
7 Theoretical Analysis of Transport in Porous Media

Multi-Equation and Hybrid Models for a Generic Transport Problem with Nonlinear Source Terms

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