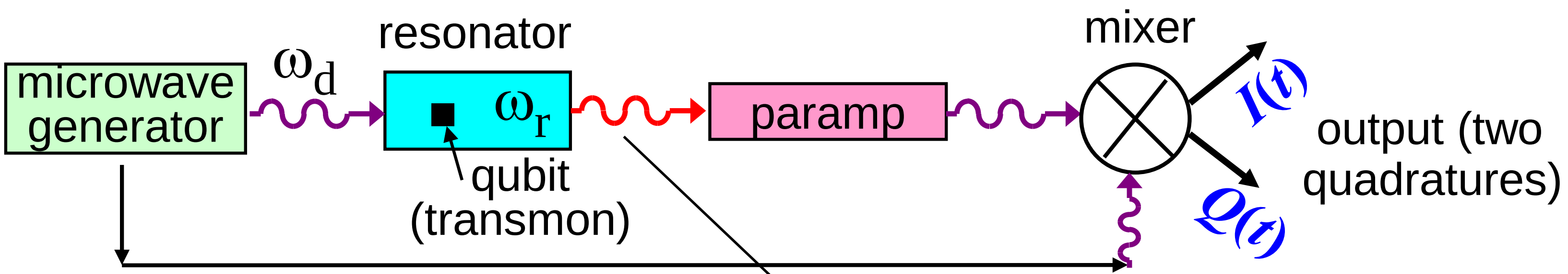
Relation between POVM and quantum Bayesian formalism

polar decomposition: $M_r = \underset{\text{unitary}}{U_r} \underbrace{\sqrt{M_r^\dagger M_r}}_{\text{Bayes}}$

Circuit QED setup for superconducting qubits

Idea: qubit state shifts resonator frequency, this affects amplitude and phase of microwave passed through (reflected from) resonator

Narrowband setup, so two signals (quadratures): $A(t)\cos(\omega_d t) + B(t)\sin(\omega_d t)$



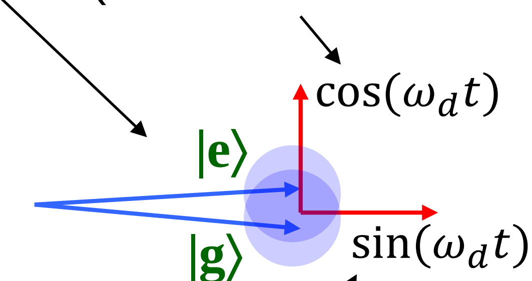
$$H = \frac{1}{2}\omega_{qb}\sigma_z + \omega_r a^\dagger a + \chi\sigma_z a^\dagger a$$

resonator frequency: $\omega_r \pm \chi$

qubit frequency: $\omega_{qb} + 2\chi\bar{n}$

(ac Stark shift)

carries information on qubit state (causes informational back-action)

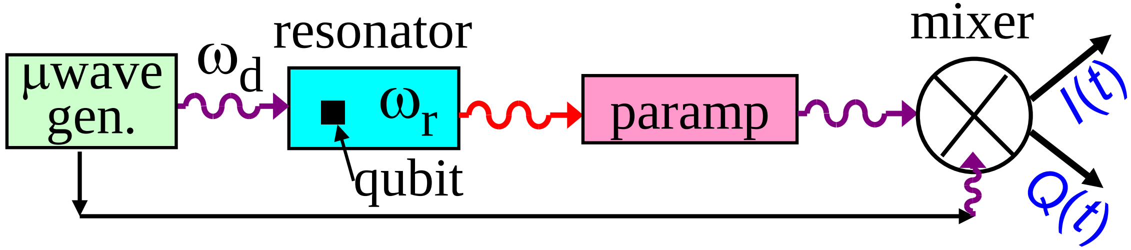


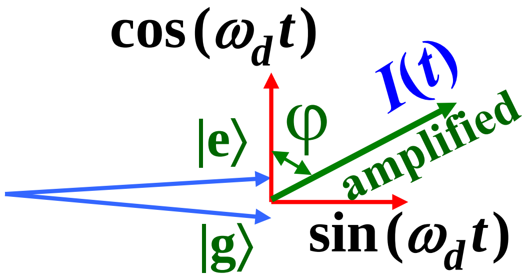
carries information on fluctuating photon number in the resonator (causes phase back-action)

Theory: A. Blais et al., PRA-2004

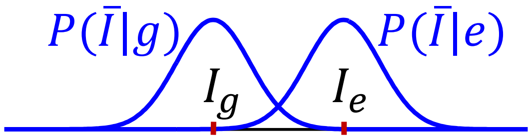
First expt.: A. Wallraff et al., Nature-2004

Phase-sensitive amplifier





get some information (~cos² φ) about qubit state and some information (~sin² φ) about photon fluctuations



$$P(\bar{I}) = \rho_{gg}(0)P(\bar{I}|g) + \rho_{ee}(0)P(\bar{I}|e)$$

$$\begin{cases} \dfrac{\rho_{gg}(\tau)}{\rho_{ee}(\tau)} = \dfrac{\rho_{gg}(0)}{\rho_{ee}(0)} \dfrac{\exp[-(\bar{I}-I_g)^2/2D]}{\exp[-(\bar{I}-I_e)^2/2D]} \\ \rho_{ge}(\tau) = \rho_{ge}(0)\sqrt{\dfrac{\rho_{gg}(\tau)\,\rho_{ee}(\tau)}{\rho_{gg}(0)\,\rho_{ee}(0)}}\exp(iK\bar{I}\tau) \end{cases}$$

Bayes

unitary

$$\bar{I} = \tau^{-1}\int_0^\tau I(t)dt \qquad D = S_I/2\tau$$

$$I_g - I_e = \Delta I\cos\varphi \qquad K = \Delta I\sin\varphi/S_I$$

$$\Gamma = \frac{(\Delta I\cos\varphi)^2}{4S_I} + K^2\frac{S_I}{4} = \frac{\Delta I^2}{4S_I} = \frac{8\chi^2\bar{n}}{\kappa}$$

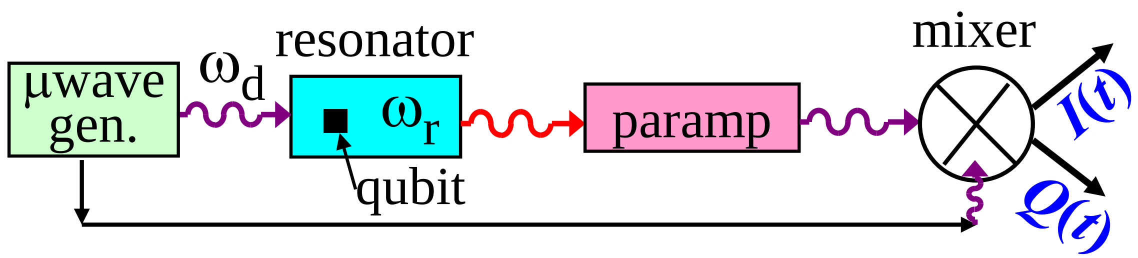
(rotating frame)

Amplified phase φ controls trade-off between informational & phase back-actions (we choose if photon number fluctuates or not)

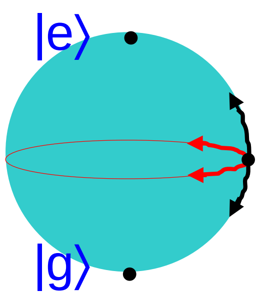
A.K., arXiv:1111.4016

Non-trivial causality

Ensemble-averaged evolution cannot be affected retroactively,
but single realizations can be affected "back in time"



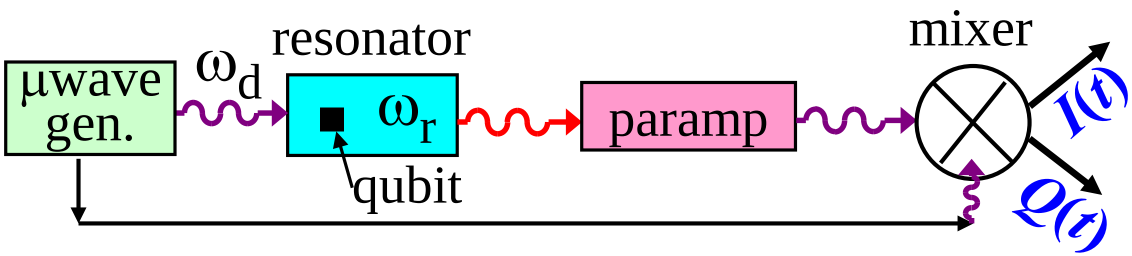
We can choose direction of qubit evolution
to be either along parallel or along meridian
or in between (delayed choice)

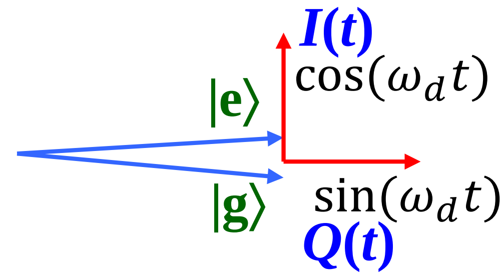


A.K., arXiv:1111.4016

Expt. confirmation: K. Murch et al., Nature-2013

Phase-preserving amplifier





$I(t)$: qubit information
$Q(t)$: photon fluct. info

Now information in both $I(t)$ and $Q(t)$

$$\begin{cases} \dfrac{\rho_{gg}(\tau)}{\rho_{ee}(\tau)} = \dfrac{\rho_{gg}(0)}{\rho_{ee}(0)} \dfrac{\exp[-(\bar{I}-I_g)^2/2D]}{\exp[-(\bar{I}-I_e)^2/2D]} \\ \rho_{ge}(\tau) = \rho_{ge}(0)\sqrt{\dfrac{\rho_{gg}(\tau)\,\rho_{ee}(\tau)}{\rho_{gg}(0)\,\rho_{ee}(0)}}\,\exp(iK\bar{Q}\tau) \end{cases}$$

Bayes   unitary

$$\bar{I} = \frac{1}{\tau}\int_0^\tau I(t)dt \qquad \bar{Q} = \frac{1}{\tau}\int_0^\tau Q(t)dt$$

$$I_g - I_e = \frac{\Delta I}{\sqrt{2}} \qquad K = \frac{\Delta I}{\sqrt{2}S_I} \qquad D = \frac{S_I}{2\tau}$$

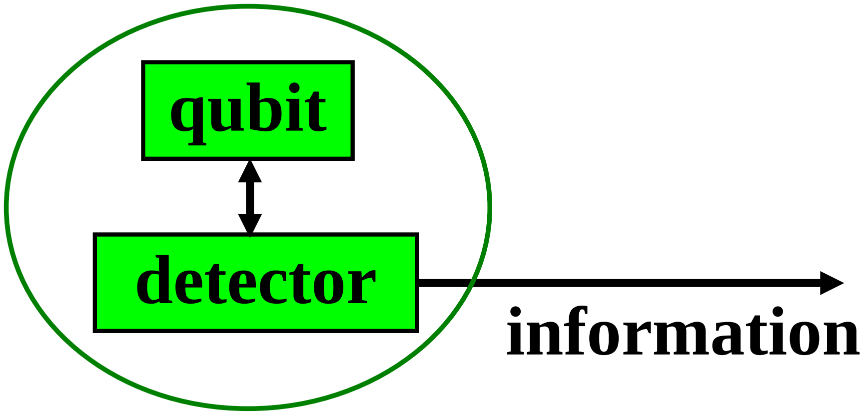
$$\Gamma = \frac{(\Delta I)^2}{8S_I} + \frac{(\Delta I)^2}{8S_I} = \frac{8\chi^2\bar{n}}{\kappa}$$

Similar to phase-sensitive case,
but separate ***I*** and ***Q*** channels

Equal contributions to ensemble dephasing from “informational” & “phase” back-actions

A.K., arXiv:1111.4016

Why not just use Schrödinger equation for the whole system?



Impossible in principle!

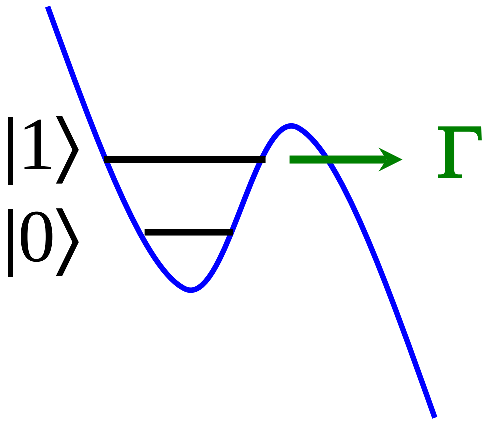
Technical reason: Outgoing information makes it an open system

Philosophical reason: Random measurement result, but deterministic Schrödinger equation

Einstein: God does not play dice (actually plays!)
Heisenberg: unavoidable quantum-classical boundary

Experiments

Partial collapse of a Josephson phase qubit



N. Katz, M. Ansmann, R. Bialczak, E. Lucero, R. McDermott, M. Neeley, M. Steffen, E. Weig, A. Cleland, J. Martinis, A. Korotkov, Science-2006

What happens if nothing happens?

