A Decentralized Scheme for Spacecraft Formation Flying via the Virtual Structure Approach

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Abstract

Built on the combined strength of decentralized control and the recently introduced virtual structure approach, a decentralized formation scheme for spacecraft formation flying is presented in this paper. Following a decentralized coordination architecture via the virtual structure approach, decentralized formation control strategies are introduced, which are appropriate when a large number of spacecraft are involved or stringent communication bandwidth limitations are exerted. The effectiveness of the proposed control strategies is demonstrated through simulation results.

1 Introduction

With regard to the benefits of using multiple vehicles, the concept of formation control has been studied extensively in the literature with application to the coordination of multiple robots, unmanned air vehicles (UAVs), autonomous underwater vehicles (AUVs), satellites, aircraft, and spacecraft (see e.g. [1, 2, 3, 4, 5, 6]). There are several advantages to using formations of multiple vehicles. These include increased feasibility, accuracy, robustness, flexibility, cost and energy efficiency, and probability of success.

Various strategies and approaches have been proposed for formation control. These approaches can be roughly categorized as leader-following (see e.g. [1, 4]), behavioral (see e.g. [2, 7, 8]), and virtual structure (see e.g. [3, 9]) approaches. Each approach has its advantages and disadvantages. The leader-following approach is easy to understand and implement. However, it is a centralized implementation, which makes the leader a single point of failure for the formation. Another weakness is that there is no explicit feedback from the followers to the leader. As an alternative to leader-following, the virtual structure approach is easy to prescribe the behavior for the group. Also it is precise and can maintain the formation very well during the maneuvers. The main disadvantage of the current virtual structure implementation is that it is centralized, which results in a single point of failure for the whole system. The behavioral approach is a decentralized implementation and only requires low bandwidth communication. Also explicit formation feedback is included via the communication between neighbors. However, the behavioral approach is hard to analyze mathematically and has limited ability for precise formation keeping, that is, the group cannot maintain formation very well during the maneuvers.

Motivated by the advantages and disadvantages of each approach discussed above, a framework which is precise, reliable, and easy to implement needs to be constructed to achieve the following characteristics. First, the framework should be decentralized when a large number of agents are involved in the formation or there are stringent limitations on the communication bandwidth. Second, formation feedback should be included in the framework to improve group robustness. Third, the group maneuvers should be easy to prescribe and direct in the framework. Finally, the framework should guarantee high precision for maintaining the formation during the maneuvers. The purpose of this paper is to propose a solution that can achieve the benefits of each approach discussed above while overcoming their limitations. The main contribution of this paper is to apply the virtual structure approach in a decentralized scheme so that both the benefits of the virtual structure approach and the decentralized scheme can be achieved simultaneously. In this paper, each spacecraft in the formation instantiates a local copy of the formation control, i.e. the coordination vector, and the local instantiation of the coordination vector in each spacecraft, which represents the states of the virtual structure, is synchronized by infrequent communication with its neighbors following a bidirectional ring topology.

2 Problem Statement

In this section, we introduce some preliminary notation and properties for spacecraft formation flying including reference frames, unit quaternions, desired states for each spacecraft, and spacecraft dynamics.

2.1 Reference Frames

Three coordinate frames are used in this paper. Reference frame $\mathcal{F}_O$ is used as an inertial frame. Reference frame $\mathcal{F}_F$ is fixed at the virtual center of the formation. Reference frame $\mathcal{F}_i$ is embedded at the center of mass of each spacecraft as a body frame, which rotates with the spacecraft and represents its orientation. Given any vector p, the representation of p in terms of its components in $\mathcal{F}_O$, $\mathcal{F}_F$, and $\mathcal{F}_i$ are represented by $[p]_O$, $[p]_F$, and $[p]_i$ respectively.

2.2 Unit Quaternions

Unit quaternions are used to represent the attitudes of rigid bodies. A unit quaternion is defined as $q = [\hat{q}^T, \bar{q}]^T$, where $\hat{q}$ is the vector part and $\bar{q}$ is the scalar part. The multiplicative identity quaternion is denoted by $\mathbf{1} = [0, 0, 0, 1]^T$. Note that a unit quaternion is not unique since q and $-q$ represent the same attitude. However, uniqueness can be achieved by restricting the Euler angle ϕ to the range $0 \leq \phi \leq \pi$ so that $\bar{q} \geq 0$ all the time [10]. In this paper, we assume that $\bar{q} \geq 0$.

2.3 The Desired States for Each Spacecraft

In the virtual structure approach, the entire formation is treated as a single rigid body. Conceptually, we can think that place holders corresponding to each spacecraft are embedded in the virtual structure to represent the desired position and attitude for each spacecraft. As the virtual structure as a whole translates or rotates in time, the place holders trace out trajectories for each corresponding spacecraft to track. The place holders in the virtual structure maintain a rigid geometric configuration with fixed relative position and orientation.

