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Satisficing Control for Multi-agent Formation Maneuvers

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Abstract

In this paper, robustly satisficing controls based on control Lyapunov functions are applied to multi-agent formation problems. We show that under certain conditions a group of satisficing control laws chosen from the robustly satisficing set can guarantee bounded formation keeping error, finite completion time, and reasonable formation velocity as well as inverse optimality and desirable stability margins. This technique is applied to a group of nonholonomic robots in simulation as a proof of concept.

1 Introduction

The coordination and control of formations of multi-agents have been a topic of interest recently [1, 2, 3, 4] with application to the coordination of multiple robots, UAVs, satellites, aircraft, and spacecraft. While the applications are different, the fundamental approaches to formation control are similiar: the common theme being the coordination of multiple vehicles to accomplish an objective. The motivation for the research of multi-agent formation control results from the following observations. First, in some situations using multiple agents is more feasible and beneficial than using one single agent, e.g. spacecraft inteferometry problems in deep space. Second, the likelihood of success may be improved to a great degree if multiple agents are used to carry out a mission, e.g. underwater disaster area search. Finally, cost and energy efficiency may be maximized if multiple agents can coordinate their movements in a certain way, e.g. multiple aircraft flying in a V-shape formation to maximize fuel efficiency. This paper addresses the issue of multi-agent formation maneuvers by combining the group control Lyapunov function (clf) approach [5] with the recently introduced satisficing paradigm [6].

A central result of this paper is to extend the application of satisficing controls from regulation problems to time-varying/tracking problems, and to provide a group of explicit state-feedback control laws based on clfs (see [7, 8, 9]) for multi-agent formation problems. The resulting controls guarantee that the multi-agent system has bounded formation error, finite completion time, and reasonable formation velocity, and that the whole system can reach its final goal eventually.

Clf-based satisficing control [6] evolved from the recently introduced notion of satisficing decision theory [10, 11] which can be seen as a formal application of cost-benefit analysis to decision making problems. When combined with the global properties of clfs, satisficing is a powerful design tool which conveniently parameterizes the entire class of continuous control laws which stabilize the closed-loop system with respect to a known clf. Additionally, robust satisficing [6] parameterizes a large class of satisficing controls which have the added benefit of desirable stability margins and which are inverse-optimal.

In [6] satisficing is used to provide a group of clf-based state-feedback control laws for affine nonlinear autonomous systems. Under certain definitions and parameterizations, these control laws are guaranteed to be inverse-optimal with desirable stability margins. However, satisficing controllers are only discussed for regulation problems. Even if tracking problems are reduced to regulation problems the original autonomous system will become nonautonomous, rendering the approach in [6] no longer valid.

In [5] clfs are used to define formation so that a constrained motion control problem of multiple systems is converted into a stabilization problem for one single system. Under certain assumptions, a team of formation constrained autonomous agents is guaranteed to maintain a given formation, however explicit control laws which satisfy these assumptions are not given. Also, the formation function is not guaranteed to converge to zero when the team reaches its final goal. This paper is aimed at connecting the satisficing approach with multi-agent formation control problems to simplify controller design as well as guarantee formation maintenance. We extend satisficing to tracking problems and show that under certain conditions robustly satisficing controllers can guarantee the assumptions in [5] such that a class of control laws is available for formation control.

The remainder of the paper is organized as follows. In section 2 we introduce the concept of satisficing controllers. In section 3 we investigate formation control ideas. In section 4 we apply satisficing approach to multi-agent formation maneuvers. In section 5 we demonstrate a simulation result of multiple robots formation maneuvers using robustly satisficing controllers.

2 Satisficing Controllers

As models for each individual agent, we will consider only affine nonlinear systems of the form

$$\dot{x} = f(x) + g(x)u, \tag{1}$$

where $x \in \mathbb{R}^n$, $f\colon \mathbb{R}^n \to \mathbb{R}^n$, $g\colon \mathbb{R}^n \to \mathbb{R}^{n\times m}$ and $u \in \mathbb{R}^m$. We will assume throughout the paper that f and g are locally Lipschitz functions. A twice continuously differentiable function (C^2) $V\colon \mathbb{R}^n \to \mathbb{R}$ is said to be a control Lyapunov function (clf) for the system if V is positive definite, radially unbounded, and if

$$\inf_u V_x^T(f + gu) < 0,$$

for all $x \neq 0$.

The basic idea of satisficing is to define two utility functions that quantify the benefits and costs of an action. At a state x, the benefits of choosing a control u are given by the "selectablity" function $p_s(u,x)$. Similiarly, at a state x, the costs associated with choosing u are given by the "rejectability" function $p_r(u,x)$. The "satisficing" set is those options for which selectability exceeds rejectability: i.e., $S_b(x) = \{u : p_s(u,x) > \frac{1}{b}p_r(u,x)\}$ where $b(x)$ is a (possibly state-dependent) parameter that can be used to control the size of the set.