Main idea:

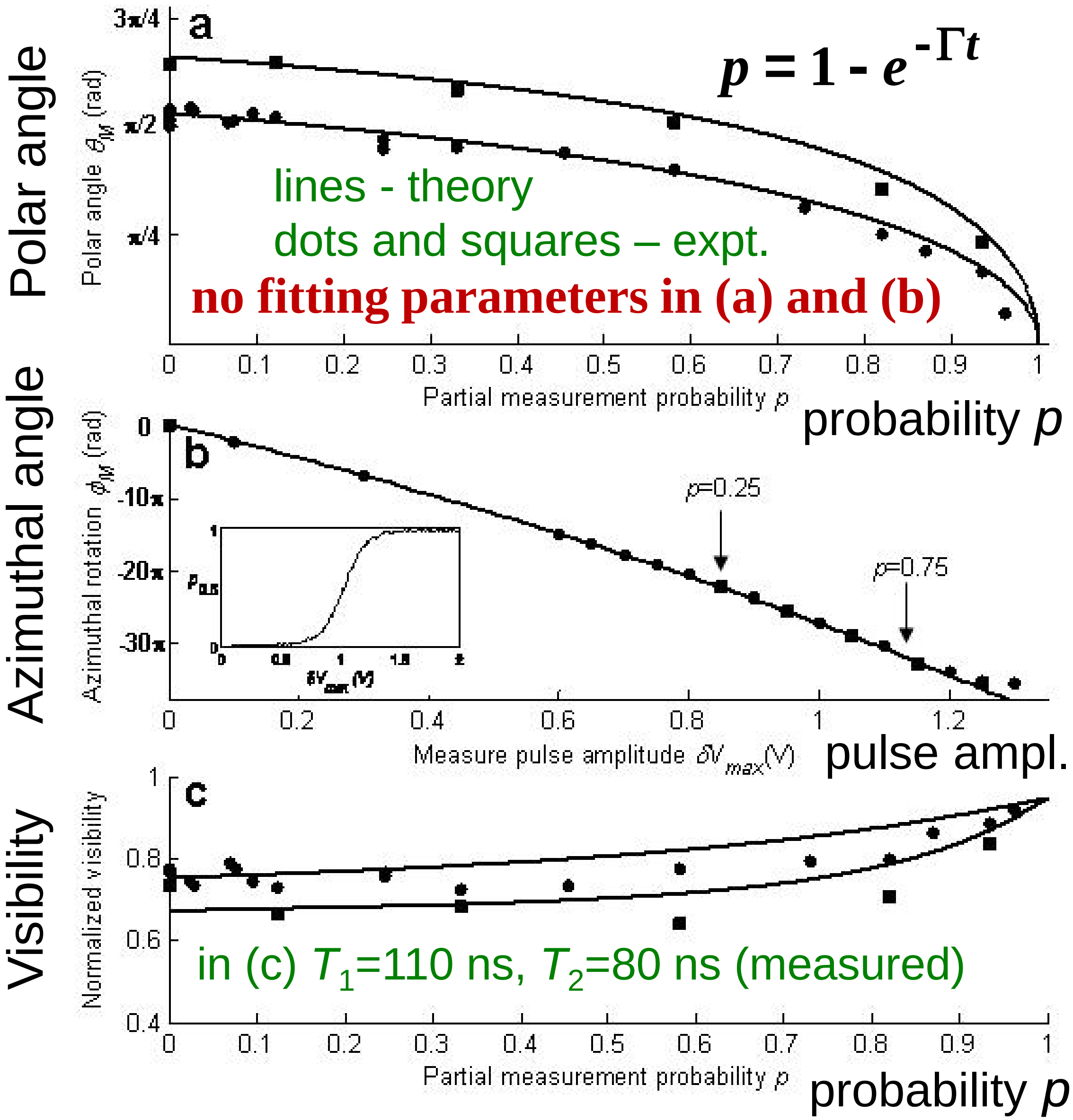
$$|\psi(0)\rangle = \alpha\,|0\rangle + \beta|1\rangle \;\to\; |\psi(t)\rangle = \begin{cases} |\mathrm{out}\rangle, & \text{if tunneled} \\ \dfrac{\alpha|0\rangle + \beta e^{-\Gamma t/2} e^{i\varphi}|1\rangle}{\mathrm{norm}}, & \text{if not tunneled} \end{cases}$$

Non-trivial:
- amplitude of state $|0\rangle$ grows without physical interaction
- finite linewidth only after tunneling

continuous null-result collapse

(idea similar to Dalibard-Castin-Molmer, PRL-1992)

Partial collapse: experimental results



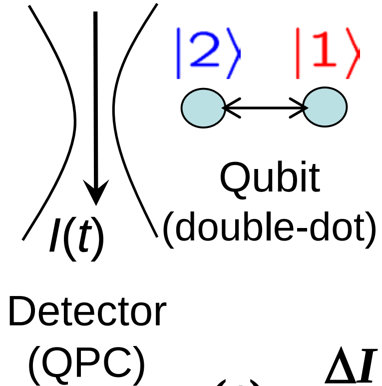
N. Katz *et al*., Science-2006

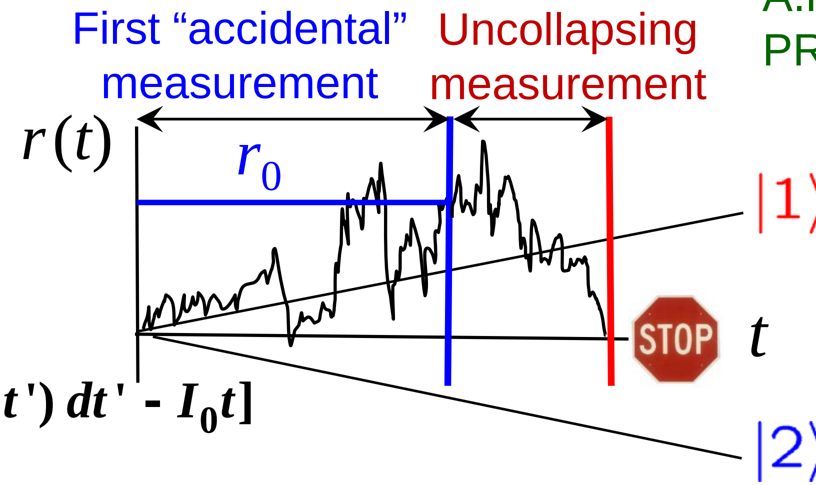
- In case of no tunneling phase qubit evolves
- Evolution is described by the Bayesian theory without fitting parameters
- Phase qubit remains coherent in the process of continuous collapse (expt. ~80% raw data, ~96% corrected for T_1,T_2)

Good confirmation of the theory

Uncollapse for qubit-QPC system (theory)

A.K. & A. Jordan, PRL-2006



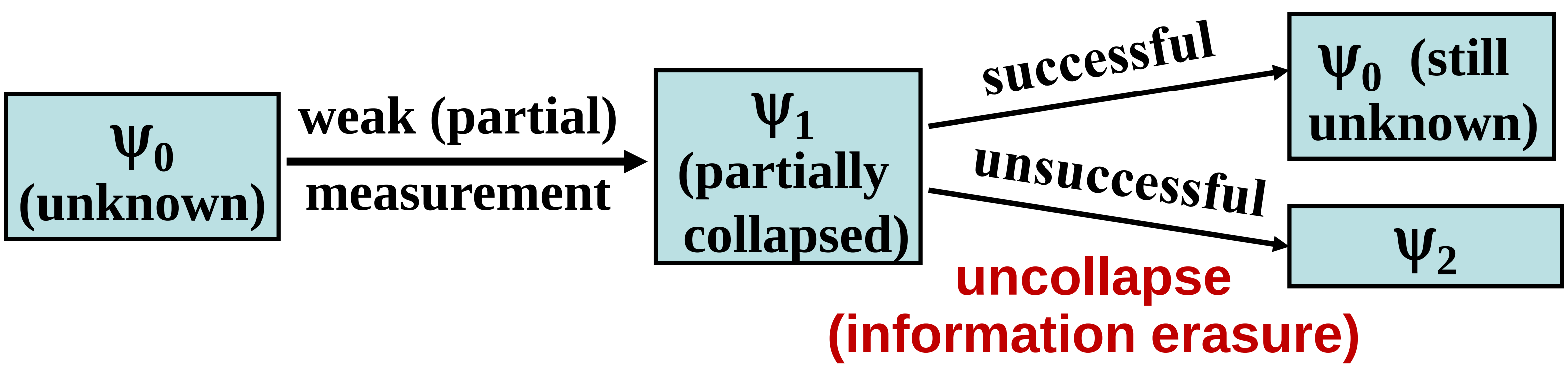


$$r(t) = \frac{\Delta I}{S_I}\left[\int_0^t I(t')\,dt' - I_0 t\right]$$

Simple strategy: continue measuring until $r(t)$ becomes zero.

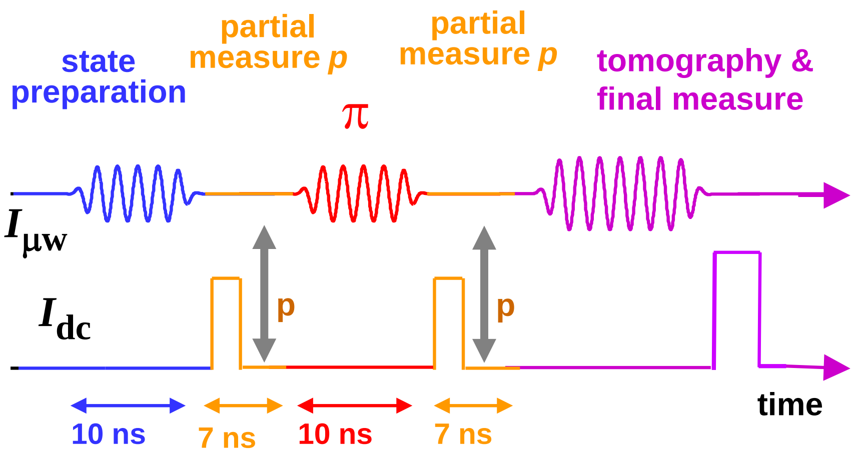
Then any unknown initial state is fully restored.

If $r = 0$ never occurs, then uncollapsing is unsuccessful.



Somewhat similar to quantum eraser of Scully and Druhl (1982)

Experiment on wavefunction uncollapse

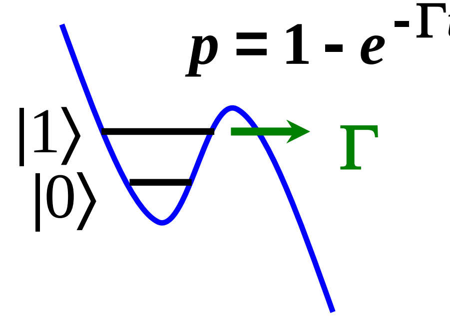


N. Katz, M. Neeley, M. Ansmann, R. Bialzak, E. Lucero, A. O'Connell, H. Wang, A. Cleland, J. Martinis, and A. Korotkov, PRL-2008

Uncollapse protocol:

- partial collapse
- π-pulse
- partial collapse (same strength)

If no tunneling for both measurements, then initial state is fully restored



$$\alpha|0\rangle + \beta|1\rangle \to \frac{\alpha|0\rangle + e^{i\phi}\beta e^{-\Gamma t/2}|1\rangle}{\text{Norm}} \to$$

$$\frac{e^{i\phi}\alpha e^{-\Gamma t/2}|0\rangle + e^{i\phi}\beta e^{-\Gamma t/2}|1\rangle}{\text{Norm}} = e^{i\phi}(\alpha|0\rangle + \beta|1\rangle)$$

phase is also restored ("spin echo")

Experimental results on the Bloch sphere

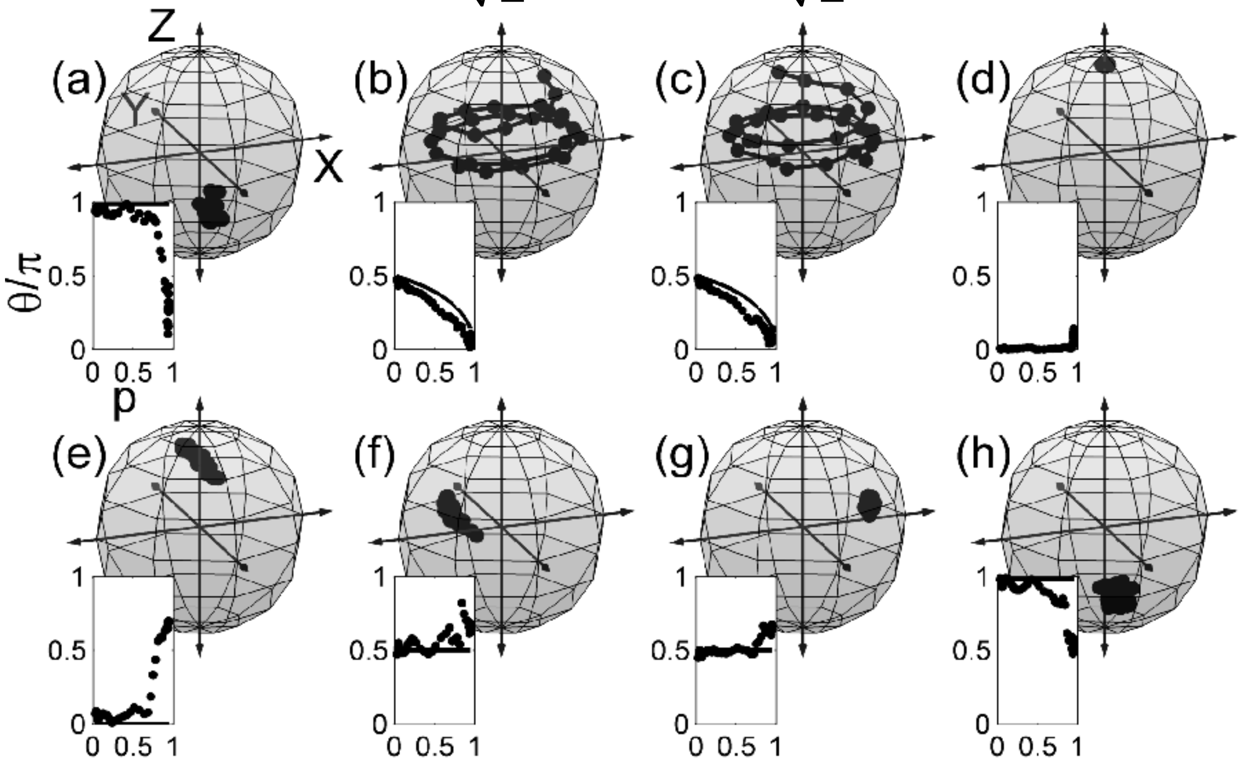
N. Katz et al., 2008

Initial state: $|1\rangle$, $\frac{|0\rangle - i|1\rangle}{\sqrt{2}}$, $\frac{|0\rangle + |1\rangle}{\sqrt{2}}$, $|0\rangle$

Partially collapsed

Uncollapsed

uncollapse works well

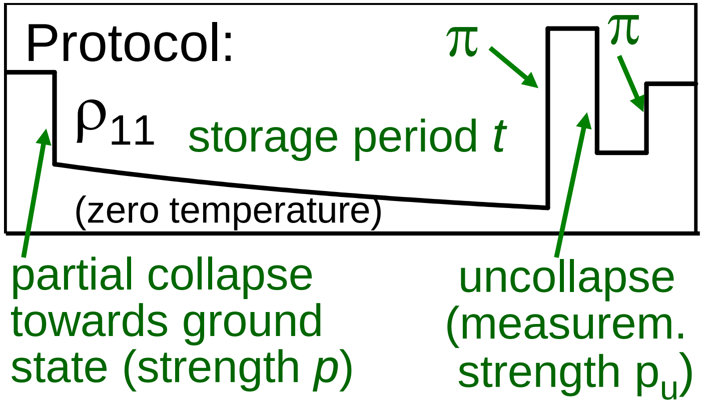


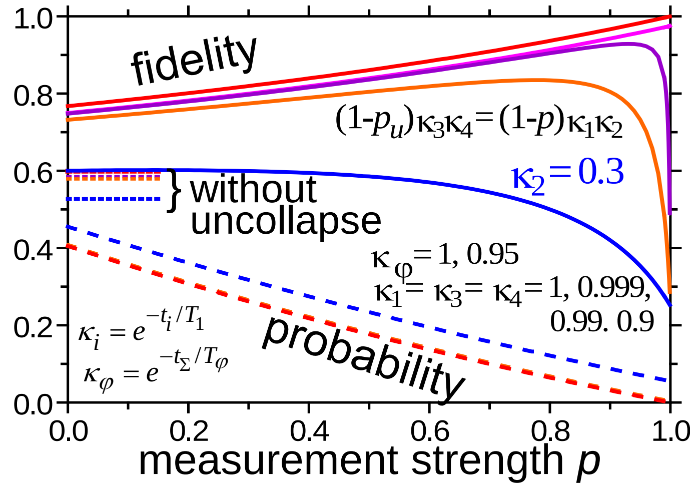
Both spin echo (azimuth) and uncollapsing (polar angle)

