Quantum measurement and control of solid-state qubits and nanoresonators

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Outline:

Introduction (Bayesian approach)

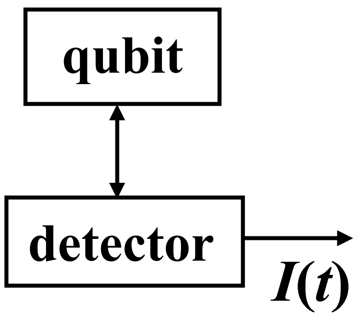
Simple quantum feedback of a solid-state qubit
(*Korotkov*, cond-mat/0404696)

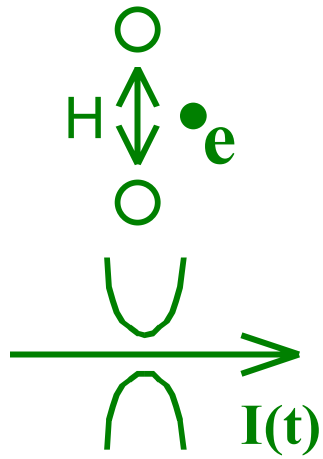
Quadratic quantum measurements
(*Mao, Averin, Ruskov, Korotkov*, PRL 93, 056803, 2004)

QND squeezing of a nanoresonator
(*Ruskov, Schwab, Korotkov*, cond-mat/0406416)

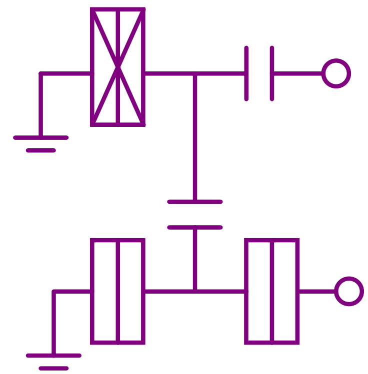
Support: ARDA

Examples of solid-state qubits and detectors

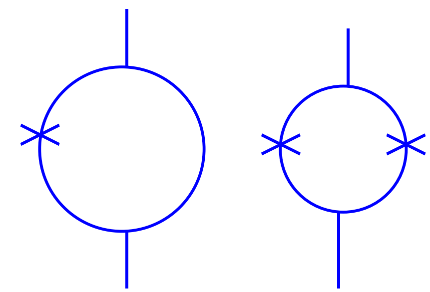




Double-quantum-qot and quantum point contact (QPC)



Cooper-pair box and single-electron transistor (SET)



Two SQUIDs

$H = H_{QB} + H_{DET} + H_{INT}$

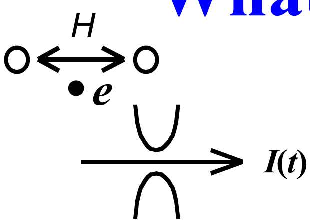
$H_{QB} = (\varepsilon/2)(c_1^+c_1 - c_2^+c_2) + H(c_1^+c_2 + c_2^+c_1)$ ε - asymmetry, H – tunneling

$\Omega = (4H^2+\varepsilon^2)^{1/2}$ – frequency of quantum coherent (Rabi) oscillations

Two levels of average detector current: I_1 for qubit state $|1\rangle$, I_2 for $|2\rangle$

Response: $\Delta I = I_1 - I_2$ Detector noise: white, spectral density S_I

What happens to a qubit state during measurement?



For simplicity (for a moment) $H=\varepsilon=0$, infinite barrier (frozen qubit), evolution due to measurement only

“Orthodox” answer

$$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \begin{matrix} \nearrow \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \\ \searrow \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \end{matrix}$$

|1> or |2>, depending on the result

“Conventional” (decoherence) answer (Leggett, Zurek)

$$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \to \begin{pmatrix} \frac{1}{2} & \frac{\exp(-\Gamma t)}{2} \\ \frac{\exp(-\Gamma t)}{2} & \frac{1}{2} \end{pmatrix} \to \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

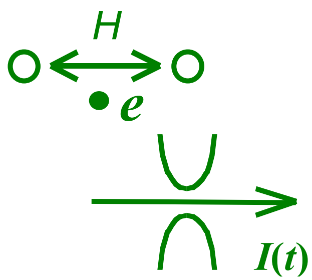
no measurement result! ensemble averaged

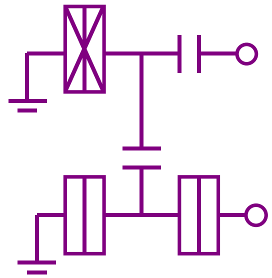
Orthodox and decoherence answers contradict each other!

applicable for:	Single quantum systems	Continuous measurements
Orthodox	yes	no
Conventional (ensemble)	no	yes
Bayesian	yes	yes

Bayesian formalism describes gradual collapse of single quantum systems
Noisy detector output $I(t)$ should be taken into account

Bayesian formalism for a single qubit





$$\hat{H}_{QB} = \frac{\varepsilon}{2}(c_1^\dagger c_1 - c_2^\dagger c_2) + H(c_1^\dagger c_2 + c_2^\dagger c_1)$$

$|1\rangle \to I_1,\ |2\rangle \to I_2$

$\Delta I = I_1 - I_2$, $I_0 = (I_1+I_2)/2$, S_I – detector noise

$$\begin{cases} d\rho_{11}/dt = -d\rho_{22}/dt = -2H\,\mathrm{Im}\,\rho_{12} + \rho_{11}\rho_{22}(2\Delta I/S_I)[I(t)-I_0] \\ d\rho_{12}/dt = i\varepsilon\rho_{12} + iH(\rho_{11}-\rho_{22}) + \rho_{12}(\rho_{11}-\rho_{22})(\Delta I/S_I)[I(t)-I_0] - \gamma\rho_{12} \end{cases}$$

A.K., 1998

$\gamma = \Gamma - (\Delta I)^2/4S_I$, Γ – ensemble decoherence

$\eta = 1 - \gamma/\Gamma = (\Delta I)^2/4S_I\Gamma$ – detector ideality (efficiency), $\eta \le 100\%$

For simulations: $I(t) - I_0 \to (\rho_{22} - \rho_{11})\Delta I/2 + \xi(t)$, $S_\xi = S_I$

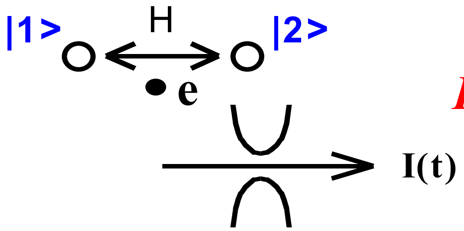
Averaging over $\xi(t) \Rightarrow$ master equation

Ideal detector (η=1) does not decohere a single qubit (pure state remains pure), then random evolution of the qubit *wavefunction* can be monitored

Similar formalisms developed earlier. Key words: Imprecise, weak, selective, or conditional measurements, POVM, Quantum trajectories, Quantum jumps, Restricted path integral, etc.

Names: E.B. Davies, K. Kraus, A.S. Holevo, C.W. Gardiner, H.J. Carmichael, C.M. Caves, M.B. Plenio, P.L. Knight, M.B. Mensky, D.F. Walls, N. Gisin, I.C. Percival, G.J. Milburn, H.M. Wiseman, R. Onofrio, S. Habib, A. Doherty, etc. (very incomplete list)

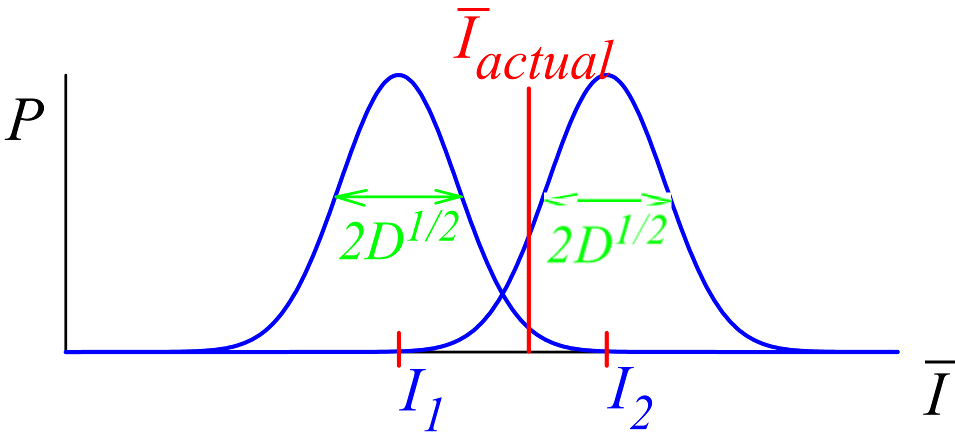
"Quantum Bayes theorem" (ideal detector assumed)



$H = \varepsilon = 0$ **(frozen qubit)**

Initial state: $\begin{pmatrix} \rho_{11}(0) & \rho_{12}(0) \\ \rho_{21}(0) & \rho_{22}(0) \end{pmatrix}$

Measurement (during time τ):



$$\bar{I} \equiv \frac{1}{\tau}\int_0^{\tau} I(t)\, dt$$

$$P(\bar{I},\tau) = \rho_{11}(0)\, P_1(\bar{I},\tau) + \rho_{22}(0)\, P_2(\bar{I},\tau)$$

$$P_i(\bar{I},\tau) = \frac{1}{\sqrt{2\pi D}} \exp[-(\bar{I}-I_i)^2/2D],$$

$$D = S_I/2\tau, \quad |I_1 - I_2| \ll I_i, \quad \tau \gg S_I/I_i^2$$

After the measurement during time τ, the probabilities can be updated using the standard Bayes formula:

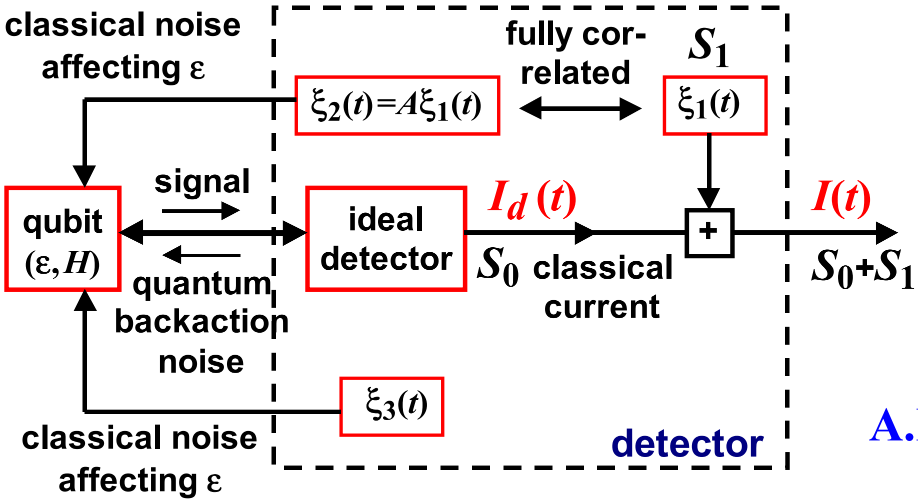
$$P(B_i \mid A) = \frac{P(B_i)P(A \mid B_i)}{\sum_k P(B_k)P(A \mid B_k)}$$

