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## *Mathematical Modeling and Applications of Turbulent Heat and Mass Transfer in Porous Media*

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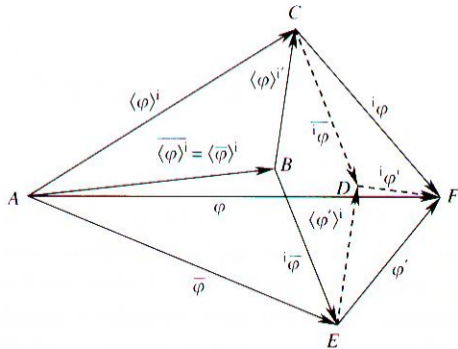
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## Summary

Engineering equipment design and environmental impact analyses can benefit from appropriate modeling of turbulent flow in porous media. Accordingly, a number of natural and engineering systems can be characterized by some sort of porous structure through which a working fluid permeates. Turbulence models proposed for such flows depend on the order of application of time and volume-average operators. Two developed methodologies, following the two orders of integration, lead to different governing equations for the statistical quantities. This chapter reviews recently published methodologies to mathematically characterize turbulent transport in porous media.

For hybrid media, involving both a porous structure and a clear flow region, difficulties arise due to the proper mathematical treatment given at the interface. This chapter also presents and discusses numerical solutions for such hybrid media, here considering a channel partially filled with a wavy porous layer through which fluid flows in turbulent regime. In addition, macroscopic forms of buoyancy terms are also considered in both the mean and the turbulent fields. Cases reviewed include heat transfer in cavities partially filled with porous material.

In summary, within this chapter local instantaneous governing equations are reviewed for clear flow before volume and time-average operators are applied to them. The double-decomposition concept is presented and thoroughly discussed prior to the derivation of macroscopic governing equations. Equations for turbulent momentum transport in porous media follow showing detailed derivation for the mean and turbulent field quantities. The statistical  $k$ - $\epsilon$  model for clear domains, used to model macroscopic turbulence effects, also serves as the basis for turbulent heat transport modeling. Turbulent mass transport in porous matrices is further reviewed in the light of the double-decomposition concept. A section on applications in hybrid media



**FIGURE 10.1**

General three-dimensional vector diagram for a quantity  $\varphi$ . (Taken from Rocamora Jr., F.D. and de Lemos, M.J.S., *Int. Commun. Heat Transfer*, 27(6), 825–834. With permission.)