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A Robust Segmented Mixed Effect Regression Model for Baseline Electricity Consumption Forecasting

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Abstract—**Renewable energy production has been surging in the United States and around the world in recent years. To mitigate increasing renewable generation uncertainty and intermittency, proactive demand response algorithms and programs are proposed and developed to improve the utilization of load flexibility further and increase power system operation efficiency. One of the biggest challenges to efficient control and operation of demand response resources is how to forecast the baseline electricity consumption and estimate the load impact from demand response resources accurately. In this paper, we propose to use a mixed-effect segmented regression model and a new robust estimate for forecasting the baseline electricity consumption in Southern California by combining the ideas of random effect regression model, segmented regression model, and the least trimmed squares estimate. Since the log-likelihood of the considered model is not differentiable at breakpoints, we propose a new backfitting algorithm to estimate the unknown parameters. The estimation performance and predictive power of the new estimation procedure have been demonstrated with both simulation studies and the real data application for the electric load baseline forecasting in Southern California.**

Index Terms—**Segmented regression model, mixed effects, trimmed maximum likelihood, demand response, electric load.**

I. Introduction

THE renewable energy sector has experienced exponential growth in the past five to ten years. The global annual growth rates of solar photovoltaic and wind energy are 42% and 17% from 2010 through 2015 [1]. The renewable penetration level in certain parts of the world is much higher than the global average penetration level. For example, the renewable energy penetration level in California reached 30% in 2017. The recently passed California Senate Bill No. 100 will further boost renewable penetration level to 60% by 2030 and to 100% by 2045. To mitigate increasing renewable generation uncertainty and intermittency, demand response resources are in critical need. In the past ten years, traditional and passive price-based and incentive-based demand response programs have been implemented throughout the United States. In recent years, proactive demand response algorithms and programs are proposed and developed to improve the utilization of load flexibility and dispatchability further [2]. Accurate load impact forecasts are needed to leverage the load flexibility from the demand response resources effectively. The load impact from a demand response resource is defined as the difference between load baselines and metered load when a demand response event is triggered. In practice, it is very challenging to develop a good estimation of the load baseline which represents the electric load that would have occurred without demand response event [3].

A sound baseline estimation methodology should represent an appropriate tradeoff between simplicity and accuracy. The existing baseline methodology can be categorized into two types. In Type-I methodology, the baseline is estimated by using a similar day-based algorithm which depends on historical interval meter data and similarity metrics such as weather and calendar. Simplicity is the most significant advantage of Type-I baseline method [4], [5]. In Type-II baseline methodology, more sophisticated statistical methods are adopted to estimate and forecast the baseline electricity consumption. Type-II methods typically yield better forecasting accuracy and are undergoing rapid developments. It can be further divided into three groups: statistical methods, machine learning/deep learning methods, and hybrid methods. In the first group, [6] proposed a refined multiple linear regression model. [7] proposed a method to coherently convert a set of lower-level node forecasts to aggregate nodes using empirical copula and Monte-Carlo sampling. In the same vein, [8] proposed an aggregation of random forests load forecasting framework. The second group of literature utilizes deep learning algorithms. Reference [9] proposed support vector regressions models to forecast the demand response baseline. In [10], an ensemble ResNet deep neural network model was proposed. The sequence to sequence recurrent neural network with attention mechanism was adopted in [11]. In the third group, hybrid methods have been developed, in which more than one forecasting algorithms serve as building

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blocks for an overall model. Reference [12] proposed a cooperative quantile regression forest and multivariate quantile regression framework. [13] proposed a two-level hybrid ensemble of deep belief networks model.

There are several limitations with the existing approaches. First, some of the methods do not exploit the structure of the forecasting problem well. For example, the segmented nature of calendar variables on the load profile is not well addressed. Second, deep learning-based forecasting algorithms are typically very computationally expensive to train, yield un-interpretable results, and can be sensitive to the selection of hyperparameters. Third, hybrid methods are generally complicated to build, thus can be error-prone to implement and benchmark. Lastly, most of the existing work build and train a separate model for each time series. This significantly limits the methods' scalability especially for large service territories operated by the electric utilities.

In this paper, we propose to use a mixed-effects segmented regression (MESR) model, a Type- II baseline methodology, to forecast the hourly electric load baseline in Southern California at the 220 kV transformer bank level. One commonly used method for electric power demand prediction at each hour is the multiple linear regression with hour as a categorical variable and weather data as continuous covariates. An alternative model for hour is to include it as a linear predictor. However, it is expected that the linear effect of hour on electric demand does not hold in the whole range of hour. To this end, we propose to model the hour effect by a segmented regression model [14]-[17], which can be considered as a compromise between modeling hour as a global linear predictor and modeling hour as a categorical variable. The nonlinear relationship with breakpoints is said to be piece-wise, segmented, broken-line, or multi-phased. The breakpoints are also called change-points, transition-points or switch-points in some applications. Using a segmented regression model for the *hour* covariate, the hour's effect on the electric consumption changes continuously across time so that we can borrow the information from other hours when estimating the hour's impact. The estimated breakpoints can also tell us how the hour's linear effect changes across different segmented areas. Segmented regression models have been widely used in many areas. In medication, the segmented regression is a powerful statistical tool for estimating intervention effects of interrupted time series studies [18]. The segmented regression is also used to identify the changes in the recent trend of cancer mortality and incidence data analysis [19]. In ecology area, the segmented regression is a widely used statistical tool to model ecological thresholds [20]. For the geometric purpose, the segmented regression statistically models the trends in groundwater levels [21]. Many other examples with piecewise linear terms have been studied in the literature including mortality studies [22], Stanford heart transplant data [23], and mouse leukemia [24]. Note that electric consumption data are essentially longitudinal/panel data. They exhibits very strong spatio-temporal dependencies [25]. To incorporate the correlation among observations and the individual-specific heterogeneity from each transformer bank, we propose to use the random effects regression model [26], [27].

Note that it is not trivial to compute the maximum likelihood estimate (MLE) for the MESR since its log-likelihood is not differentiable at breakpoints. Many standard computational algorithms, such as the Newton-Raphson algorithm, can not be used directly. In this paper, we propose a backfitting algorithm to combine the segmented regression estimation method proposed by [28] and the mixed effect regression estimation method (PWLS: penalized weighted least squares estimation method) proposed by [29] to maximize the non-differentiable log-likelihood of the mixed effects segmented regression model. Note that the MLE is sensitive to outliers, which is the case of our electric consumption data collected in the Southern California area. We further propose a robust estimation procedure for the considered model by extending the idea of the *least trimmed squares* (LTS) estimate [30]. The simulation studies demonstrate the effectiveness of the proposed estimation procedures. The LTS also provides much better prediction performance than the standard MLE for the testing data when forecasting the hourly electric power consumption in Southern California area.

