NANO'04, St. Petersburg, June 2004

Quantum feedback control of solid-state qubits and their entanglement by measurement

Rusko Ruskov and Alexander Korotkov

University of California, Riverside

Outline:

Continuous measurement of a single qubit
- **Bayesian formalism**
- **Experimental predictions and proposals**

Quantum feedback of a solid-state qubit

Entanglement of two qubits by measurement

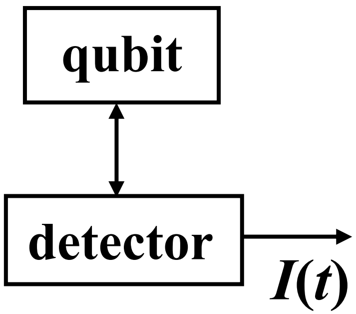
Recent developments of the Bayesian theory

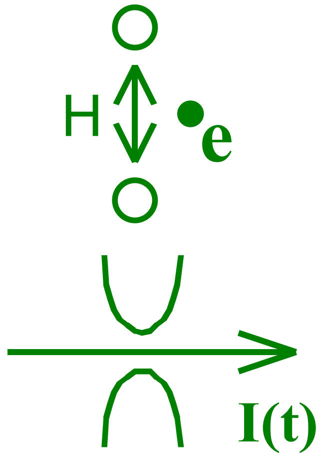
PRB 66, 041401(R) (2002), PRB 67, 231305(R) (2003), Review: cond-mat/0209629

Acknowledgement: Q. Zhang, D. Averin, W. Mao

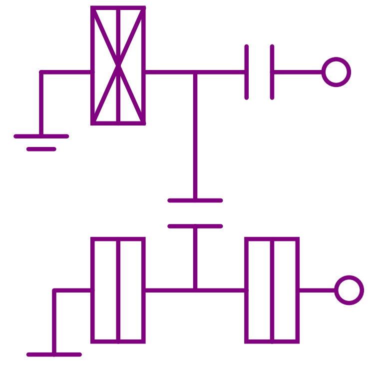
Support:

Examples of solid-state qubits and detectors

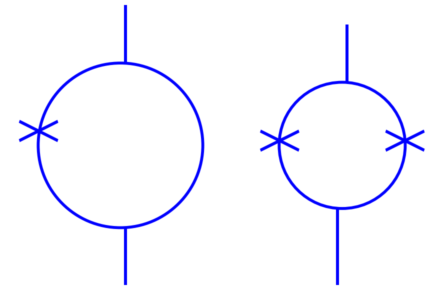




Double-quantum-qot and quantum point contact (QPC)



Cooper-pair box and single-electron transistor (SET)



Two SQUIDs

$H = H_{\mathrm{QB}} + H_{\mathrm{DET}} + H_{\mathrm{INT}}$

$H_{\mathrm{QB}} = (\varepsilon/2)(c_1^+c_1 - c_2^+c_2) + H(c_1^+c_2 + c_2^+c_1)$ ε - **asymmetry**, H – **tunneling**

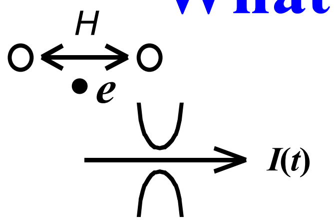
$\Omega = (4H^2+\varepsilon^2)^{1/2}/\hbar$ – **frequency of quantum coherent (Rabi) oscillations**

Two levels of average detector current: I_1 **for qubit state** $|1\rangle$, I_2 **for** $|2\rangle$

Response: $\Delta I = I_1 - I_2$ **Detector noise: white, spectral density** S_I

Alexander Korotkov — University of California, Riverside

What happens to a qubit state during measurement?



For simplicity (for a moment) ***H*=ε=0**, infinite barrier (frozen qubit), evolution due to measurement only

"Orthodox" answer

$$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \begin{matrix} \to \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \\ \to \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \end{matrix}$$

|1> or |2>, depending on the result

"Conventional" (decoherence) answer (Leggett, Zurek)

$$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \to \begin{pmatrix} \frac{1}{2} & \frac{\exp(-\Gamma t)}{2} \\ \frac{\exp(-\Gamma t)}{2} & \frac{1}{2} \end{pmatrix} \to \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

no measurement result! ensemble averaged

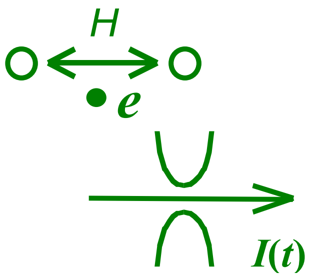
Orthodox and decoherence answers contradict each other!

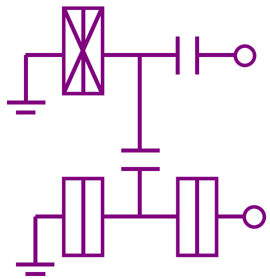
applicable for:	Single quantum systems	Continuous measurements
Orthodox	yes	no
Conventional (ensemble)	no	yes
Bayesian	yes	yes

Bayesian formalism describes gradual collapse of single quantum systems

Noisy detector output *I*(*t*) should be taken into account

Bayesian formalism for a single qubit





$$\hat{H}_{QB} = \frac{\varepsilon}{2}(c_1^\dagger c_1 - c_2^\dagger c_2) + H(c_1^\dagger c_2 + c_2^\dagger c_1)$$

$|1\rangle \to I_1, |2\rangle \to I_2$

$\Delta I = I_1 - I_2$, $I_0 = (I_1 + I_2)/2$, S_I – detector noise

$$\frac{d}{dt}\rho_{11} = -\frac{d}{dt}\rho_{22} = -2H\,\mathrm{Im}\,\rho_{12} + \rho_{11}\rho_{22}\frac{2\Delta I}{S_I}[I(t) - I_0]$$

$$\frac{d}{dt}\rho_{12} = i\varepsilon\rho_{12} + iH(\rho_{11} - \rho_{22}) + \rho_{12}(\rho_{11} - \rho_{22})\frac{\Delta I}{S_I}[I(t) - I_0] - \gamma\rho_{12}$$

A.K., 1998

$\gamma = \Gamma - (\Delta I)^2 / 4S_I$, Γ – ensemble decoherence

$\eta = 1 - \gamma/\Gamma = (\Delta I)^2 / 4S_I\Gamma$ – detector ideality (efficiency), $\eta \le 100\%$

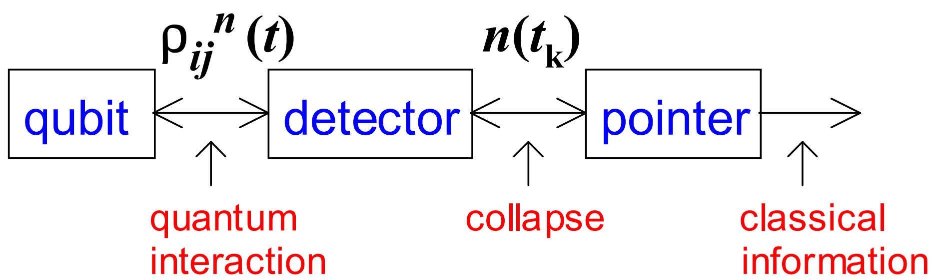
For simulations: $I(t) - I_0 \to (\rho_{22} - \rho_{11})\Delta I / 2 + \xi(t)$, $S_\xi = S_I$

Averaging over ξ(t) $\Rightarrow$ master equation

Ideal detector (η=1) does not decohere a single qubit; then the random evolution of the qubit *wavefunction* can be monitored

Similar formalisms developed earlier. Key words: Imprecise, weak, selective, or conditional measurements, POVM, Quantum trajectories, Quantum jumps, Restricted path integral, etc.

"Microscopic" derivation of the Bayesian formalism



Schrödinger evolution of "qubit + detector"
for a low-T QPC as a detector (Gurvitz, 1997)

$$\frac{d}{dt}\rho_{11}^{n} = -\frac{I_1}{e}\rho_{11}^{n} + \frac{I_1}{e}\rho_{11}^{n-1} - 2\frac{H}{\hbar}\,\mathrm{Im}\,\rho_{12}^{n}$$

$$\frac{d}{dt}\rho_{22}^{n} = -\frac{I_2}{e}\rho_{22}^{n} + \frac{I_2}{e}\rho_{22}^{n-1} + 2\frac{H}{\hbar}\,\mathrm{Im}\,\rho_{12}^{n}$$

$$\frac{d}{dt}\rho_{12}^{n} = i\frac{\varepsilon}{\hbar}\rho_{12}^{n} + i\frac{H}{\hbar}(\rho_{11}^{n} - \rho_{22}^{n}) - \frac{I_1+I_2}{2e}\rho_{12}^{n} + \frac{\sqrt{I_1 I_2}}{e}\rho_{12}^{n-1}$$

Detector collapse at $t = t_{\mathrm{k}}$
Particular n_{k} is chosen at t_{k}

$$P(n) = \rho_{11}^{n}(t_k) + \rho_{22}^{n}(t_k)$$

$$\rho_{ij}^{n}(t_k+0) = \delta_{n,nk}\,\rho_{ij}(t_k+0),$$

$$\rho_{ij}(t_k+0) = \frac{\rho_{ij}^{nk}(t_k-0)}{\rho_{11}^{nk}(t_k-0) + \rho_{22}^{nk}(t_k-0)}$$

