SPIE, Santa Fe, June 2003

Noisy quantum measurement of solid-state qubits: Bayesian approach

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Outline:

Bayesian formalism

- Continuous measurement of a single qubit
- Ideal and nonideal solid-state detectors
- Bayesian formalism for entangled qubits

Experimental predictions and proposals

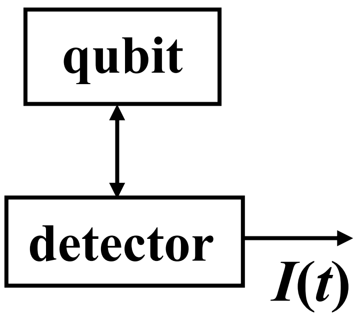
- Measured spectrum of Rabi oscillations
- Bell-type experiment
- Quantum feedback for a qubit
- Entanglement by measurement
- Quadratic quantum detection

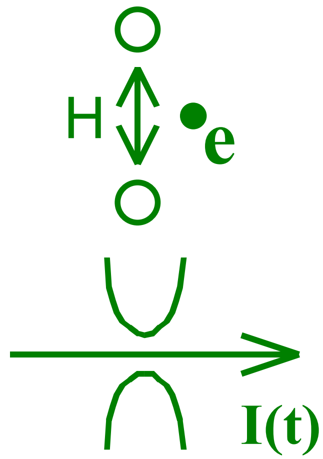
Review: cond-mat/0209629, in "Quantum Noise in Mesoscopic Physics" (Kluwer, 2003)

Acknowledgement: Rusko Ruskov

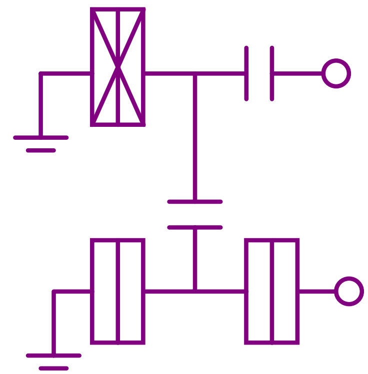
Support: ARDA

Examples of solid-state qubits and detectors

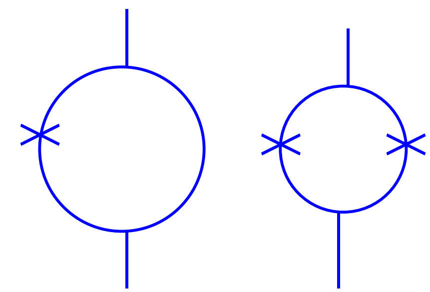




Double-quantum-qot and quantum point contact (QPC)



Cooper-pair box and single-electron transistor (SET)



Two SQUIDs

$$H = H_{\rm QB} + H_{\rm DET} + H_{\rm INT}$$

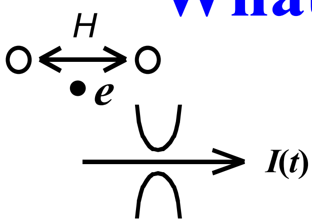
$$H_{\rm QB} = (\varepsilon/2)(c_1^+c_1 - c_2^+c_2) + H(c_1^+c_2 + c_2^+c_1)$$ ε - asymmetry, H – tunneling

$\Omega = (4H^2+\varepsilon^2)^{1/2}$ – frequency of quantum coherent (Rabi) oscillations

Two levels of average detector current: I_1 for qubit state $|1\rangle$, I_2 for $|2\rangle$

Response: $\Delta I = I_1 - I_2$ Detector noise: white, spectral density S_I

What happens to a qubit state during measurement?



For simplicity (for a moment) $H=\varepsilon=0$, infinite barrier (frozen qubit), evolution due to measurement only

"Orthodox" answer

$$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \begin{matrix} \nearrow \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \\ \searrow \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \end{matrix}$$

|1> or |2>, depending on the result

"Conventional" (decoherence) answer (Leggett, Zurek)

$$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \rightarrow \begin{pmatrix} \frac{1}{2} & \frac{\exp(-\Gamma t)}{2} \\ \frac{\exp(-\Gamma t)}{2} & \frac{1}{2} \end{pmatrix} \rightarrow \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

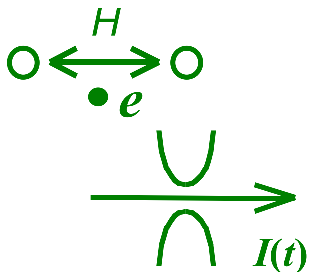
no measurement result! ensemble averaged

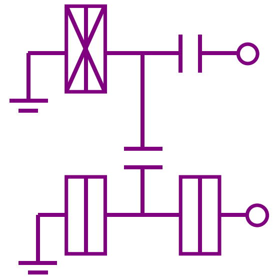
Orthodox and decoherence answers contradict each other!

applicable for:	Single quantum systems	Continuous measurements
Orthodox	yes	no
Conventional (ensemble)	no	yes
Bayesian	yes	yes

Bayesian formalism describes gradual collapse of single quantum systems
Noisy detector output $I(t)$ should be taken into account

Bayesian formalism for a single qubit





$$\hat{H}_{QB} = \frac{\varepsilon}{2}(c_1^\dagger c_1 - c_2^\dagger c_2) + H(c_1^\dagger c_2 + c_2^\dagger c_1)$$

$|1\rangle \rightarrow I_1$, $|2\rangle \rightarrow I_2$

$\Delta I = I_1 - I_2$, $I_0 = (I_1 + I_2)/2$, S_I – detector noise

$$\frac{d}{dt}\rho_{11} = -\frac{d}{dt}\rho_{22} = -2H\,\mathrm{Im}\,\rho_{12} + \rho_{11}\rho_{22}\frac{2\Delta I}{S_I}[I(t) - I_0]$$

$$\frac{d}{dt}\rho_{12} = i\varepsilon\rho_{12} + iH(\rho_{11} - \rho_{22}) + \rho_{12}(\rho_{11} - \rho_{22})\frac{\Delta I}{S_I}[I(t) - I_0] - \gamma\rho_{12}$$

A.K., 1998

$\gamma = \Gamma - (\Delta I)^2 / 4S_I$, Γ – ensemble decoherence

$\eta = 1 - \gamma/\Gamma = (\Delta I)^2 / 4S_I\Gamma$ – detector ideality (efficiency), $\eta \le 100\%$

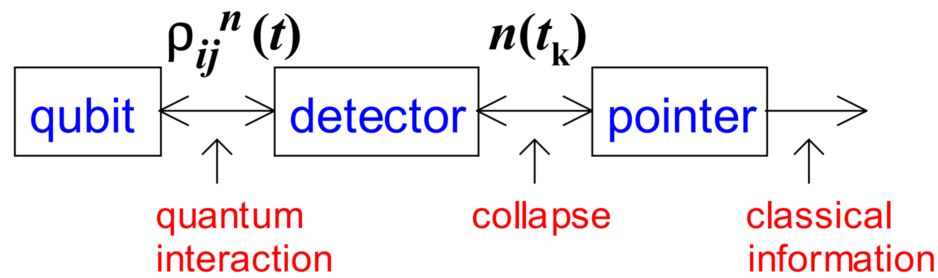
For simulations: $I(t) - I_0 \rightarrow (\rho_{22} - \rho_{11})\Delta I/2 + \xi(t)$, $S_\xi = S_I$

Averaging over $\xi(t)$ ⇒ master equation

Similar formalisms developed earlier. **Key words**: **Imprecise, weak, selective, or conditional measurements, POVM, Quantum trajectories, Quantum jumps, Restricted path integral, etc.**

Names: E.B. Davies, K. Kraus, A.S. Holevo, C.W. Gardiner, H.J. Carmichael, C.M. Caves, M.B. Plenio, P.L. Knight, M.B. Mensky, D.F. Walls, N. Gisin, I.C. Percival, **G.J. Milburn**, **H.M. Wiseman,** R. Onofrio, S. Habib, A. Doherty, etc. (very incomplete list)