The rest of the article is organized as follows. Section II introduces the MESR and describes the proposed robust estimation algorithms. Section III illustrates the finite sample performance of the proposed method using a simulation study. In Section IV, we apply the new estimation procedure to forecast the hourly electric power demand in the Southern California area. Section V concludes the paper with some discussions.

II. Model

Given a random sample $\{y_{ij}, \mathbf{x}_{ij}, \mathbf{s}_{ij}, z_{ij}, i=1, \ldots, n, j=1, \ldots, n_i\}$, where n is the number of subjects and n_i is the number of observations collected for i^{th} subject, the mixed effects segmented regression (MESR) model we propose to use for the load baseline estimation can be written as:

$$y_{ij} = \mathbf{x}_{ij}^T \phi + \mathbf{s}_{ij}^T \gamma_i + \beta_0 z_{ij} + \sum_{k=1}^{l} \beta_k \, (z_{ij} - \varphi_k)_+ + \varepsilon_{ij} \tag{1}$$

where y_{ij} is the response, $\mathbf{x}_{ij}$ is the p dimension fixed-effect covariates, $\mathbf{s}_{ij}$ is the q dimensional random-effect covariates, z_{ij} is the breakpoint variable with breakpoints $\{\varphi_k, k=1, \ldots, l\}$, t_+ equals to t if $t \geq 0$ and 0 otherwise, $\gamma_i \sim N_q(0, \Sigma_\gamma)$, and $\boldsymbol{\varepsilon}_i = (\varepsilon_{i1}, \ldots, \varepsilon_{in_i}) \sim N_{n_i}(0, \Sigma_\varepsilon)$. In this paper, we assume that $\Sigma_\varepsilon = \sigma^2 \mathrm{I}_{n_i}$. The MESR (1) consists of three parts: multiple linear regression $\mathbf{x}_{ij}^T \phi$, random-effects $\mathbf{s}_{ij}^T \gamma_i$, and segmented regression $\beta_0 z_{ij} + \sum_{k=1}^{l} \beta_k \, (z_{ij} - \varphi_k)_+$, which models the heterogeneous linear effect of z_{ij} on y_{ij} across different areas of z. β_k measures the difference of slopes (linear effect of z_{ij} on y_{ij}) before and after the breakpoint φ_k. In this paper, we mainly focus on the situation when the segmented parts are fixed effects. But the proposed estimation procedure can be extended to the situation when the segmented parts also contain random effects [31]-[33].

Let $\mathbf{y}_i = (y_{i1}, \ldots, y_{in_i})^T$, $\mathbf{X}_i = (\mathbf{x}_{i1}, \ldots, \mathbf{x}_{in_i})^T$, $\mathbf{S}_i = (\mathbf{s}_{i1}, \ldots, \mathbf{s}_{in_i})^T$ and $\mathbf{Z}_i = (\mathbf{z}_{i1}^*, \ldots, \mathbf{z}_{in_i}^*)^T$, where $\mathbf{z}_{ij}^* = (z_{ij}, (z_{ij} - \varphi_1)_+, \ldots, (z_{ij} - \varphi_l)_+)^T$.

Then (1) can be rewritten in matrix format as:

$$\mathbf{y}_i = \mathbf{X}_i\phi + \mathbf{S}_i\gamma_i + \mathbf{Z}_i\beta + \varepsilon_i \quad (2)$$

where $\beta = (\beta_0, \ldots, \beta_l)^T$. Based on (2), $E(\mathbf{y}_i|\mathbf{X}_i, \mathbf{Z}_i, \mathbf{S}_i) = \mathbf{X}_i\,\phi + \mathbf{Z}_i\,\beta$ and $\mathrm{var}(\mathbf{y}_i|\mathbf{X}_i, \mathbf{Z}_i, \mathbf{S}_i) = \mathbf{S}_i\Sigma_\gamma\mathbf{S}_i^T + \sigma_\varepsilon^2 \mathrm{I}_{n_i} \triangleq \Sigma_i$. Therefore, the random effects γ_i make the observations within each subject correlated. The log-likelihood function of $\{y_{ij}, \mathbf{x}_{ij}, \mathbf{s}_{ij}, z_{ij}, i = 1, \ldots, n, j = 1, \ldots, n_i\}$ is

$$\ell(\theta) \propto \sum_{i=1}^{n} \log\left(\left|\Sigma_i\right|^{-1/2}\right) - \frac{1}{2}\sum_{i=1}^{n}(\mathbf{y}_i - \mathbf{X}_i\phi - \mathbf{Z}_i\beta)^T\Sigma_i^{-1}(\mathbf{y}_i - \mathbf{X}_i\phi - \mathbf{Z}_i\beta) \quad (3)$$

where θ collects all the unknown parameters $\{\phi, \beta, \varphi, \sigma, \Sigma_\gamma\}$ in model (1). Unlike the traditional mixed effects model, maximizing (3) is not trivial since it is not differentiable at φ_k. We propose a backfitting algorithm to maximize (3) by alternately updating the segmented regression part and the linear mixed effects part. Next we discuss in detail how to perform such two estimation procedures.

A. Estimating Breakpoints

Given the estimate $\{\phi, \hat{\Sigma}_i\}$, the model (1) will be a segmented regression model. Breakpoints and slopes in segmented regression can be estimated through many ways such as regression spline as well as Bayesian MCMC methods [34], [35]. We will extend the linearization technique proposed by [28] to MESR (1) due to its simplicity of computation. According to the definition of breakpoints, the log-likelihood is not differentiable at φ_k. The breakpoint estimation can be performed via a first-order Taylor expansion of $(z_{ij} - \varphi_k)_+$ around an initial value $\varphi_k^{(0)}$:

$$(z_{ij} - \varphi_k)_+ \approx (z_{ij} - \varphi_k^{(0)})_+ + (\varphi_k - \varphi_k^{(0)})(-1)I(z_{ij} > \varphi_k^{(0)}) \quad (4)$$

where $I(\cdot)$ is the indicator function. It equals 1 if the condition inside the parenthesis is true and 0 otherwise; $(-1)I(z_{ij} > \varphi_k^{(0)})$ is the first derivative of $(z_{ij} - \varphi_k)_+$ assessed in $\varphi_k^{(0)}$.

