Quantum nondemolition (QND) squeezing of a nanoresonator

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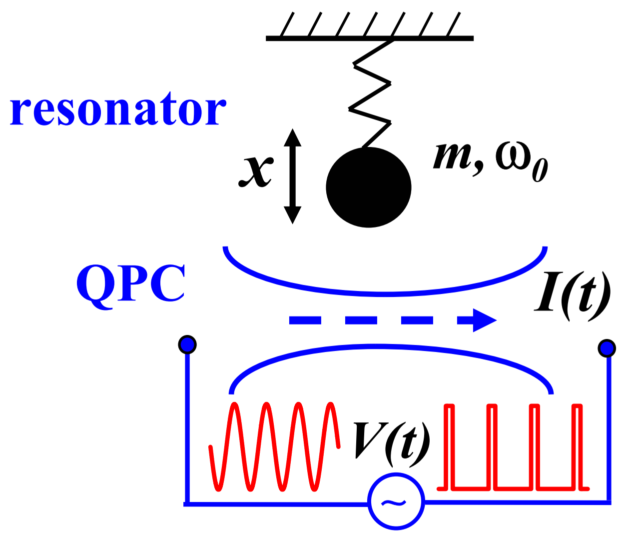
We show that the nanoresonator position an be squeezed significantly below the ground state level by measuring the nanoresonator with a quantum point contact or a single-electron transistor and applying a periodic voltage across the detector. The mechanism of squeezing is basically a generalization of quantum nondemolition measurement of an oscillator to the case of continuous measurement by a weakly coupled detector. The quantum feedback is necessary to prevent the "heating" due to measurement back-action. We also discuss a procedure of experimental verification of the squeezed state.

cond-mat/0406416

QND squeezing of a nanoresonator

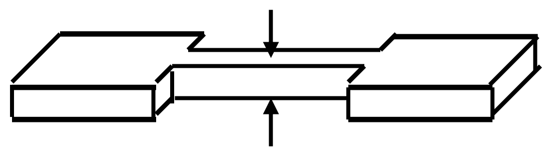
Ruskov, Schwab, Korotkov, cond-mat/0406416



$$\hat{H}_0 = \hat{p}^2/2m + m\omega_0^2\hat{x}^2/2$$

$$\hat{H}_{DET} = \sum_l E_l a_l^\dagger a_l + \sum_r E_r a_r^\dagger a_r + \sum_{l,r}(M a_l^\dagger a_r + H.c.)$$

$$\hat{H}_{INT} = \sum_{l,r}(\Delta M\, \hat{x}\, a_l^\dagger a_r + H.c.)$$



$\omega_0 \sim 1$ GHz , $T \sim 50$ mK, quantum behavior $T < \hbar\omega_0$ or $T\tau_{obs}/Q < \hbar/2$

Model similar to Hopkins, Jacobs, Habib, Schwab, PRB 2003
(continuous monitoring and quantum feedback to cool down)

New feature: Braginsky's stroboscopic QND measurement using modulation of detector voltage ⇒ **squeezing becomes possible**

Potential application: ultrasensitive force measurements

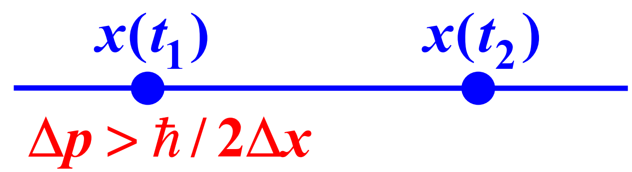
Other most important papers:
Doherty, Jacobs, PRA 1999 (formalism for Gaussian states)
Mozyrsky, Martin, PRL 2002 (ensemble-averaged evolution)

Stroboscopic QND measurements

Quantum nondemolition (QND) measurements (Braginsky-Khalili book)
(a way to suppress measurement backaction and overcome standard quantum limit)

