Forecast Load Impact from Demand Response Resources

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***Abstract*—To improve forecasting accuracy for baseline load and load impact from demand response resources, this paper develops three innovative statistical models. These models are regression spline fixed effect model, fixed effect change point model and mixed effect change point model. The models developed are applied to forecast baseline load and load impact from air conditioning cycling demand response program in Southern California. All three forecasting models yield accurate forecasts for baseline load and load impact from demand response events. Noticeable rebound effect from demand response events are observed from the dataset.**

***Index Terms*—Baseline Load, Demand Response, Load Impact, Mixed Effect Model, Rebound Effect**

I. Introduction

Renewable energy market has been surging in the United States and around the world. In particular, the recently passed California Senate Bill No. 350, Clean Energy and Pollution Reduction Act of 2015 will boost renewable penetration level in California to 50% by 2030 [1]. To mitigate increasing renewable generation uncertainty and intermittency, supply following resources such as demand response resources are in critical need. In the past ten years, traditional and passive price-based and incentive based demand response programs have been implemented throughout United States. In recent years, proactive demand response algorithms and programs are proposed and developed to further improve utilization of load flexibility and increase power system operational efficiency [2] [3]. One of the biggest challenges faced by system operators is how to accurately forecast load impact from demand response resource and control the operations of demand response resources. Many literatures focused on the control architecture and algorithm for demand response resources. The problem of load impact forecast and estimation for demand response resources has not been well studied. The load impact from a demand response resource is usually estimated as the difference between load baselines and metered load when demand response event is triggered. Although North American Energy Standards Board provided some guidelines [4] for demand response measurement and verification standard, there is still a lack of advanced methodology for load baseline estimation and forecasting. It is crucial to develop an accurate demand response load impact estimation and forecast methodology for two reasons. First, a reliable load impact estimation method gives credit to customers for the exact amount of demand response they provide. Second, an accurate demand response load impact forecast method allows market operator to deploy demand response resources with confidence to improve the efficiency and reliability of power system and electricity market.

This paper fills the knowledge gap by introducing innovative baseline load forecasting and estimation methodologies which significantly improve demand response load impact forecast accuracy. Three types of statistical forecasting methods are proposed and developed for forecasting baseline load. These models include fixed effect model, regression spline model and mixed effect change point model. The forecast methodologies' accuracy are validated through Southern California Edison's residential smart meter data and air conditioning cycling demand response program.

The remainder of this paper is organized as follows. Section II presents an overview of the demand response load impact estimation and forecasting problem. The particular problem of load impact estimation for residential loads with air conditioning cycling program is also described. Section III provides a brief review of existing baseline estimation methodology and rigorous formulations for three statistical baseline estimation and forecasting methodologies. Section IV shows the experimental set up and forecasting performance of the proposed methodologies. Section V states the conclusions.

II. Problem Description

A. Demand Response Load Impact Estimation and Forecast

Traditionally, demand response is defined as a change in the electric energy consumption by customers in response to changes in the price of electricity or direct instruction from utilities in response to a grid reliability problem [5]. Customer participation in demand response program reply upon the incentive payment which depends on the magnitude of demand response resources' load impact. The load impact is defined as the difference between baseline, the electricity that would have

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been consumed by a customer in the absence of a demand response event, and the actual metered electricity consumption. It is surprisingly difficult to estimate the load impact from a demand response resource because the baseline electricity consumption is immeasurable. A good baseline estimation methodology should represents an appropriate tradeoff between simplicity and accuracy. The existing baseline methodology can be categorized into two types [4]. In Type-I baseline methodology, the baseline is estimated by using similar day-based algorithm which depends on historical interval meter data and similarity metrics such as weather and calendar data. Simplicity is the biggest advantage of Type-I baseline method. In Type-II baseline methodology, more sophisticated statistical methods are adopted to estimate and forecast the baseline electricity consumption. Typically, Type-II baseline method yields better forecasting accuracy. Most of the existing Type-II baseline method is based on multiple linear regressions. This paper develops two new classes of baseline estimation algorithms, regression spline model and mixed effect change point model within the Type-II category. The forecasting results are compared with the fixed effect model and Type-I baseline methodology.