Suppose that the virtual structure has position r_F, velocity v_F, attitude q_F, and angular velocity ω_F relative to $\mathcal{F}_O$. Let $\lambda_F = [\lambda_1, \lambda_2, \lambda_3]$ and $\dot{\lambda}_F = [\dot{\lambda}_1, \dot{\lambda}_2, \dot{\lambda}_3]$, where the components represent the expansion/contraction rates of the virtual structure along each $\mathcal{F}_F$ axis. Therefore, the coordination vector of the virtual structure is defined as $\xi = [r_F^T, v_F^T, q_F^T, \omega_F^T, \lambda_F^T, \dot{\lambda}_F^T]^T$, which can be used to represent the states of the virtual structure.

Define the actual state of the ith spacecraft relative to $\mathcal{F}_O$ as $X_i = [r_i^T, v_i^T, q_i^T, \omega_i^T]^T$, where r_i, v_i, q_i, and ω_i represent the position, velocity, attitude, and angular velocity of the ith spacecraft. Similarly, define $X_{iF} = [r_{iF}^T, v_{iF}^T, q_{iF}^T, \omega_{iF}^T]^T$ as the actual state of the ith spacecraft relative to $\mathcal{F}_F$. A superscript "d" is also used to represent the corresponding desired state of each spacecraft, that is, the state of the place holders in the virtual structure, relative to either $\mathcal{F}_O$ or $\mathcal{F}_F$. Generally, r_{iF}^d, q_{iF}^d, v_{iF}^d, and ω_{iF}^d can vary with time, which means the formation shape is time-varying. Often, r_{iF}^d and q_{iF}^d are constant and v_{iF}^d and ω_{iF}^d are zero since each place holder needs to preserve fixed relative position and orientation in the virtual structure, that is, the formation shape is preserved during the maneuvers. In this paper, we focus on formation maneuvers that preserve the overall formation shape. Of course, the approach here can be extended to the general case easily. By defining $\Lambda(t) = diag(\lambda_F)$ and $\dot{\Lambda}(t) = diag(\dot{\lambda}_F)$ as diagonal matrices to represent expansions/contractions, we can also loosen the requirement to preserve fixed relative position between each place holder in the virtual structure to make the formation shape more flexible by allowing the place holders to expand or contract while still keeping fixed relative orientation. In this case, the position and velocity of the place holder relative to the formation frame $\mathcal{F}_F$ are given by $\Lambda(t) r_{iF}^d$ and $\dot{\Lambda}(t) r_{iF}^d$ respectively.

In the virtual structure approach, the desired states for each spacecraft are determined by the coordination vector ξ. The desired states for the ith spacecraft are defined by

$$\begin{aligned}
[r_i^d(t)]_O =& [r_F(t)]_O + C_{OF}(t)\Lambda(t)[r_{iF}^d]_F \\
[v_i^d(t)]_O =& [v_F(t)]_O + C_{OF}(t)\dot{\Lambda}(t)[r_{iF}^d]_F \\
&+ [\omega_F(t)]_O \times (C_{OF}(t)\Lambda(t)[r_{iF}^d]_F) \qquad (1)\\
[q_i^d(t)]_O =& [q_F(t)]_O[q_{iF}^d]_F \\
[\omega_i^d(t)]_O =& [\omega_F(t)]_O,
\end{aligned}$$

where $C_{OF}(t)$ is the rotation matrix of the frame $\mathcal{F}_O$ with respect to $\mathcal{F}_F$, and $[\cdot]_O$, $[\cdot]_F$, and $[\cdot]_i$ are the corresponding coordinate representations. The derivatives of the desired states can be derived correspondingly (see [11]).

2.4 Spacecraft Dynamics

The translational and rotational dynamics of each spacecraft relative to $\mathcal{F}_O$ are

$$\begin{aligned}
\frac{dr_i}{dt_o} &= v_i \\
M_i\frac{dv_i}{dt_o} &= f_i \\
\frac{d\hat{q}_i}{dt_o} &= -\frac{1}{2}\omega_i \times \hat{q}_i + \frac{1}{2}\bar{q}_i\omega_i \qquad (2)\\
\frac{d\bar{q}_i}{dt_o} &= -\frac{1}{2}\omega_i \cdot \hat{q}_i \\
J_i\frac{d\omega_i}{dt_o} &= -\omega_i \times (J_i\omega_i) + \tau_i,
\end{aligned}$$

where M_i and f_i are the mass and control force, and J_i and τ_i are the moment of inertia and control torque associated with the ith spacecraft respectively.