We will associate the notion of selectability with stability, and the notion of rejectability with instantaneous cost. In particular, let

$$p_s(u,x) = -V_x^T(f + gu),$$

where V is a known clf. Obviously, only stabilizing controls will make $p_s(u,x)$ positive. We choose the rejectability criteria to be

$$p_r(u,x) = l(x) + u^T R(x)u,$$

where $R(x) = R(x)^T > 0$ is a positive definite matrix function whose elements are locally Lipschitz and $l\colon \mathbb{R}^n \to \mathbb{R}$ is a locally Lipschitz non-negative function. For these choices the satisficing set becomes

$$S_b(x) = \left\{u \in \mathbb{R}^m\colon\ -V_x^T(f + gu) > \frac{1}{b}(l(x) + u^T R(x)u)\right\}. \tag{2}$$

Note if the value of b is too small, S_b might be empty. To ensure that the satisficing set is always nonempty we define:

$$\underline{b}(x) \triangleq \begin{cases} \frac{l}{-V_x^T f}, & \text{if } V_x^T g = 0 \\ \frac{2V_x^T f + 2\sqrt{(V_x^T f)^2 + l V_x^T g R^{-1} g^T V_x}}{V_x^T g R^{-1} g^T V_x}, & \text{otherwise} \end{cases}. \tag{3}$$

From Lemma 5 in [6], we know that for each $x \neq 0$, $\underline{b}(x) \geq 0$, and $b > \underline{b}(x)$ implies that $S_b(x) \neq \emptyset$.

Letting

$$\alpha_1(x,b) \triangleq \frac{1}{2} b R^{-1} g^T V_x \tag{4}$$

$$\alpha_2(x,b) \triangleq R^{-1/2}\sqrt{\frac{1}{4}b^2 V_x^T g R^{-1} g^T V_x - l - b V_x^T f}, \tag{5}$$

the subscript b on S can be eliminated (S is the union of the sets S_b over all $b > \underline{b}$), and the *satisficing set* for $x \neq 0$, can be characterized as

$$S(x) = \{u = -\alpha_1(x,b) + \alpha_2(x,b)\nu\colon\ b > \underline{b}(x), \|\nu\| < 1\},$$

where $\nu \in \mathbb{R}^m$. We know that $S(x)$ is nonempty for $x \neq 0$, and the satisficing set can be parameterized by the two selection functions $b(x) \in \mathbb{R}$ and $\nu(x) \in \mathbb{R}^m$.

The mapping $k : \mathbb{R}^n \to \mathbb{R}^m$ is called a *satisficing control* if $k(0) = 0$, $k(x) \in S(x)$ for each $x \in \mathbb{R}^n \setminus \{0\}$, and k is locally Lipschitz on $\mathbb{R}^n \setminus \{0\}$. It is shown in [6] that if $k(x)$ is a satisficing control then the closed loop system $\dot{x} = f + gk$ is uniformly asymptotically stable. It is also shown in [6] that if V is a clf, $\nu\colon \mathbb{R}^n \to \mathbb{R}^m$ is locally Lipschitz on $\mathbb{R}^n \setminus \{0\}$ and satisfies $\|\nu(x)\| < 1$, and $b\colon \mathbb{R}^n \to \mathbb{R}^+$ is locally Lipschitz on $\mathbb{R}^n \setminus \{0\}$ and satisfies $\underline{b}(x) < b(x)$, then

$$k(x) = \begin{cases} 0, & \text{if } x = 0 \\ -\alpha_1(x,b(x)) + \alpha_2(x,b(x))\nu(x), & \text{otherwise} \end{cases} \tag{6}$$

is a satisficing control.

Sometimes more than mere stability is desired from a control law, and for this reason a subset of satisficing controls was found which are robust with respect to input disturbances.

In [6], the *robust satisficing set*, denoted $S_R(x)$, is defined as

$$
\begin{aligned}
S_R(x) =& \{u \in S(x) \colon V_x^T g R^{-1/2} \nu \le 0\} \\
=& \{u = -\alpha_1(x,b) + \alpha_2(x,b)\nu \colon \\
& \quad b > \underline{b}(x), \|\nu\| < 1, V_x^T g R^{-1/2} \nu \le 0\}.
\end{aligned}
$$

The mapping $k_R \colon \mathbb{R}^n \to \mathbb{R}^m$ is called a *robustly satisficing control* if $k_R(0) = 0$, $k_R(x) \in S_R(x)$ for each $x \in \mathbb{R}^n \setminus \{0\}$,and k_R is locally Lipschitz on $\mathbb{R}^n \setminus \{0\}$. It is shown in [6] that if k_R is a robustly satisficing control, then it has stability margins equal to $(-\frac{1}{2}, \infty)$ and it is inverse-optimal. It is also shown in [6] that (6) is a robustly satisficing control if $V_x^T g R^{-1/2} \nu \le 0$ is also satisfied besides all the other conditions for a satisficing control.