B. *Demand Response from Air Conditioning Cycling*

A significant portion of building electricity consumption comes from air conditioning systems. The thermal storage capability of buildings allows short-term change in air conditioning system operations and electricity consumption without significant impact on occupants' comfort. Therefore, direct control of air conditioning system is an ideal demand response program. For example, in California more than half a million customers have participated in the air conditioning cycling program of the three major investor owned utilities. In these programs, direct load switches and/or programmable thermostats are installed to react to economic and reliability based demand event triggers. Depends on desired comfort level, customers may choose different duty cycle options ranging from 30% to 100%. For instance, residential customers under 50% duty cycle program are allowed to keep control of air conditioning systems up to 15 minutes of every half hour in exchange for a lower incentive compared with the 100% cycle option. This paper focuses on studying estimating load baseline and load impact of the air conditioning cycling demand response program.

III. Technical Methods

A. *Model Specifications*

The response variables and predictors in the model are described as follows.

Response variable: Hourly electricity consumption data from customers enrolled in air conditioning cycling program were recorded through the smart meters. The consumption data are aggregated to 52 220kV transformer banks from 12/31/2012 to 11/1/2013 in Southern California Edison's service territory. In this paper, the prediction model was developed based on the sum of residential customers' electricity consumption data at each 220 kV transformer bank on weekdays. Weekend data is excluded because most of the time demand response program events are not triggered during weekends and the predictors' effect on consumption is expected to be different between weekdays and weekend.

Independent variables: The two-day ahead demand response baseline and load impact prediction model are developed in this paper. The forecast model includes six predictors: daily average ambient temperature, humidity, hour/time of the day, two-day lagged electricity consumption, duty cycle percentage and total air conditioning tonnage of customers under the same transformer bank. The daily average temperature and humidity are included because they are highly correlated with electricity consumption. Two-day lagged electricity consumption variable is selected rather than one-day lagged variable because the demand response resources' load impact estimates are submitted to the independent system operator one day before the actual operations. The duty cycle option variable indicates the percentage participation rate of air conditioning load in the program and has strong influence over the load impact for air conditioning cycling demand response program.

Three prediction models are developed to estimate baseline load and load impact from demand response programs. These models are fixed effect change point model, regression spline model, and mixed effect change point model.

Regression Spline Fixed Effect Model: In traditional Type-II baseline method, the "hour" variable is treated as a categorical variable. In our proposed model, the ordinal characteristics of "hour" variable is exploited. As shown in Figure 1, the relationship between electricity consumption and hour is nonlinear. Therefore, a cubic regression spline model is developed to model the relationship between usage and hour without any parametric assumption [6]. In cubic spline, four points were chosen as knots based on percentile (20, 40, 60, 80 percentile) of hours and inserted into hour variable. Two-way and three-way interactions are also included as explanatory variables. In order to further simplify the model, variable selection method is applied to provide the best subset or combination of predictors. With stepwise selection, the final regression spline fixed effect model has the following form

$$\begin{aligned}\log(\mathrm{Usage}_{\mathrm{per,t}}) = {} & \mathrm{Transformer\ Bank} + \mathrm{s}(\mathrm{Hour_t}) + \mathrm{Temprature_t} \\ & + \mathrm{Humidity_t} + \mathrm{Ac\ tonnage}_{\mathrm{per,t}} \\ & + \log(\mathrm{Usage}_{\mathrm{per,t-48}}) + \mathrm{s}(\mathrm{Hour_t}) \\ & \times (\mathrm{Temprature_t} + \mathrm{Humidity_t}) \\ & + \{\mathrm{Temprature_t} + \mathrm{Humidity_t} + \mathrm{s}(\mathrm{Hour_t}) \\ & \times \mathrm{Humidity_t}\} \times \mathrm{Ac\ tonnage}_{\mathrm{per,t}}\,,\end{aligned}$$

where $\mathrm{s}(\mathrm{Hour_t}) = \mathrm{Hour}_t + \mathrm{Hour}_t^2 + \mathrm{Hour}_t^3 + \sum_{i=1}^{4}(\mathrm{Hour_t} - \mathrm{K_i})_+^3$ is a regression spline approximation for the nonparametric effect of hours. $(\mathrm{Hour_t} - \mathrm{K}_i)_+ = \max(\mathrm{Hour_t} - \mathrm{K}_i, 0),\ i = 1,2,3,4.$ $\mathrm{K_i}$ are the knots of hours and are taken as 20, 40, 60, 80 percentile of hours in our analysis. The transformed variables $\log(\mathrm{Usage}_{\mathrm{per,t}})$ and $\mathrm{Ac\ tonnage}_{\mathrm{per,t}}$ are defined as follows.

