Quadratic quantum measurements

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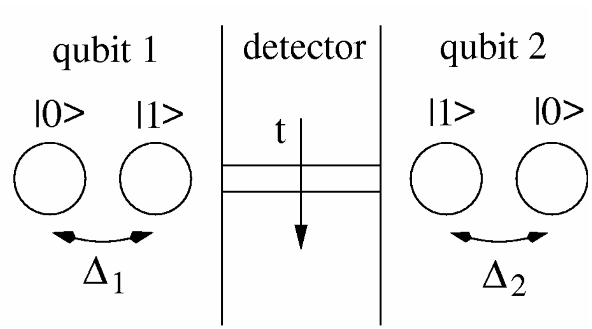
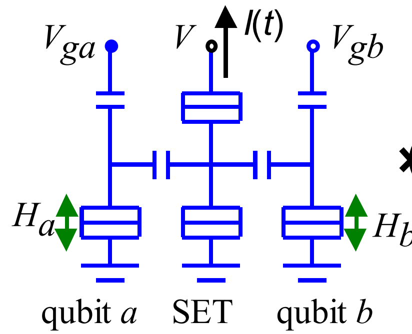
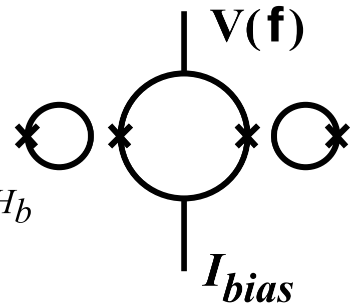
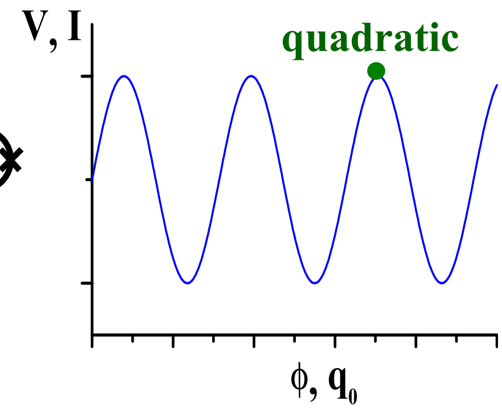
We develop a theory of quadratic quantum measurements by a mesoscopic detector. It is shown that the quadratic measurements should have non-trivial quantum information properties, providing, for instance, a simple way of entangling two non-interacting qubits. We also calculate output spectrum of a detector with both linear and quadratic response, continuously monitoring two qubits.

Studied setup: two qubits and detector









Setup is similar to Ruskov-Korotkov, PRB 67, 241305(R) (2003), but a nonlinear (instead of a linear) detector is considered

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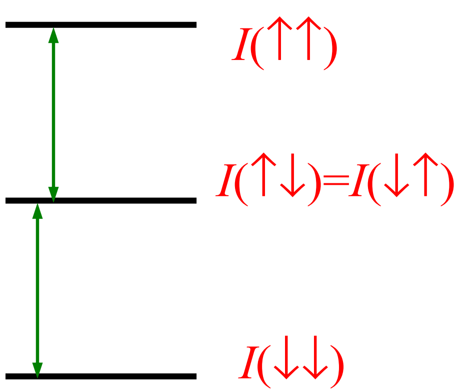
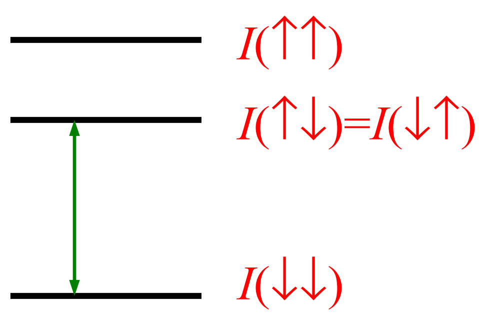
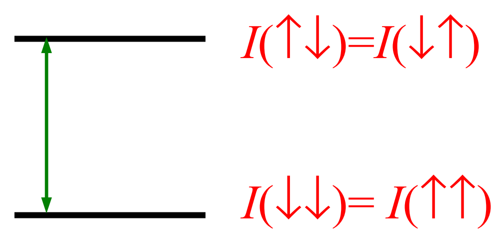
Linear detector