3 Formation Control

Generally there are three approaches to multi-agent coordination reported in the literature, namely leader-following, behavioral, and virtual structures. In this paper we use the virtual structure approach [1], where we treat the entire formation as a rigid body with place-holders fixed in the formation to represent the desired position and orientation of each agent. As the virtual structure evolves in time, the place-holders trace out desired states for each agent to track. The relative orientation of each agent within the formation is fixed with respect to each other when the formation evolves in time. Thus we only need to study the tracking performance for each agent.

Suppose each agent's dynamics can be described as

$$\dot{x}_i = f_i(x_i) + g_i(x_i)u_i, \qquad i = 1, \ldots, N, \tag{7}$$

where $f_i, g_i \in C^\infty, x_i \in \mathbb{R}^n$, and $u_i \in \mathbb{R}^{m_i}$.
Let $x_i^d(s(t))$ represent the desired state for the ith agent to track, where $s \in [s_{start}, s_{final}]$ is a parameter used to parameterize the time evolution of the desired states. In this way we can incorporate error feedback into the whole system through s.

Since we have clf-based techniques developed for the regulation problem, an intuitive way to tackle tracking is to transform it to regulation problem. For simplicity, we omit the subscript i in the following discussions. Letting $\tilde{x} = x - x^d$ we have that

$$\dot{\tilde{x}} = f(\tilde{x} + x^d) - \dot{x}^d + g(\tilde{x} + x^d)u. \tag{8}$$

It is clear that (8) is a nonautonomous system since x^d and $\dot{x}^d$ are functions of time. In this case it is hard to find a clf $V(\tilde{x}, t)$ in order to construct a state-feedback control law. However, if we want an agent to track a constant desired state x^*, (8) is an autonomous system instead. It is much easier to find a clf $V(x, x^*)$ to guarantee $x \to x^*$ asymtotically. We know that

$$\dot{V} = \frac{\partial V}{\partial x}\dot{x} < 0, \qquad x \ne x^*.$$

If $x^d(s(t))$ is a smooth desired state, we can replace x^* in the clf $V(x, x^*)$ with $x^d(s(t))$ to get $V(x, x^d(s(t)))$. If $V(x, x^d(s(t)))$ is smooth and $\frac{\partial V}{\partial x}\dot{x} < 0$ for each specific $s \in [s_{start}, s_{final}]$, we can view $V(x, x^d(s(t)))$ as a pointwise clf to regulate x to $x^d(s)$ in a pointwise fashion. In this case,

$$\dot{V} = \frac{\partial V}{\partial x}\dot{x} + \frac{\partial V}{\partial s}\dot{s}.$$

Although $\dot{V}$ is not necessarily negative, we can use satisficing control to guarantee $\frac{\partial V}{\partial x}\dot{x} < 0$, a property which is necessary for our later result.

4 Satisficing Control for Formation Maneuvers

Following the definition in [5], We define a formation measure function as

$$F(s, x) = \sum_{i=1}^{N} \beta_i V_i(x_i(t), x_i^d(s(t))), \tag{9}$$

where V_i is the pointwise clf for each agent and $\beta_i > 0$, to represent the tracking performance. The formation is defined to be preserved if $F(s, x) \le F_U$, where F_U is an upper bound on the formation measure function $F(s, x)$. The satisficing control framework for autonomous systems can be used to find a class of valid control laws.

We can define a virtual leader or virtual center in the virtual structure, which in turn defines the rest of the formation, that is, defines the desired state for each agent. We use $x_0^d(s(t))$ to represent the state of the virtual leader or virtual center. Following [5], let $\dot{s}$ be given by

$$\dot{s} = \begin{cases} \min\left\{ \frac{v_0}{\delta + \|\frac{\partial x_0^d(s)}{\partial s}\|}, \frac{-(\frac{\partial F}{\partial x})^T \dot{x}}{\delta + |\frac{\partial F}{\partial s}|} \left(\frac{\sigma(F_U)}{\sigma(F(s,x))} \right) \right\}, & s_{start} \le s < s_{final} \\ 0, & s = s_{final} \end{cases}. \tag{10}$$

In (10) $\delta > 0$ is a small positive constant, v_0 is the nominal velocity for the formation, and σ is a class K function. Therefore, formation maneuvers are performed in two steps. First, when $s_{start} \le s < s_{final}$, the formation is preserved within some boundary given by F_U. Second, when $s = s_{final}$, each agent tracks a constant desired state given by $x_i^d(s_{final})$ and reaches (eventually) its final goal.

