Quantum feedback control
of coherent oscillations in a solid-state qubit

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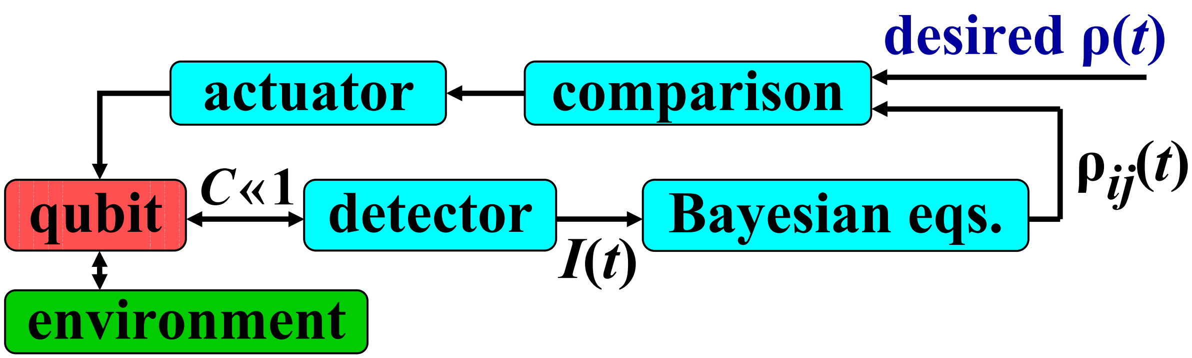
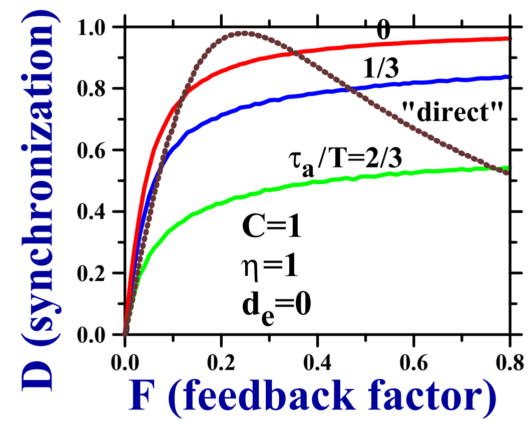




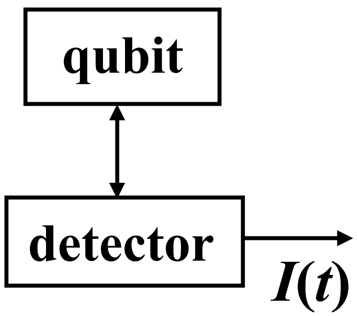
Outline:

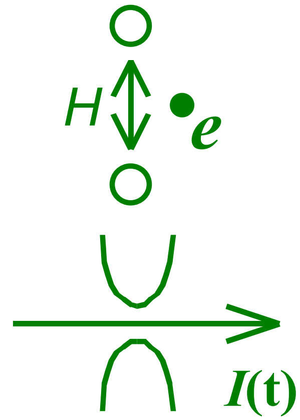
- Bayesian formalism for continuous quantum measurement of a solid-state qubit
- Quantum feedback control of a single qubit
- Generalization to entangled qubits

Support:

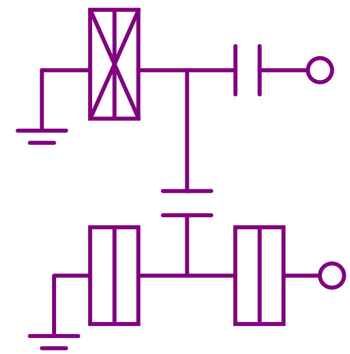
Phys. Rev. B 66, 041401(R) (2002)
Review: cond-mat/0209629

Examples of solid-state qubits and detectors

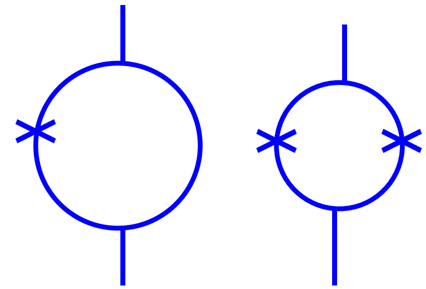




Double-quantum-qot and quantum point contact (QPC)



Cooper-pair box and single-electron transistor (SET)



Two SQUIDs

$H = H_{QB} + H_{DET} + H_{INT}$

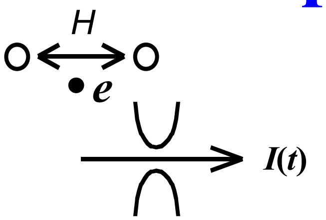
$H_{QB} = (\varepsilon/2)(c_1^+c_1 - c_2^+c_2) + H(c_1^+c_2 + c_2^+c_1)$ ε - asymmetry, H – tunneling

$\Omega = (4H^2+\varepsilon^2)^{1/2}$ – frequency of quantum coherent (Rabi) oscillations

Two levels of average detector current: I_1 for qubit state $|1\rangle$, I_2 for $|2\rangle$

Response: $\Delta I = I_1 - I_2$ Detector noise: white, spectral density S_I

What happens to a qubit state during measurement?