3 A Decentralized Architecture via the Virtual Structure Approach

In [7], a decentralized control is implemented using a bidirectional ring topology, where each robot only needs position information of its two neighbors. A formation pattern is defined to be a set composed of the desired locations for each robot. Here, instead of using a set of desired locations for each agent as a formation pattern, we take advantage of the virtual structure approach to define the formation pattern by $P = \xi^d$, where ξ^d is the desired constant coordination vector representing the desired states of the virtual structure. The coordination vector ξ can then be used to specify the desired states for each agent to track. By specifying the formation pattern for the group, the movements of each spacecraft can be completely defined. Through a sequence of formation patterns $P^{(k)}, k = 1, \cdots, K$, the group can achieve a class of formation maneuver goals.

In [7], the formation pattern is defined in such a way that each vehicle only knows its final location in the formation while the trajectory for each vehicle during the maneuver is not specified. Here the formation pattern is defined in such a way that each spacecraft will track a trajectory specified by the state of the virtual structure. Those trajectories themselves preserve a certain formation shape. From this point of view, this new formation pattern can accomplish collision avoidance more efficiently than the formation pattern defined in [7]. In this paper, each spacecraft in the formation instantiates a local copy of the formation control, i.e. the coordination vector ξ. We use a bidirectional ring topology to communicate the coordination vector implementation instead of the position or attitude information among each spacecraft.

A decentralized architecture via the virtual structure approach is shown in Figure 1. Similar to [9], the system **G** is a discrete event supervisor, which evolves with a series of formation patterns by outputting y_G and the system **F** is the formation control, which produces and broadcasts the coordination vector ξ to represent the states of the virtual structure, except that each spacecraft has a local copy of $\mathbf{F}_i$ and $\mathbf{G}_i$ in this decentralized case. The system $\mathbf{K}_i$ is the local spacecraft controller for the ith spacecraft, which receives the coordination vector ξ from the formation control, converts ξ to the desired

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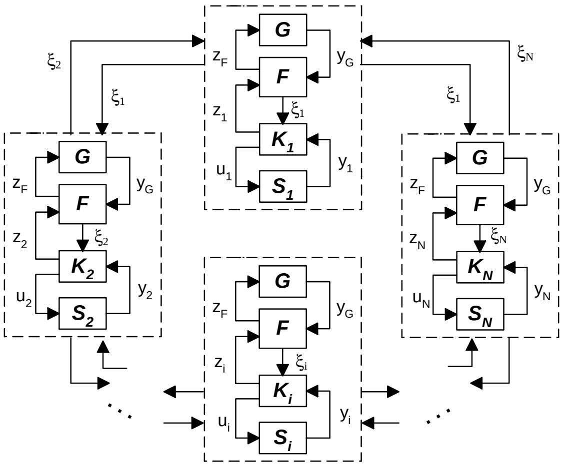

Figure 1: The decentralized architecture via the virtual structure approach.

states for the ith spacecraft, and then controls the actual states for the ith spacecraft to track its desired states. The system $\mathbf{S}_i$ is the ith spacecraft, with control input u_i representing control forces and torques, and output y_i representing the measurable outputs from the ith spacecraft. The coordination vector is instantiated in each spacecraft respectively. We use ξ_i to represent the coordination vector instantiated in the ith spacecraft. The coordination vector ξ_i in each spacecraft is synchronized through infrequent communication with its neighbors, that is, the ith spacecraft needs ξ_{i-1} and ξ_{i+1} from the $(i-1)$th and the $(i+1)$th spacecraft respectively. The ith spacecraft then forms its own control from its own coordination vector instantiation ξ_i. Formation feedback is accomplished from the ith spacecraft controller to the ith formation control through the performance measure z_i. Formation feedback from the ith formation control $\mathbf{F}_i$ to the ith instantiation of $\mathbf{G}_i$ is also included through the performance measure vector z_{Fi}.

The strength of this decentralized scheme is that only low bandwidth communication is needed. The decentralized implementation also has more flexibility, reliability, and robustness than the corresponding centralized alternative. The weakness is that each local instantiation must be synchronized, which accounts for additional complexity and inter-vehicle communications to the whole system. In addition, the decentralized scheme brings additional computation burdens to each spacecraft in the formation.

4 Decentralized Formation Control Strategies

Two major tasks need to be carried out in the decentralized formation control scheme via the virtual structure approach. One is to propose suitable control laws for each spacecraft to track its desired states defined by the virtual structure. The other is to control the virtual structure to achieve the desired formation patterns in a decentralized manner.

4.1 Formation Control Strategies for Each Spacecraft

Before proposing the control laws for each spacecraft, we assume that the desired attitude and angular velocity for each spacecraft also satisfy the rotational dynamics in (2). This is a valid assumption since in the virtual structure approach we can imagine that each place holder embedded in the virtual structure traces out the desired trajectory for each spacecraft to track, which can be thought of as a rigid body.

