

Reconfigure Distribution Network with Physics-informed Graph Neural Network

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Abstract—The reconfiguration of distribution networks is a complex problem that involves optimizing network topology to ensure efficient and reliable power delivery. Traditional approaches to this problem have relied on heuristics and optimization algorithms, which are computationally expensive and not scalable to large networks. In this paper, we propose a link prediction model based on a physics-informed graph neural network (GNN) by using the nodal and topological information of the distribution network. Numerical studies on a 119-bus distribution network show that the proposed physics-informed GNN exhibits a high level of accuracy in predicting the connectivity of tie lines. By synergistically combining the physics-informed GNN with an optimization model, the proposed algorithm significantly reduces the computation time of the network reconfiguration problem by using a subset of the link prediction results as the final tie switch connectivity.

Index Terms—Graph neural network, link prediction, distribution system, network reconfiguration.

I. INTRODUCTION

Distribution network reconfiguration (DNR) is crucial in power distribution systems, with the goal of improving the performance of the distribution network by changing its topology. The primary objectives of network reconfiguration include reducing power losses and improving the voltage profile of the distribution system.

Over the years, researchers have proposed various algorithms to solve the distribution network reconfiguration problem. Heuristic approaches [1, 2] have been extensively used for DNR due to high computational efficiency and practicality. Most of the previous work on distribution network reconfiguration adopted mathematical optimization techniques, such as mixed-integer programming (MIP) [3, 4], dynamic programming (DP) [5], benders decomposition (BD) [6], and approximated Newton method [7]. These techniques offer an effective way to optimize the network topology by rigorously modeling the operational constraints and objectives of the problem. However, they can be computationally intensive, particularly for large-scale distribution networks.

In the past few years, machine learning algorithms were applied to solve the distribution network reconfiguration problem due to their ability to learn from historical data and make predictions in unforeseen operation scenarios. In [8], a novel data augmentation method is proposed to create an additional synthetic dataset to train a reinforcement learning agent to solve

the dynamic distribution network reconfiguration problem. In [9], a batch-constrained reinforcement learning (RL) algorithm is proposed to learn the network reconfiguration control policy from a limited historical operational dataset without interacting with the distribution network. A general robust method (GRM) is proposed in [10], which combines deep learning and robust optimization to minimize the operational loss in three-phase unbalanced distribution systems. In [11], an efficient deep convolutional neural network is developed to improve the short-term voltage stability of distribution networks.

To embed the network topology information into data-driven monitoring, control, and optimization solutions in power systems, graph neural network (GNN) based algorithms are receiving increasing interest from the researchers in the field [12]. The GNN-based approach has been adopted to solve the many problems in power systems such as fault detection in distribution networks [13, 14], solar power prediction [15, 16], optimal power flow (OPF) [17, 18], distribution network parameter estimation [19], state estimation [20], and system health index prediction [21]. However, the majority of the existing applications of GNN in power systems do not explicitly take into account the change in the network topology in system operation procedures.

On one hand, the power distribution grid continues to be expanded and upgraded with tie switches to accommodate increasing penetration of distributed energy resources, its complexity and number of feasible configurations will experience further growth. This will lead to higher computation costs and time for network reconfiguration controls. On the other hand, the substantial fluctuations in power outputs from distributed renewable energy resources call for network reconfiguration to be performed at shorter time intervals. To develop a computationally efficient network reconfiguration algorithm, we first propose a GNN-based link prediction model for distribution network reconfiguration, which can capture both the dynamic topology configuration and spatial load information. Following this, we go on to solve the network reconfiguration problem, which is formulated as a MIP by selecting a subset of the link prediction results to be fixed variables. The GraphSAGE model [22] is chosen for information propagation in the distribution network-based GNN. This model is selected due to its remarkable ability to generate representations effectively through an inductive framework that leverages node features.

This approach enables GraphSAGE to produce representations for not only known nodes but also for previously unseen nodes and even entirely new input graphs and networks.

The contributions of this paper are summarized below:

- This paper proposes a physics-informed GNN to predict the status of tie switches in real-time for the power distribution network reconfiguration problem.
- By synergistically combining a fraction of the outputs of the physics-informed GNN and the MIP formulation, the proposed algorithm can find a global optimal network reconfiguration solution while reducing the computation time.