For simplicity (for a moment) $H=\varepsilon=0$, infinite barrier (frozen qubit), evolution due to measurement only

"Orthodox" answer

$$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \begin{matrix} \nearrow \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \\ \searrow \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \end{matrix}$$

$|1\rangle$ or $|2\rangle$, depending on the result

"Conventional" (decoherence) answer (Leggett, Zurek)

$$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \rightarrow \begin{pmatrix} \frac{1}{2} & \frac{\exp(-\Gamma t)}{2} \\ \frac{\exp(-\Gamma t)}{2} & \frac{1}{2} \end{pmatrix} \rightarrow \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

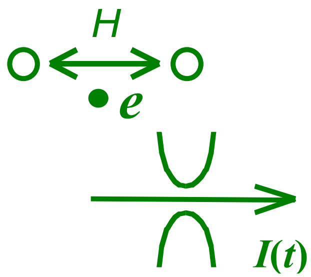
no measurement result! ensemble averaged

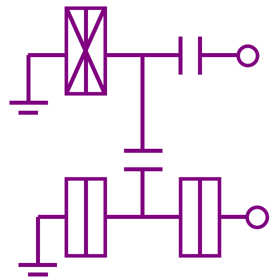
Orthodox and decoherence answers contradict each other!

applicable for:	Single quantum systems	Continuous measurements
Orthodox	yes	no
Conventional (ensemble)	no	yes
Bayesian	yes	yes

Bayesian formalism describes gradual collapse of a single quantum system
Noisy detector output $I(t)$ is taken into account

Bayesian formalism for a single qubit





$$\hat{H}_{QB} = (\varepsilon/2)(c_1^\dagger c_1 - c_2^\dagger c_2) + H(c_1^\dagger c_2 + c_2^\dagger c_1)$$

$|1\rangle \to I_1, \ |2\rangle \to I_2$

$\Delta I = I_1 - I_2$, $I_0 = (I_1 + I_2)/2$, S_I – detector noise

$$d\rho_{11}/dt = -d\rho_{22}/dt = -2H\,\mathrm{Im}\,\rho_{12} + 2\Delta I\, S_I^{-1} \rho_{11}\rho_{22}[I(t) - I_0]$$

$$d\rho_{12}/dt = i\varepsilon\rho_{12} + iH(\rho_{11} - \rho_{22}) + \Delta I\, S_I^{-1} \rho_{12}(\rho_{11} - \rho_{22})[I(t) - I_0] - \gamma\rho_{12}$$

A.K., 1998

$\gamma = \Gamma - (\Delta I)^2/4S_I$, Γ – ensemble decoherence

$\eta = 1 - \gamma/\Gamma = (\Delta I)^2/4S_I\Gamma$ – detector ideality (efficiency), $\eta \le 100\%$

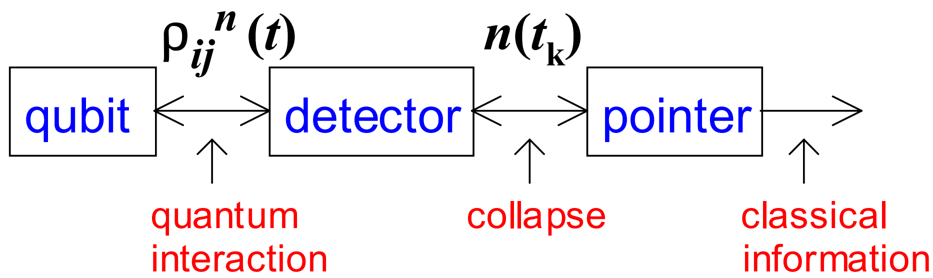
For simulations: $I(t) - I_0 \to (\rho_{22} - \rho_{11})\Delta I/2 + \xi(t)$, $S_\xi = S_I$

Averaging over $\xi(t)$ $\Rightarrow$ master equation

Similar formalisms developed earlier. Key words: Imprecise, weak, selective, or conditional measurements, POVM, Quantum trajectories, Quantum jumps, Restricted path integral, etc.