The proposed control force for the ith spacecraft is given by

$$f_i = M_i(\dot{v}_i^d - K_r(r_i - r_i^d) - K_v(v_i - v_i^d)), \qquad (3)$$

where M_i is the mass of the ith spacecraft, and K_r and K_v are symmetric positive definite matrices.

The proposed control torque for the ith spacecraft is given by

$$\begin{aligned}\tau_i =& J_i\dot{\omega}_i^d + \frac{1}{2}\omega_i \times J_i(\omega_i + \omega_i^d) - k_q \mathcal{V}(q_i^{d*}q_i) \\ &- K_\omega(\omega_i - \omega_i^d), \end{aligned} \qquad (4)$$

where J_i is the moment of inertia of the ith spacecraft, k_q is a positive scalar, K_ω is a symmetric positive definite matrix, and $\mathcal{V}(\cdot)$ represent the vector part of the quaternion.

Theorem 4.1 *If the spacecraft dynamics satisfy* (2)*, X_i^d and $\dot{X}_i^d$ are specified from* (1)*, and the control laws are given by* (3) *and* (4)*, then $\left\|X_i - X_i^d\right\| \to 0$ asymptotically.*

Proof: see [4] and [11]. ■

Note that if we define a translational tracking error as $E_{ti} = \tilde{r}_i^T K_r \tilde{r}_i + \frac{1}{2}\left\|\tilde{v}_i\right\|^2$, where $\tilde{r}_i = r_i - r_i^d$ and $\tilde{v}_i = v_i - v_i^d$, E_{ti} decreases during the maneuver and $\tilde{r}_i^T K_r \tilde{r}_i$ is bounded by $E_{ti}(0) - \frac{1}{2}\left\|\tilde{v}_i\right\|^2$. Similarly if we define a rotational tracking error as $E_{ri} = k_q\left\|\tilde{q}_i\right\|^2 + \frac{1}{2}\tilde{\omega}_i J_i \tilde{\omega}_i$, where $\tilde{q}_i = q_i - q_i^d$ and $\tilde{\omega}_i = \omega_i - \omega_i^d$, E_{ri} decreases during the maneuver and $\left\|\tilde{q}_i\right\|^2$ is bounded by $\frac{1}{k_q}(E_{ri}(0) - \frac{1}{2}\tilde{\omega}_i J_i \tilde{\omega}_i)$.

4.2 Formation Control Strategies for the Virtual Structure

There exist two objectives for the instantiation of the coordination vector ξ implemented in each spacecraft. The first objective is to reach its constant desired goal ξ^d defined by the formation pattern set. The second objective is to synchronize each instantiation ξ_i. Following the idea introduced in [7, 8], where behavior-based strategies are used to realize goal seeking and formation keeping for each agent, we apply behavior-based strategies to synchronize the coordination vector instantiation during the maneuver as well as evolve it to its desired goal at the end of the maneuver.

Let $\xi_i = [r_{Fi}^T, v_{Fi}^T, q_{Fi}^T, \omega_{Fi}^T, \lambda_{Fi}^T, \dot{\lambda}_{Fi}^T]^T$ be the ith instantiation of the coordination vector. Accordingly, let $\xi^d = [r_F^{T^d}, v_F^{T^d}, q_F^{T^d}, \omega_F^{T^d}, \lambda_F^{T^d}, \dot{\lambda}_F^{T^d}]^T$ be the desired constant goal defined in the formation pattern set. A series of formation patterns can be defined by $\xi^{d(k)}$, $k = 1, \cdots, K$. Define $E_G = \sum_{i=1}^N \left\|\xi_i - \xi^d\right\|^2$ as the goal seeking error to represent the total error between the current instantiation ξ_i and the desired goal ξ^d. Also define $E_S = \sum_{i=1}^N \left\|\xi_i - \xi_{i+1}\right\|^2$ as the synchronization error to represent the total synchronization error between neighboring instantiations, where the summation index i is defined modulo N, i.e., $\xi_{N+1} = \xi_1$. Define $E(t) = E_G(t) + E_S(t)$, then the control objective is to drive $E(t)$ to zero asymptotically.