Idea: to avoid measuring the magnitude conjugated to the magnitude of interest

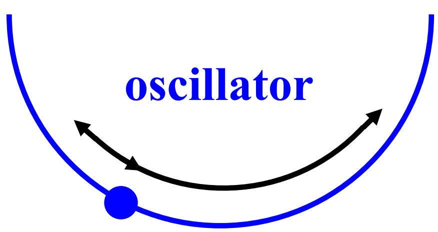
Standard quantum limit



Example: measurement of $x(t_2)$-$x(t_1)$

First measurement: $\Delta p(t_1) > \hbar/2\Delta x(t_1)$, then even for accurate second measurement inaccuracy of position difference is $\Delta x(t_1) + (t_2 - t_1)\hbar/2m\Delta x(t_1) > (t_2 - t_1)\hbar/2^{1/2}m$

Stroboscopic QND measurements (Braginsky *et al*., 1978; Thorne *et al*., 1978)

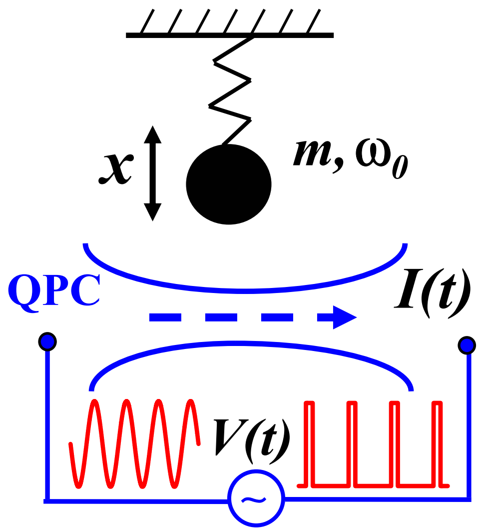


Idea: second measurement exactly one oscillation period later is insensitive to Δp
(or $\Delta t = nT/2$, $T=2\pi/\omega_0$)

Difference in our case:

- **continuous measurement**
- **weak coupling with detector**
- **quantum feedback to suppress "heating"**

Bayesian formalism for continuous measurement of a nanoresonator



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Current $I_x = 2\pi\left(M + \Delta M\, x\right)^2 \rho_l \rho_r e^2 V/\hbar = I_0 + k\,x$

Detector noise $S_x = S_0 \equiv 2eI_0$

Recipe: quantum Bayes procedure

Nanoresonator evolution (Stratonovich form), same Eqn as for qubits:

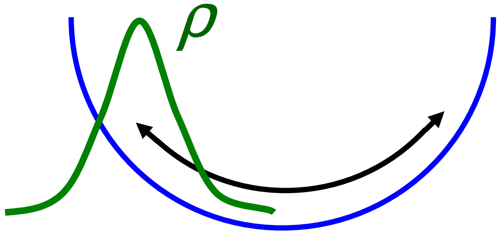
$$\frac{d\rho(x,x')}{dt} = \frac{-i}{\hbar}[\hat{H}_0,\rho] + \frac{\rho(x,x')}{S_0}\left\{ I(t)(I_x + I_{x'} - 2\langle I\rangle) - \frac{1}{2}\left(I_x^2 + I_{x'}^2 - 2\langle I^2\rangle\right)\right\}$$

$$\langle I\rangle = \sum I_x \rho(x,x), \quad I(t) = I_x + \xi(t), \quad S_\xi = S_0$$

Ito form (same as in many papers on conditional measurement of oscillators):

$$\frac{d\rho(x,x')}{dt} = \frac{-i}{\hbar}[\hat{H}_0,\rho] - \frac{k^2}{4S_0\eta}(x-x')^2\rho(x,x') + \frac{k}{S_0}(x+x'-2\langle x\rangle)\,\rho(x,x')\,\xi(t)$$

Evolution of Gaussian states



Assume Gaussian states (following Doherty-Jacobs and Hopkins-Jacobs-Habib-Schwab), then ρ(*x*,*x*') is described by only 5 magnitudes:
$\langle x \rangle$, $\langle p \rangle$ - average position and momentum (packet center),
D_x, D_p, D_{xp} – variances (packet width)
Assume large *Q*-factor (then no temperature)

Voltage modulation $f(t)V_0$: $k = f(t)\,k_0, \quad I_x = f(t)(I_{00} + k_0 x), \quad S_I = |f(t)|\, S_0$