Next, we extend satisficing control from the regulation problems to the tracking problems. Suppose

$V_i(x_i, x_i^d(s))$ is a pointwise clf for each specific $s \in [s_{start}, s_{final}]$, which will be used to regulate x_i to $x_i^d(s)$, and $l_i(x_i, x_i^d(s)) = \sigma_i(V_i(x_i, x_i^d(s)))$. We use $\frac{\partial V_i(x_i, x_i^d(s))}{\partial x_i}$ and $l_i(x_i, x_i^d(s))$ to replace V_x and l in (3), (4), and (5) respectively. A robustly satisficing control for the ith agent in the formation is now given by

$$u_i = \begin{cases} 0, & \text{if } x_i = x_i^d(s) \\ -\alpha_1(x_i, b_i) + \alpha_2(x_i, b_i)\nu_i, & \text{otherwise} \end{cases}, \tag{11}$$

where ν_i: $\mathbb{R}^n \to \mathbb{R}^{m_i}$ is locally Lipschitz on $\mathbb{R}^n \setminus \{x_i^d(s)\}$ and satisfies $\|\nu_i(x_i)\| < 1$, $(\frac{\partial V_i}{\partial x_i})^T g_i R_i^{-1/2} \nu_i \leq 0$, and b_i: $\mathbb{R}^n \to \mathbb{R}^+$ is locally Lipschitz on $\mathbb{R}^n \setminus \{x_i^d(s)\}$ and satisfies $\underline{b}_i(x_i) < b_i(x_i)$. Therefore, we know that (11) guarantees inverse optimality and desirable stability margins from a pointwise perspective.

We need the following lemmas and corollaries to show that robustly satisficing controls ensure that the formation error of the multi-agent system will be bouned by a given upper bound for any initial formation error.

Lemma 4.1 *If $s \in [s_{start}, s_{final}]$, $\dot{s}$ is given by (10), $V_i(x_i(t), x_i^d(s(t)))$ is a pointwise clf, and $u_i(s, x_i)$ is given by (11), then $\|x_i\|$ is bounded.*

Proof: Following [5], when $s \in [s_{start}, s_{final})$,

$$\begin{aligned} \dot{F}(s,x) &= (\frac{\partial F}{\partial x})^T \dot{x} + \frac{\partial F}{\partial s}\dot{s}(s,x) \\ &\leq (\frac{\partial F}{\partial x})^T \dot{x} + \frac{\partial F}{\partial s} \frac{-(\frac{\partial F}{\partial x})^T \dot{x}}{\delta + |\frac{\partial F}{\partial s}|} \left(\frac{\sigma(F_U)}{\sigma(F(s,x))} \right) \\ &= (\frac{\partial F}{\partial x})^T \dot{x} \left(1 - \frac{\frac{\partial F}{\partial s}}{\delta + |\frac{\partial F}{\partial s}|} \left(\frac{\sigma(F_U)}{\sigma(F(s,x))} \right) \right). \end{aligned} \tag{12}$$

When $s = s_{final}$, $\dot{F}(s,x) = (\frac{\partial F}{\partial x})^T \dot{x}$.
Since $u_i(s, x_i)$ is given by (11), we know that

$$(\frac{\partial F}{\partial x})^T \dot{x} = \sum_{i=1}^{N} \beta_i (\frac{\partial V_i}{\partial x_i})^T \dot{x}_i < 0. \tag{13}$$

When $s \in [s_{start}, s_{final})$, from (12), if $F(s,x) \geq F_U$, $\dot{F}(s,x) < 0$ since

$$\frac{\sigma(F_U)}{\sigma(F(s,x))} \leq 1 \qquad \text{and} \qquad \frac{\frac{\partial F}{\partial s}}{\delta + |\frac{\partial F}{\partial s}|} < 1.$$

When $s = s_{final}$, from (13), $\dot{F}(s,x) < 0$.
Thus $F(s,x)$ is ultimately bounded. Therefore $\|x_i\|$ is also bounded.

Corollary 4.2 *Under the assumptions of Lemma 4.1, $\underline{b}_i(x_i, x_i^d(s))$ is bounded, $\forall x_i \neq x_i^d(s)$.*

Proof: Following Lemma 5 in [6], $\underline{b}_i$ is locally Lipschitz on $\mathbb{R}^n \setminus \{x_i^d(s)\}$. Since $s \in [s_{start}, s_{final}]$ and $x_i^d(s)$ is smooth, $\|x_i^d\|$ is bounded. Also we know that $\|x_i\|$ is bounded from Lemma 4.1. Thus $\underline{b}_i$ is bounded, $\forall x_i \neq x_i^d(s)$.