Names: E.B. Davies, K. Kraus, A.S. Holevo, C.W. Gardiner, H.J. Carmichael, C.M. Caves, M.B. Plenio, P.L. Knight, M.B. Mensky, D.F. Walls, N. Gisin, I.C. Percival, **G.J. Milburn**, **H.M. Wiseman,** R. Onofrio, S. Habib, A. Doherty, etc. (very incomplete list)

"Microscopic" derivation of the Bayesian formalism



Schrödinger evolution of "qubit + detector"
for a low-T QPC as a detector (Gurvitz, 1997)

$$\frac{d}{dt}\rho_{11}^{n} = -\frac{I_1}{e}\rho_{11}^{n} + \frac{I_1}{e}\rho_{11}^{n-1} - 2\frac{H}{\hbar}\,\mathrm{Im}\,\rho_{12}^{n}$$

$$\frac{d}{dt}\rho_{22}^{n} = -\frac{I_2}{e}\rho_{22}^{n} + \frac{I_2}{e}\rho_{22}^{n-1} + 2\frac{H}{\hbar}\,\mathrm{Im}\,\rho_{12}^{n}$$

$$\frac{d}{dt}\rho_{12}^{n} = i\frac{\varepsilon}{\hbar}\rho_{12}^{n} + i\frac{H}{\hbar}(\rho_{11}^{n} - \rho_{22}^{n}) - \frac{I_1 + I_2}{2e}\rho_{12}^{n} + \frac{\sqrt{I_1 I_2}}{e}\rho_{12}^{n-1}$$

Detector collapse at $t = t_k$
Particular n_k is chosen at t_k

$$P(n) = \rho_{11}^{n}(t_k) + \rho_{22}^{n}(t_k)$$

$$\rho_{ij}^{n}(t_k + 0) = \delta_{n,nk}\,\rho_{ij}(t_k + 0),$$

$$\rho_{ij}(t_k + 0) = \frac{\rho_{ij}^{nk}(t_k - 0)}{\rho_{11}^{nk}(t_k - 0) + \rho_{22}^{nk}(t_k - 0)}$$

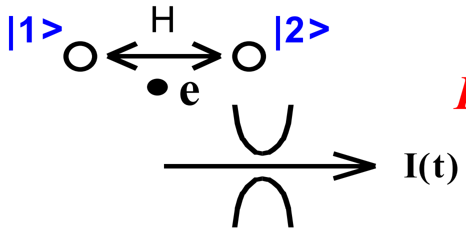
If $H = \varepsilon = 0$,
it leads to

$$\rho_{11}(t) = \frac{\rho_{11}(0)P_1(n)}{\rho_{11}(0)P_1(n) + \rho_{22}(0)P_2(n)}, \quad \rho_{22}(t) = \frac{\rho_{22}(0)P_2(n)}{\rho_{11}(0)P_1(n) + \rho_{22}(0)P_2(n)}$$

$$\rho_{12}(t) = \rho_{12}(0)\frac{[\rho_{11}(t)\rho_{22}(t)]^{1/2}}{[\rho_{11}(0)\rho_{22}(0)]^{1/2}}, \qquad P_i(n) = \frac{(I_i t/e)^n}{n!}\exp(-I_i t/e),$$

which are exactly the quantum Bayes formulas

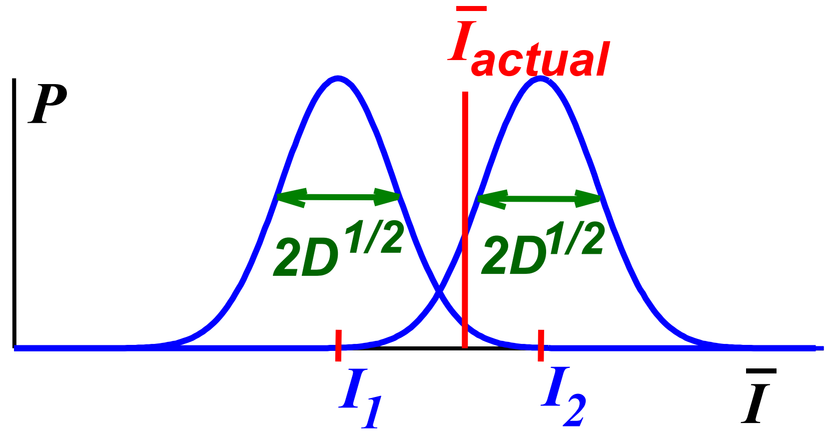
"Quantum Bayes theorem" (ideal detector assumed)



$H = \varepsilon = 0$ (frozen qubit)

