

The Stability of Darcy–Bénard Convection

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I. INTRODUCTION

Many years have passed since Horton and Rogers (1945) and Lapwood (1948) published their pioneering studies into the onset of convection in porous layers heated from below, the well-known Horton–Rogers–Lapwood or Darcy–Bénard problem. This configuration, a horizontal layer of either finite or infinite extent, has been studied in very great detail and especially so since the weakly nonlinear study of Palm et al. (1972) and the detailed numerical stability analysis of Straus (1974). The main reason for such activity lies in the fact that there are many extensions to the governing Darcy–Boussinesq equations. Commonly cited examples are boundary, inertia, local thermal non-equilibrium, anisotropic, and thermal dispersion effects. The presence of one or more of these or other modifications serves to change the nature of the resulting flow and accounts for the huge research literature. I will refer to such modified problems by the generic term ‘Darcy–Bénard’, even though further words such as ‘Forchheimer’ or ‘Brinkman’ could quite legitimately claim a place in that term.

The Darcy–Bénard problem is striking in its simplicity: a uniform-thickness horizontal porous layer with uniform temperature and impermeable upper and lower surfaces. The macroscopic equations governing the filtration of the fluid in the case where Darcy’s law applies, the fluid is Boussinesq, the matrix is rigid and isotropic, and the solid and fluid phases are in local thermal equilibrium, are so straightforward that the linearized stability analysis proceeds analytically within the space of a few lines. Even

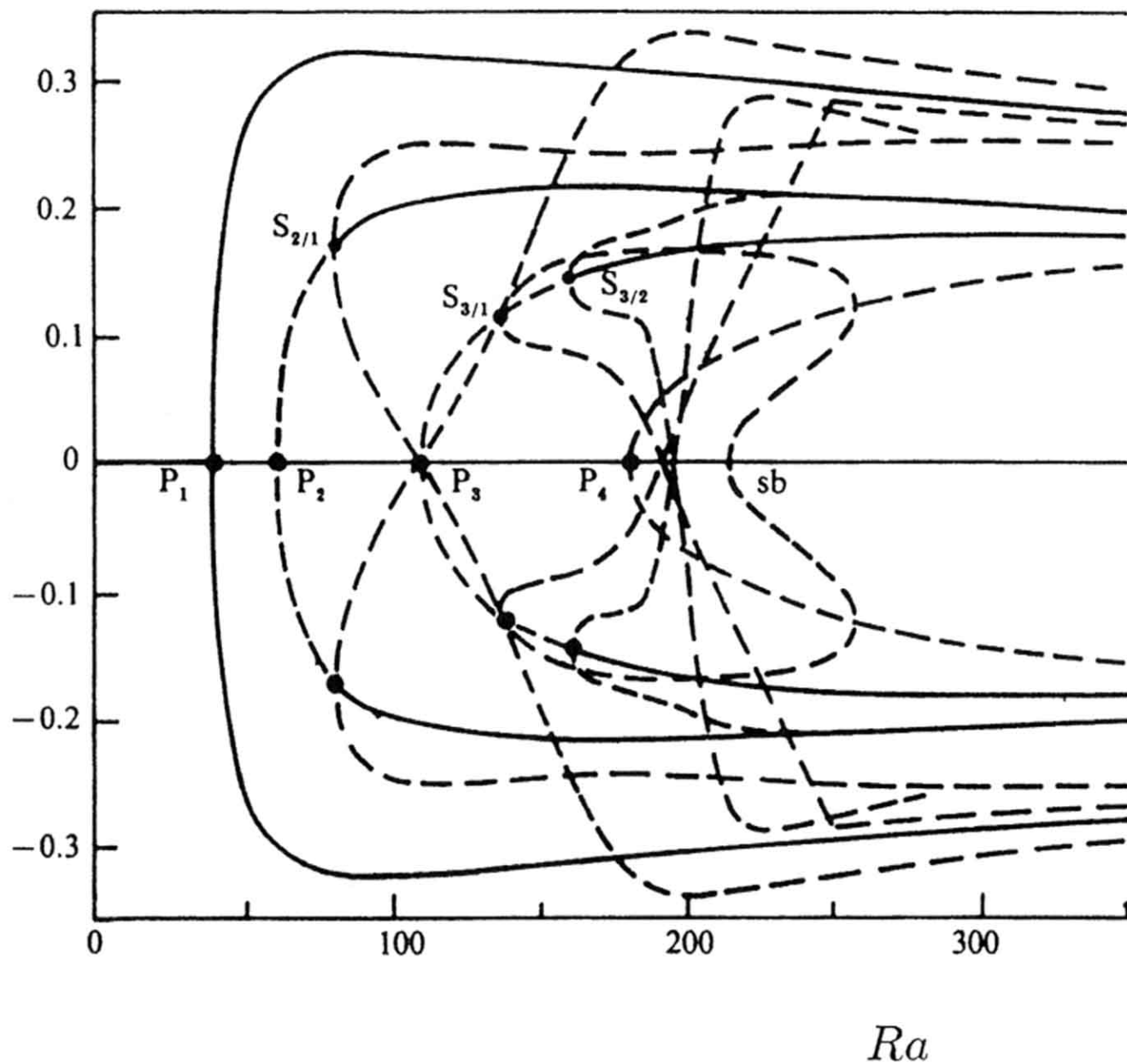


Figure 2. Computed bifurcation structure for two-dimensional flow in a square box. The stable and unstable branches are denoted by full and broken curves respectively. The vertical axis is the temperature at the midpoint of the left-hand sidewall. From Riley and Winters (1989). Reproduced by permission of Cambridge University Press.