Corollary 4.3 *Under the assumptions of Lemma 4.1, $|\frac{\partial F}{\partial s}|$ is bounded.*

Proof:

$$\frac{\partial F}{\partial s} = \sum_{i=1}^{N} \beta_i (\frac{\partial V_i}{\partial x_i^d})^T \frac{\partial x_i^d}{\partial s}.$$

Since $s \in [s_{start}, s_{final}]$ and $x_i^d(s)$ is smooth, $\|x_i^d\|$ is bounded. Thus $\|\frac{\partial V_i}{\partial x_i^d}\|$ and $\|\frac{\partial x_i^d}{\partial s}\|$ are also bounded. Therefore, $|\frac{\partial F}{\partial s}|$ is bounded.

Lemma 4.4 *Let $x = 0$ be an equilibrium point for $\dot{x} = f(t,x)$ and $D \subset \mathbb{R}^n$ be a domain containing $x = 0$. Let $V : [0, \infty) \times D \to \mathbb{R}$ be a continously differentiable function such that*

$$\begin{aligned} W_1(x) \leq V(t,x) \leq W_2(x) \\ \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f(t,x) \leq -W_3(x) \end{aligned}$$

$\forall t \geq 0$, $\forall x \in D$ where $W_1(x)$, $W_2(x)$, and $W_3(x)$ are continuously positive definite functions on D. Then, $x = 0$ is uniformly asymptotically stable.

Proof: refer to Theorem 3.8 in [12].

In [5], it is shown that if $F(s(t_0), x(t_0)) \leq F_U$, then $F(s,x) \leq F_U, \forall t \geq t_0$. Here we show that $F(s,x)$ will be bounded by F_U within finite time for arbitrary $F(s(t_0), x(t_0))$ under robustly satisficing controls (11) for each agent in the formation.

Theorem 4.5 *If $s \in [s_{start}, s_{final}]$, $\dot{s}$ is given by (10), and $u_i(s, x_i)$ is given by (11), then $F(s,x) < F_U$ for all $t > T$ for arbitrary $F(s(t_0), x(s(t_0)))$, where T is some finite constant.*

Proof: From Corollary 4.2 and 4.3, $\underline{b}_i$ and $|\frac{\partial F}{\partial s}|$ are bounded. Suppose $|\frac{\partial F}{\partial s}| < B$ and $0 \leq \underline{b}_i(x_i, x_i^d) < L_i$, where B and L_i are upper bounds. If $F(s,x) \geq F_U$ at any time t, we can get

$$1 - \frac{\frac{\partial F}{\partial s}}{\delta + |\frac{\partial F}{\partial s}|} \left(\frac{\sigma(F_U)}{\sigma(F(s,x))} \right) < \frac{B + \delta}{\delta} = M. \tag{14}$$

Since $u_i(s, x_i)$ is a robustly satisficing control, from (2), we know that

$$-(\frac{\partial V_i}{\partial x_i})^T \dot{x}_i > \frac{1}{\underline{b}_i(x_i, x_i^d)} (l_i(x_i, x_i^d) + u_i^T R_i u_i).$$

Thus

$$(\frac{\partial F}{\partial x})^T \dot{x}_i < -\sum_{i=1}^{N} \frac{\beta_i(l_i(x_i, x_i^d) + u_i^T R_i u_i)}{\underline{b}_i(x_i, x_i^d)}.$$

From (12) and (14),

$$\dot{F} < -\sum_{i=1}^{N} \frac{\beta_i l_i(x_i, x_i^d)}{\underline{b}_i(x_i, x_i^d)} M < -\sum_{i=1}^{N} \frac{\beta_i l_i(x_i, x_i^d)}{L_i} M.$$

Let $W_3(x,s) = \sum_{i=1}^{N} \frac{\beta_i l_i(x_i, x_i^d)}{L_i} M$, then $W_3(s,x) > 0, \forall x_i \neq x_i^d$. From Lemma 4.4, F will keep decreasing until $F(s,x) < F_U$. Here we assume that F will keep decreasing until $F(s,x) < F_U$ before s reaches s_{final}. If not, we know that $\dot{F} < 0$ when $s = s_{final}$ from (13). In this case, F will keep decreasing to zero.

If $F(s,x) < F_U$ at some time t_1, we assume that $F(s,x)$ will increase and exceed F_U at some time. Suppose it reaches the upper bound F_U at some time t_2, where $t_2 > t_1$. We know that $\dot{F}(s(t_2), x(t_2)) < 0$ from (12), that is, once $F(s,x)$ reaches F_U from below, it will decrease immediately and cannot exceed F_U.