Then coupling (measurement strength) is also modulated in time:

$$C = |f(t)|\, C_0, \quad C = \hbar k^2 / S_I m\omega_0^2 = 4/\omega_0 \tau_{meas}$$

Packet center evolves randomly and needs feedback (force *F*) to cool down

$$d\langle x \rangle / dt = \langle p \rangle / m + (2k_0/S_0)\,\mathrm{sgn}[f(t)]\, D_x\, \xi(t)$$

$$d\langle p \rangle / dt = -m\omega_0^2 \langle x \rangle + (2k_0/S_0)\,\mathrm{sgn}[f(t)]\, D_{xp}\, \xi(t) + F(t)$$

Packet width evolves deterministically and is QND squeezed by periodic *f*(*t*)

$$d\langle D_x \rangle / dt = (2/m) D_{xp} - (2k_0^2/S_0)\, |f(t)|\, D_x^2$$

$$d\langle D_p \rangle / dt = -2m\omega_0^2 D_{xp} + (k_0^2 \hbar^2 / 2S_0 \eta)\, |f(t)| - (2k_0^2/S_0) |f(t)|\, D_{xp}^2$$

$$d\langle D_{xp} \rangle / dt = (1/m) D_p - m\omega_0^2 D_x - (2k_0^2/S_0) |f(t)|\, D_x D_{xp}$$

Squeezing by sine-modulation, $V(t)=V_0\sin(\omega t)$

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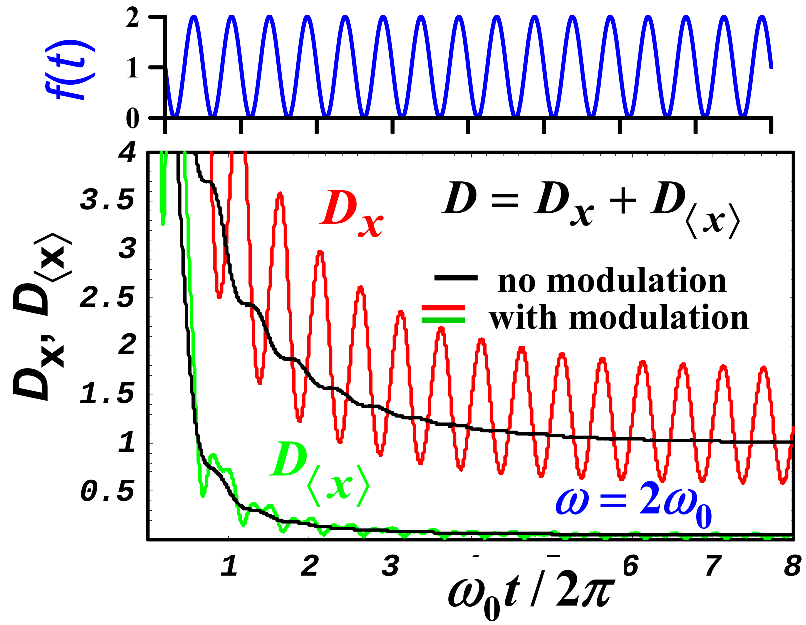
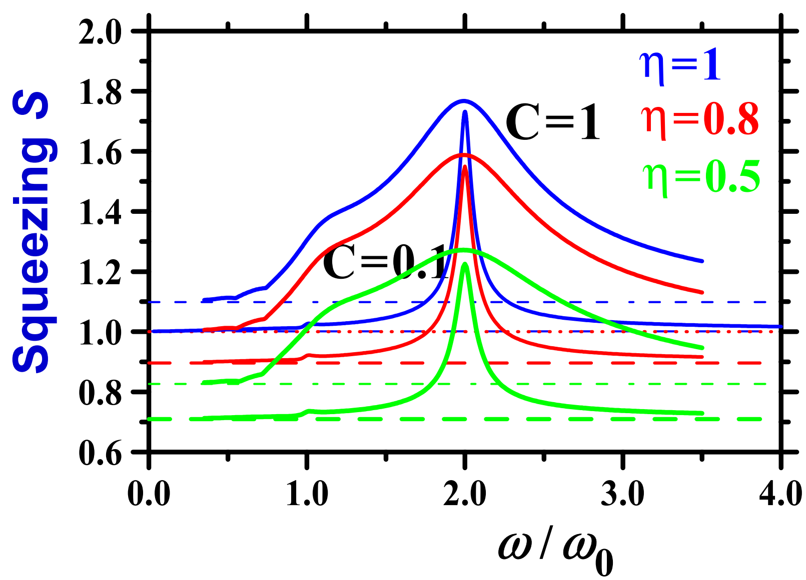