Difference: spin echo – undoing of an <u>unknown unitary</u> evolution,
uncollapsing – undoing of a <u>known, but non-unitary</u> evolution

Uncollapse in continuous qubit measurement: K. Murch et al., Nature-2013

Decoherence suppression by uncollapse





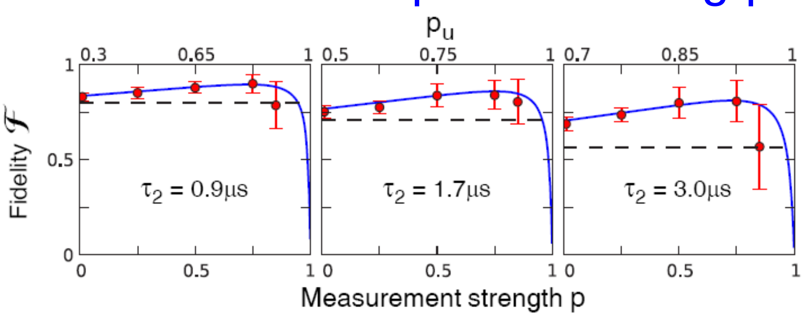
Theory: A.K & Keane, PRA-2010

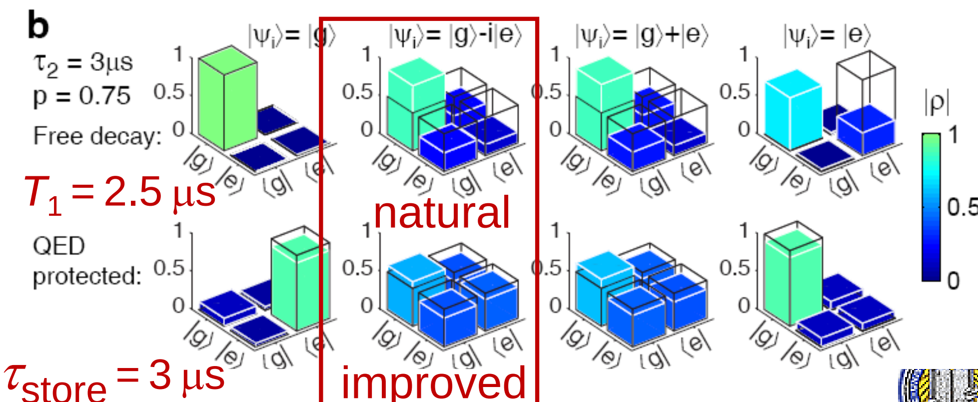
"Sleeping beauty" analogy

First realized in optics Lee et al., Opt. Expr.-2011

Also used for entanglement preservation Kim et al., Nature Phys.-2012

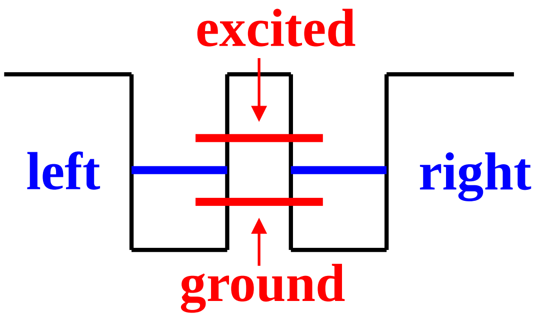
Realization with superconducting phase qubits Zhong et al., Nature Comm.-2014



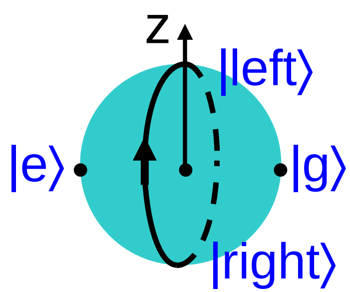


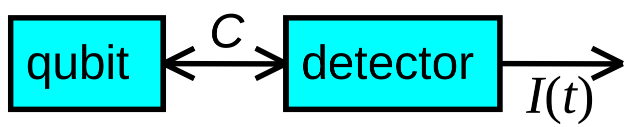
Increases effective T_1 by 3x

Non-decaying (persistent) Rabi oscillations



- Relaxes to the ground state if left alone (low-T)
- Becomes fully mixed if coupled to a high-T envir.
- Oscillates persistently between left and right if (weakly) measured continuously (“reason”: attraction to left/right states)

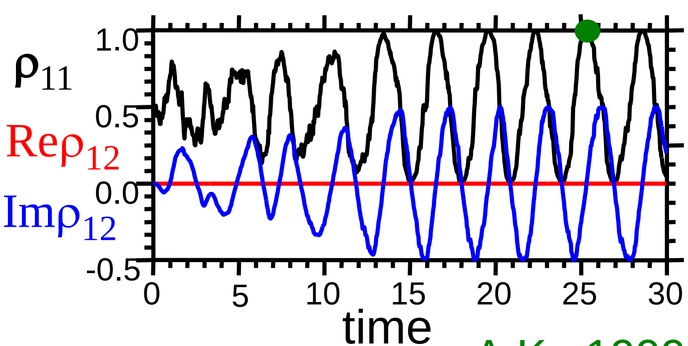




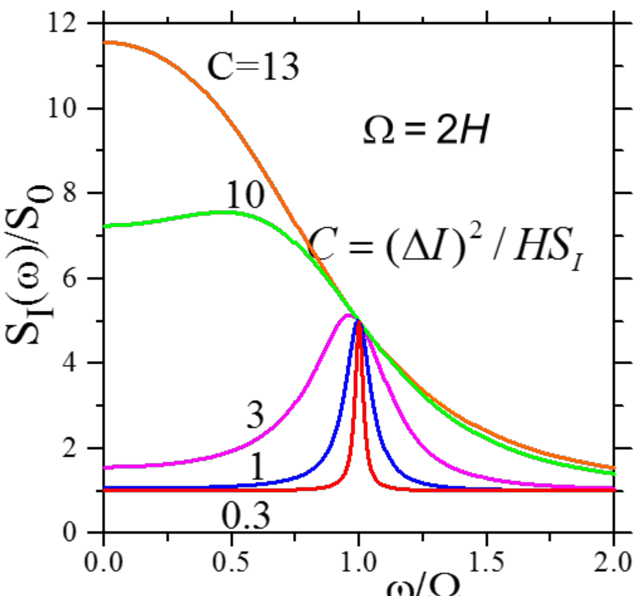
A.K., LT-1999, 2001
A.K.-Averin, 2001

Integral under peak
$\Leftrightarrow \langle z^2 \rangle$ (Bloch sph.)

perfect Rabi:
$\langle z^2\rangle = \langle \cos^2 \rangle = 1/2$
quantum: $\langle \boldsymbol{z}^2 \rangle = 1$



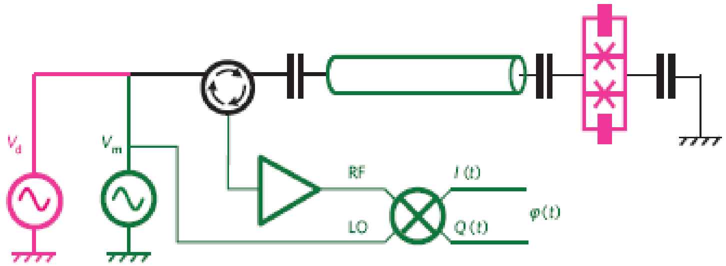
A.K., 1999

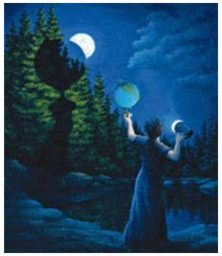


Rabi frequency Ω

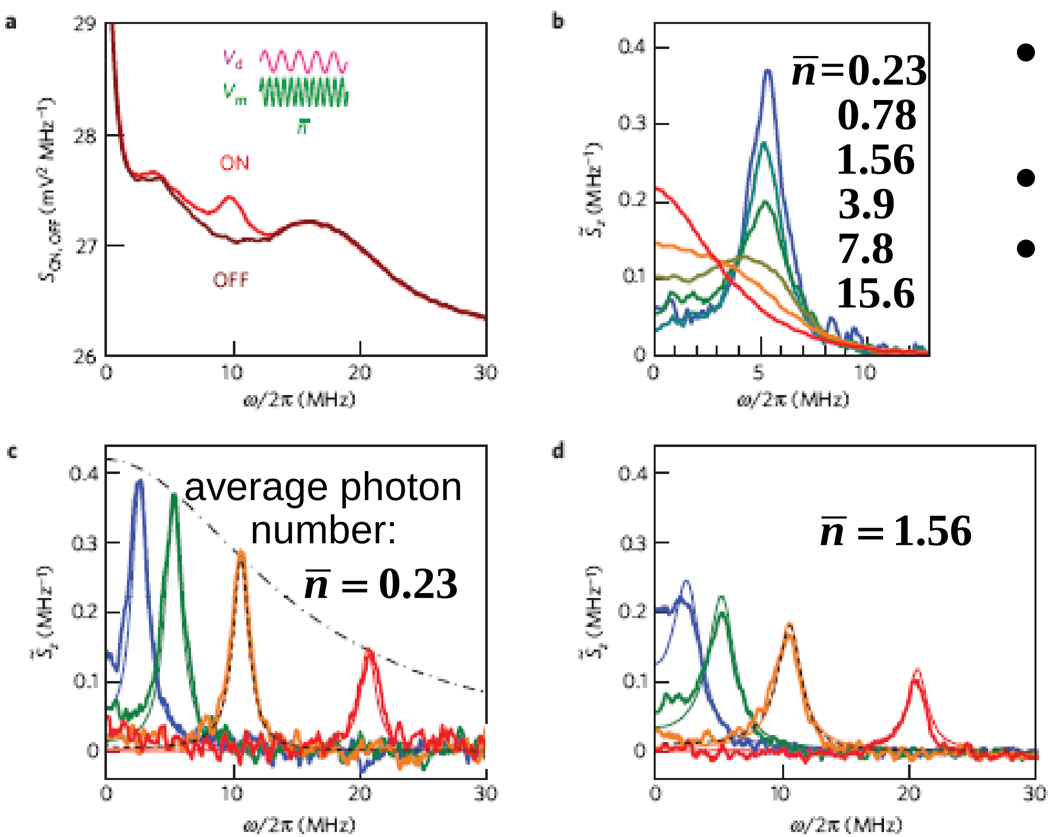
classical limit:
$$\int S_{\text{peak}} df < 8/\pi^2$$
Ruskov, A.K., Mizel (2006)

Continuous monitoring of Rabi oscillations





A. Palacios-Laloy, F. Mallet, F. Nguyen, P. Bertet, D. Vion, D. Esteve, and A. Korotkov, Nature Phys. (2010)



- superconducting qubit (transmon) in circuit QED setup
- microwave reflection from cavity
- driven Rabi oscillations ($|g\rangle \leftrightarrow |e\rangle$)

Pre-amplifier noise temperature $T_N = 4$ K

$$\frac{1}{1+\dfrac{2T_N}{\hbar\omega}} \approx 0.03$$

quantum efficiency $\quad \eta = \dfrac{\Delta S}{4S} \sim 10^{-2}$

Theory by dashed lines, very good agreement

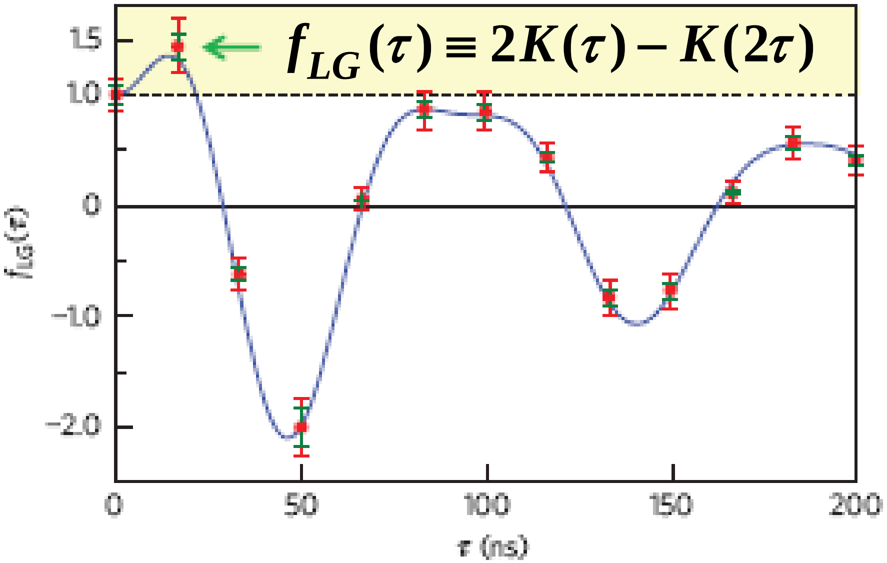
Violation of Leggett-Garg inequalities

A. Palacios-Laloy et al., 2010

In time domain

Rescaled to qubit *z*-coordinate $K(\tau) \equiv \langle z(t)\, z(t+\tau)\rangle$

$$K(\tau_1)+K(\tau_2)-K(\tau_1+\tau_2)\le 1 \quad \Rightarrow 2K(\tau)-K(2\tau)\le 1$$



$f_{LG}(0)=K(0)=\langle z^2\rangle$

$\langle z^2\rangle = 1.01 \pm 0.15$

$f_{LG}(17\,\text{ns}) = 1.44 \pm 0.12$

Ideal $f_{\text{LG,max}}$=1.5

Standard deviation $\sigma = 0.065$

$\Rightarrow$ violation by 5σ

Many later experiments on Leggett-Garg ineq. violation, incl. optics and NMR

M. Goggin et al., PNAS-2011
J. Dressel et al., PRL-2011
G. Walhder et al., PRL-2011

V. Athalye et al., PRL-2011
A. Souza et al., NJP-2011
G. Knee et al., Nat. Comm.-2011

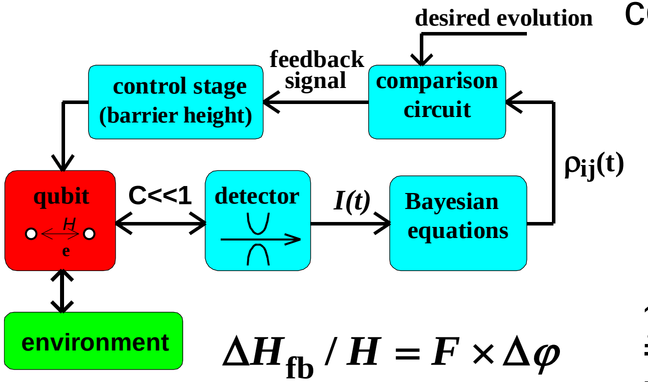
J. Groen et al., PRL-2013
(s/c, DiCarlo's group)

