State Purification and Decoherence Suppression by Continuous Measurement of a Qubit

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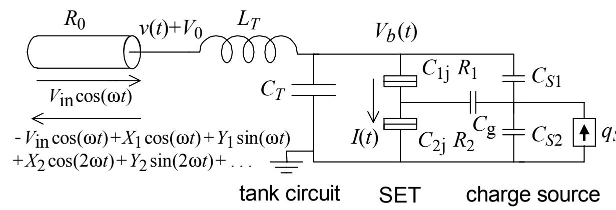
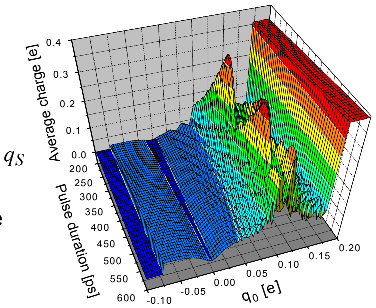
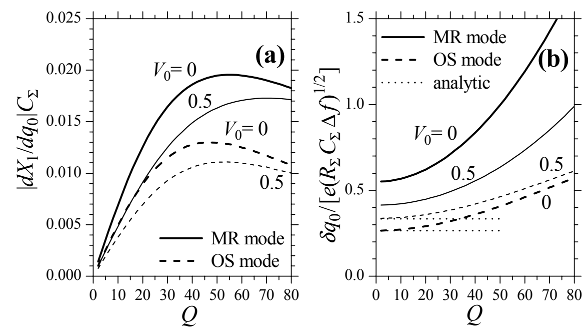
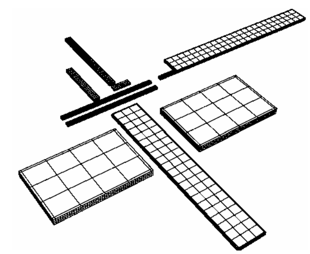
Objective

- Suppress decoherence of single solid-state qubit using quantum feedback loop
- Tune entangled qubits continuously
- Reconstruct pre-measured qubit state from the record of continuous measurement
- Analyze the feasibility of present-day experiments on quantum control of solid-state qubits









Objective Approach

- Development of Bayesian formalism for continuous quantum measurements
- Quantitative analysis of quantum feedback
- Application of Bayesian formalism to the inverse problem
- Analysis of experimental parameters

Status

- Analyzed one-qubit quantum feedback
- Developed measurement theory for entangled qubits, nonideal and quadratic detectors
- Predicted two-qubit entanglement by continuous measurement
- Analyzed RF-SET with large Q-factor
- Experimental collaboration with JPL

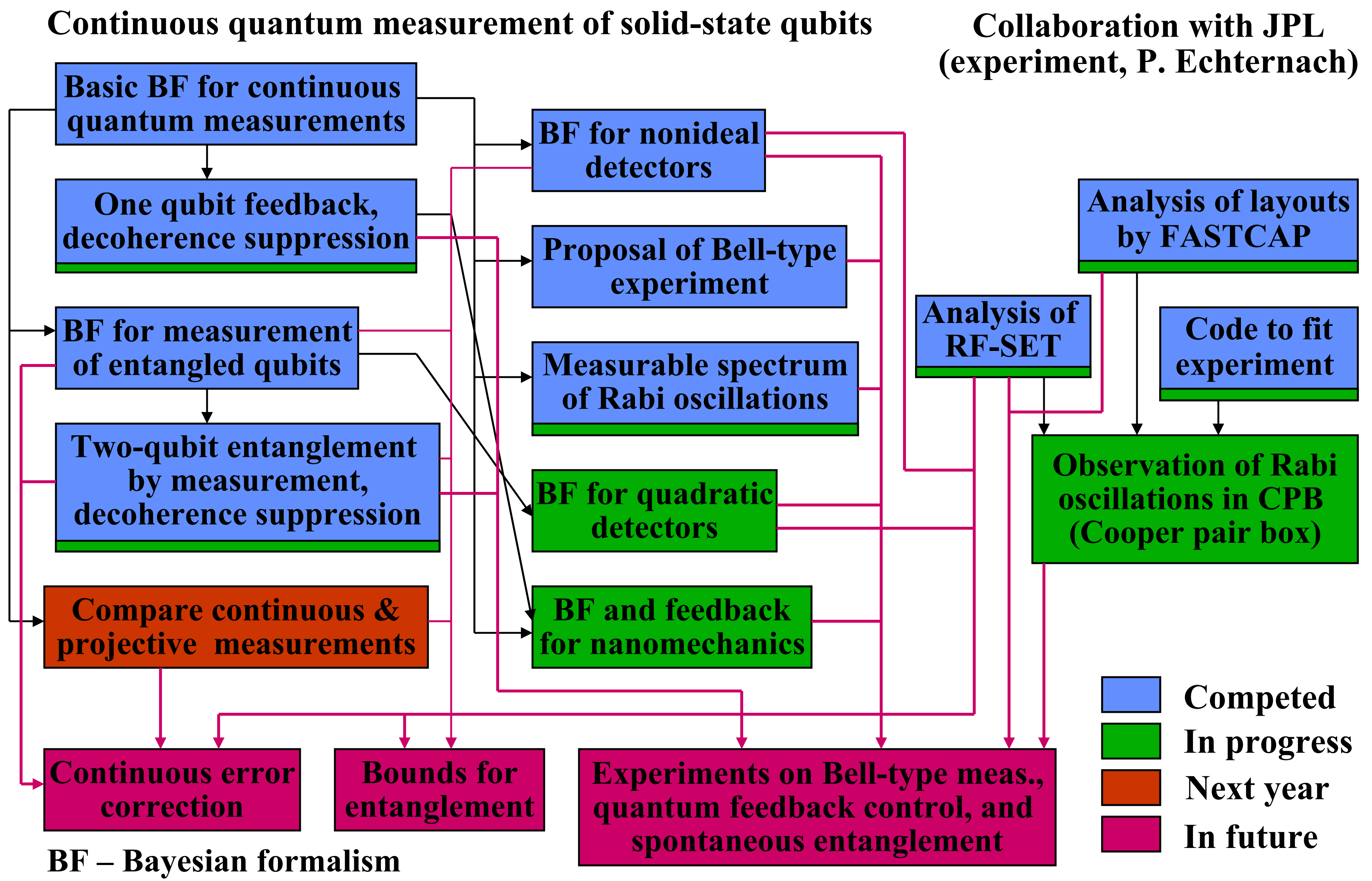
Research plan and accomplishments for the last 12 months

- study in detail preparation of entangled qubits by measurement *Completed*
- analyze two-qubit quantum feedback quantitatively *Partially completed*
- develop generalized formalism for nonideal solid-state detectors *Completed*
- compare amounts of information obtainable by continuous and instantaneous measurements of a qubit *To be done in future*

Besides:

- collaboration with JPL group (P. Echternach) on experimental observation of Rabi oscillations in a Cooper-pair box *Ongoing*
- response and sensitivity analysis for normal-metal RF-SET *Completed*
- theory of continuous quadratic quantum detection *Almost completed*
- theory of continuous quantum measurement and feedback control of a nanomechanical resonator *In progress*

Flow chart


Continuous quantum measurement of solid-state qubits
Collaboration with JPL
(experiment, P. Echternach)
Basic BF for continuous quantum measurements
One qubit feedback, decoherence suppression
BF for measurement of entangled qubits
Two-qubit entanglement by measurement, decoherence suppression
Compare continuous & projective measurements
BF for nonideal detectors
Proposal of Bell-type experiment
Measurable spectrum of Rabi oscillations
BF for quadratic detectors
BF and feedback for nanomechanics
Analysis of layouts by FASTCAP
Analysis of RF-SET
Code to fit experiment
Observation of Rabi oscillations in CPB (Cooper pair box)
Continuous error correction
Bounds for entanglement
Experiments on Bell-type meas., quantum feedback control, and spontaneous entanglement
Competed
In progress
Next year
In future
BF – Bayesian formalism

The team:

Alexander Korotkov, Associate Prof.

Rusko Ruskov, Postdoc

Qin Zhang, Grad. student

Valentin Turin, Postdoc (not supported by this project)

Abdulrahman Rafiq, Grad. student (not yet supported)

UC, Riverside

Collaborators:

Pierre Echternach, Alexandre Guillaume, JPL
Keith Schwab, LPS
Dmitri Averin, Wenjin Mao, SUNYSB

Sponsored publications since last QC review

Papers published:

- R. Ruskov and A. N. Korotkov, "Entanglement of solid-state qubits by measurement", PRB 67, 241305(R) (2003).
- R. Ruskov and A. N. Korotkov, "Spectrum of qubit oscillations from Bloch equations", PRB 67, 075303 (2003).
- A. N. Korotkov, "Nonideal quantum detectors in Bayesian formalism", PRB 67, 235408 (2003).
- A. N. Korotkov, "Noisy quantum measurement of solid-state qubits: Bayesian approach", in: *Quantum Noise in Mesoscopic Physics* (Kluwer, 2003), p. 205.
- A. N. Korotkov, "Noisy quantum measurement of solid-state qubits", Proceedings of SPIE, v. 5115, pp. 386-400.