Since the coordination vector represents the states of the virtual structure, we suppose that the ith coordination vector instantiation satisfies the following rigid body dynamics

$$\begin{pmatrix} \dot{r}_{Fi} \\ M_F \dot{v}_{Fi} \\ \dot{q}_{Fi} \\ J_F \dot{\omega}_{Fi} \\ \dot{\lambda}_{Fi} \\ \ddot{\lambda}_{Fi} \end{pmatrix} = \begin{pmatrix} \dot{v}_{Fi} \\ f_{Fi} \\ \frac{1}{2}\Omega(\omega_{Fi}) q_{Fi} \\ -\omega_{Fi} \times J_F \omega_{Fi} + \tau_{Fi} \\ \dot{\lambda}_{Fi} \\ \nu_{Fi} \end{pmatrix}, \tag{5}$$

where M_F and J_F are the virtual mass and virtual inertia of the virtual structure, f_{Fi} and τ_{Fi} are the virtual force and virtual torque exerted on the ith implementation of the virtual structure, and ν_{Fi} is the virtual control effort used to expand or contract the formation.

The tracking error for the ith spacecraft is defined as $E_{Ti} = \left\| X_i - X_i^d \right\|$. Define $\eta_G = D_G + K_F E_{Ti}$ to incorporate formation feedback (see [11]) from the ith spacecraft to the ith coordination vector implementation, where D_G and K_F are symmetric positive definite matrices. The proposed control force f_{Fi} is given by

$$\begin{aligned} f_{Fi} = & M_F(-K_G(r_{Fi} - r_F^d) - \eta_G v_{Fi} \\ & - K_S(r_{Fi} - r_{F(i+1)}) - D_S(v_{Fi} - v_{F(i+1)}) \\ & - K_S(r_{Fi} - r_{F(i-1)}) - D_S(v_{Fi} - v_{F(i-1)})), \end{aligned} \tag{6}$$

where K_G is a symmetric positive definite matrix, and K_S and D_S are symmetric positive semi-definite matrices.

The proposed control torque τ_{Fi} is given by

$$\begin{aligned} \tau_{Fi} = & -k_G \mathcal{V}(q^{d*} q_{Fi}) - \eta_G \omega_{Fi} \\ & - k_S \mathcal{V}(q^*_{F(i+1)} q_{Fi}) - D_S(\omega_{Fi} - \omega_{F(i+1)}) \\ & - k_S \mathcal{V}(q^*_{F(i-1)} q_{Fi}) - D_S(\omega_{Fi} - \omega_{F(i-1)}), \end{aligned} \tag{7}$$

where $k_G > 0$ and $k_S \geq 0$ are scalars, η_G follows the same definition as above, D_S is a symmetric positive semi-definite matrix, and $\mathcal{V}(\cdot)$ represent the vector part of the quaternion.

Similar to (6), the proposed control effort ν_{Fi} is given by

$$\begin{aligned} \nu_{Fi} = & -K_G(\lambda_{Fi} - \lambda_{Fi}^d) - \eta_G \dot{\lambda}_{Fi} \\ & - K_S(\lambda_{Fi} - \lambda_{F(i+1)}) - D_S(\dot{\lambda}_{Fi} - \dot{\lambda}_{F(i+1)}) \\ & - K_S(\lambda_{Fi} - \lambda_{F(i-1)} - D_S(\dot{\lambda}_{Fi} - \dot{\lambda}_{F(i-1)}), \end{aligned} \tag{8}$$

where K_G is symmetric positive definite matrix, η_G follows the same definition as above, and K_S and D_S are symmetric positive semi-definite matrices.

Note that the matrices in (6), (7), and (8) can be chosen differently based on specific requirements. In (6), (7), and (8), the first two terms are used to drive $E_G \to 0$, the third and fourth terms are used to synchronize the ith and $(i+1)$th coordination vector instantiations, and the fifth and sixth terms are used to synchronize the ith and $(i-1)$th coordination vector instantiations. The second term, that is, the formation feedback term is also used to slow down the ith virtual structure implementation when the ith spacecraft has a large tracking error. This strategy needs each spacecraft to know its neighboring coordination vector instantiations, which can be accomplished by infrequent communications between neighbors.

Theorem 4.2 *If the coordination vector implementations satisfy* (5) *and the control strategies are given by* (6), (7) *and* (8), *then* $E(t) \to 0$ *asymptotically.*

Proof: Since the translational, rotational, and expansion/contraction dynamics are decoupled in (5), we first show the convergence of each case and then prove the the convergence of $E(t)$.

Let $\tilde{r}_{Fi} = r_{Fi} - r_F^d$, then $v_{Fi} = \dot{r}_{Fi} = \dot{\tilde{r}}_{Fi}$. Also let $\tilde{r}_F = [\tilde{r}_{F1}^T, \cdots, \tilde{r}_{FN}^T]^T$ and $v_F = [v_{F1}^T, \cdots, v_{FN}^T]^T$.