Initial state: $\begin{pmatrix} \rho_{11}(0) & \rho_{12}(0) \\ \rho_{21}(0) & \rho_{22}(0) \end{pmatrix}$

Measurement (during time τ):



$$\bar{I} \equiv \frac{1}{\tau}\int_0^\tau I(t)\, dt$$

$$P(\bar{I},\tau) = \rho_{11}(0)\, P_1(\bar{I},\tau) + \rho_{22}(0)\, P_2(\bar{I},\tau)$$

$$P_i(\bar{I},\tau) = \frac{1}{\sqrt{2\pi D}} \exp[-(\bar{I}-I_i)^2/2D],$$

$$D = S_I/2\tau, \quad |I_1 - I_2| \ll I_i, \quad \tau \gg S_I/I_i^2$$

After measurement duration τ, the probabilities can be updated using the standard Bayes formula:

$$P(B_i|A) = \frac{P(B_i)P(A|B_i)}{\sum_k P(B_k)P(A|B_k)}$$

Quantum Bayes formulae:

$$\rho_{11}(\tau) = \frac{\rho_{11}(0)\exp[-(\bar{I}-I_1)^2/2D]}{\rho_{11}(0)\exp[-(\bar{I}-I_1)^2/2D] + \rho_{22}(0)\exp[-(\bar{I}-I_2)^2/2D]}$$

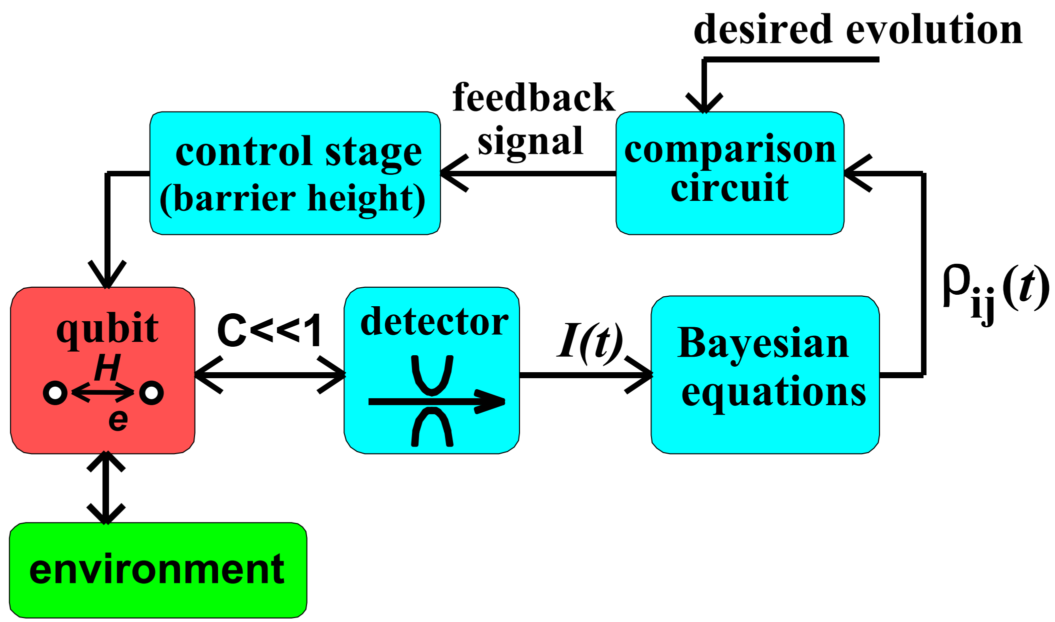
$$\frac{\rho_{12}(\tau)}{[\rho_{12}(\tau)\,\rho_{22}(\tau)]^{1/2}} = \frac{\rho_{12}(0)}{[\rho_{12}(0)\,\rho_{22}(0)]^{1/2}}, \qquad \rho_{22}(\tau) = 1 - \rho_{11}(\tau)$$

Some experimental predictions and proposals using Bayesian formalism

- Direct experiments on qubit evolution (1998)
- Measured spectral density of Rabi oscillations (1999, 2000, 2002)
- Bell-type correlation experiment (2000)
- Quantum feedback control of a qubit (2001)
- Entanglement by measurement (2002)
- Measurement by a quadratic detector (2003)
- QND squeezing of a nanoresonator by quantum feedback (2003)

Quantum feedback control of a solid-state qubit

Ruskov & A.K., 2001



Goal: maintain desired phase of Rabi oscillations in spite of environmental dephasing (keep qubit "fresh")

Idea: monitor the Rabi phase ϕ by continuous measurement and apply feedback control of the qubit barrier height, $\Delta H_{FB}/H = -F\times\Delta\phi$

To monitor phase ϕ we plug detector output $I(t)$ into Bayesian equations

Quantum feedback in quantum optics is discussed since 1993 (Wiseman-Milburn), recently first successful experiment in Mabuchi's group (Armen et al., 2002).