Papers submitted:

- V. O Turin and A. N. Korotkov, "Analysis of the RF-SET with large quality factor", cond-mat/0305012, accepted by APL (scheduled for 09.20.03 issue)
- V. O Turin and A. N. Korotkov, "Numerical analysis of the RF-SET operation', cond-mat/0308218, submitted to PRB

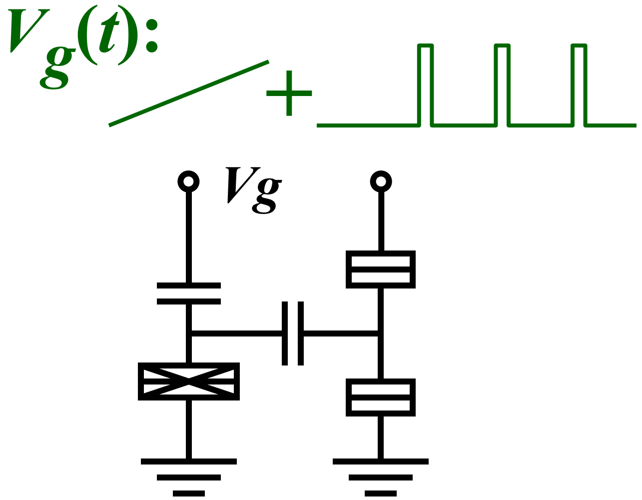
Conference talks:

- APS (Austin, TX, March 2003)
- SPIE F&N (Santa Fe, NM, June 2003)

Outline

- Collaboration with JPL (Rabi oscillations in a Cooper-pair-box qubit)
- Response and sensitivity of RF-SET
- Nonideal quantum detectors
- Entanglement of qubits by measurement
- Quadratic quantum detection
- Quantum feedback and squeezing of a nanomechanical resonator
- Future plans

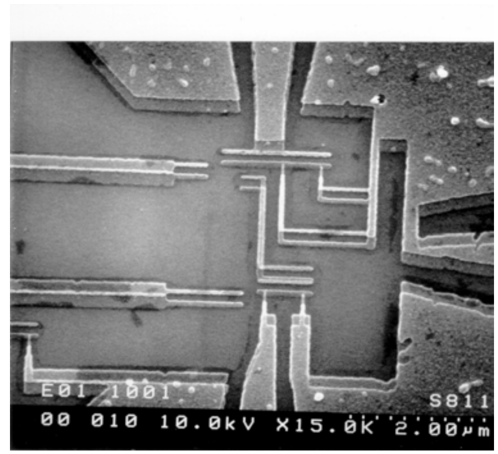
Collaboration with P. Echternach (JPL) on experimental observation of Rabi oscillations in a single-Cooper-pair-box qubit



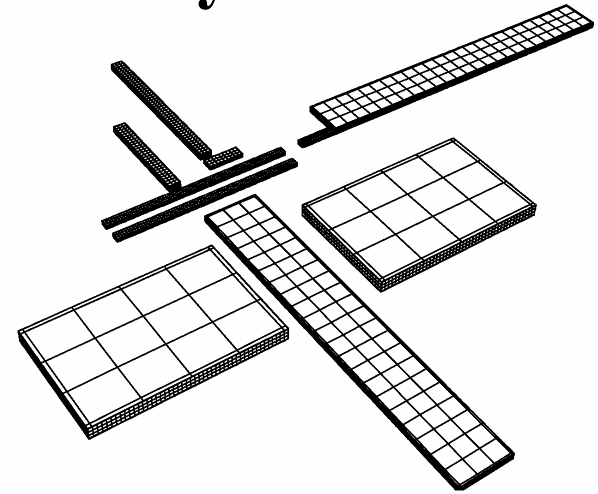
P. Echternach, 2002-2003

	Theor	Exp
Cqb,dc	5.5 aF	5 aF
Cqb,rf	47 aF	--

Simulation of layouts by FASTCAP



JPL



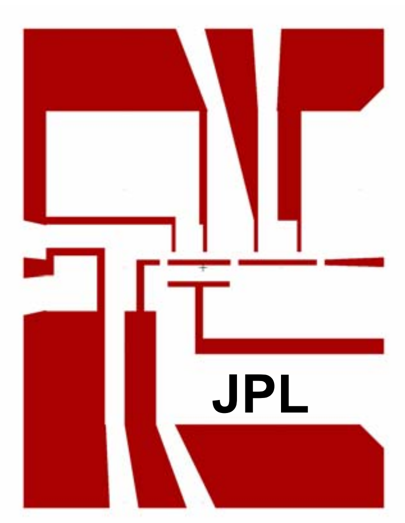
UCR

Capacitance matrix (aF):

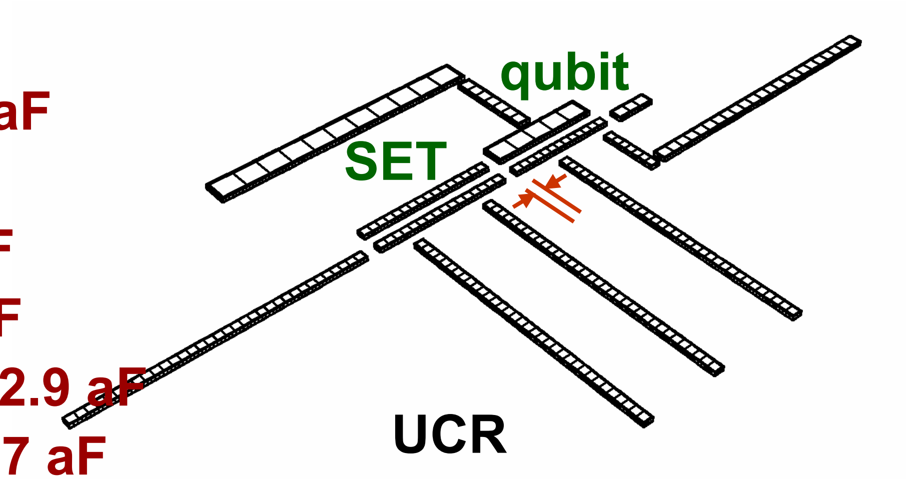
1	100.	-5.49	-47.2	-15.5	-16.7	-3.35
2	-5.49	166	-9.00	-10.4	-9.69	-27.9
3	-47.2	-9.00	158.	-7.51	-39.8	-26.3
4	-15.5	-10.4	-7.51	94.0	-22.3	-4.30
5	-16.7	-9.69	-39.8	-22.3	180.5	-10.5
6	-3.35	-27.9	-26.3	-4.30	-10.5	122.

Collaboration with P. Echternach (JPL)

Design optimization before fabrication



C SET-qb ≈ 3.1 aF
C qb,rf ≈ 3.5 aF
C qb,dc ≈ 6.9 aF
C SET,g ≈ 5.2 aF
C qb,SETbias ≈ 2.9 aF
C SET,gbdc ≈ 4.7 aF



Backaction from SET

$$\Gamma_d = \frac{(\Delta E)^2 \Gamma_1 \Gamma_2}{\hbar^2 (\Gamma_1 + \Gamma_2)^3} \approx \frac{(\Delta E)^2 e^2 R_j}{5\hbar^2 2\Delta_{sc}}$$

$$\Delta E = \frac{e^2 C_{SET-QB}}{C_{SET} C_{QB}} \qquad \frac{\Gamma_d}{\Omega_{Rabi}} \ll 1$$

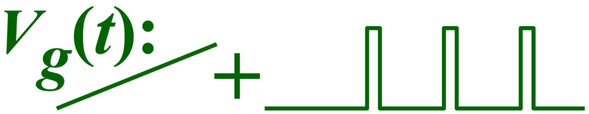
$$C_{SET-QB} \leq 0.03\, C_{SET}$$

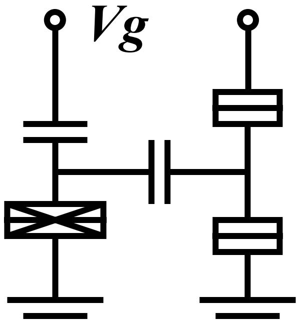
Capacitance matrix (aF):

	1	2	3	4	5	6	7	8	9
1	57.3	-3.13	-17.7	-1.78	-15.4	-2.85	-0.48	-3.49	-6.90
2	-3.13	68.2	-1.37	-25.8	-3.01	-17.	-5.19	-0.197	-4.74
3	-17.7	-1.37	81.3	-2.51	-4.24	-1.39	-0.375	-1.91	-47.3
4	-1.78	-25.8	-2.51	63.5	-2.55	-6.19	-4.71	-0.23	-10.3
5	-15.4	-3.01	-4.24	-2.55	161.	-28.7	-4.10	-8.93	-13.8
6	-2.85	-17.	-1.39	-6.19	-28.7	136.	-10.8	-0.922	-9.54
7	-0.48	-5.19	-0.375	-4.71	-4.10	-10.8	79.0	-0.321	-9.58
8	-3.49	-0.197	-1.91	-0.23	-8.93	-0.922	-0.321	25.9	-3.44
9	-6.90	-4.74	-47.3	-10.3	-13.8	-9.54	-9.58	-3.4	173.