Let $\mathbf{v}_{ij} = ((-1)I(z_{ij} > \varphi_1^{(0)}), \ldots, (-1)I(z_{ij} > \varphi_l^{(0)}))^T$, $\tilde{\mathbf{z}}_{ij} = (z_{ij}, (z_{ij} - \varphi_1^{(0)})_+, \ldots, (z_{ij} - \varphi_l^{(0)})_+)^T$, and $\delta_k = \beta_k(\varphi_k - \varphi_k^{(0)})$. Define $\mathbf{V}_i = (\mathbf{v}_{i1}, \ldots, \mathbf{v}_{in_i})^T$, $\delta = (\delta_1, \ldots, \delta_l)^T$ and $\tilde{\mathbf{Z}}_i = (\tilde{\mathbf{z}}_{i1}, \ldots, \tilde{\mathbf{z}}_{in_i})^T$. Given the estimate $\{\phi, \hat{\Sigma}_i\}$, the log-likelihood (3) can be simplified to:

$$\ell_1(\beta, \delta) \propto -\frac{1}{2}\sum_{i=1}^{n}(\tilde{\mathbf{y}}_i - \tilde{\mathbf{Z}}_i\beta - \mathbf{V}_i\delta)^T\hat{\Sigma}_i^{-1}(\tilde{\mathbf{y}}_i - \tilde{\mathbf{Z}}_i\beta - \mathbf{V}_i\delta) \quad (5)$$

where $\tilde{\mathbf{y}}_i = \mathbf{y}_i - \mathbf{X}_i\phi$. Therefore, β and δ in (5) can be easily found by weighted least squares estimate. Note that $\varphi_k = (\delta_k/\beta_k) + \varphi_k^{(0)}$. The iterative algorithm will terminate at $\delta_k = 0$. The algorithm to estimate the breakpoints, given the estimate $\{\phi, \hat{\Sigma}_i\}$, is summarized as follows.

Algorithm 1: Segmented regression estimation

1. Set initial values of all breakpoints $\varphi_k^{(0)}$, for $k = 1, \ldots, l$ and calculate the variable $\tilde{\mathbf{Z}}_i$ and the variable $\mathbf{V}_i$.
2. Repeat
3. Fit the regression model of $\tilde{\mathbf{y}}_i$ on $\tilde{\mathbf{Z}}_i$ and $\mathbf{V}_i$ by maximizing the log-likelihood (5). Update the breakpoint with equation $\varphi_k^{(s+1)} = (\delta_k^{(s)}/\beta_k^{(s)}) + \varphi_k^{(s)}$, where $\varphi_k^{(s)}$ is the estimate of φ_k at s^{th} iteration.
4. Until converge

B. Estimating Mixed-effects Regression Models

In this section, we discuss how to maximize (3) given the estimate β and φ, where $\varphi = (\varphi_1, \ldots, \varphi_l)^{\mathrm{T}}$. Let $\hat{\mathbf{Z}}_i$ be the estimate of $\mathbf{Z}_i$ after replacing φ_k by $\hat{\varphi}_k$. Plugging in the estimate $\{\hat{\mathbf{Z}}_i, \beta\}$ into the model (1), we have:

$$\mathbf{y}_i^* = \mathbf{X}_i\phi + \mathbf{S}_i\gamma_i + \varepsilon_i, \quad (6)$$

where $\mathbf{y}_i^* = \mathbf{y}_i - \hat{\mathbf{Z}}_i\beta$. Therefore, the model (6) is simply a traditional mixed-effects regression model. We propose to employ the penalized weighted least squares (PWLS) method developed by [29] to estimate the unknown parameters in (6). Please refer to [29] for more detail about how to compute linear mixed-effects regression model, which is also implemented in R package *lme4*.

C. Mixed-effects Breakpoint Estimation

By combining the estimation procedures in Section II-A and II-B, we propose the following backfitting algorithm to maximize the log-likelihood (3) for the model (2).

Algorithm 2: MLE

1. Set initial values of breakpoint $\varphi_k^{(0)}$ and $\beta^{(0)}$.
2. Repeat
3. Given current breakpoint values $\varphi_k^{(s)}$ and slopes $\beta^{(s)}$, calculate $\mathbf{y}_i^{*(s)} = \mathbf{y}_i - \hat{\mathbf{Z}}_i^{(s)}\beta^{(s)}$.
4. Fit mixed-effects regression model by the PWLS estimation procedure introduced in Section II-B to obtain covariance matrix $\Sigma_r^{(s)}$ and the fixed effect regression estimate $\phi^{(s)}$.
5. Calculate $\tilde{y}_i^{(s)} = \mathbf{y}_i - \mathbf{X}_i\phi^{(s)}$.
6. Fit segmented regression model with $\tilde{y}_i^{(s)}$ and $\Sigma_r^{(s)}$ using Algorithm 1 and update segmented regression parameter estimate to $\varphi^{(s+1)}$ and $\beta^{(s+1)}$.
7. Until converge

D. Robust Mixed-effects Segmented Regression Estimation

It is well known that the MLE is sensitive to outliers and might give misleading results when there are outliers in the data, which is the case for our collected electric power demand data in Southern California area. Please see Section IV for more detail. The issue of outlier is well recognized in the field of load forecasting, and is typically solved using robust regression algorithms. For example, [36] considered Huber's robust regression; [37] advocated the use of L_1 regression model. In the statistics literature, many robust regression methods have been proposed, although not all of them have been investigated in the load forecasting literature. See, for example, M-estimates [38], R-estimates [39], Least Median of Squares (LMS) estimates [40], Least Trimmed Squares (LTS) estimates [30], S-estimates [41], MM-estimates [42], robust and efficient weighted least squares estimator [43, REWLSE], mean shift method [44]-[46]. [47] provided a good review of some commonly used robust regression estimation methods. Next we propose to use the idea of least trimmed squares estimate [30] to provide a robust estimate of the model (1). Given an integer trimming parameter $h \le N$ where N is the total number of training samples, the least trimmed squares minimizes the sum of the smallest h squared residuals with objective function:

$$\sum_{k=1}^{h} r_{(k)}^2 \quad (7)$$

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where $r_{(1)}^2 \le \ldots \le r_{(N)}^2$ are the ordered squared residuals $\{y_{ij} - \hat{y}_{ij}, i=1,\ldots,n; j=1,\ldots,n_i\}$ with $\hat{y}_{ij} = \mathbf{x}_{ij}^T \phi + \mathbf{s}_{ij}^T \gamma_i + \hat{\beta}_0 z_{ij} + \sum_{k=1}^{l} \hat{\beta}_k (z_{ij} - \hat{\varphi}_k)_+$. The robust MESR estimation based on LTS is described in the following table.