Analysis of one-qubit quantum feedback

Desired evolution: $\rho_{11}(t) = 1 - \rho_{22}(t) = (1 + \cos\Omega t)/2$ $\quad \Omega = \sqrt{4H^2 + \varepsilon^2}$

$\rho_{12}(t) = \rho_{21}^*(t) = i(\sin\Omega t)/2$ $\quad$ assume ε=0

Monitored evolution:

$$\dot\rho_{11} = -\dot\rho_{22} = -2H_{FB}\,\mathrm{Im}\,\rho_{12} + 2\Delta I\, S_I^{-1}\rho_{11}\rho_{22}[I_a(t) - I_0]$$

$$\dot\rho_{12} = i\varepsilon\rho_{12} + iH_{FB}(\rho_{11} - \rho_{22}) + \Delta I\, S_I^{-1}\rho_{12}(\rho_{11} - \rho_{22})[I_a(t) - I_0] - \gamma\rho_{12}$$

$$I_a(t) = \int_0^\infty I(t-\tau)G(\tau)d\tau, \;\; I(t) = I_0 + (\rho_{11} - \rho_{22})\Delta I/2 + \xi(t)$$

Feedback:

$$H_{FB} = (1 - F \times \Delta\phi)H, \quad \Delta\phi = \phi(t) - \Omega t - \tau_{comp} \pmod{2\pi}$$

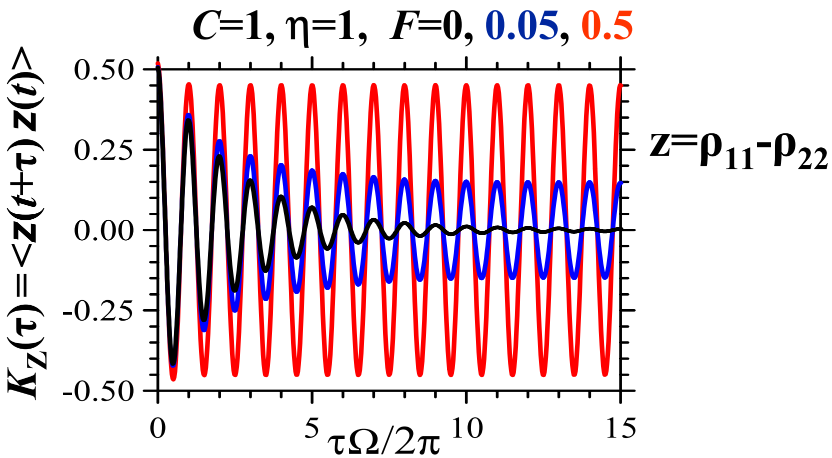
$$\phi(t) = \arctan(2\,\mathrm{Im}\,\rho_{12}/(\rho_{11} - \rho_{22}))$$

Simple case (γ=0, infinite bandwidth, no delay, etc.):

$$\frac{d}{dt}\Delta\phi = -\sin\phi\frac{\Delta I}{S_I}\left(\frac{\Delta I}{2}\cos\phi + \xi\right) - \frac{2FH}{\hbar}\Delta\phi$$

Performance of quantum feedback
(no extra environment)

Qubit correlation function



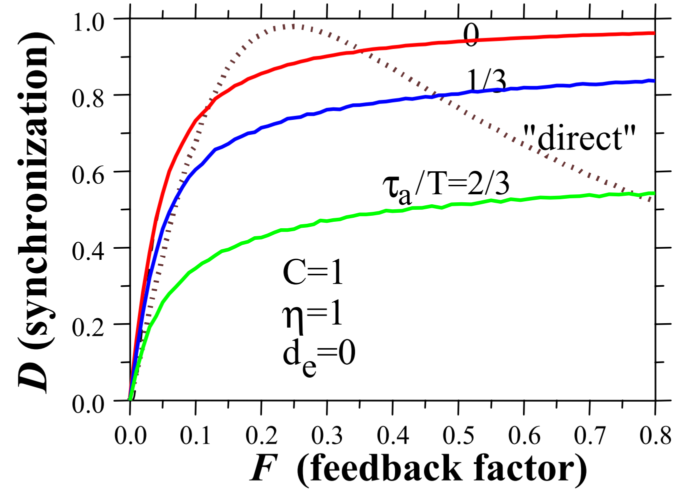
$$K_Z(\tau) = \frac{\cos\Omega t}{2} \exp\left[\frac{C}{16F}(e^{-2FH\tau/\hbar} - 1)\right]$$