Quantum Bayes formulas:

$$\rho_{11}(\tau) = \frac{\rho_{11}(0)\exp[-(\bar{I}-I_1)^2/2D]}{\rho_{11}(0)\exp[-(\bar{I}-I_1)^2/2D] + \rho_{22}(0)\exp[-(\bar{I}-I_2)^2/2D]}$$

$$\frac{\rho_{12}(\tau)}{[\rho_{12}(\tau)\,\rho_{22}(\tau)]^{1/2}} = \frac{\rho_{12}(0)}{[\rho_{12}(0)\,\rho_{22}(0)]^{1/2}}, \qquad \rho_{22}(\tau) = 1 - \rho_{11}(\tau)$$

Nonideal detectors with input-output noise correlation



$$K = \frac{AS_1 + \theta S_0}{\hbar S_I}, \quad S_I = S_0 + S_1$$

K – correlation between output and backaction noises

A.K., 2002

$$\frac{d}{dt}\rho_{11} = -\frac{d}{dt}\rho_{22} = -2H\,\mathrm{Im}\,\rho_{12} + \rho_{11}\rho_{22}\frac{2\Delta I}{S_I}[I(t) - I_0]$$

$$\frac{d}{dt}\rho_{12} = i\tilde{\varepsilon}\rho_{12} + iH(\rho_{11} - \rho_{22}) + \rho_{12}(\rho_{11} - \rho_{22})\frac{\Delta I}{S_I}[I(t) - I_0] + iK[I(t) - I_0] - \tilde{\gamma}\rho_{12}$$

Fundamental limits for ensemble decoherence

$$\Gamma = \gamma + (\Delta I)^2/4S_I, \quad \gamma \geq 0 \;\Rightarrow\; \Gamma \geq (\Delta I)^2/4S_I$$

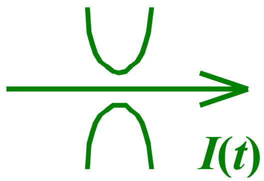
$$\Gamma = \gamma + (\Delta I)^2/4S_I + K^2 S_I/4, \quad \gamma \geq 0 \;\Rightarrow\; \Gamma \geq (\Delta I)^2/4S_I + K^2 S_I/4$$

Translated into energy sensitivity: $(\epsilon_I\, \epsilon_{BA})^{1/2} \geq \hbar/2$ **or** $(\epsilon_I\, \epsilon_{BA} - \epsilon_{I,BA})^{1/2} \geq \hbar/2$

Ideality of realistic solid-state detectors

(ideal detector does not cause single qubit decoherence)

1. Quantum point contact



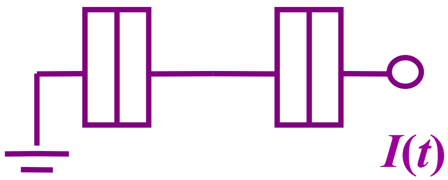
Theoretically, **ideal quantum detector, η=1**

A.K., 1998 (Gurvitz, 1997; Aleiner et al., 1997)

Experimentally, η > 80%

(using Buks et al., 1998)

2. SET-transistor



Very non-ideal in usual operation regime, η ‹‹ 1

Shnirman-Schőn, 1998; A.K., 2000, Devoret-Schoelkopf, 2000

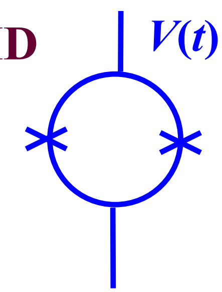
However, reaches ideality, η = 1 if:

- in deep cotunneling regime **(Averin, 2000, van den Brink, 2000)**
- S-SET, using supercurrent **(Zorin, 1996)**
- S-SET, double-JQP peak **(Clerk et al., 2002)**

??? S-SET, usual JQP **(Johansson et al.)**, onset of QP branch **(?)**

- resonant-tunneling SET, low bias **(Averin, 2000)**

3. SQUID



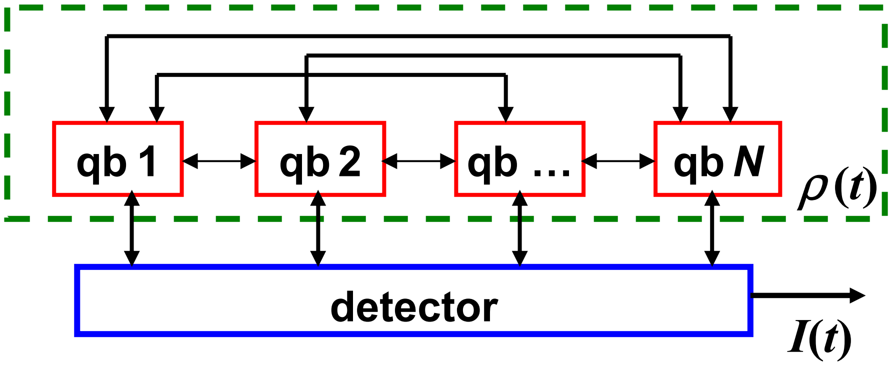
Can reach ideality, η = 1

(Danilov-Likharev-Zorin, 1983; Averin, 2000)

4. FET ?? HEMT ??

ballistic FET/HEMT ??

Bayesian formalism for *N* entangled qubits



Up to 2^N levels of current

$$\frac{d}{dt}\rho_{ij} = \rho_{ij}\frac{1}{S_I}[I(t)\,(I_i + I_j - 2\sum_k \rho_{kk} I_k) - \frac{1}{2}(I_i^2 + I_j^2 - 2\sum_k \rho_{kk} I_k^2)]$$
$$+\frac{-i}{\hbar}[\hat{H}_{qb},\rho]_{ij} + i\frac{\Delta\varepsilon_{ij}}{\hbar}\rho_{ij} + iK_{ij}[I(t) - \frac{I_i + I_j}{2}]\rho_{ij} - \tilde{\gamma}_{ij}\rho_{ij}$$

(Stratonovich form)

$$I(t) = \sum_i \rho_{ii}(t) I_i + \xi(t)$$

Averaging over ξ(t) î master equation

A.K., PRA 65, 052304 (2002); PRB 67, 235408 (2003)

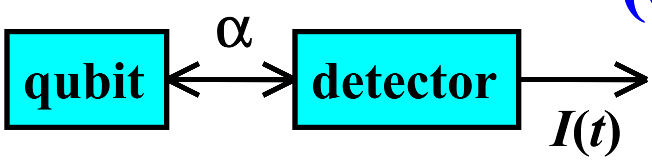
Stratonovich: $$\frac{df(t)}{dt} \equiv \lim_{\Delta t\to 0}\frac{f(t+\Delta t/2) - f(t-\Delta t/2)}{\Delta t}$$ **(easy derivatives and physical meaning)**

Ito: $$\frac{df(t)}{dt} \equiv \lim_{\Delta t\to 0}\frac{f(t+\Delta t) - f(t)}{\Delta t}$$ **(easy averaging over noise)**

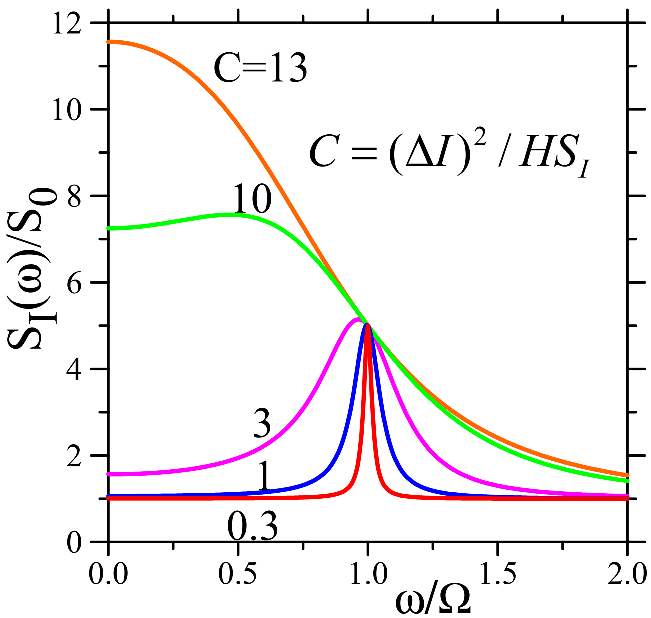
Experimental predictions and proposals based on the Bayesian formalism

- Direct experiments on Bayesian evolution (1998)
- Measured spectral density of Rabi oscillations (1999, 2000, 2002)
- Bell-type correlation experiment (2000)
- Quantum feedback control of a qubit (2001)
- Entanglement by measurement (2002)
- Measurement and entanglement by a quadratic detector (2004)
- Simple quantum feedback via quadratures (2004)
- QND squeezing of a nanoresonator (2004)

Measured spectrum of qubit coherent oscillations (or spin precession)



What is the spectral density $S_I(\omega)$ of detector current?