“Microscopic” derivation of the Bayesian formalism



Schrödinger evolution of “qubit + detector”
for a low-T QPC as a detector (Gurvitz, 1997)

$$\frac{d}{dt}\rho_{11}^{n} = -\frac{I_1}{e}\rho_{11}^{n} + \frac{I_1}{e}\rho_{11}^{n-1} - 2\frac{H}{\hbar}\,\mathrm{Im}\,\rho_{12}^{n}$$

$$\frac{d}{dt}\rho_{22}^{n} = -\frac{I_2}{e}\rho_{22}^{n} + \frac{I_2}{e}\rho_{22}^{n-1} + 2\frac{H}{\hbar}\,\mathrm{Im}\,\rho_{12}^{n}$$

$$\frac{d}{dt}\rho_{12}^{n} = i\frac{\varepsilon}{\hbar}\rho_{12}^{n} + i\frac{H}{\hbar}(\rho_{11}^{n} - \rho_{22}^{n}) - \frac{I_1+I_2}{2e}\rho_{12}^{n} + \frac{\sqrt{I_1 I_2}}{e}\rho_{12}^{n-1}$$

Detector collapse at $t = t_k$
Particular n_k is chosen at t_k

$$P(n) = \rho_{11}^{n}(t_k) + \rho_{22}^{n}(t_k)$$

$$\rho_{ij}^{n}(t_k+0) = \delta_{n,nk}\,\rho_{ij}(t_k+0),$$

$$\rho_{ij}(t_k+0) = \frac{\rho_{ij}^{nk}(t_k-0)}{\rho_{11}^{nk}(t_k-0) + \rho_{22}^{nk}(t_k-0)}$$

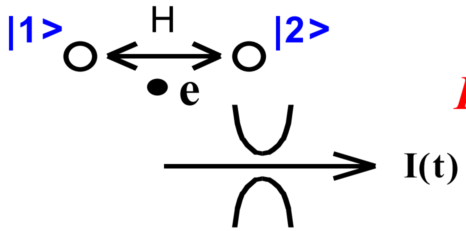
If $H = \varepsilon = 0$,
it leads to

$$\rho_{11}(t) = \frac{\rho_{11}(0)P_1(n)}{\rho_{11}(0)P_1(n) + \rho_{22}(0)P_2(n)}, \quad \rho_{22}(t) = \frac{\rho_{22}(0)P_2(n)}{\rho_{11}(0)P_1(n) + \rho_{22}(0)P_2(n)}$$

$$\rho_{12}(t) = \rho_{12}(0)\frac{[\rho_{11}(t)\rho_{22}(t)]^{1/2}}{[\rho_{11}(0)\rho_{22}(0)]^{1/2}}, \qquad P_i(n) = \frac{(I_i t/e)^n}{n!}\exp(-I_i t/e),$$

which are exactly quantum Bayes formulas

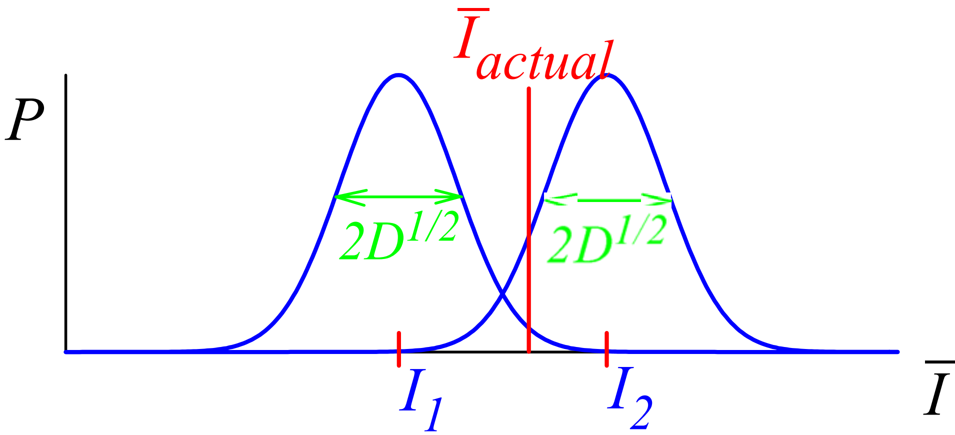
"Quantum Bayes theorem" (ideal detector assumed)



$H = \varepsilon = 0$ **(frozen qubit)**

Initial state: $\begin{pmatrix} \rho_{11}(0) & \rho_{12}(0) \\ \rho_{21}(0) & \rho_{22}(0) \end{pmatrix}$

Measurement (during time τ):



$$\bar{I} \equiv \frac{1}{\tau}\int_0^\tau I(t)\, dt$$

$$P(\bar{I},\tau) = \rho_{11}(0)\, P_1(\bar{I},\tau) + \rho_{22}(0)\, P_2(\bar{I},\tau)$$

$$P_i(\bar{I},\tau) = \frac{1}{\sqrt{2\pi D}} \exp[-(\bar{I} - I_i)^2 / 2D],$$

$$D = S_I / 2\tau, \quad |I_1 - I_2| \ll I_i, \quad \tau \gg S_I / I_i^2$$

After the measurement during time τ, the probabilities can be updated using the standard Bayes formula:

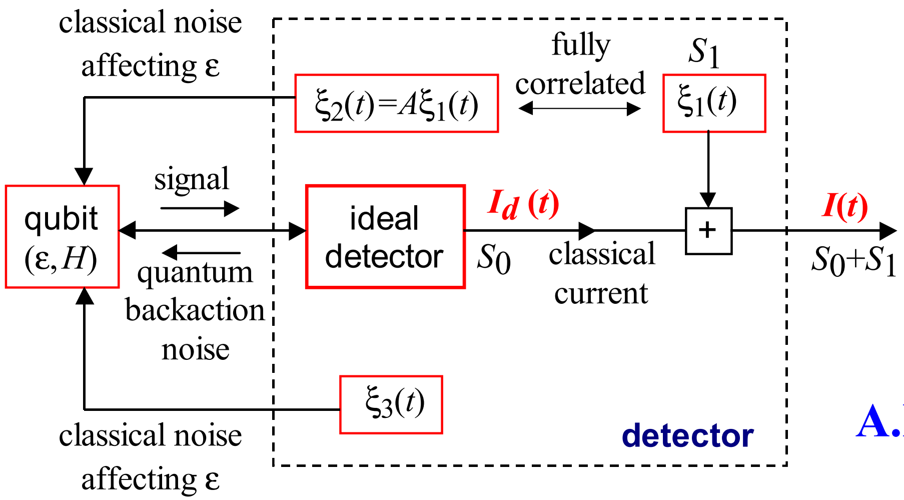
$$P(B_i \mid A) = \frac{P(B_i)P(A \mid B_i)}{\sum_k P(B_k)P(A \mid B_k)}$$

Quantum Bayes formulas:

$$\rho_{11}(\tau) = \frac{\rho_{11}(0)\exp[-(\bar{I} - I_1)^2/2D]}{\rho_{11}(0)\exp[-(\bar{I} - I_1)^2/2D] + \rho_{22}(0)\exp[-(\bar{I} - I_2)^2/2D]}$$

$$\frac{\rho_{12}(\tau)}{[\rho_{12}(\tau)\,\rho_{22}(\tau)]^{1/2}} = \frac{\rho_{12}(0)}{[\rho_{12}(0)\,\rho_{22}(0)]^{1/2}}, \qquad \rho_{22}(\tau) = 1 - \rho_{11}(\tau)$$

Nonideal detectors with input-output noise correlation



$$K=\frac{AS_1+\theta S_0}{\hbar S_I},\quad S_I=S_0+S_1$$

K – correlation between output and backaction noises

A.K., 2002

$$\frac{d}{dt}\rho_{11}=-\frac{d}{dt}\rho_{22}=-2H\,\mathrm{Im}\,\rho_{12}+\rho_{11}\rho_{22}\frac{2\Delta I}{S_I}[I(t)-I_0]$$

$$\frac{d}{dt}\rho_{12}=i\varepsilon\rho_{12}+iH(\rho_{11}-\rho_{22})+\rho_{12}(\rho_{11}-\rho_{22})\frac{\Delta I}{S_I}[I(t)-I_0]+iK[I(t)-I_0]-\tilde{\gamma}\rho_{12}$$