Quantum feedback to stabilize Rabi oscillations

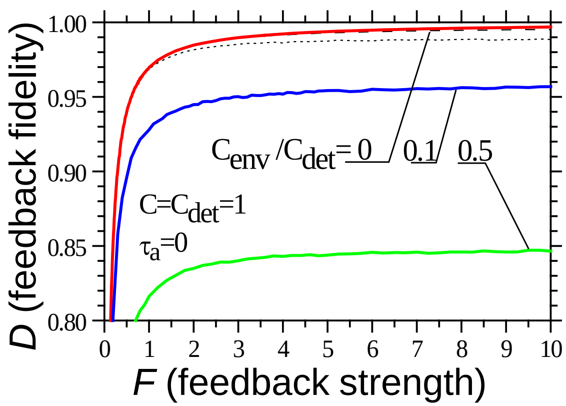
Bayesian

Best but very difficult

(monitor quantum state and control deviation)



$$\Delta H_{fb} / H = F \times \Delta\varphi$$



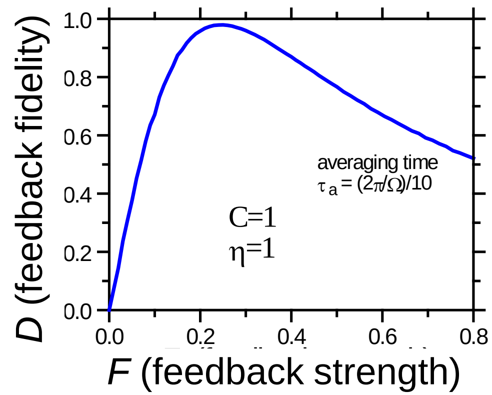
R. Ruskov & A.K., 2002

"Direct"

Similar to Wiseman-Milburn (1993, optics)

(apply measurement signal to control with minimal processing)

$$\Delta H_{fb} / H = F \sin(\Omega t)$$
$$\times \left(\frac{I(t) - I_0}{\Delta I / 2} - \cos \Omega t \right)$$



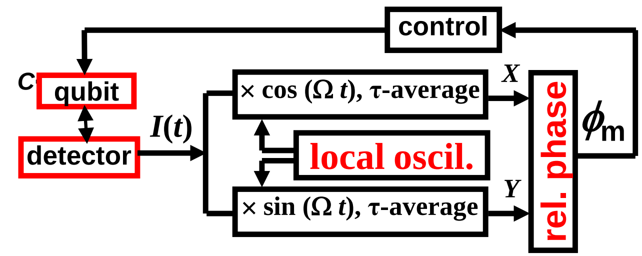
R. Ruskov & A.K., 2002

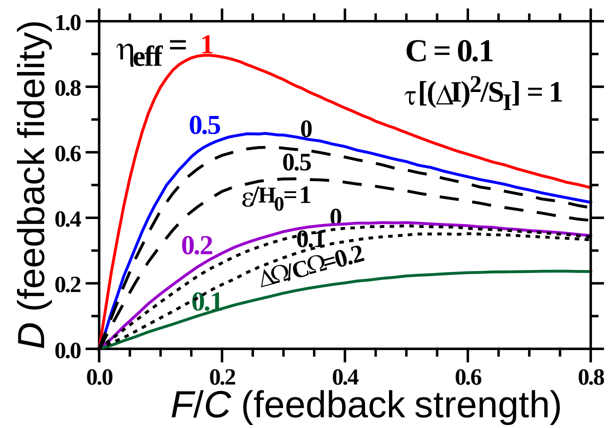
"Simple"

Imperfect but simple

(do as in usual classical feedback)

$$\frac{\Delta H_{fb}}{H} = F \times \phi_m$$





A.K., 2005

Berkeley-2012 experiment: "direct" and "simple"

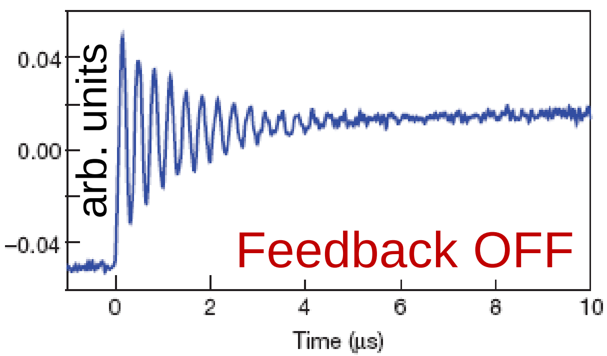
Quantum feedback of Rabi oscillations

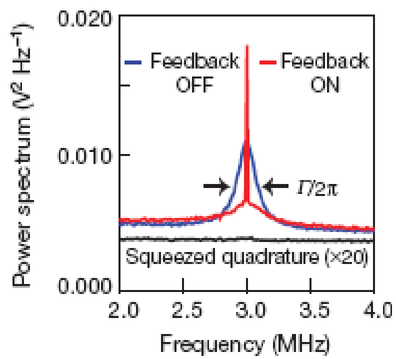
R. Vijay, C. Macklin, D. Slichter, S. Weber, K. Murch, R. Naik, A. Korotkov, and Irfan Siddiqi, Nature-2012

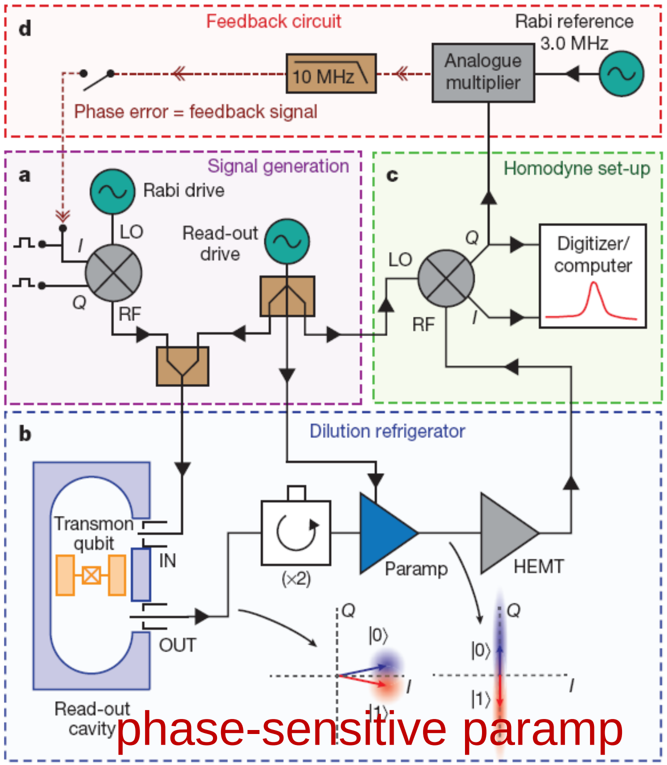
(quantum feedback with atoms, stabilizing photon number: C. Sayrin, … S. Haroche, Nature-2011)

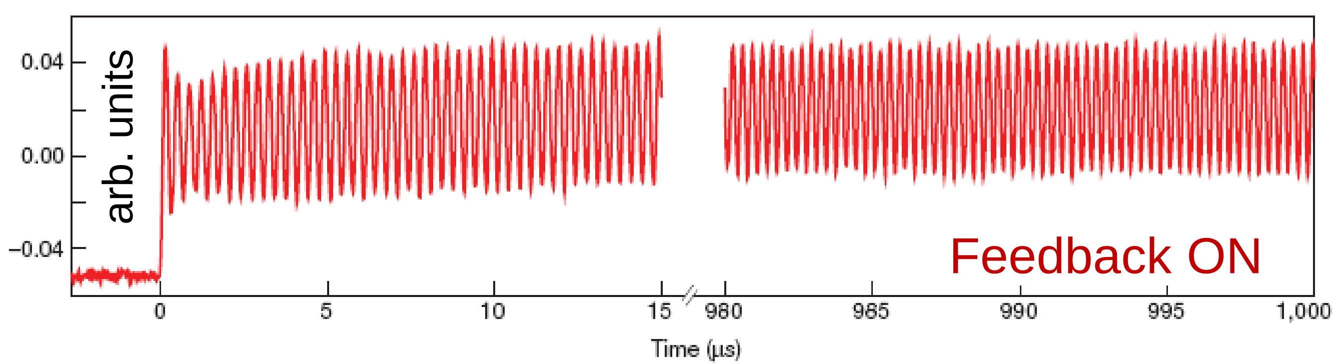
Simple idea: $I(t) \sim \cos(\Omega_R t - \theta_{ERR}) + \text{noise}$

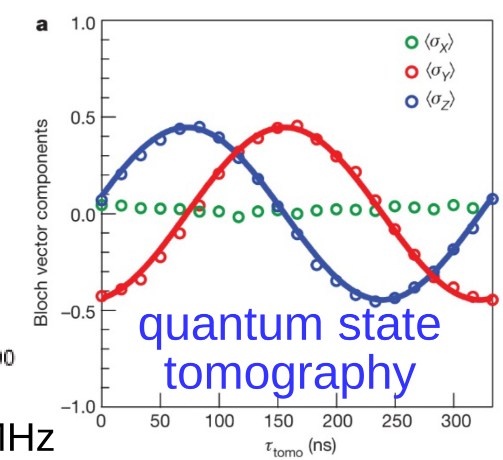
$$\Delta\Omega_R / \Omega_R = -F \sin(\theta_{ERR}), \;\; \sin(\theta_{ERR}) \sim \overline{I(t)\sin(\Omega_R t)}$$





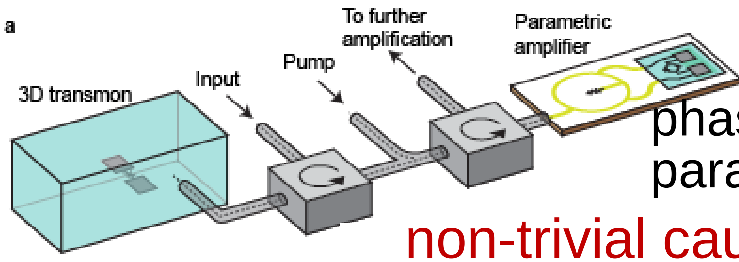






Rabi freq. 3 MHz, paramp BW 10 MHz, cavity LW 8 MHz, env. deph. 0.05 MHz

Quantum trajectories of transmon qubit



phase-sensitive paramp $\eta = 0.49$

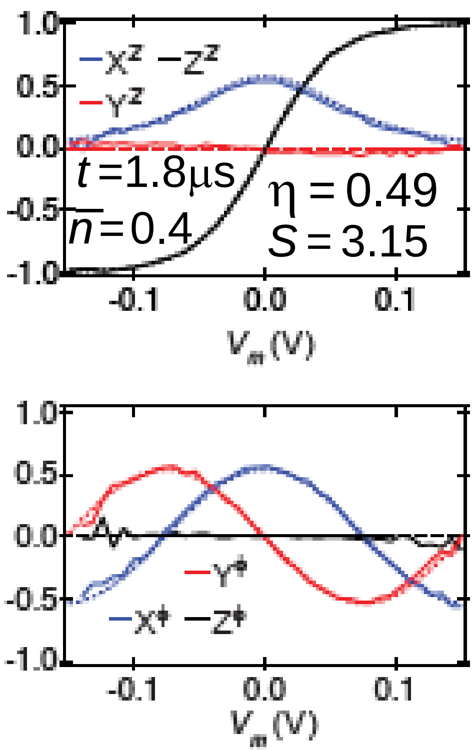
non-trivial causality

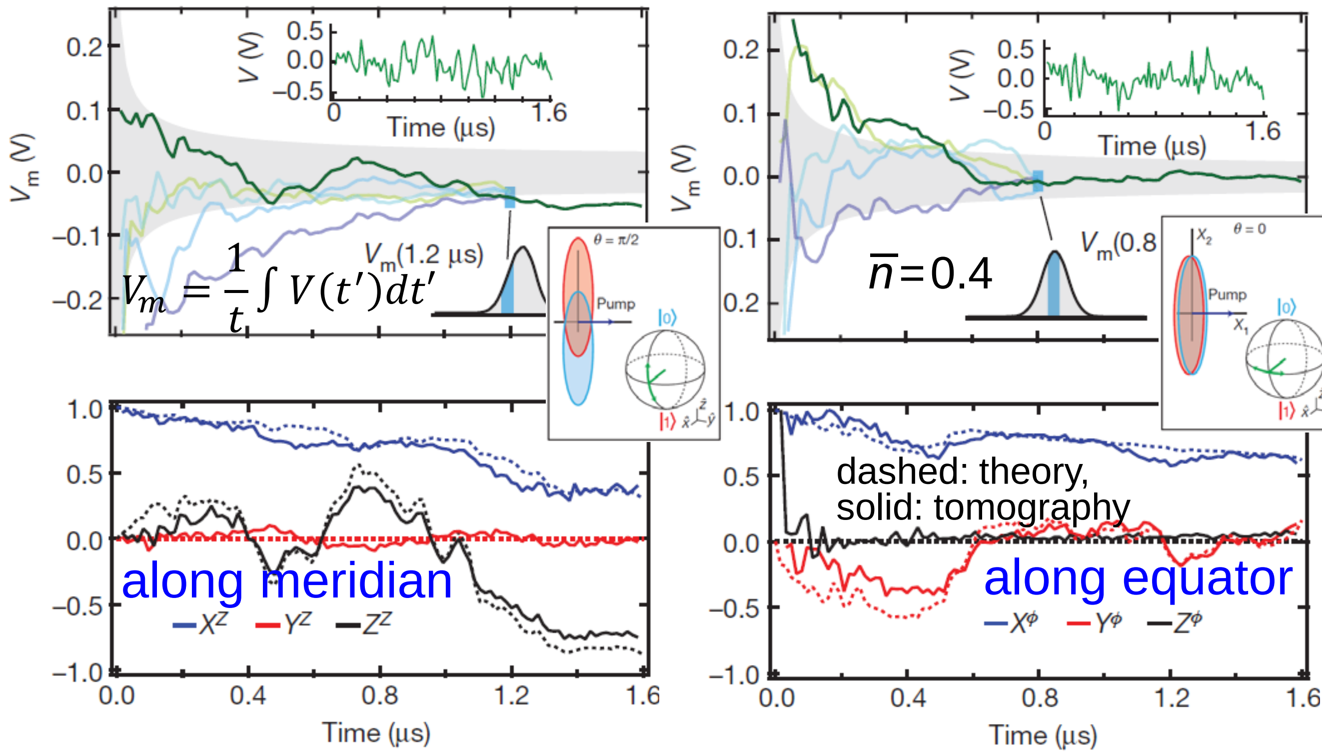
K. Murch, S. Weber, C. Macklin, & I. Siddiqi, Nature-2013