The remainder of the paper is organized as follows. Section II presents the mathematical formulation of the distribution network reconfiguration problem. Section III introduces the proposed GNN-based link prediction model. Numerical studies are performed in Section IV and conclusions are given in Section V.

II. PROBLEM FORMULATION

In this section, the mathematical formulation of the distribution network reconfiguration problem is presented. Here we consider the static network reconfiguration problem at an operating point. The mixed-integer conic programming (MICP) formulation from [4] is adopted. As shown in (1), the objective of the network reconfiguration problem is to minimize the distribution network loss. (2) and (3) are the active and reactive power balance constraints. Constraint (4) and (5) express the relation between any two connected buses. (6) to (9) are the constraints for network radiality and connectivity. (10) is the conic constraint. (11) is the current magnitude constraint. (12) and (13) are the voltage limit constraints.

$$\min \sum_{(i,j) \in \mathbf{E}} r_{ij} l_{ij} \quad (1)$$

$$PG_i - PD_i = \sum_{(i,j) \in \mathbf{E}} P_{ij} - \sum_{(k,i) \in \mathbf{E}} (P_{ki} - r_{ki} l_{ki}) + g_i v_i, i \in \mathbf{N} \quad (2)$$

$$QG_i - QD_i = \sum_{(i,j) \in \mathbf{E}} Q_{ij} - \sum_{(k,i) \in \mathbf{E}} (Q_{ki} - x_{ki} l_{ki}) + b_i v_i, i \in \mathbf{N} \quad (3)$$

$$v_j \leq M(1 - \alpha_{ij}) + v_i - 2(r_{ij} P_{ij} + x_{ij} Q_{ij}) + (r_{ij}^2 + x_{ij}^2) l_{ij}, \quad (i, j) \in \mathbf{E} \quad (4)$$

$$v_j \geq -M(1 - \alpha_{ij}) + v_i - 2(r_{ij} P_{ij} + x_{ij} Q_{ij}) + (r_{ij}^2 + x_{ij}^2) l_{ij}, \quad (i, j) \in \mathbf{E} \quad (5)$$

$$\sum_{(i,j) \in \mathbf{E}} \alpha_{ij} = n - 1, \quad (6)$$

$$\beta_{ij} + \beta_{ji} = \alpha_{ij}, (i, j) \in \mathbf{E} \quad (7)$$

$$\sum_{j \in \mathcal{N}(i)} \beta_{ij} = 1, i \in \mathbf{N} \text{ and } i \notin \mathbf{N}_s \quad (8)$$

$$\beta_{ij} = 0, i \in \mathbf{N}_s, j \in \mathcal{N}(i) \quad (9)$$

$$l_{ij} \geq \frac{P_{ij}^2 + Q_{ij}^2}{v_i} \quad (10)$$

$$l_{ij} \leq \alpha_{ij} \bar{I}_{ij}^2 \quad (11)$$

$$v_i = 1, i \in \mathbf{N}_s, (i, j) \in \mathbf{E} \quad (12)$$

$$\underline{V}_i^2 \leq v_i \leq \bar{V}_i^2, \quad (13)$$

where PG_i and QG_i are the active and reactive power generation at bus i . PD_i and QD_i are the active and reactive power demands at bus i . P_{ij} and Q_{ij} are the active and reactive power flow of line ij . r_{ij} and x_{ij} are the resistance and reactance of line ij . g_i and b_i are the conductance and susceptance of bus i . l_{ij} is the squared current magnitude of line ij . v_i is the squared voltage magnitude at bus i . \mathbf{N} and \mathbf{N}_s are the set of buses and substation buses. \mathbf{E} is the set of lines. α_{ij} is a binary variable which equals to 1 if line ij are connected and 0 otherwise. β_{ij} is a binary variable which equals to 1 if bus j is the parent of bus i and 0 otherwise. M is a large constant number. \bar{I}_{ij} is the maximum current magnitude of line ij . \underline{V}_i and \bar{V}_i are the minimum and maximum voltage magnitude at bus i .