Fundamental limits for ensemble decoherence

$$\Gamma=\gamma+(\Delta I)^2/4S_I,\quad \gamma\geq 0\ \Rightarrow\ \Gamma\geq(\Delta I)^2/4S_I$$

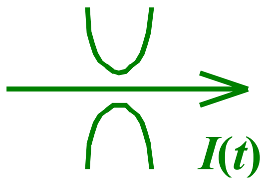
$$\Gamma=\gamma+(\Delta I)^2/4S_I+K^2S_I/4,\quad \gamma\geq 0\ \Rightarrow\ \Gamma\geq(\Delta I)^2/4S_I+K^2S_I/4$$

Translated into energy sensitivity: $(\epsilon_I\,\epsilon_{BA})^{1/2}\geq\hbar/2$ **or** $(\epsilon_I\,\epsilon_{BA}-\epsilon_{I,BA})^{1/2}\geq\hbar/2$

Ideality of realistic solid-state detectors

(ideal detector does not cause single qubit decoherence)

1. Quantum point contact



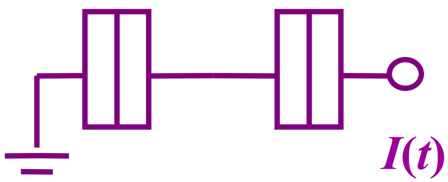
Theoretically, **ideal quantum detector, η=1**

A.K., 1998 (Gurvitz, 1997; Aleiner et al., 1997)

Experimentally, η > 80%

(using Buks et al., 1998)

2. SET-transistor



Very non-ideal in usual operation regime, η ‹‹ 1

Shnirman-Schőn, 1998; A.K., 2000, Devoret-Schoelkopf, 2000

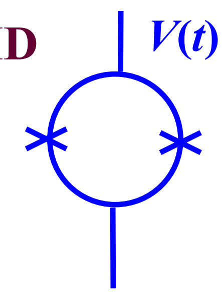
However, reaches ideality, η = 1 if:

- in deep cotunneling regime **(Averin, 2000, van den Brink, 2000)**
- S-SET, using supercurrent **(Zorin, 1996)**
- S-SET, double-JQP peak **(Clerk et al., 2002)**

??? S-SET, usual JQP **(Johansson et al.)**, onset of QP branch **(?)**

- resonant-tunneling SET, low bias **(Averin, 2000)**

3. SQUID



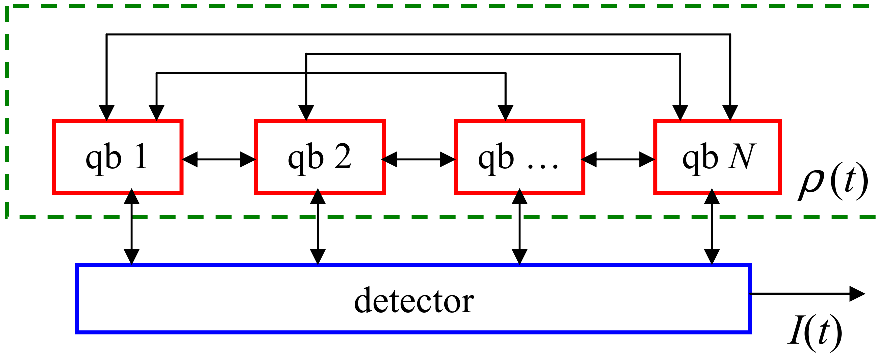
Can reach ideality, η = 1

(Danilov-Likharev-Zorin, 1983; Averin, 2000)

4. FET ?? HEMT ??

ballistic FET/HEMT ??

Bayesian formalism for *N* entangled qubits measured by one detector



Up to 2^N levels of current

$$\frac{d}{dt}\rho_{ij} = \frac{-i}{\hbar}[\hat{H}_{qb},\rho]_{ij} + \rho_{ij}\frac{1}{S}\sum_k \rho_{kk}\Big[\big(I(t)-\frac{I_k+I_i}{2}\big)(I_i-I_k) + \big(I(t)-\frac{I_k+I_j}{2}\big)(I_j-I_k)\Big] - \gamma_{ij}\rho_{ij}$$

(Stratonovich form)

$$\gamma_{ij} = (\eta^{-1}-1)(I_i-I_j)^2/4S_I \qquad I(t) = \sum_i \rho_{ii}(t) I_i + \xi(t)$$

Averaging over ξ(t) $\Rightarrow$ master equation

No measurement-induced dephasing between states $|i\rangle$ and $|j\rangle$ if $I_i = I_j$!

A.K., PRA 65 (2002), PRB 67 (2003)

Some experimental predictions and proposals

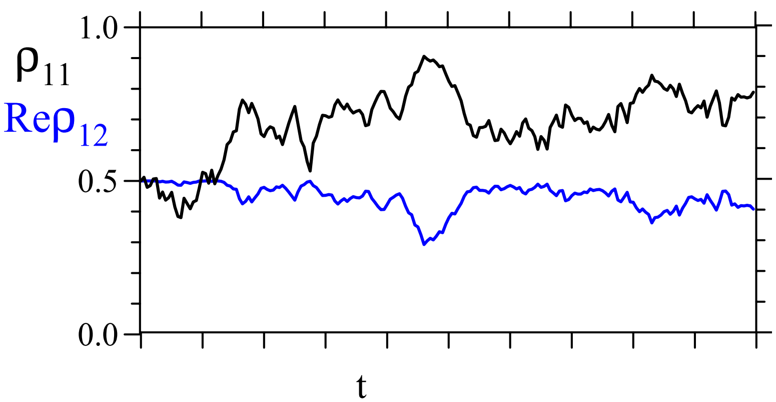
- Direct experiments (1998)
- Measured spectral density of Rabi oscillations (1999, 2000, 2002)
- Bell-type correlation experiment (2000)
- Quantum feedback control of a qubit (2001)
- Entanglement by measurement (2002)
- Measurement by a quadratic detector (2003)

Direct Bayesian experiments

(A.K., 1998)

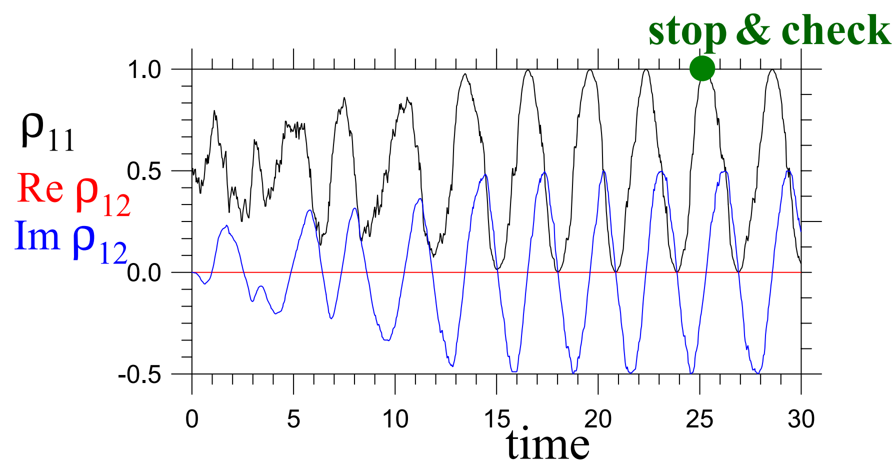
Idea: check the evolution of (almost pure!) qubit state given by Bayesian equations

Evolution from 1/2-alive to 1/3-alive Schrödinger cat



1. Prepare coherent state and make H=0.
2. Measure for a finite time t.
3. Check the predicted wavefunction (using evolution with $H\neq 0$ to get the state |1›.

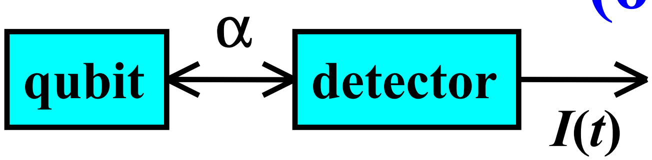
Density matrix purification by measurement



1. Start with completely mixed state.
2. Measure and monitor the Rabi phase.
3. Stop evolution (make H=0) at state |1›.
4. Measure.