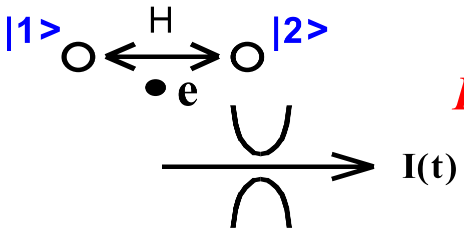
If $H = \varepsilon = 0$,
it leads to

$$\rho_{11}(t) = \frac{\rho_{11}(0)P_1(n)}{\rho_{11}(0)P_1(n) + \rho_{22}(0)P_2(n)}, \quad \rho_{22}(t) = \frac{\rho_{22}(0)P_2(n)}{\rho_{11}(0)P_1(n) + \rho_{22}(0)P_2(n)}$$

$$\rho_{12}(t) = \rho_{12}(0)\frac{[\rho_{11}(t)\rho_{22}(t)]^{1/2}}{[\rho_{11}(0)\rho_{22}(0)]^{1/2}}, \quad P_i(n) = \frac{(I_i t/e)^n}{n!}\exp(-I_i t/e),$$

which are exactly quantum Bayes formulas

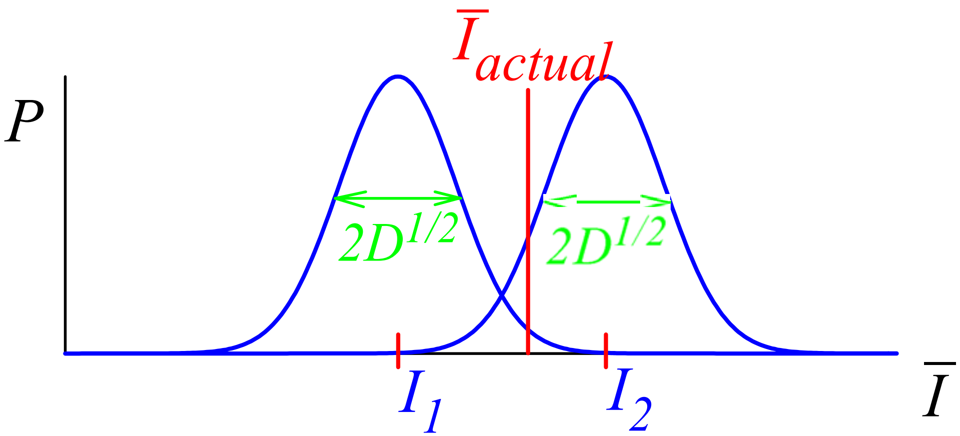
"Quantum Bayes theorem" (ideal detector assumed)



$H = \varepsilon = 0$ (frozen qubit)

Initial state: $\begin{pmatrix} \rho_{11}(0) & \rho_{12}(0) \\ \rho_{21}(0) & \rho_{22}(0) \end{pmatrix}$

Measurement (during time τ):



$$\bar{I} \equiv \frac{1}{\tau}\int_0^\tau I(t)\,dt$$

$$P(\bar{I},\tau) = \rho_{11}(0)\,P_1(\bar{I},\tau) + \rho_{22}(0)\,P_2(\bar{I},\tau)$$

$$P_i(\bar{I},\tau) = \frac{1}{\sqrt{2\pi D}}\exp[-(\bar{I}-I_i)^2/2D],$$

$$D = S_I/2\tau, \quad |I_1 - I_2| \ll I_i, \quad \tau \gg S_I/I_i^2$$

After the measurement during time τ, the probabilities can be updated using the standard Bayes formula:

$$P(B_i|A) = \frac{P(B_i)P(A|B_i)}{\sum_k P(B_k)P(A|B_k)}$$

Quantum Bayes formulas:

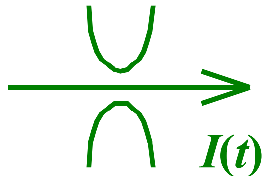
$$\rho_{11}(\tau) = \frac{\rho_{11}(0)\exp[-(\bar{I}-I_1)^2/2D]}{\rho_{11}(0)\exp[-(\bar{I}-I_1)^2/2D] + \rho_{22}(0)\exp[-(\bar{I}-I_2)^2/2D]}$$

$$\frac{\rho_{12}(\tau)}{[\rho_{12}(\tau)\,\rho_{22}(\tau)]^{1/2}} = \frac{\rho_{12}(0)}{[\rho_{12}(0)\,\rho_{22}(0)]^{1/2}}, \qquad \rho_{22}(\tau) = 1 - \rho_{11}(\tau)$$

Ideality of realistic solid-state detectors

(ideal detector does not cause single qubit decoherence)

1. Quantum point contact



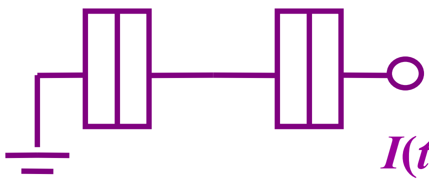
Theoretically, **ideal quantum detector, η=1**

A.K., 1998 (Gurvitz, 1997; Aleiner et al., 1997)

Experimentally, η > 80%

(using Buks et al., 1998)

2. SET-transistor



Very non-ideal in usual operation regime, η ‹‹ 1

Shnirman-Schőn, 1998; A.K., 2000, Devoret-Schoelkopf, 2000

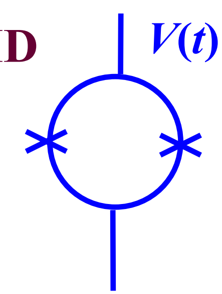
However, reaches ideality, η = 1 if:

- in deep cotunneling regime **(Averin, 2000, van den Brink, 2000)**
- S-SET, using supercurrent **(Zorin, 1996)**
- S-SET, double-JQP peak **(Clerk et al., 2002)**

??? S-SET, usual JQP **(Johansson et al.)**, onset of QP branch **(?)**

- resonant-tunneling SET, low bias **(Averin, 2000)**

3. SQUID



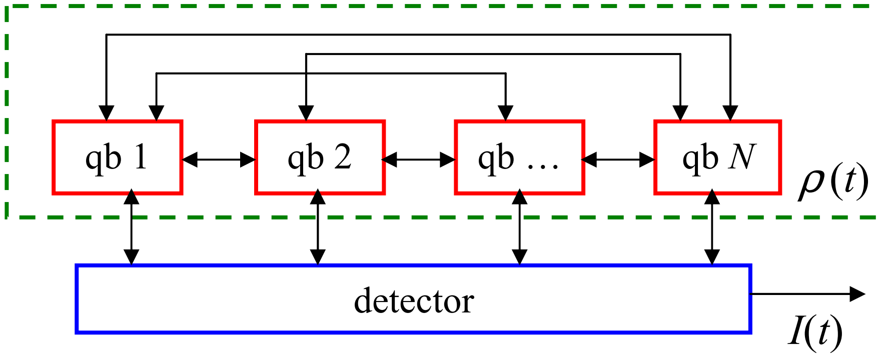
Can reach ideality, η = 1

(Danilov-Likharev-Zorin, 1983; Averin, 2000)

4. FET ?? HEMT ??

ballistic FET/HEMT ??

Bayesian formalism for N entangled qubits measured by one detector



Up to 2^N levels of current

$$\frac{d}{dt}\rho_{ij} = \frac{-i}{\hbar}[\hat{H}_{qb},\rho]_{ij} + \rho_{ij}\frac{1}{S}\sum_k \rho_{kk}\Big[\big(I(t)-\frac{I_k+I_i}{2}\big)(I_i-I_k) + \big(I(t)-\frac{I_k+I_j}{2}\big)(I_j-I_k)\Big] - \gamma_{ij}\rho_{ij}$$

(Stratonovich form)

$$\gamma_{ij} = (\eta^{-1}-1)(I_i-I_j)^2/4S_I \qquad I(t) = \sum_i \rho_{ii}(t) I_i + \xi(t)$$

Averaging over ξ(t) ⇒ master equation

No measurement-induced dephasing between states $|i\rangle$ and $|j\rangle$ if $I_i = I_j$!

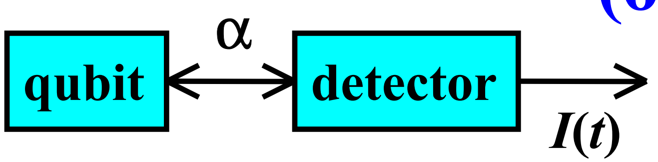
A.K., PRA 65 (2002), PRB 67 (2003)

Some experimental predictions and proposals

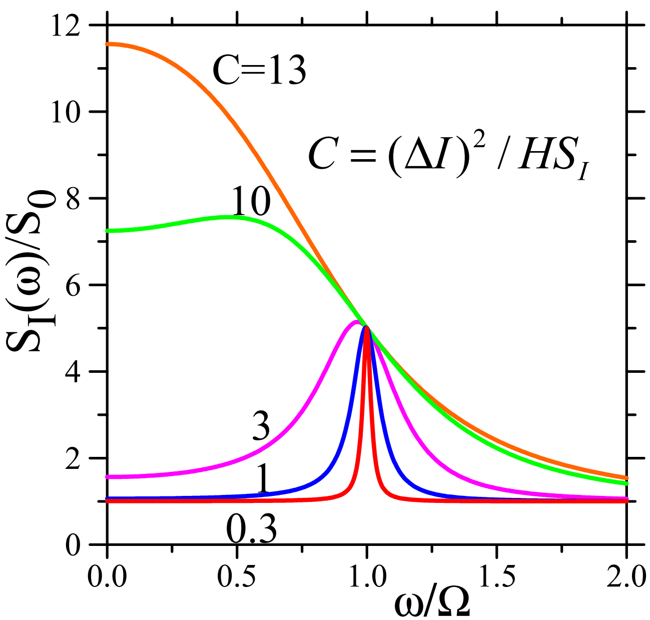
- Direct Bayesian experiments (1998)
- Measured spectral density of Rabi oscillations (1999, 2000, 2002)
- Bell-type correlation experiment (2000)
- Quantum feedback control of a qubit (2001, 2004)
- Entanglement by measurement (2002)
- Measurement and entanglement by a quadratic detector (2003)
- Squeezing of a nanomechanical resonator (2004)

A. Korotkov, R. Ruskov, D. Averin, W. Mao, Q. Zhang, K. Schwab

Measured spectrum of Rabi oscillations (or spin precession)



What is the spectral density $S_I(\omega)$ of detector current?