For the translational dynamics, following [7], consider the Lyapunov function candidate $V_1 = \frac{1}{2}\sum_{i=1}^N (r_{Fi} - r_{F(i+1)})_T K_S (r_{Fi} - r_{F(i+1)}) + \frac{1}{2}\sum_{i=1}^N \tilde{r}_{Fi}^T K_G \tilde{r}_{Fi} + \frac{1}{2}\sum_{i=1}^N v_{Fi}^T v_{Fi}$. Differentiating it, we can get $\dot{V}_1 = \sum_{i=1}^N v_{Fi}^T (K_S(r_{Fi} - r_{F(i+1)}) + K_S(r_{Fi} - r_{F(i-1)}) + K_G \tilde{r}_{Fi} + \frac{f_{Fi}}{M_F})$.

According to (6), $\dot{V}_1 = -v_F^T(I_N \otimes \eta_G + C \otimes D_S) v_F$, where I_N is a $N \times N$ identity matrix, C is a circular matrix with the first row given by $[2, -1, 0, \cdots, 0, -1] \in \mathbb{R}^N$, and $\otimes$ denotes the Kronecker product. Based on Lemma IV.1 in [7], $I_N \otimes D_G$ is positive definite and $C \otimes D_S$ is positive semi-definite. Thus $\dot{V}_1$ is negative semi-definite.

Let $\sum = \{(\tilde{r}_F, v_F) | \dot{V}_1 = 0\}$, and let $\bar{\sum}$ be the largest invariant set in $\sum$. On $\bar{\sum}$, $\dot{V}_1 = 0$, which implies that $v_F = 0$. Then from (6), we know that $K_G \tilde{r}_{Fi} + K_S(r_{Fi} - r_{F(i+1)}) + K_S(r_{Fi} - r_{F(i-1)}) = 0$, $i = 1, \cdots, N$. That is, $(I_N \otimes K_G + C \otimes K_S)\tilde{r}_F = 0$.

Similarly, $I_N \otimes K_G$ is positive definite and $C \otimes K_S$ is positive semi-definite. Thus we know that $\tilde{r}_F = 0$.

Therefore, by LaSalle's invariance principle, $\left\| r_{Fi} - r_F^d \right\| \to 0$, $\|v_{Fi}\| \to 0$, and $\left\| r_{Fi} - r_{F(i+1)} \right\| \to 0$, $i = 1, \cdots, N$. Accordingly, $\left\| v_{Fi} - v_{F(i+1)} \right\| \to 0$, $i = 1, \cdots, N$.

For the rotational dynamics, following [8], consider the Lyapunov function candidate $V_2 = k_G \sum_{i=1}^N \left\| q_{Fi} - q_F^d \right\|^2 + k_S \sum_{i=1}^N \left\| q_{Fi} - q_{F(i+1)} \right\|^2 + \frac{1}{2}\sum_{i=1}^N \omega_{Fi}^T J_F \omega_{Fi}$.

In [12], it is shown that

$$\frac{d}{dt}\|q - p\|^2 = \mathcal{V}(p^* q)^T (\omega_q - \omega_p), \tag{9}$$

where ω_q and ω_p are the angular velocities corresponding to q and p respectively.

Applying (9), the derivative of V_2 is $\dot{V}_2 = \sum_{i=1}^N \omega_{Fi}^T k_G \mathcal{V}(q_F^{d*} q_{Fi}) + \sum_{i=1}^N (\omega_{Fi} - \omega_{F(i+1)})^T k_S \mathcal{V}(q^*_{F(i+1)} q_{Fi}) + \sum_{i=1}^N \omega_{Fi}^T (-\frac{1}{2}\omega_{Fi} \times J_{Fi}\omega_{Fi} + \tau_{Fi})$.

According to (7) and with some manipulations, $\dot{V}_2 = -\sum_{i=1}^N \left(\omega_{Fi}^T \eta_G \omega_{Fi} + (\omega_{Fi} - \omega_{F(i+1)})^T D_S (\omega_{Fi} - \omega_{F(i+1)})\right)$, which is negative semi-definite.

Let $\sum = \{(q_{F1}, \cdots, q_{FN}, \omega_{F1}, \cdots, \omega_{FN}) | \dot{V}_2 = 0\}$, and let $\bar{\sum}$ be the largest invariant set in $\sum$. On $\bar{\sum}$, $\dot{V}_2 = 0$, which implies that $\omega_{Fi} = 0$, $i = 1, \cdots, N$.

