4 Thin Porous Media

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CONTENTS

4.1 INTRODUCTION

Thin porous media are common and of great importance for various industries and products. These include papers and cartons, filters and filtration cakes, porous coatings, fuel cells, textiles, and diapers and wipes, to name only a few. A thin porous medium is obviously characterized by lateral dimensions much greater than its thickness. As an example, the thickness of the so-called gas diffusion layers (GDLs) of proton exchange membrane fuel cells is typically on the order of 200 μm, whereas its lateral dimension is on the order of 20 cm (see Section 4.3), leading here to a ratio lateral dimension/thickness on the order of 10^3. Suppose we wish to compute some transfer within such a system assuming that the transfer in question is governed by a known PDE within the framework of the continuum approach to porous media. Consider a uniform computational mesh with a minimum of 10 grid points over the thickness. This leads to a grid containing about 10^9 computational nodes over the whole layer. Thus, clearly adapted modeling and adequate approximations are needed to determine the transfer over the whole domain. Furthermore, relying on the traditional continuum approach to porous media can of course lead to poor results when the traditional length scale separation criterion is not met over the thickness. As an example, a GDL is only a few pore sizes thick. This type of situation, where classical length scale separation criteria are not met, is one of the distinguishing features of thin porous media. Actually, this leads to a possible and convenient stricter definition of thin porous media. Accordingly, a thin porous medium could be defined as a medium whose thickness is on the order of the pore dimension, typically less than 10 pore sizes. However, introducing such a purely geometrical definition can be considered as a bit too simplistic since the limit between a thin porous medium and a thick porous medium can be in fact transport process dependent. This will be illustrated in Section 4.3. Thus, we believe that further studies are still needed before everyone agrees with the same definition.

At first sight, thin porous media research area can be seen as a disparate collection of quite different situations. For instance, fuel cells are electrochemical devices that seem to have very little in common with paper processing technology or paper printing or the biomedical porous coatings used at a bone–implant interface. Yet an increasing number of people agreed that sharing modeling or experimental experiences coming from the various thin porous media applications can be beneficial. In other terms, thin porous media are emerging as a possible research area where specific questions of common interest to many applications can be addressed.

This chapter is in line with this tendency. The objectives are first to show that thin porous media can behave differently from ordinary porous media. Then the idea is to identify generic aspects from the consideration of selected examples.

4.2 BRINKMAN EQUATION AND THIN POROUS MEDIA

4.2.1 Brinkman Equation

Brinkman equation is a variant of momentum conservation equation that reads, for example (Nield and Bejan 1992),

$$\nabla P = -\frac{\mu}{K}\mathbf{v} + \tilde{\mu}\nabla^2\mathbf{v}, \tag{4.1}$$

where

- P is the pressure
- K is the permeability
- $\mathbf{v}$ is the filtration velocity
- μ is the fluid dynamic viscosity
- $\tilde{\mu}$ is an effective viscosity

This equation can be derived using homogenization, for example, Levy (1981), but the conclusion of the theory is that keeping the Laplacian term is only justified for unrealistically highly porous material, that is, $\varepsilon \approx 1$, where ε is the porosity. However, the homogenization technique considers spatially periodic medium far from walls. Although the Laplacian term in Equation 4.1 cannot be rigorously justified, the Brinkman equation can be an interesting approximation in the context of thin porous media because of the influence of the walls that often border the thin porous layer. This is illustrated in what follows through two examples.

4.2.2 Flow through an Array of Cylinders Confined between Two Plates

As sketched in Figure 4.1, consider a periodic array of short cylinders confined between two parallel walls as model thin porous medium. The flow within this system was studied by Tsay and Weinbaum (1991) in relation with the capillary filtration through the clefts between endothelial cells in continuous capillaries; see Tsay and Weinbaum (1991) for more details. It can be also a good model for flow in a single fracture partially occupied by particles or trapped droplets or bubbles, for example, Chauvet et al. (2012). The objective is to compute the flow through the fracture, taking into account both the presence of the cylinders and the friction on the confinement walls. As shown in details by Tsay and Weinbaum (1991), the Brinkman equation can be used to obtain a reasonably good estimate of the viscous resistance in this system.