$$\log(\mathrm{Usage}_{\mathrm{per,t}}) = \log(\mathrm{Usage}_t/\mathrm{Total\ Ac\ tonnage})$$

$$\mathrm{Ac\ tonnage}_{\mathrm{per,t}} = \mathrm{Duty\ cycling\ tonnage}_t / \mathrm{Total\ Ac\ tonnage}$$

$Usage_t$ in the above equation denotes aggregated residential customer electricity consumption at the transformer bank level. The transformed response variable is derived by dividing the aggregated usage by total air conditioning tonnage of residential customer in the air conditioning cycling problem under the transformer bank and applying the log-transformation. The new response variable indicates electricity consumption level of customer with one unit of air conditioning tonnage. The transformed explanatory variable $\text{Ac tonnage}_{\text{per,t}}$ shows the percentage of occupying air conditioning tonnage and can be used to estimate the demand response from air conditioning cycling program. The $\text{Ac tonnage}_{\text{per,t}}$ is set to 1, when estimating the baseline load and set to 1 minus the percentage participation rate (for example 0 if the cycling percentage is 100%) when the Air Conditioning Cycling program is operated. Then the difference of two predicted consumptions is the estimate of load impact from demand response programs.

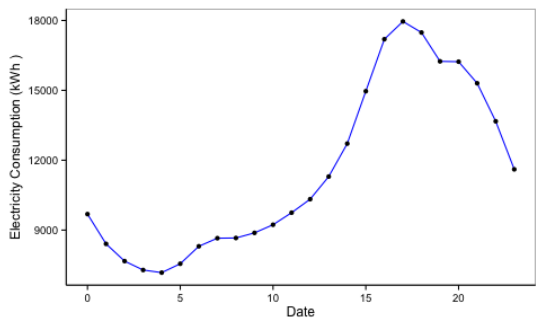

Figure 1. Hourly electricity usage on Aug 9, 2013.

In our model, transformer bank is included as a fixed effect factor to allow the inherent difference among transformers.

Fixed Effect Change Point Model: Based on Figure 1, the hourly curve can be separated into three segments with two change points of hours (one in the early morning and the other one in the late afternoon). Each segment can be approximated by a linear function. To simplify the regression spline model, a change point model for the hour variable is proposed [7]. Our proposed change point model is more homogeneous with fewer parameters than traditional Type-II baseline method. It possesses better prediction power by borrowing consumption information from neighboring hours. After running variable selection procedure, the fixed effect change point model has the following form.

$$\begin{aligned}\log(\text{Usage}_{\text{per,t}}) = {} & \text{Transformer Bank} + \text{Hour}_\text{t} + (\text{Hour}_\text{t} - 8)_+ \\ & + (\text{Hour}_\text{t} - 17)_+ + \text{Temprature}_\text{t} + \text{Humidity}_\text{t} \\ & + \text{Ac tonnage}_{\text{per,t}} + \log(\text{Usage}_{\text{per,t-48}}) \\ & + [(\text{Hour}_\text{t} - 8)_+ + (\text{Hour}_\text{t} - 17)_+] \\ & \times (\text{Temprature}_\text{t} + \text{Humidity}_\text{t}) \\ & + [\text{Temprature}_\text{t} + \text{Humidity}_\text{t} + (\text{Hour} - 8)_+ \\ & \times \text{Humidity}_\text{t}] \times \text{Ac tonnage}_{\text{per,t}},\end{aligned}$$

$$\text{where } (\text{Hour}_\text{t} - \text{h})_+ = \max(\text{Hour}_\text{t} - \text{h}, 0)\,, \text{t} = 1, \ldots 24.$$

Mixed Effect Change Point Model: Note that the collected hourly electricity consumption data are essentially longitudinal/panel data, since the data are frequently measured across time [8]. The observations collected over time within the same transformer bank are correlated. Ignoring such correlation by fixed effect model would result in inefficient estimates and lose prediction power. In order to incorporate such correction, we further propose a mixed effect change point model by treating transformer banks as random-effects. Using a random effects model can also drastically reduce the number of unknown parameters in the model and thus has more efficient parameter estimates.