Squeezing obviously oscillates in time,
maximum squeezing at maximum voltage,
momentum squeezing shifted in phase by π/2.

$$S \equiv \max_t (\Delta x_0)^2 / D_x$$

Analytics (weak coupling):

$$S(2\omega_0) = \sqrt{3\eta}, \quad \Delta\omega = 0.36\omega_0 C_0 / \sqrt{\eta}$$

η - detector efficiency, C_0 – coupling

$\Delta x_0 = (\hbar/2m\omega_0)^{1/2}$ – ground state width

$D_x = (\Delta x)^2$, $D_{\langle x\rangle} = \langle\langle x\rangle^2\rangle - \langle\langle x\rangle\rangle^2$

Quantum feedback:

$$F = -m\omega_0\gamma_x\langle x\rangle - \gamma_p\langle p\rangle$$

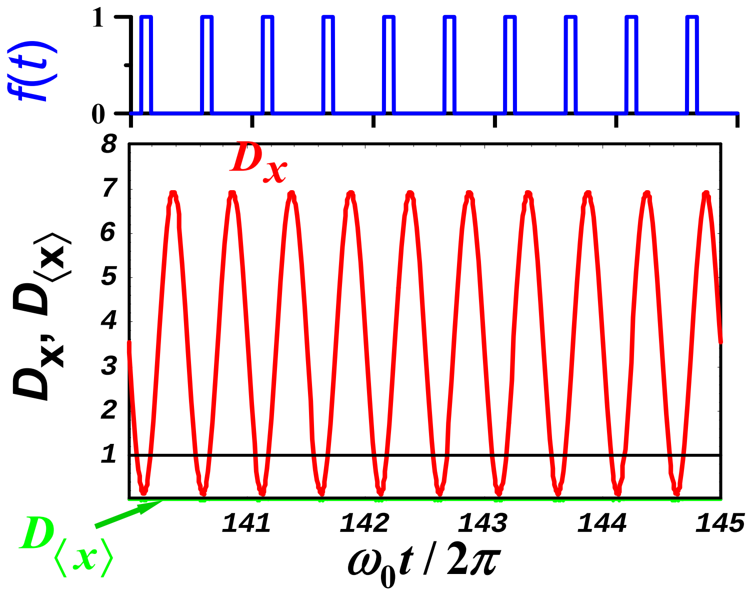
(same as in Hopkins *et al*.; without modulation it cools the state down to the ground state)

Feedback is sufficiently efficient, $D_{\langle x\rangle}$ Ü D_x

Squeezing up to 1.73 at $\omega=2\omega_0$

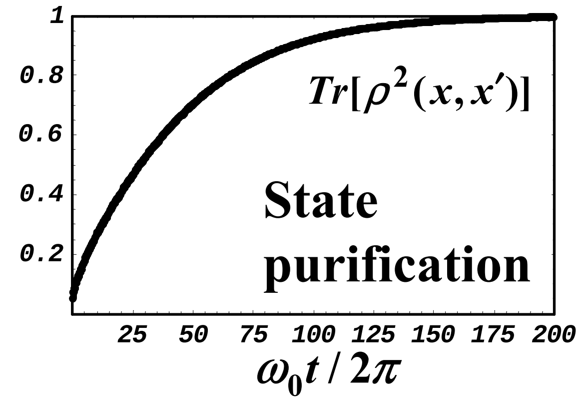
Squeezing by stroboscopic (pulse) modulation