Therefore, $F(s,x) < F_U$ within finite time for arbitrary initial formation error.

Corollary 4.6 *If $F(s(t_0), x(t_0)) \leq F_U$, then $F(s,x) \leq F_U$, $\forall x \geq t_0$.*

Corollary 4.7 *$F(s,x) \to 0$ as $t \to \infty$ when $s = s_{final}$.*

Proof: When $s = s_{final}$, $\dot{s} = 0$ and $x_i^d(s_{final})$ is constant. From (13), $\dot{F}(s,x) < 0$. Based on Lyapunov theory for autonomous systems, $F(s,x) \to 0$ as $t \to \infty$.

In [5], it is shown that under some assumptions the system has an upper bound on the completion time and certain formation velocity. Here we show that by properly choosing a robustly satisficing control (via the selection functions $b(x)$ and $\nu(x)$) all of the assumptions in [5] are satisfied.

Theorem 4.8 *If $s \in [s_{start}, s_{final}]$, $\dot{s}$ satisfies (10), $x_0^d(s)$ is smooth, $l_i(x_i, x_i^d) = \sigma_i(V_i(x_i, x_i^d))$, and $u_i(s, x_i)$ is given by (11), then*
(1) the multi-agent system has finite completion time,
(2) the formation velocity is v_0 if $F(s,x) \ll F_U$ and $\delta v_0 \ll \|\frac{\partial x_0^d(s)}{\partial s}\|$.

Proof: Following [5], let the completion time be defined as

$$\int_{T_s}^{T_f} \dot{s} = s_{final} - s_{start}.$$

From Corollary 4.2, $\underline{b}_i(x_i, x_i^d)$ is bounded. Suppose $\underline{b}_i(x_i, x_i^d) < \frac{1}{L_i}$, where $L_i > 0$. Since u_i is a robustly satisficing control, from (2), we know that

$$-(\frac{\partial V_i}{\partial x_i})^T \dot{x}_i > \frac{1}{\underline{b}_i}(l_i + u_i^T R_i u_i).$$

Since $l_i = \sigma_i(V_i)$, we can get

$$-(\frac{\partial V_i}{\partial x_i})^T \dot{x}_i > \frac{\sigma_i(V_i)}{\underline{b}_i} > L_i \sigma_i(V_i).$$

Thus

$$\frac{-(\frac{\partial V_i}{\partial x_i})^T \dot{x}_i}{\sigma_i(V_i)} > L_i.$$

According to Lemma 3.1 in [5], we can get

$$\frac{-(\frac{\partial F}{\partial x})^T \dot{x}}{\sigma(F(s,x))} > L,$$

where $L = min(\beta_i L_i)$ and $\sigma(F(s,x)) = min\left(\sigma_i(\frac{F}{\beta_{max} N})\right)$.

(1) Applying the above results and following the proof for Theorem 3.2 in [5], we can show that T_f is bounded by a constant.
(2) We can choose a class K function σ_i in such a way to make $\frac{-(\frac{\partial V_i}{\partial x_i})^T \dot{x}_i}{\sigma_i(V_i)} \to \infty$ as $V_i \to 0$, for example, let $\sigma(z) = z^2$. Applying the above results and following the proof for Theorem 3.3 in [5], we can show that $\|\frac{dx_0}{dt}\| \approx v_0$ when the above assumptions are satisfied.

Note that the upperbound F_U is not necessarily a constant throughout the formation maneuvers. We can set F_U as a function of time, for example, a decreasing stairstep function or an exponentially decreasing function etc. based on the requirement for formation keeping. By choosing a smaller F_U, the formation will evolve correspondingly more slowly.

5 Application Example

In this section we use robustly satisficing controllers to perform formation maneuvers for a group of nonholonomic robots. The kinematic equations for these robots are:

$$\dot{x} = vcos(\phi)$$
$$\dot{y} = vsin(\phi)$$
$$\dot{\phi} = \omega,$$

where (x,y) is the Cartesian position of the robot, ϕ is the orientation of the robot, and (v,ω) is the control input. The translational motion of the robot in the direction perpendicular to the drive axis is restricted, which is known as the nonholonomic constraint. A common technique is to use feedback linearization [13, 14] to simplify the dynamics for a fixed point off the center of the wheel axis which can be donoted as (x_h, y_h). The disadvantage of doing this is that the angular information about the robot is lost. The off axis position is given by the equations

$$x_h = x + Lcos(\phi)$$

$$y_h = y + Lsin(\phi).$$

Thus the output dynamics are

$$\begin{pmatrix} \dot{x}_h \\ \dot{y}_h \end{pmatrix} = R(\phi) \begin{pmatrix} v \\ L\omega \end{pmatrix}.$$