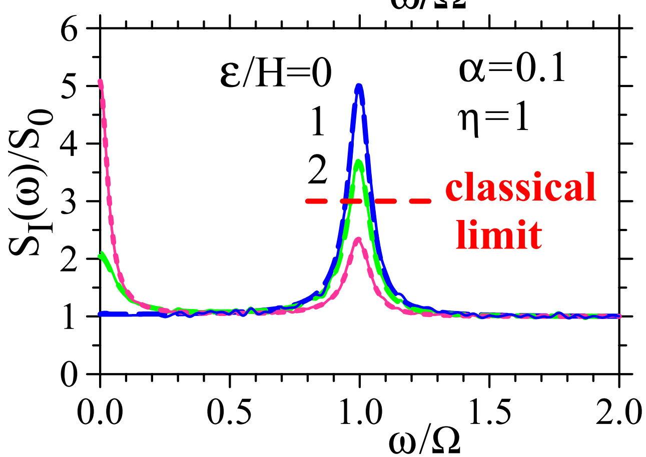
Assume classical output, eV » $\hbar\Omega$

$$\varepsilon = 0, \quad \Gamma = \eta^{-1}(\Delta I)^2 / 4S_0$$

$$S_I(\omega) = S_0 + \frac{\Omega^2 (\Delta I)^2 \Gamma}{(\omega^2 - \Omega^2)^2 + \Gamma^2 \omega^2}$$

Spectral peak can be seen, but peak-to-pedestal ratio ≤ 4η ≤ 4

(result can be obtained using various methods, not only Bayesian method)



Weak coupling, $\alpha = C/8$ « 1

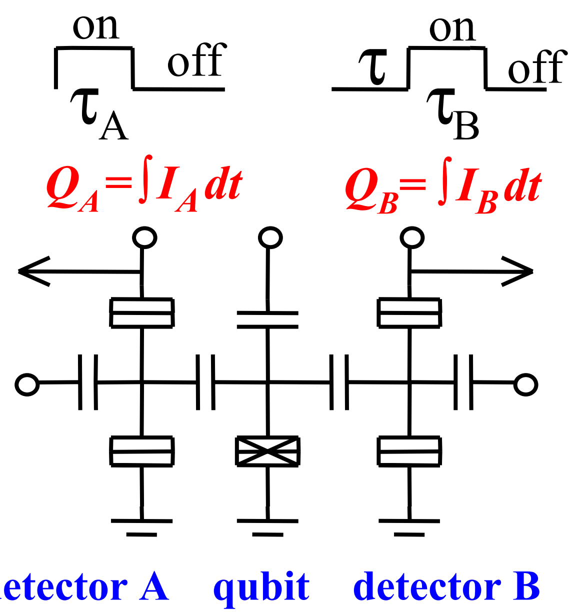
$$S_I(\omega) = S_0 + \frac{\eta S_0 \varepsilon^2 / H^2}{1 + (\omega\hbar^2\Omega^2 / 4H^2\Gamma)^2} + \frac{4\eta S_0 (1 + \varepsilon^2 / 2H^2)^{-1}}{1 + [(\omega - \Omega)\Gamma(1 - 2H^2/\hbar^2\Omega^2)]^2}$$

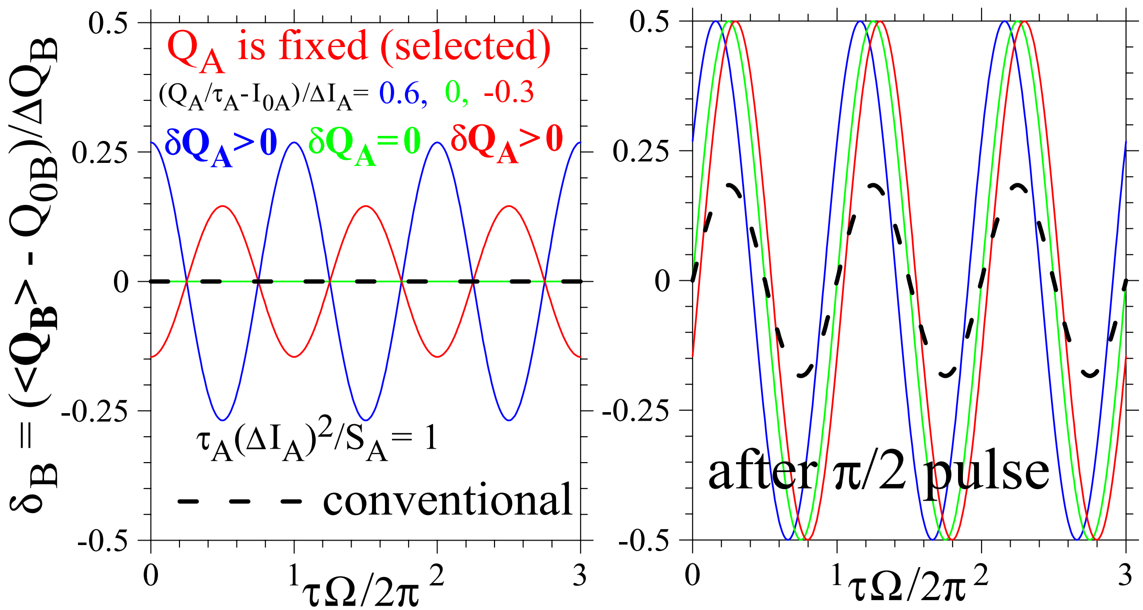
A.K., LT'99
A.K.-Averin, 2000
A.K., 2000
Averin, 2000
Goan-Milburn, 2001
Makhlin et al., 2001
Balatsky-Martin, 2001
Ruskov-A.K., 2002
Mozyrsky et al., 2002
Balatsky et al., 2002
Bulaevskii et al., 2002
Shnirman et al., 2002
Bulaevskii-Ortiz, 2003
Shnirman et al., 2003

Contrary:
Stace-Barrett, 2003

Bell-type correlation experiment

A.K., 2000





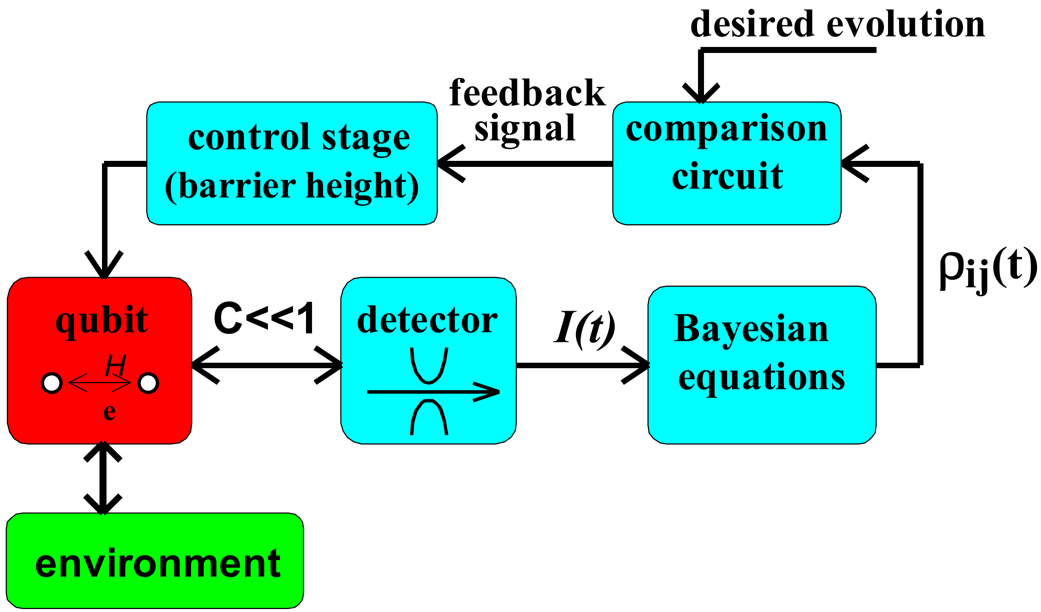
Idea: two consecutive finite-time (imprecise) measurements of a qubit by two detectors; probability distribution $\mathbf{P}(Q_A, Q_B, \tau)$ shows the effect of the first measurement on the qubit state.

Proves that the qubit remains in a pure state during the measurement (for $\eta=1$).

Advantage: no need to record noisy detector output with GHz bandwidth; instead, we use two detectors and fast ON/OFF switching.

Quantum feedback control of a solid-state qubit

Ruskov & A.K., 2001



Goal: maintain desired phase of Rabi oscillations in spite of environmental dephasing (keep qubit "fresh")

Idea: monitor the Rabi phase ϕ by continuous measurement and apply feedback control of the qubit barrier height, $\Delta H_{FB}/H = -F\times\Delta\phi$

To monitor phase ϕ we plug detector output $I(t)$ into Bayesian equations

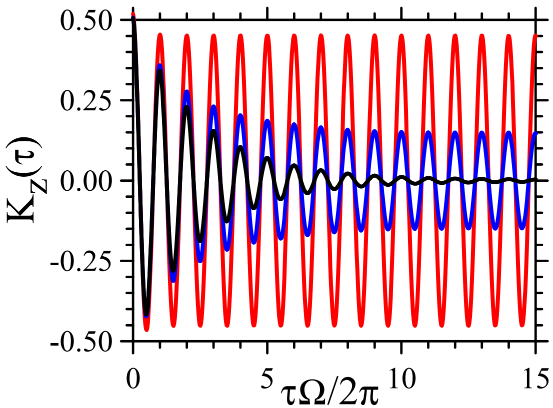
Quantum feedback in optics is discussed since 1993 (Wiseman-Milburn), recently first experiments (Armen et al., 2002; Geremia et al., 2004).