pulse modulation $\omega = 2\omega_0 / n$

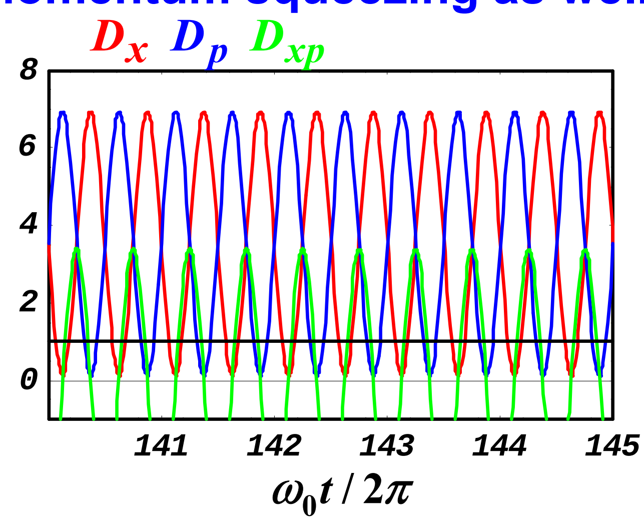


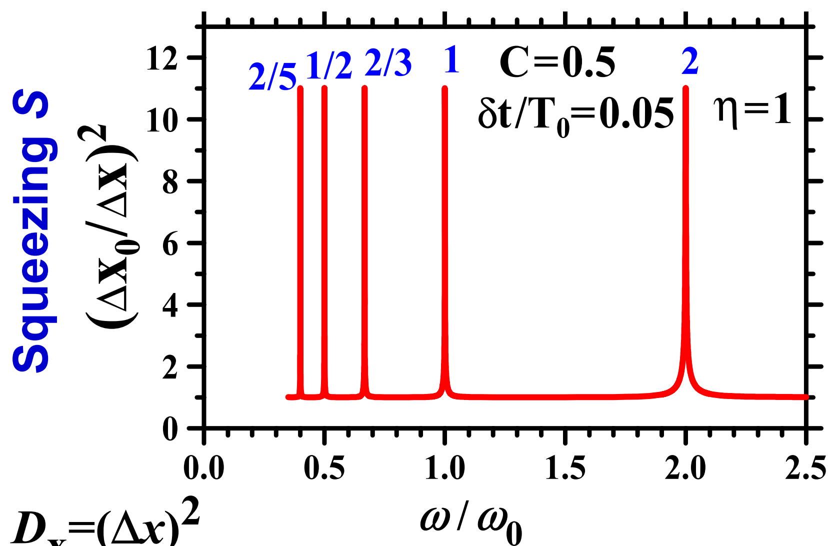
$D_{\langle x \rangle} << D_x$

using feedback



Momentum squeezing as well





$D_x = (\Delta x)^2$

S á 1

Efficient squeezing at $\omega = 2\omega_0 / n$

(natural QND condition)

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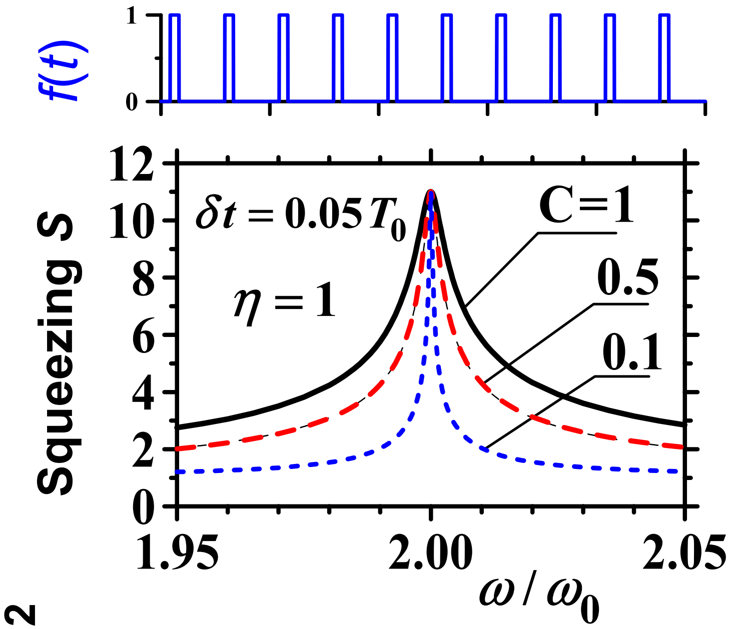
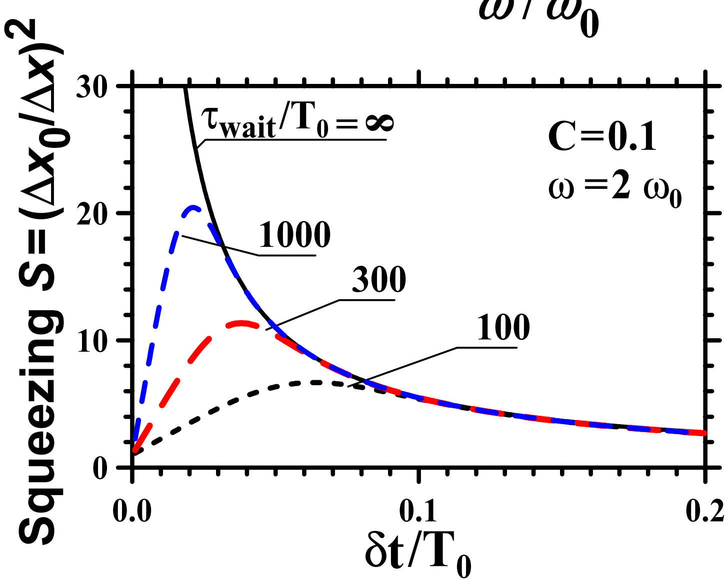
Squeezing by stroboscopic modulation