(for weak coupling and good fidelity)

Detector current correlation function

$$K_I(\tau) = \frac{(\Delta I)^2}{4}\frac{\cos\Omega t}{2}(1+e^{-2FH\tau/\hbar}) \times \exp\left[\frac{C}{16F}(e^{-2FH\tau/\hbar}-1)\right] + \frac{S_I}{2}\delta(\tau)$$

Fidelity (synchronization degree)



$C = \hbar(\Delta I)^2 / S_I H$ – coupling

τ_a^{-1} – available bandwidth

F – feedback strength

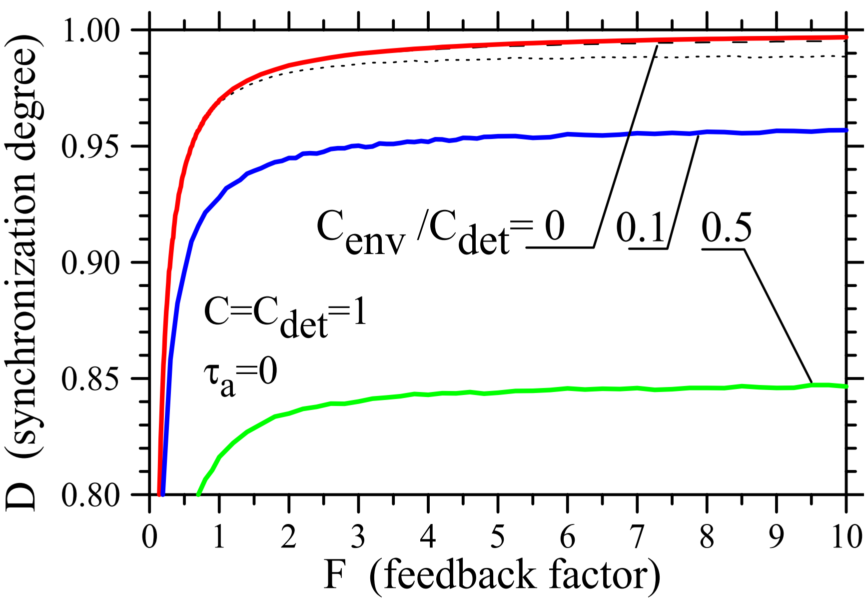
$$D = 2\langle \mathrm{Tr}\rho\rho_{desir}\rangle - 1$$

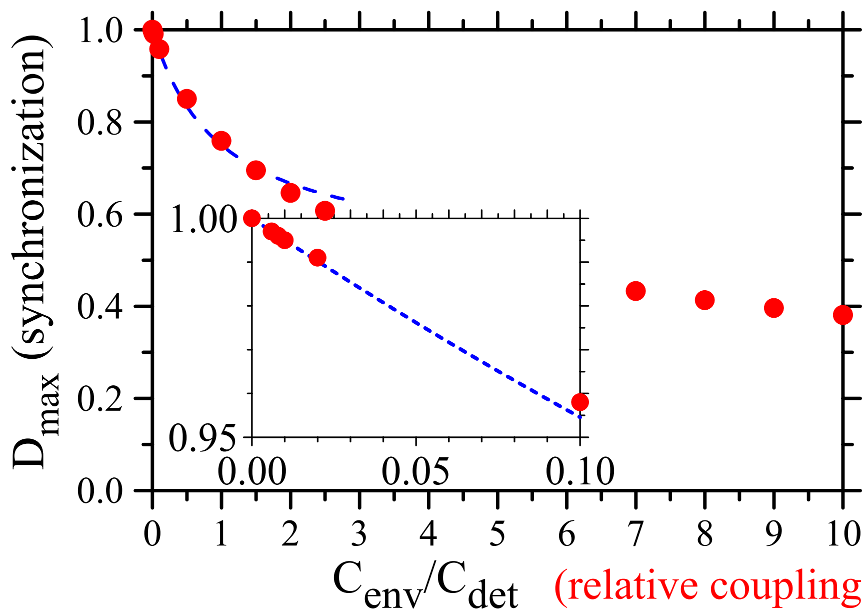
For ideal detector and wide bandwidth, fidelity can be arbitrary close to 100%

$$D = \exp(-C/32F)$$

Ruskov & Korotkov, PRB 66, 041401(R) (2002)