Algorithm 3: LTS

1. A subsample of size h^* is selected randomly from the data and then the model (1) is fitted to the subsample using Algorithm MLE of Section II-C. Let $\theta^{(0)}$ be the initial parameter estimate.
2. Repeat
3. Based on current model parameter estimate $\theta^{(s)}$, make prediction of N responses: $\hat{y}_{ij}^{(s)}$, and calculate the residuals $r_{ij}^{(s)} = y_{ij} - \hat{y}_{ij}^{(s)}$. Rank the squared residuals from smallest to largest and select the first h observations that correspond to the smallest h squared residuals.
4. Fit the model (1) to the subsample selected in Step 3 using Algorithm MLE and get the model parameter estimate $\theta^{(s+1)}$.
5. Until converge

To increase the chance of finding the global minimum, one might run Algorithm LTS from many random subsamples and choose the solution which has the smallest trimmed squares. Let r be the dimension of θ. The initial sample size h^* can be any small number larger than r as long as the initial parameter estimate $\theta^{(0)}$ can be computed based on the subsample. The maximum breakpoint [48], i.e., the smallest fraction of contamination that can cause the estimator to take arbitrary large values, of LTS is 0.5 and is attained when $h = [(N+r+1)/2]$. If we have the prior that the proportion of outliers is no more than α, we can also set $h = [N(1-\alpha)+1]$, where α is called the trimming proportion. In practice, one might also try several α values to evaluate LTS and check how the estimate behaves with different trimming proportions [49]-[51]. In our real data application, we use a validation data to choose the trimming proportion.

III. Simulation Study

In this section, we use a simulation study to illustrate the performance of the proposed estimation procedure for the MESR. All the computations are implemented in R. We use R package *segmented::segmented* for breakpoint estimation and *lme4::lmer* for random-effect estimation. We generate observations $\{y_{ij}, \mathbf{x}_{ij}, \mathbf{s}_{ij}, z_{ij}, i=1,\ldots,n, j=1,\ldots,n_i\}$, from the following model

$$y_{ij} = \phi_0 + \phi_1 x_{ij} + \gamma_{i0} + s_{ij}\gamma_{i1} + \beta_0 z_{ij} + \beta_1 (z_{ij} - \varphi_1)_+ + \beta_2 (z_{ij} - \varphi_2)_+ + \varepsilon_{ij}, \quad (8)$$

where $x_{ij} \sim Pois(10)$, $s_{ij} \sim Uniform(5,10)$, z_{ij}'s are n_i arithmetic sequence ranging from $(0,20)$, $\varepsilon_{ij} \sim \mathcal{N}(0, 0.5)$, $\begin{pmatrix}\gamma_{i0}\\ \gamma_{i1}\end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix}0\\0\end{pmatrix}, \begin{pmatrix}\sigma_{r1}^2 & \rho\sigma_{r1}\sigma_{r2}\\ \rho\sigma_{r1}\sigma_{r2} & \sigma_{r2}^2\end{pmatrix}\right)$, with $\sigma_{r1} = \sigma_{r2} = 1, \rho = 0.5$, $i = 1,\ldots,n, j=1,\ldots,n_i$. The other parameters in (8) are set to be: $\phi_0 = -2.5$; $\phi_1 = 1.5$; $\beta_0 = 1.5$; $\beta_1 = 1.5$; $\beta_2 = -2.5$; $\varphi_1 = 6.67$; $\varphi_2 = 13.33$.

We consider the following four simulation scenarios:

1) $n=50, n_i$ is randomly chosen from $(90,110)$.
2) $n=50, n_i$ is randomly chosen from $(18,22)$.
3) $n=200, n_i$ is randomly chosen from $(450,550)$.
4) $n=200, n_i$ is randomly chosen from $(18,22)$.

First, we utilize model (8) to simulate dataset without outliers. The model is estimated with Algorithm MLE. In Tables I-IV, we report the Mean, Median, and Standard Deviation for the estimates of fixed-effects regression parameters, breakpoints, segmented regression parameters, and random-effects covariance matrix, respectively based on 500 replications.

TABLE I
SIMULATION RESULTS OF FIXED-EFFECT PARAMETER ESTIMATES BY ALGORITHM MLE FOR SITUATION WITHOUT OUTLIERS

MLE	$\phi_0 = -2.5$			$\phi_1 = 1.5$		
	Mean	Median	SD	Mean	Median	SD
$n=50, n_i \sim U(90,110)$	-2.505	-2.508	0.125	1.500	1.499	0.002
$n=50, n_i \sim U(18,22)$	-2.500	-2.502	0.169	1.500	1.500	0.005
$n=200, n_i \sim U(450,550)$	-2.498	-2.497	0.064	1.500	1.500	0.001
$n=200, n_i \sim U(18,22)$	-2.491	-2.498	0.125	1.500	1.500	0.004

TABLE II
SIMULATION RESULTS OF BREAKPOINTS ESTIMATES WITH ALGORITHM MLE FOR SITUATION WITHOUT OUTLIERS

MLE	$\varphi_1 = 6.667$			$\varphi_2 = 13.333$		
	Mean	Median	SD	Mean	Median	SD
$n=50, n_i \sim U(90,110)$	6.667	6.666	0.022	13.334	13.332	0.012
$n=50, n_i \sim U(18,22)$	6.667	6.664	0.053	13.334	13.333	0.034
$n=200, n_i \sim U(450,550)$	6.667	6.667	0.006	13.333	13.333	0.003
$n=200, n_i \sim U(18,22)$	6.667	6.666	0.023	13.332	13.332	0.023

From Tables I-IV, we can see that the proposed MLE algorithm performs well when the dataset does not contain any outliers. Also, when the sample size increases, standard deviation of each parameter estimate decreases.

Next, we simulate dataset with outliers based on model (8). Model parameters are estimated by both Algorithm MLE and Algorithm LTS. In order to check how robust each estimate is against the outliers, we randomly choose 5% of each simulated data and add 30 to the response Y (the range of Y is $(15,69)$) and 10 to the value of X (the range of X is $(0,10)$). When applying LTS, we need to choose the trimming proportion α, which has long been a difficult problem. However, LTS can provide a robust model estimate as long as the proportion of outliers is less than α but with low efficiency if the α is too large. Usually a conservative choice of α is recommended in practice. For our examples, we report the results for both $\alpha = 0.1$ and $\alpha = 0.2$. Note that the results of LTS will be better if $\alpha = 0.05$ is used.