Collaboration with P. Echternach (JPL)

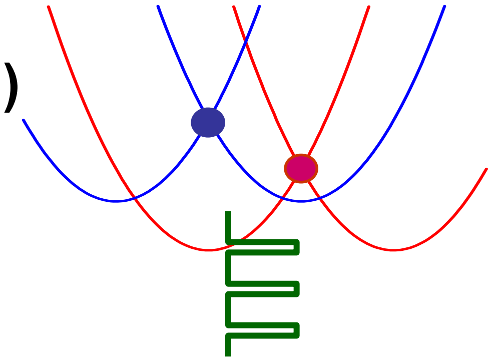
Numerical simulation of Rabi oscillations in single-Cooper-pair-box qubit

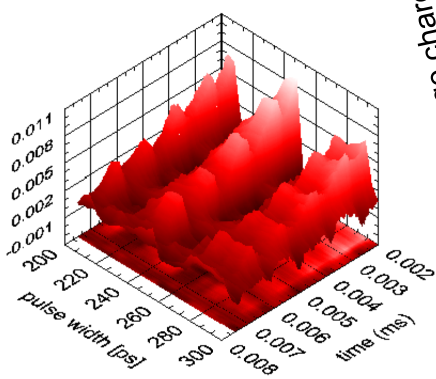




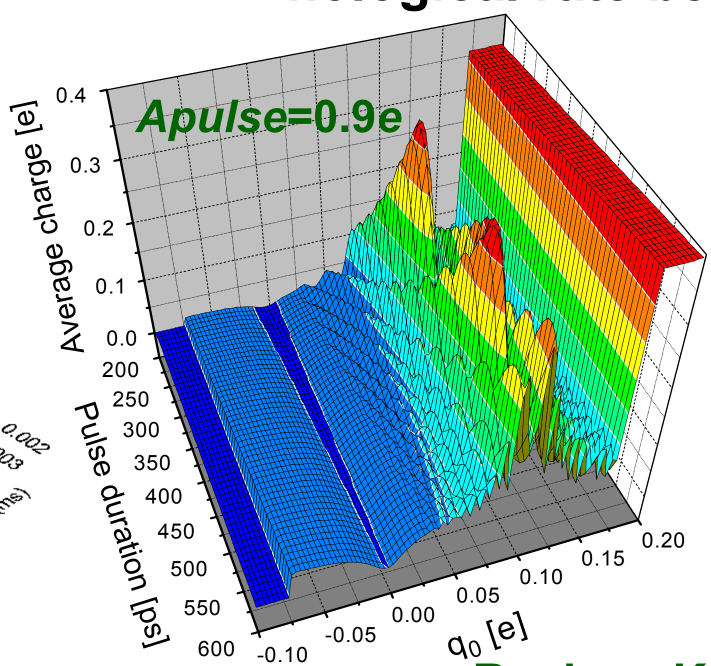
Model:

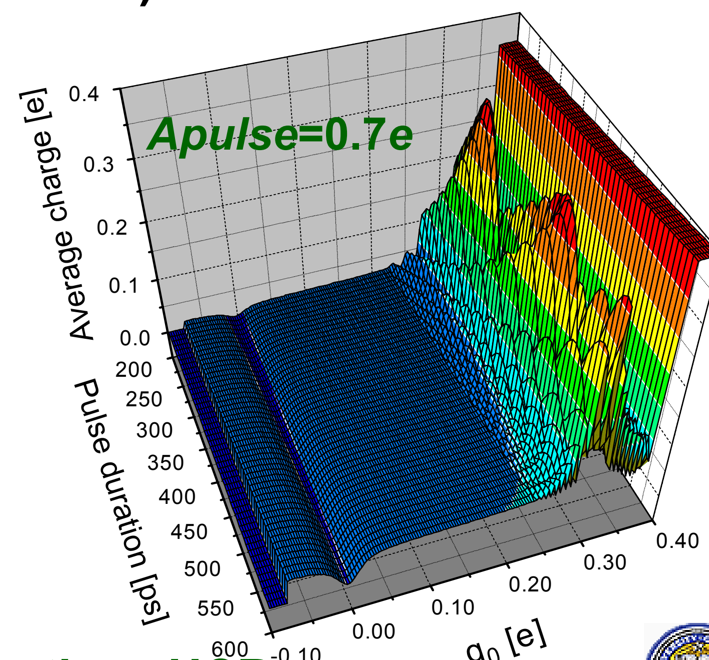
- four charge states (-1,0,1,2)
- coherent coupling of states 0/2 and 1/-1
- quasiparticle tunneling (BCS above 2Δ, phenomenological rate below 2Δ)





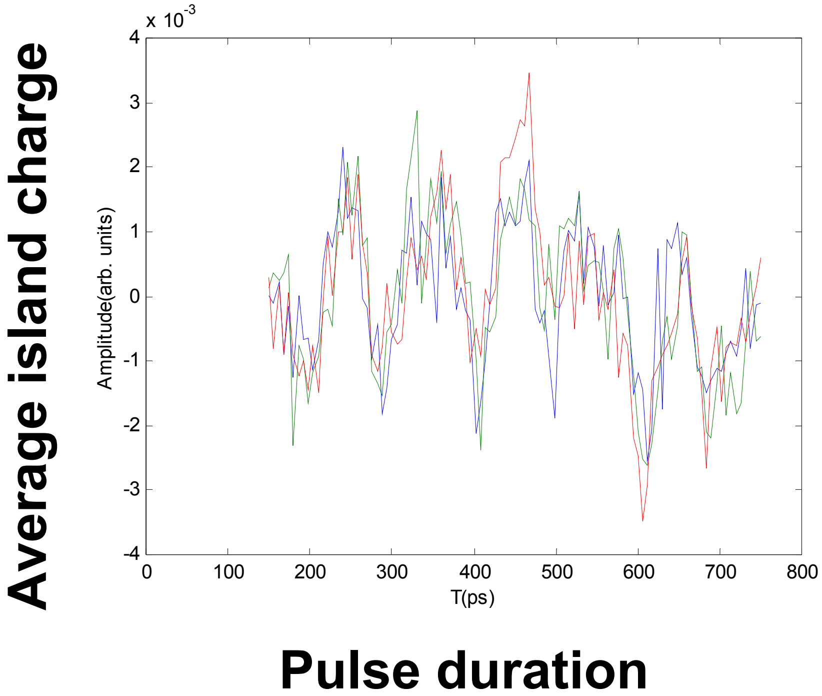
JPL

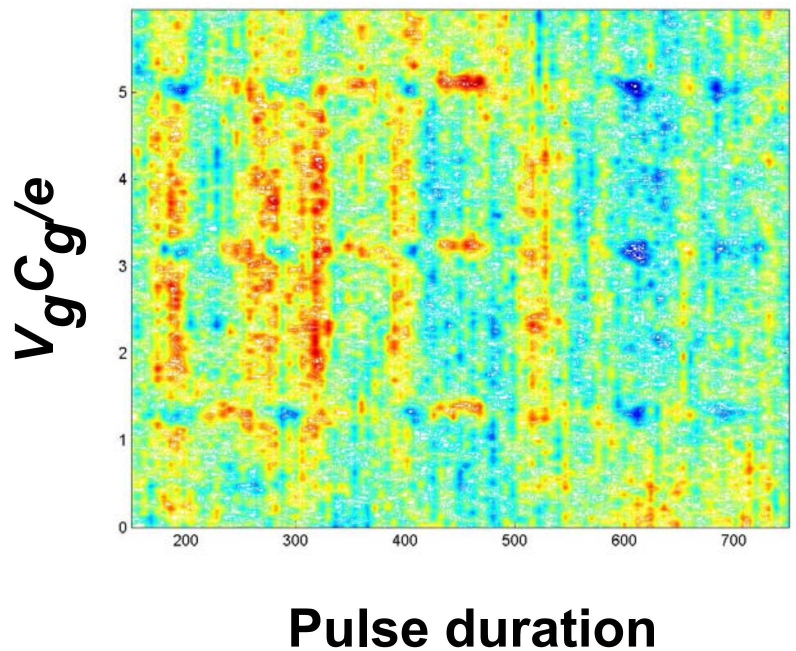




Ruskov, Korotkov, UCR

Recent experimental results of A. Guillaume and P. Echternach (JPL)