Difficulty: need to record noisy detector current $I(t)$, typical required bandwidth ~ 1-10 GHz.

Measured spectrum of Rabi oscillations (or spin precession)



What is the spectral density $S_I(\omega)$ of detector current?

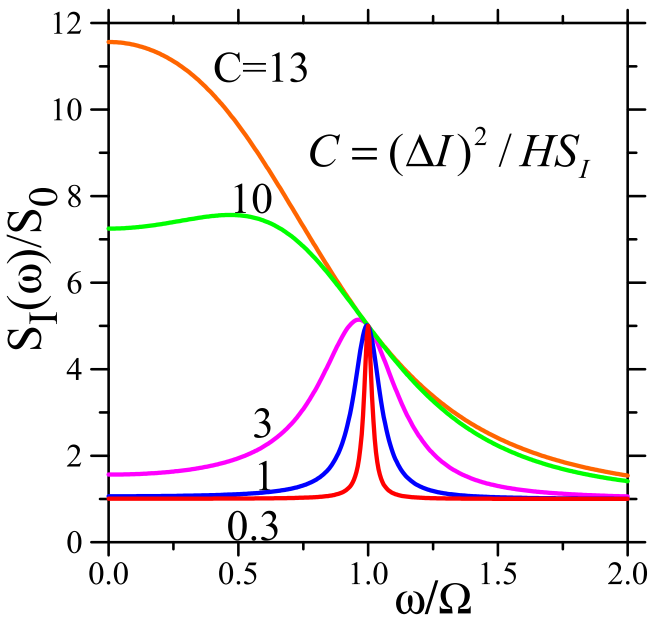
Assume classical output, eV » $\hbar\Omega$

$$\varepsilon = 0, \quad \Gamma = \eta^{-1}(\Delta I)^2/4S_0$$

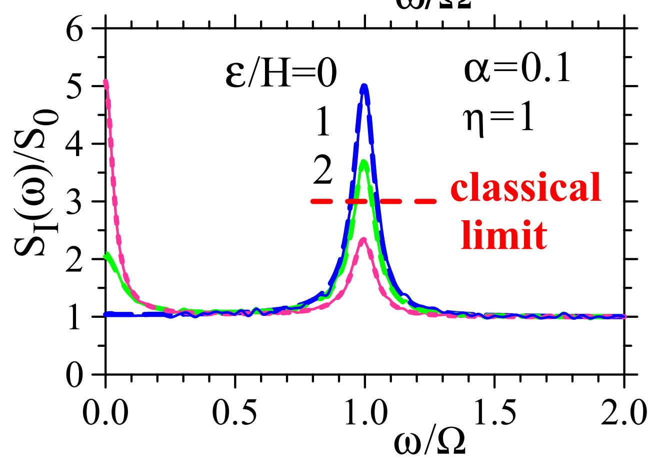
$$S_I(\omega) = S_0 + \frac{\Omega^2(\Delta I)^2\Gamma}{(\omega^2-\Omega^2)^2+\Gamma^2\omega^2}$$

Spectral peak can be seen, but peak-to-pedestal ratio ≤ 4η ≤ 4

(result can be obtained using various methods, not only Bayesian method)



A.K., LT'99
Averin-A.K., 2000
A.K., 2000
Averin, 2000
Goan-Milburn, 2001
Makhlin et al., 2001
Balatsky-Martin, 2001
Ruskov-A.K., 2002
Mozyrsky et al., 2002
Balatsky et al, 2002
Bulaevskii et al., 2002
Shnirman et al., 2002
Bulaevskii-Ortiz, 2003



Weak coupling, $\alpha = C/8 \ll 1$

$$S_I(\omega) = S_0 + \frac{\eta S_0\varepsilon^2/H^2}{1+(\omega\hbar^2\Omega^2/4H^2\Gamma)^2} + \frac{4\eta S_0(1+\varepsilon^2/2H^2)^{-1}}{1+[(\omega-\Omega)\Gamma(1-2H^2/\hbar^2\Omega^2)]^2}$$

Possible experimental confirmation?

(STM-ESR experiment similar to Manassen-1989)

APPLIED PHYSICS LETTERS VOLUME 80, NUMBER 3 21 JANUARY 2002

Electronic spin detection in molecules using scanning-tunneling-microscopy-assisted electron-spin resonance

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(Received 8 May 2001; accepted for publication 8 November 2001)

By combining the spatial resolution of a scanning-tunneling microscope (STM) with the electronic spin sensitivity of electron-spin resonance, we show that it is possible to detect the presence of localized spins on surfaces. The principle is that a STM is operated in a magnetic field, and the resulting component of the tunnel current at the Larmor (precession) frequency is measured. This component is nonzero whenever there is tunneling into or out of a paramagnetic entity. We have

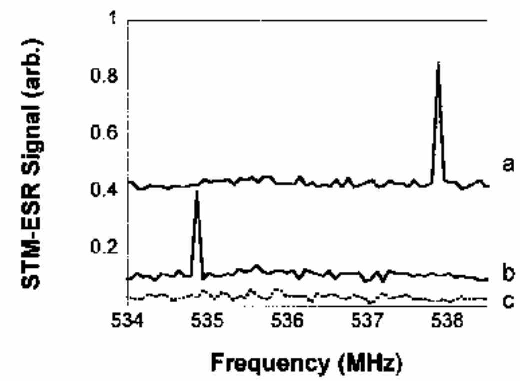


FIG. 3. STM-ESR spectra of (a), (b) two different areas (a few nm apart) of the molecule-covered sample and (c) bare HOPG. The graphs are shifted vertically for clarity.

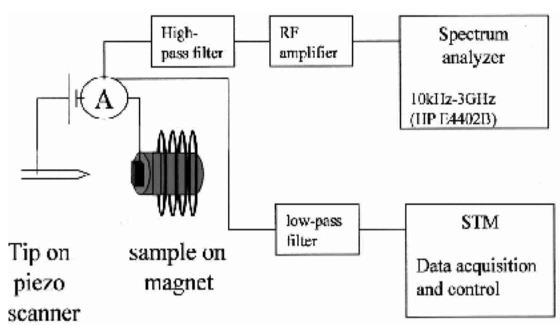


FIG. 1. Schematic of the electronics used in STM-ESR.

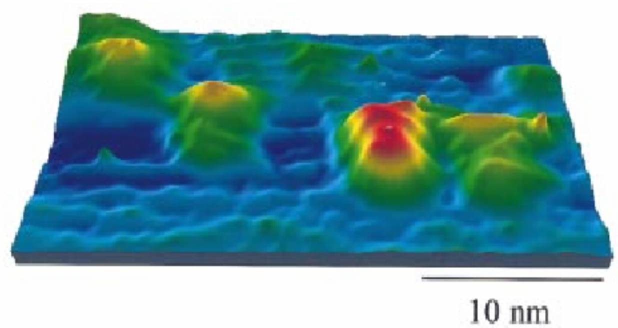


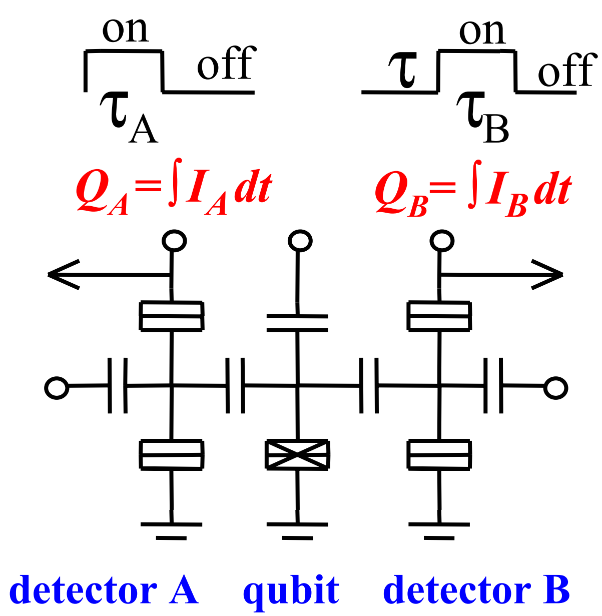
FIG. 2. (Color) STM image of a 250 Å×150 Å area of HOPG with four adsorbed BDPA molecules.