In Tables VI-IX, we report the simulation results for the estimates of fixed-effects regression parameters, breakpoints, segmented regression parameters, and random-effects covariance matrix, respectively based on 200 replications. From the tables, we can see that the standard MLE fails to provide reasonable estimates of fixed-effects regression parameters and random-effects covariance matrix when the data contain 5% outliers while LTS can provide reasonable estimates for all parameters with both $\alpha = 0.1$ and $\alpha = 0.2$.

TABLE III
SIMULATION RESULTS OF BREAKPOINTS SLOPE ESTIMATES WITH ALGORITHM MLE FOR SITUATION WITHOUT OUTLIERS

MLE	$\beta_0 = 1.5$			$\beta_1 = 1.5$			$\beta_2 = -2.5$		
	Mean	Median	SD	Mean	Median	SD	Mean	Median	SD
$n = 50,\ n_i \sim U(90, 110)$	1.499	1.500	0.006	1.499	1.499	0.008	–2.499	–2.499	0.008
$n = 50,\ n_i \sim U(18, 22)$	1.499	1.500	0.013	1.502	1.502	0.020	–2.502	–2.502	0.019
$n = 200,\ n_i \sim U(450, 550)$	1.500	1.500	0.001	1.500	1.500	0.001	–2.500	–2.500	0.002
$n = 200,\ n_i \sim U(18, 22)$	1.499	1.499	0.010	1.502	1.501	0.014	–2.501	–2.502	0.014

TABLE IV
SIMULATION RESULTS OF RANDOM-EFFECT ESTIMATES WITH ALGORITHM MLE FOR SITUATION WITHOUT OUTLIERS

MLE	$\sigma_{r1} = 1$			$\sigma_{r2} = 1$			$\rho = 0.5$		
	Mean	Median	SD	Mean	Median	SD	Mean	Median	SD
$n = 50,\ n_i \sim U(90, 110)$	0.969	0.959	0.108	0.978	0.976	0.101	0.504	0.503	0.121
$n = 50,\ n_i \sim U(18, 22)$	0.988	0.988	0.141	0.993	0.990	0.096	0.490	0.501	0.150
$n = 200,\ n_i \sim U(450, 550)$	0.990	0.991	0.050	0999	0.999	0.056	0.499	0.499	0.001
$n = 200,\ n_i \sim U(18, 22)$	0.987	0.990	0.101	0.997	1.000	0.068	0.500	0.506	0.094

TABLE V
SEVEN EXPLANATORY VARIABLES IN REAL DATA APPLICATION

Notation	Explanatory variable
$log(Usage_{per,t-48})$	two-day lagged electricity consumption
$Temperature_t$	Daily average ambient temperature
$Humidity_t$	Humidity of the day
$Hour_t$	Hour/Time of the day
$AC\ tonnage_{per,t}$	Duty cycle percentage
Total AC tonnage	Total AC tonnage under the same transformer bank
A Bank	The indicator variable of transformer bank

Note: Variable *A Bank* is the random-effect variable; variable *Hour* is the segmented variable.

IV. REAL DATA ANALYSIS

In this Section, we illustrate the application of the proposed estimation procedure of MESR to forecast the electric load in Southern California.

A. Data

The electric consumption data are aggregated to 52 220 kV transformer banks from 12/31/2012 to 11/1/2013 in Southern California Edison's service territory. The task is to build a prediction model for the total residential customer electricity consumption at each 220 kV transformer bank on weekdays. The data cleansing of the raw dataset is done in two steps. First, we exclude daily observations for commercial customers and remove zero-usage records from electric consumption data file. Second, we add daily temperature and humidity information for each Bank according to Banks' zipcodes.

The response variable $Usage_t$ is the aggregated customers' hourly electricity consumption recorded through the smart meters. We use the following transformation to make it comparative among 52 subgroups.

TABLE VI
SIMULATION RESULTS OF FIXED-EFFECT ESTIMATES VIA ALGORITHM MLE AND ALGORITHM LTS WITH DIFFERENT α LEVELS FOR SITUATION WITH OUTLIERS

Performance		$\phi_0 = -2.5$			$\phi_1 = 1.5$		
		Mean	Median	SD	Mean	Median	SD
$n=50,\ n_i \sim U(90, 110)$	MLE	3.332	3.334	0.663	1.019	1.017	0.026
	LTS $\alpha = 0.2$	–2.535	–2.539	0.283	1.500	1.500	0.004
	LTS $\alpha = 0.1$	–2.521	–2.522	0.210	1.500	1.500	0.003
$n=50,\ n_i \sim U(18, 22)$	MLE	3.293	3.298	1.069	1.490	1.487	0.045
	LTS $\alpha = 0.2$	–2.471	–2.438	0.616	1.500	1.500	0.015
	LTS $\alpha = 0.1$	–2.496	–2.485	0.539	1.499	1.500	0.014
$n=200,\ n_i \sim U(450, 550)$	MLE	3.310	3.314	0.180	1.017	1.017	0.006
	LTS $\alpha = 0.2$	–2.502	–2.507	0.130	1.500	1.500	0.001
	LTS $\alpha = 0.1$	–2.502	–2.505	0.089	1.500	1.500	0.001
$n=200,\ n_i \sim U(18, 22)$	MLE	3.359	3.417	1.049	1.493	1.496	0.046
	LTS $\alpha = 0.2$	–2.464	–2.437	0.290	1.500	1.500	0.007
	LTS $\alpha = 0.1$	–2.488	–2.491	0.267	1.500	1.499	0.007

This article has been accepted for publication in a future issue of this journal, but has not been edited. Content will change prior to final publication.