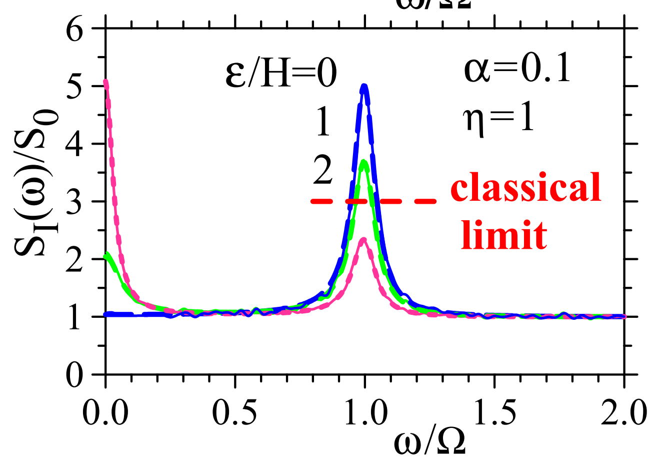
Assume classical output, eV » ħΩ

$$\varepsilon = 0, \quad \Gamma = \eta^{-1}(\Delta I)^2/4S_0$$

$$S_I(\omega) = S_0 + \frac{\Omega^2(\Delta I)^2\Gamma}{(\omega^2-\Omega^2)^2+\Gamma^2\omega^2}$$

Spectral peak can be seen, but peak-to-pedestal ratio ≤ 4η ≤ 4

(result can be obtained using various methods, not only Bayesian method)



Weak coupling, $\alpha = C/8 \ll 1$

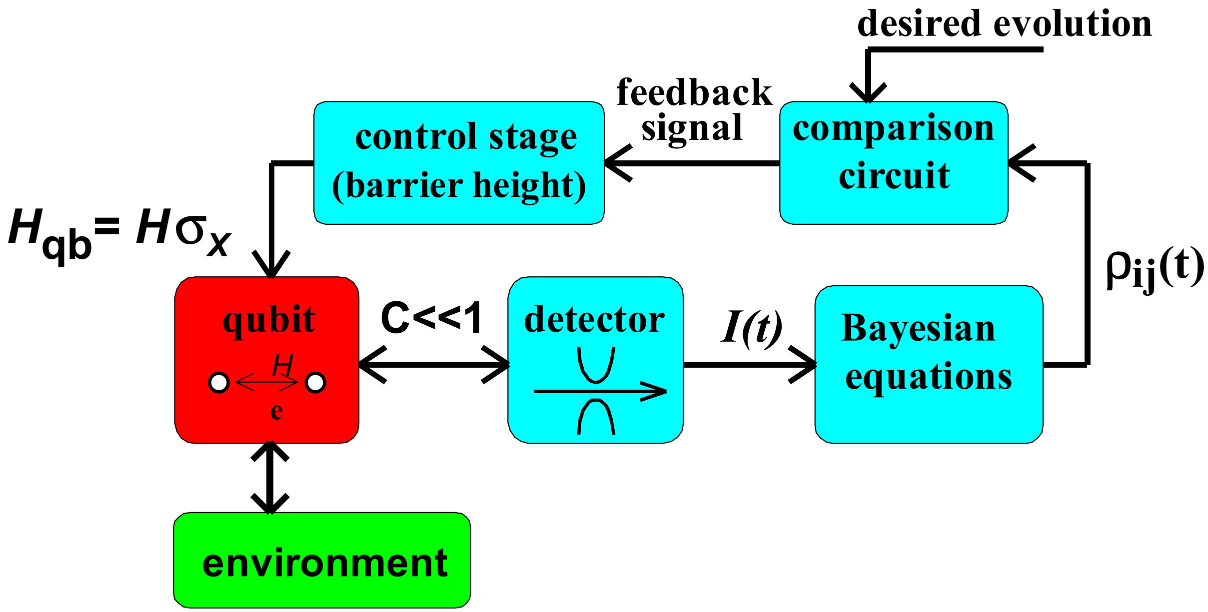
$$S_I(\omega) = S_0 + \frac{\eta S_0\varepsilon^2/H^2}{1+(\omega\hbar^2\Omega^2/4H^2\Gamma)^2} + \frac{4\eta S_0(1+\varepsilon^2/2H^2)^{-1}}{1+[(\omega-\Omega)\Gamma(1-2H^2/\hbar^2\Omega^2)]^2}$$

- A.K., LT'99
- Averin-A.K., 2000
- A.K., 2000
- Averin, 2000
- Goan-Milburn, 2001
- Makhlin et al., 2001
- Balatsky-Martin, 2001
- Ruskov-A.K., 2002
- Mozyrsky et al., 2002
- Balatsky et al., 2002
- Bulaevskii et al., 2002
- Shnirman et al., 2002
- Bulaevskii-Ortiz, 2003
- Shnirman et al., 2003

Contrary:
Stace-Barrett, 2003 (PRL 2004)

Quantum feedback control of a solid-state qubit

Ruskov & A.K., 2001



Goal: maintain desired phase of coherent (Rabi) oscillations in spite of environmental dephasing (keep qubit "fresh")

Idea: monitor the Rabi phase ϕ by continuous measurement and apply feedback control of the qubit barrier height, $\Delta H_{FB}/H = -F\times\Delta\phi$

To monitor phase ϕ we plug detector output $I(t)$ into Bayesian equations

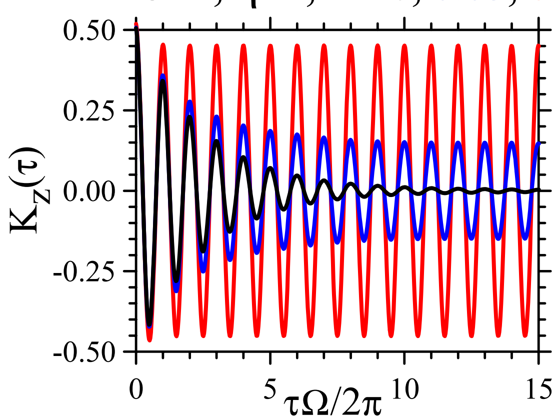
Quantum feedback in quantum optics is discussed since 1993 (Wiseman-Milburn), recently first successful experiments in Mabuchi's group (2002, 2004).

Performance of quantum feedback

(no extra environment)

Qubit correlation function

C=1, η=1, F=0, 0.05, 0.5



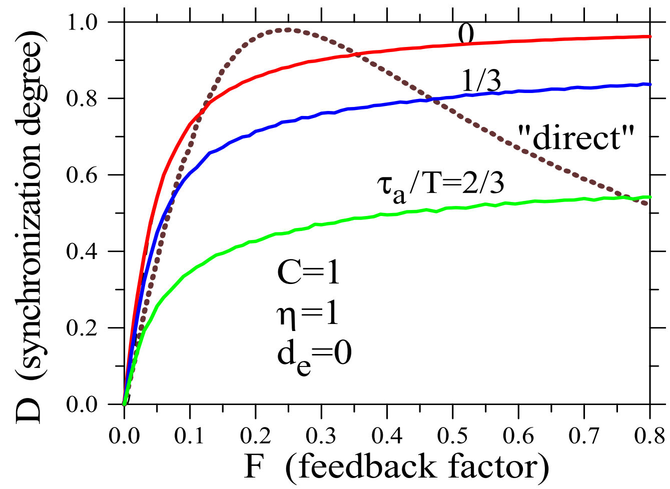
$$K_z(\tau)=\frac{\cos\Omega t}{2}\ \exp\left[\frac{C}{16F}(e^{-2FH\tau/\hbar}-1)\right]$$

(for weak coupling and good fidelity)

Detector current correlation function

$$K_I(\tau)=\frac{(\Delta I)^2}{4}\frac{\cos\Omega t}{2}\ (1+e^{-2FH\tau/\hbar}) \times\exp\left[\frac{C}{16F}(e^{-2FH\tau/\hbar}-1)\right]+\frac{S_I}{2}\delta(\tau)$$

Fidelity (synchronization degree)



$C=\hbar(\Delta I)^2/S_I H$ – **coupling**

τ_a^{-1} – **available bandwidth**

F – feedback strength

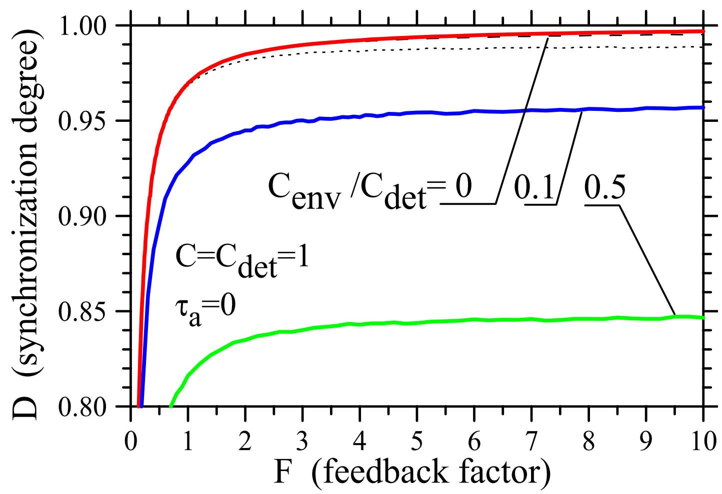
$$D=2\langle \mathrm{Tr}\rho\rho_{desir}\rangle-1$$

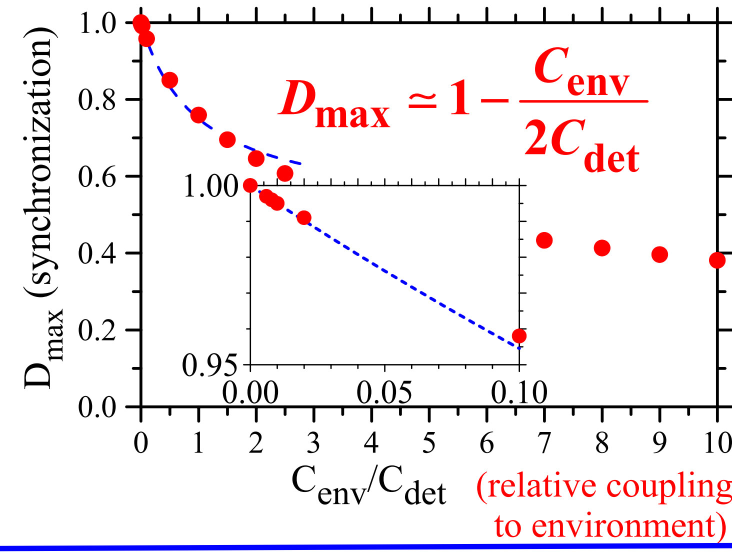
For ideal detector and wide bandwidth, fidelity can be arbitrary close to 100%

$$D=\exp(-C/32F)$$

Ruskov & Korotkov, PRB 66, 041401(R) (2002)

Suppression of environment-induced decoherence by quantum feedback



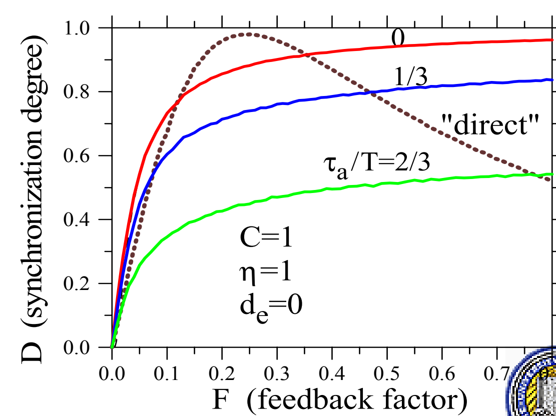


Big experimental problem: necessity of very fast (>>Ω, GHz-range) real-time solution of the Bayesian equations; therefore wide bandwidth