Coupling 0.52 MHz
Cavity LW 10.8 MHz
Paramp BW 20 MHz

Partial measurement: expt. vs. Bayesian theory

Individual quantum trajectories: experiment vs. Bayesian theory

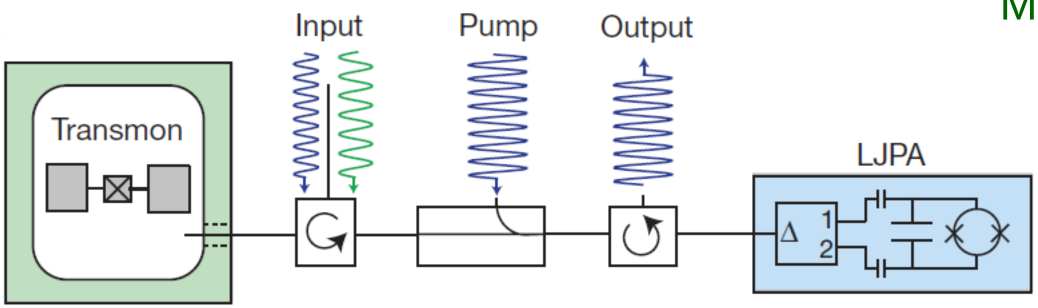




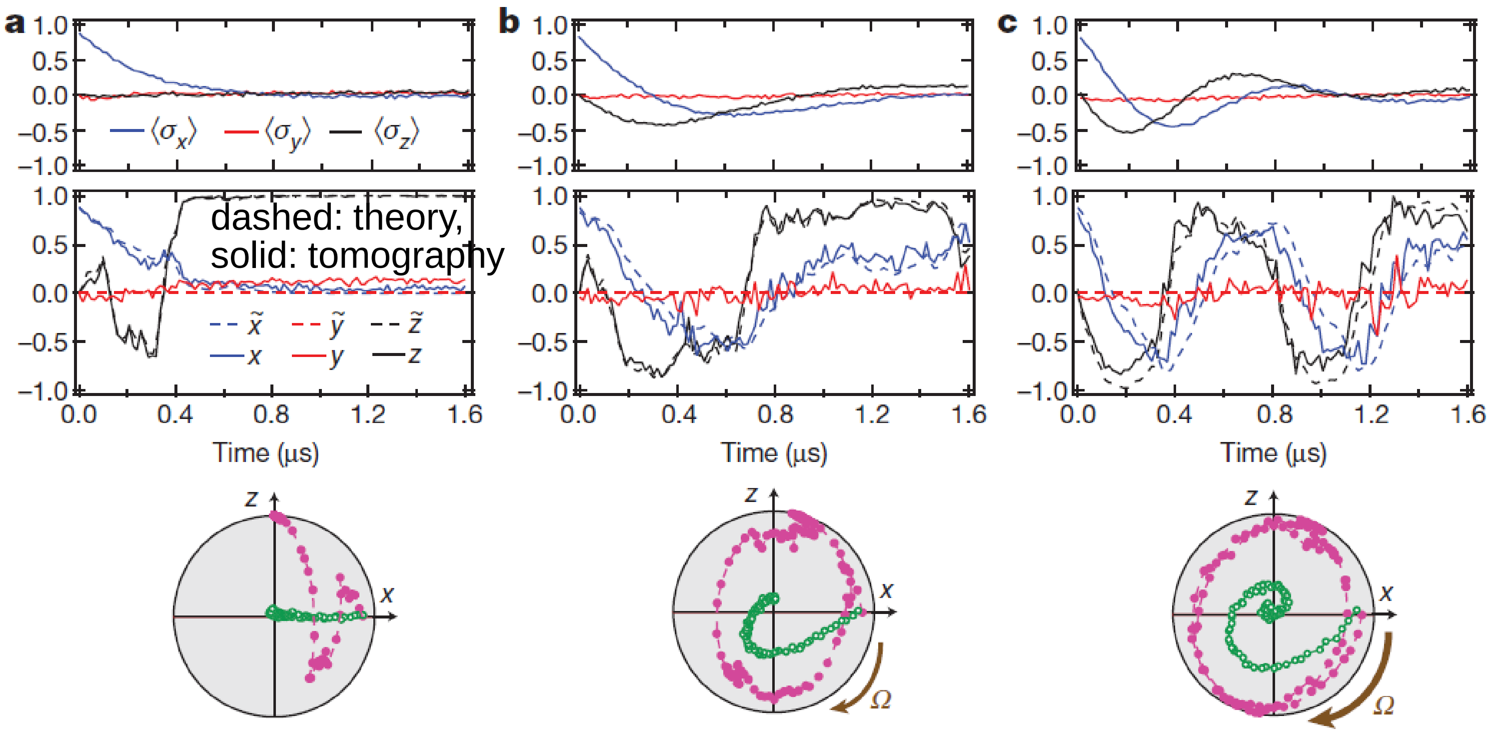
Good agreement with simple Bayesian theory (dashed)

Quantum trajectories with Rabi drive

S. Weber, Chantasri, Dressel, Jordan, Murch, and I. Siddiqi, Nature-2014



$\eta_{tot} = 0.4$

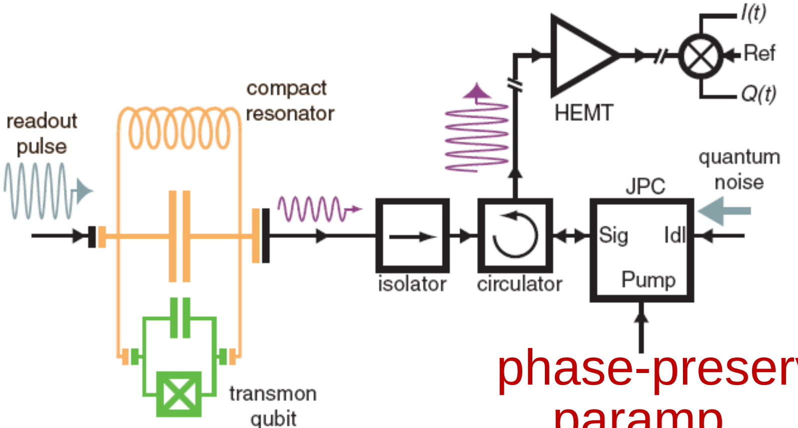


Ensemble-averaged

Individual

Good agreement with simple Bayesian theory

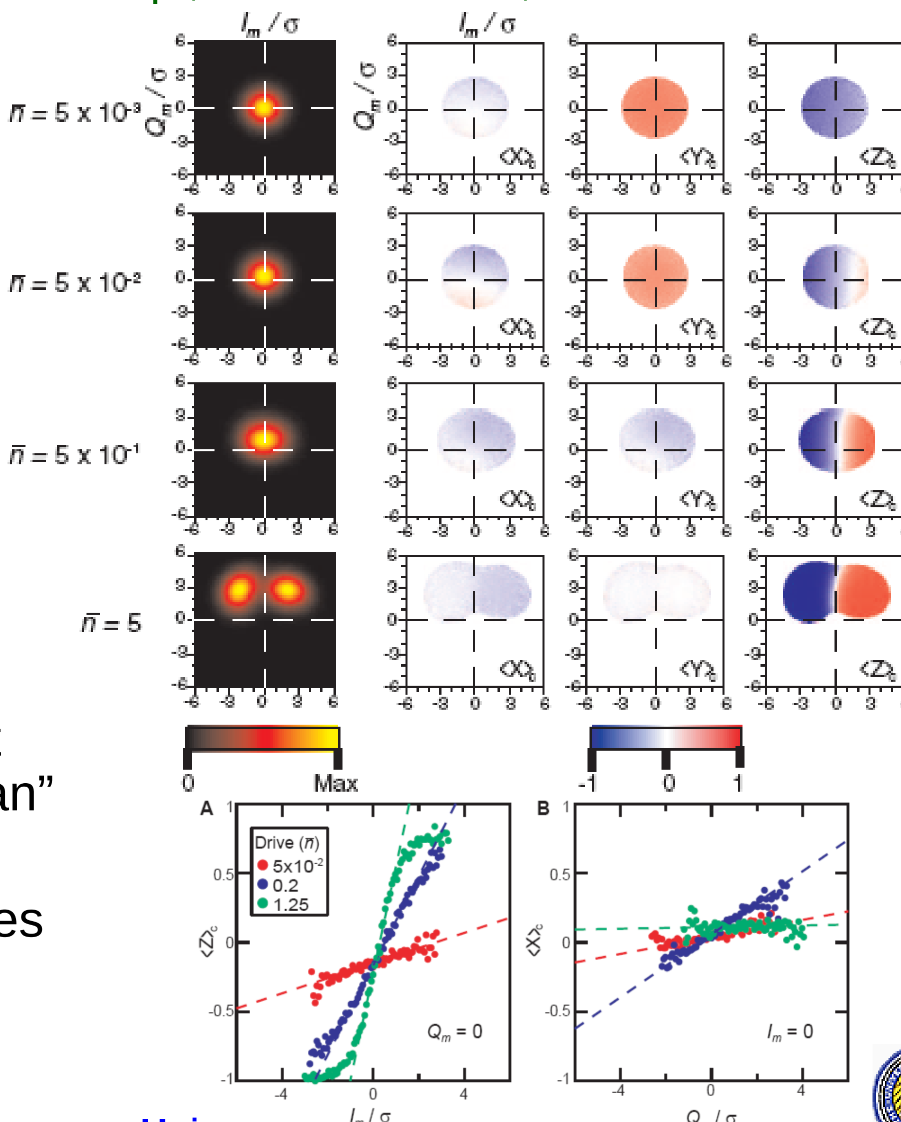
Phase-preserving continuous measurement



phase-preserving paramp

$\eta = 0.2$

M. Hatridge, Shankar, Mirrahimi, Schackert, Geerlings, Brecht, Sliwa, Abdo, Frunzio, Girvin, Schoelkopf, and M. Devoret, Science-2013



Protocol:

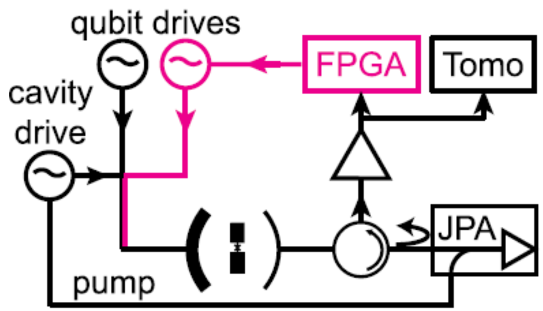
1) Start with |0>+|1>
2) Measure with controlled strength
3) Tomography of resulting state

Experimental findings:

- Result of *I*-quadrature measurement determines state shift along “meridian” of the Bloch sphere
- *Q*-quadrature meas. result determines shift along “parallel” (within equator)
- Agrees well with the theory

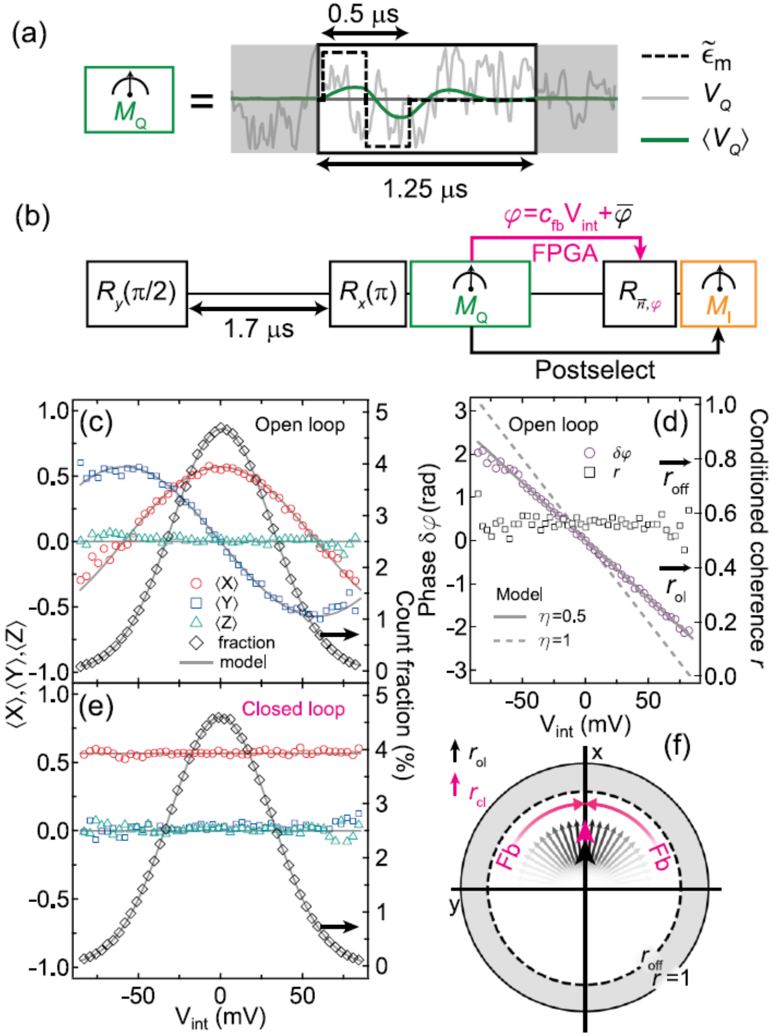
Suppression of measurement-induced dephasing by feedback (undoing motion along equator)

G. de Lange, Riste, Tiggelman, Eichler, Tornberg, Johansson, Wallraff, Schouten, and L. DiCarlo, PRL-2014



$\eta = 0.5$

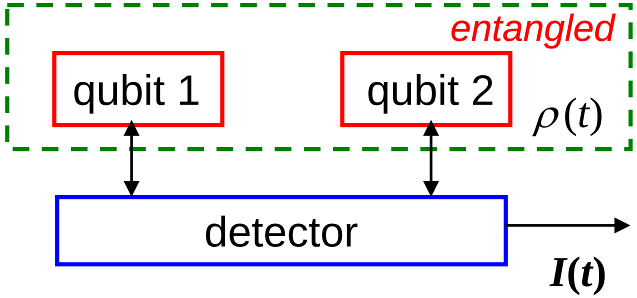
Phase-sensitive amplifier, measure non-informational quadrature (back-action is along parallels)