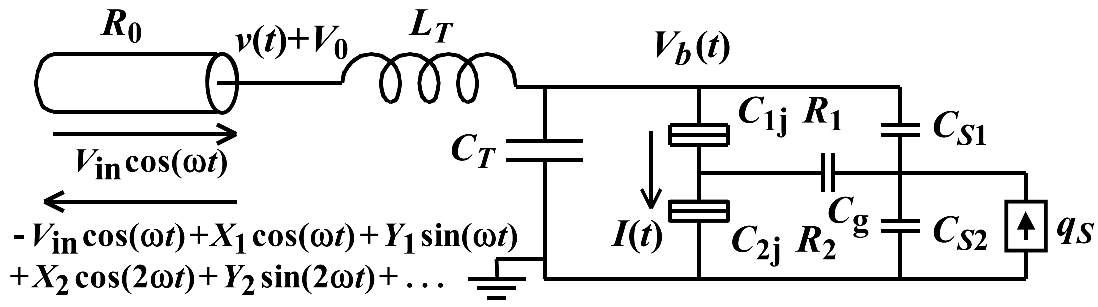




Rabi oscillations in a Cooper-pair-box qubit

Response and sensitivity of RF-SET

Turin-Korotkov, 2003



$$Q_L = (1/Q + 1/Q_{SET})^{-1}$$

$$Q = \sqrt{L_T / C_T} / R_0$$

Matching: $Q \approx \sqrt{R_{SET} / R_0}$

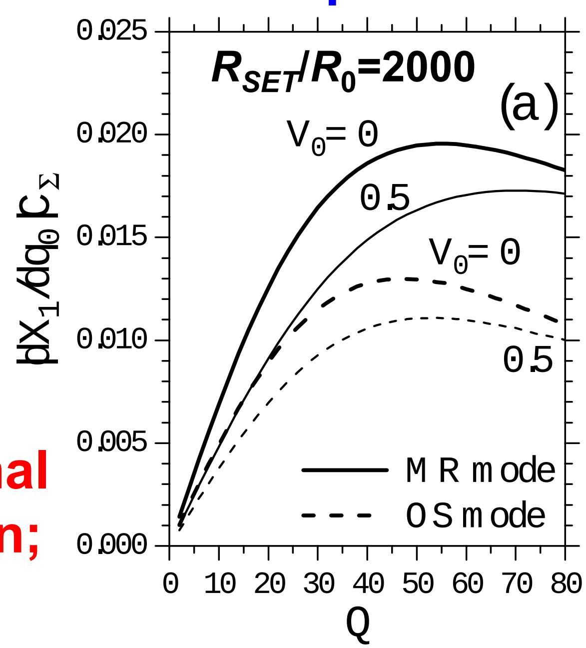
Previous theoretical papers:

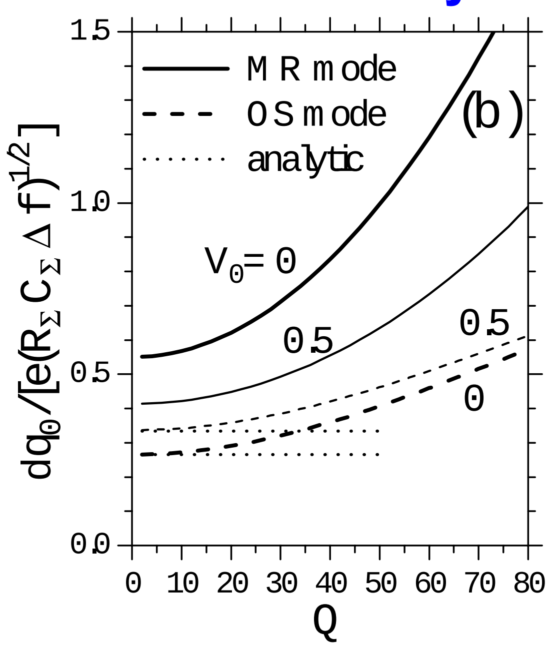
- Korotkov-Paalanen, 1999
- Blencowe-Wybourne, 2000
- Zhang-Blencowe, 2002

All of them assumed low *Q*-factor (<< matching)

response

sensitivity



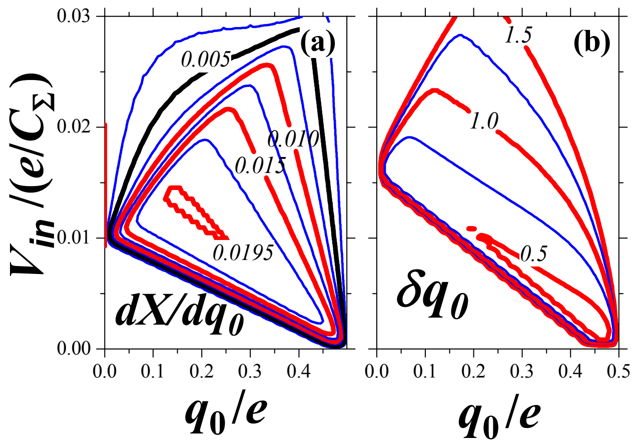


RF-SET response is maximal close to matching condition; however, large *Q*-factor worsens RF-SET sensitivity (shot-noise-limited)

MR – maximum response mode
OS – optimized sensitivity mode

Response and sensitivity of RF-SET

response **sensitivity**



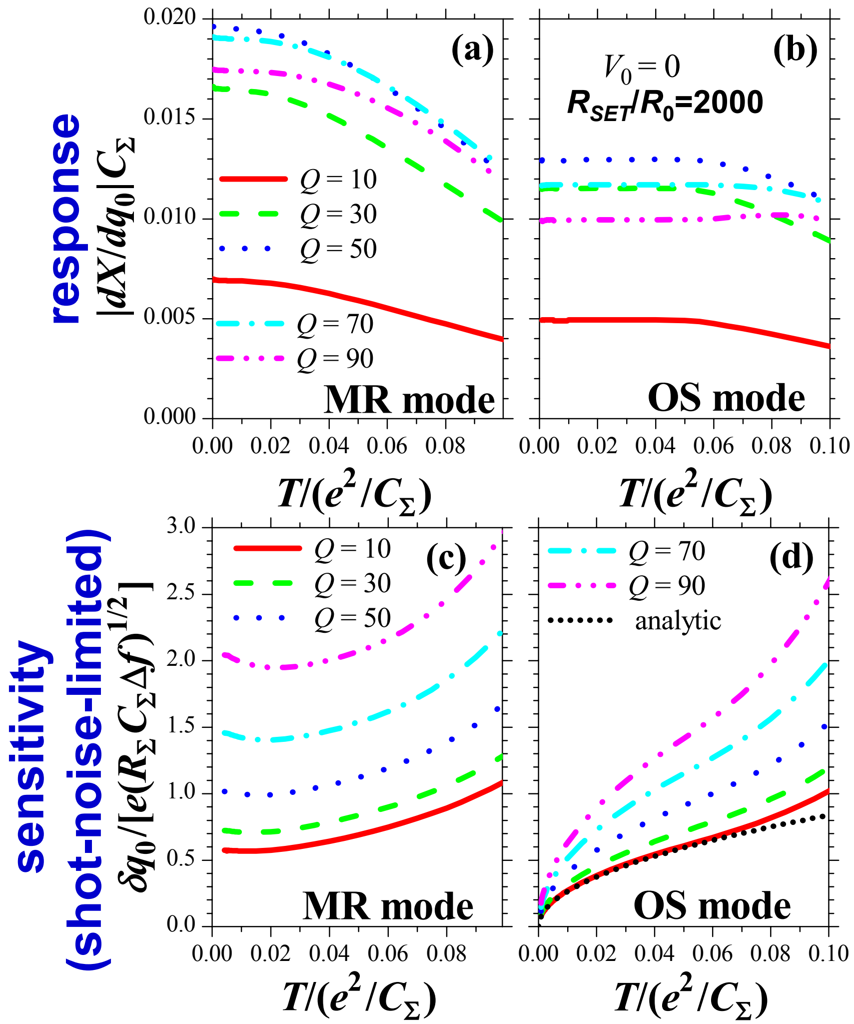
Optimizations of response and sensitivity are different (rf amplitude is much smaller for optimal sensitivity)

Model:

- full nonlinear analysis
- several overtones

- normal metal SET only
- no cotunneling
- low frequency signal
- no backaction analyzed