Performance of quantum feedback

(no extra environment)

Qubit correlation function

C=1, η=1, F=0, 0.05, 0.5



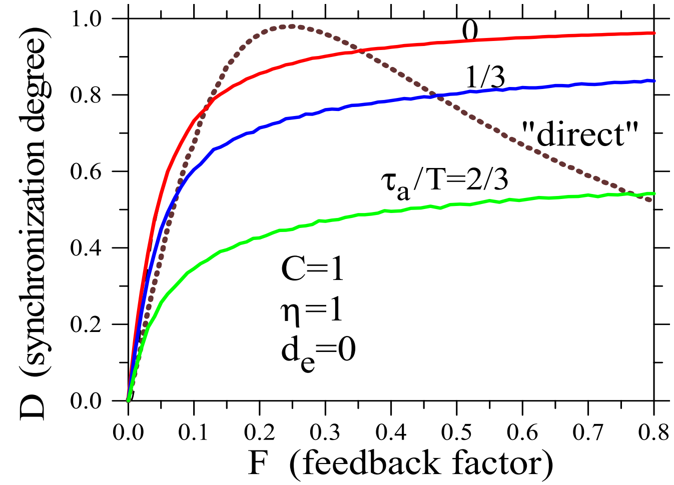
$$K_z(\tau)=\frac{\cos\ \Omega t}{2}\ \exp\left[\frac{C}{16F}(e^{-2FH\tau/\hbar}-1)\right]$$

(for weak coupling and good fidelity)

Detector current correlation function

$$K_I(\tau)=\frac{(\Delta I)^2}{4}\frac{\cos\ \Omega t}{2}\ (1+e^{-2FH\tau/\hbar})$$
$$\times\exp\left[\frac{C}{16F}(e^{-2FH\tau/\hbar}-1)\right]+\frac{S_I}{2}\delta(\tau)$$

Fidelity (synchronization degree)



$C=\hbar(\Delta I)^2/S_I H$ – coupling

τ_a^{-1} – available bandwidth

F – feedback strength

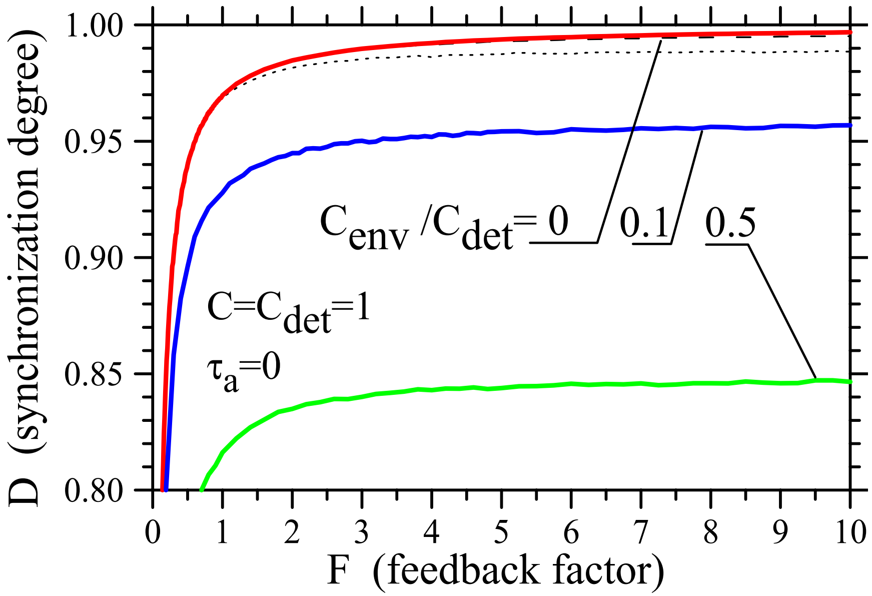
$$D=2\langle \mathrm{Tr}\rho\rho_{desir}\rangle-1$$

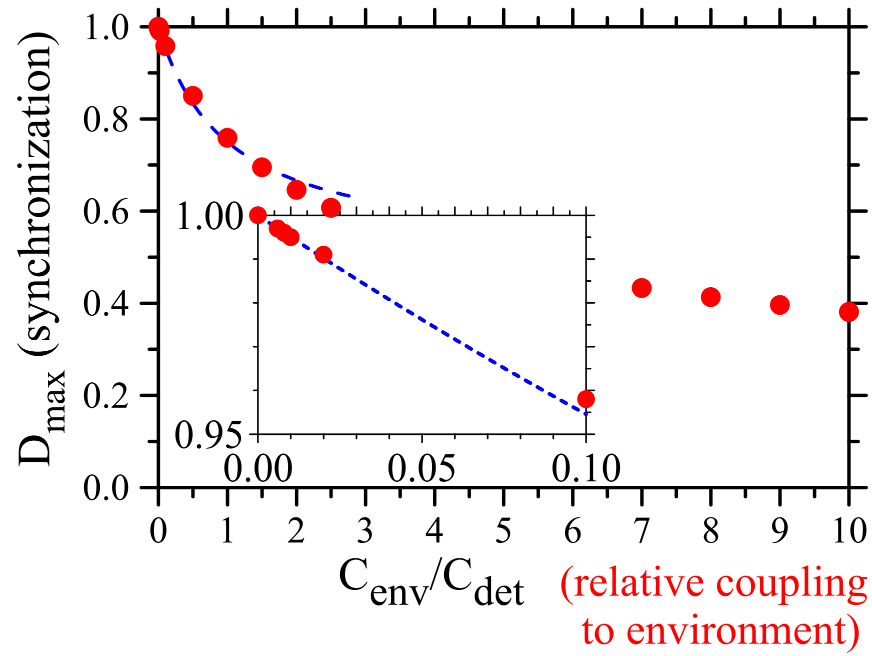
For ideal detector and wide bandwidth, fidelity can be arbitrary close to 100%

$$D=\exp(-C/32F)$$

Ruskov & Korotkov, PRB 66, 041401(R) (2002)

Suppression of environment-induced decoherence by quantum feedback



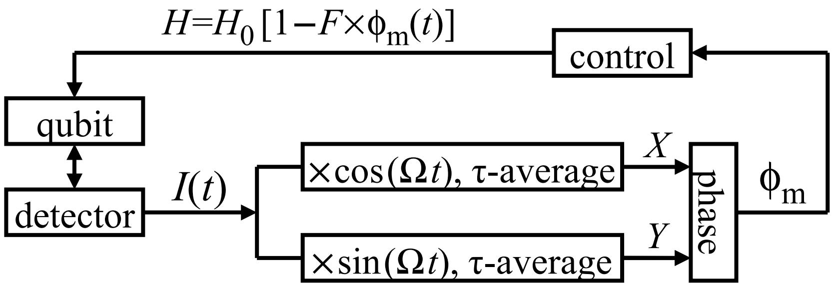


$$D_{max} = 1 - \frac{C_{env}}{2C_{det}} \quad \text{for} \quad C_{env} \ll C_{det}$$

If qubit coupling to the environment is 100 times weaker than to the detector, then D_{max} = 99.5% and qubit fidelity 99.75%. (D = 0 without feedback.)

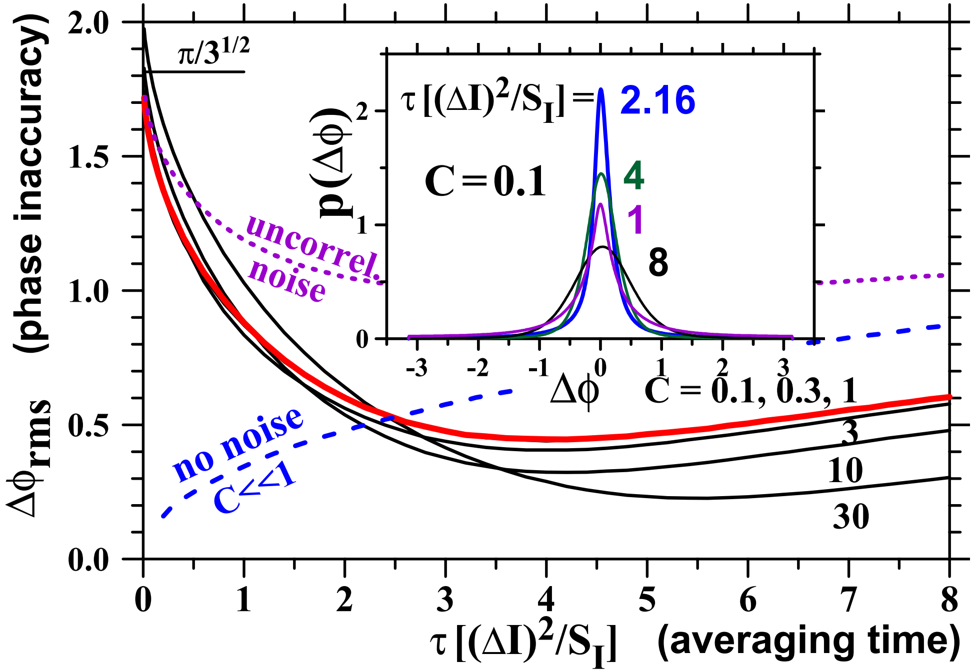
Simple quantum feedback of a qubit

A.K., 2004



Idea: two quadrature components of the detector current *I*(*t*) carry information on the phase of qubit Rabi oscillations

Quadratures are extracted by mixing *I*(*t*) with a local oscillator or using a tank circuit

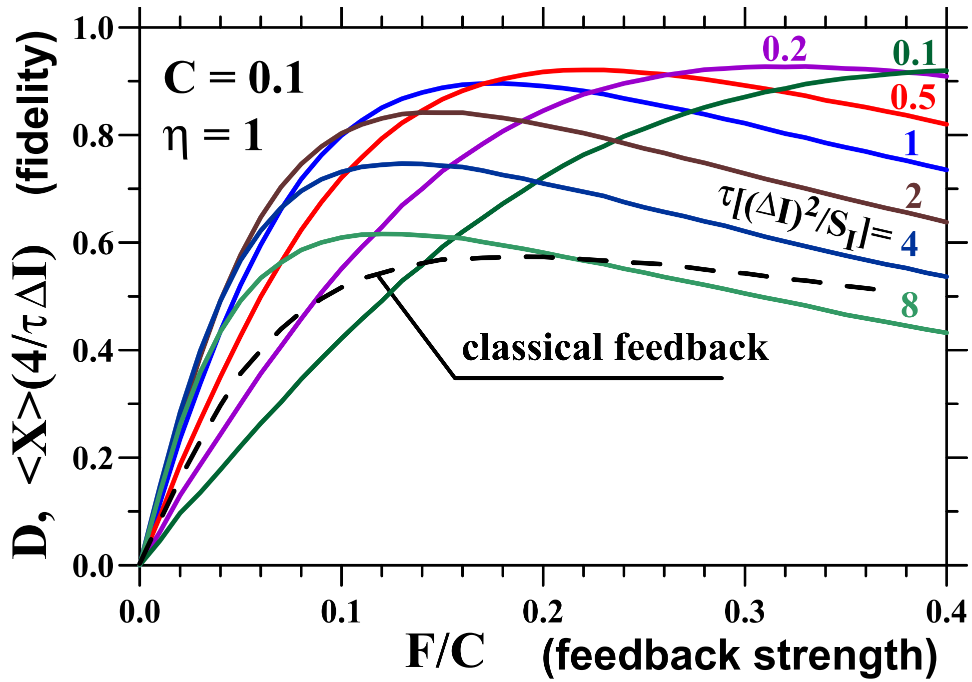


Surprisingly, the phase calculated from two quadratures is very close to actual phase of Rabi oscillations