III. DISTRIBUTION NETWORK RECONFIGURATION WITH PHYSICS-INFORMED GRAPH NEURAL NETWORK

The overall framework of our proposed algorithm, which combines GNN and optimization model is shown in Fig. 1. A spatial graph is constructed using the topology and load information of the distribution network, which is then fed into a GNN along with a link classifier to predict the status of tie switches. Subsequently, a subset of the link prediction results is used by a MIP solver to determine the distribution network reconfiguration results.

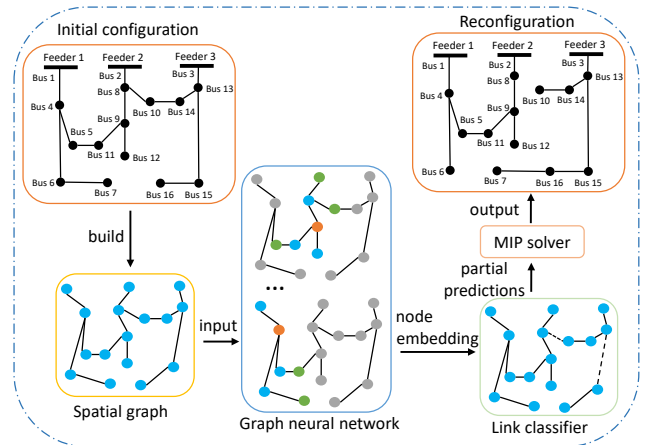


Fig. 1. Overall framework for distribution network reconfiguration

A. Representing the power distribution network as a graph

The distribution network can be represented as a graph, denoted by $\mathcal{G} = (\mathcal{V}, \mathcal{A})$. The graph should not only encapsulate bus information but also capture the interconnections among them. The set of nodes \mathcal{V} of the graph corresponds to the buses in the distribution system. $n = |\mathcal{V}|$ is the number of buses. The adjacent matrix \mathcal{A} delineates the connections between them, $\mathcal{A} \in \{0, 1\}^{n \times n}$. Specifically, \mathcal{A}_{ij} takes the value of 1 if bus i and bus j are connected.

The operation status of the distribution system is highly influenced by the power injection, withdrawal, voltage upper and lower bounds, and conductance and susceptance information at each node. To this end, the features of a node, denoted by v_i , are defined as $v_i = [PG_i, QG_i, PD_i, QD_i, \underline{V}_i, \bar{V}_i, g_i, b_i]$. Note that PG_i and QG_i are set to zero if bus i does not have a generator. Similarly, PD_i and QD_i are set to zero if bus i does not have a load. The training label of the tie switch y_{ij} for line ij is the connectivity of the tie switch. In other words, $y_{ij} = \alpha_{ij}$.

B. Proposed Network Architecture

As aforementioned, GraphSAGE is an inductive framework that utilizes nodal attributes to generate representations effectively, even for previously unseen nodes or entirely new input graphs. The nodal features will be first processed by several GraphSAGE layers and then passed to the link classifier layer to generate predictions for tie switch connections.

1) *GraphSAGE*: The node features v_i are propagated to v'_i with (14).

$$v'_i = \mathbf{W}_1 v_i + \mathbf{W}_2 \frac{1}{|\mathcal{N}(i)|} \sum_{j \in \mathcal{N}(i)} v_j, \forall i \in \mathbf{N}, \quad (14)$$

where \mathbf{W}_1 and \mathbf{W}_2 are the weight matrices, and $\mathcal{N}(i)$ is the set of neighboring nodes of node i .

2) *Link Classifier*: The propagated node features are mapped to the link and tie switch level with (15).

$$e_{i,j} = h_{\Theta}(v_i) \odot h_{\Theta}(v_j), \forall (i, j) \in \mathbf{E}, \quad (15)$$

where h_{Θ} is a neural network, which processes the node features. Here we use a multi-layer perceptron.