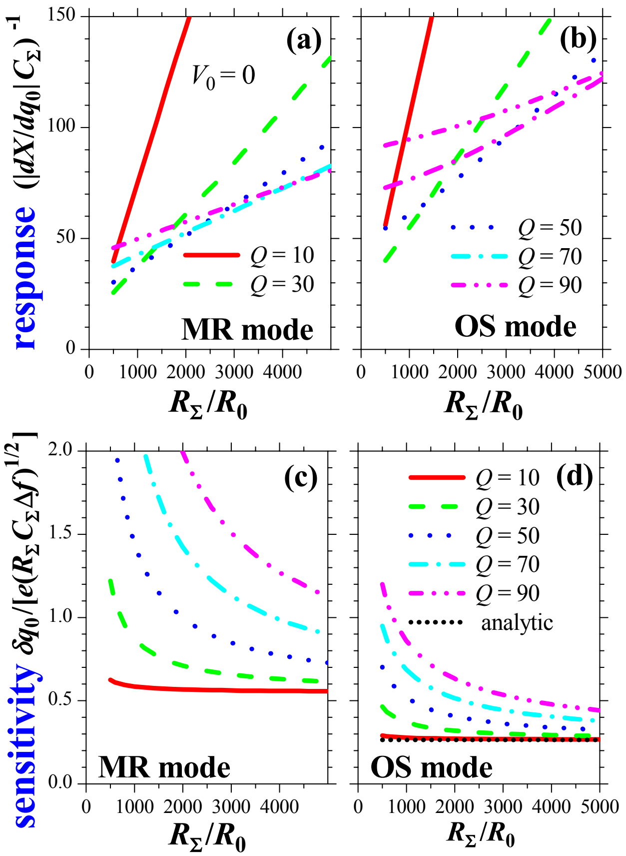
Temperature dependence



MR – maximum response mode
OS – optimized sensitivity mode

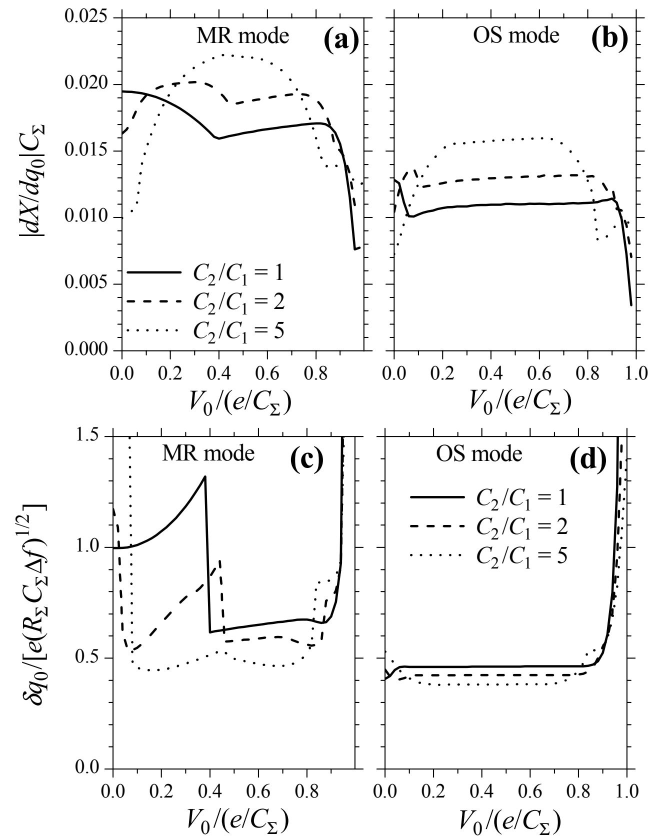
Response and sensitivity of RF-SET

Dependence on SET resistance



MR – maximum response mode
OS – optimized sensitivity mode

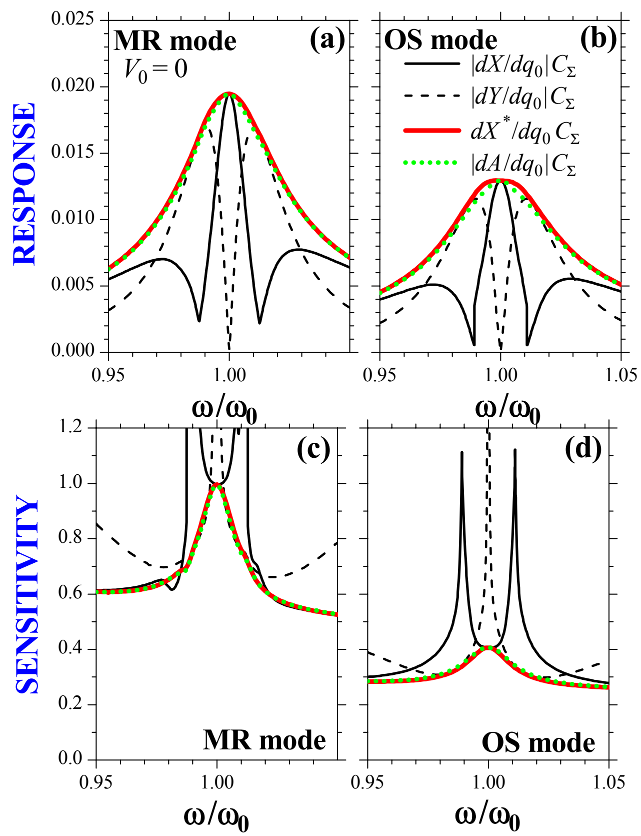
Effect of asymmetric rf biasing



Asymmetric rf biasing does not worsen the RF-SET performance

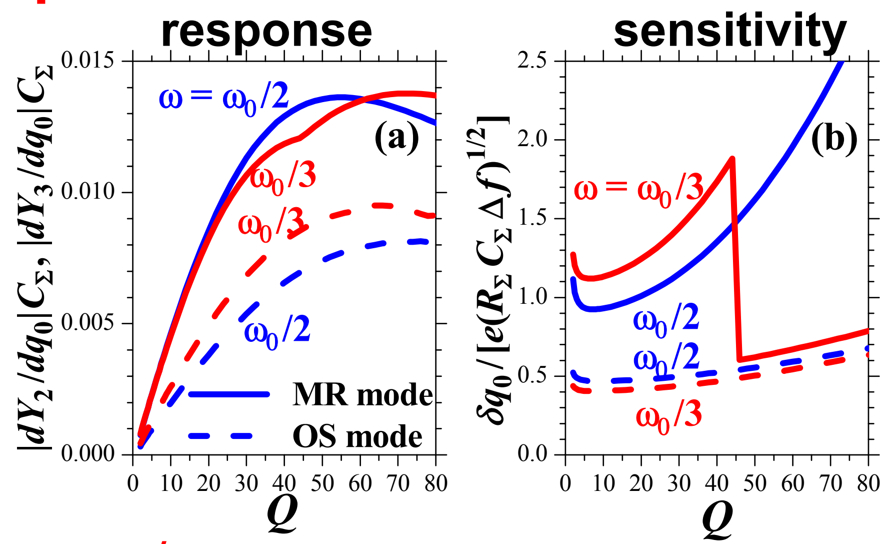
Response and sensitivity of RF-SET

Dependence on rf detuning



- sensitivity does not worsen with detuning
- monitoring by rectification is as good as homodyne detection

Proposal of resonant overtone mode



$\omega = \omega_0/n$, reflected wave due to SET nonlinearity, in resonance with tank

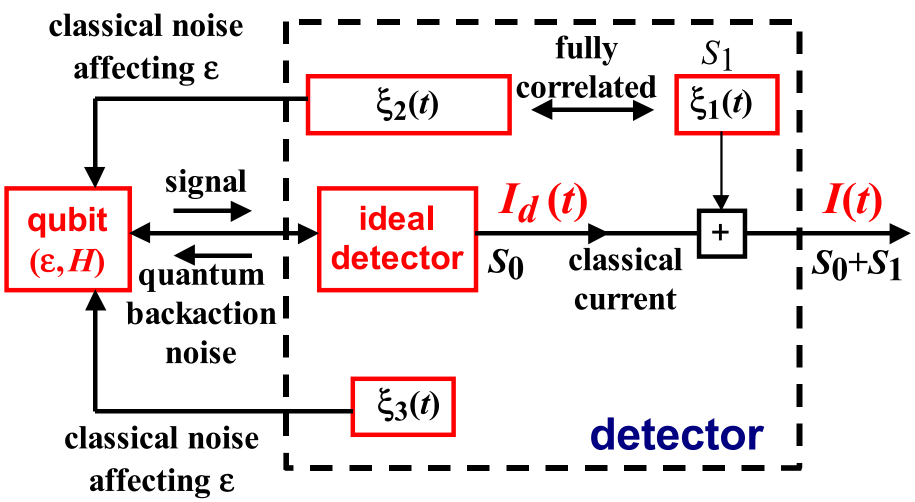
Advantage: different frequencies of incident and reflected waves

RF-SET performance in the mode of resonant overtone is comparable to performance in the usual regime

Recent experimental realization:
Keith Schwab, similar performance in the proposed and usual modes

Nonideal quantum detectors of solid-state qubits

Korotkov, PRB 67, 235408 (2003)



$$\frac{d}{dt}\rho_{11} = -2H\,\mathrm{Im}\,\rho_{12} + \rho_{11}\rho_{22}\frac{2\Delta I}{S_I}[I(t)-I_0]$$

$$\frac{d}{dt}\rho_{12} = i\tilde{\varepsilon}\rho_{12} + iH(\rho_{11}-\rho_{22}) + iK[I(t)-I_0]$$

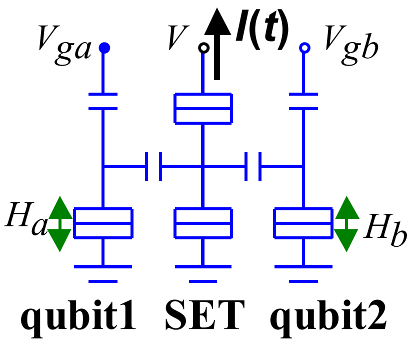
$$+\rho_{12}(\rho_{11}-\rho_{22})\frac{\Delta I}{S_I}[I(t)-I_0] - \tilde{\gamma}\rho_{12}$$