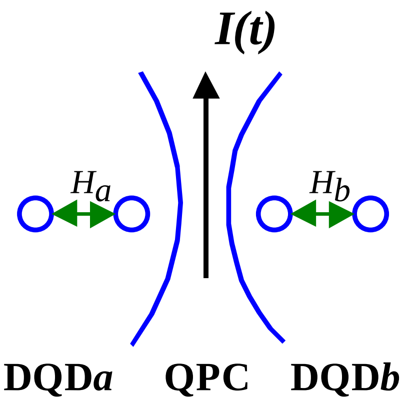


Idea: collect measurement signal (with weight function) to find back-action; then undo

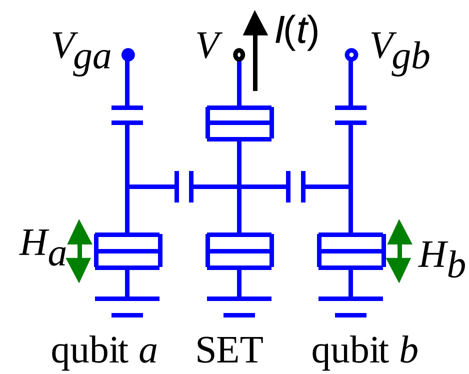
"refocusing" (feedback) increases qubit coherence $2|\rho_{01}|$ from 0.40 to 0.56

Entanglement by measurement (theory)





R. Ruskov & A.K., 2003



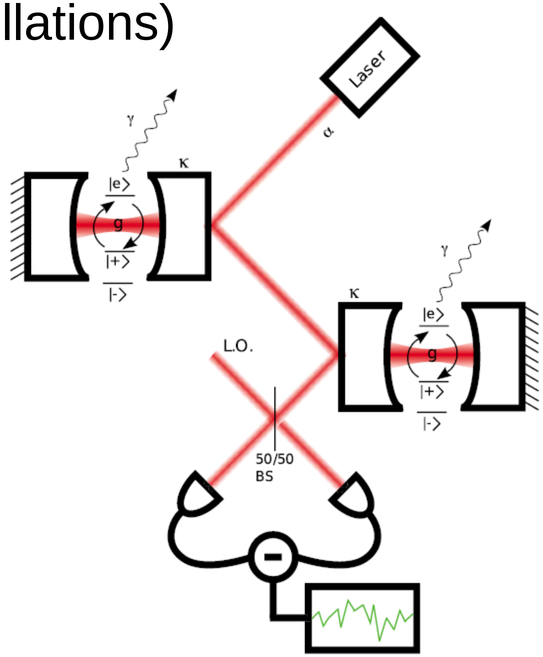
same current for |01⟩ and |10⟩
⇒ entangles gradually

$$\frac{|0\rangle+|1\rangle}{\sqrt{2}}\otimes\frac{|0\rangle-|1\rangle}{\sqrt{2}}\to\frac{|10\rangle-|01\rangle}{\sqrt{2}}$$

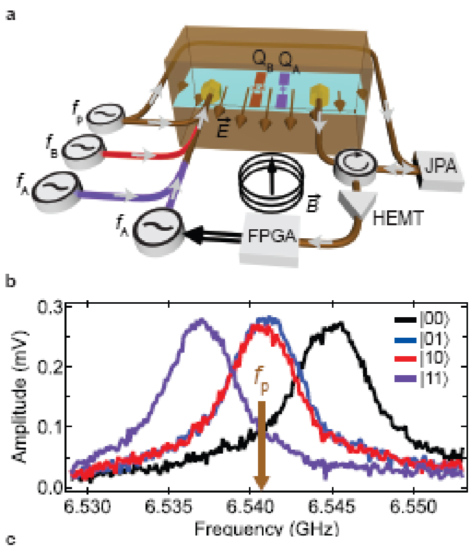
(probabilistically, even with Rabi oscillations)

Similar proposal in optics

J. Kerckhoff, L. Bouten, A. Silberfarb, and H. Mabuchi, 2009

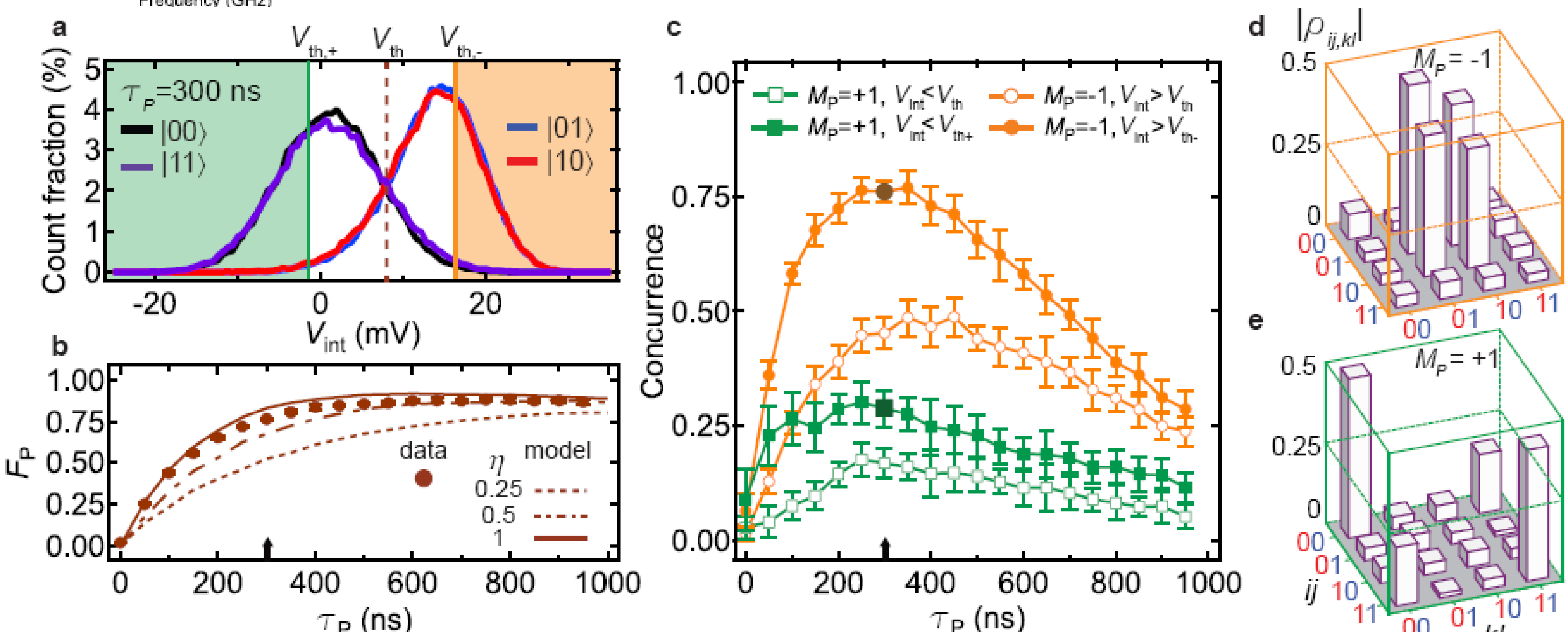


Entanglement by measurement (expt.)



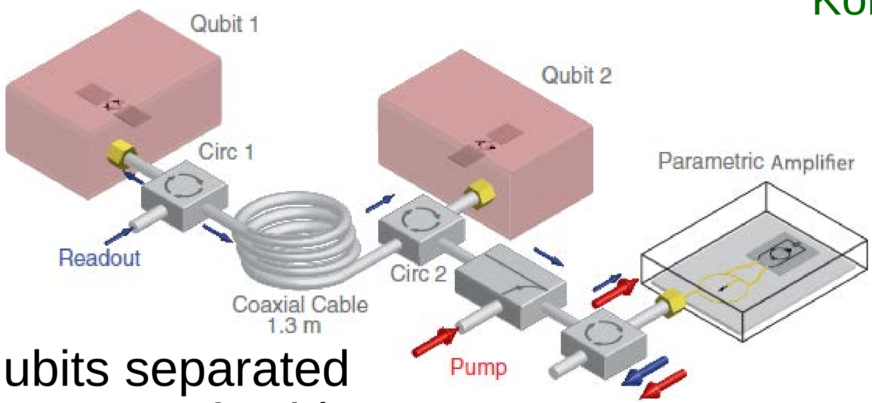
D. Riste, Dukalski, Watson, de Lange, Tiggelman, Blanter, Lehnert, Schouten, and L. DiCarlo, Nature-2013

- Two superconducting qubits in the same resonator, indistinguishable $|01\rangle$ and $|10\rangle$
- Max. concurrence 0.77
- Trick: $|00\rangle$ and $|11\rangle$ are only slightly distinguishable
- Max. deterministic concurrence 0.34
- Race against decoherence (η is not very important)

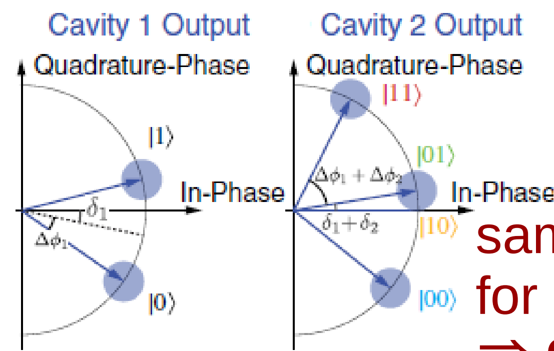


Measurement-induced entanglement of remote qubits

N. Roch, Schwartz, Motzoi, Macklin, Vijay, Eddins, Korotkov, Whaley, Sarovar, I. Siddiqi, PRL- 2014

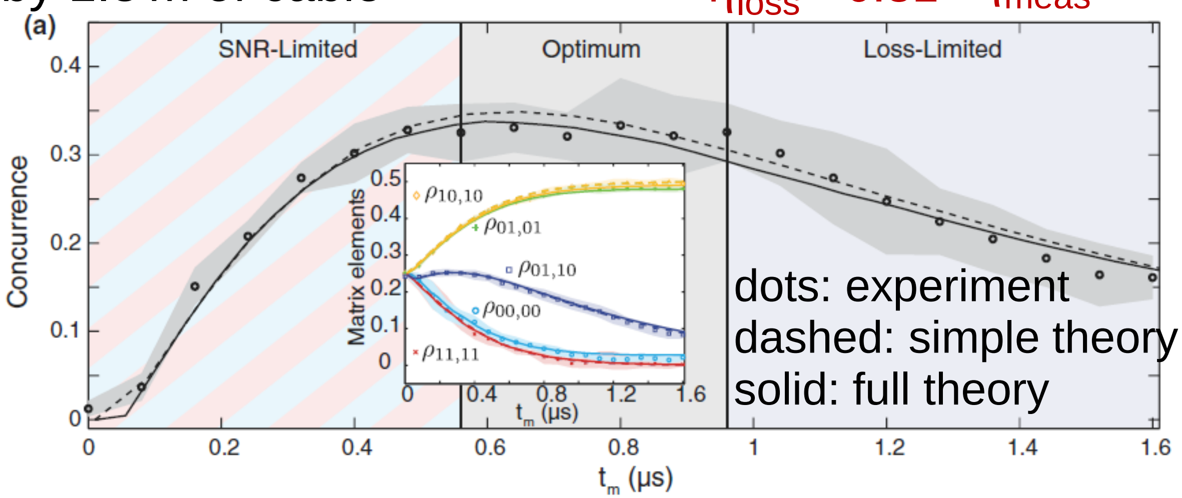


qubits separated by 1.3 m of cable

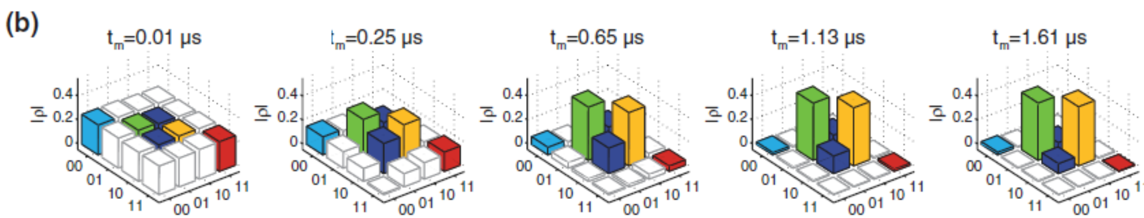


same output signal for $|01\rangle$ and $|10\rangle$
$\Rightarrow$ **entanglement**

$\eta_{loss} = 0.81$ $\eta_{meas} = 0.4$



dots: experiment
dashed: simple theory
solid: full theory



- First demonstration of remote entanglement of superconducting qubits
- Remote entanglement is much more difficult than local (η_{loss} is very important)
- Simple theory is close to full quantum trajectory, fits well experimental data.

Simultaneous measurement of non-commuting observables of a qubit

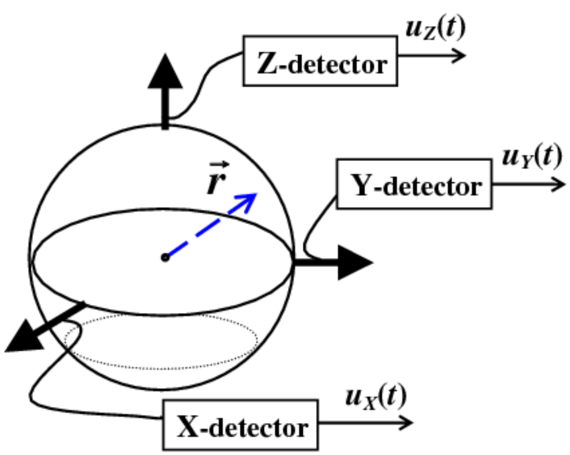
For continuous measurement, nothing forbids simultaneous measurement of non-commuting observables

Very simple quantum Bayesian description: just add terms for evolution

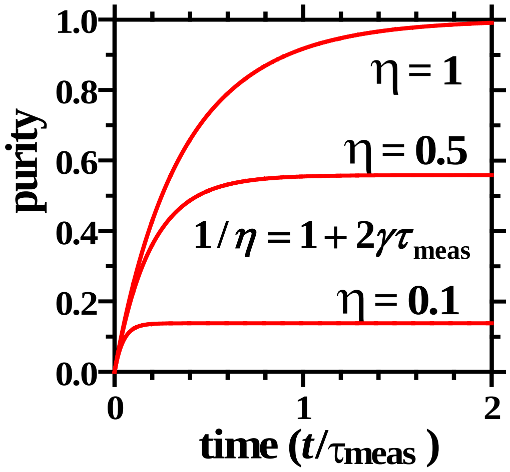
Measurement of three complementary observables for a qubit

