

Double-Diffusive Convection in Porous Media

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7.1 Introduction

7.1.1 Definitions

Natural convection flow in porous media, due to thermal buoyancy alone, has been widely studied (Combarous and Bories, 1975) and well-documented in the literature (Cheng, 1978; Bejan, 1984; Nield and Bejan, 1992, 1998) while only a few works have been devoted to double-diffusive convection in porous media. This type of convection concerns the processes of combined (simultaneous) heat and mass transfer which are driven by buoyancy forces. Such phenomena are usually referred to as thermohaline, thermosolutal, double-diffusive, or combined heat and mass transfer natural convection, in this case the mass fraction gradient and the temperature gradient are independent (no coupling between the two). Double-diffusive convection frequently occurs in seawater flow and mantle flow in the earth's crust, as well as in many engineering applications.

Soret-driven thermosolutal convection results from the tendency of solute to diffuse under the influence of a temperature gradient. The concentration gradient is created by the temperature field and is not the result of a boundary condition (see De Groot and Mazur, 1961; Patil and Rudraiah, 1980). For saturated porous media, the phenomenon of cross-diffusion is further complicated due to the interaction between fluid and porous matrix, and accurate values of cross-diffusion coefficients are not available. This makes it impossible to proceed to a practical quantitative study of cross-diffusion

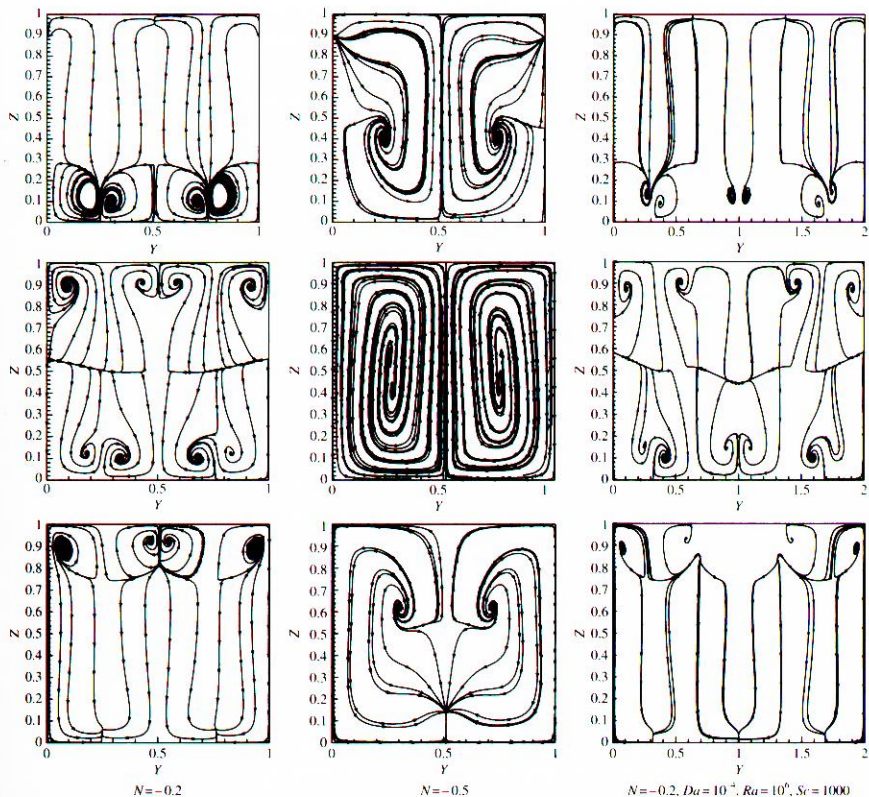


FIGURE 7.1

Stream functions at $X = 0.25$ (upper), $X = 0.5$ (middle), and $X = 0.75$ (bottom) for aspect ratio of unity (left) with $N = -0.2$ and $N = -0.5$ (middle) and aspect ratio of two (right) with $N = -0.2$. $Ra = 10^6$, $Da = 10^{-4}$, $Sc = 1000$. (Taken from I. Sezai and A.A. Mohamad. *J. Fluid Mech.* 400: 333–353, 1999. With permission.)