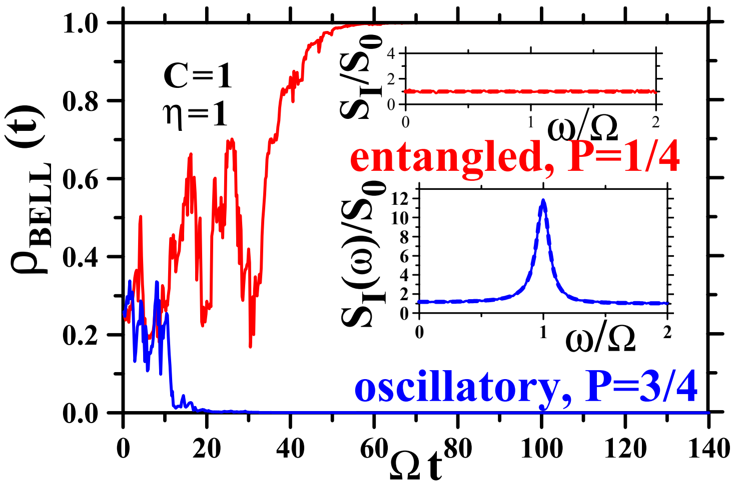
phenomenological model combining quantum and classical noises

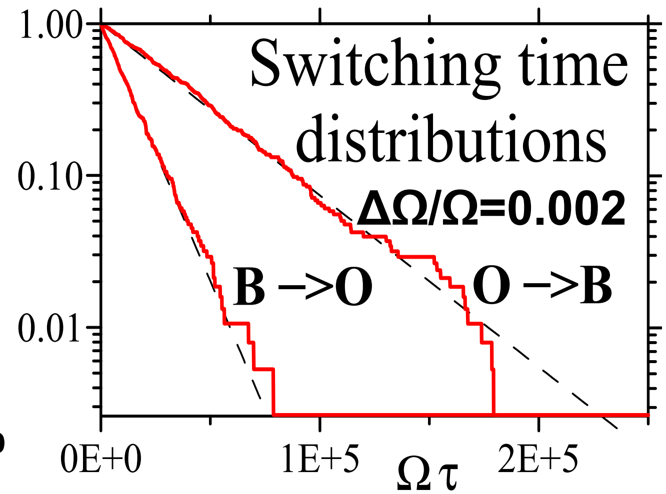
- Bayesian formalism is extended to a model, which takes into account several factors of the detector nonideality
- "asymmetric" coupling and correlation between output and backaction noises are taken into account
- formalism for a single qubit measurement is generalized to measurement of entangled qubits

Making and keeping two qubits entangled by continuous measurement

Ruskov-Korotkov, PRB 67, 241305(R) (2003)





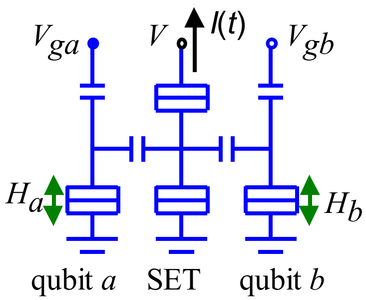


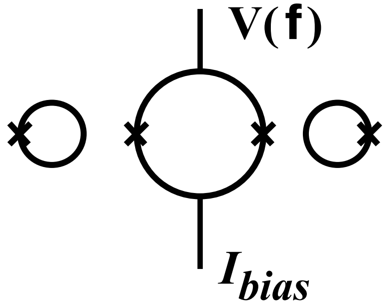
- **With probability 1/4, two qubits measured by an equally coupled detector become fully entangled (Bell state); the entangled state can be distinguished from the other (oscillating) state by a flat spectrum of detector noise.**
- **Deviations from symmetry and environmental dephasing lead to switching between the entangled and oscillating states. Switching rates have been calculated numerically and analytically (for rare switching).**
- **Entangled state can be maintained by feedback.**

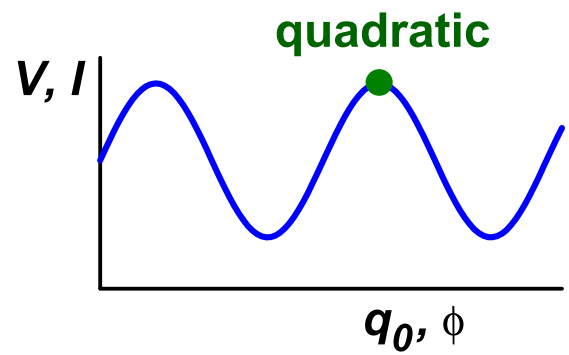
Short paper published, long (detailed) paper almost finished.

Quadratic quantum detection

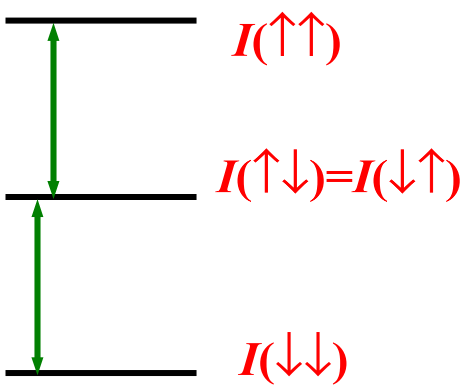
Mao, Averin, Ruskov, Korotkov, 2003



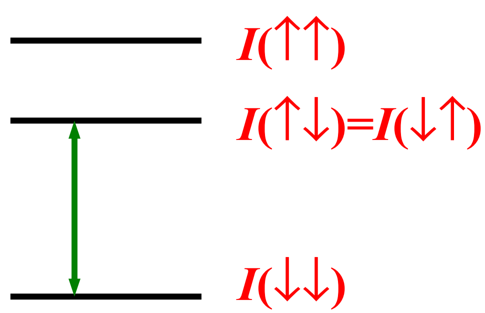




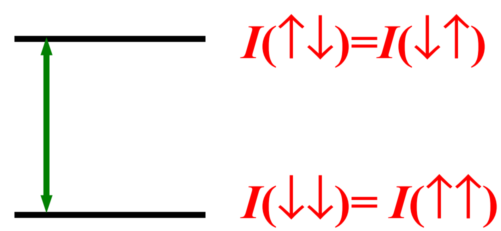
Linear detector



Noninear detector

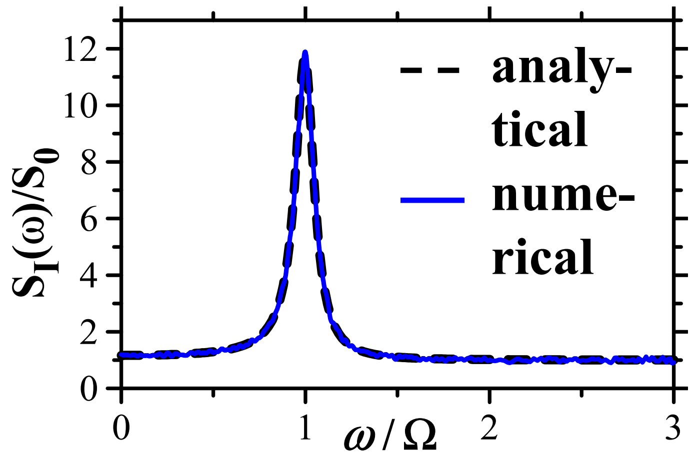


Quadratic detector

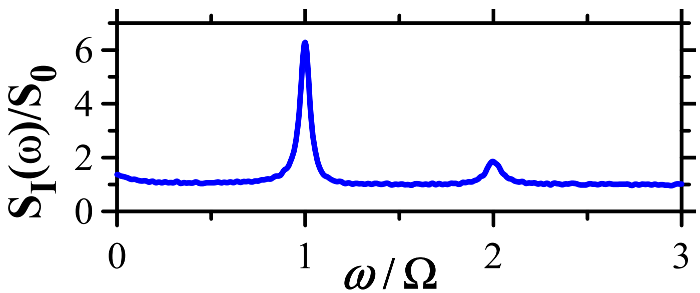


Quadratic detection is useful for quantum error correction (Averin-Fazio, 2002)

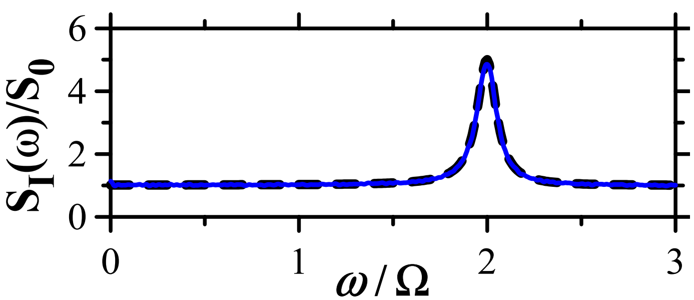
Linear detector



Nonlinear detector



Quadratic detector



Two-qubit detection

(oscillatory subspace)

$$S_I(\omega) = S_0 + \frac{8}{3}\frac{\Omega^2(\Delta I)^2\Gamma}{(\omega^2-\Omega^2)^2+\Gamma^2\omega^2}$$

$$\Gamma = \eta^{-1}(\Delta I)^2/4S_0,\ \Delta I = I_1 - I_{23} = I_{23} - I_4$$

Spectral peak at Ω, peak/noise = (32/3)η

(Ω is the Rabi frequency)