3) *Loss Function*: The output of the proposed physics-informed GNN is a prediction for the connection of tie switch ij , which is calculated by (16):

$$\hat{y}_{ij} = \frac{1}{1 + \exp(-\sum_{d=1}^D e_{i,j}[d])}, \forall (i, j) \in \mathbf{E}, \quad (16)$$

where \hat{y}_{ij} is the probability that tie switch ij is connected. Note that $\hat{y}_{ij} \in (0, 1)$ and D is the dimension of $e_{i,j}$.

Since this is a binary classification problem, the binary cross entropy loss shown in (17) is used.

$$l(\mathbf{y}, \hat{\mathbf{y}}) = - \sum_{(i,j) \in \mathbf{E}} (y_{ij} \log(\hat{y}_{ij}) + (1 - y_{ij}) \log(1 - \hat{y}_{ij})) \quad (17)$$

C. Solving a Sub-problem of the Original MIP

Upon training the GNN, the learned probability distribution p_{θ} is utilized to predict the connectivity for lines and tie switches. Subsequently, a fraction of binary variables α_{ij} are set to either 0 or 1 based on these predictions. Next, we leverage a MIP algorithm to solve a sub-problem of the original MIP (sub-MIP), which significantly reduces the search space. The detailed steps can be found in Algorithm 1. First, the spatial graph \mathcal{G} is constructed based on the inputs to the MIP problem I . Then a transformed probability distribution \hat{p}_{θ} is calculated using the outputs of the trained GNN's model p_{θ} . The transformed probability distribution \hat{p}_{θ} is subsequently sorted in descending order. For tie switches that have a probability of connectivity closer to 0 or 1, we sample the predictions \hat{y}_{ij} according to p_{θ} and incorporate constraints $\alpha_{ij} = \hat{y}_{ij}$ into the MIP model I . Finally, we solve the resulting sub-MIP I' and obtain the tie switches' connectivity α .

Algorithm 1 Sub-MIP Solving

Input: learned distribution p_{θ} , MIP model I , sample ratio ρ .
Output: line/tie switch connection variables α and losses.

- 1: Build graph \mathcal{G} based on I .
 - 2: Calculate $\hat{p}_{\theta}(y|\mathcal{G}) = |p_{\theta}(y|\mathcal{G}) - 0.5|$.
 - 3: $\Psi = \text{argsort } \hat{p}_{\theta}(y|\mathcal{G})$.
 - 4: **for** $(i, j) \in \Psi[0 : \text{round}(\rho \cdot |\mathbf{E}|)]$ **do**
 - 5: Sample $\hat{y}_{ij} \sim p_{\theta}(y|\mathcal{G})$
 - 6: Add constraint $\alpha_{ij} = \hat{y}_{ij}$ to I .
 - 7: Solve obtained sub-MIP I' and retrieve α
 - 8: Output the result of network reconfiguration and losses
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IV. NUMERICAL STUDIES

In this section, we first provide the setup for the distribution network and the training dataset. To showcase the effectiveness of GNNs, we introduce a baseline model where we replace the GraphSAGE layers in the proposed model with fully-connected layers (FNN). This comparison is conducted to demonstrate the superior performance of the physics-informed GNN. The prediction accuracy for the connectivity of the tie switches on the testing dataset will be presented. Finally, we compare the computation time of the commercial MIP solver, the baseline algorithm, and our proposed method.

A. Setup of Numerical Studies

The 119-bus distribution network described in reference [23] is selected as the test system, which is depicted in Fig. 2. The distribution system tie switches are indexed to facilitate detailed analysis and visualization of results. The historical load data is adopted from the smart meter energy consumption data in the London Households project [24]. The dataset contains half-hourly energy consumption readings in kWh for 5,567 households in London, covering the period from November 2011 to February 2014. In order to better mimic the load consumption of the distribution network, we aggregate the power consumption of individual households to

derive the nodal real power injections. It is assumed that the power factors remain constant at 0.95 lagging.

The dataset for the numerical studies is produced through simulation. 10,000 training samples/graphs with different network configurations and node features are created using the historical load data. We used Gurobi as the MICP problem's solver and set the MIPGap to 0.01%. Among all the created datasets, 8,000 samples are used for training, 1,000 for model validation, and the remaining 1,000 for testing purposes.