Noninear detector



Quadratic detector



Quadratic detection is useful for quantum error correction (Averin-Fazio, 2002)

Bayesian formalism for a nonlinear detector

$$H = H_{QBs} + H_{DET} + \sum_{j=1,2} [t(\{\sigma_z^j\})\xi + t^\dagger(\{\sigma_z^j\})\xi^\dagger]$$

$$t(x) = t_0 + \delta_1\sigma_z^1 + \delta_2\sigma_z^2 + \lambda\sigma_z^1\sigma_z^2 \qquad \delta_j = 0 \Rightarrow \text{quadratic detector}$$

Assumed: 1) weak tunneling in the detector, 2) large detector voltage (fast detector dynamics, and 3) weak response.
The model describes an ideal detector (no extra noises).

Recipe: Coupled detector-qubits evolution and frequent collapses of the number *n* of electrons passed through the detector

Two-qubit evolution (Ito form):

$$\frac{d}{dt}\rho_{kl} = -i[H_{QBs},\rho]_{kl} + [I(t)-\langle I\rangle][\frac{1}{S_0}(I_k + I_l - 2\langle I\rangle) - i\varphi_{kl}]\rho_{kl} - \gamma_{kl}\rho_{kl}$$

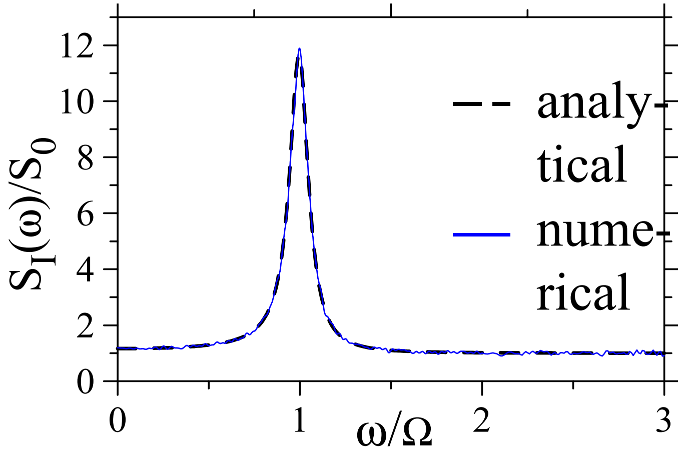
$$\gamma_{kl} = (1/2)(\Gamma_+ + \Gamma_-)[(|t_k| - |t_l|)^2 + \varphi_{kl}^2 |t_0|^2], \quad \varphi_{kl} = \arg(t_k t_l^*)$$

$$\langle I\rangle = \sum_j \rho_{jj} I_j, \quad I_k = (\Gamma_+ - \Gamma_-)|t_k|^2, \quad S_0 = 2(\Gamma_+ + \Gamma_-)|t_0|^2$$

Two-qubit detection

(oscillatory subspace)

Linear detector



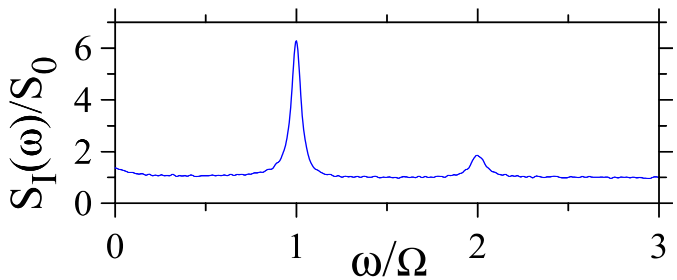
$$S_I(\omega) = S_0 + \frac{8}{3}\frac{\Omega^2(\Delta I)^2\Gamma}{(\omega^2-\Omega^2)^2+\Gamma^2\omega^2}$$