Extra spectral peaks at 2Ω and 0

$$S_I(\omega) = S_0 + \frac{4\Omega^2(\Delta I)^2\Gamma}{(\omega^2-4\Omega^2)^2+\Gamma^2\omega^2}$$

$$(\Delta I = I_{23} - I_{14},\ \ I_1 = I_4,\ \ I_2 = I_3)$$

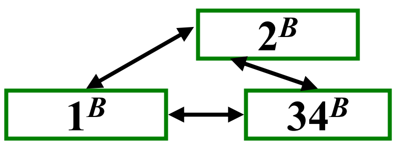
Peak only at 2Ω, peak/noise = 4η

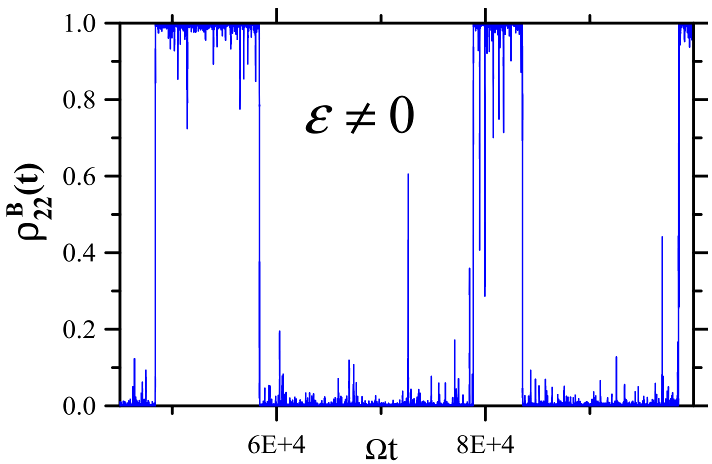
Mao, Averin, Ruskov, Korotkov, 2003

Two-qubit quadratic detection: scenarios and switching

Three scenarios:
(distinguishable by average current)

1) collapse into $|\uparrow\downarrow - \downarrow\uparrow\rangle = |1\rangle^B$, current $I_{\neq\emptyset}$, flat spectrum
2) collapse into $|\uparrow\uparrow - \downarrow\downarrow\rangle = |2\rangle^B$, current $I_{\neq\neq}$, flat spectrum
3) collapse into remaining subspace $|34\rangle^B$, current $(I_{\neq\emptyset}+I_{\neq\neq})/2$, spectral peak at 2Ω, peak/pedestal = 4η.





3) Slightly asymmetric qubits, $\varepsilon \neq 0$

$$\Gamma_{2B\to 34B} = 2\varepsilon^2\Gamma/\Omega^2$$

Switching between states due to imperfections

1) Slightly different Rabi frequencies, $\Delta\Omega=\Omega_1-\Omega_2$

$$\Gamma_{1B\to 2B} = \Gamma_{2B\to 1B} = (\Delta\Omega)^2/2\Gamma, \quad \Gamma = \eta^{-1}(\Delta I)^2/4S_0$$

$$S_I(\omega) = S_0 + \frac{(\Delta I)^2\Gamma}{(\Delta\Omega)^2}\,\frac{1}{1+\left[\omega\Gamma/(\Delta\Omega)^2\right]^2}$$

2) Slightly nonquadratic detector, $I_1 \neq I_4$

$$\Gamma_{2B\to 34B} = [(I_1-I_4)/\Delta I]^2\Gamma/2$$

$$S_I(\omega) = S_0 + \frac{2}{3}\,\frac{4\Omega^2(\Delta I)^2\Gamma}{(\omega^2-4\Omega^2)^2+\Gamma^2\omega^2} + \frac{8(\Delta I)^4}{27\Gamma(I_1-I_4)^2}\,\frac{1}{1+[4\omega(\Delta I)^2/3\Gamma(I_1-I_4)^2]^2}$$

Mao, Averin, Ruskov, Korotkov, 2003

Measurement of multi-qubit operators

Measurement of one qubit – natural
Measurement of a multi-qubit function – not trivial

***Problem*: measurement tends to collapse each qubit separately**
***Solution*: not distinguishable states (equal coupling)**

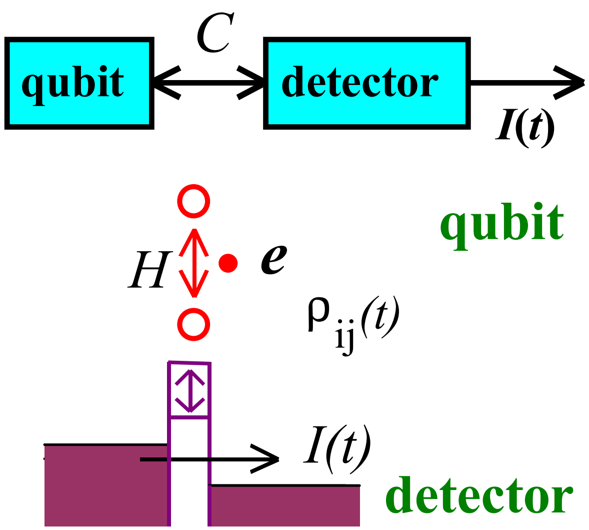
1. **Measurement of** $(\vec{\sigma}_1 + \vec{\sigma}_2)^2$ ***(Ruskov-Korotkov, 2002)***
   Linear equally coupled detector, continuous measurement

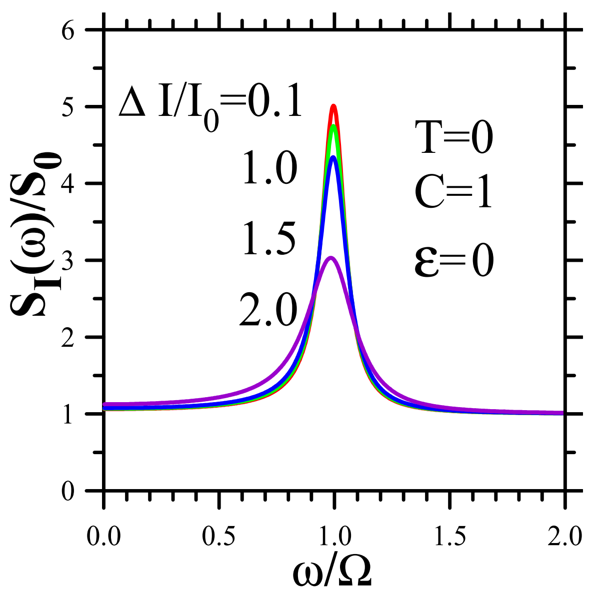
2. **Measurement of** $\sigma_{1X}\sigma_{2X}$ ***(Averin-Fazio, 2002)***
   Quadratic equally coupled detector, projective measurement

3. **Operator** $\sigma_{1Z}\sigma_{2Z} + \sigma_{1Y}\sigma_{2Y}$ **(*Mao-Averin-Ruskov-Korotkov, 2003*)**
   Quadratic equally coupled detector; continuous measurement

• • •

Measurable spectrum of Rabi oscillations





Ruskov-Korotkov, PRB 67, 075303 (2003)

Rabi oscillations in a single qubit lead to the spectral peak in the detector current; however, peak-to-noise ratio £ 4.

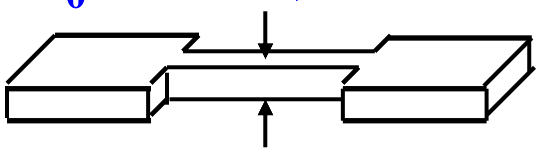
Many approaches to this problem:

Korotkov, LT'99
Korotkov-Averin, 2000
Korotkov, 2000
Averin, 2000
Goan-Milburn, 2001
Makhlin et al., 2001
Balatsky-Martin, 2001
Ruskov-A.K., 2002
Mozyrsky et al., 2002
Balatsky et al, 2002
Bulaevskii et al., 2002
Shnirman et al., 2002
Zhu-Balatsky, 2002
Bulaevskii-Ortiz, 2003
Nussinov et al., 2003
Gurvitz et al., 2003
Stace-Barrett, 2003

Bayesian approach to continuous position measurement of a nanomechanical resonator