TABLE VII
SIMULATION RESULTS OF BREAKPOINT ESTIMATES VIA ALGORITHM MLE AND ALGORITHM LTS WITH DIFFERENT α LEVELS FOR SITUATION WITH OUTLIERS

Performance		$\varphi_1=6.667$			$\varphi_2=13.333$		
		Mean	Median	SD	Mean	Median	SD
$n=50,\ n_i\sim U(90,110)$	MLE	6.647	6.679	0.546	13.322	13.317	0.324
	LTS $\alpha=0.2$	6.670	6.380	0.755	13.351	13.342	0.519
	LTS $\alpha=0.1$	6.670	6.677	0.415	13.323	13.332	0.283
$n=50,\ n_i\sim U(18,22)$	MLE	6.319	6.317	0.204	12.656	12.651	0.139
	LTS $\alpha=0.2$	6.425	6.445	0.185	13.027	13.037	0.237
	LTS $\alpha=0.1$	6.603	6.616	0.170	13.116	13.117	0.145
$n=200,\ n_i\sim U(450,550)$	MLE	6.671	6.673	0.172	13.333	13.337	0.098
	LTS $\alpha=0.2$	6.670	6.685	0.293	13.325	13.330	0.166
	LTS $\alpha=0.1$	6.670	6.677	0.166	13.328	13.330	0.094
$n=200,\ n_i\sim U(18,22)$	MLE	6.307	6.316	0.209	12.643	12.649	0.144
	LTS $\alpha=0.2$	6.427	6.433	0.082	13.030	13.039	0.118
	LTS $\alpha=0.1$	6.606	6.610	0.076	13.190	13.116	0.067

TABLE VIII
SIMULATION RESULTS OF THE BREAKPOINT SLOPE ESTIMATES VIA ALGORITHM MLE AND ALGORITHM LTS WITH DIFFERENT α LEVELS FOR THE SITUATION WITH OUTLIERS

Performance		$\beta_0=1.5$			$\beta_1=1.5$			$\beta_2=-2.5$		
		Mean	Med	SD	Mean	Med	SD	Mean	Med	SD
$n=50,\ n_i\sim U(90,110)$	MLE	1.493	1.494	0.109	1.521	1.518	0.154	-2.516	-2.522	0.163
	LTS $\alpha=0.2$	1.501	1.502	0.123	1.510	1.500	0.166	-2.519	-2.505	0.168
	LTS $\alpha=0.1$	1.506	1.503	0.070	1.505	1.501	0.083	-2.509	-2.507	0.071
$n=50,\ n_i\sim U(18,22)$	MLE	1.590	1.590	0.039	1.544	1.536	0.074	-2.610	-2.602	0.135
	LTS $\alpha=0.2$	1.556	1.556	0.027	1.584	1.564	0.102	-2.602	-2.580	0.109
	LTS $\alpha=0.1$	1.575	1.576	0.029	1.557	1.557	0.069	-2.590	-2.588	0.070
$n=200,\ n_i\sim U(450,550)$	MLE	1.500	1.502	0.035	1.500	1.499	0.040	-2.500	-2.502	0.042
	LTS $\alpha=0.2$	1.500	1.502	0.054	1.502	1.500	0.051	-2.499	-2.502	0.054
	LTS $\alpha=0.1$	1.495	1.497	0.022	1.506	1.505	0.026	-2.506	-2.499	0.028
$n=200,\ n_i\sim U(18,22)$	MLE	1.587	1.586	0.037	1.549	1.544	0.075	-2.605	-2.611	0.138
	LTS $\alpha=0.2$	1.574	1.574	0.014	1.557	1.556	0.048	-2.499	-2.502	0.054
	LTS $\alpha=0.1$	1.553	1.553	0.014	1.571	1.566	0.035	-2.506	-2.499	0.028

TABLE IX
SIMULATION RESULTS OF RANDOM-EFFECT ESTIMATES VIA ALGORITHM MLE AND ALGORITHM LTS WITH DIFFERENT α LEVELS FOR SITUATION WITH OUTLIERS

Performance		$\sigma_{r1}=1$			$\sigma_{r2}=1$			$\rho=0.5$		
		Mean	Med	SD	Mean	Med	SD	Mean	Med	SD
$n=50,\ n_i\sim U(90,110)$	MLE	0.518	0.379	0.582	1.019	1.026	0.117	0.843	0.999	0.612
	LTS $\alpha=0.2$	1.000	0.993	0.097	0.993	1.003	0.097	0.509	0.510	0.115
	LTS $\alpha=0.1$	0.992	0.998	0.111	0.994	0.999	0.097	0.511	0.519	0.107
$n=50,\ n_i\sim U(18,22)$	MLE	0.529	0.529	0.111	0.533	0.523	0.318	0.815	0.816	0.158
	LTS $\alpha=0.2$	0.823	0.825	0.121	0.779	0.779	0.066	0.498	0.485	0.066
	LTS $\alpha=0.1$	0.746	0.744	0.114	1.080	1.073	0.079	0.494	0.493	0.079
$n=200,\ n_i\sim U(450,550)$	MLE	0.769	0.897	0.435	1.013	1.015	0.056	0.619	0.592	0.272
	LTS $\alpha=0.2$	0.992	0.990	0.048	0.998	0.998	0.047	0.495	0.496	0.050
	LTS $\alpha=0.1$	0.992	0.989	0.048	0.998	0.997	0.047	0.496	0.496	0.049
$n=200,\ n_i\sim U(18,22)$	MLE	0.535	0.531	0.118	0.553	0.550	0.315	0.787	0.787	0.148
	LTS $\alpha=0.2$	0.884	0.848	0.060	0.781	0.782	0.033	0.476	0.473	0.085
	LTS $\alpha=0.1$	0.763	0.763	0.056	1.082	1.082	0.038	0.493	0.493	0.071

$$logUsage_{per,t} = log(Usage_t / Total\ AC\ tonnage). \quad (9)$$

In equation (9), the transformed response variable is derived through dividing the aggregated usage by total air conditioning tonnage of a residential customer in the air conditioning cycling program under the transformer bank and applying the log-transformation. The electricity consumption is divided by the total AC tonnage because the latter determines the numerical magnitude of the load measurements. Since the new response variable represents the electricity consumption level per unit of air conditioning tonnage, it makes the effects of other explanatory variables comparable across different transformer banks, which allows us to use common slopes to simplify the model. Fig. 1 depicts the transformed response variable for a few sample banks over 5 days (120 hours).

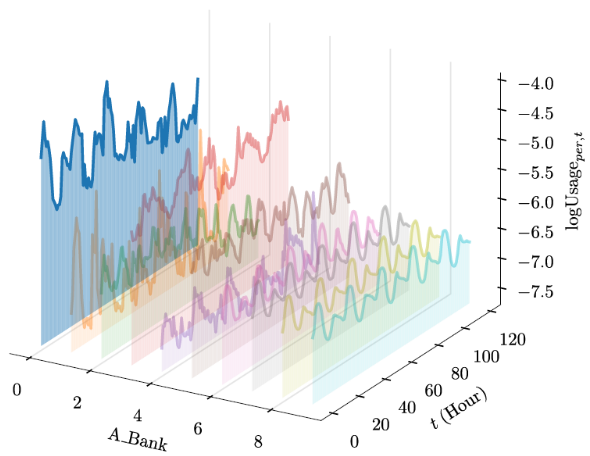

Fig. 1. Data visualization of transformed response variable $logUsage_{per,t}$.