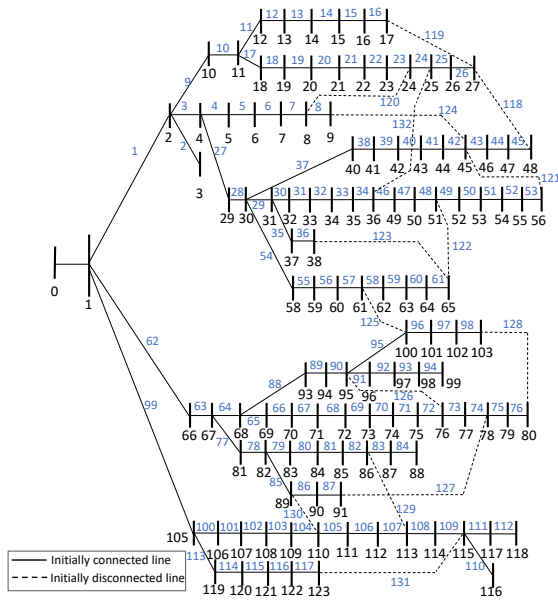


Fig. 2. The 119-bus distribution network

The hyperparameters of the proposed physics-informed GNN model and baseline FNN model are shown in Table I, which are tuned separately to reach their best performances.

TABLE I
HYPERPARAMETERS OF THE GNN AND THE FNN

GraphSAGE layers	number of layers	2
	hidden layer size	64
	activation function	ReLU
Fully-connected layers	number of layers	2
	hidden layer size	64
	activation function	ReLU
Link classification layers	number of layers	3
	hidden layer size	64
	activation function	ReLU
Training parameters	number of epochs	2000
	training batch size	128
	validation batch size	128
	learning rate	5×10^{-5}
	patience	50
	early stop	50

B. Training Process

Fig. 3 illustrates the validation losses of our proposed model and the baseline model throughout the training process. It can be observed that both models experience a significant reduction in validation losses in the first 800 epochs. However, after

this point, the validation loss of the FNN model stabilizes at around 0.35, while the GNN model's validation loss continues to decrease steadily until it reaches approximately 0.25.

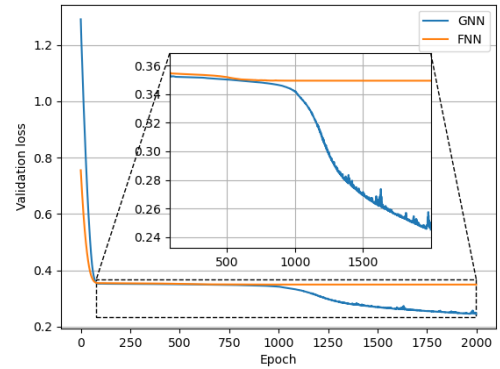


Fig. 3. Validation losses in the training process

The prediction accuracy for the tie switch status on the validation dataset of both models is shown in Fig. 4, from which we can see that the accuracy of both models initially increases rapidly to approximately 88% and then stabilizes for a certain period. Subsequently, the accuracy of the GNN model gradually increases to approximately 92%, whereas the accuracy of the FNN model remains constant.

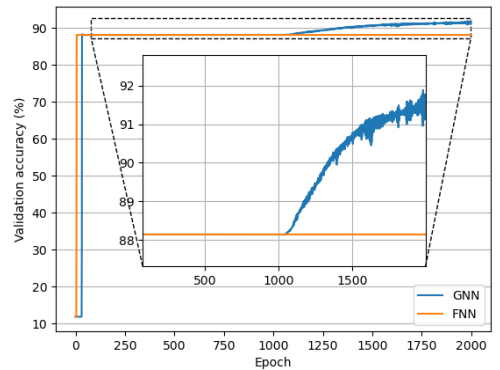


Fig. 4. Validation accuracy of the proposed and baseline models

C. Link Prediction Accuracy on Testing Dataset

The link prediction accuracy for all tie switches in the testing dataset is reported in this subsection. As illustrated in the histogram of Fig. 5, our proposed and baseline models both yield prediction accuracy greater than 90% for the majority of the lines and tie switches. The prediction accuracy for lines or tie switches with an accuracy below 90% is also highlighted. It is obvious that the proposed GNN model significantly outperforms the FNN model in terms of accuracy for the lines whose prediction accuracy is below 90%.