Setting v and $L\omega$ in the control to

$$\begin{pmatrix} v \\ L\omega \end{pmatrix} = R(-\phi) \begin{pmatrix} u_x \\ u_y \end{pmatrix},$$

we obtain

$$\begin{pmatrix} \dot{x}_h \\ \dot{y}_h \end{pmatrix} = \begin{pmatrix} u_x \\ u_y \end{pmatrix}.$$

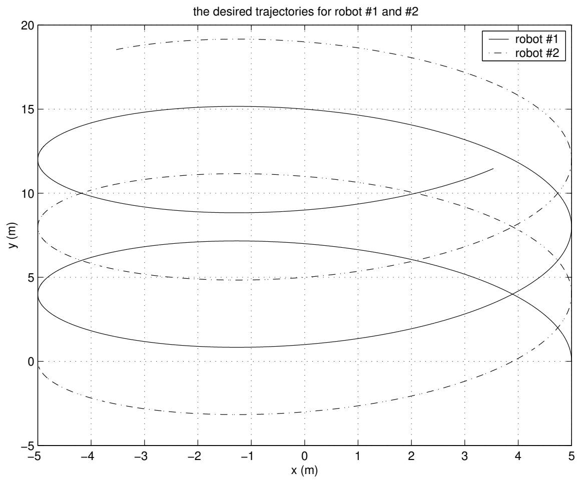

Figure 1: The desired trajectories for robot #1 and #2.

It is obvious that $f(x) = 0$, $g(x) = diag\{1,1\}$, and $u = (u_x, u_y)$. Let $V_i = \frac{1}{2}(x_{hi} - x_{hi}^d)^2 + \frac{1}{2}(y_{hi} - y_{hi}^d)^2$, which is a valid (pointwise in s) clf for the ith robot. Define $F(s,x) = 2\sum_{i=1}^N V_i(x_{hi}, x_{hi}^d(s))$ as the formation measure function. Let $\sigma(V_i) = V_i^2$ and use robustly satisficing control for the robots. Although we use a very simple model here, the general idea of using robustly satisficing control for formation maneuvers is feasible for any affine nonlinear system for which a pointwise clf can be found.

We will simulate two robots moving in spiral formation. Set $F_u = 0.2$ and let $s \in [0, 5]$. The desired distance between these two robots is 10 meters. The center of the line connecting the desired positions of the two robots, i.e., the virtual center of the formation, tracks a desired trajectory (x_{h0}^d, y_{h0}^d) which is given by $(0, s)$. Also the line connecting the desired positions of the two robots rotates about its center counterclockwise with an angle given by $\omega_0 s$. The desired states for the two robots are (x_{hi}^d, y_{hi}^d) which are given by $(5cos(\omega_0 s), s + 5sin(\omega_0 s))$ and $(-5cos(\omega_0 s), s - 5sin(\omega_0 s))$ respectively. The two robots start from rest with some initial errors.

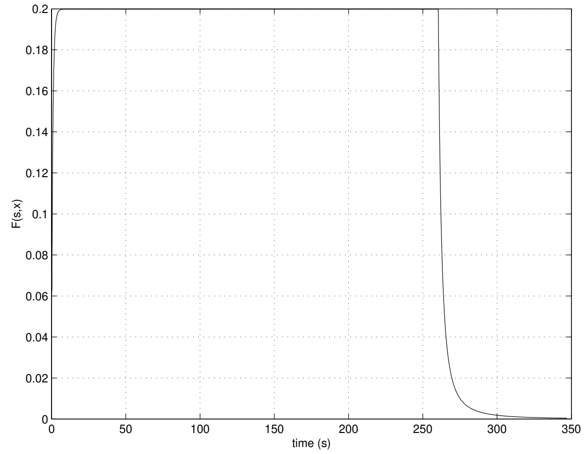

Figure 2: The formation function $F(s,x)$ with $F(s(t_0), x(t_0)) = 0.1$.

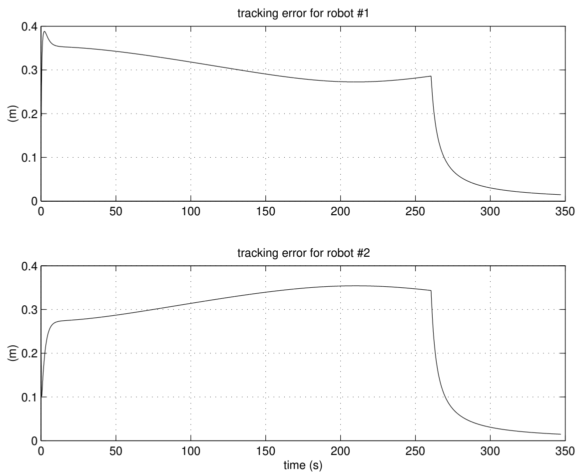

Figure 3: The tracking errors for robot #1 and #2 with $F(s(t_0), x(t_0)) = 0.1$.