With stepwise selection, the final model is of the form

$$\begin{aligned}\log(\text{Usage}_{\text{per,t}}) = {} & \text{Transformer Bank} + \text{Hour}_\text{t} + (\text{Hour}_\text{t} - 8)_+ \\ & + (\text{Hour}_\text{t} - 17)_+ + \text{Temprature}_\text{t} + \text{Humidity}_\text{t} \\ & + \text{Ac tonnage}_{\text{per,t}} + \log(\text{Usage}_{\text{per,t-48}}) \\ & + [(\text{Hour}_\text{t} - 8)_+ + (\text{Hour}_\text{t} - 17)_+] \\ & \times (\text{Temprature}_\text{t} + \text{Humidity}_\text{t}) \\ & + [\text{Temprature}_\text{t} + \text{Humidity}_\text{t} + (\text{Hour} - 8)_+ \\ & \times \text{Humidity}_\text{t}] \times \text{Ac tonnage}_{\text{per,t}},\end{aligned}$$

$$\text{where Transformer Bank} \sim \text{N}(0, \sigma^2_{\text{Transformer Bank}}\mathbf{I})$$

Two change points "8" and "17" are identified for the variable "hours" by maximizing the profile log-likelihood. Therefore, the change point model can well capture the usage changing pattern by noting that people tend to leave for work after 8:00 in the morning and come back from work after 17:00 in the afternoon.

B. Model Prameter Estimation Process

All the parameters in mixed-effect change point model are estimated via restricted maximum likelihood (REML) [10]. REML includes an adjustment for degrees of freedom used in estimating fixed effects from the general linear mixed model. Suppose the mixed-effect change point model has the following form

$$\log(\text{Usage}_{\text{per,t}}) = \mathbf{y} = \mathbf{X\tau} + \mathbf{Z\mu} + \mathbf{e},$$

where $\mathbf{X}$ is the first design matrix containing all fixed-effect parameters' observations and $\boldsymbol{\tau}$ is unknown fixed-effect for the parameters in $\mathbf{X}$. The relationship between $\log(\text{Usage}_{\text{per,t}})$ and $\mathbf{X\tau}$ is the same as linear model. The second design matrix $\mathbf{Z}$ includes random-effect information for each observation. $\boldsymbol{\mu}$ represents random block effects for all transformer banks and is normally distributed $\boldsymbol{\mu} \sim \text{N}(\mathbf{0}, \sigma^2_{\text{Transformer Bank}}\mathbf{I})$. $\mathbf{e}$ is a vector of residual errors with $\mathbf{e} \sim \text{N}(\mathbf{0}, \sigma^2\mathbf{I})$. Estimates of fixed and random effects can be achieved from solving the mixed model equations

$$\begin{bmatrix}\mathbf{X'X} & \mathbf{X'Z} \\ \mathbf{Z'X} & \mathbf{X'X} + \mathbf{G}^{-1}\end{bmatrix}\begin{pmatrix}\boldsymbol{\tau} \\ \boldsymbol{\mu}\end{pmatrix} = \begin{pmatrix}\mathbf{X'y} \\ \mathbf{Z'y}\end{pmatrix}, \text{where } \mathbf{G} = \text{var}(\boldsymbol{\mu})/\sigma^2.$$

IV. FORECAST MODEL PERFORMANCE

A. Model Coefficents, p-value and Goodness-of-fit

The performance of the proposed three models are very similar for our collected data. To save the space, we only report the results for mixed effect change point model. R package "lme4" is applied to estimate the model parameters. Table 1 presents fixed effect parameter estimates and Table 2 shows random effect parameter estimates.

Based on the above fitting results, all variables have significant effects on usage. In addition, we can see that the main effects of temperature, humidity, $\text{Ac tonnage}_{\text{per,t}}$, and two-day lagged usage are all positive.

In addition, the hours' effect on usage is positive before 8:00 am, negative between 8:00 am and 17:00 pm, and positive again after 17:00 pm. Based on the interaction terms, we can also see that the hours' effects depend on temperature, humidity, and $\text{Ac tonnage}_{\text{per,t}}$. The effect of $\text{Ac tonnage}_{\text{per,t}}$ also depends on temperature and humidity.