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Then from (7), we know that

$$k_G \mathcal{V}(q_F^{d*} q_{Fi}) + k_S \mathcal{V}(q_{F(i+1)}^* q_{Fi}) + k_S \mathcal{V}(q_{F(i-1)}^* q_{Fi}) = 0. \quad (10)$$

Applying the property of the unit quaternion, (10) is equivalent to

$$k_G \mathcal{V}(q_F^{d*} q_{Fi}) + k_S \mathcal{V}(q_{F(i+1)}^* q_F^d q_F^{d*} q_{Fi} + k_S \mathcal{V}(q_{F(i-1)}^* q_F^d q_F^{d*} q_{Fi}) = 0. \quad (11)$$

Following the similar procedure in [8], (11) can be written as $\mathcal{V}(p_i^* q_{Fi}) = 0$, where $p_i = k_G \mathbf{1} + k_S q_F^{d*} q_{F(i+1)} + k_S q_F^{d*} q_{F(i-1)}$.

Then following the similar proof in [8], we can show that $\mathcal{V}(q_F^{d*} q_{Fi}) = 0$, which implies that $q_{Fi} = q_F^d$. Therefore, by LaSalle's principle, $\left\| q_{Fi} - q_F^d \right\| \to 0$, $\|\omega_{Fi}\| \to 0$, and $\left\| q_{Fi} - q_{F(i+1)} \right\| \to 0$, $i = 1, \cdots, N$. Accordingly, $\left\| \omega_{Fi} - \omega_{F(i+1)} \right\| \to 0$, $i = 1, \cdots, N$.

For the expansion/contraction dynamics, the proof is the same as the translational dynamics.

Combining three parts of the proof, $E(t) \to 0$ asymptotically. ∎

Combined with the control law for each spacecraft, we can see that the virtual structure will achieve its final goal asymptotically and each spacecraft will also track its desired state specified by the virtual structure asymptotically during the maneuver. Therefore, the formation maneuver can be achieved asymptotically.

5 Simulation Results

In this section, we consider a scenario with nine spacecraft. In the scenario, a mothership spacecraft with mass 1500 Kg is located one kilometer away from a plane where eight daughter spacecraft each with mass 150 Kg are distributed equally along a circle with a diameter one kilometer in the plane. We assume that the nine spacecraft evolves like a rigid structure, that is, the formation shape is preserved and each spacecraft preserves a fixed relative orientation within the formation throughout the formation maneuvers. The configuration of the nine spacecraft is shown in Figure 2, where the spacecraft off the plane is labeled as #1 and the rest are labeled from #2 to #9 clockwise around the circle.

We simulate a scenario when the nine spacecraft start from rest with some initial position and attitude errors and then rotate 45 degrees about the inertial z axis as a whole. Here we assume that each place holder in the formation has the same orientation, that is, q_{iF}^d is the same for each spacecraft. In simulation, we instantiate a local copy of the coordination vector ξ in each spacecraft and synchronize them using the control strategy introduced in section 4.2. To show the robustness of the control strategy, we start the coordination vector implementation in each spacecraft at a different time instance and introduce a different sample time for each coordination vector instantiation. Three cases will be compared in this section. These include cases without actuator saturation and formation feedback (case 1), with actuator saturation but without formation feedback (case 2), with both actuator saturation and formation feedback (case 3).

In this section, the average coordination error is defined as $\frac{1}{N}\sum_{i=1}^N \left\|\xi_i - \bar{\xi}\right\|$, where $\bar{\xi} = \frac{1}{N}\sum_{i=1}^N \xi_i$. The

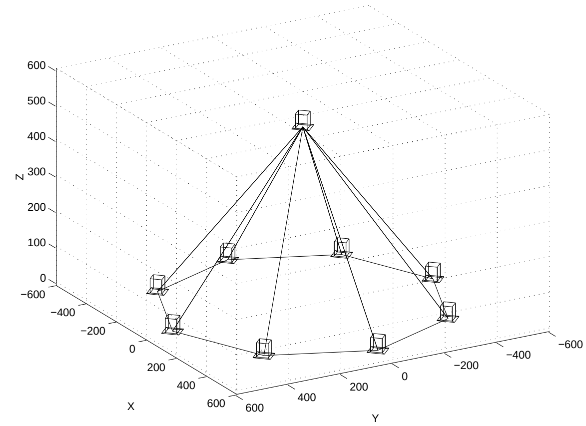

Figure 2: The geometric configuration of nine spacecraft.