$$\Gamma = \eta^{-1}(\Delta I)^2/4S_0,\ \Delta I = I_1 - I_{23} = I_{23} - I_4$$

Spectral peak at Ω, peak/noise = (32/3)η

(Ω is the Rabi frequency) **(Ruskov-Korotkov, 2002)**

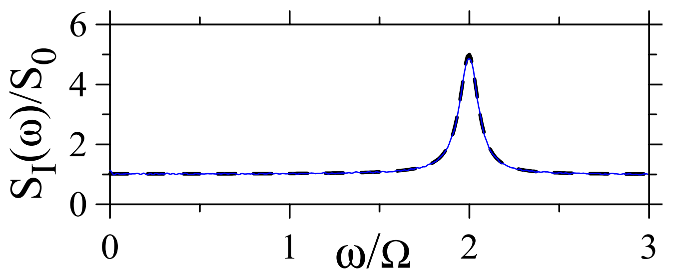
Nonlinear detector



Extra spectral peaks at 2Ω and 0

(analytical formula for weak coupling case)

Quadratic detector



$$S_I(\omega) = S_0 + \frac{4\Omega^2(\Delta I)^2\Gamma}{(\omega^2-4\Omega^2)^2+\Gamma^2\omega^2}$$

$$(\Delta I = I_{23} - I_{14},\ \ I_1 = I_4,\ \ I_2 = I_3)$$

Peak only at 2Ω, peak/noise = 4η

1=↑↑, 2=↑↓, 3=↓↑, 4=↓↓

Mao, Averin, Ruskov, Korotkov, 2004

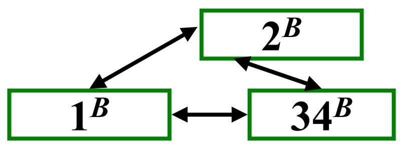
Two-qubit quadratic detection: scenarios and switching

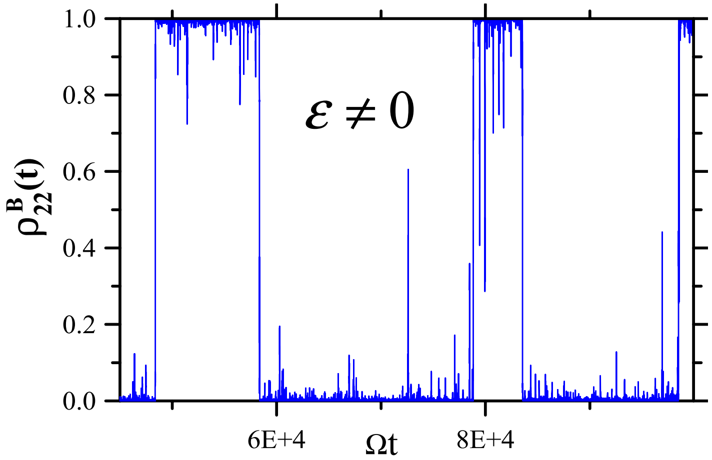
Three scenarios:
(distinguishable by average current)

1) collapse into |↑↓ - ↓↑Ú = |1Ú***B*** , current $I_{Æ∞}$, flat spectrum

2) collapse into |↑↑ - ↓↓Ú = |2Ú***B*** , current $I_{ÆÆ}$, flat spectrum

3) collapse into remaining subspace |34Ú***B*** , current ($I_{Æ∞}$+ $I_{ÆÆ}$)/2, spectral peak at 2Ω, peak/pedestal = 4η.