In Fig 1, we plot the desired trajectories for robot #1 and #2. To see the pattern clearly, we let $s \in [0, 15]$. In Fig 2, we plot $F(s,x)$ when the initial formation error is below the upper bound F_U. We can see that the formation error is always bounded by F_U. The tracking errors for robot #1 and #2 in this case are shown in Fig 3. In Fig 4, we plot $F(s,x)$ when the initial formation error is above the upper bound F_U. To see the change of $F(s,x)$ clearly, we plot $F(s,x)$ during the initial short time period in Fig 5. We can see that even if the initial formation error is above the upper bound, $F(s,x)$ decreases quickly until it is bounded by F_U. The tracking errors for robot #1 and #2 in this case are shown in Fig 6. From Fig 2 and Fig 4, we can also see that $F \approx F_U$ when the multi-agent system is far away from its final goal. But when $s = s_{final}$, $F(s,x)$ will decrease to zero so that each agent can be regulated to its final state.

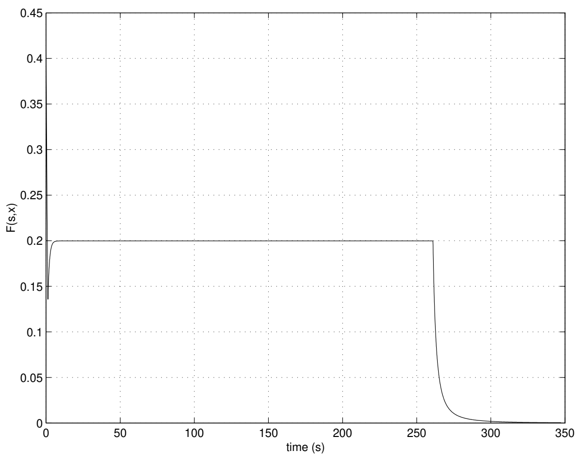

Figure 4: The formation function $F(s,x)$ with $F(s(t_0), x(t_0)) = 0.43$.

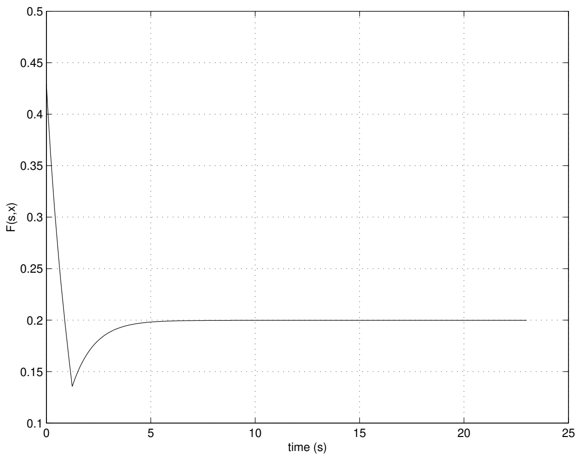

Figure 5: The formation function $F(s,x)$ with $F(s(t_0), x(t_0)) = 0.43$ during the initial short time period.

6 Conclusion

In this paper, we apply the satisficing technique to multi-agent formation control problems. We show that as long as a pointwise clf exists for an affine nonlinear system, a group of explicit state-feedback control laws can be found using satisficing control, which guarantee formation maintenance throughout the maneuvers. By properly parameterizing and selecting certain functions

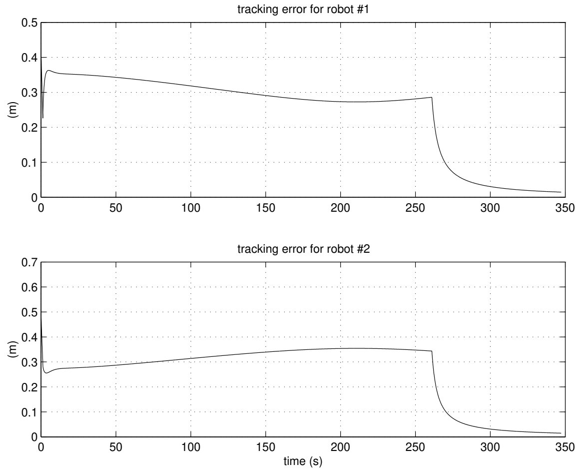

Figure 6: The tracking errors for robot #1 and #2 with $F(s(t_0), x(t_0)) = 0.43$.

in the satisficing set, satisficing control provides a nice solution to multi-agent formation problems, where the formation error can be specified in terms of clfs.

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