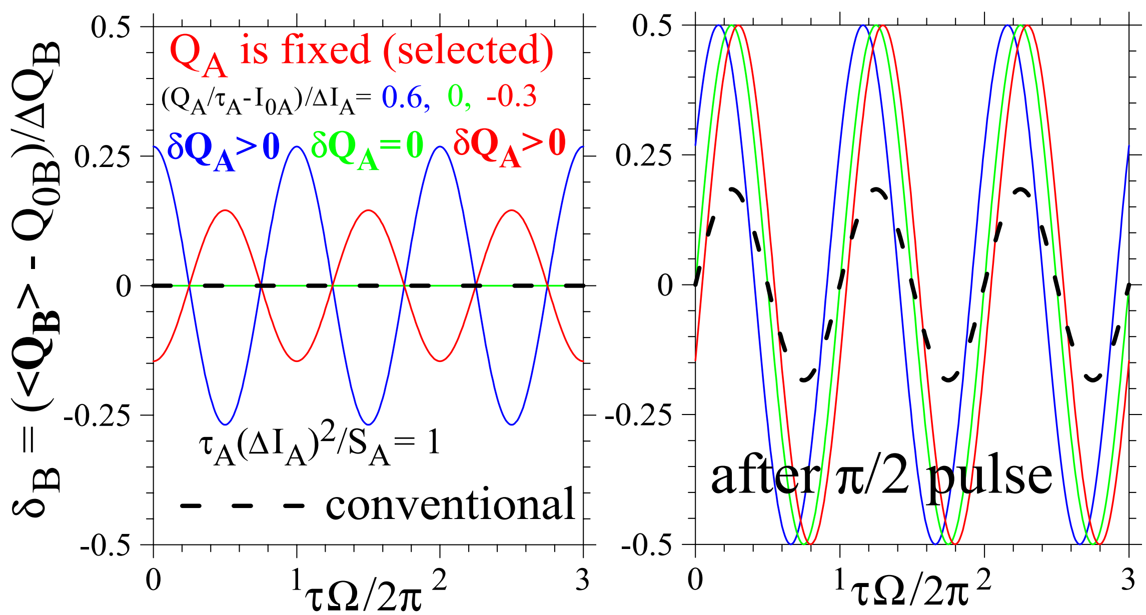
$$\frac{\text{peak}}{\text{noise}} \leq 3.5$$

(Colm Durkan, private comm.)

Bell-type correlation experiment

A.K., 2000





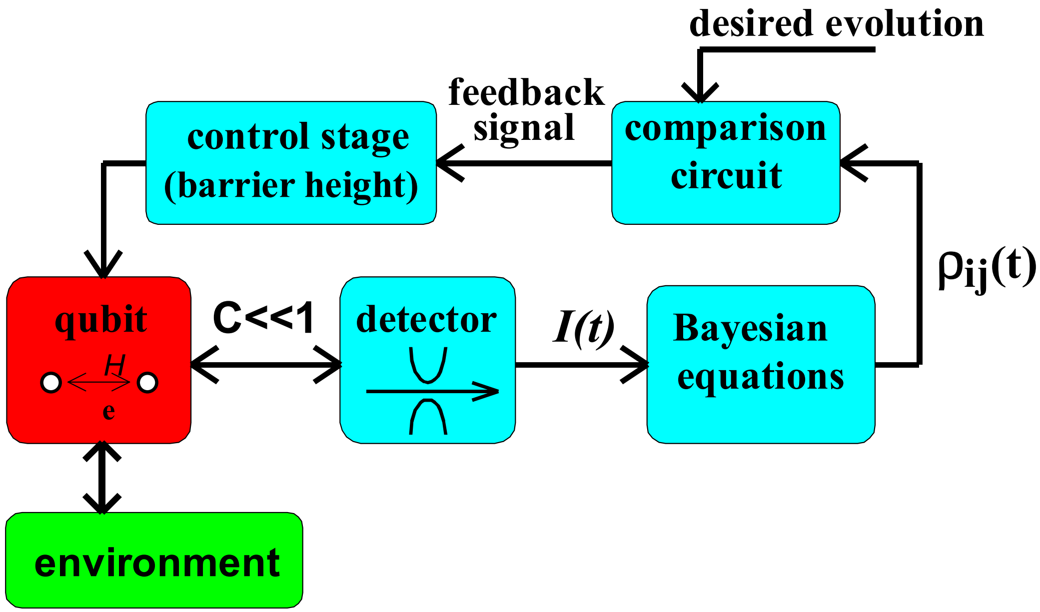
Idea: two consecutive finite-time (imprecise) measurements of a qubit by two detectors; probability distribution $\mathbf{P}(Q_A, Q_B, \tau)$ shows the effect of the first measurement on the qubit state.

Proves that the qubit remains in a pure state during the measurement (for $\eta = 1$).

Advantage: no need to record noisy detector output with GHz bandwidth; instead, we use two detectors and fast ON/OFF switching.

Quantum feedback control of a solid-state qubit

Ruskov & A.K., 2001



Goal: maintain desired phase of Rabi oscillations in spite of environmental dephasing (keep qubit "fresh")

Idea: monitor the Rabi phase ϕ by continuous measurement and apply feedback control of the qubit barrier height, $\Delta H_{FB}/H = -F\times\Delta\phi$

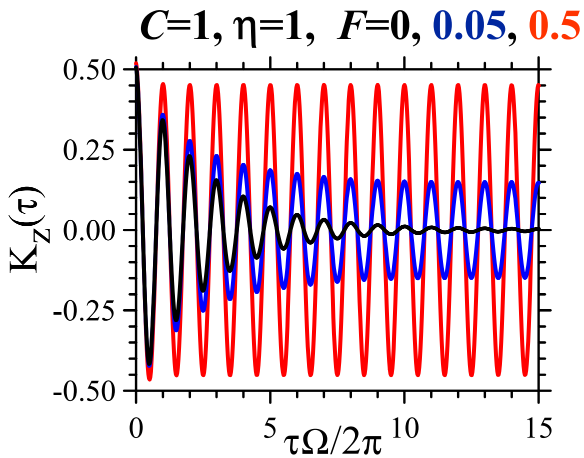
To monitor phase ϕ we plug detector output $I(t)$ into Bayesian equations

Quantum feedback in quantum optics is discussed since 1993 (Wiseman-Milburn), recently first successful experiment in Mabuchi's group (Armen et al., 2002).

Performance of quantum feedback

(no extra environment)

Qubit correlation function



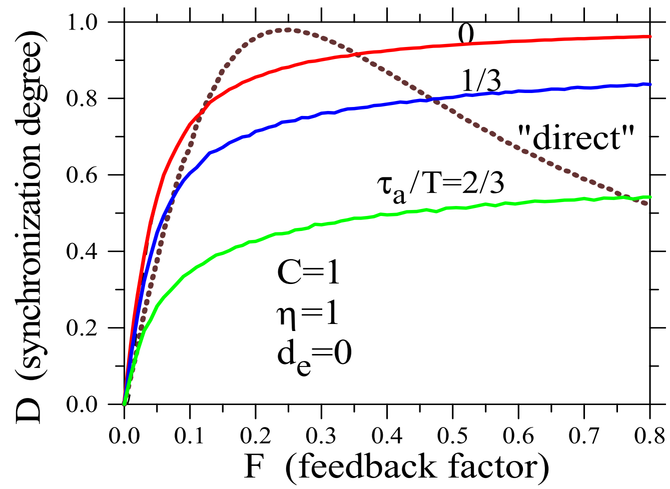
$$K_z(\tau) = \frac{\cos\ \Omega t}{2}\ \exp\left[\frac{C}{16F}(e^{-2FH\tau/\hbar} - 1)\right]$$

(for weak coupling and good fidelity)

Detector current correlation function

$$K_I(\tau) = \frac{(\Delta I)^2}{4}\frac{\cos\ \Omega t}{2}\ (1 + e^{-2FH\tau/\hbar}) \times \exp\left[\frac{C}{16F}(e^{-2FH\tau/\hbar} - 1)\right] + \frac{S_I}{2}\delta(\tau)$$

Fidelity (synchronization degree)