The explanatory variables we collected are listed in Table V. Two-day lagged electricity consumption variable is selected rather than the one-day lagged variable because the demand response resources load impact estimates need to be submitted to the independent system operator one day before the actual operations. The average temperature and humidity are included because they are highly correlated with electricity consumption. The duty cycle option variable indicates the percentage of participation rate of air conditioning load in the program and has substantial influence over the load impact for air conditioning cycling demand response program. The transformer bank indicator variable A Bank is chosen as the random effect, because it contains information about the data from different geographic areas and thus is expected to be heterogeneous with different baselines. A random effect model, assuming that A Banks are sampled from a larger population, is able to incorporate the individual-specific heterogeneity of A Banks while allowing to borrow information across A Banks with much smaller number of parameters (compared to one fixed effect parameter for each of 52 A Banks). In addition, this allows us to extend the model to additional transformer banks.

In this work, the training dataset is chosen as the samples in the first 205 observed weekdays for all transformer banks. The testing dataset consists of the samples from the 10 observed weekdays immediately following the training dataset. The total number of testing sample is 12480.

B. Model and Result

We apply the proposed estimation procedure of MESR to forecast the electricity consumption. Figure 2 displays the hourly trend for average electric consumption and its prediction for a typical forecasting day. There are two breakpoints. The first breakpoint locates between 2am and 3am. The second breakpoints locates between 6pm and 8pm.

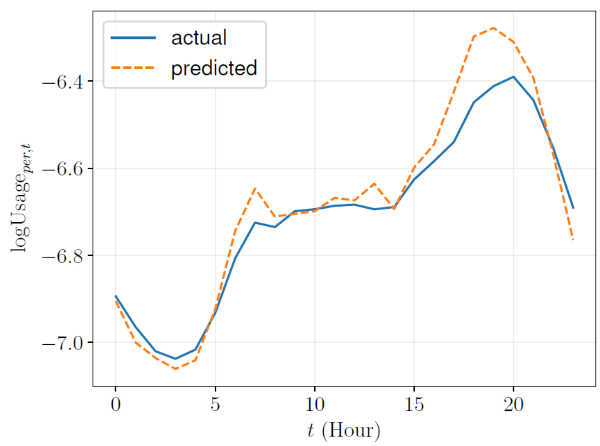

Fig. 2. Trend between average hourly electric consumption $LogUsage_{per,t}$ with variable *Hour* averaged over all *A Bank*.

It seems that the curve corresponding to the actual consumption (after the log transformation) indicates three segments with two breakpoints. The first breakpoint locates between 2am and 3am and the second breakpoint lies between 6pm and 8pm. We also tried the model with three breakpoints (one more breakpoint in the middle segmented area) but the BIC for two breakpoints is smaller. The prediction curve in Fig. 2 corresponds to the predicted values across all Banks for the same forecasting day. It can be seen that the proposed model can predict the actual values very well and the fitted values also matched the breakpoint relationship.

The observations collected over time within the same transformer bank are correlated. To see this, we plot the auto-correlation function of the observation time series of each transformer bank. In Fig. 3, the time series demonstrates strong auto-correlation patterns.

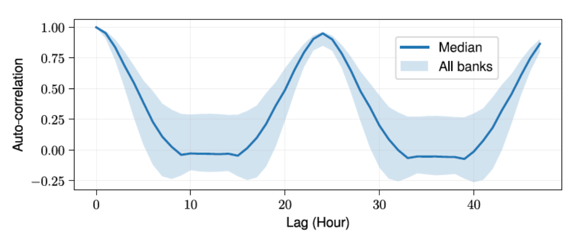

Fig. 3. Auto-correlation function of the observation time series of each transformer bank.

Ignoring such correlation by fixed effect model would result in inefficient estimates and lose prediction power. In order to incorporate such correction, the transformer bank is treated as random-effects. Using a random-effects model can also drastically reduce the number of unknown parameters in the model and thus has more efficient parameter estimates.

Next we describe the construction of the fixed effects. The first six explanatory variables described in Table V, along with their two-way and three-way interactions, are considered as potential explanatory variables. The model variables are selected via LASSO regression method [52], which improves prediction accuracy and model interpretability. The final selected MESR is shown in (10).

$$\begin{aligned}&log(Usage_{per,t}) = A\,Bank + Hour_t + (Hour_t - \varphi^1)_+ + \\&(Hour_t - \varphi^2)_+ + Temperature_t + Humidity_t + \\&AC\ tonnage_{per,t} + log(Usage_{per,t-48}) + [(Hour + \\&(Hour_t - \varphi^1)_+ + (Hour_t - \varphi^2)_+] \cdot Temperature_t + \\&[(Hour + (Hour_t - \varphi^2)_+] \cdot Humidity_t + [(Hour + \\&(Hour_t - \varphi^1)_+ + (Hour_t - \varphi^2)_+] \cdot AC\ tonnage_{per,t} + \\&[Temperature_t + Humidity_t] \cdot AC\ tonnage_{per,t} + \\&Temperature_t \cdot Humidity_t\end{aligned} \tag{10}$$

where $A\,Bank \sim \mathcal{N}(0, \sigma^2_{ABank}\mathrm{I})$. We apply both MLE and LTS algorithm to estimate the model and compare their forecasting performance. Since the true proportion of outliers is unknown, we choose three proportions $\alpha = 0.15, 0.10, 0.05$ for LTS to fit the model (10). In addition, we compare our proposed algorithms with two benchmarks: the multiple linear regression model discussed in [53] and the cooperative quantile regression forest (QRF)/multivariate quantile regression (MQR) method described in [12]. We set the training/testing data partitioning of these benchmarks to be the same as our setup discussed in Section IV-A. We compare their performance by mean absolute percentage error (MAPE) and root mean squared errors (RMSE) on the testing dataset. The formula for MAPE and RMSE are given by $MAPE = \frac{1}{N}\sum\frac{|y_{ij} - \hat{y}_{ij}|}{y_{ij}}$ and $RMSE = \sqrt{\frac{\sum(y_{ij} - \hat{y}_{ij})^2}{N}}$, where N is the total number of testing samples. For better comparison, we also report three quartiles of absolute percentage error (APE) and absolute error (AE).