Parameter	Estimate	p-Value
Intercept	-3.50E+00	<0.001
Hour	9.26E-03	<0.001
$(\text{Hour}_t - 8)_+$	-6.18E-02	<0.001
$(\text{Hour}_t - 17)_+$	1.41E-01	<0.001
Temperature	9.95E-03	0.003
Humidity	9.46E-03	0.002
$\text{Ac tonnage}_{\text{per,t}}$	8.99E-01	0.005
$\log(\text{Usage}_{\text{per,t}-48})$	6.51E-01	<0.001
$(\text{Hour}_t - 8)_+ \times \text{Temprature}_t$	1.12E-03	<0.001
$(\text{Hour}_t - 17)_+ \times \text{Temprature}_t$	-2.78E-03	<0.001
$(\text{Hour}_t - 8)_+ \times \text{Humidity}_t$	-9.86E-04	0.002
$(\text{Hour}_t - 17)_+ \times \text{Humidity}_t$	2.16E-04	<0.001
$\text{Temprature}_t \times \text{Ac tonnage}_{\text{per,t}}$	-6.85E-03	0.046
$\text{Humidity}_t \times \text{Ac tonnage}_{\text{per,t}}$	-9.56E-03	0.001
$(\text{Hour} - 8)_+ \times \text{Humidity}_t \times \text{Ac tonnage}_{\text{per,t}}$	9.66E-04	0.002

Table 1. Fixed effects estimates and p-value in mixed effect change point model.

Groups	Std. Dev.
Transformer	0.1127
Residual	0.2329

Table 2. Random-effects standard deviation estimates in mixed effect change point model.

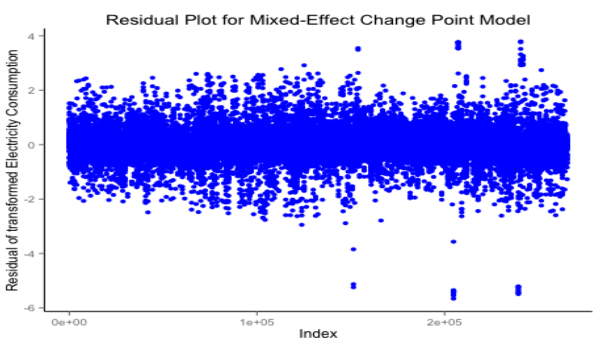

Figure 2. Residual plot for mixed-effect change point model

B. Baseline Load Forecast Accuracy

We employ two typical performance metrics, mean absolute percentage error (MAPE) and mean absolute percentage error (RMSE), as evaluation criteria. MAPE is a measure of prediction accuracy, which accounts for the scale effect of the measures and has the following form

$$\text{MAPE} = \frac{1}{n}\sum_i \sum_j \sum_k \frac{|y_{ijk} - \hat{y}_{ijk}|}{y_{ijk}},$$

$$i = 1,2, \dots ,52, j = 1, \dots , N_d, k = 1,2, \dots , 24$$

where y_{ijk} is observed electricity consumption of transformer bank i, on date j at hour k and $\hat{y}_{ijk}$ stands for the predicted electricity consumption based on the mixed effect change point model. The RMSE is another measure of prediction accuracy, which has the following form

$$\text{RMSE} = \sqrt{\frac{\sum_i \sum_j \sum_k \left(y_{ijk} - \hat{y}_{ijk}\right)^2}{n}},$$

$$i = 1,2, \dots ,52, j = 1, \dots , N_d, k = 1,2, \dots , 24$$

When performing model prediction, the electricity consumption data in the last fifteen observed days are chosen as testing sample. The corresponding training dataset for each testing date is collected from the beginning to two-day before the testing date.

It is known that large transformer banks have most impact on the Air Conditioning Cycling Program. Therefore, we mainly report the prediction results for the top 80% transformer banks (42 transformer banks out of 52). Table 3 and 4 present the performance of baseline load prediction in terms of MAPE and RMSE for both training sample and testing sample.

	Mixed Effect Change Point	Fixed Effect Change Point	Regression Spline Fixed Effect
Training average MAPE	9.76%	9.76%	9.37%
Testing average MAPE	6.96%	6.96%	6.66%

Table 3. Average MAPE for baseline prediction

	Mixed Effect Change Point	Fixed Effect Change Point	Regression Spline Fixed Effect
Training average RMSE	2162.3	2162.3	2148.0
Testing average RMSE	642.3	642.7	634.0

Table 4. Average RMSE for baseline prediction

From the previous tables, these three prediction models have very similar forecasting average MAPEs for both training dataset and testing dataset. Note that the average RMSE of training data is higher than the testing data. One possible reason is that the RMSE depends on the scale of observations and the testing data which are collected at the end of October have relatively smaller usage than the average of training data. In addition, note that Tables 3 and 4 report the prediction accuracy for individual transformer Banks. If we want to estimate the overall usage, MAPE will be much smaller.