Ruskov, A.K., Molmer, PRL-2010

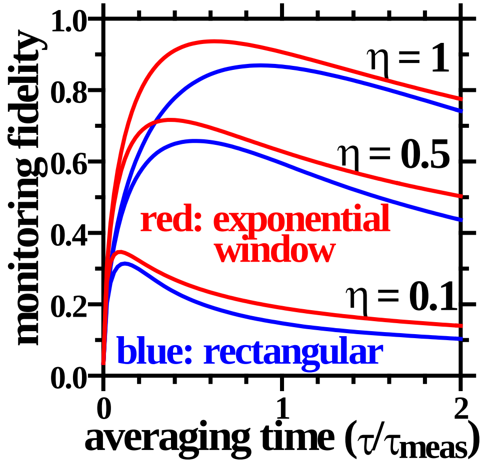
Evolution: $$\frac{d\vec{r}}{dt} = -2\gamma\,\vec{r} + a\{\vec{u}(t)(1-r^2) - [\vec{r}\times[\vec{r}\times\vec{u}(t)]]\}$$ diffusion over Bloch sphere



state purification



simple monitoring

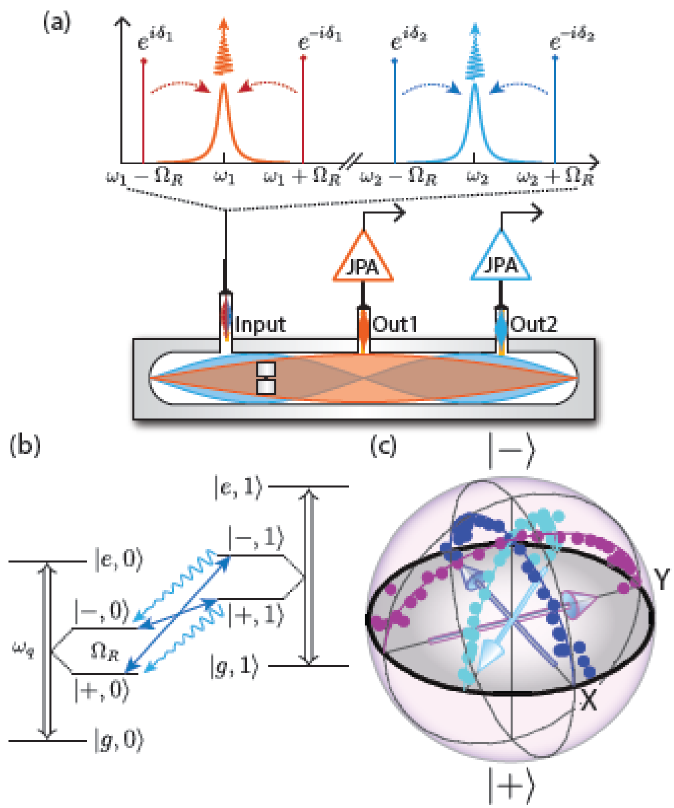


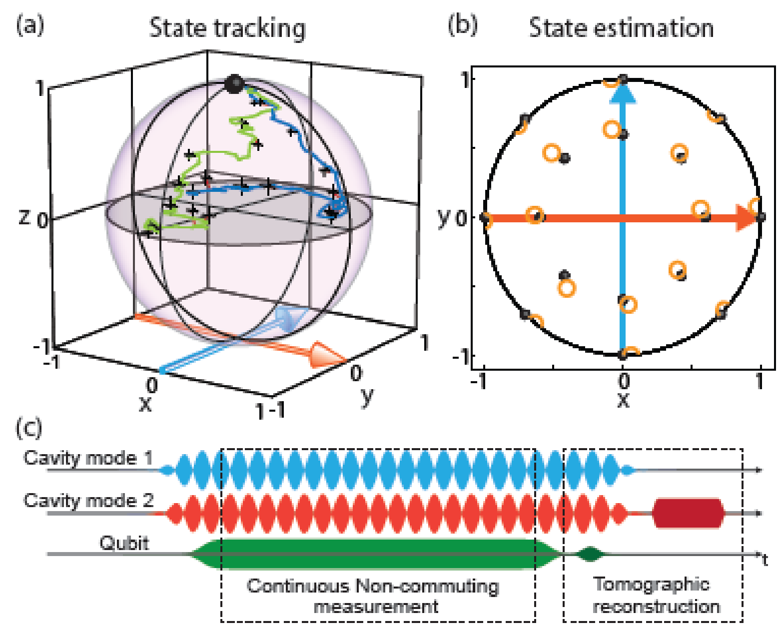
Until recently it was unclear how to realize experimentally

Simultaneous measurement of σ_x and σ_z

Actually, any $\sigma_z \cos\varphi + \sigma_x \sin\varphi$

S. Hacohen-Gourgy, L. Martin, E. Flurin, V. Ramasesh, B. Whaley, and I. Siddiqi, Nature-2016





quantum trajectory theory for simulations

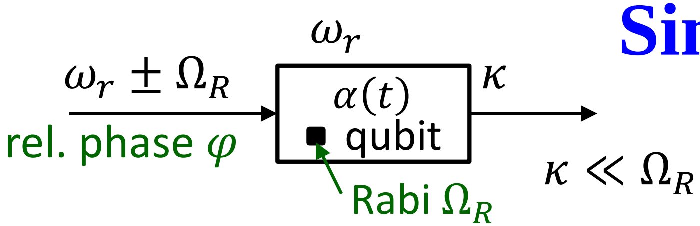
- Measurement in rotating frame of fast Rabi oscillations (40 MHz)
- Double-sideband rf wave modulation with the same frequency
- Two resonator modes for two channels

$$\Omega_{\text{Rabi}} = \Omega_{\text{SB}} = 2\pi \times 40\ \text{MHz}$$

$$\kappa/2\pi = 4.3 \text{ and } 7.2\ \text{MHz}$$

$$\Gamma_1^{-1} = \Gamma_2^{-1} = 1.3\ \mu\text{s}$$

$$\boldsymbol{\Gamma \ll \kappa \ll \Omega_{\text{Rabi}}}$$



Simple physical picture

Physical qubit (Rabi Ω_R)

$$z_{\rm ph}(t) = r_0 \cos(\Omega_R t + \phi_0)$$
$$x_{\rm ph}(t) = r_0 \sin(\Omega_R t + \phi_0)$$
$$y_{\rm ph}(t) = y_0$$

This modulates resonator frequency

$$\omega_r(t) = \omega_r^b + \chi r_0 \cos(\Omega_R t + \phi_0)$$

Drive with modulated amplitude

$$A(t) = \varepsilon \sin(\Omega_R t + \varphi)$$

Then evolution of field $\alpha(t)$ is

$$\dot{\alpha} = -i\chi r_0 \cos(\Omega_R t + \phi_0)\, \alpha - i\varepsilon \sin(\Omega_R t + \varphi) - \frac{\kappa}{2}\alpha$$

Now solve this differential equation

Fast oscillations (neglect κ)

$$\Delta\alpha(t) = i\frac{\varepsilon}{\Omega_R}\cos(\Omega_R t + \varphi)$$

Insert, then slow evolution is

$$\dot{\alpha}_s = \frac{\chi\varepsilon}{2\Omega_R}\underbrace{r_0 \cos(\phi_0 - \varphi)} - \frac{\kappa}{2}\alpha_s$$

Thus, slow evolution is determined by <u>effective</u> qubit (in rotating frame),

$$z = r_0 \cos(\phi_0),\ x = r_0 \sin(\phi_0),\ y = y_0,$$

measured along axis φ (basis $|1_\varphi\rangle$, $|0_\varphi\rangle$)

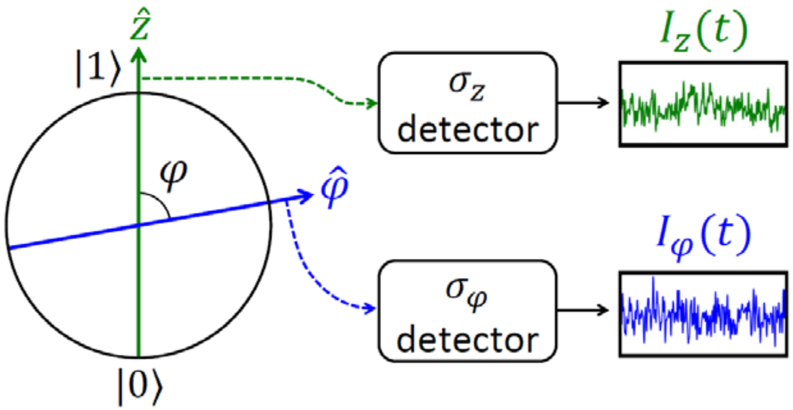
$$r_0 \cos(\phi_0 - \varphi) = {\rm Tr}[\sigma_\varphi \rho]$$
$$\sigma_\varphi = \sigma_z \cos\varphi + \sigma_x \sin\varphi$$

Stationary state $\alpha_{\rm st,1} = -\alpha_{\rm st,0} = \dfrac{\chi\varepsilon}{\Omega_R \kappa}$

From this point, usual Bayesian theory

J. Atalaya, S. Hacohen-Gourgy, L. Martin, I. Siddiqi, and A.K., arXiv:1702.08077

Correlators in simultaneous measurement of non-commuting qubit observables



J. Atalaya, S. Hacohen-Gourgy, L. Martin, I. Siddiqi, and A.K., arXiv:1702.08077

$$K_{ij}(\tau) = \langle I_j(t+\tau)\, I_i(t)\rangle$$

"Collapse recipe": replace continuous measurement with projective at t and $t+\tau$, use ensemble-averaged evolution in between

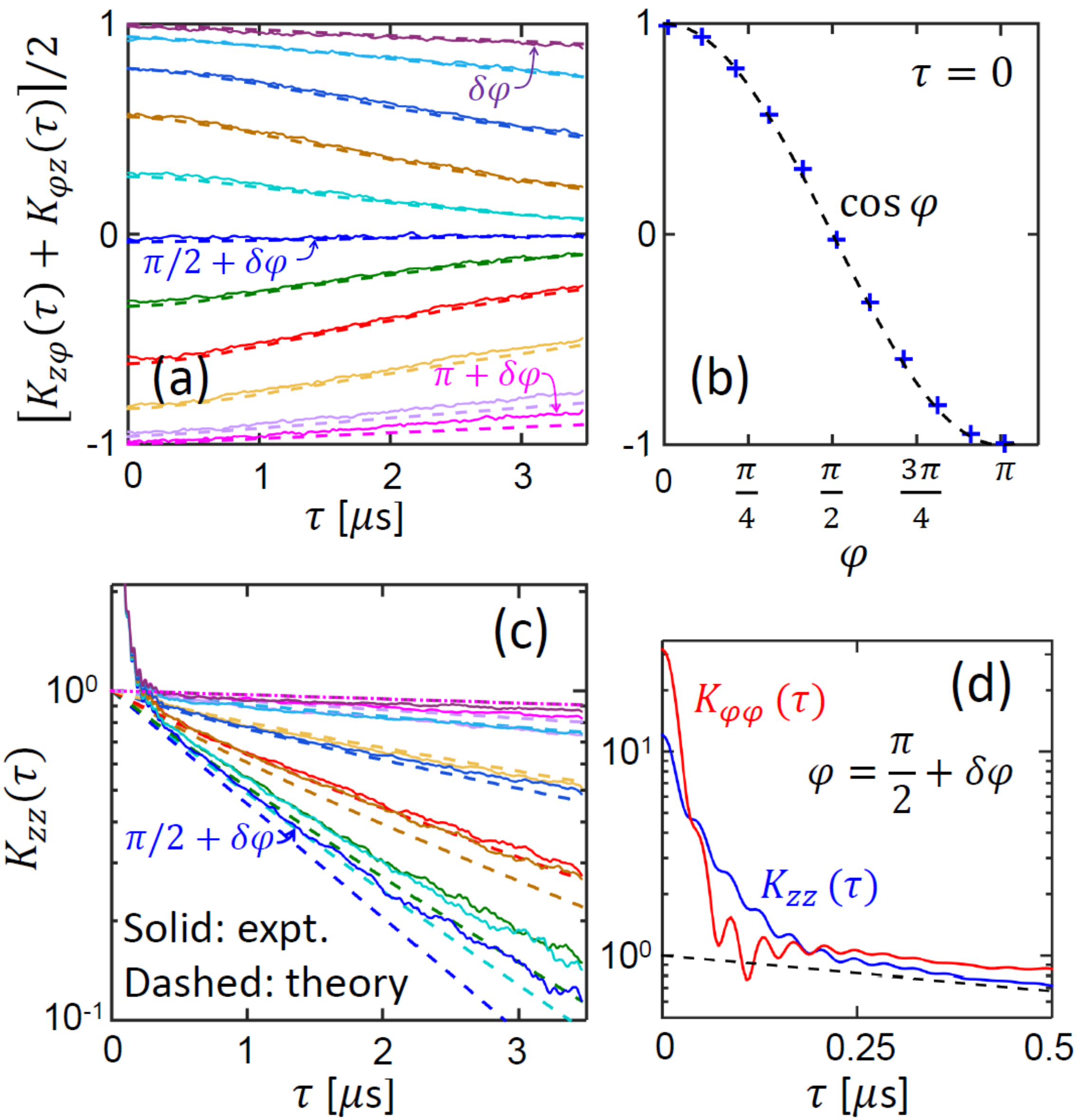
$$K_{zz}(\tau) = \frac{1}{2}\left[1 + \frac{\Gamma_z + \cos(2\varphi)\Gamma_\varphi}{\Gamma_+ - \Gamma_-}\right] e^{-\Gamma_-\tau} + \frac{1}{2}\left[1 - \frac{\Gamma_z + \cos(2\varphi)\Gamma_\varphi}{\Gamma_+ - \Gamma_-}\right] e^{-\Gamma_+\tau}$$

$$K_{z\varphi}(\tau) = \frac{(\Gamma_z + \Gamma_\varphi)\cos\varphi + 2\tilde{\Omega}_R \sin\varphi}{\Gamma_+ - \Gamma_-}\left(e^{-\Gamma_-\tau} - e^{-\Gamma_+\tau}\right) + \frac{\cos\varphi}{2}\left(e^{-\Gamma_-\tau} + e^{-\Gamma_+\tau}\right)$$

$$\Gamma_\pm = \frac{1}{2}\left(\Gamma_z + \Gamma_\varphi \pm \left[\Gamma_z^2 + \Gamma_\varphi^2 + 2\Gamma_z\Gamma_\varphi\cos(2\varphi) - 4\tilde{\Omega}_R^2\right]^{1/2}\right) + \frac{1}{2T_1} + \frac{1}{2T_2}$$

Γ_z, Γ_φ: measurement-induced decoherence rates, $\tilde{\Omega}_R$: residual Rabi frequency