$C = \hbar(\Delta I)^2 / S_I H$ – coupling

τ_a^{-1} – available bandwidth

F – feedback strength

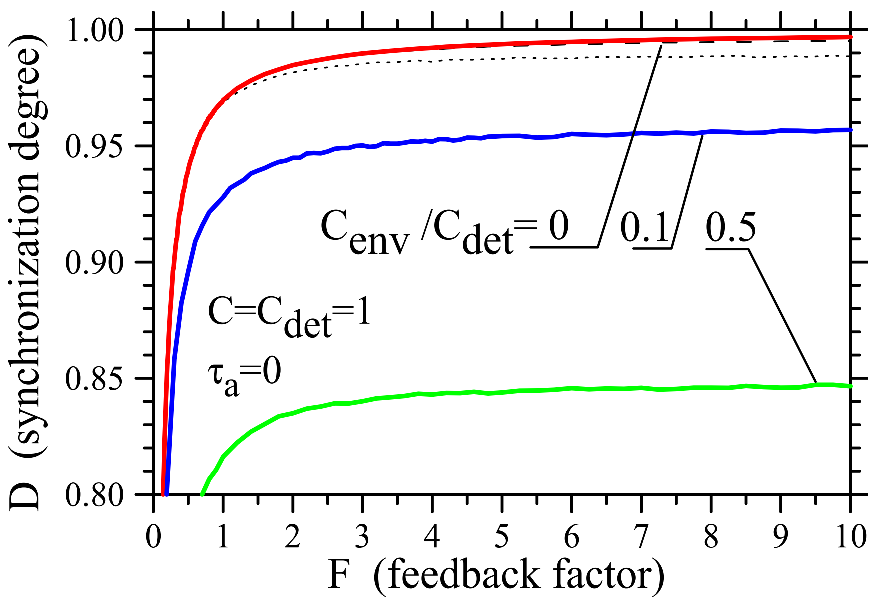
$D = 2\langle \mathrm{Tr}\rho\rho_{desir}\rangle - 1$

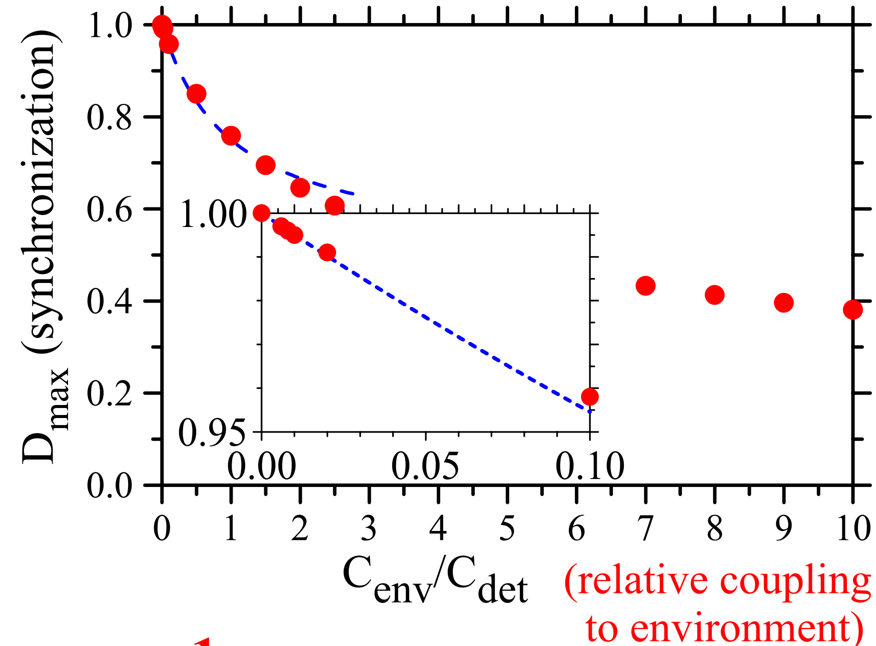
For ideal detector and wide bandwidth, fidelity can be arbitrary close to 100%

$D = \exp(-C/32F)$

Ruskov & Korotkov, PRB 66, 041401(R) (2002)

Suppression of environment-induced decoherence by quantum feedback



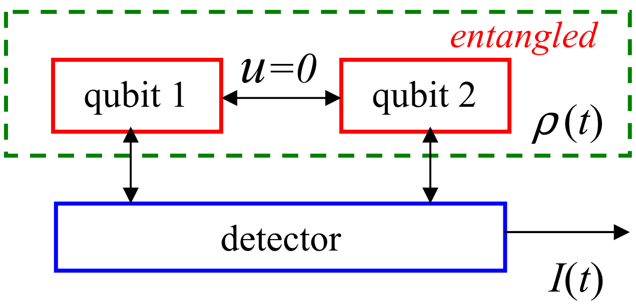


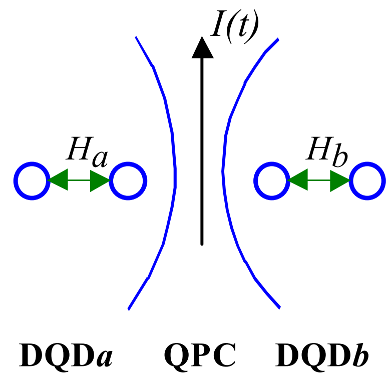
$$D_{max} \simeq 1 - \frac{C_{env}}{2C_{det}} \frac{1}{1 + C_{env}/C_{det}}$$

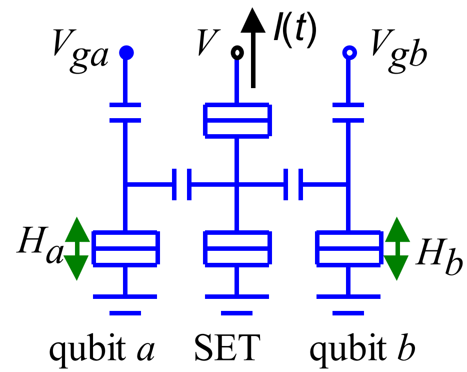
If qubit coupling to the environment is 100 times weaker than to the detector, then D_{max} = 99.5% and qubit fidelity 99.75%. (D = 0 without feedback.)

Two-qubit entanglement by measurement

Ruskov & A.K., 2002







Assume symmetric case: equal symmetric qubits, $\varepsilon_a=\varepsilon_b=0$, $H_a=H_b$, $\Omega_a=\Omega_b$, equal coupling, $C_a=C_b$, no direct interaction, u=0

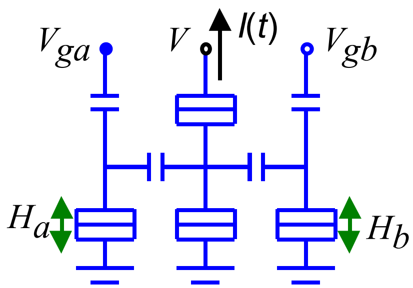
$$\hat{H} = \hat{H}_{QB} + \hat{H}_{DET} + \hat{H}_{INT}$$

$$\hat{H}_{QB} = \varepsilon_a(a_\downarrow^\dagger a_\downarrow - a_\uparrow^\dagger a_\uparrow) + H_a(a_\uparrow^\dagger a_\downarrow + a_\downarrow^\dagger a_\uparrow) + \varepsilon_b(b_\downarrow^\dagger b_\downarrow - b_\uparrow^\dagger b_\uparrow) + H_b(b_\uparrow^\dagger b_\downarrow + b_\downarrow^\dagger b_\uparrow)$$