From Tables X and XI, our proposed robust mixed effects segmented regression model (LTS in the tables) outperforms the multiple linear regression model in [53] and the QRF/MQR model in [53]. The improvements are more significant in terms of the RMSE and AE. The reason why the LTS has a slightly higher MAPE compared to the QRF/MQR baseline is that the LTS produces a bit larger estimation errors for some transformer banks with lighter loading. This results in a higher MAPE due to the small denominator. Within the LTS method, each evaluation criterion reaches the lowest value when $\alpha = 0.1$ and is much smaller than those of MLE. The breakpoint estimates shown in Table XII confirm the locations of breakpoints plotted in Fig. 2.

Table XIII displays the fixed-effects and breakpoints slope estimates for LTS with $\alpha = 0.1$. The variance estimates of the random effects and the error term are 0.0015 and 0.0052, respectively.

TABLE X
PREDICTION RESULTS EVALUATED BY ABSOLUTE PERCENTAGE ERROR FOR THE LAST 10 DAYS IN OCTOBER 2013 WITH ALGORITHM MLE COMPARED WITH ALGORITHM LTS AT DIFFERENT α LEVELS

Performance	MAPE	25% APE	50% APE	75% APE
GEFcom2012[53]	17.97%	5.95%	12.06%	19.80%
QRF/MQR[12]	10.15%	2.36%	5.25%	10.70%
MLE	13.94%	4.55%	8.48%	13.66%
LTS $\alpha = 0.05$	11.08%	2.78%	5.45%	9.37%
LTS $\alpha = 0.1$	10.75%	2.46%	4.95%	8.77%
LTS $\alpha = 0.15$	10.88%	2.55%	5.10%	9.01%

TABLE XI
PREDICTION RESULTS EVALUATED BY ROOT MEAN SQUARE ERROR FOR LAST 10 DAYS IN OCTOBER 2013 WITH ALGORITHM MLE COMPARED WITH ALGORITHM LTS AT DIFFERENT α LEVELS

Performance	RMSE	25% AE	50% AE	75% AE
GEFcom2012[53]	1305.65	26.92	168.96	684.95
QRF/MQR[12]	742.73	10.01	67.77	298.06
MLE	672.88	5.78	42.19	164.08
LTS $\alpha = 0.05$	449.68	3.98	27.01	97.45
LTS $\alpha = 0.1$	414.42	3.65	24.73	86.33
LTS $\alpha = 0.15$	420.00	4.75	25.26	88.84

TABLE XII
BREAKPOINTS ESTIMATION FOR ELECTRIC POWER DEMAND DATASET VIA ALGORITHM MLE AND ALGORITHM LTS AT DIFFERENT α LEVELS

Breakpoint	φ_1	φ_2
MLE	2.64	20.47
LTS $\alpha = 0.05$	2.27	20.47
LTS $\alpha = 0.1$	2.27	20.77
LTS $\alpha = 0.15$	2.27	20.47

According to Table XIII, all the parameters are significant at level $\alpha = 0.05$. When calculating p-values, Satterthwaite method is used for approximating degrees of freedom of the t-distribution for the *t*-test statistics. The variable *Hour* and its breakpoints have both positive and negative slopes and the signs match the plot in Fig. 2. Also, there is sensible positive relationship between *AC tonnage* and electric load *Usage*.

V. CONCLUSION

In this paper, we propose to use a mixed-effects segmented regression model to forecast the electric load baseline in Southern California. When estimating unknown parameters, we propose a backfitting algorithm by combining the ideas of the penalized least square method for random-effects regression model and the linearization technique [28] for segmented regression.

In addition, we extend the idea of LTS to MESR to provide a robust model estimate. Both simulation study and real data application demonstrate the effectiveness of the proposed new estimation procedures.

TABLE XIII
PARAMETER ESTIMATES FOR ELECTRIC POWER DEMAND DATASET EVALUATED WITH ALGORITHM LTS METHOD AT $\alpha = 0.1$ WITH ALL PARAMETER ESTIMATES SIGNIFICANT AT SIGNIFICANCE LEVEL 0.05

Parameter	Estimate	p-value
Intercept	1.063	< 0.0001
$Hour_t$	2.100×10^{-2}	0.0003
$(Hour_t - \varphi_1)_+$	-4.713×10^{-2}	< 0.0001
$(Hour_t - \varphi_2)_+$	6.665×10^{-2}	< 0.0001
$Temperature_t$	-2.198×10^{-2}	< 0.0001
$Humidity_t$	2.556×10^{-2}	0.0010
$AC\ tonnage_{per,t}$	8.838×10^{-1}	< 0.0001
$\log(Usage_{per,t-48})$	-2.223×10^{0}	< 0.0001
$Hour_t \times Humidity_t$	-2.493×10^{-5}	< 0.0001
$(Hour_t - \varphi_2)_+ \times Humidity_t$	2.017×10^{-4}	< 0.0001
$Hour_t \times Temperature_t$	-8.165×10^{-4}	< 0.0001
$(Hour_t - \varphi_1)_+ \times Temperature_t$	1.018×10^{-3}	< 0.0001
$(Hour_t - \varphi_2)_+ \times Temperature_t$	-1.606×10^{-3}	< 0.0001
$Hour_t \times AC\ tonnage_t$	2.012×10^{-2}	0.0002
$Temperature_t \times Humidity_t$	-6.684×10^{-5}	< 0.0001
$Temperature_t \times ACtonnage_{per,t}$	2.763×10^{-2}	< 0.0001
$Humidity_t \times ACtonnage_{per,t}$	2.763×10^{-2}	< 0.0001

Since the model was built up with hourly data, we could also aggregate the data and construct a daily electric load model. In this paper, we assume that the number of breakpoints is known. If the number of breakpoints is unknown, one could apply the selection techniques proposed by [54]-[57] to our model. In the model (1), all random effects are assumed to have a multivariate normal distribution. It will be interesting to extend the work of [58] to relax the normality assumption of the random effects in (1). In addition, for LTS, although an conservation α or serval α values can be used in practice, it requires more research to data adaptively choose the optimal α so that LTS can have both the robustness property and the high efficiency. Since we normalized the response variable by AC tonnage, it is expected that there is not too much heterogeneity for the effects of other variables after we controlled the heterogeneity of A Banks. But it is worthy of more research to try some more complicated models such as random slopes for all other variables and their interactions, as well as nonparametric regression for hour, humidity, or temperature variables.

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