Comparison with experiment



Cross-correlators for 11 values of φ between 0 and π

Maximally non-commuting: $\varphi = \pi/2$

200,000 experimental traces

Very good agreement

Self-correlators

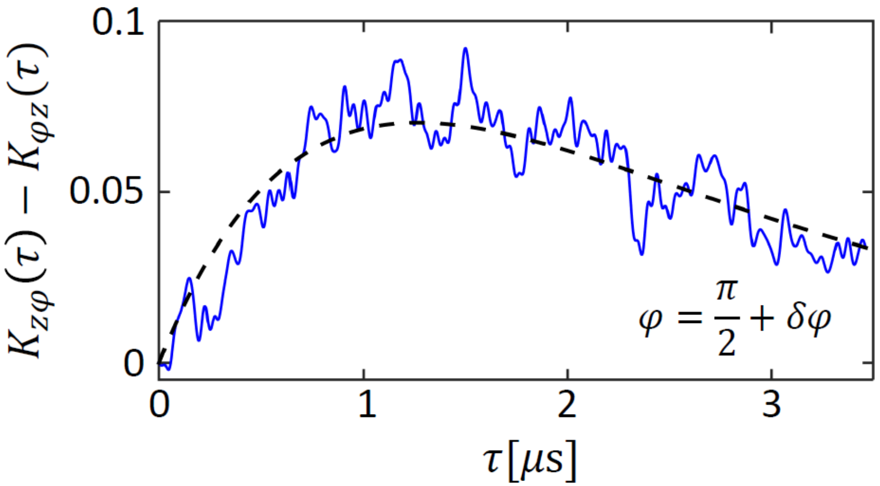
$$\delta\varphi = \frac{\kappa_\varphi - \kappa_z}{2\Omega_R}$$

(correction to angle)

Parameter estimation via correlators

Rabi frequency mismatch: $\widetilde{\Omega}_R = \Omega_R - \Omega_{\text{sideband}}$

$$K_{z\varphi}(\tau) - K_{\varphi z}(\tau) = \frac{\widetilde{\Omega}_R \sin\varphi}{\Gamma_+ - \Gamma_-}\left(e^{-\Gamma_+\tau} - e^{-\Gamma_-\tau}\right)$$



Fitting: $\widetilde{\Omega}_{\mathrm{R}} = \Omega_R - \Omega_{\text{sideband}} \approx 2\pi \times 12\ \text{kHz}$

Very sensitive technique

$(\Omega_R/2\pi = 40\ \text{MHz})$

J. Atalaya, S. Hacohen-Gourgy, L. Martin, I. Siddiqi, and A.K., arXiv:1702.08077

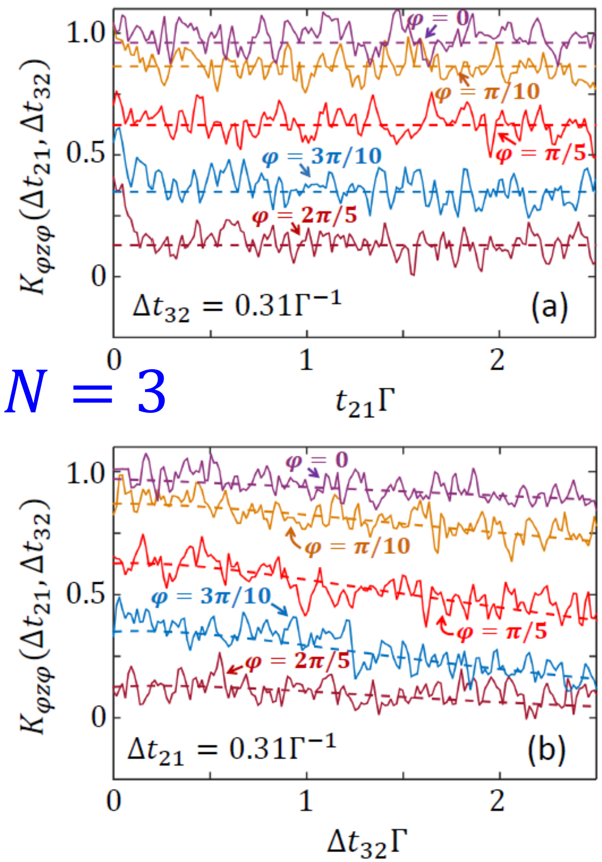
Generalization to N-time correlators

J. Atalaya, S. Hacohen-Gourgy, L. Martin, I. Siddiqi, and A.K., PRA-2018

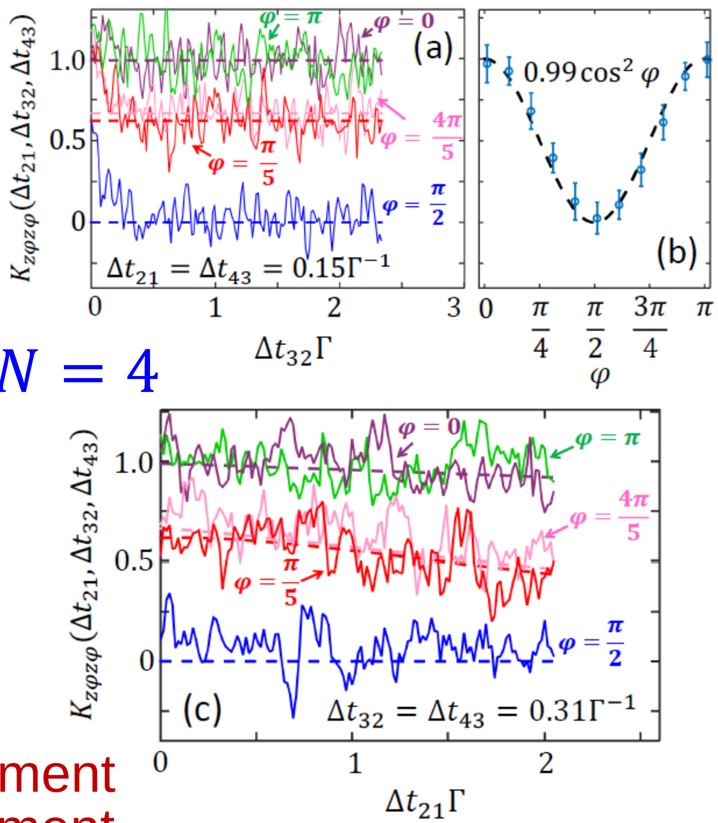
Many detectors

$$K_{l_1 l_2 \dots l_N}(t_1, t_2 \dots t_N) = \langle I_{L_N}(t_N) \dots I_{l_2}(t_2)\, I(t_1) \rangle$$

Surprising factorization: $K_{l_1 \dots l_N}(t_1, \dots t_N) = K_{l_1 .. l_{N-2}}(t_1, \dots t_{N-2})\, K_{l_{N-1} l_N}(t_{N-1}, t_N)$



$N = 3$



$N = 4$

good agreement with experiment

Arrow of time for continuous measurement

J. Dressel. A. Chantasri, A. Jordan, and A. Korotkov, PRL-2017

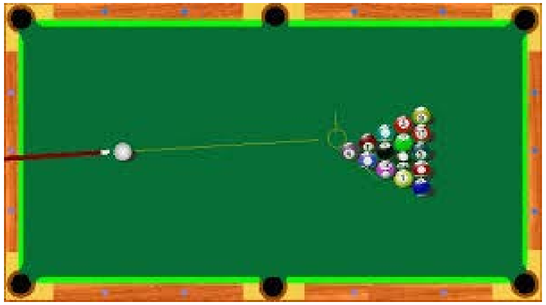
Unitary evolution is time-reversible.

Is continuous quantum measurement time-reversible?

If yes, can we distinguish forward and backward evolutions?

Classical mechanics

Dynamics is time-reversible. However, for more than a few degrees of freedom, one time direction is much more probable than the other.



Posing of the problem: a game

We are given a “movie”, showing quantum evolution $|\psi(t)\rangle$ of a qubit due to continuous measurement and Hamiltonian, together with “soundtrack”, representing noisy measurement record. We need to tell if the movie is played forward of backward.

Reversing qubit evolution

Hamiltonian: $H = \hbar\Omega\sigma_y/2$

Measurement output: $r(t) = z(t) + \sqrt{\tau}\,\xi(t)$,

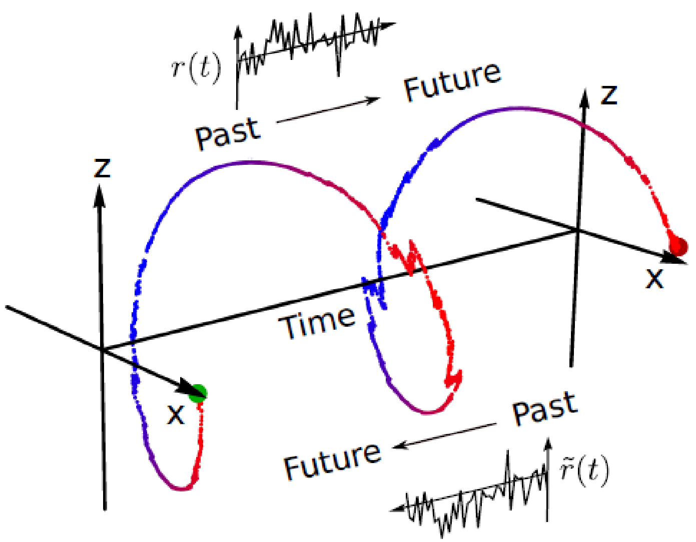
"measurement" (collapse) time τ, white noise $\langle\xi(t)\,\xi(0)\rangle = \delta(t)$

Quantum Bayesian equations (Stratonovich form, $\eta = 1$)

$$\dot{x} = -\Omega z - xzr/\tau, \quad \dot{y} = -yz\,r/\tau, \quad \dot{z} = \Omega x + (1 - z^2)\,r/\tau$$

Time-reversal symmetry: $t \to -t, \ \Omega \to -\Omega, \ r \to -r$

(so, need to flip Rabi direction and measurement record)



This quantum movie, played backwards, is fully legitimate (soundtrack is flipped)

Is there a way to distinguish forward from backward?

Emergence of an arrow of time

Use classical Bayes rule to distinguish forward from backward movie

$$P[F|r(t)] = \frac{P_F[r(t)]}{P_F[r(t)] + P_B[r(t)]} = \frac{R}{1+R}, \qquad R = \frac{P_F[r(t)]}{P_B[r(t)]}$$

Since the measurement record ("soundtrack") is flipped, the particular noise realization becomes less probable (usually)

$$\left.\begin{aligned} r(t) &= z(t) + \sqrt{\tau}\,\xi(t) \\ -r(t) &= z(t) + \sqrt{\tau}\,\xi_B(t) \end{aligned}\right\} \Rightarrow \quad \xi_B(t) = -\xi(t) - \frac{2z(t)}{\sqrt{\tau}}$$

$\xi_B(t)$ is less probable than $\xi(t)$

$$\boxed{\ln R = \frac{2}{\tau}\int_0^T r(t)\, z(t)\, dt}$$

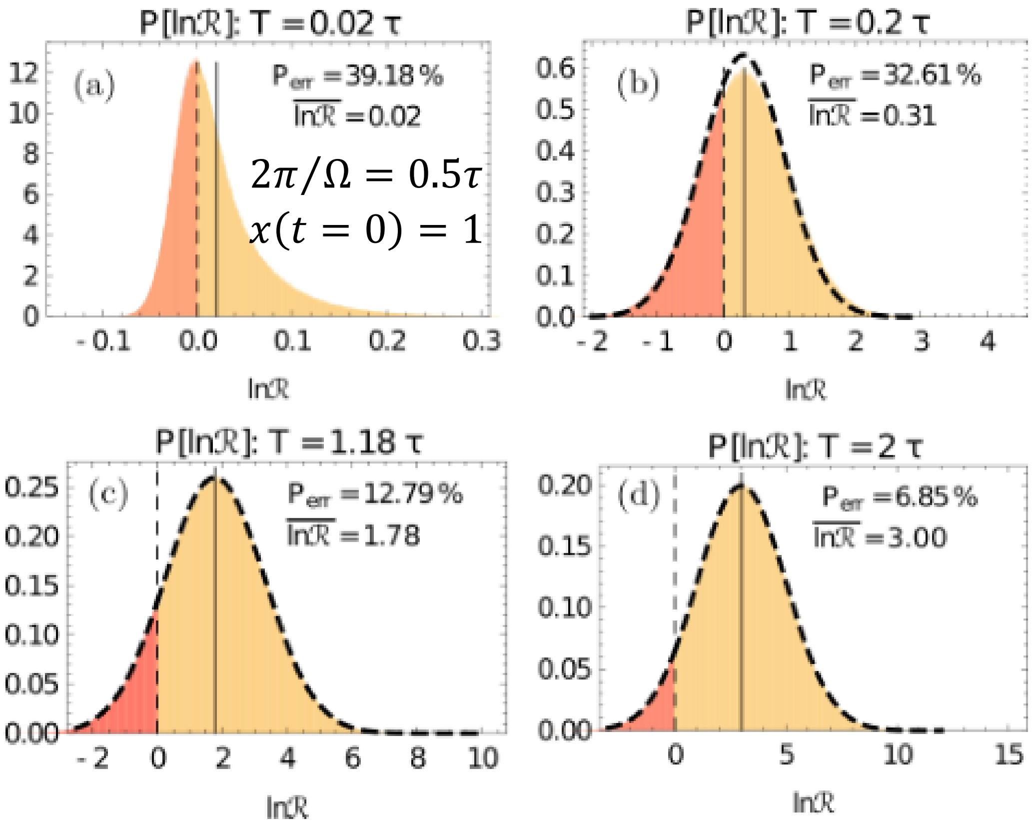
Relative log-likelihood, distinguishing time running forward or backward

For a long movie time T, almost certainly $\ln R > 0$, so we will know the direction of time. For a short T, we will often make a mistake in guessing the time direction.

Numerical results

$$R = \frac{P_F[r(t)]}{P_B[r(t)]}$$

Probability distribution for $\ln R$



$$\ln R = \frac{2}{\tau}\int_0^T r(t)\, z(t)\, dt$$

Asymptotic behavior (long T)

$$R \approx \frac{3T}{2\tau} \pm \sqrt{\frac{2T}{\tau}}$$

For a long trajectory, probability of guessing the direction of time incorrectly is

$$P_{err} \approx \frac{1}{2}\left[1 - \mathrm{Erf}\left(\frac{3}{4}\sqrt{T/\tau}\right)\right]$$

$$\approx \frac{2}{3}\sqrt{\frac{\tau}{\pi T}}\exp\left(-\frac{9\, T}{16\, \tau}\right)$$

(decreases exponentially with the ratio T/τ)

Statistical arrow of time emerges at timescale of "measurement time" τ

(seemingly backward-in-time trajectories are still possible at $T > \tau$)

Conclusions

- It is easy to see what is "inside" collapse: simple Bayesian framework works for many solid-state setups
- Measurement back-action necessarily has a "spooky" part (informational, without a physical mechanism); it may also have a unitary part (with a physical mechanism)
- Quantum Bayesian theory is similar to Quantum Trajectory theory, though looks quite different; also equivalent to POVM
- Many experiments with superconducting qubits have demonstrated what is "inside" collapse (most of our proposals already realized)
- Possibly useful (especially quantum feedback)
- Simultaneous measurement of non-commuting observables has become possible experimentally
- Continuous measurement of a qubit is time-reversible (with flipped record), but a statistical arrow of time emerges

Thank you!