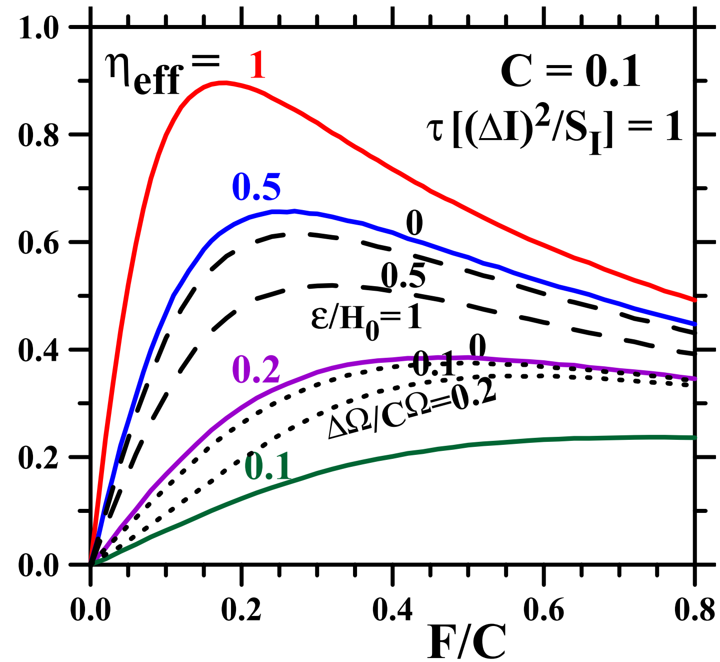
(noise improves the monitoring accuracy, real evolution follows the observed behavior)

C- dimensionless coupling
τ – averaging time
Δφ - inaccuracy of phase monitoring

Efficiency of the simple quantum feedback



fidelity for several values of coupling



fidelity for nonideal detectors

The price for simplicity is the limitation of fidelity by about 90%

Experimental verification by positive average in-phase quadrature *X*

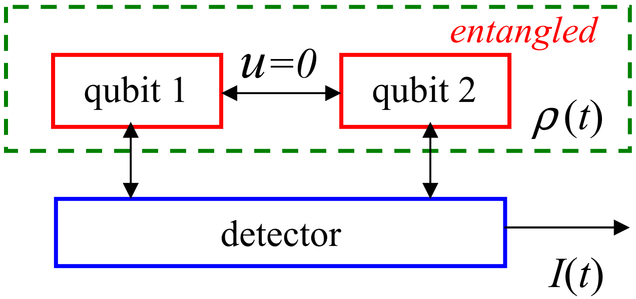
$\langle X\rangle = D\tau\Delta I/4$

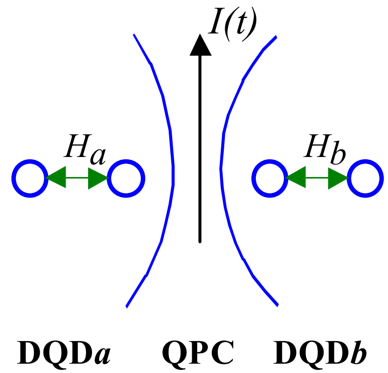
Works well for nonideal detectors, $D = 0.25$ for $\eta = 0.1$

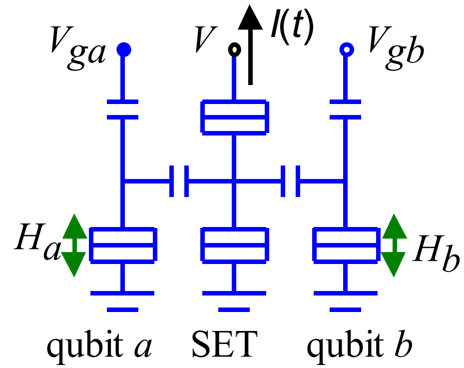
Relatively simple experiment!

Two-qubit entanglement by measurement

Ruskov & A.K., 2002







Assume symmetric case: equal symmetric qubits, $\varepsilon_a=\varepsilon_b=0$, $H_a=H_b$, $\Omega_a=\Omega_b$,
equal coupling, $C_a=C_b$, no direct interaction, $u=0$

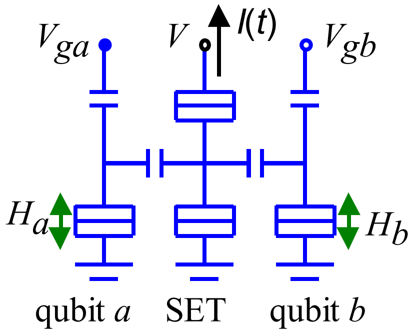
$$\hat{H} = \hat{H}_{QB} + \hat{H}_{DET} + \hat{H}_{INT}$$

$$\hat{H}_{QB} = \varepsilon_a(a_\downarrow^\dagger a_\downarrow - a_\uparrow^\dagger a_\uparrow) + H_a(a_\uparrow^\dagger a_\downarrow + a_\downarrow^\dagger a_\uparrow) + \varepsilon_b(b_\downarrow^\dagger b_\downarrow - b_\uparrow^\dagger b_\uparrow) + H_b(b_\uparrow^\dagger b_\downarrow + b_\downarrow^\dagger b_\uparrow)$$

$I(\uparrow\downarrow)=I(\downarrow\uparrow)$, states indistinguishable by measurement

$|\text{Bell}\rangle = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$ does not evolve

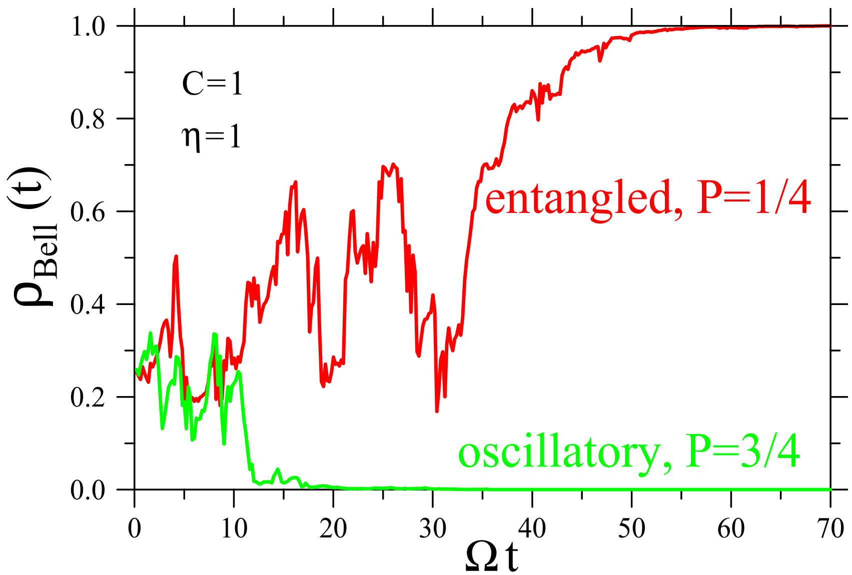
Collapse into |Bell⟩ state (spontaneous entanglement) with probability 1/4 starting from fully mixed state

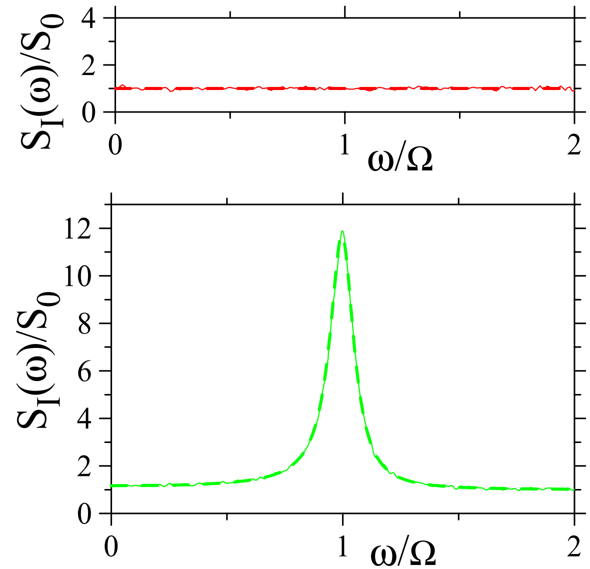


Continuous measurement

(detector is ON all the time)

Two scenarios of evolution from mixed state





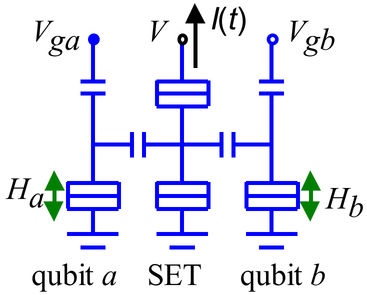
Peak/noise = (32/3) η

1) $\rho_{in} \rightarrow \rho_{Bell}$, probability $\rho_{11}^{B}(0)$ (1/4 for fully mixed state)

2) $\rho_{in} \rightarrow$ oscillatory state, probability $1-\rho_{11}^{B}(0)$ (3/4 for fully mixed state)
spectral peak at Rabi frequency Ω, $S_{peak}/S_0 \leq 32/3$