Some help: "direct" ("naïve") feedback

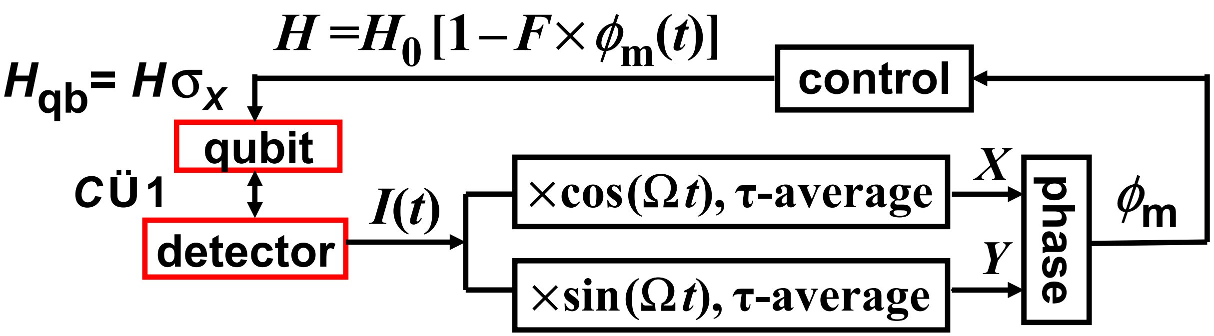
$$H_{fb}/H - 1 = F \times \{2[I(t) - I_0]/\Delta I - \cos(\Omega t)\}\sin(\Omega t)$$

However, still wide bandwidth (>> Ω) required



Simple quantum feedback of a solid-state qubit

(A.K., cond-mat/0404696)



We want to maintain coherent (Rabi) oscillations for arbitrary long time,
$\rho_{11}-\rho_{22}=\cos(\Omega t)$, $\rho_{12}=i\sin(\Omega t)/2$

Idea: **use two quadrature components of the detector current $I(t)$ to monitor approximately the phase of qubit oscillations**
(a very natural way for usual classical feedback!)

$$X(t) = \int_{-\infty}^{t} [I(t') - I_0] \cos(\Omega t') \exp[-(t-t')/\tau]\, dt$$
$$Y(t) = \int_{-\infty}^{t} [I(t') - I_0] \sin(\Omega t') \exp[-(t-t')/\tau]\, dt$$

$$\phi_m = -\arctan(Y/X)$$

(similar formulas for a tank circuit instead of mixing with local oscillator)

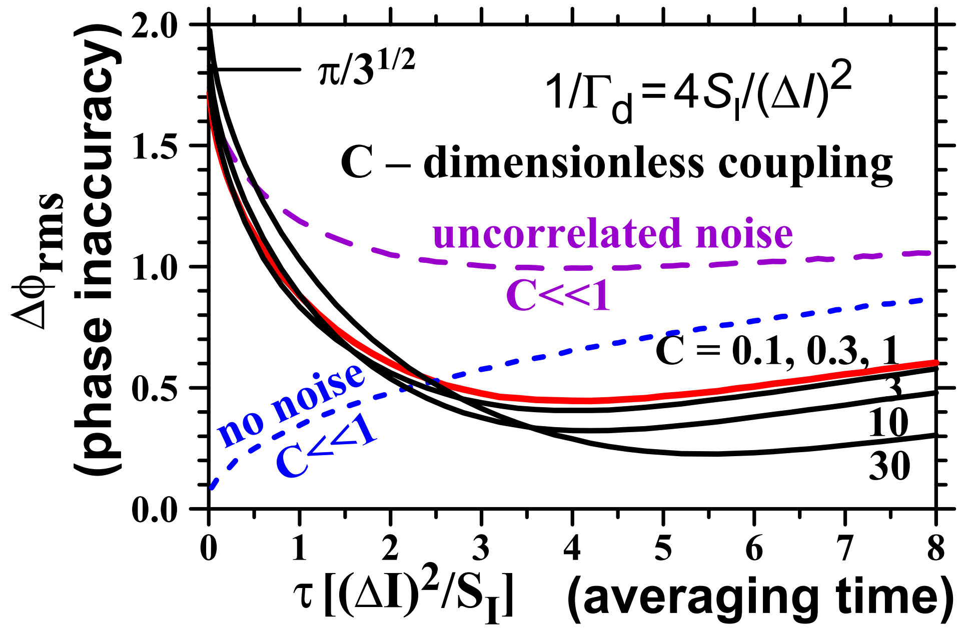
Advantage: **simplicity and relatively narrow bandwidth** $(1/\tau \sim \Gamma_d \ll \Omega)$

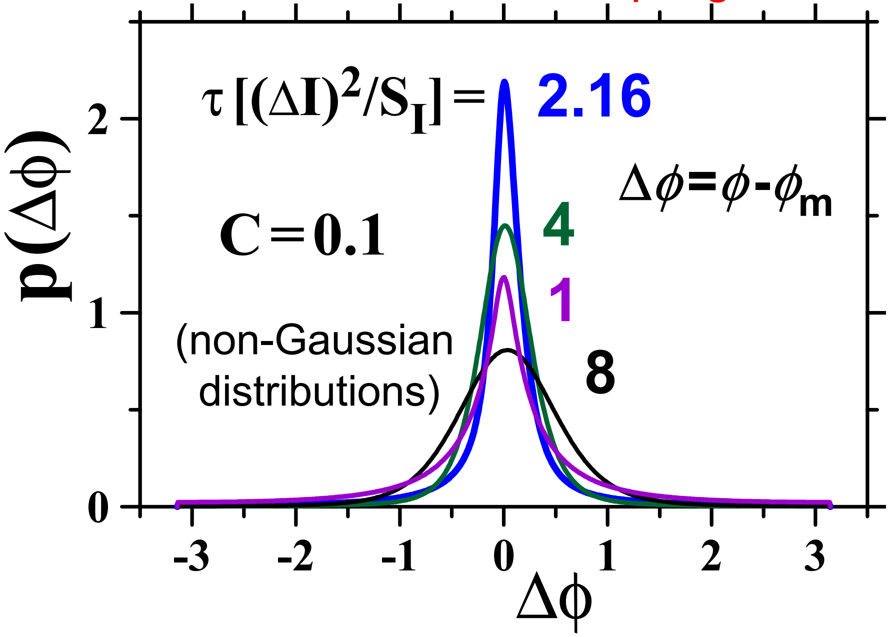
Anticipated problem: without feedback the spectral peak-to-pedestal ratio <4, therefore not much information in quadratures

(surprisingly, situation is much better than anticipated!)

Accuracy of phase monitoring via quadratures
(no feedback yet)

weak coupling C á 1





Best approximation
$\langle X^2+Y^2\rangle=(S_I/\Delta I)^2$
$(2/5)(41^{1/2}-1)\approx 2.16$

Noise improves the monitoring accuracy!
(purely quantum effect, “reality follows observations”)

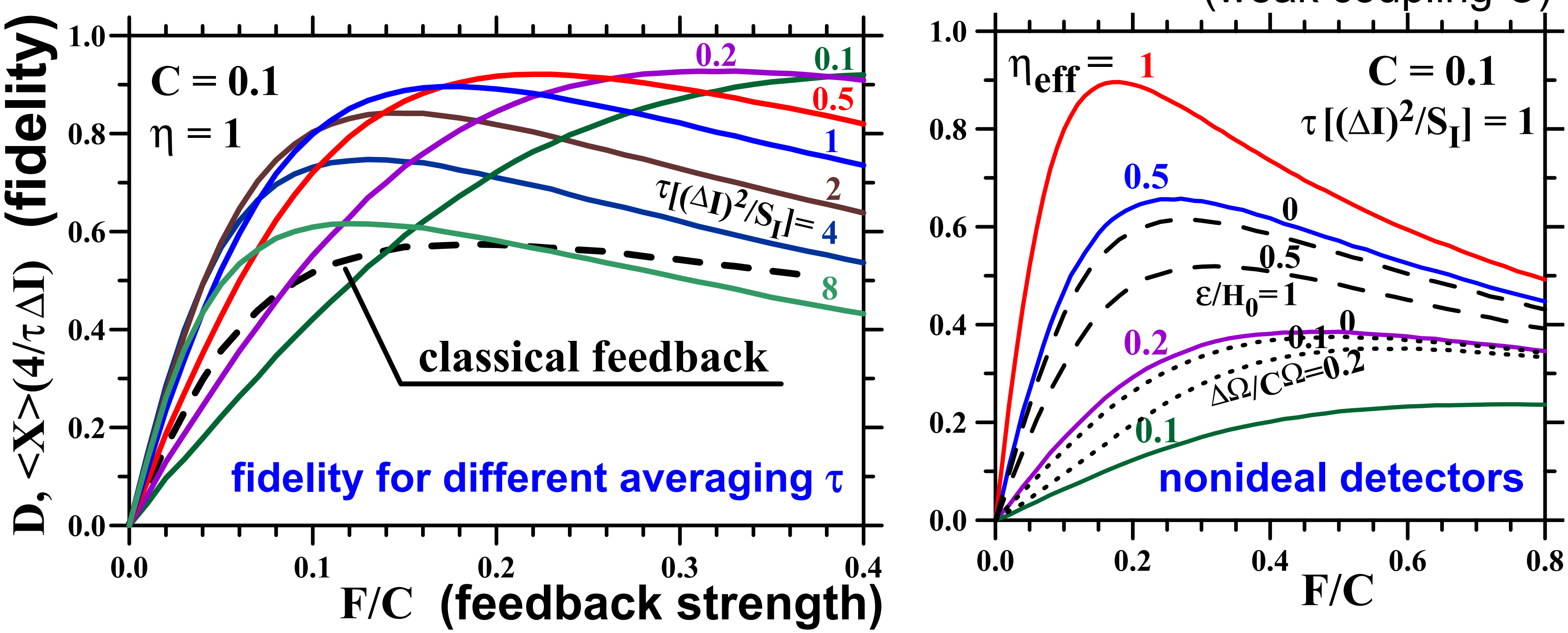
$$d\phi/dt = -[I(t)-I_0]\sin(\Omega t+\phi)(\Delta I/S_I)$$ (actual phase shift, ideal detector)

$$d\phi_m/dt = -[I(t)-I_0]\sin(\Omega t+\phi_m)/(X^2+Y^2)^{1/2}$$ (observed phase shift)

Noise enters the actual and observed phase evolution in a similar way

Simple quantum feedback

(weak coupling *C*)



- Fidelity *F* up to ~95% achievable (*D*~90%)
- Natural, practically classical feedback setup
- Averaging τ~1/Γ>>1/Ω (narrow bandwidth!)
- Detector efficiency (ideality) η≤0.1 still OK
- Robust to asymmetry ε and frequency shift ΔΩ
- Very simple verification – just positive in-phase quadrature $\langle X \rangle$

$D \equiv 2F - 1$

$F \equiv \langle \mathrm{Tr}\, \rho(t)\, \rho_{des}(t) \rangle$

$D \simeq \langle X \rangle (4/\tau \Delta I)$

X – in-phase quadrature of the detector current

Simple experiment?!