$I(\uparrow\downarrow)=I(\downarrow\uparrow)$, states indistinguishable by measurement

|BellÚ = (|↑↓Ú - |↓↑Ú)/◊2 does not evolve

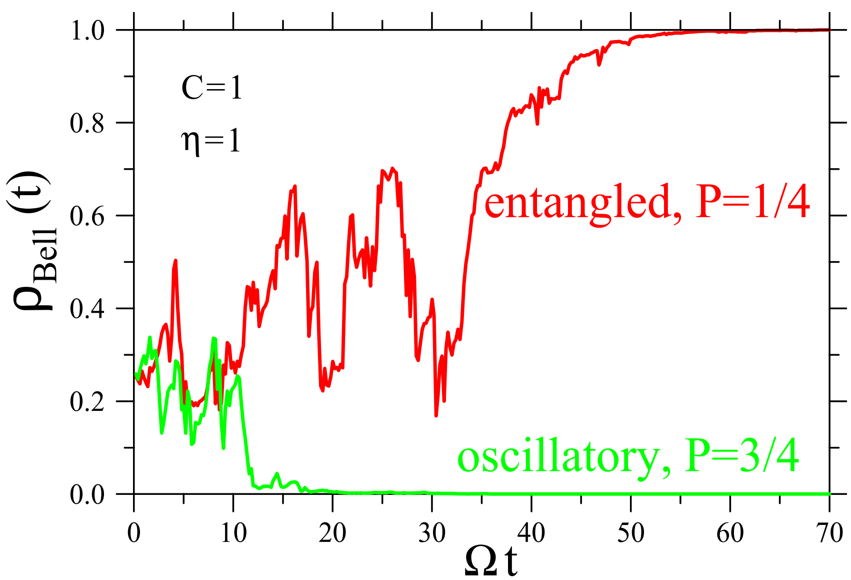
Collapse into |BellÚ state (spontaneous entanglement) with probability 1/4 starting from fully mixed state

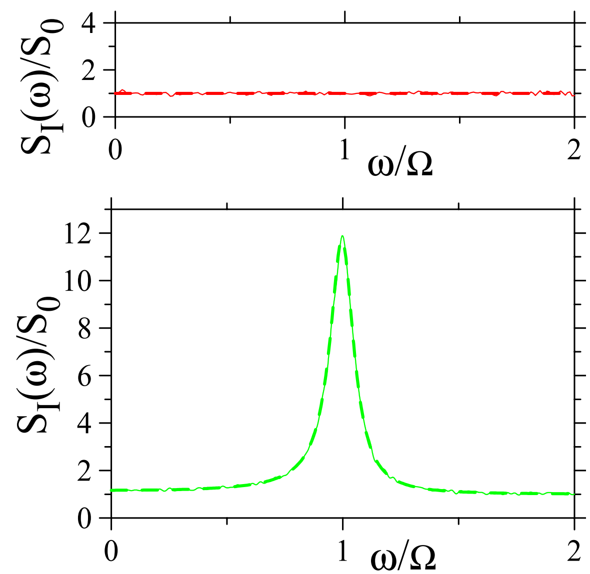


Continuous measurement

(detector is ON all the time)

Two scenarios of evolution from mixed state





Peak/noise = (32/3) η

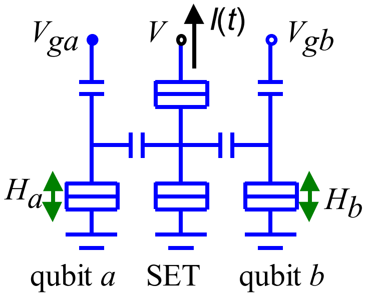
1) $\rho_{in} \rightarrow \rho_{Bell}$, probability $\rho_{11}{}^{B}(0)$ (1/4 for fully mixed state)

2) $\rho_{in} \rightarrow$ oscillatory state, probability $1 - \rho_{11}{}^{B}(0)$ (3/4 for fully mixed state)

spectral peak at Rabi frequency Ω=2*H*/Ñ, S_{peak}/S_0 § 32/3

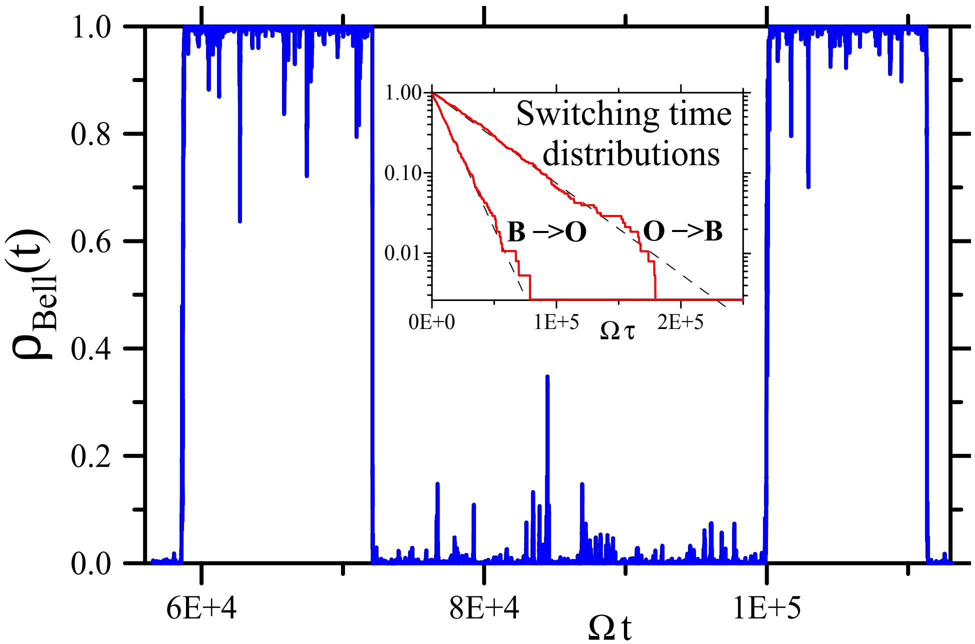
Entanglement due to common quantum noise; however, detector is needed

Ruskov & A.K., PRB (2003)



Small imperfections:

switching between entangled and oscillatory states



Different Rabi frequencies:

$$\Gamma_{B\to 0} = (\Delta\Omega)^2 / 2\Gamma$$

Different coupling:

$$\Gamma_{B\to 0} = (\Delta C / C)^2 \Gamma / 8$$

Environmental dephasing:

$$\Gamma_{B\to 0} = (\gamma_a + \gamma_b) / 2$$

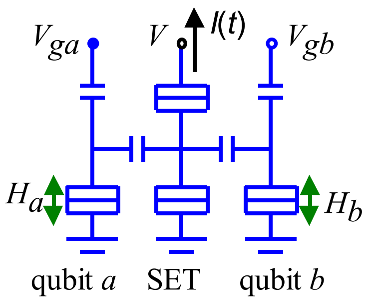
$$\Gamma_{O\to B} = \Gamma_{B\to O} / 3$$

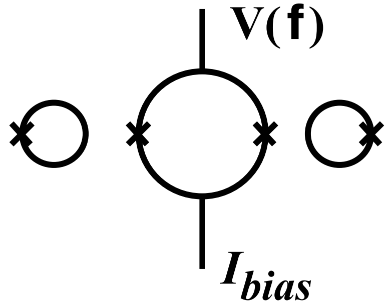
Using trivial feedback procedure (applying noise in undesirable state), we can keep two qubits entangled

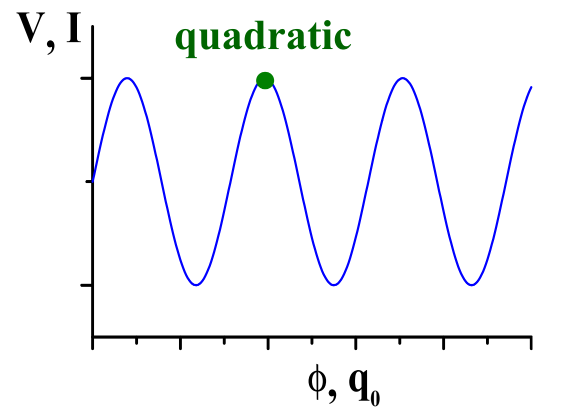
Detector may be nonideal (just 32η/3 peak), so SET is OK