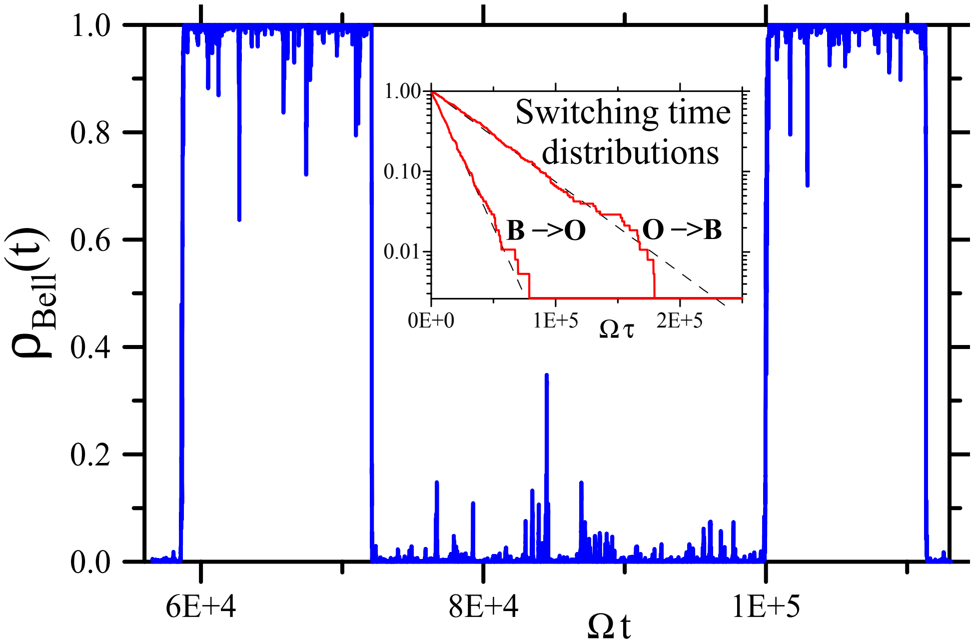
Entanglement due to common quantum noise; however, detector is needed

Ruskov & A.K., PRB (2003)



Small imperfections:

switching between entangled and oscillatory states



Different Rabi frequencies:

$$\Gamma_{B\to 0} = (\Delta\Omega)^2 / 2\Gamma$$

Different coupling:

$$\Gamma_{B\to 0} = (\Delta C / C)^2 \Gamma / 8$$

Environmental dephasing:

$$\Gamma_{B\to 0} = (\gamma_a + \gamma_b) / 2$$

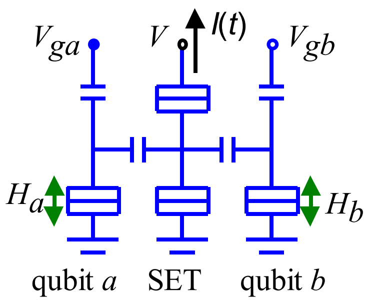
$$\Gamma_{O\to B} = \Gamma_{B\to O} / 3$$

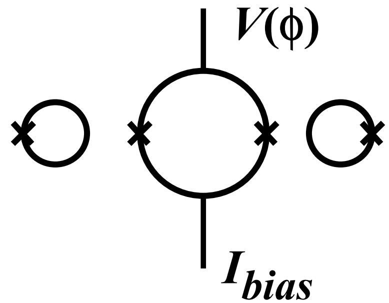
Using trivial feedback procedure (applying noise in undesirable state), we can keep two qubits entangled

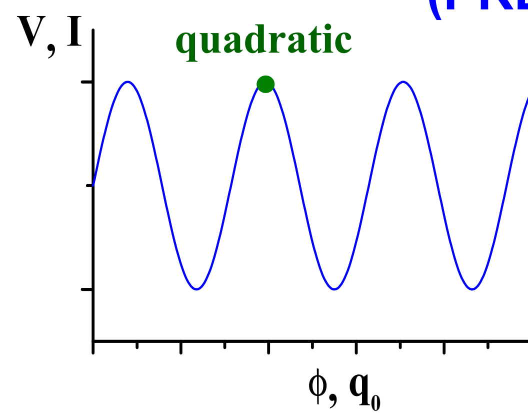
Detector may be nonideal (just 32η/3 peak), so SET is OK

Quadratic Quantum Detection

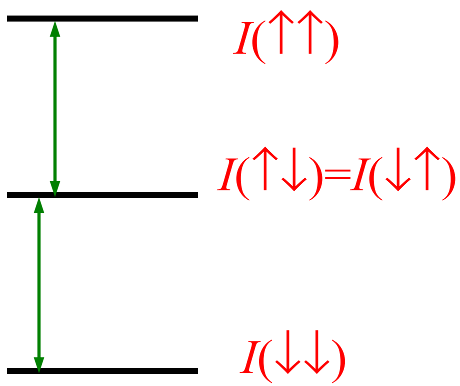
Mao, Averin, Ruskov, A.K., 2003
(PRL, 2004)



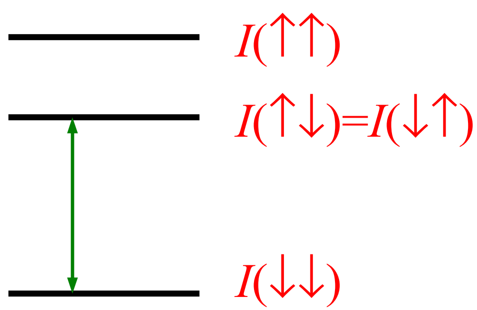




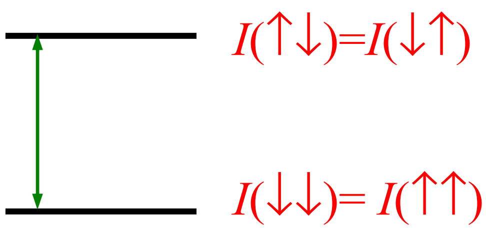
Linear detector



Noninear detector



Quadratic detector

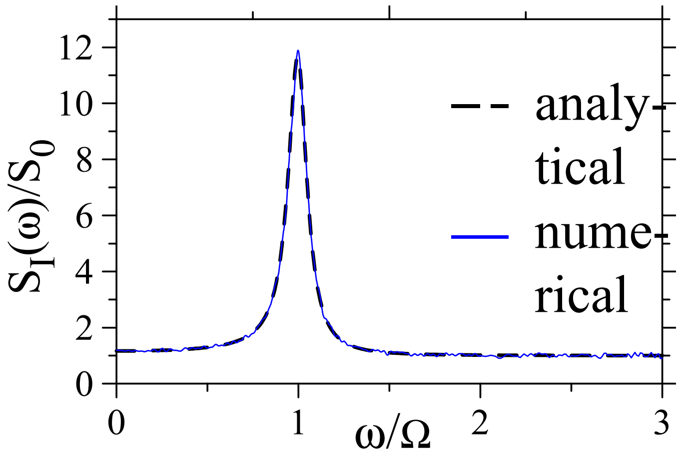


Quadratic detection is useful for quantum error correction (Averin-Fazio, 2002)

Two-qubit detection

(oscillatory subspace)

Linear detector



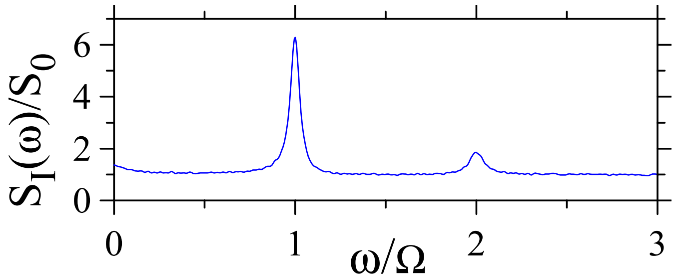
$$S_I(\omega) = S_0 + \frac{8}{3}\frac{\Omega^2(\Delta I)^2\Gamma}{(\omega^2-\Omega^2)^2+\Gamma^2\omega^2}$$

$$\Gamma = \eta^{-1}(\Delta I)^2/4S_0, \; \Delta I = I_1 - I_{23} = I_{23} - I_4$$

Spectral peak at Ω, peak/noise = (32/3)η

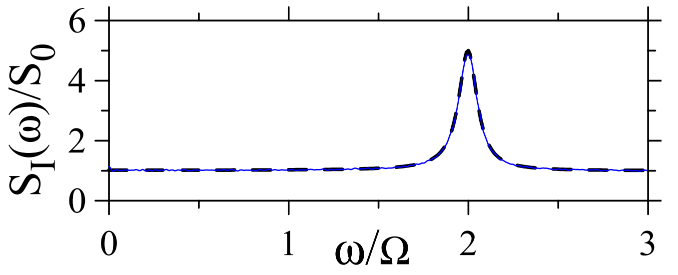
(Ω is the Rabi frequency)

Nonlinear detector



Extra spectral peaks at 2Ω and 0

Quadratic detector



$$S_I(\omega) = S_0 + \frac{4\Omega^2(\Delta I)^2\Gamma}{(\omega^2-4\Omega^2)^2+\Gamma^2\omega^2}$$

$$(\Delta I = I_{23} - I_{14}, \;\; I_1 = I_4, \;\; I_2 = I_3)$$

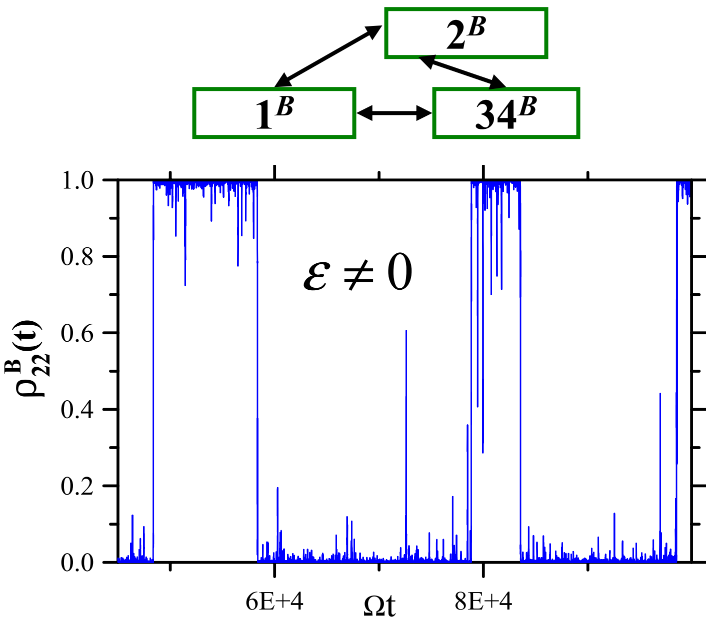
Peak only at 2Ω, peak/noise = 4η

Mao, Averin, Ruskov, A.K., 2003

Two-qubit quadratic detection: scenarios and switching

Three scenarios:
(distinguishable by average current)

1) collapse into $|\uparrow\downarrow - \downarrow\uparrow\rangle = |1\rangle^B$, current $I_{\uparrow\downarrow}$, flat spectrum
2) collapse into $|\uparrow\uparrow - \downarrow\downarrow\rangle = |2\rangle^B$, current $I_{\uparrow\uparrow}$, flat spectrum
3) collapse into remaining subspace $|34\rangle^B$, current $(I_{\uparrow\downarrow}+I_{\uparrow\uparrow})/2$, spectral peak at 2Ω, peak/pedestal = 4η.