Suppression of environment-induced decoherence by quantum feedback



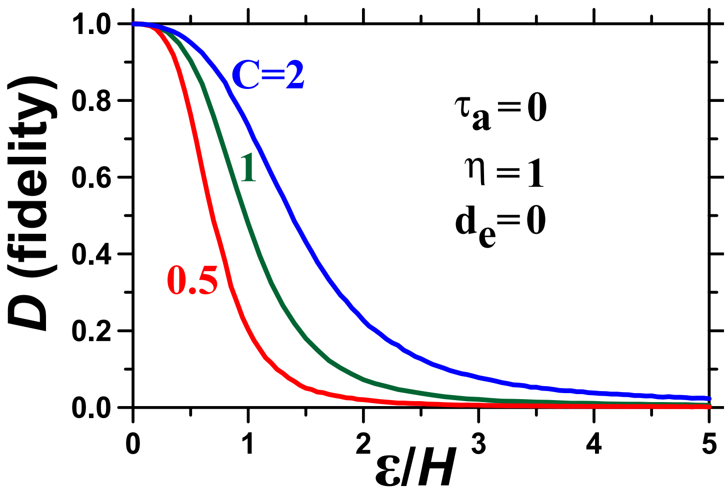


$$D_{max} \simeq 1 - \frac{C_{env}}{2C_{det}} \frac{1}{1 + C_{env}/C_{det}}$$

If qubit coupling to the environment is 100 times weaker than to the detector, then D_{max} = 99.5% and qubit fidelity 99.75%. (D = 0 without feedback.)

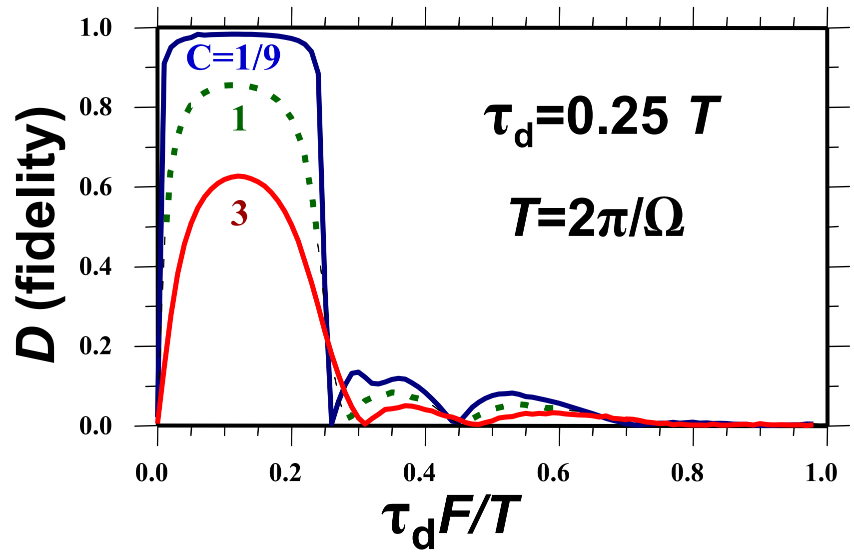
Some other issues of one-qubit feedback

Finite energy asymmetry ε



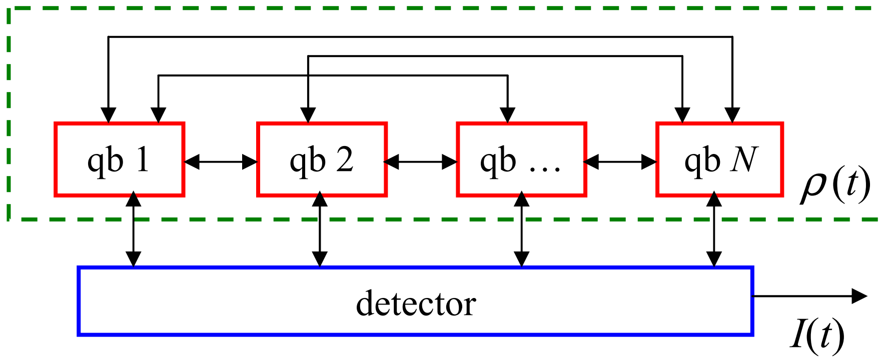
Feedback designed for ε=0 works still well for small energy asymmetry ε

Additional delay τ_d



Feedback becomes unstable when $\tau_d F > T/4$ (too strong feedback with delay)

Bayesian formalism for *N* entangled qubits measured by one detector



Up to 2^N levels of detector current

$$\frac{d}{dt}\rho_{ij} = \frac{-i}{\hbar}[\hat{H}_{qb},\rho]_{ij} + \rho_{ij}\frac{1}{S}\sum_k \rho_{kk}\Big[\big(I(t)-\frac{I_k+I_i}{2}\big)(I_i-I_k)+$$

$$+\big(I(t)-\frac{I_k+I_j}{2}\big)(I_j-I_k)\Big]-\gamma_{ij}\rho_{ij}$$

(Stratonovich form)

$$\gamma_{ij} = (\eta^{-1}-1)(I_i-I_j)^2/4S_I \qquad I(t)=\sum_i \rho_{ii}(t)I_i+\xi(t)$$