C. Forecast Results: Load Reduction Forecasting

The air conditioning cycling program triggered on 10 days during the third quarter of 2013. The demand response event information is presented in Table 5. Since the reduced duty cycle program resulted in short-term change in air conditioning system operations and lower electricity consumption, we expect to see the reduction of electricity usages compared with 100%

duty cycle option. We can use mixed-effect change point model to forecast electricity consumption under reduced duty cycle and 100% duty cycle options and then use their difference for load reduction forecasting.

Date	Start Time	End Time
06/28/13	4:00 PM	6:00 PM
07/02/13	4:00 PM	6:00 PM
07/19/13	4:00 PM	5:00 PM
08/22/13	3:00 PM	5:00 PM
08/28/13	3:00 PM	5:00 PM
08/29/13	2:00 PM	5:00 PM
09/04/13	3:00 PM	5:00 PM
09/05/13	4:00 PM	5:00 PM
09/06/13	2:00 PM	6:00 PM
09/09/13	3:00 PM	5:00 PM

Table 5. Execuation dates of air conditioning cycling program

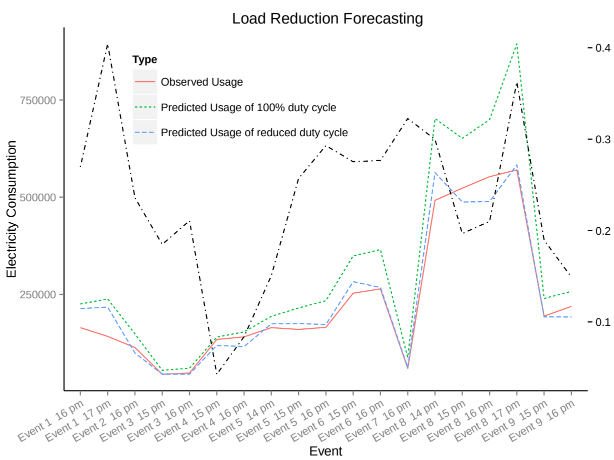

Figure 3. Load impact from demand response events

When predicting reduction influence, we skip the first executing day (6/28/2013), since there is no relevant information about reduced duty cycling. The last 9 days' hourly load reductions are plotted in Figure 3. The green dotted line represents predicted 100% duty cycle; red solid line indicates observed reduced electricity consumption and blue dash line means the predicted values. Since 100% duty cycle option is the same as baseline load, the load reduction is reflected by the difference between green line and red line. In order to present load reduction effect visually, the secondary vertical axis in Figure 3 shows the reduction percentage. Note that the load reduction effects for the first few events might not be very accurate due to lack of previous event data. However, for the last few dates, the predictions are very accurate and the MAPEs ranges from 0.002 to 0.019. If more event data is available, the prediction accuracy could be further improved.

D. Rebound Effect

After a demand response event, utilities may observe a phenomenon known as the "rebound effect" where the customer air conditioning loads typical overshoots normal load baseline level. The difference between the observed electricity consumption and the load baseline is defined as the load rebound. The rebound effect for the air conditioning cycling program is quantified based on the baseline forecast method proposed in this paper. The aggregated rebound effects at the transformer bank level are demonstrated in Figure 4. As shown in the Figure, following most of the demand response events, a positive rebound phenomenon can be observed for at least 3 hours.

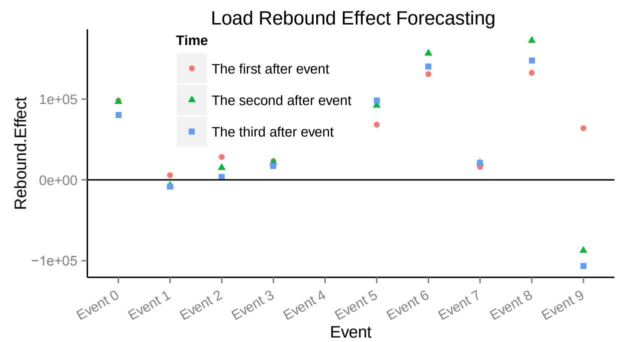

Figure 4. Rebound effect after demand response events

V. CONCLUSIONS

Three innovative statistical models are developed in this paper to forecast load baseline and load impact from demand response resources. The out-of-sample forecast results show that the proposed forecasting models performed well. The estimated load impact and air conditioning load rebound effects are significant based on the baseline load forecast model.

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