Although the proposed GNN-based link prediction approach yields high accuracy, it may make a rare incorrect prediction, which will lead to isolated nodes. For example, the proposed model may recommend disconnecting line 94. However, as shown in Fig. 2, this prediction would cause bus 99 to be

disconnected from the rest of the distribution network. To avoid this undesirable outcome, we need to carefully select the ratio ρ in Algorithm 1 for the proposed model and the benchmark model. A larger value of ρ will result in a greater number of fixed binary variables, but it may also increase the likelihood of the sub-MIP being infeasible.

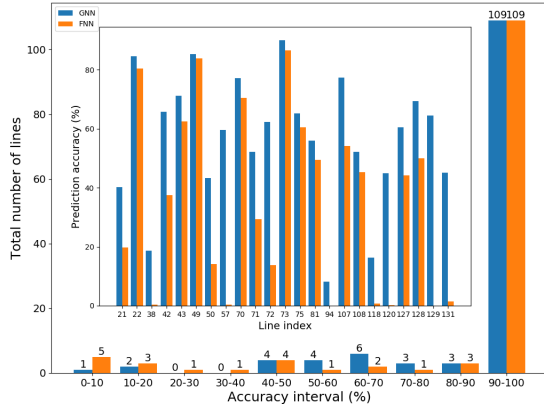


Fig. 5. Prediction accuracy of the proposed and baseline methods

D. Performance Comparison

In order to demonstrate the effectiveness of our proposed GNN model, we use the trained models for online testing on 20 randomly selected instances. The average distribution network loss of 20 instances and the corresponding computation time are depicted in Fig. 6. The results were obtained using three models: MICP, MICP + FNN (the baseline model), and MICP + GNN (the proposed model). The results for MICP were obtained by setting different computation time limits for each instance, while the results for MICP + FNN and MICP + GNN were obtained by setting different ratios ρ in Algorithm 1.

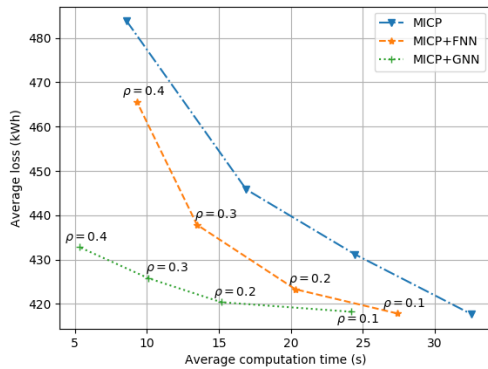


Fig. 6. Performance comparison

As shown in the figure, the proposed model significantly outperforms both the baseline model and MICP. When setting ρ to 0.1, the average system loss obtained by the three models is nearly identical, but the proposed model has a shorter average computation time than the other two models. We chose these ratios because when ρ exceeds 0.5, some instances become infeasible for both the baseline and proposed models.

The number of infeasible instances for both the baseline and proposed models is given in Table II. We can see that our proposed model has fewer infeasible instances than the baseline model due to its higher accuracy in predicting the connectivity of tie switches.

TABLE II
NUMBER OF INFEASIBLE INSTANCES OF TWO MODELS

Model	$\rho = 0.5$	$\rho = 0.6$	$\rho = 0.7$	$\rho = 0.8$	$\rho = 0.9$
MIP+FNN	3	14	19	20	20
MIP+GNN	0	0	4	13	20

V. CONCLUSION

This paper proposes a physics-informed graph neural network-based link prediction model for distribution network reconfiguration. By capturing both network topology information and spatial load distribution, our proposed model can accurately predict the connection of tie switches in power distribution networks. By seamlessly integrating the proposed graph neural network model with a mixed integer conic programming approach, the complexity and computation time of the network reconfiguration task are greatly reduced. Numerical study results on a 119-bus distribution network demonstrate that our proposed approach outperforms the pure optimization-based method and the baseline neural network model in terms of computation time and accuracy.

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