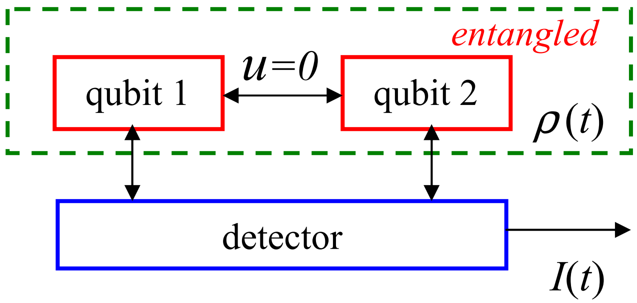
Averaging over ξ(t) î master equation

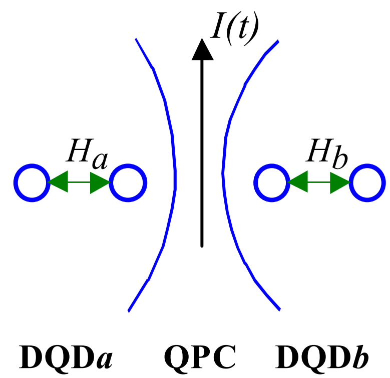
No measurement-induced dephasing between states |*i*Ò and |*j*Ò if $I_i = I_j$!

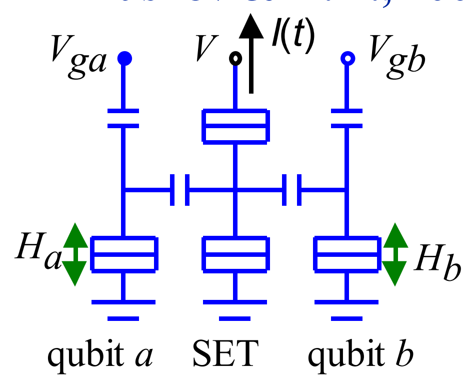
A.K., PRA 65 (2002), PRB 67 (2003)

Two-qubit entanglement by measurement

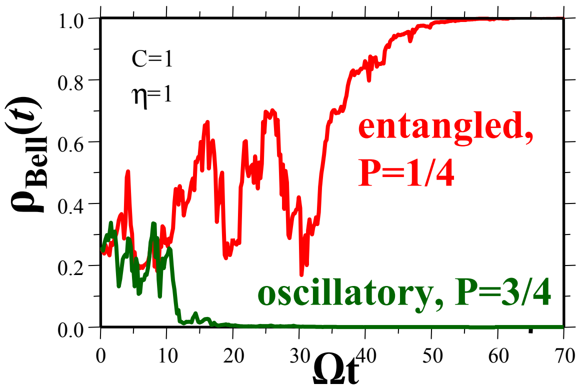
Ruskov & A.K., 2002

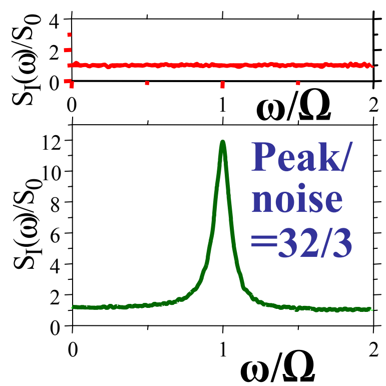






$I(\uparrow\downarrow)=I(\downarrow\uparrow)$





Collapse into Bell state (|↑↓ò - |↓↑ò)/÷2 (spontaneous entanglement) with probability 1/4 starting from fully mixed state

Monitoring two qubits by an equally coupled detector, it is possible to produce fully entangled state. Imperfections lead to switching into non-entangled state.

Can quantum feedback maintain entanglement?

(still open question!)

Conclusions

- Bayesian formalism describes continuous quantum measurement of single quantum systems (no ensemble averaging); directly applicable to quantum feedback

- Quantum feedback control can maintain Rabi oscillations in a qubit for arbitrary long time, even in presence of dephasing environment

- Theory of quantum feedback control is a new, interesting, useful, and almost unexplored field;
  Many simple questions not answered yet;
  No attempts of general theory yet (*s*-plane, etc.)

No solid-state experiments yet; hopefully coming soon (~ 5 years)
(one quantum feedback experiment in optics already; one more soon)