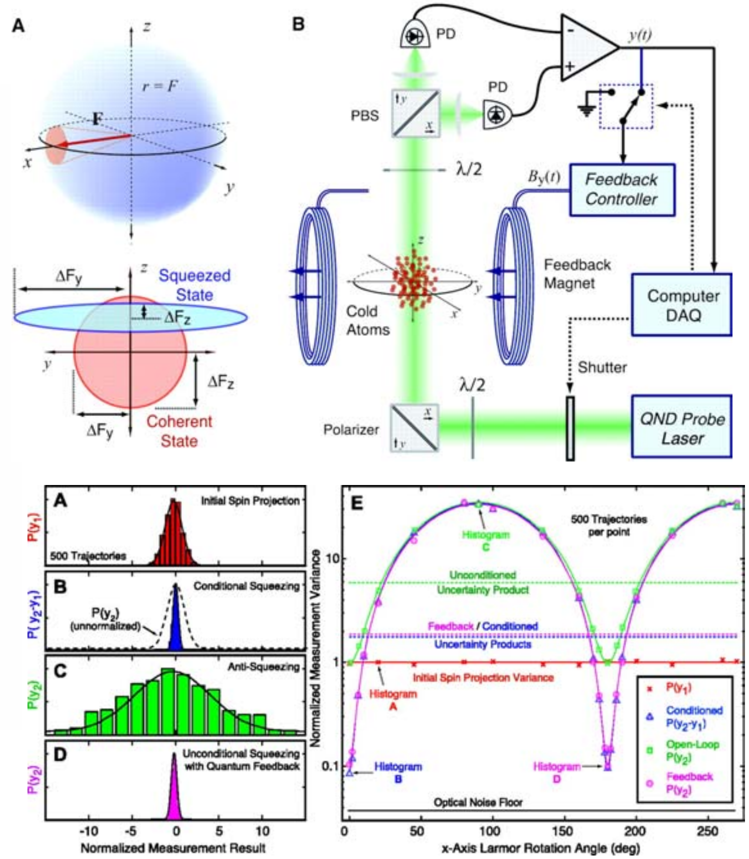
Quantum feedback in optics

Recent experiment: Science 304, 270 (2004)

Real-Time Quantum Feedback Control of Atomic Spin-Squeezing

JM Geremia,* John K. Stockton, Hideo Mabuchi

Real-time feedback performed during a quantum nondemolition measurement of atomic spin-angular momentum allowed us to influence the quantum statistics of the measurement outcome. We showed that it is possible to harness measurement backaction as a form of actuation in quantum control, and thus we describe a valuable tool for quantum information science. Our feedback-mediated procedure generates spin-squeezing, for which the reduction in quantum uncertainty and resulting atomic entanglement are not conditioned on the measurement outcome.



First detailed theory:

H.M. Wiseman and G. J. Milburn,
Phys. Rev. Lett. 70, 548 (**1993**)

No experimental attempts of quantum feedback in solid-state yet
(even theory is still considered controversial)

Experiments soon?

Summary on simple quantum feedback of a solid-state qubit

- Very straightforward, practically classical feedback idea (monitoring the phase of oscillations via quadratures) works well for the qubit coherent oscillations

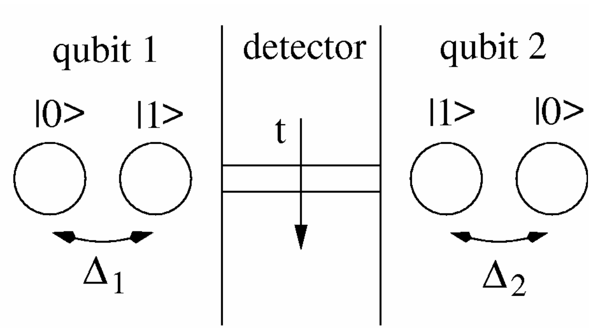
- Price for simplicity is a less-then-ideal operation (fidelity is limited by ~95%)

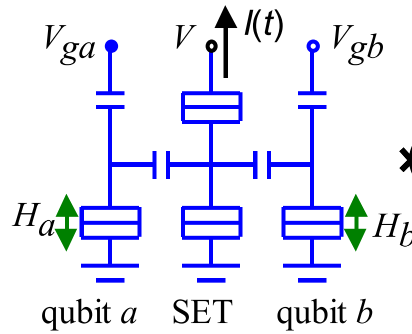
- Feedback operation is much better than expected

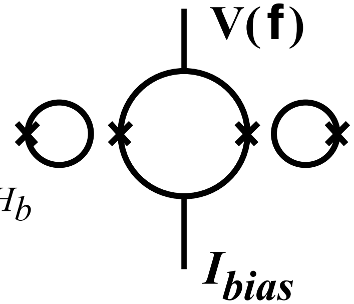
- Relatively simple experiment (simple setup, narrow bandwidth, inefficient detectors OK, simple verification)

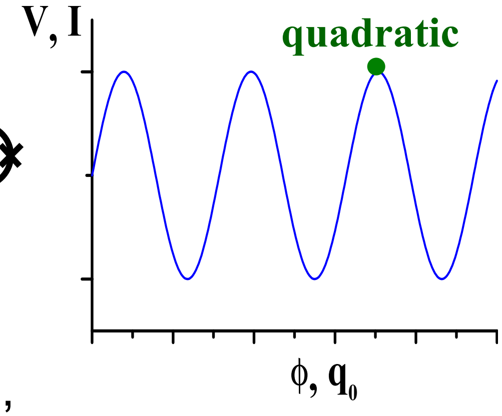
Quadratic Quantum Measurements

Mao, Averin, Ruskov, Korotkov; Phys. Rev. Lett. 93, 056803 (2004)



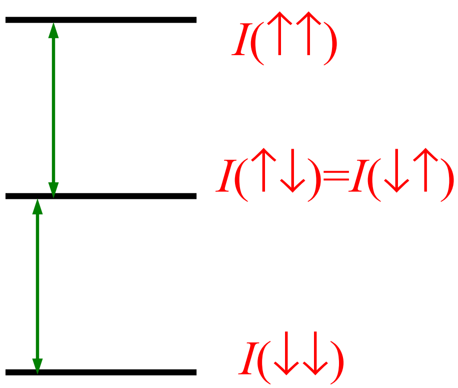




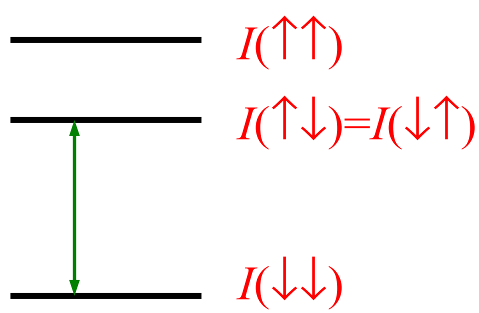


Setup similar to Ruskov-Korotkov, PRB 67, 241305(R) (2003), but a nonlinear (instead of a linear) detector is considered

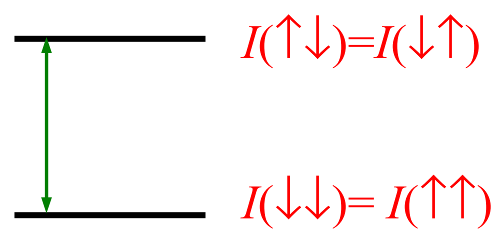
Linear detector



Noninear detector



Quadratic detector



Quadratic detection is useful for quantum error correction (Averin-Fazio, 2002)

Bayesian formalism for a nonlinear detector

$$H = H_{QBs} + H_{DET} + \sum_{j=1,2} [t(\{\sigma_z^j\})\xi + t^\dagger(\{\sigma_z^j\})\xi^\dagger]$$

$$t(x) = t_0 + \delta_1\sigma_z^1 + \delta_2\sigma_z^2 + \lambda\sigma_z^1\sigma_z^2$$

$\delta_j = 0 \Rightarrow$ **quadratic detector**

Assumed: 1) weak tunneling in the detector, 2) large detector voltage (fast detector dynamics, and 3) weak response.

The model describes an ideal detector (no extra noises).

Recipe: Coupled detector-qubits evolution and frequent collapses of the number *n* of electrons passed through the detector

Two-qubit evolution (Ito form):

$$\frac{d}{dt}\rho_{kl} = -i[H_{QBs}, \rho]_{kl} + [I(t) - \langle I\rangle][\frac{1}{S_0}(I_k + I_l - 2\langle I\rangle) - i\varphi_{kl}]\rho_{kl} - \gamma_{kl}\rho_{kl}$$

$$\gamma_{kl} = (1/2)(\Gamma_+ + \Gamma_-)[(|t_k| - |t_l|)^2 + \varphi_{kl}^2 |t_0|^2], \quad \varphi_{kl} = \arg(t_k t_l^*)$$

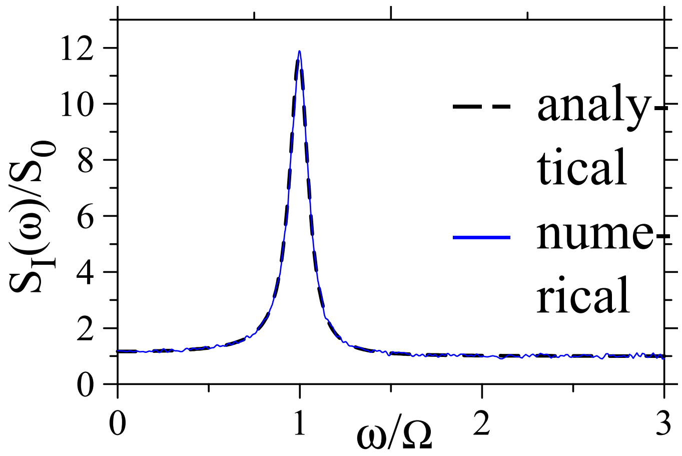
$$\langle I\rangle = \sum_j \rho_{jj} I_j, \quad I_k = (\Gamma_+ - \Gamma_-)|t_k|^2, \quad S_0 = 2(\Gamma_+ + \Gamma_-)|t_0|^2$$

(The formula happens to be the same as for linear detector)

Two-qubit detection

(oscillatory subspace)

Linear detector



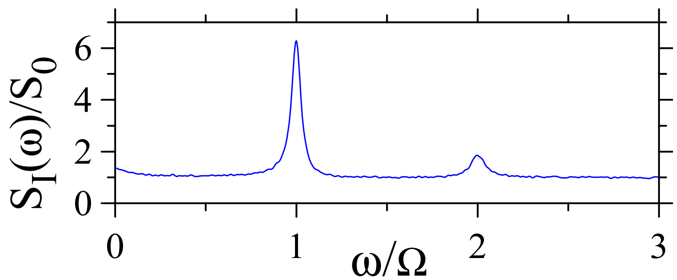
$$S_I(\omega) = S_0 + \frac{8}{3}\frac{\Omega^2(\Delta I)^2\Gamma}{(\omega^2-\Omega^2)^2+\Gamma^2\omega^2}$$

$$\Gamma = \eta^{-1}(\Delta I)^2/4S_0, \; \Delta I = I_1 - I_{23} = I_{23} - I_4$$