3) Slightly asymmetric qubits, $\varepsilon \neq 0$

$$\Gamma_{2B\to 34B} = 2\varepsilon^2\Gamma/\Omega^2$$

Switching between states due to imperfections

1) Slightly different Rabi frequencies, $\Delta\Omega = \Omega_1 - \Omega_2$

$$\Gamma_{1B\to 2B} = \Gamma_{2B\to 1B} = (\Delta\Omega)^2/2\Gamma, \quad \Gamma = \eta^{-1}(\Delta I)^2/4S_0$$

$$S_I(\omega) = S_0 + \frac{(\Delta I)^2\Gamma}{(\Delta\Omega)^2}\frac{1}{1+\left[\omega\Gamma/(\Delta\Omega)^2\right]^2}$$

2) Slightly nonquadratic detector, $I_1 \neq I_4$

$$\Gamma_{2B\to 34B} = [(I_1 - I_4)/\Delta I]^2\Gamma/2$$

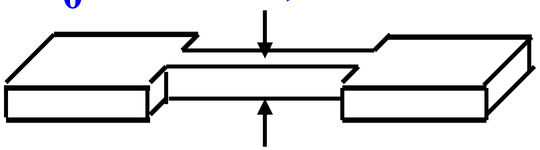
$$S_I(\omega) = S_0 + \frac{2}{3}\frac{4\Omega^2(\Delta I)^2\Gamma}{(\omega^2-4\Omega^2)^2+\Gamma^2\omega^2} + \frac{8(\Delta I)^4}{27\Gamma(I_1-I_4)^2}\frac{1}{1+[4\omega(\Delta I)^2/3\Gamma(I_1-I_4)^2]^2}$$

Mao, Averin, Ruskov, Korotkov, 2003

Bayesian approach to continuous position measurement of a nanomechanical resonator

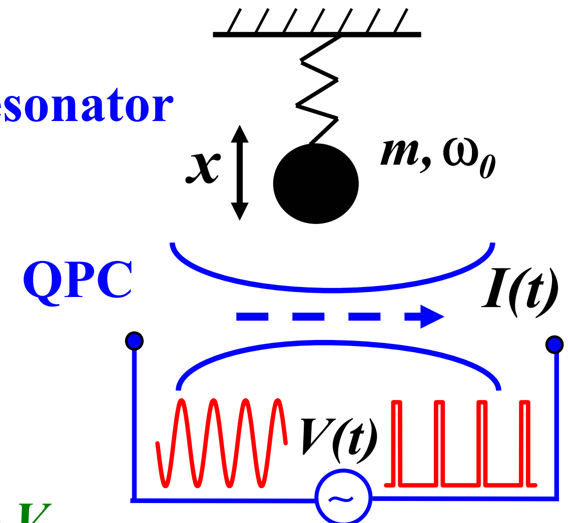
Ruskov- Korotkov, 2004

$\omega_0 \sim 1$ GHz , $T \sim 20$ mK , quantum behavior



$$\hat{H}_0 = \frac{\hat{p}^2}{2m} + \frac{m\omega_0^2 \hat{x}^2}{2}$$

resonator



$$\hat{H}_{DET} = \sum_l E_l a_l^\dagger a_l + \sum_r E_r a_r^\dagger a_r + \sum_{l,r} (M a_l^\dagger a_r + H.c.)$$

$$\hat{H}_{INT} = \sum_{l,r} (\Delta M \, \hat{x} \, a_l^\dagger a_r + H.c.)$$

Detector noise

$$S_X = S_0 \equiv 2eI_0$$

Coupling

$$C \equiv \frac{\hbar k^2}{S_0 m \omega_0^2} \propto \frac{T^{osc}}{\tau^{meas}}$$

$$I_X = 2\pi \left(M + \Delta M \, x\right)^2 \rho_l \rho_r e^2 \frac{V}{\hbar} = I_0 + k\,x$$

Evolution equation (Stratonovich form)

$$\frac{d}{dt}\rho(x,x') = \frac{-i}{\hbar}[\hat{H}_0, \rho] + \rho(x,x')\frac{1}{S_0}\left\{ \left(I(t) - I_0\right) k(x + x' - 2\bar{x}) - k^2 \left(\frac{x^2 + x'^2}{2} - \overline{x^2} \right) \right\}$$

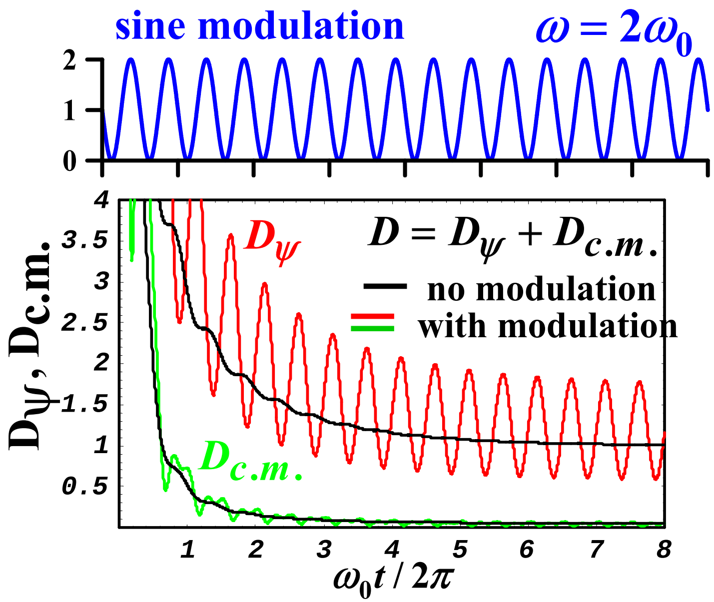
Cooling by feedback $\quad \hat{H}^{fb} = -F\,\hat{x}, \quad F = -\gamma\left(m\omega_0 \bar{x} + \bar{p}\right)$

Formalism similar to A. Hopkins et al., 2003 & Doherty-Jacobs, 1999

QND squeezing of a nanoresonator by feedback

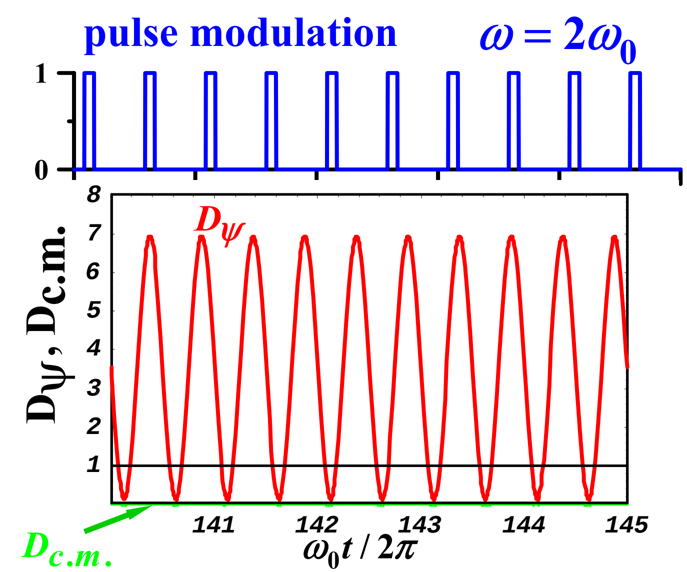
Constant voltage – no squeezing at $C \ll 1$ (Hopkins, Jacobs, Habib, Schwab, 2003)