Quadratic Quantum Detection

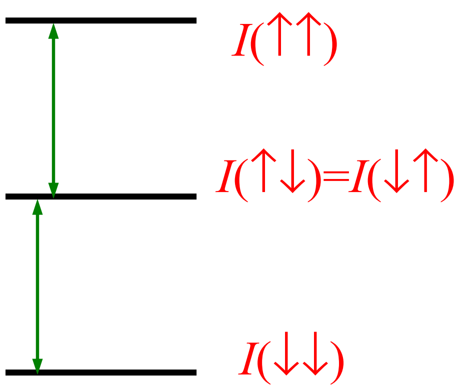
Mao, Averin, Ruskov, A.K., 2003



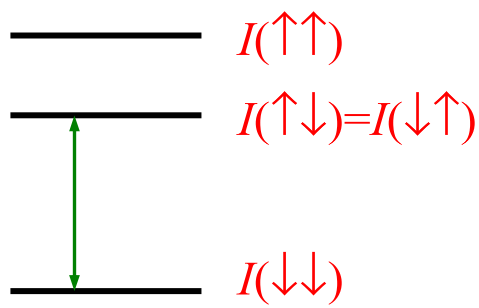




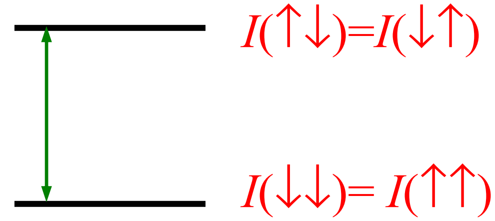
Linear detector



Noninear detector



Quadratic detector

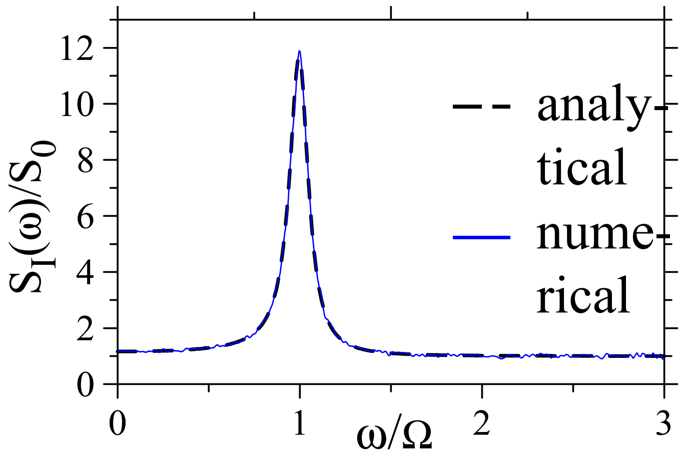


Quadratic detection is useful for quantum error correction (Averin-Fazio, 2002)

Two-qubit detection

(oscillatory subspace)

Linear detector



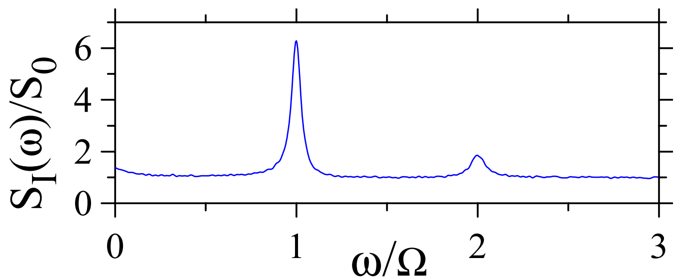
$$S_I(\omega) = S_0 + \frac{8}{3}\frac{\Omega^2(\Delta I)^2\Gamma}{(\omega^2-\Omega^2)^2+\Gamma^2\omega^2}$$

$$\Gamma = \eta^{-1}(\Delta I)^2/4S_0,\ \Delta I = I_1 - I_{23} = I_{23} - I_4$$

Spectral peak at Ω, peak/noise = (32/3)η

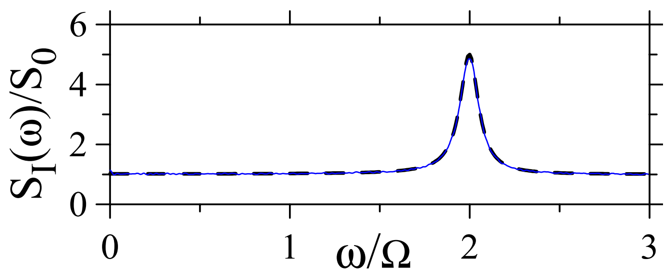
(Ω is the Rabi frequency)

Nonlinear detector



Extra spectral peaks at 2Ω and 0

Quadratic detector



$$S_I(\omega) = S_0 + \frac{4\Omega^2(\Delta I)^2\Gamma}{(\omega^2-4\Omega^2)^2+\Gamma^2\omega^2}$$

$$(\Delta I = I_{23} - I_{14},\ \ I_1 = I_4,\ \ I_2 = I_3)$$

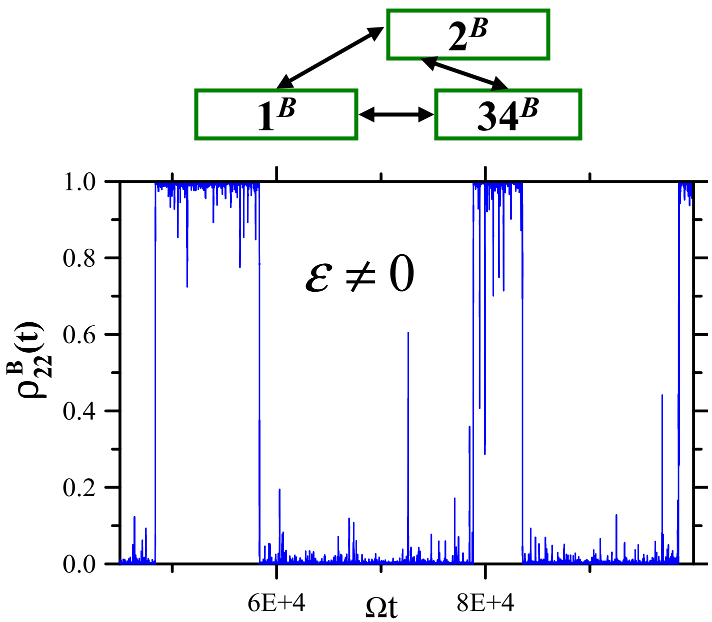
Peak only at 2Ω, peak/noise = 4η

Mao, Averin, Ruskov, A.K., 2003

Two-qubit quadratic detection: scenarios and switching

Three scenarios:
(distinguishable by average current)

1) collapse into |↑↓ - ↓↑Ú = |1Ú^B , current $I_{Æ∞}$, flat spectrum
2) collapse into |↑↑ - ↓↓Ú = |2Ú^B , current $I_{ÆÆ}$, flat spectrum
3) collapse into remaining subspace |34Ú^B , current $(I_{Æ∞}+I_{ÆÆ})/2$, spectral peak at 2Ω, peak/pedestal = 4η.



3) Slightly asymmetric qubits, **ε≠0**

$$\Gamma_{2B\to 34B} = 2\varepsilon^2\Gamma/\Omega^2$$

Switching between states due to imperfections

1) Slightly different Rabi frequencies, $\Delta\Omega=\Omega_1-\Omega_2$

$$\Gamma_{1B\to 2B} = \Gamma_{2B\to 1B} = (\Delta\Omega)^2/2\Gamma, \quad \Gamma = \eta^{-1}(\Delta I)^2/4S_0$$

$$S_I(\omega) = S_0 + \frac{(\Delta I)^2\Gamma}{(\Delta\Omega)^2}\frac{1}{1+\left[\omega\Gamma/(\Delta\Omega)^2\right]^2}$$

2) Slightly nonquadratic detector, $I_1 \neq I_4$

$$\Gamma_{2B\to 34B} = [(I_1 - I_4)/\Delta I]^2\Gamma/2$$

$$S_I(\omega) = S_0 + \frac{2}{3}\frac{4\Omega^2(\Delta I)^2\Gamma}{(\omega^2-4\Omega^2)^2+\Gamma^2\omega^2}$$

$$+\frac{8(\Delta I)^4}{27\Gamma(I_1-I_4)^2}\frac{1}{1+[4\omega(\Delta I)^2/3\Gamma(I_1-I_4)^2]^2}$$

Mao, Averin, Ruskov, Korotkov, 2003

Conclusions

- Bayesian formalism for continuous quantum measurement is simple (almost trivial); but still a new interesting subject in solid-state mesoscopics

- Several experimental predictions have been already made; however, many problems not studied yet

- No experiments yet (except one); hopefully, coming soon