Spectral peak at Ω, peak/noise = (32/3)η

(Ω is the Rabi frequency) **(Ruskov-A.K., 2002)**

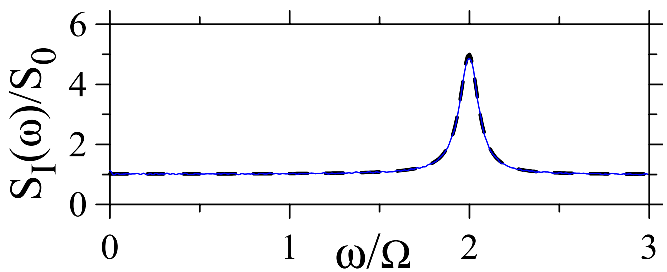
Nonlinear detector



Extra spectral peaks at 2Ω and 0

(analytical formula for weak coupling case)

Quadratic detector



$$S_I(\omega) = S_0 + \frac{4\Omega^2(\Delta I)^2\Gamma}{(\omega^2-4\Omega^2)^2+\Gamma^2\omega^2}$$

$$(\Delta I = I_{23} - I_{14}, \;\; I_1 = I_4, \;\; I_2 = I_3)$$

Peak only at 2Ω, peak/noise = 4η

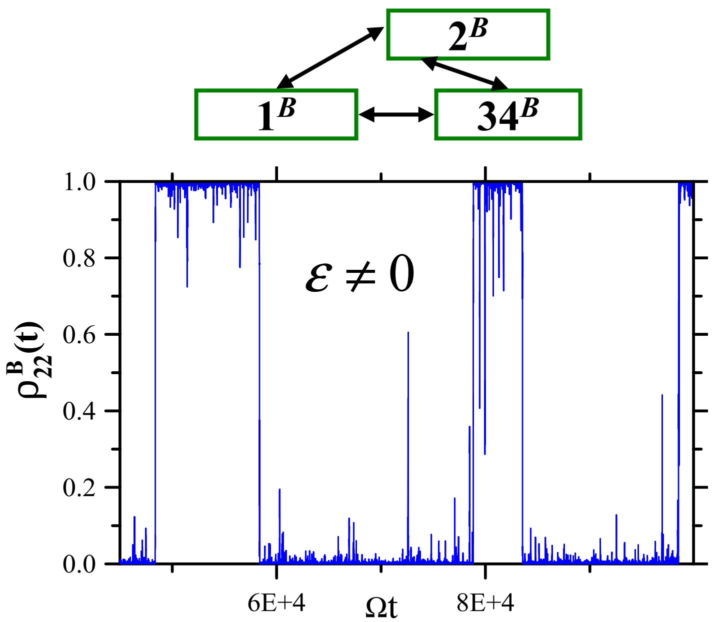
1=↑↑, 2=↑↓, 3=↓↑, 4=↓↓

Mao, Averin, Ruskov, A.K., 2004

Two-qubit quadratic detection: scenarios and switching

Three scenarios:
(distinguishable by average current)

1) collapse into |↑↓ - ↓↑Ú = |1Ú^B , current $I_{Æ∞}$, flat spectrum
2) collapse into |↑↑ - ↓↓Ú = |2Ú^B , current $I_{ÆÆ}$, flat spectrum
3) collapse into remaining subspace |34Ú^B , current $(I_{Æ∞}+I_{ÆÆ})/2$, spectral peak at 2Ω, peak/pedestal = 4η.



3) Slightly asymmetric qubits, $\varepsilon \neq 0$

$$\Gamma_{2B\to 34B} = 2\varepsilon^2\Gamma/\Omega^2$$

Switching between states due to imperfections

1) Slightly different Rabi frequencies, $\Delta\Omega = \Omega_1 - \Omega_2$

$$\Gamma_{1B\to 2B} = \Gamma_{2B\to 1B} = (\Delta\Omega)^2/2\Gamma, \quad \Gamma = \eta^{-1}(\Delta I)^2/4S_0$$

$$S_I(\omega) = S_0 + \frac{(\Delta I)^2\Gamma}{(\Delta\Omega)^2}\,\frac{1}{1+\left[\omega\Gamma/(\Delta\Omega)^2\right]^2}$$

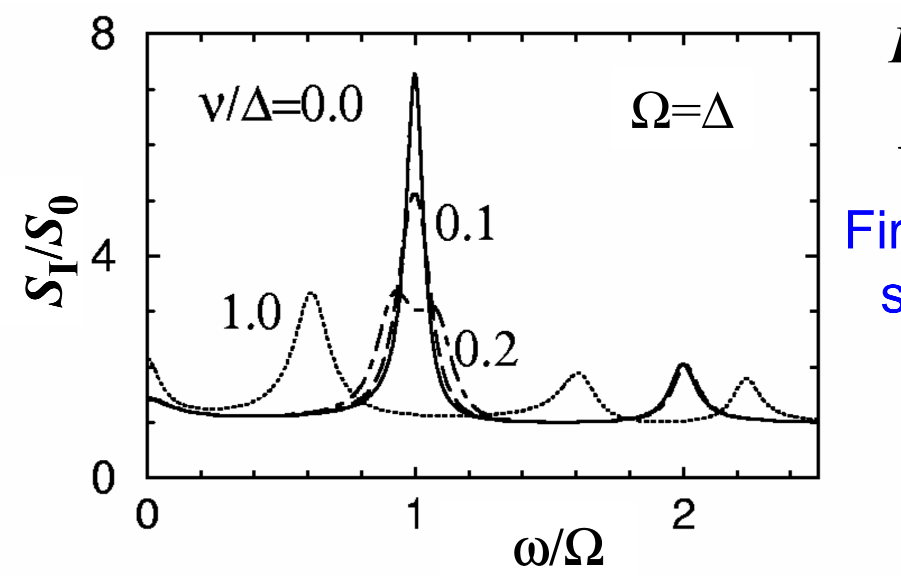
2) Slightly nonquadratic detector, $I_1 \neq I_4$

$$\Gamma_{2B\to 34B} = [(I_1 - I_4)/\Delta I]^2\,\Gamma/2$$

$$S_I(\omega) = S_0 + \frac{2}{3}\,\frac{4\Omega^2(\Delta I)^2\Gamma}{(\omega^2 - 4\Omega^2)^2 + \Gamma^2\omega^2} + \frac{8(\Delta I)^4}{27\Gamma(I_1 - I_4)^2}\,\frac{1}{1+[4\omega(\Delta I)^2/3\Gamma(I_1 - I_4)^2]^2}$$

Mao, Averin, Ruskov, Korotkov, 2004

Effect of qubit-qubit interaction



$$H_{QBs} = -\sum_j (\varepsilon_j \sigma_z^j + \Delta_j \sigma_x^j)/2 + \frac{\nu}{2}\sigma_z^1 \sigma_z^2$$

ν - interaction between two qubits

First spectral peak splits (first order in ν), second peak shifts (second order in ν)

$$\omega_{1-} = [\Delta^2 + (\nu/2)]^{1/2} - \nu/2$$

$$\omega_{1+} = [\Delta^2 + (\nu/2)]^{1/2} + \nu/2$$

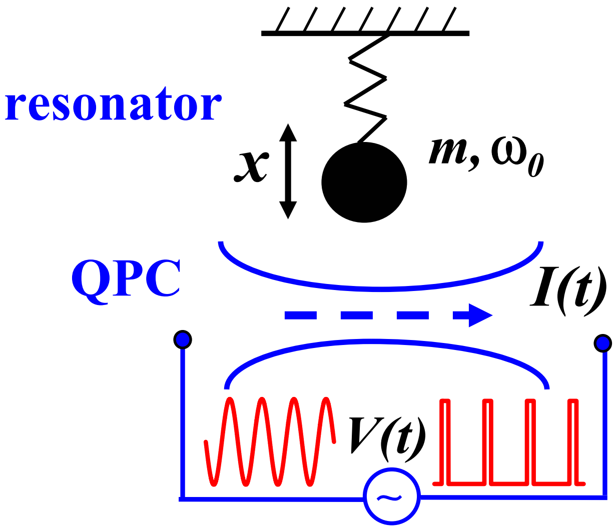
$$\omega_2 = 2[\Delta^2 + (\nu/2)]^{1/2} = \omega_{1-} + \omega_{1+}$$

Summary on quadratic quantum measurements

- **Bayesian formalism is the same as for linear detectors**
- **Detector nonlinearity leads to the second peak in the spectrum (at 2Ω), in purely quadratic case there is no peak at Ω (very similar to classical nonlinear and quadratic detectors)**
- **Qubits become entangled (with some probability) due to measurement, detection of entanglement is easier than for a linear detector (current instead of spectrum), imperfections lead to switching to/from entanglement**

QND squeezing of a nanoresonator

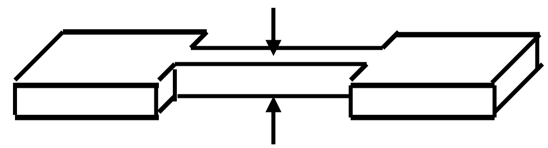
Ruskov, Schwab, Korotkov, cond-mat/0406416



$$\hat{H}_0 = \hat{p}^2/2m + m\omega_0^2 \hat{x}^2/2$$

$$\hat{H}_{DET} = \sum_l E_l a_l^\dagger a_l + \sum_r E_r a_r^\dagger a_r + \sum_{l,r} (M a_l^\dagger a_r + H.c.)$$

$$\hat{H}_{INT} = \sum_{l,r} (\Delta M \, \hat{x} \, a_l^\dagger a_r + H.c.)$$



$\omega_0 \sim 1$ GHz , $T \sim 50$ mK,
quantum behavior $T < \hbar\omega_0$
or $T\tau_{obs}/Q < \hbar/2$

Quite similar to Hopkins, Jacobs, Habib, Schwab, PRB 2003
(continuous monitoring and quantum feedback to cool down)

New feature: Braginsky's stroboscopic QND measurement using modulation of detector voltage ⇒ **squeezing becomes possible**

Potential application: ultrasensitive force measurements

Other most important papers:
Doherty, Jacobs, PRA 1999 (formalism for Gaussian states)
Mozyrsky, Martin, PRL 2002 (ensemble-averaged evolution)

Stroboscopic QND measurements

Quantum nondemolition (QND) measurements (Braginsky-Khalili book)
(a way to suppress measurement backaction and overcome standard quantum limit)

Idea: to avoid measuring the magnitude conjugated to the magnitude of interest

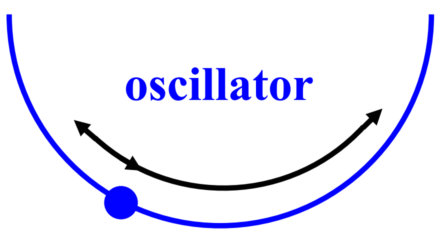
Standard quantum limit

$\Delta p > \hbar / 2\Delta x$

Example: measurement of $x(t_2)$-$x(t_1)$

First measurement: $\Delta p(t_1) > \hbar/2\Delta x(t_1)$, then even for accurate second measurement inaccuracy of position difference is $\Delta x(t_1) + (t_2 - t_1)\hbar/2m\Delta x(t_1) > (t_2 - t_1)\hbar/2^{1/2}m$