Analytics (weak coupling, short pulses)

Maximum squeezing

$$S(2\omega_0/n) = \frac{2\sqrt{3\eta}}{\omega_0 \delta t}$$

Linewidth

$$\Delta\omega = \frac{4C_0(\delta t)^3 \omega_0^4}{\pi n^2 \sqrt{3\eta}}$$

C_0 – dimensionless coupling with detector
δt – pulse duration, $T_0 = 2\pi/\omega_0$
η – quantum efficiency of detector
(long formula for the line shape)

Finite *Q*-factor limits the time we can afford to wait before squeezing develops, $\tau_{wait}/T_0 \sim Q/\pi$

Squeezing saturates as ~exp(-n/n_0) after $n_0 = \sqrt{3\eta}\,/\,C_0(\omega_0\delta t)^2$ measurements

Therefore, squeezing cannot exceed

$$S \simeq \sqrt{C_0 Q}\,\sqrt[4]{\eta}$$

Observability of nanoresonator squeezing

Procedure: 1) prepare squeezed state by stroboscopic measurement,
2) switch off quantum feedback
3) measure in the stroboscopic way $X_N = \frac{1}{N}\sum_{j=1}^{N} x_j$

For instantaneous measurements ($\delta t \to 0$) the variance of X_N is

$$D_{X,N} = \frac{\hbar}{2m\omega_0}\left(\frac{1}{S} + \frac{1}{NC_0\omega_0\delta t}\right) \to \frac{1}{S}(\Delta x_0)^2 \quad \text{at } N \to \infty$$

S – squeezing,
Δx_0 – ground state width

Then distinguishable from ground state (S=1)
in one run for S à 1 (error probability ~$S^{-1/2}$)

Not as easy for continuous measurements because of extra “heating”.
$D_{X,N}$ has a minimum at some N and then increases.
However, numerically it seems $\min_N D_{X,N} \sim 2(\Delta x_0)^2 / S$ (only twice worse)

Example: $\min_N D_{X,N} / (\Delta x_0) = 0.078$ for C_0=0.1, η=1, $\delta t/T_0$=0.02, $1/S$=0.036

Squeezed state is distinguishable in one run (with small error probability), therefore suitable for ultrasensitive force measurement beyond standard quantum limit

Conclusions

- **Periodic modulation of the detector voltage modulates measurement strength and periodically squeezes the width of the nanoresonator state ("breathing mode")**
- **Packet center oscillates and is randomly "heated" by measurement; quantum feedback can cool it down (keep it near zero in both position and momentum)**
- **Sine-modulation leads to a small squeezing (<1.73), stroboscopic (pulse) modulation can lead to a strong squeezing (>>1) even for a weak coupling with detector**
- **Still to be done: correct account of *Q*-factor and temperature**
- **Potential application: ultrasensitive force measurement beyond standard quantum limit**