Ruskov- Korotkov, 2003

$\omega_0 \sim 1$ GHz , $T \sim 20$ mK , quantum behavior

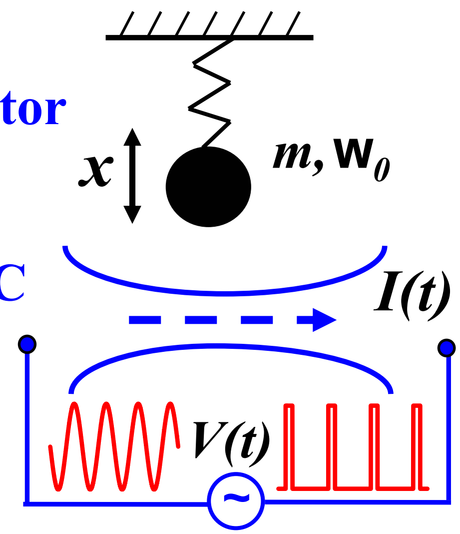


$$\hat{H}_0 = \frac{\hat{p}^2}{2m} + \frac{m\omega_0^2 \hat{x}^2}{2}$$

$$\hat{H}_{DET} = \sum_l E_l a_l^\dagger a_l + \sum_r E_r a_r^\dagger a_r + \sum_{l,r} (M a_l^\dagger a_r + H.c.)$$

$$\hat{H}_{INT} = \sum_{l,r} (\Delta M \, \hat{x} \, a_l^\dagger a_r + H.c.)$$

resonator



Detector noise

$$S_X \simeq S_0 \equiv 2eI_0$$

Coupling

$$C \equiv \frac{\hbar k^2}{S_0 m \omega_0^2} \propto \frac{T^{osc}}{\tau^{meas}}$$

$$I_X = 2\pi \left(M + \Delta M \, x \right)^2 \rho_l \rho_r e^2 \frac{V}{\hbar} \simeq I_0 + k\,x$$

Evolution equation (Stratonovich form)

$$\frac{d}{dt}\rho(x,x') = \frac{-i}{\hbar}[\hat{H}_0, \rho] + \rho(x,x') \frac{1}{S_0} \left\{ \left(I(t) - I_0 \right) k(x + x' - 2\bar{x}) - k^2 \left(\frac{x^2 + x'^2}{2} - \overline{x^2} \right) \right\}$$

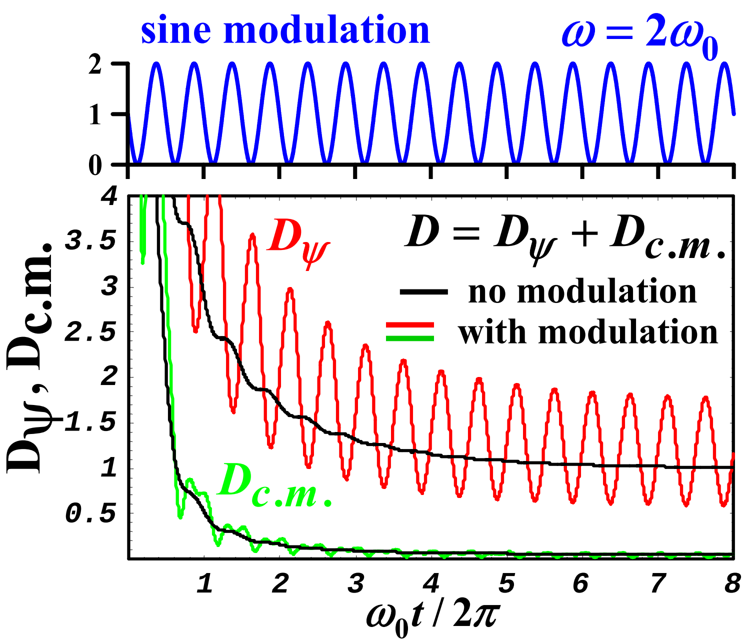
Cooling by feedback $\hat{H}^{fb} = -F\,\hat{x}, \quad F = -\gamma \left(m\omega_0 \bar{x} + \bar{p} \right)$

Formalism similar to A. Hopkins et al., 2003 & Doherty-Jacobs, 1999

QND squeezing of a nanoresonator by feedback

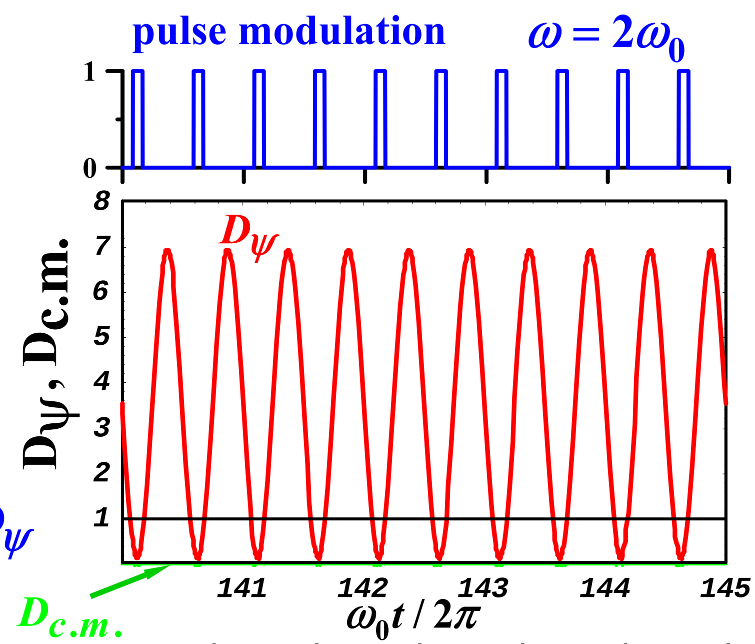
Constant voltage – no squeezing at *C* Ü 1 (Hopkins, Jacobs, Habib, Schwab, 2003)

We consider periodic *V(t)* – squeezing possible! **(Ruskov-Korotkov, 2003)**

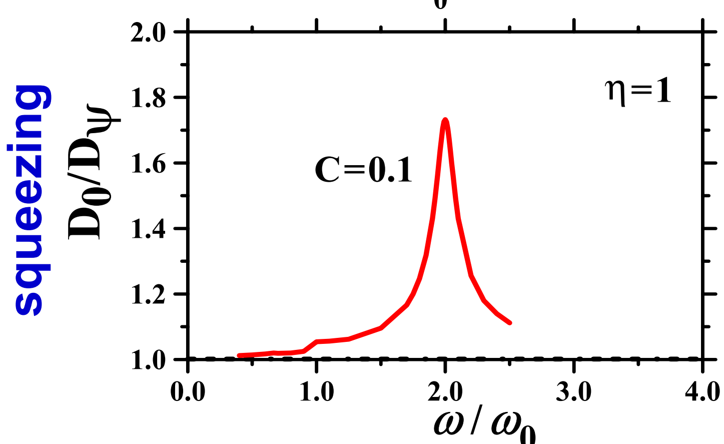


$C = 0.1$

$\dfrac{\gamma}{\omega_0} = 10$

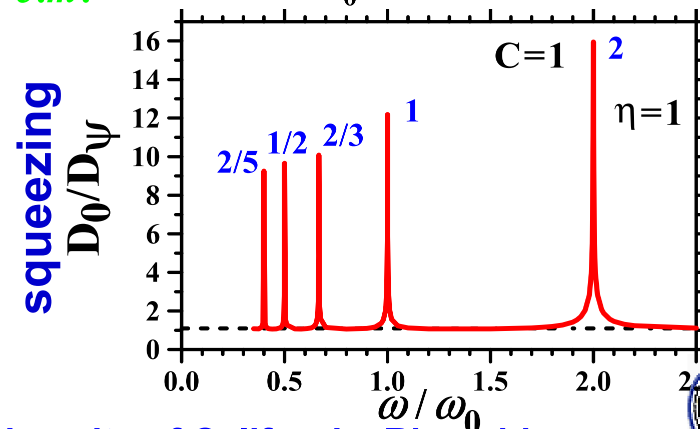


$D_{c.m.} \ll D_\psi$



Squeezing resonances

at $\omega = \dfrac{2\omega_0}{n}$



Summary

- Tight experimental collaboration with JPL group (P. Echternach) established
- Response and sensitivity of a normal-metal RF-SET calculated within "orthodox" theory; new operation regime proposed
- Bayesian formalism for nonideal detectors developed
- Initialization of two-qubit entanglement by measurement studied; de-entanglement rates due to imperfections analyzed
- Continuous quadratic quantum measurement studied; spectral peak at doubled Rabi frequency predicted
- Quantum feedback of nanomechanical resonators studied, QND squeezing shown to be possible

Research Plan

State Purification and Decoherence Suppression by Continuous Measurement of a Qubit

A. N. Korotkov, UC Riverside

- Progress on last year's objectives

- studied preparation of entangled two-qubit state by continuous measurement
- developed theory of continuous quantum measurement by a nonideal detector
- preliminary results on quantum monitoring of micromechanical oscillators
- analyzed two-qubit continuous measurement by a quadratic quantum detector
- analyzed performance of an RF-SET with realistically large Q-factor
- 3 journal papers and 2 proceedings papers published
- collaboration with P. Echternach (JPL) on experimental observation of Rabi oscillations

- **Research plan for the next 12 months**

- **continue detailed analysis of one-qubit and two-qubit quantum feedback**
- **analyze QND measurement and feedback control of micromechanical resonators**
- **compare quantitatively continuous and instantaneous measurements of a qubit**
- **analyze in more detail RF-SET response and sensitivity in a realistic setup**
- **collaboration with JPL group on coherent one-qubit and two-qubit manipulations**

- **Long term objectives**

- **error correction and qubit initialization by continuous measurement and feedback**
- **calculation of necessary bounds for entanglement demonstration**
- **RF-SET as a quantum detector (for a quantum feedback)**
- **experiments on Bell-type measurement, quantum feedback, spontaneous entanglement**