Stroboscopic QND measurements (Braginsky *et al*., 1978; Thorne *et al*., 1978)

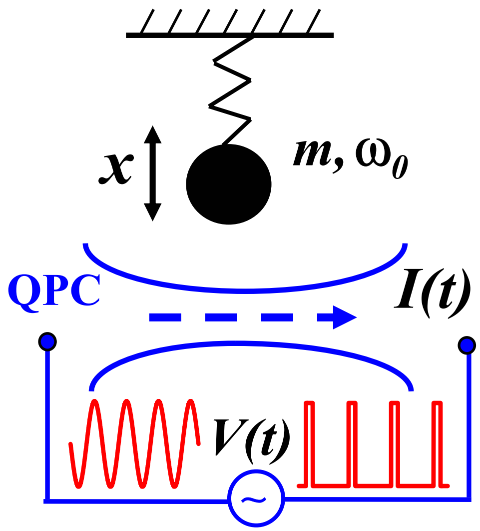


Idea: second measurement exactly one oscillation period later is insensitive to Δp
(or $\Delta t = nT/2$, $T=2\pi/\omega_0$)

Difference in our case:

- **continuous measurement**
- **weak coupling with detector**
- **quantum feedback to suppress "heating"**

Bayesian formalism for continuous measurement of a nanoresonator



$$\hat{H}_0 = \hat{p}^2/2m + m\omega_0^2\hat{x}^2/2$$

$$\hat{H}_{DET} = \sum_l E_l a_l^\dagger a_l + \sum_r E_r a_r^\dagger a_r + \sum_{l,r}(M a_l^\dagger a_r + H.c.)$$

$$\hat{H}_{INT} = \sum_{l,r}(\Delta M\, \hat{x}\, a_l^\dagger a_r + H.c.)$$

Current $I_x = 2\pi\left(M + \Delta M\, x\right)^2 \rho_l \rho_r e^2 V/\hbar = I_0 + k\,x$

Detector noise $S_x = S_0 \equiv 2eI_0$

Recipe: frequent collapses of the number of QPC electrons

Nanoresonator evolution (Stratonovich form), same Eqn as for qubits:

$$\frac{d\rho(x,x')}{dt} = \frac{-i}{\hbar}[\hat{H}_0,\rho] + \frac{\rho(x,x')}{S_0}\left\{ I(t)(I_x + I_{x'} - 2\langle I\rangle) - \frac{1}{2}\left(I_x^2 + I_{x'}^2 - 2\langle I^2\rangle\right)\right\}$$

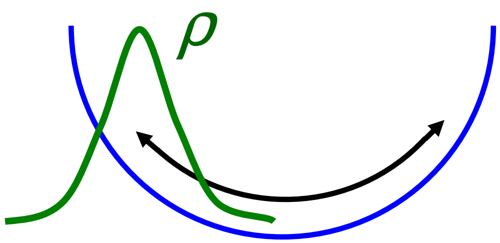
$$\langle I\rangle = \sum I_x \rho(x,x), \quad I(t) = I_x + \xi(t), \quad S_\xi = S_0$$

Ito form (same as in many papers on conditional measurement of oscillators):

$$\frac{d\rho(x,x')}{dt} = \frac{-i}{\hbar}[\hat{H}_0,\rho] - \frac{k^2}{4S_0\eta}(x-x')^2\rho(x,x') + \frac{k}{S_0}(x + x' - 2\langle x\rangle)\,\rho(x,x')\,\xi(t)$$

After that we practically follow Doherty-Jacobs (1999) and Hopkins *et al*. (2003)

Evolution of Gaussian states



Assume Gaussian states (following Doherty-Jacobs and Hopkins-Jacobs-Habib-Schwab), then $\rho(x,x')$ is described by only 5 magnitudes:

$\langle x \rangle$, $\langle p \rangle$ - average position and momentum (packet center),

D_x, D_p, D_{xp} – variances (packet width)

Assume large Q-factor (then no temperature)

Voltage modulation $f(t)V_0$: $\quad k = f(t)\,k_0, \quad I_x = f(t)(I_{00} + k_0 x), \quad S_I = |f(t)|\,S_0$

Then coupling (measurement strength) is also modulated in time:

$$C = |f(t)|\,C_0, \quad C = \hbar k^2 / S_I m\omega_0^2 = 4/\omega_0\tau_{meas}$$

Packet center evolves randomly and needs feedback (force *F*) to cool down

$$d\langle x\rangle/dt = \langle p\rangle/m + (2k_0/S_0)\,\mathrm{sgn}[f(t)]\,D_x\,\xi(t)$$

$$d\langle p\rangle/dt = -m\omega_0^2\langle x\rangle + (2k_0/S_0)\,\mathrm{sgn}[f(t)]\,D_{xp}\,\xi(t) + F(t)$$

Packet width evolves deterministically and is QND squeezed by periodic *f*(*t*)

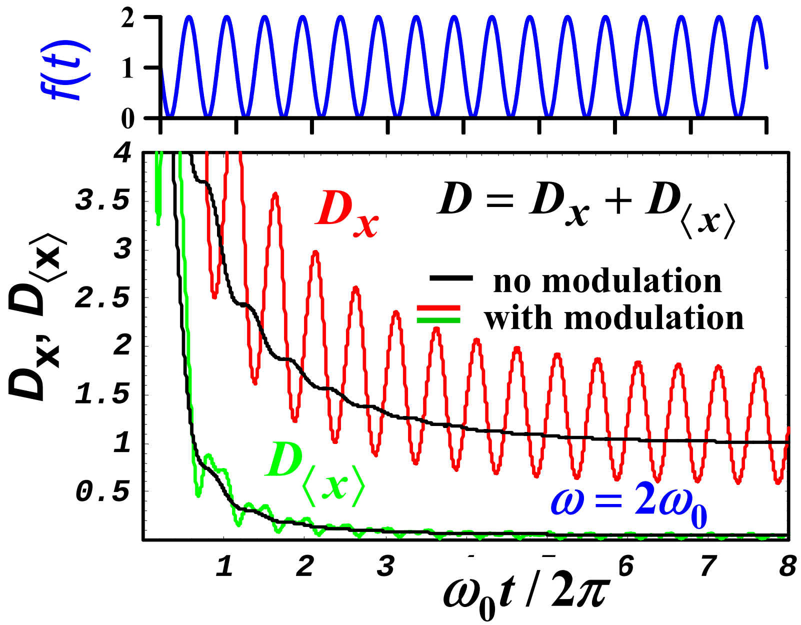
$$d\langle D_x\rangle/dt = (2/m)D_{xp} - (2k_0^2/S_0)\,|f(t)|\,D_x^2$$

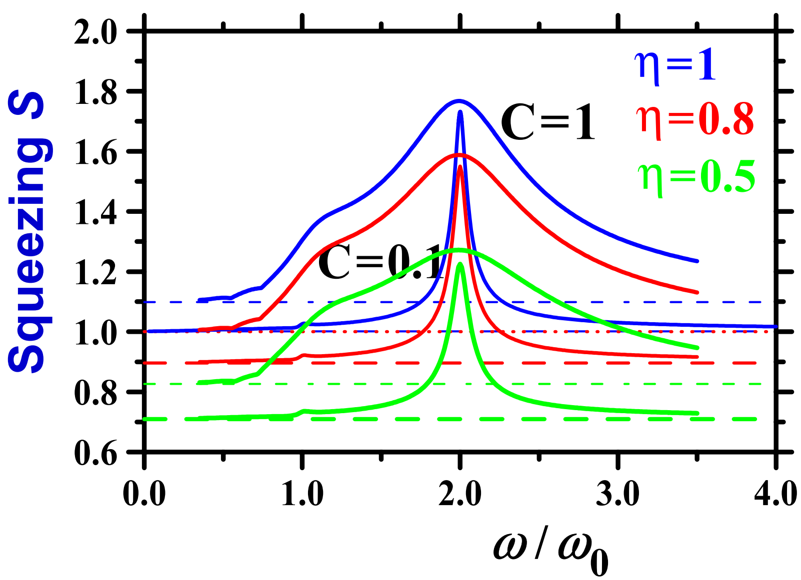
$$d\langle D_p\rangle/dt = -2m\omega_0^2 D_{xp} + (k_0^2\hbar^2/2S_0\eta)\,|f(t)| - (2k_0^2/S_0)\,|f(t)|\,D_{xp}^2$$

$$d\langle D_{xp}\rangle/dt = (1/m)D_p - m\omega_0^2 D_x - (2k_0^2/S_0)\,|f(t)|\,D_x D_{xp}$$

Squeezing by sine-modulation, $V(t)=V_0\sin(\omega t)$

Ruskov-Schwab-Korotkov





Squeezing obviously oscillates in time, maximum squeezing at maximum voltage, momentum squeezing shifted in phase by π/2.

$$S \equiv \max_t(\Delta x_0)^2 / D_x$$

Analytics (weak coupling):

$$S(2\omega_0) = \sqrt{3\eta}, \quad \Delta\omega = 0.36\,\omega_0 C_0 / \sqrt{\eta}$$

η - detector efficiency, C_0 – coupling

$\Delta x_0 = (\hbar/2m\omega_0)^{1/2}$ – ground state width

$D_x=(\Delta x)^2$, $D_{\langle x\rangle}=\langle\langle x\rangle^2\rangle-\langle\langle x\rangle\rangle^2$

Quantum feedback:

$$F = -m\omega_0\gamma_x\langle x\rangle - \gamma_p\langle p\rangle$$

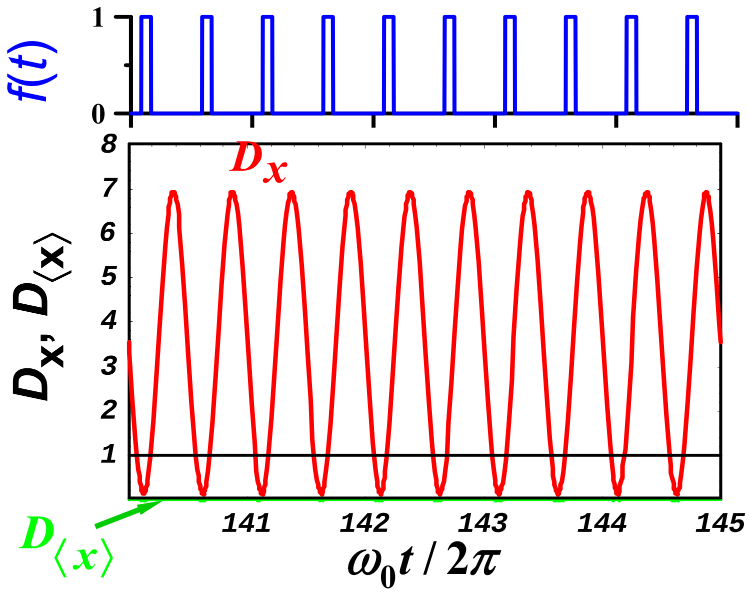
(same as in Hopkins *et al*.; without modulation it cools the state down to the ground state)