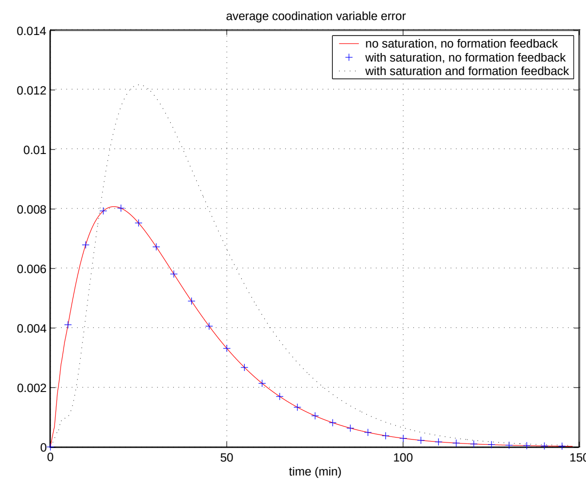

Figure 3: The average coordination error of the coordination vector instantiations.

average coordination error in these three cases is plotted in Figure 3. We can see that each instantiation of the coordination vector is synchronized asymptotically in all these cases. Also, the average coordination error is large during the initial time interval since each local instantiation starts at a different time instance. Case 1 and 2 are identical since the actuator saturation for each spacecraft does not affect the dynamics of the virtual structure when there is no formation feedback from each spacecraft to its coordination vector instantiation. Case 3 has a larger maximum average coordination error than the other two cases since formation feedback is introduced for each coordination vector instantiation, which may add some unsynchronization between different instantiations.

In Figure 4, we plot the absolute position and attitude tracking errors for spacecraft #7 in each case. The position tracking error is defined as $\left\| r_i - r_i^d \right\|$ while the attitude tracking error is defined as $\left\| q_i - q_i^d \right\|$. We can see the tracking errors in each case will decrease to zero asymptotically by using the control law given in section 4.1. The absolute position tracking error in case 2 is much larger than that in the other two cases due to the

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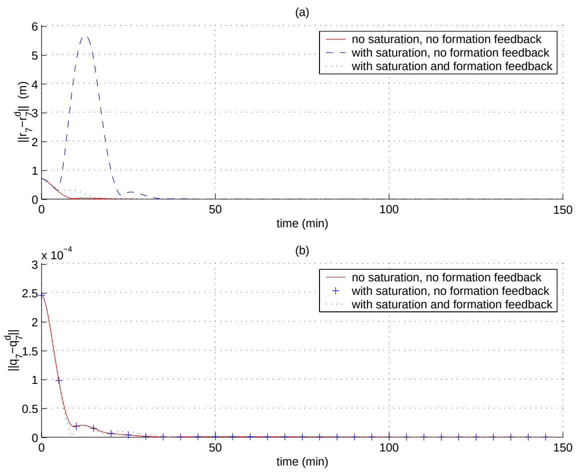

Figure 4: The absolute position and attitude tracking error for the 7th spacecraft.

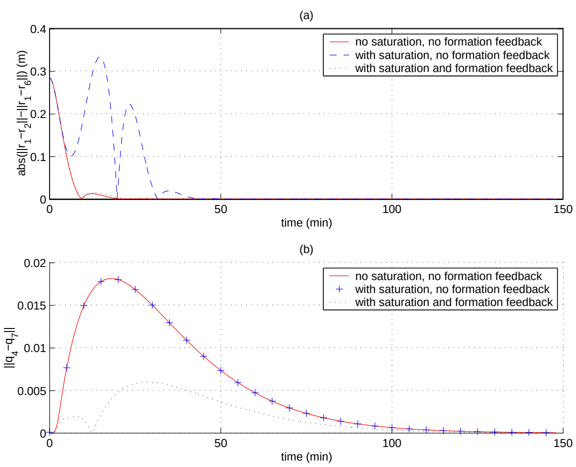

Figure 5: The relative position and attitude errors.

control force saturation. With formation feedback, the absolute position tracking errors in case 3 is similar to that in case 1 even if there is control force saturation. When we increase the formation feedback gains, the absolute tracking errors can be decreased further.

In Figure 5, we plot the relative position and attitude errors between some spacecraft in each case respectively. Based on the configuration, the desired relative distance between spacecraft #1 and #2 and the desired relative distance between spacecraft #1 and #6 should be equal. We plot $|\,\|r_1 - r_2\| - \|r_1 - r_6\|\,|$ in part (a) as an example to see how well the formation shape is preserved. The desired relative attitude between each spacecraft should be equal based on our previous assumption. We plot $\|q_4 - q_7\|$ in part (b) as an example to see how well the relative orientation relationship is preserved. Similarly, the relative position tracking error in case 2 is larger than that in the other two cases due to the control force saturation. With formation feedback, case 3 has a similar relative position error to case 1. The relative attitude error in case 3 is even smaller than that in the other two cases due to the formation feedback. When we increase the formation feedback gains, the relative errors can be decreased further.

6 Conclusion

In this paper, we proposed a decentralized scheme for spacecraft formation control via the virtual structure approach. Through low bandwidth communication between neighboring spacecraft, the instantiation of the coordination vector in each spacecraft can be synchronized and then be used to define the desired states for each spacecraft to track. Decentralized formation control strategies were presented for each spacecraft to synchronize the coordination vector instantiation and track its desired states. An example has demonstrated the effectiveness of the proposed strategies.

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