We consider periodic $V(t)$ – squeezing possible! **(Ruskov-Korotkov, 2004)**

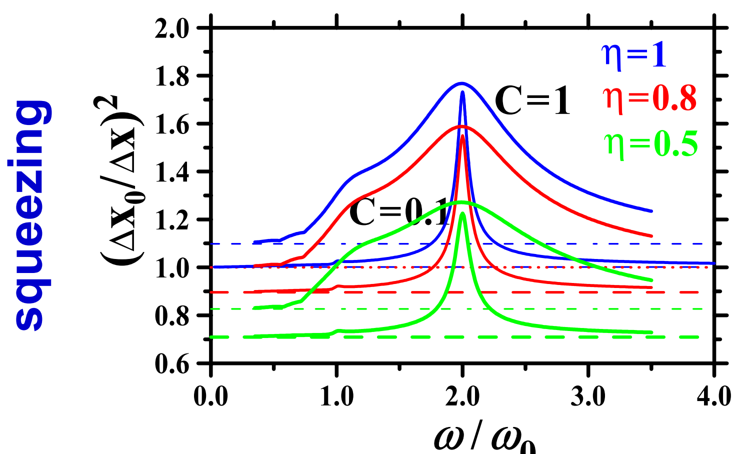


$C = 0.1$

$\dfrac{\gamma}{\omega_0} = 10$

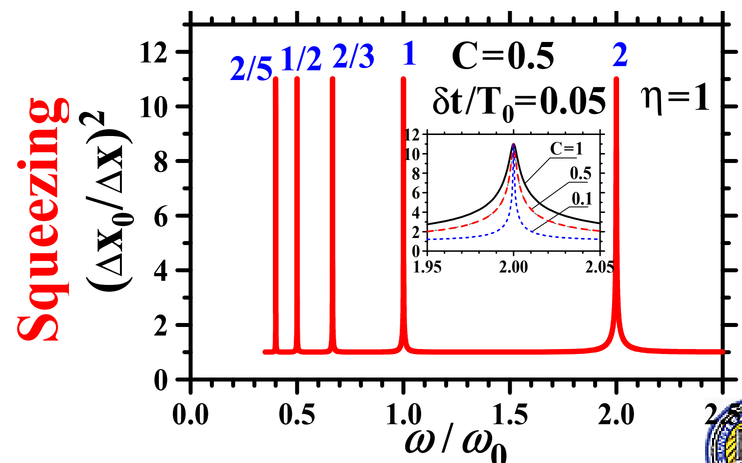


$D_{c.m.} \ll D_\psi$



Squeezing resonances

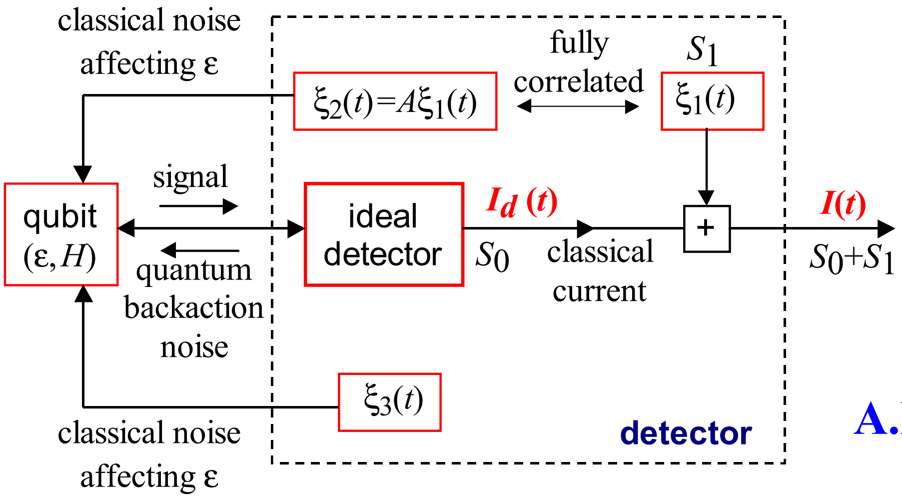
at $\omega = \dfrac{2\omega_0}{n}$



Conclusions

- Bayesian formalism for continuous quantum measurement is simple (almost trivial); but still a new interesting subject in solid-state mesoscopics
- Quantum feedback of a single qubit can keep the Rabi oscillations in a qubit for arbitrary long time; feedback can be realized in a simple way via quadratures
- Two qubits can be made fully entangled just by measuring them with an equally coupled detector
- Various experimental predictions have been made using the Bayesian formalism; the range of applications still expanding
- No experiments yet; hopefully, coming soon

Nonideal detectors with input-output noise correlation



$$K = \frac{AS_1 + \theta S_0}{\hbar S_I}, \quad S_I = S_0 + S_1$$

K – correlation between output and backaction noises

A.K., 2002

$$\frac{d}{dt}\rho_{11} = -\frac{d}{dt}\rho_{22} = -2H\,\mathrm{Im}\,\rho_{12} + \rho_{11}\rho_{22}\frac{2\Delta I}{S_I}[I(t) - I_0]$$

$$\frac{d}{dt}\rho_{12} = i\varepsilon\rho_{12} + iH(\rho_{11} - \rho_{22}) + \rho_{12}(\rho_{11} - \rho_{22})\frac{\Delta I}{S_I}[I(t) - I_0] + iK[I(t) - I_0] - \gamma_m\rho_{12}$$

Fundamental limits for ensemble decoherence

$$\Gamma = \gamma + (\Delta I)^2/4S_I, \quad \gamma \geq 0 \;\Rightarrow\; \Gamma \geq (\Delta I)^2/4S_I$$

$$\Gamma = \gamma + (\Delta I)^2/4S_I + K^2 S_I/4, \quad \gamma \geq 0 \;\Rightarrow\; \Gamma \geq (\Delta I)^2/4S_I + K^2 S_I/4$$

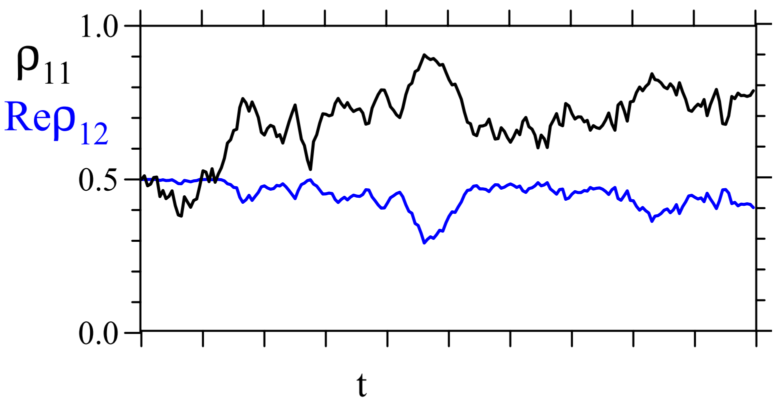
Translated into energy sensitivity: $(\epsilon_I\, \epsilon_{BA})^{1/2} \geq \hbar/2$ **or** $(\epsilon_I\, \epsilon_{BA} - \epsilon_{I,BA})^{1/2} \geq \hbar/2$

Direct Bayesian experiments

(A.K., 1998)

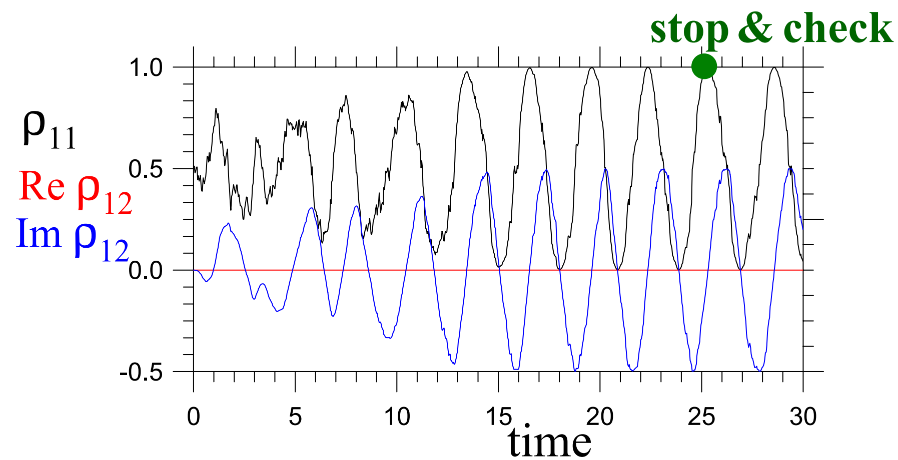
Idea: check the evolution of (almost pure!) qubit state given by Bayesian equations

Evolution from 1/2-alive to 1/3-alive Schrödinger cat



1. Prepare coherent state and make H=0.
2. Measure for a finite time t.
3. Check the predicted wavefunction (using evolution with $H\neq 0$ to get the state |1›.

Density matrix purification by measurement



1. Start with completely mixed state.
2. Measure and monitor the Rabi phase.
3. Stop evolution (make H=0) at state |1›.
4. Measure.

Difficulty: need to record noisy detector current $I(t)$, typical required bandwidth ~ 1-10 GHz.

Possible experimental confirmation?

(STM-ESR experiment similar to Manassen-1989)

APPLIED PHYSICS LETTERS VOLUME 80, NUMBER 3 21 JANUARY 2002

Electronic spin detection in molecules using scanning-tunneling-microscopy-assisted electron-spin resonance

C. Durkan[a] and M. E. Welland
Nanoscale Science Laboratory, Department of Engineering, University of Cambridge, Trumpington Street, Cambridge CB2 1PZ, United Kingdom

(Received 8 May 2001; accepted for publication 8 November 2001)

By combining the spatial resolution of a scanning-tunneling microscope (STM) with the electronic spin sensitivity of electron-spin resonance, we show that it is possible to detect the presence of localized spins on surfaces. The principle is that a STM is operated in a magnetic field, and the resulting component of the tunnel current at the Larmor (precession) frequency is measured. This component is nonzero whenever there is tunneling into or out of a paramagnetic entity. We have

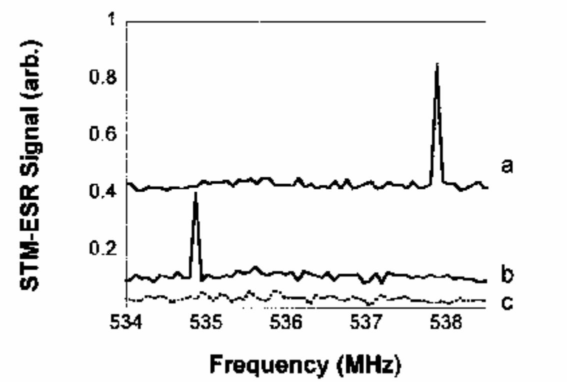


FIG. 3. STM-ESR spectra of (a), (b) two different areas (a few nm apart) of the molecule-covered sample and (c) bare HOPG. The graphs are shifted vertically for clarity.

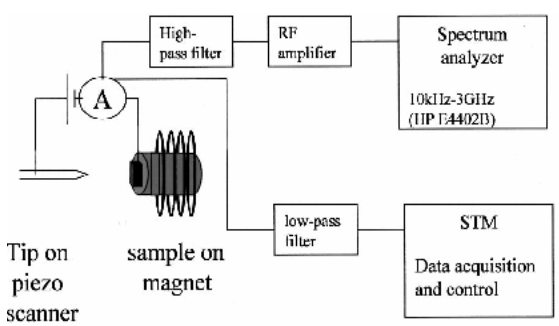


FIG. 1. Schematic of the electronics used in STM-ESR.

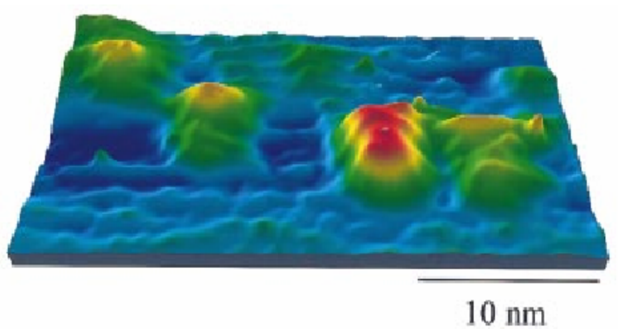


FIG. 2. (Color) STM image of a 250 Å × 150 Å area of HOPG with four adsorbed BDPA molecules.

$$\frac{\text{peak}}{\text{noise}} \leq 3.5$$

(Colm Durkan, private comm.)