Feedback is sufficiently efficient, $D_{\langle x\rangle}$ Ü D_x

Squeezing up to 1.73 at $\omega=2\omega_0$

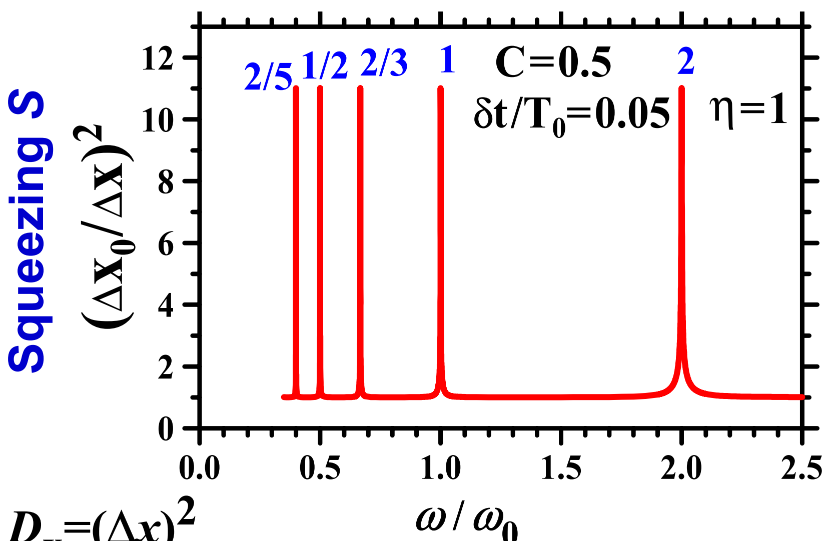
Squeezing by stroboscopic (pulse) modulation

pulse modulation $\omega = 2\omega_0 / n$

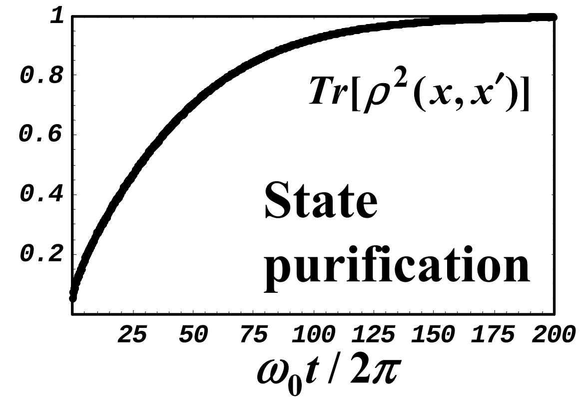


$D_{\langle x\rangle} \ll D_x$

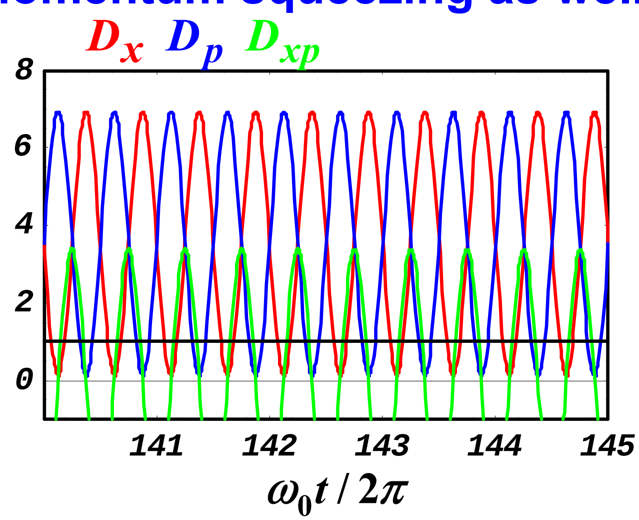
using feedback



$D_X = (\Delta x)^2$



Momentum squeezing as well



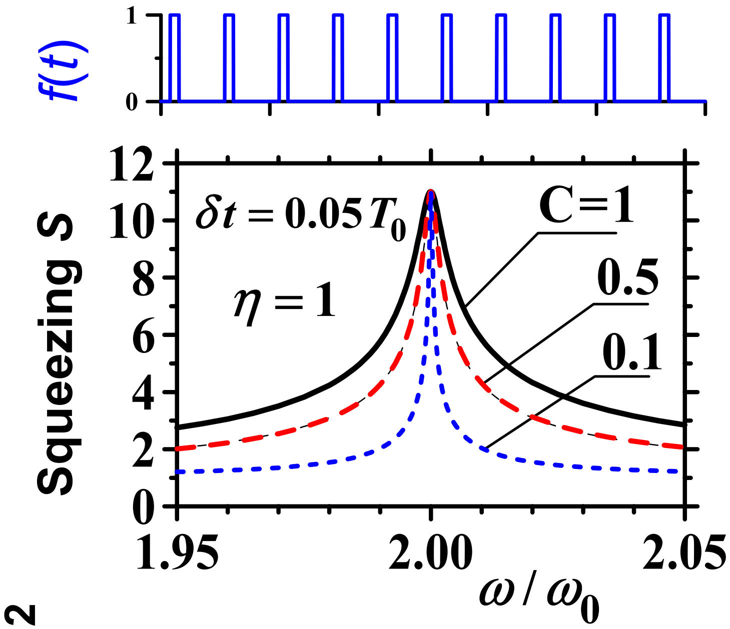
***S* á 1**

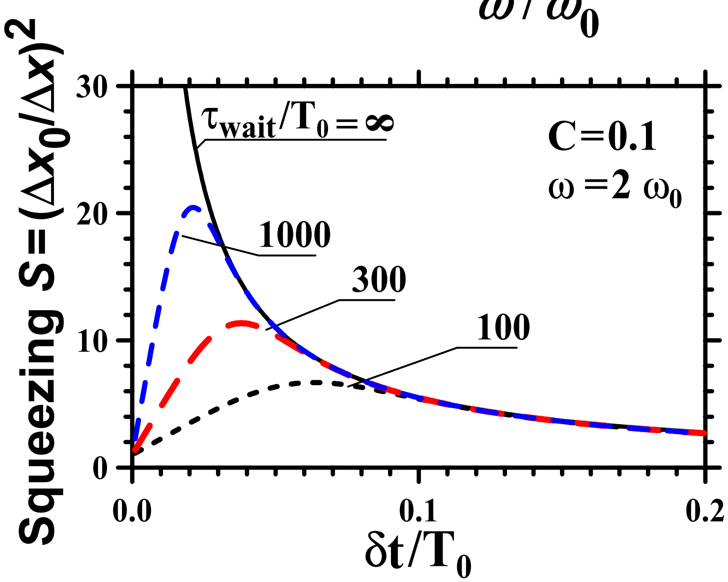
Efficient squeezing at $\omega = 2\omega_0/n$

(natural QND condition)

Ruskov-Schwab-Korotkov

Squeezing by stroboscopic modulation





Analytics (weak coupling, short pulses)

Maximum squeezing

$$S(2\omega_0/n) = \frac{2\sqrt{3\eta}}{\omega_0 \delta t}$$

Linewidth

$$\Delta\omega = \frac{4C_0(\delta t)^3\omega_0^4}{\pi n^2\sqrt{3\eta}}$$

C_0 – dimensionless coupling with detector
δt – pulse duration, $T_0 = 2\pi/\omega_0$
η – quantum efficiency of detector
(long formula for the line shape)

Finite *Q*-factor limits the time we can afford to wait before squeezing develops, $\tau_{wait}/T_0 \sim Q/\pi$

Squeezing saturates as ~exp(-n/n_0) after $n_0 = \sqrt{3\eta}/C_0(\omega_0\delta t)^2$ measurements

Therefore, squeezing cannot exceed

$$S \simeq \sqrt{C_0 Q}\,\sqrt[4]{\eta}$$

Observability of nanoresonator squeezing

Ruskov-Schwab-Korotkov

Procedure: 1) prepare squeezed state by stroboscopic measurement,
2) switch off quantum feedback
3) measure in the stroboscopic way $X_N = \frac{1}{N}\sum_{j=1}^{N} x_j$

For instantaneous measurements ($\delta t \to 0$) the variance of X_N is

$$D_{X,N} = \frac{\hbar}{2m\omega_0}\left(\frac{1}{S} + \frac{1}{NC_0\omega_0\delta t}\right) \to \frac{1}{S}(\Delta x_0)^2 \quad \text{at } N \to \infty$$

S – squeezing,
Δx_0 – ground state width

Then distinguishable from ground state (S=1)
in one run for S à 1 (error probability ~$S^{-1/2}$)

Not as easy for continuous measurements because of extra “heating”.
$D_{X,N}$ has a minimum at some N and then increases.
However, numerically it seems $\min_N D_{X,N} \sim 2(\Delta x_0)^2 / S$ (only twice worse)

Example: $\min_N D_{X,N} /(\Delta x_0) = 0.078$ for C_0=0.1, η=1, $\delta t/T_0$=0.02, 1/S=0.036

Squeezed state is distinguishable in one run (with small error probability), therefore suitable for ultrasensitive force measurement beyond standard quantum limit

Summary on QND squeezing of a nanoresonator

- **Periodic modulation of the detector voltage modulates measurement strength and periodically squeezes the width of the nanoresonator state ("breathing mode")**
- **Packet center oscillates and is randomly "heated" by measurement; quantum feedback can cool it down (keep it near zero in both position and momentum)**
- **Sine-modulation leads to a small squeezing (<1.73), stroboscopic (pulse) modulation can lead to a strong squeezing (>>1) even for a weak coupling with detector**
- **Still to be done: correct account of *Q*-factor and temperature**
- **Potential application: force measurement beyond standard quantum limit**

Conclusions

- **Bayesian formalism for solid-state quantum measurements is being used to produce various experimental predictions (though still not well-accepted in solid-state community)**

- **Simple, practically classical feedback using quadratures of the detector current should work well for qubit oscillations; relatively simple experiment**

- **Measurements by nonlinear (quadratic) detectors are described by the Bayesian formalism (same formulas as for linear detector), nonlinearity leads to the spectral peak at double frequency and makes easier qubit entanglement by measurement**

- **Measurement of a nanoresonator with strength modulated in time (modulating detector voltage) can produce a squeezed state; squeezed state is measurable and potentially useful**

- **No solid-state experiments yet; hopefully, reasonably soon**