3) Slightly asymmetric qubits, $\varepsilon \neq 0$

$$\Gamma_{2B\to 34B} = 2\varepsilon^2\Gamma/\Omega^2$$

Switching between states due to imperfections

1) Slightly different Rabi frequencies, $\Delta\Omega = \Omega_1 - \Omega_2$

$$\Gamma_{1B\to 2B} = \Gamma_{2B\to 1B} = (\Delta\Omega)^2/2\Gamma, \quad \Gamma = \eta^{-1}(\Delta I)^2/4S_0$$

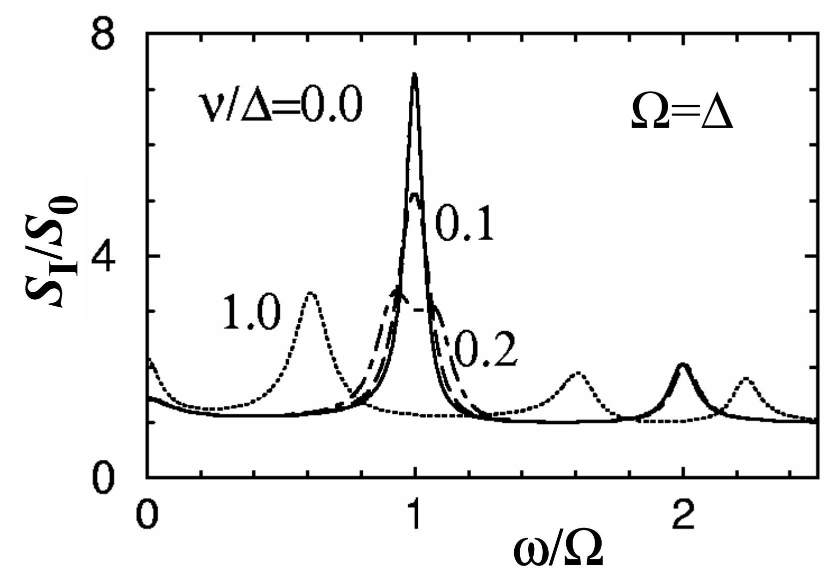
$$S_I(\omega) = S_0 + \frac{(\Delta I)^2\Gamma}{(\Delta\Omega)^2}\,\frac{1}{1+\left[\omega\Gamma/(\Delta\Omega)^2\right]^2}$$

2) Slightly nonquadratic detector, $I_1 \neq I_4$

$$\Gamma_{2B\to 34B} = [(I_1 - I_4)/\Delta I]^2\Gamma/2$$

$$S_I(\omega) = S_0 + \frac{2}{3}\,\frac{4\Omega^2(\Delta I)^2\Gamma}{(\omega^2-4\Omega^2)^2+\Gamma^2\omega^2} + \frac{8(\Delta I)^4}{27\Gamma(I_1-I_4)^2}\,\frac{1}{1+[4\omega(\Delta I)^2/3\Gamma(I_1-I_4)^2]^2}$$

Effect of qubit-qubit interaction



$$H_{QBs} = -\sum_j (\varepsilon_j \sigma_z^j + \Delta_j \sigma_x^j)/2 + \frac{\nu}{2}\sigma_z^1 \sigma_z^2$$

ν - interaction between two qubits

First spectral peak splits (first order in ν), second peak shifts (second order in ν)

$$\omega_{1-} = [\Delta^2 + (\nu/2)]^{1/2} - \nu/2$$

$$\omega_{1+} = [\Delta^2 + (\nu/2)]^{1/2} + \nu/2$$

$$\omega_2 = 2[\Delta^2 + (\nu/2)]^{1/2} = \omega_{1-} + \omega_{1+}$$

Conclusions

- **Conditional (Bayesian) formalism for a nonlinear detector is developed**
- **Detector nonlinearity leads to the second peak in the spectrum (at 2Ω), in purely quadratic case there is no peak at Ω (very similar to classical nonlinear and quadratic detectors)**
- **Qubits become entangled (with some probability) due to measurement, detection of entanglement is easier than for a linear detector (current instead of spectrum)**